Degrees of Freedom for Design and Control

Design Degree of Freedom

Design Degree of Freedom (DDF) is the difference between the total number of variables and the number of chemical and physical equations. This number is important because it gives the number of optimizing design variables available for optimizing some appropriate measure of profitability.

How to determine DDF:
DDF = N_v (No. of design variables that describe the system) **-** N_c (No. of constaints)

The design variables fall into three basic categories:

a. Intensive variables (such as stream concentrations, temperature, and pressure.)
b. Extensive variables (such as magnitudes of either material streams or energy streams.)
c. Repetition variable (It states that if an element is repeated. For example, distillation columns are often represented as being composed of a series of equilibrium stages.)

The restriction constraints are categorized as:

a. Inherent restrictions (For examples, a stream is split into two streams, the two exiting streams must have identical intensive properties, such as concentrations, and they must be the same as those of the entering stream.)
b. Material balance restrictions
c. Energy balance restrictions
d. Phase distribution restrictions

The practical difficulties in determining the DDF:

1. It requires that a detail steady-state model of the entire process be available. Deriving such models can be quite difficult.
2. Typical industrial chemical processes have many hundreds of variables and many hundreds of equations. It is easy to get incorrect accounting to give wrong

number. Equations can be given not independent. Variables can be forgotten.

Example:

Analysis of a Simple Element

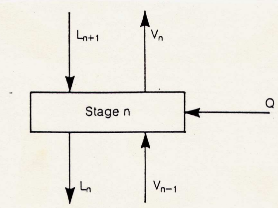


FIGURE Equilibrium stage

$N_V^e = 4 \times (N_s + 2) + 1$

$N_C^e = 2 + N_s + N_s + 1 = 2N_s + 3$

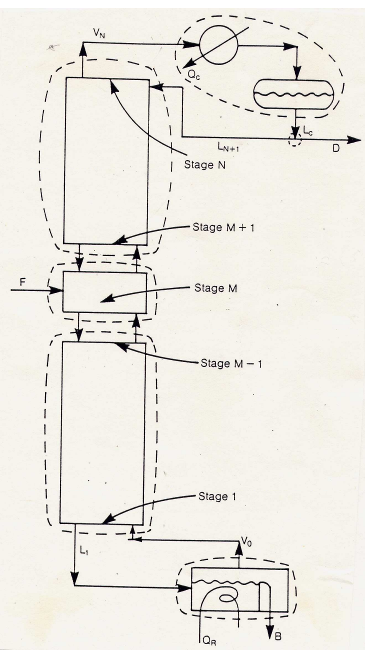
2 : leaving streams have the same T & P

N_s : Component balances

N_s : V-L-E relations

1 : Overall energy balances

$N_D^e = N_V^e - N_C^e = (4N_s + 9) - (2N_s + 3) = 2N_s + 6$



Number of Variables for Simple Distillation Column

Basic Element	$N_V^u = \sum N_D^e$
Total condenser	$N_S + 4$
Reflux splitter	$N_S + 5$
N–M equilibrium stages above feed stage	$2N_s + 2(N - M) + 5$
Feed stage	$3N_S + 8$
$M - 1$ equilibrium stages between reboiler and feed stage	$2N_S + 2(M - 1) + 5$
Reboiler	$N_S + 4$
	$N_V^u = 10N_S + 2N + 29$

From Figure , nine connecting streams are produced when the basic elements are combined. These are L_1, V_o, V_N, L_{N+1}, L_c, and the four streams leaving and entering the feed stage. Hence the number of new restricting relationships for the unit is

$$N_C^u = 9(N_S + 2) = 9N_S + 18$$

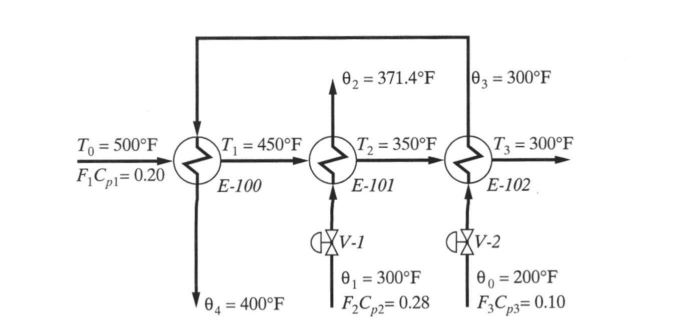
As a result the number of degrees of freedom for the distillation column is

$$\begin{aligned} N_D^u &= N_V^u - N_C^u \\ &= (10N_S + 2N + 29) - (9N_S + 18) \\ &= N_S + 2N + 11 \end{aligned}$$

