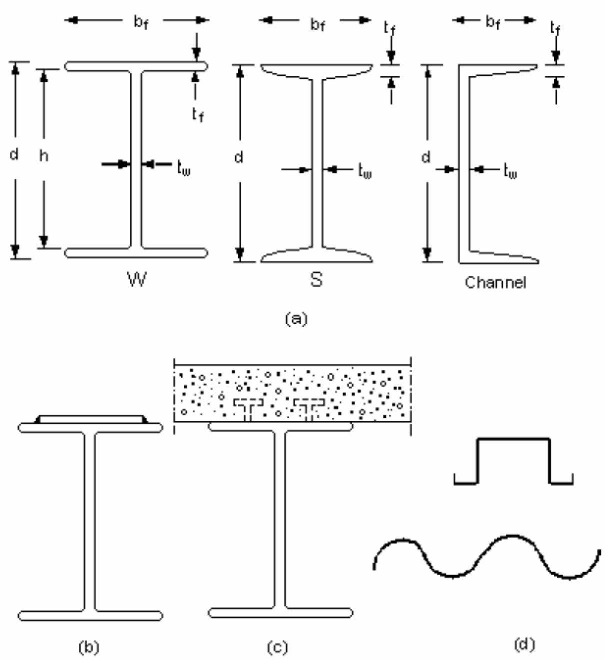
Design of Beams (Flexural Members)
(Part 5 of AISC/LRFD)



References

1. Part 5 of the AISC LRFD Manual
2. Chapter F and Appendix F of the AISC LRFD Specifications (Part 16 of LRFD Manual)
3. Chapter F and Appendix F of the Commentary of the AISC LRFD Specifications (Part 16 of LRFD Manual)

Basic Theory

If the axial load effects are negligible, it is a *beam*; otherwise it is a *beam-column.*

Shapes that are built up from plate elements are usually called *plate girders*; the difference is the height-thickness ratio $\frac{h}{t_w}$ of the web.

$$\begin{cases} \dfrac{h}{t_w} \le 5.70\sqrt{\dfrac{E}{F_y}} & \text{beam} \\ \dfrac{h}{t_w} > 5.70\sqrt{\dfrac{E}{F_y}} & \text{plate girder} \end{cases}$$

Bending

M = bending moment at the cross section under consideration
y = perpendicular distance from the neutral plane to the point of interest
I_x = moment of inertia with respect to the neutral axis
S_x = elastic section modulus of the cross section

For *elastic analysis*, from the elementary mechanics of materials, the bending stress at any point can be found

$$f_b = \frac{My}{I_x}$$

The maximum stress

$$f_{\max} = \frac{Mc}{I_x} = \frac{M}{I_x / c} = \frac{M}{S_x}$$

This is valid as long as the loads are small and the material remains linearly elastic. For steel, this means $f_{\max}$ must not exceed F_y and the bending moment must not exceed

$$\boxed{M_y = F_y S_x}$$

M_y = the maximum moment that brings the beam to the point of yielding

For *plastic analysis*, the bending stress everywhere in the section is F_y, the plastic moment is

$$M_p = F_y\left(\frac{A}{2}\right)a = F_y Z$$

M_p = plastic moment

A = total cross-sectional area

a = distance between the resultant tension and compression forces on the cross-section

$Z = \left(\frac{A}{2}\right)a$ = plastic section modulus of the cross section

Shear

Shear stresses are usually not a controlling factor in the design of beams, except for the following cases: 1) The beam is very short. 2) There are holes in the web of the beam. 3) The beam is subjected to a very heavy concentrated load near one of the supports. 4) The beam is coped.

f_v = shear stress at the point of interest

V = vertical shear force at the section under consideration

Q = first moment, about the neutral axis, of the area of the cross section between the point of interest and the top or bottom of the cross section

