NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM
SYNTHESIS OF HIGHWAY PRACTICE 42

DESIGN OF PILE FOUNDATIONS

TRANSPORTATION RESEARCH BOARD
NATIONAL RESEARCH COUNCIL

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SYNTHESIS OF HIGHWAY PRACTICE **42**

DESIGN OF PILE FOUNDATIONS

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RESEARCH SPONSORED BY THE AMERICAN ASSOCIATION OF STATE HIGHWAY AND TRANSPORTATION OFFICIALS IN COOPERATION WITH THE FEDERAL HIGHWAY ADMINISTRATION

AREAS OF INTEREST:
- BRIDGE DESIGN
- CONSTRUCTION
- FOUNDATIONS (SOILS)
- RAIL TRANSPORT

TRANSPORTATION RESEARCH BOARD
NATIONAL RESEARCH COUNCIL
WASHINGTON, D.C. 1977

NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM

Systematic, well-designed research provides the most effective approach to the solution of many problems facing highway administrators and engineers. Often, highway problems are of local interest and can best be studied by highway departments individually or in cooperation with their state universities and others. However, the accelerating growth of highway transportation develops increasingly complex problems of wide interest to highway authorities. These problems are best studied through a coordinated program of cooperative research.

In recognition of these needs, the highway administrators of the American Association of State Highway and Transportation Officials initiated in 1962 an objective national highway research program employing modern scientific techniques. This program is supported on a continuing basis by funds from participating member states of the Association and it receives the full cooperation and support of the Federal Highway Administration, United States Department of Transportation.

The Transportation Research Board of the National Research Council was requested by the Association to administer the research program because of the Board's recognized objectivity and understanding of modern research practices. The Board is uniquely suited for this purpose as: it maintains an extensive committee structure from which authorities on any highway transportation subject may be drawn; it possesses avenues of communications and cooperation with federal, state, and local governmental agencies, universities, and industry; its relationship to its parent organization, the National Academy of Sciences, a private, nonprofit institution, is an insurance of objectivity; it maintains a full-time research correlation staff of specialists in highway transportation matters to bring the findings of research directly to those who are in a position to use them.

The program is developed on the basis of research needs identified by chief administrators of the highway and transportation departments and by committees of AASHTO. Each year, specific areas of research needs to be included in the program are proposed to the Academy and the Board by the American Association of State Highway and Transportation Officials. Research projects to fulfill these needs are defined by the Board, and qualified research agencies are selected from those that have submitted proposals. Administration and surveillance of research contracts are responsibilities of the Academy and its Transportation Research Board.

The needs for highway research are many, and the National Cooperative Highway Research Program can make significant contributions to the solution of highway transportation problems of mutual concern to many responsible groups. The program, however, is intended to complement rather than to substitute for or duplicate other highway research programs.

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The members of the technical committee selected to monitor this project and to review this report were chosen for recognized scholarly competence and with due consideration for the balance of disciplines appropriate to the project. The opinions and conclusions expressed or implied are those of the research agency that performed the research, and, while they have been accepted as appropriate by the technical committee, they are not necessarily those of the Transportation Research Board, the National Research Council, the National Academy of Sciences, or the program sponsors.

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PREFACE

There exists a vast storehouse of information relating to nearly every subject of concern to highway administrators and engineers. Much of it resulted from research and much from successful application of the engineering ideas of men faced with problems in their day-to-day work. Because there has been a lack of systematic means for bringing such useful information together and making it available to the entire highway fraternity, the American Association of State Highway and Transportation Officials has, through the mechanism of the National Cooperative Highway Research Program, authorized the Transportation Research Board to undertake a continuing project to search out and synthesize the useful knowledge from all possible sources and to prepare documented reports on current practices in the subject areas of concern.

This synthesis series attempts to report on the various practices, making specific recommendations where appropriate but without the detailed directions usually found in handbooks or design manuals. Nonetheless, these documents can serve similar purposes, for each is a compendium of the best knowledge available on those measures found to be the most successful in resolving specific problems. The extent to which they are utilized in this fashion will quite logically be tempered by the breadth of the user's knowledge in the particular problem area.

FOREWORD

By Staff
Transportation Research Board

This synthesis will be of special interest and usefulness to bridge engineers and others seeking information on pile foundations. Detailed information is presented on pile design principles and criteria.

Administrators, engineers, and researchers are faced continually with many highway problems on which much information already exists either in documented form or in terms of undocumented experience and practice. Unfortunately, this information often is fragmented, scattered, and unevaluated. As a consequence, full information on what has been learned about a problem frequently is not assembled in seeking a solution. Costly research findings may go unused, valuable experience may be overlooked, and due consideration may not be given to recommended practices for solving or alleviating the problem. In an effort to correct this situation, a continuing NCHRP project, carried out by the Transportation Research Board as the research agency, has the objective of synthesizing and reporting on common highway problems. Syntheses from this endeavor constitute an NCHRP report series that collects and assembles the various forms of information into single concise documents pertaining to specific highway problems or sets of closely related problems.

Pile foundations are used by all state highway agencies and by other organizations involved in civil engineering projects. However, present procedures for design vary considerably among agencies and in some cases do not reflect the best available information. This report of the Transportation Research Board reviews design principles and construction problems and recommends criteria based on current knowledge.

To develop this synthesis in a comprehensive manner and to ensure inclusion of significant knowledge, the Board analyzed available information assembled from numerous sources, including a large number of state highway and transportation departments. A topic panel of experts in the subject area was established to guide the researchers in organizing and evaluating the collected data, and to review the final synthesis report.

This synthesis is an immediately useful document that records practices that were acceptable within the limitations of the knowledge available at the time of its preparation. As the processes of advancement continue, new knowledge can be expected to be added to that now at hand.

CONTENTS

FIGURES

TABLES

ACKNOWLEDGMENTS

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DESIGN OF PILE FOUNDATIONS

SUMMARY

The first problem facing the designer of a foundation is to determine whether or not the site conditions are such that piles must be used. Piles are used where upper soil strata are compressible or weak; where footings cannot transmit inclined, horizontal, or uplift forces; where scour is likely to occur; where future excavation may be adjacent to the structure; and where expansive or collapsible soils extend for a considerable depth.

Piles may be classified by material type or by method of placement. The choice of pile type is influenced by subsurface conditions, location and topography of the site, and structural and geometric characteristics of the structure to be supported.

The designer of a deep foundation must possess a variety of skills, much experience, and considerable knowledge of engineering sciences. No set of simple rules and procedures can be expected to cover the variety of conditions and forms of instability that can endanger a deep foundation.

The ultimate load on a pile is the load that can cause failure of either the pile or the soil. The pile failure condition may govern design where pile points penetrate dense sand or rock, but in most situations, ultimate load is determined by the soil failure. The soil always fails in the same manner: punching shear under the point, accompanied or preceded by direct-shear failure along the shaft. Because the ultimate load is often not well defined, various empirical ultimate load criteria have to be used. Most often these have been based on considerations of plastic (irrecoverable) or total (plastic and elastic) settlements of the pile under a test load. Unless the load-settlement curve shows a definite peak load, the most acceptable criterion would define the ultimate load as that causing total pile settlement equal to 10 percent of the point diameter for driven piles and 25 percent for bored piles.

Computation of the ultimate load is quite difficult and a "general solution" is not yet available. For design purposes, the ultimate load is separated into two components: the base or point load, and the shaft or skin load. Theories for determination of point load based on the plasticity theory are now considered inadequate and are being replaced by linear or nonlinear elasto-plastic theories. The theoretical approach for evaluation of skin resistance is similar to that used to analyze resistance to sliding of a rigid body in contact with the soil. Equations are available to calculate the point and skin resistances. However, the calculations require detailed knowledge of strength and deformation characteristics of the soil strata and also of the variation of density and water content within those strata. For most structures, the cost of obtaining this information is prohibitive; in addition, it is normally preferable to estimate unit resistance directly from such field tests as the static-cone-penetration test, standard penetration test, or pressuremeter test.

The displacement needed to mobilize skin resistance is small compared to that for point resistance. Thus, ultimate skin resistance is reached much sooner than point resistance and the portion of the load carried by the point is smaller in working conditions than at failure.

In situations where soil around a pile moves downward (e.g., because of water removal from aquifer strata), it exerts a negative friction (downdrag) on the pile

and may cause severe damage to the structure. Several methods are available to reduce negative skin friction, including installing piles in casing or coating them with bitumen.

The mechanics of load transfer between pile and soil is very complex. Some parameters are difficult to express in numerical terms. Yet numerical assessment is essential, and several analytical methods are available, based either on a transfer-function approach or an elastic-solid approach.

Settlement analysis of pile foundations is somewhat similar to that of shallow foundations. However, there are differences because the soil has been disturbed by pile placement and thus may exhibit sharp variations of stiffness. Furthermore, a driven pile may retain residual stresses, the exact position of load transfer to the soil is unknown, and adjacent piles may have an effect. Thus, only approximate solutions to pile settlement are available.

