Design of a Balanced-Cantilever Bridge



 $L = 80 \text{ ft} \Rightarrow Bridge \text{ Span} = 2.6 \text{ } L = 2.6 \times 80' = 208'$

Bridge Width = 30'

No. of girders = 6, Width of each girder = 15''

Loads: LL = HS20, Wearing Surface = 30 psf

Material Properties: $f'_c = 3 \text{ ksi}$, $f_s = 20 \text{ ksi}$

1. Design of Slab

Bridge width = 30', Number of girders = 6@15'': Clear span between girders $S = (30' - 6 \times 15'')/5 = 4.5'$ \therefore The c/c distance between girders = 4.5' + 15'' = 5.75'Assuming slab thickness = $6'' \Rightarrow$ Slab weight = 75 psf = 0.075 ksf : Wearing Surface = 30 psf \Rightarrow w_{DL} = 75+30 = 105 psf = 0.105 ksf : Dead-load moment, $M_{DL} = \pm w_{DL}S^2/10 = 0.105 \times 4.5^2/10 = 0.213 \text{ k'/'}$ AASHTO specifies Live-load moment, $M_{LL} = \pm 0.8 (S+2)/32 P_{20}$ $=\pm 0.8 [(4.5+2)/32] \times 16 = \pm 2.6 \text{ k'/'}$ Impact factor, $I = 50/(S+125) = 0.386 > 0.3 \implies I = 0.3$ Impact moment, $M_{IMP} = M_{LL} \times I = \pm 2.6 \times 0.3 = \pm 0.78 \text{ k'/'}$: Total moment, $M_T = M_{DL} + M_{LL} + M_{IMP} = \pm (0.213 + 2.6 + 0.78) = \pm 3.593 \text{ k'/'}$ For design, $f'_c = 3 \text{ ksi} \implies f_c = 0.4 f'_c = 1.2 \text{ ksi}$ \therefore n = 9, k = 9/(9+20/1.2) = 0.351, j = 1-k/3 = 0.883 \therefore R = $\frac{1}{2} \times 1.2 \times 0.351 \times 0.883 = 0.186$ ksi $\Rightarrow d_{reg} = \sqrt{(M_T/Rb)} = \sqrt{(3.593/0.186 \times 1)} = 4.40''$ $t = 6'' \Longrightarrow d = 6'' - 1.5'' = 4.5'' > d_{reg}$ \Rightarrow d = 4.5", t = 6" (OK). : Required reinforcement, $A_s = M_T/(f_s jd) = 3.593 \times 12/(20 \times 0.883 \times 4.5) = 0.543 \text{ in}^{2/7}$:. Use #5 @7" c/c [or #6 @10" c/c] Also, $2.2/\sqrt{S} = 2.2/\sqrt{4.5} = 1.04 > 0.67$:. Distribution steel, $A_{s(dist)} = 0.67 A_s = 0.67 \times 0.543 = 0.364 \text{ in}^{2/7}$ \therefore A_{s(dist)} per c/c span = 5.75' × 0.364 in²/'





2. Dead Load Analysis of Interior Girders



Girder depths remain constant between A-D and vary parabolically between D-I and I-N. The variation is symmetric about I. If the girder depths at D and N are both 40" (L/2 in inches) and that at I is 70" (about 70-80% larger), the depths at the other sections can be calculated easily. The depths calculated are the following

$$\begin{split} h_A &= 40^{\prime\prime}, \, h_B = 40^{\prime\prime}, \, h_C = 40^{\prime\prime}, \, h_D = 40^{\prime\prime}, \, h_E = 41.2^{\prime\prime}, \, h_F = 44.8^{\prime\prime}, \, h_G = 50.8^{\prime\prime}, \, h_H = 59.2^{\prime\prime}, \, h_I = 70^{\prime\prime}, \\ h_J &= 59.2^{\prime\prime}, \, h_K = 50.8^{\prime\prime}, \, h_L = 44.8^{\prime\prime}, \, h_M = 41.2^{\prime\prime}, \, h_N = 40^{\prime\prime} \end{split}$$

Using these dimensions (with an additional 30 psf; i.e., 2.4" concrete layer) for the analysis of the girder for self-weight using the software GRASP, the following results are obtained.

Section	x from left (')	h (")	V (k)	M (k')
А	0	40	27.40	0.00
В	8	40	18.32	182.84
С	16	40	9.24	293.04
D	24	40	0.16	330.59
Е	32	41.2	-9.00	295.21
F	40	44.8	-18.46	185.39
G	48	50.8	-28.51	-2.48
Н	56	59.2	-39.47	-274.39
I (L)	64	70	-51.62	-638.72
I (R)	64	70	51.78	-638.72
J	72	59.2	39.62	-273.14
K	80	50.8	28.67	0.00
L	88	44.8	18.61	189.10
М	96	41.2	9.16	300.16
N	104	40	0.00	336.78

Table 2.1 Dead Load Shear Forces and Bending Moments

3. Live Load Analysis of Interior Girders

The live load analysis of interior girders is carried out for HS20 loading with wheel loads of 4 k, 16 k, and 16 k at 14' distances, as shown below.



For live-load analysis, each wheel load (4 k, 16 k, 16 k) needs to be multiplied by a factor

S/5 ≥1.0

In this case, S = 5.75'; \therefore Factor = 5.75/5 = 1.15 > 1.0, OK

.: Wheel Loads for live load analysis are

 $(16 \times 1.15 =)$ 18.4 k, $(16 \times 1.15 =)$ 18.4 k and $(4 \times 1.15 =)$ 4.6 k

Also, the impact factor I = $50/(L_0+125) \le 0.30$, where L_0 = Loaded length

Assuming $L_0 = 0.6L = 48'$ (conservatively), I = 50/(48+125) = 0.289

... The impact shear forces and bending moments can be obtained by multiplying live load shears and moments by I (= 0.289).

As an alternative to using separate moments for live load and impact, one can do them simultaneously by multiplying the wheel loads by (1 + I) = 1.289; i.e., taking wheel loads to be $(18.4 \times 1.289 =) 23.72 \text{ k}$, $(18.4 \times 1.289 =) 23.72 \text{ k}$ and $(4.6 \times 1.289 =) 5.93 \text{ k}$.

The combined (live load + impact) shears and bending moments can be obtained by moving the wheels from A to N (keeping W_1 or W_3 in front) and recording all the shear forces (V) and bending moments (M). The software GRASP can be used for this purpose.

Instead of such random wheel movements, Influence Lines can be used to predict the critical position of wheels in order to get the maximum forces. This can considerably reduce the computational effort. The subsequent discussions follow this procedure.

The IL for V and M at the 'simply supported span' K-L-M-N and the critical wheel arrangements are as follows.



Using $\langle x \rangle = x$, if $x \ge 0$, or = 0 otherwise

 $V_{LL+IMP}(x_s) = (23.72/L_s) [(L_s-x_s) + (L_s-x_s-14) + \langle L_s-x_s-28 \rangle/4]$ $M_{LL+IMP}(x_s) = x_s V_{LL+IMP}(x_s); \text{ if } x_s \leq L_s/3.$

=
$$(23.72/L_s) [x_s (L_s-x_s) + x_s (L_s-x_s-14) + (x_s-14) (L_s-x_s)/4];$$
 otherwise

Using these equations, with $L_s = 48'$, the following values are obtained

[These calculations can be carried out conveniently in EXCEL]

Section	x _s (')	V (k)	M (k')
K	0	42.99	0.00
L	8	34.10	272.78
М	16	25.20	403.24
N	24	16.81	432.89

Table 3.1 V_{LL+IMP} and M_{LL+IMP} for K-N

The IL for V and M at the span 'cantilever span' I(R)-J-K and the critical wheel arrangements are as follows.