I = moment of inertia with respect to the neutral axis

b = width of the cross section at the point of interest

From the elementary mechanics of materials, the shear stress at any point can be found

$$f_v = \frac{VQ}{Ib}$$

This equation is accurate for small b. Clearly the web will completely yield long before the flange begins to yield. Therefore, *yield of the web represents one of the shear limit states*. Take the shear yield stress as 60% of the tensile yield stress, for the web at failure

$$f_v = \frac{V_n}{A_w} = 0.60F_y$$

A_w = area of the web

The nominal strength corresponding to the limit state is

$$V_n = 0.60F_y A_w$$

This will be the nominal strength in shear provided that there is no shear buckling of the web. This depends on $\frac{h}{t_w}$, the width-thickness ratio of the web. Three cases:

No web instability: $\frac{h}{t_w} \le 2.45\sqrt{\frac{E}{F_y}}$

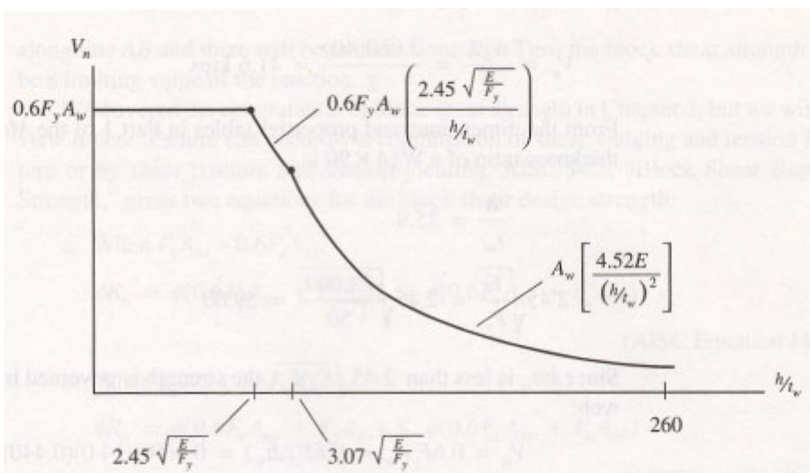
$$V_n = 0.60F_y A_w \qquad \text{AISC Eq. (F2-1)}$$

Inelastic web buckling: $2.45\sqrt{\frac{E}{F_y}} < \frac{h}{t_w} \le 3.07\sqrt{\frac{E}{F_y}}$

$$V_n = 0.60 F_y A_w \left(\frac{2.45\sqrt{E / F_y}}{h / t_w} \right) \qquad \text{AISC Eq. (F2-2)}$$

Elastic web buckling: $3.07\sqrt{\frac{E}{F_y}} < \frac{h}{t_w} \le 260$

$$V_n = A_w \left[\frac{4.52E}{(h / t_w)^2} \right] \qquad \text{AISC Eq. (F2-3)}$$



Failure Modes

Shear: A beam can fail due to violation of its shear design strength.

Flexure: Several possible failure modes must be considered. A beam can fail by reaching M_p (fully plastic), or it can fail by

- Lateral torsional buckling (LTB), elastically or inelastically
- Flange local buckling (FLB), elastically or inelastically

- Web local buckling (WLB), elastically or inelastically

If the maximum bending stress is less than the proportional limit when *buckling* occurs, the failure is *elastic*. Otherwise, it is *inelastic*.

Lateral Torsional Buckling

The compressive flange of a beam behaves like an axially loaded column. Thus, in beams covering long spans the compression flange may tend to buckle. However, this tendency is resisted by the tensile flange to certain extent. The overall effect is a phenomenon known as *lateral torsional buckling*, in which the beam tends to twist and displace laterally. Lateral torsional buckling may be prevented by: 1) Using lateral supports at intermediate points. 2) Using torsionally strong sections (e.g., box sections). 3) Using I-sections with relatively wide flanges.