Pile foundations are normally constructed as groups of closely spaced piles. Pile spacing is based on stability and economy; ideally, the spacing should be such that the group capacity is not less than the sum of the capacities of the individual piles. However, this is not always possible. The ultimate load of a group may be less than the sum depending on soil type, size and shape of the group, spacing and length of piles, and construction procedures. In addition, settlement of a group is usually more than that of a single pile with comparable working load. There is currently no acceptable rational theory for the bearing capacity of groups. A number of empirical "efficiency formulae" have been used, as well as other methods. However, current knowledge of the factors affecting bearing capacity and settlement of pile groups is limited and research is needed for safe, economical design.

When a pile is required to transmit lateral loads into the ground, it is necessary to determine lateral deflection, slope of the pile axis, and position and magnitude of maximum bending moment. If certain assumptions are made, the solutions are relatively simple and reasonably accurate.

Horizontal movements of the ground can cause a dangerous lateral loading on piles that can be large enough to break a pile in bending or shear. Piles in areas where this type of loading can occur must be designed with a sufficient safety factor to prevent failure.

Buckling of fully embedded piles is rare as long as the soil is capable of supporting a pile in friction. Partially embedded piles may be analyzed to determine the buckling load.

Horizontal loads on pile groups may require that some piles be battered to avoid excessive movement of the foundation or bending in the piles. Design solutions are available that show a reasonable agreement with observed behavior.

Pile driving is most commonly done by impact hammers wherein a ram falls or is driven repeatedly against the pile head. Vibratory drivers are also used. At the driving site the engineer needs to know whether a pile can be driven with a given hammer; if so, what is the set in the final blows, what is the maximum stress in the pile, and should a hard or soft cushion be used? To answer these questions, a rational analysis may be useful. Computations show that both compressive and tensile stresses in piles can be reduced by using a heavier ram with lower impact velocity. The stresses can also be reduced by the use of cushions. The wave-equation approach, despite some shortcomings, is superior to the conventional "pile-formula" approach, although the latter may work well for many sites.

Because of the many uncertainties in analysis of pile foundations, it has become customary to perform full-scale pile load tests at the site of important projects to verify actual pile response to load. Various arrangements are used to load the test pile, all incorporating a hydraulic jack to transmit the load and gauges to measure movement. A strain rod arrangement built into the test pile allows measurement of settlement at points along the pile. Two principal methods of loading are used. In the load-controlled mode, the load is applied in increments of the design load and maintained until settlements cease. In the displacement-controlled mode, small increments of settlement are imposed and maintained until the load reaches equilibrium. In a variation of the latter mode settlement, increments are imposed at specified time intervals. Load testing can not be used as a substitute for an engineering analysis of anticipated performance under load. Such an analysis should be based on the principles outlined in this synthesis and must include an adequate soil exploration and testing program.

CHAPTER ONE

INTRODUCTION

ESTABLISHMENT OF NEED FOR A PILE FOUNDATION

The first difficult problem confronting the designer of a foundation is to establish whether or not the site conditions are such that piles must be used. Principal situations in which piles may be needed are shown in Figure 1. The most common case is that in which the upper soil strata are too compressible or generally too weak to support heavy vertical reactions transmitted by the superstructure. In this instance, piles serve as extensions of columns or piers to carry the loads to a deep, rigid stratum such as rock (point-bearing piles, Fig. 1a). If such a rigid stratum does not exist within a reasonable depth, the loads must be gradually transferred, mainly by friction, along pile shafts (friction piles, Fig. 1b).

Piles are also frequently required because of the relative inability of shallow footings to transmit inclined, horizontal, or uplift forces and overturning moments. Such situations are common in design of earth-retaining structures (walls and bulkheads) and tall structures subjected to high wind and earthquake forces. Piles resist upward forces by negative friction around their shafts (uplift piles, Fig. 1c). Horizontal forces are resisted either by vertical piles in bending (Fig. 1d) or by groups of vertical and battered piles that act as a structural system, combining the axial and lateral resistances of all piles in the group (Fig. 1e).

Although the use of pile foundations is normally associated with the presence of some weak strata in the soil profile, it is by no means restricted to such soil conditions. For example, pile foundations are often required when scour around footings could cause erosion, in spite of the presence of strong, incompressible strata (such as sand or gravel) at shallow depths. In such conditions, piles serve as the ultimate assurance that the foundation loads will be transmitted to the ground even if the currents erode the soil surrounding the base of the structure (Fig. 1f). A similar use of piles is found occasionally in urban areas, where future construction operations may present serious problems for the planned structure if shallow foundations are used. In the example shown in Figure 1g, where future excavation is anticipated, it may be advisable to use piles to carry foundation loads below the level of expected excavation. This may deter costly damages and eliminate the need for future underpinning.

In areas where expansive or collapsible soils extend to considerable depth below the soil surface, pile foundations may be needed to assure safety against undesirable seasonal movements of the foundations. The function of piles in such situations is to transfer the foundation loads, including uplift from an expanding soil (Fig. 1h) or downdrag from a collapsing soil, to a level where the soil is not affected by seasonal movements of moisture. Similar need for pile foundations can arise in areas of discontinuous permafrost to ensure transmission of loads to the part of the ground not affected by surface thaw.

To establish whether or not given structural loads can be transmitted by a shallow foundation, a reasonably detailed analysis of bearing capacity and settlement or, more generally, displacement of a hypothetical shallow foundation as an alternate solution is often needed. Such an analysis provides the dimensions and depth of shallow footings needed to ensure desired safety against shear failure in the soil around the foundation, as well as the magnitude and time-rate of displacement of these footings under anticipated loads. This analysis should follow established pro-

* The distinction between point-bearing piles and friction piles, common among practitioners, is somewhat arbitrary in the sense that all piles transmit loads simultaneously along their shafts and at their points. It would be more accurate to distinguish between predominantly point-bearing and predominantly friction piles. Strictly speaking, the use of these terms is justified only when either shaft load or point load is neglected in design.

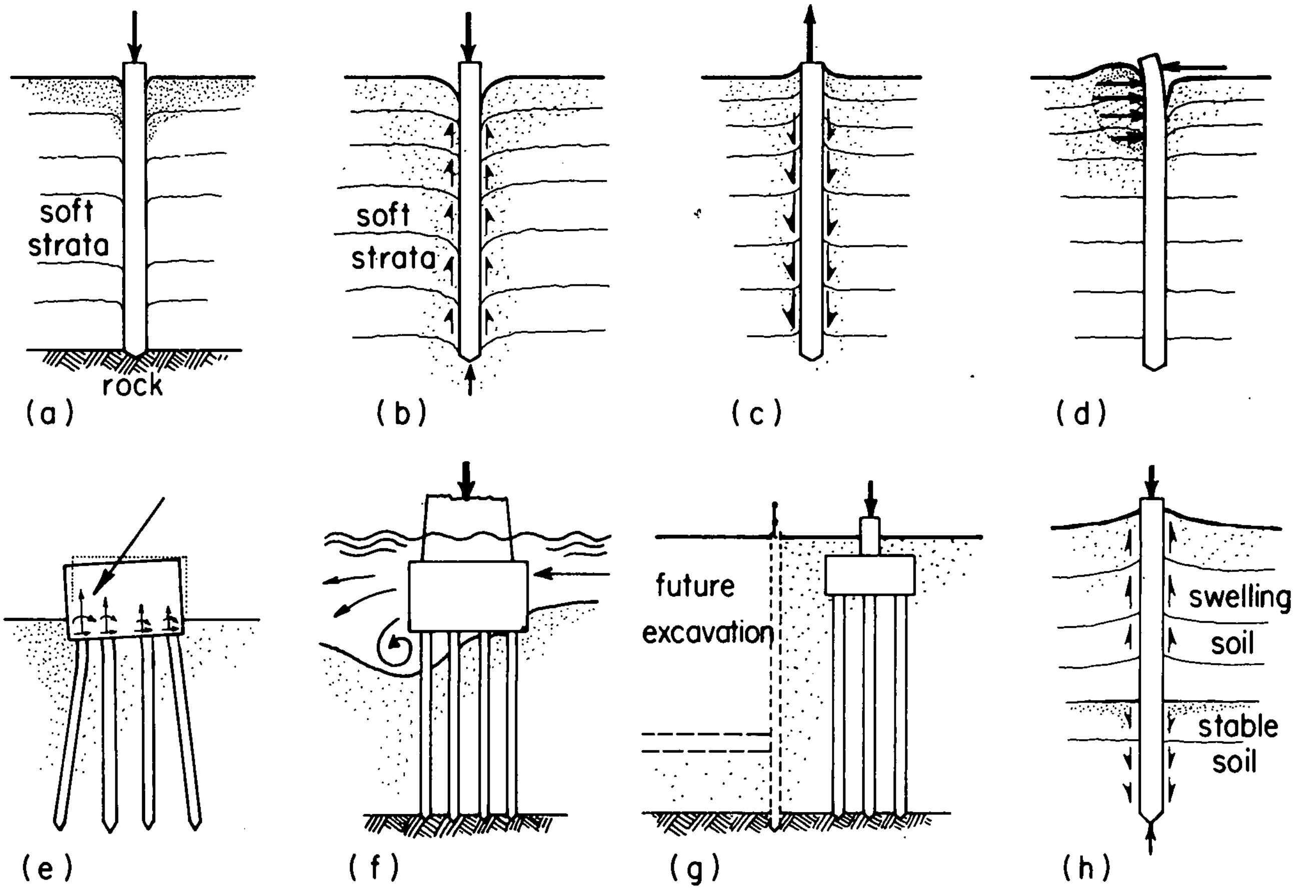


Figure 1. Situations in which piles may be needed.

cedures based on principles of soil mechanics (see Refs. *1*, *2*). A comparative analysis of the pile foundation alternative is then made following the methods presented in this synthesis. An approximate cost analysis of both alternatives may also be needed if both a shallow foundation and a pile foundation are technically feasible. Such an analysis follows the principles of engineering economic analysis (*3*) and may include such factors as construction time and uncertainties. For examples of comparative cost analyses of shallow and deep foundations, see References *4*, *5*.