Using $\langle x \rangle = x$, if $x \ge 0$, or = 0 otherwise

$$V_{LL+IMP}(x_c) = 23.72 [1 + \{1 - \langle 14 - x_c \rangle / L_s\} + \{1 - (28 - x_c) / L_s\} / 4]$$

= 23.72 [2.25 - {<14 - x_c > + (28 - x_c) / 4} / L_s]
Moreover (x_i) = 23.72 (x_i / L_i) [L_i + (L_i - 14) + (L_i - 28) / 4]

 $M_{LL+IMP} (x_c) = -23.72 (x_c / L_s) [L_s + (L_s - 14) + (L_s - 28)/4]$

$$= -(23.72 \text{ x}_{c}) [2.25 - 21/L_{s}]$$

Using these equations, with $L_s = 48'$, the following values are obtained

Section	x _c (')	V (k)	M (k')
I (R)	16	51.89	-687.88
J	8	47.93	-343.94

Table 3.2 V_{LL+IMP} and M_{LL+IMP} for $I(R)\mbox{-}J$

The IL for V and M at the 'end span' A-B-C-D-E-G-H-I(L) and the critical wheel arrangements are as follows.



Here the results for $x_e \le L_e/2$ will be calculated and symmetry will be used for the other half. Using $\langle x \rangle = x$, if $x \ge 0$, or = 0 otherwise

For positive shear and moment

$$\begin{aligned} V^{+}_{LL+IMP}(x_{e}) &= (23.72/L_{e}) \left[(L_{e}-x_{e}) + (L_{e}-x_{e}-14) + (L_{e}-x_{e}-28)/4 \right] \geq 0 \\ M^{+}_{LL+IMP}(x_{e}) &= x_{e} V^{+}_{LL+IMP}(x_{e}); \text{ if } x_{e} \leq L_{e}/3. \\ &= (23.72/L_{e}) \left[x_{e} (L_{e}-x_{e}) + x_{e} (L_{e}-x_{e}-14) + (x_{e}-14) (L_{e}-x_{e})/4 \right]; \text{ otherwise.} \end{aligned}$$

For negative shear and moment

$$\begin{split} V^-_{LL+IMP}(x_e) &= -(23.72/L_e) \left[x_e + (x_e - 14) + (x_e - 28)/4 \right] \leq 0 \\ & \text{or} = -(23.72 \ L_c/L_e) \left[1 + (1 - 14/L_s) + (1 - 28/L_s)/4 \right] \\ M^-_{LL+IMP}(x_e) &= -(23.72 \ x_e \ L_c/L_e) \left[1 + (1 - 14/L_s) + (1 - 28/L_s)/4 \right] \end{split}$$

Using the derived equations, with $L_c=16^\prime$ & $L_e=64^\prime,$ the following values are obtained

Section	x _e (')	$V^+(k)$	M ⁺ (k')	$V^{-}(k)$	$M^{-}(k')$
А	0	45.59	0.00	-10.75	0.00
В	8	38.92	311.33	-10.75	-85.99
C	16	32.24	515.91	-10.75	-171.97
D	24	25.57	624.13	-12.23	-257.96
E	32	18.90	646.37	-18.90	-343.94
F	40	*	624.13	-25.57	-429.93
G	48	*	515.91	-32.24	-515.91
Н	56	*	311.33	-38.92	-601.90
I(L)	64	*	0.00	-45.59	-687.88

Table 3.3 V_{LL+IMP} and M_{LL+IMP} for A-I(L)

 $\begin{array}{l} \underline{\text{The 'simply supported span' K-L-M-N}} \\ V_{LL+IMP}(x_s) &= (23.72/L_s) \left[(L_s \! - \! x_s) + (L_s \! - \! x_s \! - \! 14) + < L_s \! - \! x_s \! - \! 28 \! > \! /4 \right] \\ M_{LL+IMP}\left(x_s\right) &= x_s \ V_{LL+IMP}(x_s); \ \text{if} \ x_s \leq L_s \! / \! 3. \\ &= (23.72/L_s) \left[x_s \ (L_s \! - \! x_s) + x_s \ (L_s \! - \! x_s \! - \! 14) + (x_s \! - \! 14) \ (L_s \! - \! x_s) \! / \! 4 \right]; \ \text{otherwise.} \\ \text{Using these equations, with} \ L_s = 48', \ \text{the following values are obtained} \end{array}$

Table 3.1 V_{LL+IMP} and M_{LL+IMP} for K-N

Section	x _s (')	V (k)	M (k')
K	0	42.99	0.00
L	8	34.10	272.78
М	16	25.20	403.24
N	24	16.81	432.89





The 'cantilever span' I(R)-J-K

$$\begin{split} V_{LL+IMP}(x_c) &= 23.72 \; [2.25 - \{<\!\!14\!-\!x_c\!\!>\!+(28\!-\!x_c)\!/\!4\}/L_s] \\ M_{LL+IMP}\left(x_c\right) &= -(23.72\;x_c)\; [2.25 - 21/L_s] \end{split}$$

Using these equations, with $L_s = 48'$, the following values are obtained

Table 3.2 V_{LL+IMP} and M_{LL+IMP} for I(R)-J

Section	x _c (')	V (k)	M (k')					
I (R)	16	51.89	-687.88					
J	8	47.93	-343.94					





The 'end span' A-B-C-D-E-G-H-I(L)

Here the results for $x_e \le L_e/2$ will be calculated and symmetry will be used for the other half. For positive shear and moment

$$\begin{split} V^{^{+}}_{LL+IMP}(x_e) &= (23.72/L_e) \left[(L_e-x_e) + (L_e-x_e-14) + (L_e-x_e-28)/4 \right] \geq 0 \\ M^{^{+}}_{LL+IMP}(x_e) &= x_e \, V^{^{+}}_{LL+IMP}(x_e); \, \text{if} \, x_e \leq L_e/3. \\ &= (23.72/L_e) \left[x_e \, (L_e-x_e) + x_e \, (L_e-x_e-14) + (x_e-14) \, (L_e-x_e)/4 \right]; \, \text{otherwise}. \end{split}$$

For negative shear and moment

$$\begin{split} V_{\text{LL+IMP}}^{-}(x_e) &= -(23.72/L_e) \left[x_e + (x_e - 14) + (x_e - 28)/4 \right] \leq 0 \\ \text{or} &= -(23.72 \ L_c/L_e) \left[1 + (1 - 14/L_s) + (1 - 28/L_s)/4 \right] \\ M_{\text{LL+IMP}}^{-}(x_e) &= -(23.72 \ x_e L_c/L_e) \left[1 + (1 - 14/L_s) + (1 - 28/L_s)/4 \right] \end{split}$$

Using the derived equations, with $L_c = 16'$ and $L_e = 64'$, the following values are obtained

Table 3.3 V_{LL+IMP} and M_{LL+IMP} for A-I(L)

	23				
Section	x _e (')	$V^{+}(k)$	M ⁺ (k')	$V^{-}(k)$	$M^{-}(k')$
Α	0	45.59	0.00	-10.75	0.00
В	8	38.92	311.33	-10.75	-85.99
С	16	32.24	515.91	-10.75	-171.97
D	24	25.57	624.13	-12.23	-257.96
Е	32	18.90	646.37	-18.90	-343.94
F	40	*	624.13	-25.57	-429.93
G	48	*	515.91	-32.24	-515.91
Н	56	*	311.33	-38.92	-601.90
I(L)	64	*	0.00	-45.59	-687.88



4. Combination of Dead and Live Loads

The dead load and (live load + Impact) shear forces and bending moments calculated earlier at various sections of the bridge are now combined to obtain the design (maximum positive and/or negative) shear forces and bending moments.