Distillation Column Specification

	N_D^u
Pressure in each stage	N
Pressure in reboiler	1
Pressure in condenser	1
Pressure in reflux splitter	1
Heat leak from each stage	N
Heat leak in reflux splitter	1
Feed stream	$N_S + 2$
Reflux temperature	1
Total number of stages, N	1
Number of stages below feed stage, M	1
Distillate rate, D/F	1
Maximum allowable vapor rate, V/F	1
	$N_S + 2N + 11$

1. Heat exchanger network:



Targets:

Hot stream: 500oF to 300oF

Cold stream one: 200oF to 400oF

Cold stream two: 300o to 371.4oF

Equations:

$$Q_i = F_{i,hot}\Delta T_{i,hot}$$
$$Q_i = F_{i,cold}\Delta T_{i,cold}$$
$$Q_i = U_i A_i \frac{\Delta T_{i,hot} - \Delta T_{i,cold}}{\ln \Delta T_{i,hot} / \Delta T_{i,cold}}; \quad i = 1,2,3$$

Variables: $F_1, F_2, F_3, T_o, T_1, T_2, T_3, \theta_o, \theta_1, \theta_2, \theta_3, \theta_4, Q_1, Q_2, Q_3$

Parameters: U_i, A_i, C_{pi}

No. of Design DOF = 15- 3×3 =6

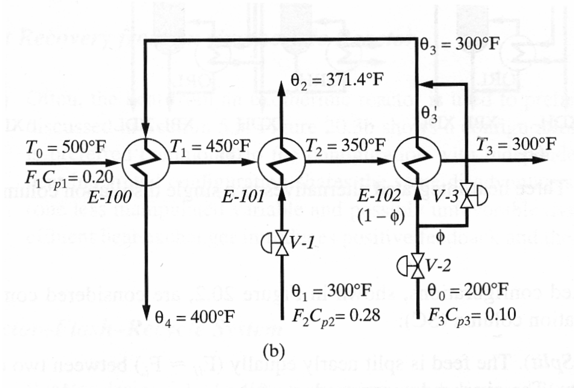
Externally defined variables: $F_o, T_o, \theta_1, \theta_3$

DOF left for control = 6 - 4 = 2

Variables to be Controlled: θ_2, θ_4

Manipulation variables used: F_2, F_3

2. Heat exchanger network no.2:



Equations:

$$Q_i = F_{i,hot}\Delta T_{i,hot}$$

$$Q_i = F_{i,cold}\Delta T_{i,cold}$$

$$Q_i = U_i A_i \frac{\Delta T_{i,hot} - \Delta T_{i,cold}}{\ln \Delta T_{i,hot} / \Delta T_{i,cold}}; \quad i = 1,2,3$$

$$\theta_3 = (1-\phi)\theta_o + \phi\theta_3^{'}$$

Variables: $F_1, F_2, F_3, T_o, T_1, T_2, T_3, \theta_1, \theta_2, \theta_3, \theta_3^{'}, \theta_4, \phi$

Parameters: U_i, A_i, C_{pi}

No. of Design DOF = 17- 3×3-1 =7

Externally defined variables: $F_o, T_o, \theta_1, \theta_3$

DOF left for control = 7-4 = 3

Variables to be Controlled: $\theta_2, \theta_4, T_3,$

Manipulation variables used: F_2, F_3, ϕ

Concept of Control Degree of Freedom:

1. The CDF is the number of variables that can be controlled in the process, and it is important to know this number when developing a control system for the process.
2. At the design stage, we can set equipment sizes in addition to setting stream flow rates, compositions, temperatures, and/or pressures. At the control stage, we can only set stream flow rates.
3. The CDF will be used :
   a. to set production rate
   b. to control all liquid levels
   c. to control all gas pressures

d. to control product qualities
e. to satisfy safety, environmental, and regulatory constraints.
f. to achieve optimum operation(e.g., minimize energy consumption, maximize yield, etc.)
g. to improve dynamic performance.