To determine the depth of pile penetration needed as protection against scour, a study of the hydraulic regime of the particular stream or beach may be necessary in addition to collection of records on local experience. For important high cost structures, expert advice in river or coastal hydraulics is needed. References *6*, *7*, and *8* contain simple guidelines for analysis of this type of problem.

In expansive soils, it is essential for good design to ensure the transfer of foundation loads through the most active upper layers of swelling soil to the more stable deeper strata. It is necessary first to establish, by appropriate soil testing and analysis, (a) the anticipated variation of moisture conditions on the site and (b) the depth of the active zone. Such a study can be made following some of the proposed approaches for analysis of soil moisture and suction profiles on the site (*9-12*). Guidelines for pile design in such conditions can be found in References *13*, *14*, and *15*. The design problems of piles subjected to uplift by frost heave of soils or by movements of a frozen ice sheet surrounding a pile in water are somewhat similar (see Refs. *16*, *17*).

SELECTION OF PILE TYPE

There are many different types of piles (at least 100 have been described in the engineering literature) and they can be classified in different ways. Classification on the basis of material divides piles into timber, steel, and concrete. These materials are sometimes combined to form composite piles. Timber and steel piles are always prefabricated; concrete piles can be precast of reinforced or prestressed concrete, or cast *in situ*. Casting *in situ* is usually done under the protection of a casing, which can be removed to be re-used or can be left in the ground as an integral part of the pile. Occasionally, in firm and stable ground, piles can be cast without any casing. A review of the primary advantages and disadvantages of pile types classified on the basis of material is given in Table 1. More details, including structural features, durability, and construction characteristics of different types, can be found in References *4*, *18*, *19*, and *20*.

Structural behavior of piles is greatly affected by the method of their placement into the ground. When classified on this basis, piles can be divided into displacement piles and bored (nondisplacement) piles. Displacement

TABLE 1

PRINCIPAL ADVANTAGES AND DISADVANTAGES OF DIFFERENT PILE TYPES

PILE TYPE	ADVANTAGES	DISADVANTAGES
Timber	Easy to handle or cut-off. Relatively inexpensive material. Readily available (USA). Naturally tapered.	Decay above water table, especially in marine environment. Limited in size and bearing capacity. Prone to damage by hard driving. Difficult to extend. Noisy to drive.
Steel	Easy to handle, cut off, extend. Available in any length or size. Can penetrate hard strata, boulders, soft rock. Convenient to combine with steel superstructure.	Subject to corrosion, require protection in marine environment. Flexible H-piles may deviate from axis of driving. Relatively expensive material. Noisy to drive.
Concrete: Precast	Durability in almost any environment. Convenient to combine with concrete superstructure.	Cumbersome to handle and drive. Difficult to cut off or extend. Noisy to drive.
Cast-in-place: Casing left in ground	Allows inspection before concreting. Easy to cut off or extend.	Casing cannot be re-used. Thin casing may be damaged by impact or soil pressure.
Casing withdrawn or no casing	No storage space required. Can be finished at any elevation. Can be made before excavation. Some types allow larger displacements in weaker soils. Some types have no driving operation suitable where noise and vibration are prohibited (downtown).	In soft soils shaft may be squeezed by soil cave-in. In case of heavy compaction of concrete previously completed piles may be damaged. If concrete is placed too fast there is danger of creation of a void.

piles are placed in the ground by operations such as driving, jacking, or vibration. In all cases, the pile displaces the surrounding soil and causes some increase in lateral ground stress, along with possible densification of the soil. The degree to which the effects of displacement are felt depends on the geometry of the cross section of the pile, as well as on the method of pile placement. Thus, a closed-end pipe or a full-section square pile causes much larger lateral-stress increase than an open-ended pipe or a steel H-pile. (The latter two are examples of low-displacement piles.) In loose, cohesionless deposits, driven piles may cause considerably more soil densification than vibrated or jacked piles and the degree of densification will, to a certain extent, depend also on the type of hammer used, as well as on the magnitude and frequency of driving strokes.

Nondisplacement piles are driven prefabricated, or cast in a hole created by removing an equal volume of soil from the ground. This is usually done by operations such as augering (drilling, rotary boring) or grabbing (percussion boring), often under protection of a casing, or drilling mud, or both, which are used to prevent soil cave-in and large-scale disturbance. In very dense, cohesionless strata, jetting with water can be used, at least part of the way, to remove an equivalent volume of soil and to ease driving to the desired depth. Piles so formed would be classified as partially or fully nondisplaced. Their placement causes little or no change in lateral ground stress, and consequently, such piles develop less shaft friction than displacement piles of the same size and shape.

The choice of appropriate pile type in any given circumstance is influenced mainly by subsurface conditions, location and topography of the site, and structural and geometrical characteristics of the proposed superstructure. Subsurface soil and water conditions usually represent the most significant factors. For example, if the bearing stratum is hard rock covered by boulders or disintegrated rock, it is essential to select a pile type that can penetrate to the desired elevation. The presence of thin strata of hard rock or of loose boulders, coarse gravel, or simply landfill containing debris of old buildings also poses special problems of penetration that can preclude the use of several pile types and give special advantage to others. Loose cohesionless soils develop much greater shaft-bearing capacities if driven, high-displacement piles are used. The displacement effect can be enhanced by the use of tapered shafts; however, the potential increase in shaft capacities is undesirable if negative friction is to be feared. High-displacement piles may also be undesirable in stiffer cohesive soils because they can cause excessive soil heaving. The presence of groundwater and its flow condition can be a deciding factor in whether or not to use casing for open-ended, cast-*in-situ* concrete piles. Sometimes, as in the case of artesian pressures in water, uncased piles should be excluded from consideration.

Location and topography of the site may sometimes play an important role in selection of pile type. In congested urban locations, driven piles may be undesirable because of noise or the potential damage caused by vibrations, soil densification, or ground heave. Prefabricated piles may be undesirable if storage space is not available. In remote, rural areas, characteristics of access roads (bridge capacity and headroom) may restrict the use of certain kinds of pile construction equipment and thus eliminate certain pile types. Local availability of a certain material, such as timber or steel, may often have a decisive effect on the economy of a certain pile type. An offshore or in-river location sometimes dictates the use of prefabricated as opposed to cast-*in-situ* piles. Steep terrain may make the use of certain pile-driving equipment more costly or even impossible.

Structural and general characteristics of the proposed superstructure may also sometimes make certain pile types less desirable. For example, heavy structures with large reactions require high-capacity piles rather than timber and small-diameter cast-*in-situ* piles. For small, isolated structures the cost of bringing the pile construction equipment to the site can be significant; thus, pile types that can be placed by light, locally available equipment are advantageous.

Although one pile type may emerge as the only logical choice for a given set of conditions, more often several different types may meet all of the requirements for a particular structure. In such cases the final choice is made on the basis of a cost analysis that should assess the over-all cost of alternatives, including uncertainties in execution, time delays, and cost of load testing programs, as well as differences in the cost of pile caps and other elements of the structure that may differ among alternatives.