[These calculations can be conveniently done in EXCEL, and subsequent columns should be kept for shear & flexural design]

а <i>и</i> :	V _{DL}	V _{LL+IMP}	V _{Design}	M _{DL}	M _{LL+IMP}	M^+_{Design}	M ⁻ _{Design}
Section	(k)	(k)	(k)	(k')	(k')	(k')	(k')
А	27.40	45.59 -10.75	72.99	0.00	0.00	0.00	0.00
В	18.32	38.92 -10.75	57.24	182.84	311.33 -85.99	494.17	0.00
С	9.24	32.24 -10.75	41.48	293.04	515.91 -171.97	808.95	0.00
D	0.16	25.57 -12.23	25.73	330.59	624.13 -257.96	954.72	0.00
Е	-9.00	18.90 -18.90	-27.90	295.21	646.37 -343.94	941.58	-48.75
F	-18.46	-25.57	-44.03	185.39	624.13 -429.93	809.52	-244.24
G	-28.51	-32.24	-60.75	-2.48	515.91 -515.91	513.43	-518.39
Н	-39.47	-38.92	-78.39	-274.39	311.33 -601.90	36.94	-876.29
I(L)	-51.62	-45.59	-97.21	-638.72	-687.88	0.00	-1326.60
I(R)	51.78	51.89	103.67	-638.72	-687.88	0.00	-1326.60
J	39.62	47.93	87.55	-273.14	-343.94	0.00	-617.08
K	28.67	42.99	71.66	0.00	0.00	0.00	0.00
L	18.61	34.10	52.71	189.10	272.78	461.88	0.00
М	9.16	25.20	34.36	300.16	403.24	703.40	0.00
N	0.00	16.81	16.81	336.78	432.89	769.67	0.00

Table 4.1 Combination of DL & LL+IMP to get V_{Design} and M_{Design}

5. Design of Interior Girders

Shear Design

The shear design of interior girders is performed by using the conventional shear design equations of RCC members. The stirrup spacing is given by the equation

$$\begin{split} S_{(req)} &= A_s \; f_s \; d/(V-V_c) \\ \text{where} \; f_s &= 20 \; \text{ksi. If 2-legged \#5 stirrups are used, } A_s &= 0.62 \; \text{in}^2. \\ V_c &= 0.95 \sqrt{f'_c} \; \text{bd} \; = 0.95 \sqrt{(0.003)} \; 15 \; \text{d} = 0.7805 \; \text{d} \\ S_{(req)} &= 12.4 \; d/(V-0.7805 \; \text{d}) \\ d_{(req)} &= V/(2.95 \sqrt{f'_c} \; \text{b}) = V/2.4237 \end{split}$$

where d and V vary from section to section.

ACI recommends that the maximum stirrup spacing (S) shouldn't exceed

d/2, or 24" or A_s/0.0015b = 0.62/0.0225 = 27.56"

The calculations are carried out in tabular form and listed below.

It is convenient to perform these calculations in EXCEL.

Section	x from left (')	h (")	d (")	V (kips)	d _(req) (")	S _(req) from formula (")	S _(provided) (")
А	0.00	40.00	33.50	72.99	30.12	8.87	8
В	8.00	40.00	33.50	57.24	23.62	13.36	13
С	16.00	40.00	33.50	41.48	17.11	16.75	16
D	24.00	40.00	33.50	25.73	10.62	16.75	16
Е	32.00	41.20	34.70	-27.90	11.51	17.35	16
F	40.00	44.80	38.30	-44.03	18.17	19.15	16
G	48.00	50.80	44.30	-60.75	25.06	20.99	16
Н	56.00	59.20	52.70	-78.39	32.34	17.54	16
I(L)	64.00	70.00	63.50	-97.21	40.11	16.53	16
I(R)	64.00	70.00	63.50	103.67	42.77	14.55	14
J	72.00	59.20	52.70	87.55	36.12	14.08	14
K(*)	80.00	50.80	44.30 (*)	71.66	Articulation	*	*
L	88.00	44.80	38.30	52.71	21.75	19.15	16
М	96.00	41.20	34.70	34.36	14.18	17.35	16
N	104.00	40.00	33.50	16.81	6.94	16.75	16

Table 5.1 Design for Shear Force

Flexural Design

The flexural design of interior girders is performed by using the conventional flexural design equations for singly/doubly reinforced RCC members (rectangular or T-beam section).

For positive moments, the girders are assumed singly reinforced T-beams with

$$A_s = M/[f_s (d-t/2)]$$

However, the compressive stresses in slab should be checked against f_c (= 1.2 ksi here).

For negative moments, the girders are rectangular beams. For singly reinforced beams, the depth $d \ge d_{(req)} = \sqrt{(M/Rb)}$, and the required steel area (A_s) at top is

$$A_s = M/(f_s j d)$$

For doubly reinforced beams, $d < d_{(req)}$; i.e., $M > M_c$ (= Rbd²). The moment is divided into two parts; i.e., $M_1 = M_c$ and $M_2 = M - M_c$. The required steel area (A_s) at top is given by

$$A_s = A_{s1} + A_{s2} = M_1/(f_s j d) + M_2/\{f_s(d-d')\}$$

Here d' is the depth of compression steel from the compression edge of the beam. In addition, compressive steels are necessary (in the compression zone at bottom), given by

$$A_{s}' = M_{2}/\{f_{s}'(d-d')\}$$
, where $f_{s}' = 2 f_{s}(k-d'/d)/(1-k) \le f_{s}$

Development Length of #10 bars = $0.04 \times 1.27 \times 40/\sqrt{0.03 \times 1.4} = 51.94''$

Section	x from left (')	d (")	M ⁺ (k')	A_s^+ (in ²)	M ⁻ (k')	M _c ⁻ (k')	$\begin{array}{c} A_{s1}^{-}\\ (in^2) \end{array}$	A_{s2}^{-} (in ²)	$\begin{array}{c} A_{s}^{-}\\ (in^{2}) \end{array}$	A's (in ²)
Α	0.00	33.50	0.00	0.00	0.00	260.92	0.00	0.00	0.00	0.00
В	8.00	33.50	494.17	9.72	0.00	260.92	0.00	0.00	0.00	0.00
С	16.00	33.50	808.95	15.91	0.00	260.92	0.00	0.00	0.00	0.00
D	24.00	33.50	954.72	18.78	0.00	260.92	0.00	0.00	0.00	0.00
Е	32.00	34.70	941.58	17.82	-48.75	279.95	0.95	0.00	0.95	0.00
F	40.00	38.30	809.52	13.76	-244.24	341.05	4.33	0.00	4.33	0.00
G	48.00	44.30	513.43	7.46	-518.39	456.28	7.00	0.89	7.89	0.98
Н	56.00	52.70	36.94	0.45	-876.29	645.72	8.32	2.76	11.08	2.95
I(L)	64.00	63.50	0.00	0.00	-1326.60	937.50	10.03	3.83	13.86	3.99
I(R)	64.00	63.50	0.00	0.00	-1326.60	937.50	10.03	3.83	13.86	3.99
J	72.00	52.70	0.00	0.00	-617.08	645.72	7.96	0.00	7.96	0.00
K (*)	80.00	44.30 (*)	0.00	0.00	0.00	456.28	Articulation	*	*	*
L	88.00	38.30	461.88	7.85	0.00	341.05	0.00	0.00	0.00	0.00
М	96.00	34.70	703.40	13.31	0.00	279.95	0.00	0.00	0.00	0.00
N	104.00	33.50	769.67	15.14	0.00	260.92	0.00	0.00	0.00	0.00