4. The CDF are stream flow rates, it simply count up the number of control valves in the plantwide process.
5. Through a number of plantwide studies, the number of DDF was observed to equal to the number of CFD.
6. Question: How we determine the number of CFD?

Answer: CDF= $N_i + N_o + N_e - P_e$

N_i = Number of input streams

N_o = Number of output streams

N_e = Number of energy streams

P_e = number of extra phase in each sub-unit.

Explanation for the result:

Total number of Df associate with each stream:

Single phase System:

IF there are C components, then in the case of design degree freedom, C+2 are assigned to each input. While this may be true in design, it is not the case in control, where in general the only manipulation which can be performed on a stream is to change its flow. Thus, a feed stream contributes only one item to the unit's degree of freedom. Thus, the total degrees of freedom associated with one stream is

$$N_v = n_i + n_o(C+2) + n_e$$

As for the constaints, all outlet streams have the same composition, implies $(C-1)(n_o-1)$ constraints. Also, all the outlet streams have the same temperature and pressure. This implies there are $2(n_0-1)$ constraints

$$\begin{aligned} N_c &= C+1+(C-1)(n_o-1)+2(n_o-1) \\ &= (C+1)+(Cn_o - C - n_o + 1) + 2(n_o-1) \\ &= Cn_o + n_o \end{aligned}$$

$$N_{d\,f} = n_i + n_e + 2n_o + Cn_o - Cn_o - n_o = n_i + n_e + n_o$$

Multiple phase System:

$N_v = N_i + N_o\,(C+2) + N_e + P(C-1)$

Total constraints:

C+1 for balance equations (C for mass and 1 for energy)

$2(N_o - 1)$ for output temperature and pressure equalities.

$N_o (C-1)$ for composition equalities of each phases.

P-1 for sets of phase equilibrium relations.

$N_C = (C+1) + 2(N_o - 1) + N_o (C-1) + C(P-1)$

And, for each unit:

$N_{CDF} = N_v - N_C = N_i + N_o + N_e - P + 1 = N_i + N_o + N_e - P_e$

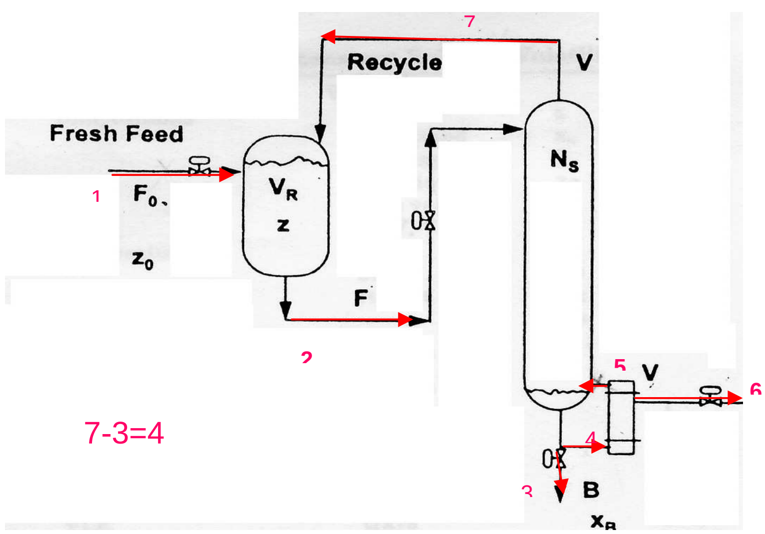
To compute CDF for connected units:

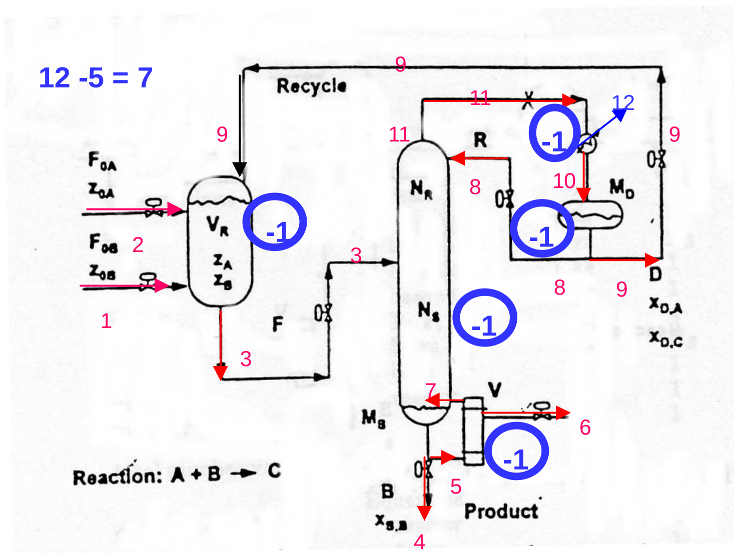
a. Calculated CDF for each sub-unit.

b. Sum the resulting CDF's.

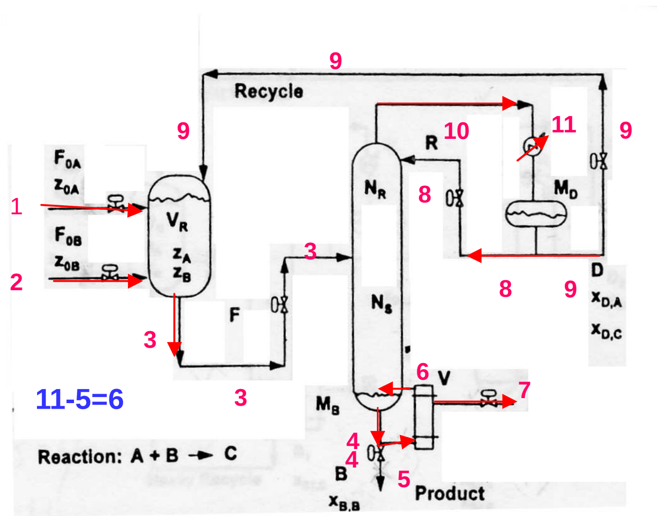
c. Subtract number of connected streams to give the total CDF

Examples:

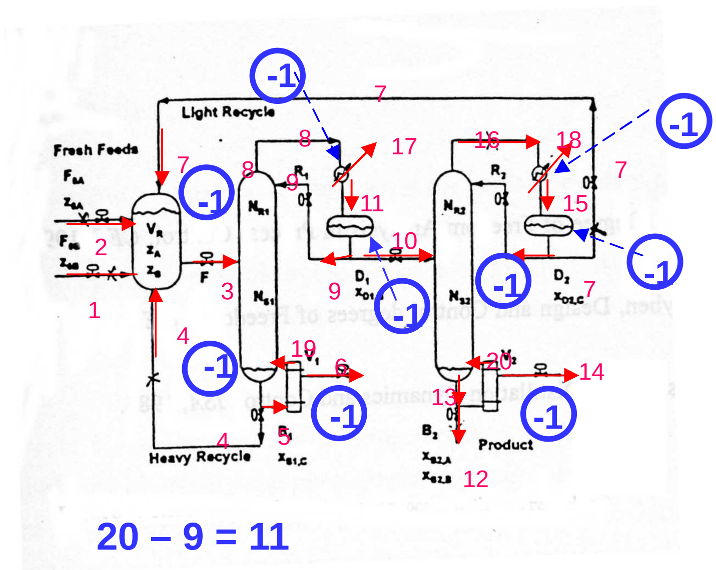




12 -5 = 7
Recycle
F_{0A}
z_{0A}
F_{0B}
z_{0B}
V_R
z_A
z_B
F
N_R
N_S
R
M_D
D
$x_{D,A}$
$x_{D,C}$
M_B
V
B
$x_{B,B}$
Product
Reaction: A + B → C



Recycle
F_{0A}
z_{0A}
F_{0B}
z_{0B}
V_R
z_A
z_B
F
N_R
N_S
R
M_D
D
$x_{D,A}$
$x_{D,C}$
11-5=6
M_B
V
B
$x_{B,B}$
Product
Reaction: A + B → C



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2. William L. Luyben, Design and Control degrees of Freedom, *I&EC Res.*, **1996**, 35, 2204-2214
3. Pradeep B. Deshpande, Distillation Dynamics and Control, *ISA*, **1985**.