NOTATIONS AND DEFINITIONS

A = cross-sectional area of pile shaft (L^2).
A_c = cross-sectional area of the cushion (L^2).
A_p = bearing area of pile point (L^2).
A_s = bearing area of pile shaft (L^2).
B = pile diameter (L).
$\overline{B}$ = pile group width (L).
C_{qh}, C_{qv}, C_{ph}, C_{pv} = pile reaction factors, Eq. 68 (dimensionless).
C_{qm}, C_{pm}, C_{mm} = pile reaction factors, Eq. 68 (L^{-1}).
C_p, C_s = pile settlement coefficients (dimensionless).
D = embedded pile length (L).
D_r = relative density of the soil (dimensionless).
E_c = modulus of elasticity of the cushion (FL^{-2}).
E_p = modulus of elasticity of pile shaft (FL^{-2}).
E_s = modulus of deformation of the soil (FL^{-2}).
$E_s{}^*$ = plane strain of deformation modulus of the soil (FL^{-2}).
H = horizontal component of external load (F).
I = moment of inertia of pile section (L^4).
I_{ij}, I_{ip}, I_{pj} = transfer coefficient for pile displacement (dimensionless).
I_{rr} = rigidity index of the soil.
I_{pp} = transfer coefficient for point displacement caused by point load (dimensionless).
I_{ps} = transfer coefficient for point displacement caused by shaft load (dimensionless).
K_c = cushion stiffness (FL^{-1}).
K_h = soil reaction modulus (FL^{-2}).
K_l = soil reaction modulus for zero deflection (FL^{-2}).
K_o = coefficient of at-rest lateral earth pressure (dimensionless).
K_p = coefficient of passive earth pressure (dimensionless).
K_s = coefficient of shaft pressure (dimensionless).
L = pile length (L).
L_s = stress wave length (L).
L_f = depth of fixity of the pile (L).
M = bending moment (FL).
M_1, M_2 = pile group constants (L).
M_v = compression modulus of the soil (FL^{-2}).
N = standard penetration blow count (blows/foot).
$\overline{N}$ = corrected standard penetration blow count (blows/foot).
$N_c{}^*$ = bearing capacity factor for cohesion term (dimensionless).
N_h = lateral soil reaction constant, Eq. 58 (dimensionless).
$N_q{}^*$ = bearing capacity factor for overburden term (dimensionless).
N_σ = bearing capacity factor for mean normal ground stress term (dimensionless).
N_s = bearing capacity factor for shaft resistance (dimensionless).
N'_s = bearing capacity factor for shaft resistance in terms of mean normal ground stress (dimensionless).
P = lateral pile load (F).
P = pile perimeter (L).
Q = axial pile load (F).
Q_a = nominal design load.
Q_{crit} = critical buckling load of the pile (F).
Q_o = ultimate load (F).
Q_p = ultimate point load (F).
Q_s = ultimate shaft load (F).
R = pile reaction during driving (F).
R_g = resultant of external loads acting on pile group (F).
S_1, S_2, S_3 = pile group constants (dimensionless).
T = characteristic length of the pile-soil system, Eq. 54 (L).
T_h = time factor, Eq. 12 (dimensionless).
V = vertical component of external load (F).
W_r = weight of ram (F).
$\mathcal{M}$ = external load couple acting on pile group (FL).

c = constant in pile driving formula (L).
c = velocity of wave propagation through the pile (LT^{-1}).
c = strength intercept-cohesion (FL^{-2}).
c_a = adhesion between pile shaft and soil (FL^{-2}).
c_h = coefficient of radial consolidation of the soil (L^2T^{-1}).
f_s = ultimate shaft resistance (FL^{-2}).
$\overline{f_s}$ = average shaft resistance (FL^{-2}).
g = acceleration due to gravity LT^{-2}.
h = hammer stroke (L).
h_c = thickness of the cushion (L).
k_n = coefficient of axial pile reaction (FL^{-1}).
k_t = coefficient of lateral pile reaction (FL^{-1}).
n_h = coefficient of soil reaction (FL^{-3}).
p = distance between pile axis and group center (L).
p_z = reactive lateral pressure on the pile (FL^{-2}).
p_u = ultimate cavity expansion pressure (FL^{-2}).
p_o = ultimate soil reaction pressure (FL^{-2}).
q = distance between pile head and the point on pile axis closest to center of pile group (L).
q_c = cone point resistance (FL^{-2}).
q_o = ultimate point resistance (FL^{-2}).
q_s = normal stress acting on pile shaft (FL^{-2}).
q_v = effective vertical ground stress (FL^{-2}).
q_{va} = average vertical ground stress over pile length (FL^{-2}).
s = pile set during driving (L).
s = pile spacing (L).
s_u = undrained shear strength (FL^{-2}).
t = elapsed time (T).
t = characteristic length, Eq. 64 (L).
u = lateral deflection of pile head (L).
u_p = lateral pile deflection (L).
u_s = lateral soil displacement (L).
v = velocity of any point along the pile (LT^{-1}).
v_o = ram impact velocity (LT^{-1}).
w = settlement of a single pile (L).
w_c = critical displacement between pile shaft and soil (L).
w_o = vertical displacement of pile head (L).
w_p = vertical displacement of pile point (L).
w_z = vertical displacement of a point on pile axis at depth z.
w_{pp} = point displacement caused by point load (L).
w_{ps} = point displacement caused by skin load (L).
w_s = axial deformation of pile shaft (L).
x = coordinate of pile head (L).
x_c = coordinate of pile group center (L).
z = distance along pile axis (L).
z_c = coordinate of pile group center (L).
α = adhesion coefficient $= f_s/s_u$ (dimensionless).
α_s = skin load distribution coefficient, Eq. 37 (dimensionless).
α = inclination angle of pile axis to the horizontal (dimensionless).
β = correlation coefficient between q_o in t/s_f and N.
γ = effective unit weight of the soil (FL^{-3}).
γ_p = effective unit weight of the pile (FL^{-3}).
γ_w = effective unit weight of water (FL^{-3}).
δ = angle of friction between the soil and pile shaft (dimensionless).
η = hammer efficiency (dimensionless).
η_g = pile group efficiency (dimensionless).
ζ = pile reaction coefficient, Eq. 63 (dimensionless).
ζ_g = group settlement factor (dimensionless).
θ = slope of deflected pile axis at ground line (dimensionless).
λ = correlation coefficient between q_o and p_u (dimensionless).
λ = lateral stiffness ratio of the piles (dimensionless).
ν_s = Poisson's ratio of the soil (dimensionless).
ρ = friction ratio, Eq. 21 (dimensionless).
ρ_p = mass density of the pile.
σ = pile driving stress (FL^{-2}).
σ_c = compressive stress in the pile (FL^{-2}).
σ_t = tensile stress in the pile (FL^{-2}).
σ_o = mean normal ground stress (FL^{-2}).
ϕ = angle of shearing resistance of the soil (dimensionless).
ϕ' = angle of shearing resistance in terms of effective stress (dimensionless).
Σ_m = pile group constant, Eq. 70 (FL).

CHAPTER TWO

DESIGN CRITERIA

BASIC DESIGN CONSIDERATIONS

The designer of a deep foundation must possess a variety of skills, much experience, and considerable knowledge of engineering sciences. No set of simple rules and procedures (such as those developed in some areas of structural design) can be expected to cover the variety of conditions and forms of instability that can affect a deep foundation. The following discussion outlines some basic design criteria that a design engineer may find useful in meeting the basic requirements of safety, dependability, functionality, and economy.

As in the case of shallow foundations, principal dimensions of deep foundations are determined so as to satisfy two basic requirements of safety: (a) the foundation must possess sufficient safety against failure and (b) the foundation should not undergo excessive displacements under working loads. Thus, the nominal design load (Q_a) should not exceed a specified fraction of the ultimate load (Q_o), so that at any time

$$Q_a \leq Q_o/F_s \quad (1)$$

in which F_s is a safety factor. At the same time, the settlements and horizontal displacements of the foundation under working loads should not exceed specified limits set by the usage requirements and structural tolerances of the supported structure. (For details on safety factors, working loads for settlement analysis, and settlement tolerances, see Refs. *1, 18, 21, 22.*)

The ultimate load (Q_o) is the load that can cause either the structural failure of the foundation itself or the bearing-capacity failure of the soil. Excluding buckling and bending under the action of lateral loads and failure caused by excessive stresses during pile driving, which is discussed later, structural failure is assumed to occur when the maximum axial stress in the foundation shaft equals the critical stress for the shaft material (yield stress for steel, compressive or tensile strength for concrete or timber). This condition may govern design where pile points penetrate into very dense sand or rock. In most other situations, the ultimate load is determined from considerations of bearing-capacity failure of the soil.

THE ULTIMATE-LOAD CRITERION

Although the mode of shear failure of soil under a shallow foundation varies with the soil type, rate of loading, and other factors (cf. Ref. *1*), experience shows that soil under a deep foundation always fails in the same manner; i.e., in punching shear under the foundation point, accompanied or preceded by direct-shear failure of the soil along the foundation shaft (Fig. 2). As in the case of punching shear of shallow foundations, the ultimate load is rarely well defined; in many cases there is no visible collapse of the foundation and no clearly defined peak load (Fig. 3). To decide, on the basis of visual examination alone, on the magnitude of ultimate load in such cases can be quite deceiving. Figure 4 (*23*) shows the same load-to-settlement relationship of a test pile drawn in two different scales. Although the upper diagram may suggest that an "ultimate load" of 100 tons (90 metric tons) is reached, the lower indicates that the pile still has unused capacity at that load. Thus an unambiguous criterion is needed to establish the nominal magnitude of the "ultimate load."