Table 5.2 Design for Bending Moment

6. Design of Articulation

The width of the girder will be doubled at the articulation; i.e., $b_a = 30''$, the gradual widening will start at a distance = 6b = 90''. The design parameters are the following,

Weight of the cross-girder = $0.15 \times 2 \times (50.8''/12) \times 5.75' = 7.30$ k

Design shear force = $V_K = (71.66 + 7.30) k = 78.96 k$

Length of the articulation, $A_L = 2'$; \therefore Design moment $M_{K(a)} = 78.96 \times 2'/2 = 78.96 \text{ k}'$

A bearing plate or pad will be provided to transfer the load.

Assume bearing strength = 0.5 ksi \Rightarrow Required bearing area = 78.96/0.5 = 157.92 in²

: The bearing area is $(12'' \times 16'')$, with thickness = 6'' (assumed for pad)

The depth of girder at K is = 50.8" \therefore Design depth at articulation = (50.8–6)/2 = 22.4" \Rightarrow Effective depth d_K \cong 19.4"

The required depth from shear $d_{(req)} = V/(2.95\sqrt{f'_c b_a}) = 16.29''$, which is <19.4", OK. ∴ Stirrup spacing, $S_{(req)} = A_s f_s d/(V-V_c)$ where $f_s = 20$ ksi. If 2-legged #5 stirrups are used, $A_s = 0.62$ in². $V_c = 0.95\sqrt{f'_c bd_K} = 0.95\sqrt{(0.003)} \ 30 \ d_K = 1.561 \ d_K$ $\Rightarrow S_{(req)} = 12.4 \ d_K / (V-1.561 \ d_K) = 12.4 \times 19.4 / (76.22-1.561 \times 19.4) = 4.94''$ ∴ Provide 2-legged #5 stirrups @4.5" c/c

In addition, inclined bars will be provided for the diagonal cracks. These will be the same size as the main bars and their spacing will be governed by d/2 (of the main girder).

Here, $d = 44.3'' \Longrightarrow$ Spacing = 22.15"

Since the length of articulation is 2' = 24'', provide 2 #10 bars @ 12''c/c

The required depth from bending, $d_{(req)} = \sqrt{(M/Rb_a)} = 13.04''$, which is <19.4"

 \Rightarrow Singly reinforced section, with required steel,

 $A_s = M/(f_s j d) = 78.96 \times 12/(20 \times 0.883 \times 19.4) = 2.77 in^2$

These will be adjusted with the main reinforcements in design.







Section I



Longitudinal Section of Articulation



Cross-Girder at Articulation

7. Design of Railings and Kerb



The following arrangement is chosen for the railing

Assume (for the assignment)

- Span of Railing, $S_r = L_s/8$, Width (b) of railing section = $(S_r + 2)$ in
- Height of Railpost = $S_r/4 + S_r/4 + S_r/6$, Width (b) of railpost section = $(S_r + 4)$ in

Railing

The assumed load on each railing = 5 k

: Design bending moment $M_{(\pm)} = 0.8 (PL/4) = 0.8 (5 \times 6/4) = 6.0 k'$

If the width b = 8'', $d_{(req)}$ from bending $= \sqrt{(M_{(\pm)}/Rb)} = \sqrt{\{6 \times 12/(0.197 \times 8)\}} = 6.76''$

Shear force V = 5.0 k \Rightarrow d_(req) from shear = V/2.95 $\sqrt{f'_c}$ b = 5.0/(2.95 $\sqrt{(0.003) \times 8)}$ = 3.87"

: Assume d = 7'', h = 8.5''

: $A_s = M_{(\pm)}/(f_s j d) = 6 \times 12/(20 \times 0.883 \times 7) = 0.59 \text{ in}^2$; i.e., use 2 #5 bars at top and bottom

 $V_c = 0.95 \sqrt{f'_c} \ bd = 0.95 \sqrt{(0.003) \times 8 \times 7} = 2.91 \ k$

 $\therefore \text{Spacing of 2-legged #3 stirrups, } S_{(req)} = A_s f_s d/(V-V_c) = 0.22 \times 20 \times 7/(5.0-2.91) = 14.76''$ $\therefore \text{Provide 2-legged #3 stirrups @3.5''c/c (i.e., d/2)}$



Rail Post

Design bending moment $M_{(-)} = 5 \times 1.5 + 5 \times 3.0 = 22.5 \text{ k'}$ If the width b = 10", the d_(req) from bending = $\sqrt{(M_{(-)}/\text{Rb})} = \sqrt{\{22.5 \times 12/(0.197 \times 10)\}} = 11.71"$ Shear force V = 10.0 k ⇒ d_(req) from shear = V/2.95 $\sqrt{f'_c}$ b = 10.0/(2.95 $\sqrt{(0.003)} \times 10) = 6.19"$ ∴ Assume d = 12", h = 13.5"

$$\therefore A_s = M_{(-)}/(f_s jd) = 22.5 \times 12/(20 \times 0.883 \times 12) = 1.27 \text{ in}^2$$
; i.e., use 3 #6 bars inside

 $V_c = 0.95 \sqrt{f'_c} \text{ bd} = 0.95 \sqrt{(0.003) \times 10 \times 12} = 6.24 \text{ k}$ If 2-legged #3 stirrups are used, $A_s = 0.22 \text{ in}^2$ ∴ Stirrup spacing, $S_{(req)} = A_s f_s d/(V-V_c) = 0.22 \times 20 \times 12/(10.0-6.24) = 14.06''$ ∴ Provide 2-legged #3 stirrups @6''c/c (i.e., d/2)



Edge Slab and Kerb

Edge Slab

Design load = 16 k, assumed width = 4 ft

Design bending moment for edge slab $M_{(-)} = 16/4 \times 18/12 = 6.0$ k'/ft

 $d_{(req)} = \sqrt{(M_{(-)}/Rb)} = \sqrt{\{6.0/0.197\}} = 5.52''$

Assuming d = 5.5", t = 7.5"

 $\therefore A_s = M_{(-)}/(f_s jd) = 6.0 \times 12/(20 \times 0.883 \times 5.5) = 0.74 \text{ in}^2/\text{ft},$

which is greater than the reinforcement (= $0.54 \text{ in}^2/\text{ft}$) for the main slab.

There are two alternatives, using

(a) d = 5.5'', t = 7'', with #6 @10'' c/c (like main slab) + one extra #6 after 2 main bars

(b) d = 7.5'', t = 9'', with #6 @10'' c/c (like main slab)

Use $A_{s(temp)} = 0.03t = 0.21 \text{ in}^2/\text{ft}$, or 0.27 in²/ft, i.e., #5 @14" c/c or 12" c/c transversely Kerb

Design load = 10 k/4′ ⇒ Design bending moment for kerb $M_{(-)} = 10/4 \times 10/12 = 2.08$ k′/ft ∴ The required depth, $d_{(req)} = \sqrt{(M_{(-)}/Rb)} = \sqrt{\{2.08/0.197\}} = 3.25'' << assumed d = 20'', t = 24''$ ∴ $A_s = M_{(-)}/(f_sjd) = 2.08 \times 12/(20 \times 0.883 \times 20) = 0.071$ in²/ft, which is not significant ∴ $A_{s(temp)} = 0.03$ h = 0.03×17.5 = 0.525 in²/ft

:. Provide #6 bars @10"c/c over the span (i.e., consistent with #6 bars @10"c/c for the slab) and = $0.525 \times 24/12 = 1.05 \text{ in}^2$; i.e., 4 #5 bars within the width of kerb.