Local Buckling

The hot-rolled steel sections are thin-walled sections consisting of a number of thin plates. When normal stresses due to bending and/or direct axial forces are large, each plate (for example, flange or web plate) may buckle locally in a plane perpendicular to its plane. In order to prevent this undesirable phenomenon, the *width-to-thickness ratios of the thin flange and the web plates are limited by the code*.

AISC classifies cross-sectional shapes as compact, noncompact and slender ones, depending on the value of the width-thickness ratios. (LRFD-Specification Table B5.1)

λ = width-thickness ratio

λ_p = upper limit for compact category

λ_r = upper limit for noncompact category

Then the three cases are

$\lambda \le \lambda_p$ and the flange is continuously connected to the web, the shape is *compact*.

$\lambda_p < \lambda \le \lambda_r$ the shape is *noncompact*

$\lambda > \lambda_r$ the shape is *slender*

The above conditions are based on the worst width-thickness ratio of the elements of the cross section. The following table summarizes the width-thickness limits for rolled I-, H- and C- sections (for C-sections, $\lambda = b_f / t_f$. The web criterion is met by all standard I- and C- sections listed in the Manual. Built-up welded I- shapes (such as plate girders can have noncompact or slender elements).

Element	λ	λ_p	λ_r
Flange	$\dfrac{b_f}{2t_f}$	$0.38\sqrt{\dfrac{E}{F_y}}$	$0.83\sqrt{\dfrac{E}{F_y - 10}}$
Web	$\dfrac{h}{t_w}$	$3.76\sqrt{\dfrac{E}{F_y}}$	$5.70\sqrt{\dfrac{E}{F_y}}$

Design Requirements

1. Design for flexure (LRFD SPEC F1)

L_b unbraced length, distance between points braced against lateral displacement of the compression flange (in.)

L_p limiting laterally unbraced length for full plastic bending capacity (in.) – a property of the section

L_r limiting laterally unbraced length for inelastic lateral-torsional buckling (in.) – a property of the section

E modulus of elasticity for steel (29,000 ksi)

G shear modulus for steel (11,200 ksi)

J torsional constant (in.^{4})

C_w warping constant (in.^{6})

M_r limiting buckling moment (kip-in.)

M_p plastic moment, $\boxed{M_p = F_y Z \le 1.5 M_y}$

M_y moment corresponding to the onset of yielding at the extreme fiber from an elastic stress distribution $M_y = F_y S_x$

M_u controlling combination of factored load moment

M_n nominal moment strength

ϕ_b resistance factor for beams (0.9)

The limit of $1.5M_y$ for M_p is to prevent excessive working-load deformation that is satisfied when

$$M_p = F_y Z \leq 1.5 M_y \text{ or } F_y Z \leq 1.5 F_y S \text{ or } \frac{Z}{S} \leq 1.5$$

Design equation

Applied factored moment ≤ moment capacity of the section

OR

Required moment strength ≤ design strength of the section

$$M_u \leq \phi_b M_n$$

In order to calculate the nominal moment strength M_n, first calculate L_p, L_r, and M_r for I-shaped members including hybrid sections and channels as

$L_p = 1.76 r_y \sqrt{\dfrac{E}{F_y}}$ - a section property AISC Eq. (F1-4)

$L_r = \dfrac{r_y X_1}{F_L} \sqrt{1 + \sqrt{1 + X_2 F_L^2}}$ - a section property AISC Eq. (F1-6)

$M_r = F_L S_x$ - section property AISC Eq. (F1-7)

$F_L = F_y - F_r$ for nonhybrid member, otherwise it is the smaller of $F_{yf} - F_r$ or F_{yw} (subscripts *f* and *w* mean flange and web)

F_r compressive residual stress in flange, 10 ksi for rolled shapes; 16.5 ksi for welded built-up shapes

$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}}$$ AISC Eq. (F1-8)

$$X_2 = \frac{4C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2$$ AISC Eq. (F1-9)

S_x section modulus about the major axis (in.^{3})

I_y moment of inertia about the minor y-axis (in.^{4})

r_y radius of gyration about the minor y-axis (in.^{4})

Nominal Bending Strength of Compact Shapes

If the shape is *compact* $(\lambda \le \lambda_p)$, no need to check FLB (flange local buckling) and WLB (web local buckling).