Various ultimate-load criteria, all empirical in nature, have been proposed and used by different researchers and design organizations (*19*). As given in Table 2, such criteria are most often based on considerations of plastic (irrecoverable) or total (plastic plus elastic) settlements of the pile under the test load. A comparison of ultimate loads

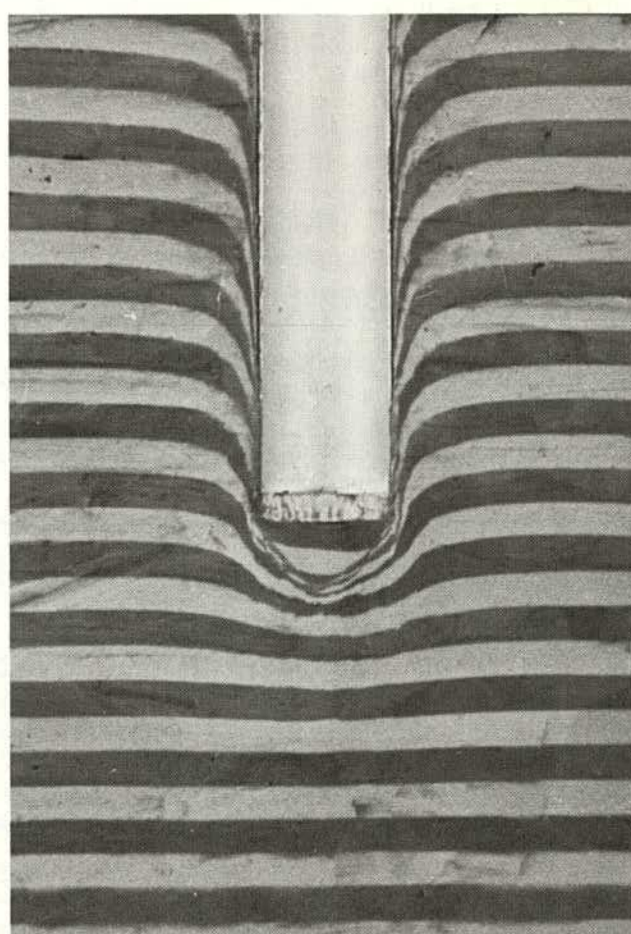


Figure 2. Failure pattern under a model pile in soft clay (33).

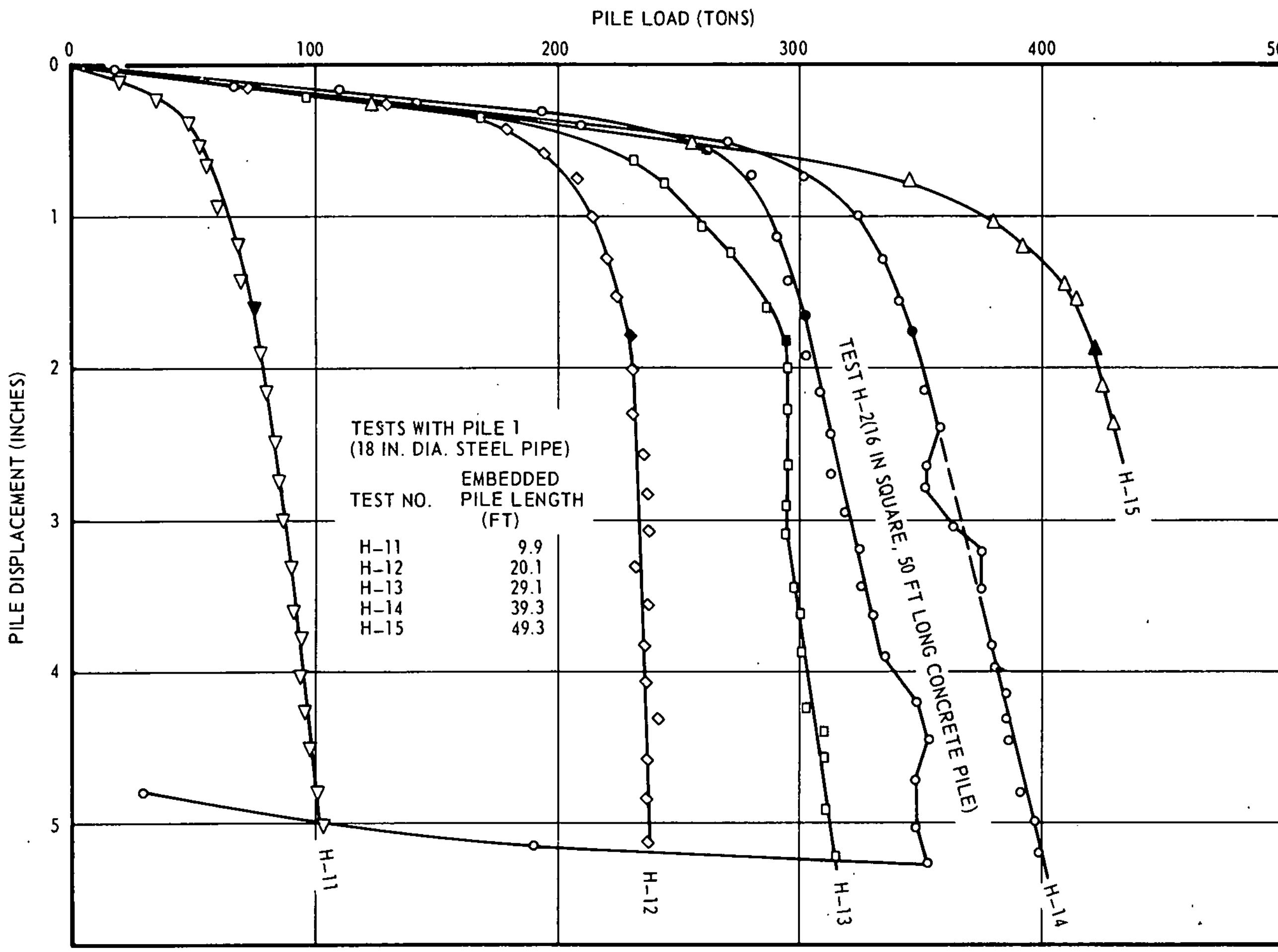


Figure 3. Load-displacement diagrams for series of test piles in sand (44).

obtained by applying these criteria to results of actual load tests shows relatively little difference (±10 percent) as long as the piles are not greater than 12 in. (300 mm) in diameter. However, substantial differences between ultimate loads obtained by various criteria can be found from results of load tests of large-diameter or very long piles (*221*).

To properly interpret these differences, it is essential to recall some basic facts about the mechanism of load transfer between a pile and surrounding soil. Modern research on pile behavior has established that full mobilization of skin resistance requires a relative displacement between the pile shaft and surrounding soil of 0.25 to 0.40 in. (6 to 10 mm), regardless of pile size and length (*24*). At the same time, mobilization of ultimate point resistance of a pile requires a displacement of approximately 10 percent of pile-tip diameter for driven piles and as much as 30 percent of the pile-tip diameter for bored piles. With these facts it is not difficult to understand that 1 in. (25 mm) of total settlement or 0.5 in. (13 mm) of plastic settlement may indeed nearly mobilize the ultimate load of a 6-in. (50-mm)-diameter pile but only a fraction of the ultimate load of a 96-in. (2 400-mm)-diameter bored pile. A simple calculation based on knowledge of basic load-settlement relationships of loaded areas demonstrates that 0.03 in./ton (0.8 mm/metric ton) of deformation may be indicative of failure stage for a small pile and still represent a normal deformation rate of a large pile in the safe-load range. Thus, it follows that certain ultimate-load criteria given in Table 2 (1a, 2, 5, or 6), containing absolute magnitudes of plastic or total limit settlement can not generally be valid. As such, they should be eliminated or substituted by analogous pile-diameter-dependent criteria.* It should be also equally easy to prove that Criterion 9 of Table 2 can not be generally valid because it assumes that the ultimate load can be reached only after an infinitely large displacement.** The remaining criteria (1b, 3, 4, 7, and 8) appear to be equally dependable, particularly if 1b is corrected to exclude the elastic (recoverable) deformation of the pile shaft from the total settlement. From the practical point of view, Criteria 3 and 4 have the disadvantage of being tied to the traditional time-consuming maintained-load testing method whereas Criteria 7 and 8 require load testing to very large

* The widely used AASHTO criterion of defining failure load as plastic settlement of 0.25 in. (6 mm) may be in order for small piles; it is definitely overconservative for piles exceeding 12 in. (300 mm) in diameter. A good substitute criterion would be limiting plastic settlement to, perhaps, 2 percent of the pile diameter.

** This criterion can offer some service in situations where load test was terminated before reaching the ultimate load; it offers a consistent approach to extrapolation of a load-settlement diagram toward failure.

TABLE 2

RULES FOR DETERMINATION OF ULTIMATE LOAD

1. *Limiting total settlement*
   - (a) Absolute — 1.0 in. (Holland, New York City Code)
   - (b) Relative — 10% of pile tip diameter (England)
2. *Limiting plastic settlement*
   - 0.25 in. (AASHTO)
   - 0.33 in. (Magnel)
   - 0.50 in. (Boston Code)
3. *Limiting ratio plastic settlement/elastic settlement*
   - 1.5 (Christiani and Nielsen)
4. Maximum ratio $\frac{\text{elastic settlement increment}}{\text{plastic settlement increment}}$
   - (Széchy, 1961, Ref. 25)
5. *Limiting ratio setttlement/load*
   - (a) Total — 0.01 in./ton (California, Chicago)
   - (b) Incremental — 0.03 in./ton (Ohio)
   - 0.05 in./ton (Raymond Co.)
6. *Limiting ratio plastic settlement/load*
   - (a) Total — 0.01 in./ton (New York City Code)
   - (b) Incremental — 0.03 in./ton (Raymond Co.)
7. Maximum ratio $\frac{\text{settlement increment}}{\text{load increment}}$
   - (Vesić, 1963, Ref. 26)
8. *Maximum curvature of log w/log Q line*
   - (De Beer, 1967, Ref. 27)
9. *Van der Veen postulate*

$$w = \beta \ln \left(1 - \frac{Q}{Q_{max}}\right)$$

(Van der Veen, 1953, Ref. 23)

displacements, normally not less than one half of the pile-tip diameter. Thus, Criterion 1b is probably the most acceptable for general engineering practice. It should be used in the following corrected form: Unless the load-settlement curve of a pile shows a definite peak load, the ultimate load is defined as the load causing total pile settlement equal to 10 percent of the point diameter for driven piles and 25 percent of the point diameter for bored piles.