8. Design of Substructure

Design of Abutment and Wing Walls

Stability Analysis



The effect of front soil is ignored and wall geometry is simplified (denoted by the dotted lines).

Sliding

Approximate Vertical load = $12.69 + 0.12 \times \{(3+14) \times 8 + 4 \times 8.75\} + 0.15 \times \{14.5 \times 2 + 2 \times 14\}$ = 12.69 + 16.32 + 4.2 + 4.35 + 4.2 = 41.76 k/'

: Horizontal resistance $H_R = 0.45 \times 41.76 = 18.79 \text{ k/}'$

Horizontal load H = 1.90 + 0.12×3/3×20 + 0.12×20/3×20/2 = 1.90 + 2.4 + 8.0 = 12.30 k/′

: Factor of Safety against sliding = $H_R/H = 18.79/12.30 = 1.53 > 1.5$, OK

Overturning

Approximate resisting moment

$$M_{R} = 12.69 \times (4.5+0.625) + 16.32 \times (6.5+4) + 4.2 \times (5.75+4.375) + 4.35 \times 7.25 + 4.2 \times 5.5$$

= 336.18 k'/'

Overturning moment M = $1.90 \times 16 + 2.4 \times 20/2 + 8.0 \times 20/3 = 107.78 \text{ k'/'}$

: Factor of Safety against overturning = $M_R/M = 336.18/107.78 = 3.12 > 1.5$, OK

Design of Back-wall

Design wheel load = 16 k and assumed loaded length $\cong 4'$ \therefore Load per unit width V = 16/4 = 4 k/' Moment per unit width M = 4×2/2 = 4 k'/' $\therefore d_{(req)}$ for moment = $\sqrt{(4/0.186)} = 4.64''$ $\therefore d = 18-3 = 15'' >> d_{(req)}$ $A_s = 4 \times 12/(20 \times 0.883 \times 15) = 0.18 \text{ in}^2/'$ $A_{s(temp)} = 0.03 \times 18 = 0.54 \text{ in}^2/'$

This should be adjusted with stem reinforcement.

Design of Stem

Design V/length = $1.9 + 0.12 \times 18 + 0.72 \times 18/2 = 1.9 + 2.16 + 6.48 = 10.54 \text{ k/'}$ Design M/length = $1.9 \times 14 + 2.16 \times 18/2 + 6.48 \times 18/3 = 84.92 \text{ k'/'}$ $\therefore d_{(req)}$ for shear = $10.54/(0.95 \times \sqrt{(0.003)} \times 12) = 16.88''$ and $d_{(req)}$ for moment = $\sqrt{(84.92/0.186)} = 21.38''$ $d = 24-3.5 = 20.5'' < d_{(req)} \Rightarrow t = 25'', d = 21.5''$ $A_s = 84.92 \times 12/(20 \times 0.883 \times 21.5) = 2.68 \text{ in}^{2}/'$ $\therefore \text{Use } \#10 @ 5.5'' \text{ c/c}$ (along the length near soil) 14' $A_{s(temp)} = 0.03 \times 25 = 0.75 \text{ in}^{2}/'$ $\therefore \text{Use } \#5 @ 5'' \text{ c/c}$ (along the width and length farther from soil)

This should also be adequate for the back-wall (both along length and width)

Design of Toe and Heel

Total vertical force on the soil below the wall = 41.76 k/'

The resultant moment about the far end to the toe = M_R -M = 336.18-107.78 = 228.40 k'/'

 $\therefore e_0 = 228.40/41.76 = 5.47'$, which is >14.5/3 and <14.5×2/3 \Rightarrow uplift avoided.

The maximum soil pressure = $(41.76/14.5) \times [1+6(14.5/2-5.47)/14.5] = 5.00$ ksf, which is > 2 ksf.

...Pile foundation is suggested, and the toe and heel should be designed as pile cap.



0.12 + 0.72

 p_h (ksf)

Design of Piles and Pile-cap

The following arrangement of piles is assumed for the width of the toe and heel (i.e., 14.5') and within the c/c distance of girders (i.e., width = 5.75').



Pile Forces

For the total width of one row of piles, $V = 41.76 \text{ k/} \times 5.75' = 240.10 \text{ k}$

and M = $(41.76 \times 7.25 - 228.40) \text{ k'/'} \times 5.75' = 483.19 \text{ k'}$ Pile reactions are given by $F_i = V/n + My_i/\sum y_i^2$ where n = Number of piles = 4, $\sum y_i^2 = 2 \times 1.75^2 + 2 \times 5.25^2 = 61.25 \text{ ft}^2$ \therefore Here, $F_i = 240.10/4 + 483.19 \text{ y}_i/61.25 = 60.03 + 7.89 \text{ y}_i$ $\therefore F_1 = 60.03 + 7.89 \times 5.25 = 101.44 \text{ k}, F_2 = 60.03 + 7.89 \times 1.75 = 73.83 \text{ k}$ $F_3 = 60.03 + 7.89 \times (-1.75) = 46.22 \text{ k}, F_4 = 60.03 + 7.89 \times (-5.25) = 18.61 \text{ k}$

Design of Piles

Using 3% steel, P = 0.85 (0.25 f_c' + 0.03 f_s) ($\pi/4$) D²

 $\Rightarrow 101.44 = 0.85 \ (0.25 \times 3 + 0.03 \times 20) \ (\pi/4) \ D^2 \Rightarrow D = 10.61''$

: Use 11" diameter piles with 6 #5 bars and #5 ties @11" c/c

Smaller sections can be used for the piles other than Pile 1

The piles should be long enough to transfer the axial loads safely to the surrounding soil.

Assuming the entire pile load for pile 1 to be resisted by skin friction, $F_1 = \alpha_2 (\pi DL) q_a/2$

where α_2 = Reduction factor for soil disturbance $\cong 0.8$, D = Pile diameter = 11"

L = Pile length, q_a = Allowable compressive stress on soil = 2 ksf

 $\therefore 101.44 = 0.8 \times (\pi \times 11/12 \times L) \times 2/2 \implies L = 44.03', \therefore \text{Provide 45' long piles.}$

Design of Pile-cap

For the design of pile-cap, the pile loads per girder are considered along with the soil pressure converted to soil load per girder width. The pile loads on the pile-cap for c/c girder width (5.75') are shown below.



1. Design of Toe

The assumed thickness = 2' = 24''.