- **Lateral torsional buckling (LTB)**

  If $L_b \le L_p$, no LTB:

  $$M_n = M_p \le 1.5M_y$$ AISC Eq. (F1-1)

  If $L_p < L_b \le L_r$, inelastic LTB:

  $$M_n = C_b\left[M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$ AISC Eq. (F1-2)

  Note that M_n is a linear function of L_b.

  If $L_b > L_r$ (slender member), elastic LTB:

  $$M_n = M_{cr} \le M_p$$ AISC Eq. (F1-12)

$$M_{cr} = C_b \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w} \le M_p$$

$$= \frac{C_b S_x X_1 \sqrt{2}}{L_b / r_y}\sqrt{1 + \frac{X_1^2 X_2}{2\left(L_b / r_y\right)^2}}$$ AISC Eq. (F1-13)

Note that M_{cr} is a nonlinear function of L_b

C_b is a factor that takes into account the nonuniform bending moment distribution over an unbraced length L_b

M_A absolute value of moment at quarter point of the unbraced segment

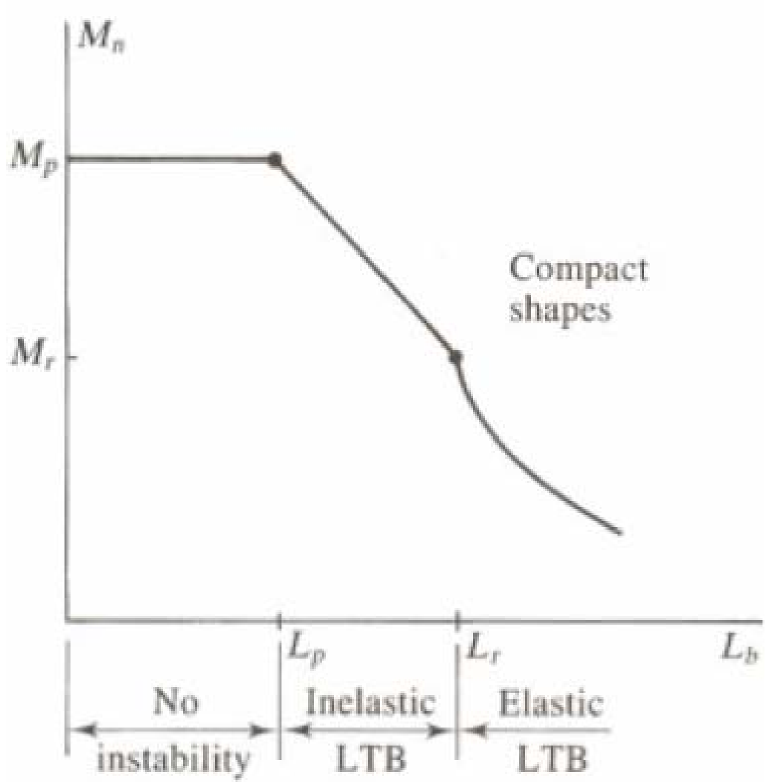
M_B absolute value of moment at mid-point of the unbraced segment

M_C absolute value of moment at three-quarter point of the unbraced segment

$M_{\max}$ absolute value of maximum moment in the unbraced segment

$$C_b = \frac{12.5 M_{\max}}{2.5 M_{\max} + 3M_A + 4M_B + 3M_C}$$ AISC Eq. (F1-3)

If the bending moment is uniform, all moment values are the same giving $C_b = 1$. This is also true for a conservative design.