COMPUTATION OF THE ULTIMATE LOAD

The basic problem of computation of ultimate load of a deep foundation can be formulated as follows: A cylindrical shaft of diameter B (Fig. 5) is placed to depth D inside a soil mass of known physical properties. A static, vertical, central load (Q) is applied at the top and increased until a shear failure in the soil is produced. The problem is to determine the ultimate load (Q_o) that this foundation can support.

Although an obvious similarity exists between this problem and the analogous problem for a shallow foundation, there are some distinct differences that must be kept in mind from the outset. In the case of a shallow foundation, the bearing soil, which is under the foundation base, has normally not been disturbed, except for changes in effective ground stresses caused by excavation, placing of the footing, and, possibly, backfilling. However, in the case of a deep foundation, the bearing soil, which is normally both above and below the foundation base, is almost always disturbed. The degree of disturbance depends on soil type and the method of placement of the foundation. In the case of bored piles (Fig. 6a) most of the change occurs around the foundation shaft, where a relatively narrow zone of soil surrounding the pile must undergo some remolding because of soil removal by augering or other means. At the same time, depending on the construction procedure, some lateral-stress relief usually takes place before installation of the foundation. In the case of driven piles, however, substantial soil remolding both above and below the foundation base is unavoidable. If the surrounding soil is clay (Fig. 6b), a zone extending about one pile diameter around the pile may experience significant changes in structure and, depending on clay sensitivity, may lose considerable shear strength, which is partially or totally regained over an extended period of time.

In the case of piles driven into saturated stiff clay, there are significant changes in secondary structure (closing of fissures), extending to a distance of several diameters around the pile, with remolding and complete loss of effects of previous stress history in the immediate vicinity of the pile. If the surrounding soil is cohesionless silt, sand, or partially saturated clay (Fig. 6c), pile driving may cause soil densification, which is most pronounced in the immediate vicinity of the pile shaft and extends in gradually diminishing intensity over a zone extending between one to two pile diameters around the pile shaft. The driving process is also accompanied by increases in horizontal ground stress and changes in vertical stress in the pile vicinity, some or all of which can be lost by relaxation in creep-prone soils. In dense, cohesionless soils (such as sand or gravel), loosening may take place in some zones, along with substantial grain crushing and densification in the immediate vicinity of the pile. [According to Kérisel and Adam (*28*), some of the test piles in dense sand were excavated and pulled out with a hull of highly densified, crushed material that resembled a fine-grained sandstone.] In such soils there are permanent changes in horizontal as well as in vertical ground stress that can be highly pronounced. Hard driving can leave large residual stresses in both the pile and the soil, consideration of which may be essential for understanding the behavior of the pile-soil system (Fig. 6c). Because piles are often designed in groups, the situation is further complicated by the complex and not always well-understood effect of placing of adjacent piles. For these and other reasons the problem under consideration poses difficulties unparalleled in other common soil mechanics problems. A general solution to the problem is not yet available and will be difficult to formulate.

For design purposes the ultimate load is conventionally separated into two components, the shaft or skin load (Q_s) and the base or point load (Q_p), which are superimposed as follows:

$$Q_o = Q_p + Q_s = q_o A_p + f_s A_s \quad (2)$$

A_p and A_s represent, respectively, the bearing areas of the

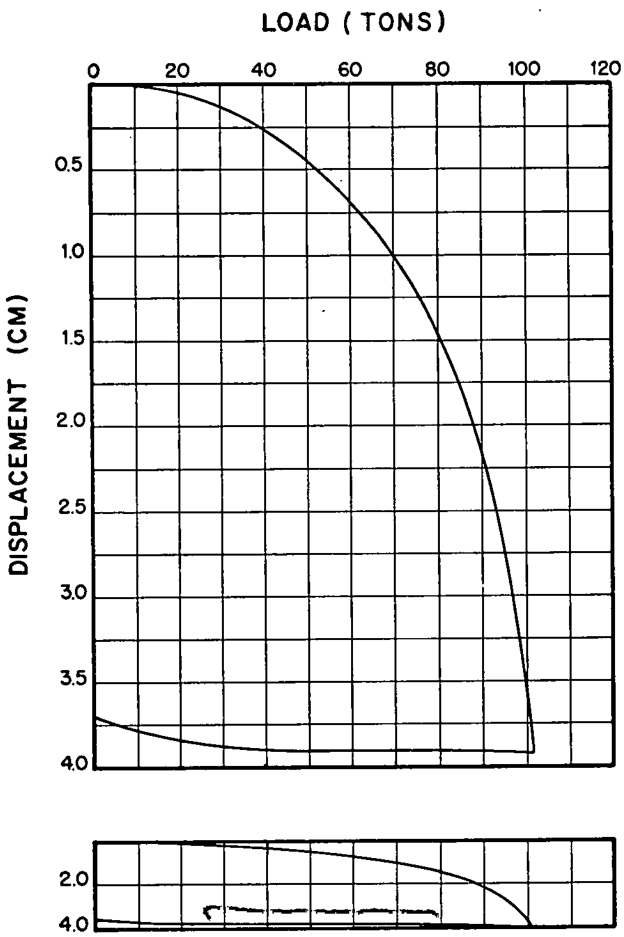


Figure 4. Load-displacement diagram of a test pile drawn in two different scales (23).

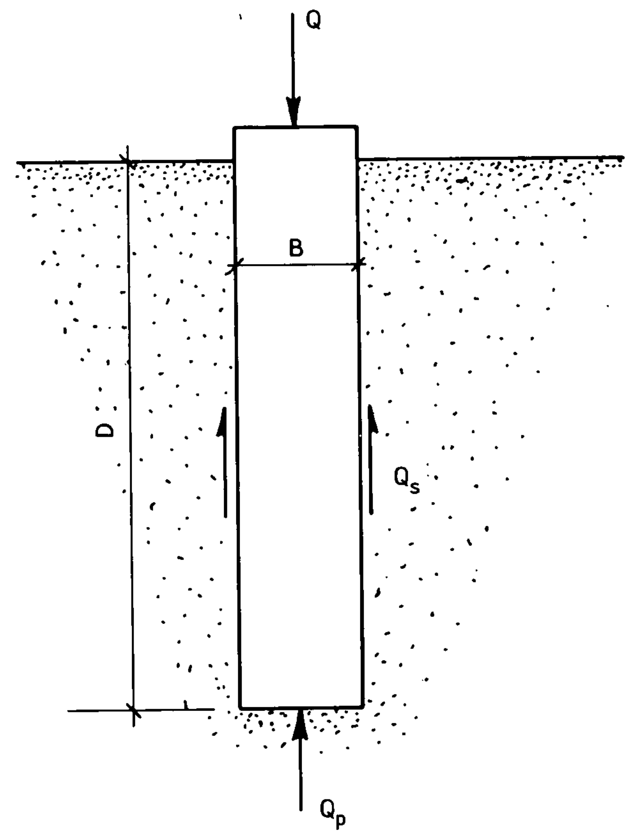


Figure 5. Basic problem of a deep foundation.

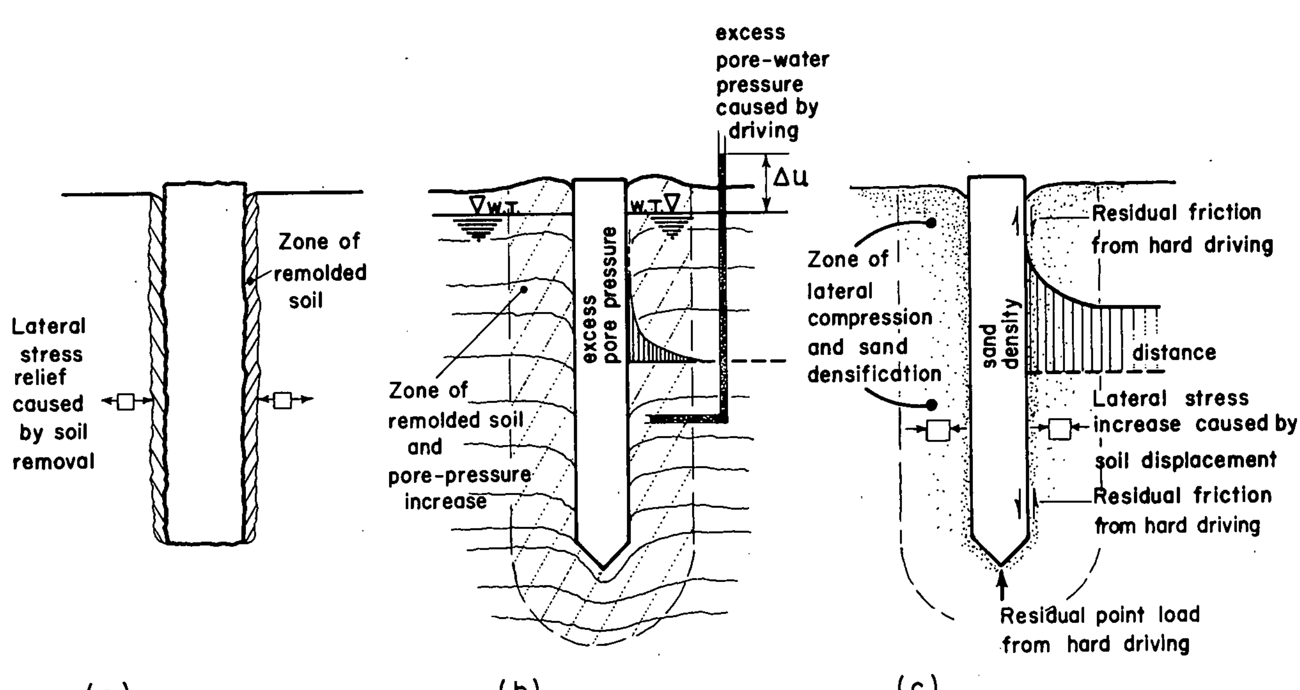


Figure 6. Effects of placement of pile into a soil mass.