The pile-cap is assumed to have 3" effective cover and 6" embedment for piles. Maximum Punching shear =101.44 k, allowable punching stress = $1.9 \times \sqrt{(0.003)} = 0.104$ ksi Punching area around a 12" pile = π (11+d) d = 101.44/0.104 \Rightarrow d_(req) \cong 13.3"; i.e., t_(req) = 22.5" Maximum flexural shear =101.44 k, allowable shear stress = $0.95 \times \sqrt{(0.003)} = 0.052$ ksi Shearing area = 5.75×12 d = $101.44/0.052 \Rightarrow$ d_(req) = 28.25"; i.e., t_(req) = 37.25" Maximum bending moment =101.44×2.5 = 253.60 k' \Rightarrow d_(req) = $\sqrt{\{253.60/(0.186 \times 5.75)\}} = 15.41$ "; i.e., t_(req) = 24.41" \therefore t = 38", d = 29" \Rightarrow A_s = $253.60 \times 12/(20 \times 0.883 \times 29) = 5.94$ in²; i.e., 5.94/5.75 = 1.03 in²/ft A_{s(temp)} = $0.03 \times 38 = 1.14$ in²/ft

2. Design of Heel

Maximum flexural shear =14.49×7- 46.22 -18.61= 36.60 k, allowable shear stress = 0.052 ksi Shearing area = $5.75 \times 12 \text{ d} = 36.60/0.052 \Rightarrow d_{(req)} = 10.19''$; i.e., $t_{(req)} = 19.19''$ Maximum bending moment = $-14.49 \times 8 \times 8/2 + 46.22 \times 2.5 + 18.61 \times 6 = -236.47 \text{ k'}$ $\Rightarrow d_{(req)} = \sqrt{\{236.47/(0.186 \times 5.75)\}} = 14.88''$; i.e., $t_{(req)} = 23.88''$ $\therefore t = 38'', d = 29'' \Rightarrow$ It will be OK for punching shear also $\therefore A_s = 236.47 \times 12/(20 \times 0.883 \times 29) = 5.54 \text{ in}^2$; i.e., $0.96 \text{ in}^2/\text{ft}$, $A_{s(temp)} = 0.03 \times 38 = 1.14 \text{ in}^2/\text{ft}$



5. Design of Interior Girders (USD)

Shear Design

The shear design of interior girders is performed by using the conventional shear design equations of RCC members. The stirrup spacing is given by the equation

$$\begin{split} S_{(req)} &= A_s f_y \, d/(V_u/\phi - V_c) \\ \text{where } f_y &= 40 \text{ ksi. If 2-legged \#5 stirrups are used, } A_s = 0.62 \text{ in}^2. \\ V_c &= 1.9 \sqrt{f'_c} \text{ bd } = 1.9 \sqrt{(0.003)} \text{ 15 } \text{ d} = 1.561 \text{ d} \\ S_{(req)} &= 24.8 \text{ d}/(V_u/0.85 - 1.561 \text{ d}) \\ \text{d}_{(req)} &= V/(5.9 \phi \sqrt{f'_c} \text{ b}_w) = V/4.120 \end{split}$$

where d and V vary from section to section.

ACI recommends that the maximum stirrup spacing (S) shouldn't exceed

$$d/2$$
, or 24" or $A_s f_y/0.06b_w = 24.8/0.90 = 27.56$ "

The calculations are carried out in tabular form and listed below.

It is convenient to perform these calculations in EXCEL.

Section	x from left (')	h (")	d (")	V _u (kips)	d _(req) (")	S _(req) from formula (")	S _(provided) (")
Α	0.00	40.00	33.50	115.86	28.12	9.89	8.0
В	8.00	40.00	33.50	91.81	22.28	14.91	13.0
С	16.00	40.00	33.50	67.74	16.44	16.75	16.0
D	24.00	40.00	33.50	43.69	10.60	16.75	16.0
E	32.00	41.20	34.70	-44.73	10.86	17.35	16.0
F	40.00	44.80	38.30	-69.31	16.82	19.15	16.0
G	48.00	50.80	44.30	-94.72	22.99	22.15	20.0
Н	56.00	59.20	52.70	-121.42	29.47	21.57	20.0
I(L)	64.00	70.00	63.50	-149.77	36.35	20.43	20.0
I(R)	64.00	70.00	63.50	160.70	39.00	17.51	16.0
J	72.00	59.20	52.70	136.95	33.24	16.57	16.0
K(*)	80.00	50.80	44.30 (*)	113.22	Articulation	*	*
L	88.00	44.80	38.30	84.02	20.39	19.15	16.0
Μ	96.00	41.20	34.70	55.66	13.51	17.35	16.0
N	104.00	40.00	33.50	28.58	6.94	16.75	16.0

Table 5.1 Design for Shear Force

Flexural Design



For T-beams (possible for positive moments), b_f is the minimum of

(i)
$$16t_f + b_w = 111''$$
, (ii) Simple Span/4 = $0.6L \times 12/4 = 1.8L = 144''$, (iii) c/c = 69''

$$\Rightarrow$$
 b_f = 69"; i.e.,

$$\begin{split} A_{s(+)} &= (f_c/f_y)[1 - \sqrt{\{1 - 2M_{(+)}/(\phi f_c b_f d^2)\}}]b_f d \\ &= (2.55/40) \times [1 - \sqrt{\{1 - 2M_{(+)} \times 12/(0.9 \times 2.55 \times 69 \times d^2)\}}] \times 69 \times d = 4.40 \text{ d } [1 - \sqrt{\{1 - 0.152M_{(+)}/d^2\}}] \\ \text{Development Length of #10 bars} &= 0.04 \times 1.27 \times 40/\sqrt{0.03} \times 1.4 = 51.94'' \end{split}$$

These calculations, performed in EXCEL, are listed below.

Section	x from left (')	d (")	M+ (k')	$\begin{array}{c} A_{s+} \\ (in^2) \end{array}$	M. (k')	M _{c-} (k')	A _{s-} (in ²)	N _{s+}	N _{s-}	N _{bot}	N _{top}
Α	0.00	33.50	0.00	0.00	0.00	1095.60	0.00	0.00	0.00	6	4
В	8.00	33.50	785.24	8.03	0.00	1095.60	0.00	6.32	0.00	8	4
С	16.00	33.50	1287.30	13.42	0.00	1095.60	0.00	10.57	0.00	12	4
D	24.00	33.50	1523.85	16.04	0.00	1095.60	0.00	12.63	0.00	14	4
E	32.00	34.70	1512.12	15.29	-171.44	1175.49	1.69	12.04	1.33	14	4
F	40.00	38.30	1320.57	11.91	-470.83	1432.05	4.36	9.38	3.43	12	6
G	48.00	44.30	873.58	6.69	-880.52	1915.88	7.24	5.27	5.70	8	8
Н	56.00	52.70	145.12	0.92	-1407.38	2711.33	9.87	0.72	7.77	4	10
I(L)	64.00	63.50	0.00	0.00	-2063.60	3936.48	12.02	0.00	9.47	4	10
I(R)	64.00	63.50	0.00	0.00	-2063.60	3936.48	12.02	0.00	9.47	4	10
J	72.00	52.70	0.00	0.00	-967.09	2711.33	6.54	0.00	5.15	4	10
K (*)	80.00	44.30 (*)	*	*	*	*	*	*	*	*	*
L	88.00	38.30	728.47	6.46	0.00	1432.05	0.00	5.09	0.00	8	4
Μ	96.00	34.70	1105.73	11.02	0.00	1175.49	0.00	8.68	0.00	10	4
N	104.00	33.50	1207.41	12.55	0.00	1095.60	0.00	9.88	0.00	10	4

Table 5.2 Design for Bending Moment



Fig. 1: Design for Shear (USD)





Sections	А	В	С	D	E	F	G	Н	Ι	J	Κ	L	М	Ν
Top #10	4	4	4	4	4	6	8	10	10	10	*	4	4	4
Bottom #10	6	8	12	14	14	12	8	4	4	4	*	8	10	10
#5 Stirrup S	8	13	16	16	16	16	20	20	16	16	*	16	16	16





6. Design of Articulation (USD)

The width of the girder will be doubled at the articulation; i.e., $b_a = 30''$, the gradual widening will start at a distance = 6b = 90''. Length of the articulation, $A_L = 2'$. Reaction from interior girder = 113.22 k

Weight of the cross-girder = $0.15 \times 2 \times (50.8''/12) \times 5.75' \times 1.4 = 10.23$ k

Design shear force = $V_K = (113.22 + 10.23) k = 123.45 k$

 $\therefore A_L = 2' \Rightarrow$ Design moment $M_{K(a)} = 123.45 \times 2'/2 = 123.45 \text{ k}'$

A bearing plate or pad will be provided to transfer the load.