Nominal Bending Strength of Noncompact Shapes

If the shape is *noncompact* $(\lambda_p < \lambda \le \lambda_r)$ because of the flange, the web or both, the nominal moment strength will be the smallest of the following:

- **Lateral torsional buckling (LTB)**

  If $L_b \le L_p$, no LTB:

  $$M_n = M_p \le 1.5M_y \qquad \text{AISC Eq. (F1-1)}$$

  If $L_p < L_b \le L_r$, inelastic LTB:

  $$M_n = C_b\left[M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{AISC Eq. (F1-2)}$$

Note that M_n is a linear function of L_b

If $L_b > L_r$, elastic LTB:

$$M_n = M_{cr} \le M_p \qquad \text{AISC Eq. (F1-12)}$$

$$M_{cr} = C_b \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w} \le M_p \qquad \text{AISC Eq. (F1-13)}$$

Note that M_{cr} is a nonlinear function of L_b

- **Flange local buckling (FLB)**

If $\lambda \le \lambda_p$, no FLB.

If $\lambda_p < \lambda \le \lambda_r$, the flange is noncompact:

$$M_n = M_p - \left(M_p - M_r\right)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right) \le M_p \qquad \text{AISC Eq. (A-F1-3)}$$

Note that M_n is a linear function of λ

- **Web local buckling (WLB)**

If $\lambda \le \lambda_p$, no WLB.

If $\lambda_p < \lambda \le \lambda_r$, the web is noncompact:

$$M_n = M_p - \left(M_p - M_r\right)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right) \le M_p \qquad \text{AISC Eq. (A-F1-3)}$$

Note that M_n is a linear function of λ

Slender sections $\lambda > \lambda_r$: For laterally stable slender sections

$$M_n = M_{cr} = SF_{cr} \le M_p$$

M_{cr} critical (buckling) moment

F_{cr} critical stress

2. *Design for shear (LRFD SPEC F2)*

ϕ_v resistance factor for shear (0.9)

V_u controlling combination of factored shear

V_n nominal shear strength

F_{yw} yield stress of the web (ksi)

A_w web area, the overall depth *d* times the web thickness t_w

Design equation for $\frac{h}{t_w} \le 260$:

$$V_u \le \phi_v V_n$$

The design shear strength of unstiffened web is $\phi_v V_n$, where

$$V_n = \begin{cases} 0.60 F_{yw} A_w & \dfrac{h}{t_w} \le 2.45\sqrt{\dfrac{E}{F_{yw}}} \\ 0.60 F_{yw} A_w \left(\dfrac{2.45\sqrt{E/F_{yw}}}{h/t_w} \right) & 2.45\sqrt{\dfrac{E}{F_{yw}}} < \dfrac{h}{t_w} \le 3.07\sqrt{\dfrac{E}{F_{yw}}} \\ A_w \left[\dfrac{4.52E}{(h/t_w)^2} \right] & 3.07\sqrt{\dfrac{E}{F_{yw}}} < \dfrac{h}{t_w} \le 260 \end{cases}$$

These are Eqs. (F2-1), (F2-2) and (F2-3) in Chapter F of LRFD Specifications. For $\frac{h}{t_w} > 260$, web stiffeners are required, and the provision of Appendix F2 must be consulted. Note that shear is rarely a problem in rolled steel beams; the usually practice is to design a beam for flexural and check for shear.

3. Design for serviceability

Deflection of beam should be checked with service loads. This is the serviceability requirement of a structure. (LRFD-Specification L).

Design Procedure

- Compute the factored load moment M_u (required moment strength); it should be less than or equal to the design strength, $\phi_b M_n$. The weight of the beam is part of the dead load but is unknown at this point. A value may be assumed, or ignored temporarily.
- Select a shape that satisfies the flexural strength requirement. This can be done in one of the following two ways:

  Assume a shape, compute the design strength and compare it with the factored load moment. Revise if necessary.

  Use the beam design charts in LRFD Part 5.
- Check the shear strength.
- Check the deflection (serviceability requirement).