foundation base and shaft; q_o and f_s are the unit base and shaft resistances expressed in stress units (ton/ft^2 or kPa). These unit resistances depend on a number of parameters, most significant of which are the strength and deformation characteristics and the initial state of stress of the soil strata involved, as well as the shape, size, material properties, and method of placement of the foundation.

The bearing areas of the foundation base (A_p) and shaft (A_s) are, by definition, the exposed areas of the pile body in contact with soil. In the case of piles with developed profiles, such as open pipe or H-pile, the unit resistances (q_0 and f_s) are usually expressed in terms of fictitious bearing areas (A_p and A_s) defined as areas contained within the outer perimeter of the profile. This is associated with observations of formation of a soil plug within the interior of the pipe or between the outer flanges of the H-profile. In the case of cohesionless soils, where the q_o/f_s ratio may be very high, it is advisable to verify that the assumed plug can transmit the reaction from pile point to the main body of the pile by friction (*226*).

In the case of piles with enlarged bases (such as belled piers), the effective bearing area of the shaft may be reduced by the long-term downward movement of the soil immediately above the base. Although there is little indication from experimental data that this reduction is significant, some designers (*4*) recommend that a length of the shaft above the base equal to base diameter be disregarded in computing the bearing area of the skin. (The side of the enlarged base is almost always disregarded in computing skin resistance.)

CHAPTER THREE

PILE RESISTANCE

POINT RESISTANCE

The theories seeking to relate the unit resistances (q_o and f_s) to known pile and soil characteristics are still under development. Classical theories for determination of q_o, based on plasticity theory alone, are considered inadequate (*29*) and are being replaced by more refined, linear or nonlinear elasto-plastic theories. Conventional theories present the solutions for q_o in the well-known form

$$q_o = cN_c^* + q_v N_q^* \qquad (3)$$

in which c represents the strength intercept (cohesion) of the assumed straight-line Mohr envelope and q_v, the effective vertical stress in the ground at the foundation level. N_c^* and N_q^* are dimensionless bearing-capacity factors, related to each other by the equation

$$N_c^* = (N_q^* - 1) \cot \phi \qquad (4)$$

which remains valid when dealing with a nominally linear Mohr envelope defined by a constant angle of shearing resistance (ϕ).

Most recent research on this subject (*30-32*) shows beyond doubt that the point resistance is governed not by the vertical ground stress (q_v) but by the mean normal ground stress (σ_0), which is related to q_v by the expression

$$\sigma_0 = \frac{1 + 2K_o}{3} q_v \qquad (5)$$

in which K_o represents the coefficient of at-rest lateral pressure. Thus, the bearing-capacity equation (Eq. 3) should be used in the following revised form

$$q_o = cN_c^* + \sigma_0 N_\sigma \qquad (6)$$

in which N_c^* and N_σ are appropriate factors, related to each other by the correspondence theorem (Eq. 4), and σ_0 represents the mean normal ground stress given by Eq. 5.

The computation of N_σ can, in principle, be made by any of the established methods of geotechnical analysis that takes into account soil deformability prior to failure. It is essential, however, that the computation be based on a realistic failure pattern. According to observations on models and full-size piles, there always exists under the pile tip a highly compressed conical wedge I (Fig. 7). In relatively loose soil, this wedge forces its way through the mass without producing other visible slip surfaces. However, in relatively dense soil wedge I pushes the radial-shear zone II sideways into the plastic zone III. Thus, the advancement of the pile into dense soil is made possible by lateral expansion of the soil along the circular ring (*BD*), as well as by any compression possible within zones I and II. This characteristic pattern in a soft bentonite clay (*33*) and in dense sand (*26, 34*) is presented in Figures 2 and 8.

Figure 9b (*35*) shows a scale drawing of the shape of a highly compressed wedge under a pile in dense sand, along with variations of sand density (Fig. 9a) and displacement pattern around the pile tip (Fig. 9c). In these examples, the base angle of wedge I is approximately equal to 45 + ϕ/2, if ϕ is taken as secant angle at the appropriate stress level. However, the sides of the wedge appear to have a concave curvature, forming an obtuse, rounded tip instead of a sharp apex.

Details of a recommended procedure to obtain N_σ as a function of angle of shearing resistance (ϕ) and rigidity index (I_{rr}) of the soil can be found in Appendix A, which also contains a table of values of the bearing-capacity factors N_σ and $N_c{}^*$.

A chart of N_σ-values is shown in Figure 10. If a comparison with $N_q{}^*$-values given in conventional theories is made, it is important to keep in mind that these theories related q_0 with vertical ground stress (q_v), which is related to the mean, normal ground stress (σ_0) by Eq. 5. It follows from Eqs. 3, 5, and 6 that

$$N_q{}^* = \tfrac{1}{3}(1 + 2K_0)N_\sigma \qquad (7)$$

Thus, for the total range of K_0 between 0.4 and, for example, 2.5, the "conventional" $N_q{}^*$ should be compared with 0.6 to 2 N_σ. A review of experimental values of $N_q{}^*$ observed in different pile investigations is shown in Figure 11 and summarized in Table 3. The available evidence suggests that the $N_q{}^*$-values for driven piles in ordinary quartz sands of alluvial and marine origin do not exceed those for shallow square footings. Thus, a good approximate formula for $N_q{}^*$ expressed in terms of ϕ alone is (*1*):

$$N_q{}^* = (1 + \tan\phi)e^{\tan\phi}\tan^2(45 + \phi/2) \qquad (8)$$

In applying this expression or chart in Fig. 10 it is essential to consider ϕ-angles corresponding to the stress level at failure in the vicinity of pile point. For medium-to-dense sands, these angles may be substantially lower than ϕ-angles determined from triaxial tests performed at conventional low pressures (*37*).

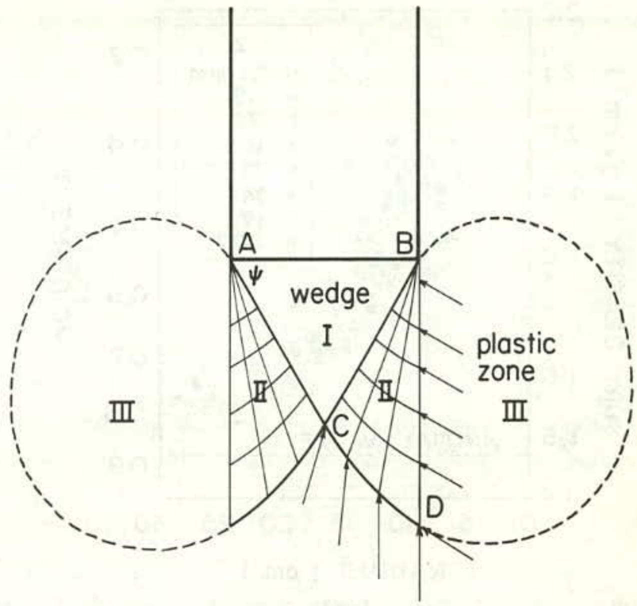
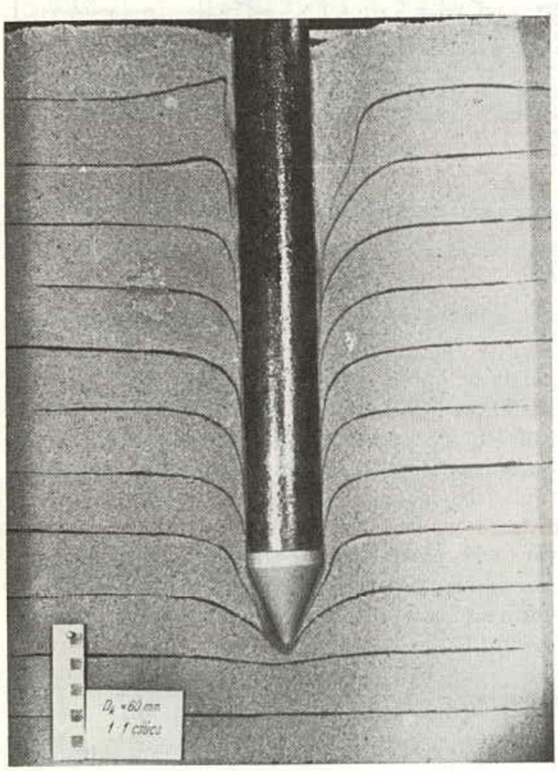
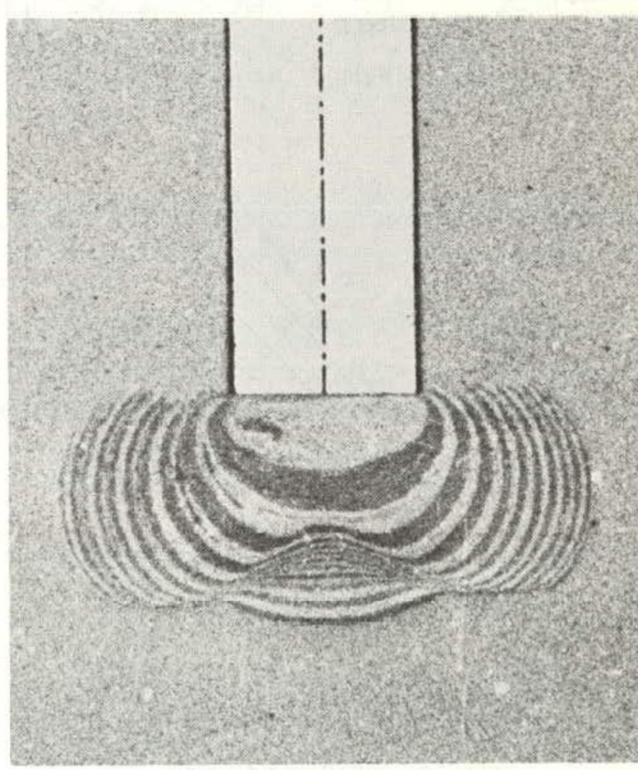