Assume bearing strength = 1.0 ksi \Rightarrow Required bearing area = 123.45/1.0 = 123.45 in²

:. The bearing area is $(12'' \times 12'')$, with thickness = 6'' (assumed for pad)

The depth of girder at K is = 50.8''

:. Design depth at articulation = $(50.8-6)/2 = 22.4'' \Rightarrow$ Effective depth $d_K \cong 19.4''$

The required depth from shear $d_{(req)} = V_K / (5.9 \phi \sqrt{f'_c b_a}) = 12.73''$, which is <19.4'', OK.

: Stirrup spacing, $S_{(req)} = A_s f_y d/(V_u/\phi - V_c)$

If 2-legged #5 stirrups are used, $A_s = 0.62 \text{ in}^2$.

 $V_c = 1.9 \sqrt{f'_c} \ bd_K = 1.9 \sqrt{(0.003)} \ 30 \ d_K = 3.122 \ d_K$

$$\Rightarrow$$
 S_(req) = 24.8 d_K /(V_K/0.85-3.122 d_K) = 24.8 × 19.4/(145.23-3.122×19.4) = 5.68'

∴ Provide 2-legged #5 stirrups @5.5"c/c

In addition, inclined bars will be provided for the diagonal cracks. These will be the same size as the main bars and their spacing will be governed by d/2 (of the main girder).

Here, $d = 44.3'' \Longrightarrow$ Spacing = 22.15"

Since the length of articulation is 2' = 24'', provide 2 #10 bars @ 12''c/c

The required depth from bending, $d_{(req)} = \sqrt{(M_{K(a)}/R_ub_a)} = 7.95''$, which is <19.4"

 \Rightarrow Singly reinforced section, with required steel,

 $A_s = (f_c/f_y)[1 - \sqrt{\{1 - 2M_{K(a)}/(\phi f_c b_a d^2)\}}] \ b_a d = 2.19 \ in^2$

These will be adjusted with the main reinforcements in design.



Longitudinal Section of Articulation



Cross-Girder at Articulation

7. Design of Railing and Rail Post (USD)



The following arrangement is chosen for the railing

Railing

The assumed load on each railing = 5 k

: Design bending moment $M_{(\pm)} = 0.8 (PL/4) = 0.8 (1.7 \times 5 \times 6/4) = 10.2 \text{ k}'$

If the width b = 6'', the required depth from bending is

 $d_{(req)} = \sqrt{(M_{(\pm)}/R_u b)} = \sqrt{\{10.2 \times 12/(0.781 \times 6)\}} = 5.11''; i.e., assume d = 6'', t = 7.5''$

: $A_s = (f_c/f_y)[1 - \sqrt{\{1 - 2M_{(\pm)}/(\phi f_c b d^2)\}}]$ bd = 0.662 in²; i.e., use 2 #6 bars at top and bottom

Shear force $V_u = 1.7 \times 5 = 8.5$ k, $V_c = 1.9 \sqrt{f'_c}$ bd $= 1.9 \sqrt{(0.003)} \times 6 \times 6 = 3.75$ k

If 2-legged #3 stirrups are used, $A_s = 0.22 \text{ in}^2$

: Stirrup spacing, $S_{(req)} = A_s f_y d/(V_u/\phi - V_c) = 0.22 \times 40 \times 6/(8.5/0.85 - 3.75) = 8.44''$

∴ Provide 2-legged #3 stirrups @3"c/c (i.e., d/2)



Rail Post

Design bending moment $M_{(-)} = 1.7 \times 5 \times 1.5 + 1.7 \times 5 \times 3.0 = 38.25$ k'

If the width b = 6'', the required depth from bending is

 $d_{(req)} = \sqrt{(M_{(-)}/R_ub)} = \sqrt{\{38.25 \times 12/(0.781 \times 8)\}} = 8.57''; i.e., assume \ d = 10'', \ t = 11.5''$

: $A_s = (f_c/f_y)[1 - \sqrt{\{1 - 2M_{(-)}/(\phi f_c b d^2)\}}]$ bd = 1.494 in²; i.e., use 3 #6 bars inside

Shear force $V_u = 1.7 \times 10 = 17$ k, $V_c = 1.9 \sqrt{f'_c}$ bd $= 1.9 \sqrt{(0.003)} \times 8 \times 10 = 8.33$ k

If 2-legged #3 stirrups are used, $A_s = 0.22 \text{ in}^2$

:. Stirrup spacing,
$$S_{(req)} = A_s f_y d/(V_u/\phi - V_c) = 0.22 \times 40 \times 10/(17/0.85 - 8.33) = 7.54''$$

∴ Provide 2-legged #3 stirrups @5"c/c (i.e., d/2)



Kerb

Design bending moment $M_{(-)} = 1.7 \times 5/4' \times (12/12) = 2.13 \text{ k'/'}$

If the width b = 12'', the required depth from bending is

 $d_{(req)} = \sqrt{(M_{(-)}/R_ub)} = \sqrt{\{2.13 \times 12/(0.781 \times 12)\}} < assumed \ d = 7.5'', \ t = 9''$

: $A_s = (f_c/f_y)[1 - \sqrt{\{1 - 2M_{(-)}/(\phi f_c b d^2)\}}] bd = in^2$; i.e., use 3 #6 bars inside

Shear force $V_u = 1.7 \times 10 = 17$ k, $V_c = 1.9\sqrt{f'_c}$ bd $= 1.9\sqrt{(0.003)} \times 8 \times 10 = 8.33$ k

If 2-legged #3 stirrups are used, $A_s = 0.22 \text{ in}^2$

:. Stirrup spacing,
$$S_{(req)} = A_s f_y d/(V_u/\phi - V_c) = 0.22 \times 40 \times 10/(17/0.85 - 8.33) = 7.54''$$

∴Provide 2-legged #3 stirrups @5"c/c (i.e., d/2)



8. Design of Substructure

Design of Abutment and Wing Walls



Stability Analysis (Using Working Load and Allowable Pressure)

The effect of front soil is ignored and wall geometry is simplified (denoted by the dotted lines).

Sliding

Approximate Vertical load = $12.69 + 0.12 \times \{(3+14) \times 8 + 4 \times 8.75\} + 0.15 \times \{14.5 \times 2 + 2 \times 14\}$ = 12.69 + 16.32 + 4.2 + 4.35 + 4.2 = 41.76 k/'

: Horizontal resistance $H_R = 0.45 \times 41.76 = 18.79 \text{ k/}'$

Horizontal load H = $1.90 + 0.12 \times (3 \times 20 + 20 \times 20/2)/3 = 1.90 + 2.4 + 8.0 = 12.30 \text{ k/}'$

: Factor of Safety against sliding = $H_R/H = 18.79/12.30 = 1.53 > 1.5$, OK

Overturning

Approximate resisting moment

$$\begin{split} M_{R} &= 12.69 \times (4.5 + 0.625) + 16.32 \times (6.5 + 4) + 4.2 \times (5.75 + 4.375) + 4.35 \times 7.25 + 4.2 \times 5.5 \\ &= 329.36 \ k'/' \end{split}$$

Overturning moment M = $1.90 \times 16 + 2.4 \times 20/2 + 8.0 \times 20/3 = 107.73 \text{ k'/'}$

: Factor of Safety against overturning = M_R/M = 329.36/107.73 = 3.06>2.5, OK

Design of Back-wall



This should be adjusted with stem reinforcement

Design of Stem

Design shear/length = $(0.12 \times 18 + 0.72 \times 18/2) \times 1.4 = 3.02 + 9.07 = 12.09 \text{ k/}'$ 0.12 Design moment/length = $3.02 \times 18/2 + 9.07 \times 18/3 = 81.60 \text{ k'/'}$ 4' $\therefore d_{(reg)}$ for shear = 12.09/1.062 = 11.39" and $d_{(reg)}$ for moment = $\sqrt{(81.60/0.781)} = 10.22''$ $d = 24 - 3.5 = 20.5'' > d_{(reg)}$: $A_s = (f_c/f_y)[1 - \sqrt{\{1 - 2M/(\phi f_c bd^2)\}}] bd = 1.39 in^{2/7}$ \therefore Use #10 @ 10.5" c/c (along the length near soil) 14' $A_{s(temp)} = 0.03 \times 24 = 0.72 \text{ in}^{2/2}$: Use #5 @ 5" c/c (along the width and length farther from soil) This should also be adequate for the back-wall , and the second se (both along length and width) 0.12 + 0.72

p_h (ksf)

6.8 k/′

9″

15"

18"

4′

1′

2′

Design of Toe and Heel

Total vertical force on the soil below the wall = 41.76 k/'

The resultant moment about the far end to the toe = $M_R - M = 329.36 - 107.73 = 221.63 \text{ k'/'}$

 \therefore e₀ = 221.63/41.76 = 5.31', which is >14.5/3 and <14.5×2/3 \Rightarrow uplift avoided.

The maximum soil pressure = $(41.76/14.5) \times [1+6(14.5/2-5.31)/14.5] = 5.20$ ksf, which is >2 ksf.

...Pile foundation is suggested, and the toe and heel should be designed as pile cap.

Design of Piles and Pile-cap

The following arrangement of piles is assumed for the width of the toe and heel (i.e., 14.5') and within the c/c distance of girders (i.e., width = 5.75').



Pile Forces

Vertical force and resultant moment in USD are = 60.84 k/' and = 473.27 - 156.61 = 316.67 k'/'For the total width of one row of piles, V = $60.84 \text{ k/'} \times 5.75' = 349.82 \text{ k}$

and M =
$$(60.84 \times 7.25 - 316.67) \text{ k'/'} \times 5.75' = 715.38 \text{ k'}$$

Pile reactions are given by $F_i = V/n + My_i/\sum y_i^2$
where n = Number of piles = 4, $\sum y_i^2 = 2 \times 1.75^2 + 2 \times 5.25^2 = 61.25 \text{ ft}^2$
 \therefore Here, $F_i = 349.82/4 + 715.38 y_i/61.25 = 87.46 + 11.68 y_i$
 \therefore $F_1 = 87.46 + 11.68 \times 5.25 = 148.77 \text{ k}$, $F_2 = 87.46 + 11.68 \times 1.75 = 107.90 \text{ k}$
 $F_3 = 87.46 + 11.68 \times (-1.75) = 67.02 \text{ k}$, $F_4 = 87.46 + 11.68 \times (-5.25) = 26.14 \text{ k}$
Design of Piles

For the load F₁, 148.77 = 0.80 ϕ A_g{f_c+ p (f_y - f_c)} = 0.80 × 0.70 A_g {2.55 + 0.03×(40-2.55)} \therefore A_g = 72.32 in² \Rightarrow D = 9.60"; i.e., Provide 10"-dia piles with 4 #6 bars and #3 ties @10" c/c The same pile will be used for all piles though smaller sections can be used for the others The piles should be long enough to transfer the axial loads safely to the surrounding soil by skin friction and end bearing.

Assuming the entire pile load for Pile1 to be resisted by skin friction, $F_{1(WSD)} = \alpha_2 (\pi DL) q_a/2$ where $\alpha_2 =$ Reduction factor for soil disturbance $\cong 0.8$, D = Pile diameter = 10" = 0.833', L = Pile length, q_a = Allowable compressive stress on soil = 2 ksf $\therefore 100.02 = 0.8 \times (\pi \times 0.83 \times L) \times 2/2 \implies L = 47.76', \therefore$ Provide 48' long piles

Design of Pile-cap

For the design of pile-cap, the pile loads per girder are considered along with the soil pressure converted to soil load per girder width. The pile loads on the pile-cap for c/c girder width (5.75') are shown below.



1. Design of Toe

The pile-cap is assumed to have 3" effective cover and 6" embedment for piles.

Maximum punching shear = 148.77 k, shear strength = $3.8 \times 0.85 \times \sqrt{(0.003)} = 0.177$ ksi

Punching area around a 10" pile = π (10+d) d = 148.77/0.177 \Rightarrow d_(req) \cong 12.1"; i.e., t_(req) = 21.1"

Maximum flexural shear = 148.77 k, shear strength = $1.9 \times 0.85 \times \sqrt{(0.003)} = 0.088$ ksi

Shearing area = $5.75 \times 12 \text{ d} = 148.77/0.088 \Rightarrow d_{(req)} = 24.4''; \text{ i.e., } t_{(req)} = 33.4''$

Maximum bending moment = $148.77 \times 2.5 = 371.93$ k'

 $\Rightarrow d_{(req)} = \sqrt{\{371.93/(0.781 \times 5.75)\}} = 9.1''; i.e., t_{(req)} = 18.1''$ $\therefore t = 34'', d = 25'' \Rightarrow A_s = (f_c/f_y)[1 - \sqrt{\{1 - 2M/(\phi f_c b d^2)\}}] bd = 7.69 in^2; i.e., 7.69/5.75 = 1.34 in^2/ft$ $A_{s(temp)} = 0.03 \times 34 = 1.02 in^2/ft$

2. Design of Heel

Maximum flexural shear =20.29×7- 67.02 -26.14 = 48.85 k, allowable shear stress = 0.088 ksi Shearing area = $5.75 \times 12 \text{ d} = 48.85/0.088 \Rightarrow d_{(req)} = 8''$; i.e., $t_{(req)} = 17''$ Maximum bending moment = $-20.29 \times 8 \times 8/2 + 67.02 \times 2.5 + 26.14 \times 6 = -324.78 \text{ k'}$ $\Rightarrow d_{(req)} = \sqrt{\{324.78/(0.781 \times 5.75)\}} = 8.5''$; i.e., $t_{(req)} = 17.5''$ $\therefore t = 34'', d = 25'' \Rightarrow$ It will be OK for punching shear also $\therefore A_s = (f_c/f_y)[1 - \sqrt{\{1-2M/(\phi f_c b d^2)\}}]bd = 5.62 \text{ in}^2$; i.e., 0.98 in²/ft, $A_{s(temp)} = 0.03 \times 34 = 1.02 \text{ in}^2/\text{ft}$