Figure 7. Assumed failure pattern under pile point.

It is also important to note that the $N_q{}^*$-values for a homogeneous deposit of dense sand decrease quite drastically with depth, as both ϕ and I_{rr} decrease substantially with mean normal stress (*37*). For example, at 80 percent

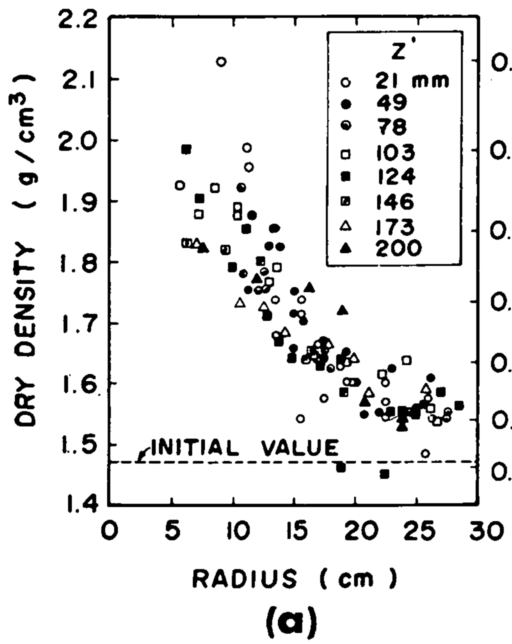


(a)



(b)

Figure 8. Failure patterns under pile point in dense sand (220, 34).



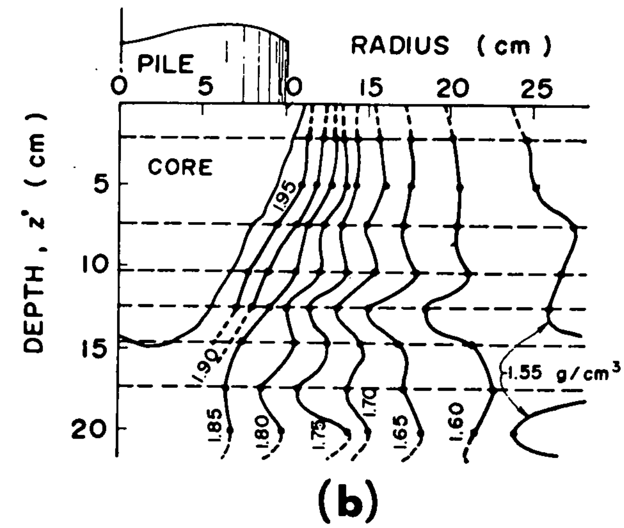
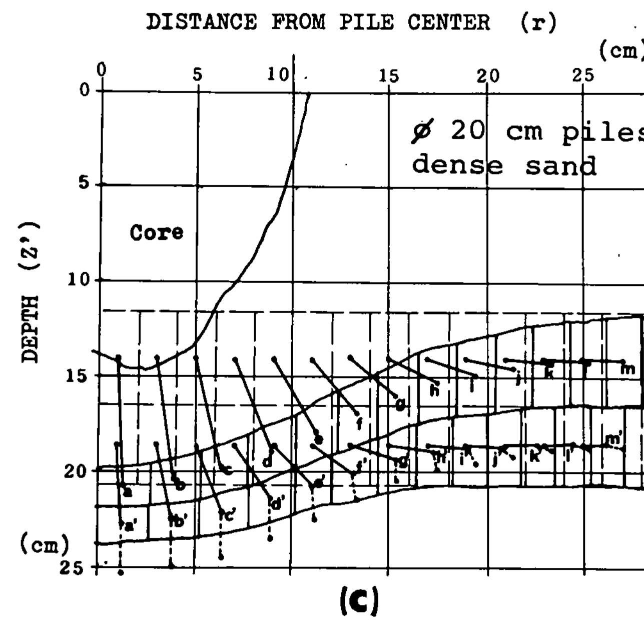


Figure 9. The shape of wedge, variations of sand density, and displacement pattern under the tip of a ϕ 20-cm pile in dense sand (35).



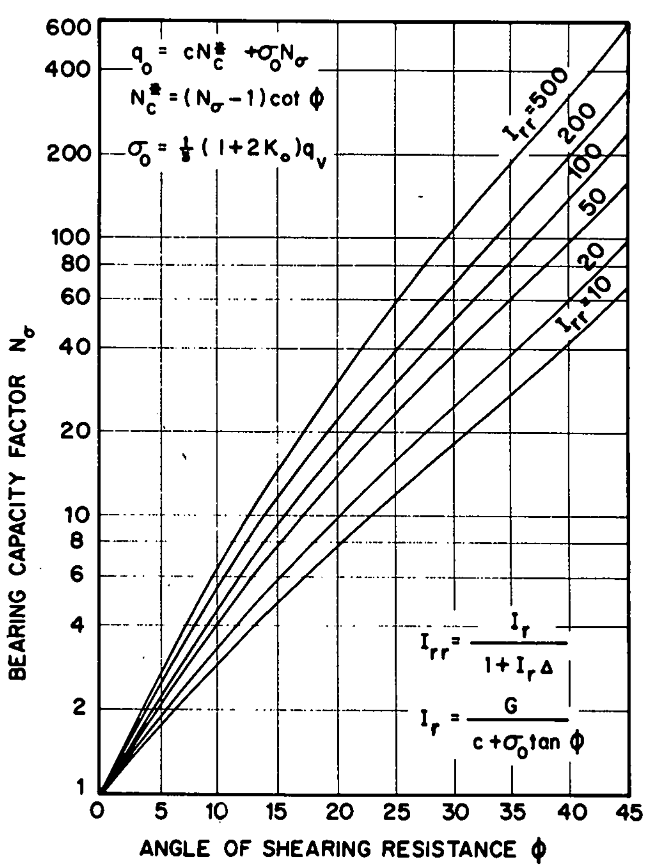


Figure 10. Variation of bearing capacity factor N_σ with I_{rr} and ϕ. (For an explanation of symbols, see Appendix A.)

relative density the Chattahoochee sand at 10 psi (70 kPa) has a ϕ-angle of 45° and an I_{rr}-value of 122. The same sand at 1,500 psi (10 mPa) has a ϕ-angle of 32.5° and an I_{rr}-value of 10. The corresponding N_q*-values for a normally consolidated deposit are, respectively, 125 and 14. In view of this, the increase of penetration resistance with depth in dense sand is not linear. The fact that both ϕ and I_{rr} vary with pressure is the source of important scale effects (not immediately visible from Eq. 6) that are manifested in decrease of q_o with increase in pile size at the same relative depth (38).

SKIN RESISTANCE

The theoretical approach for evaluation of unit skin resistance (f_s) is generally similar to that used to analyze the resistance to sliding of a rigid body in contact with soil. It is assumed that f_s consists of two parts: *adhesion* (c_a), which should be independent of the normal stress (q_s) acting on foundation shaft, and *friction*, which should be proportional to that normal stress. Thus in any particular stratum in contact with the foundation shaft:

$$f_s = c_a + q_s \tan\delta \qquad (9)$$

In this equation tanδ represents the coefficient of friction between the soil and the shaft, which, according to experience with piles of normal roughness, can be taken equal to tan ϕ', the coefficient of friction of the remolded soil in terms of effective stresses. The pile-soil adhesion (c_a) is

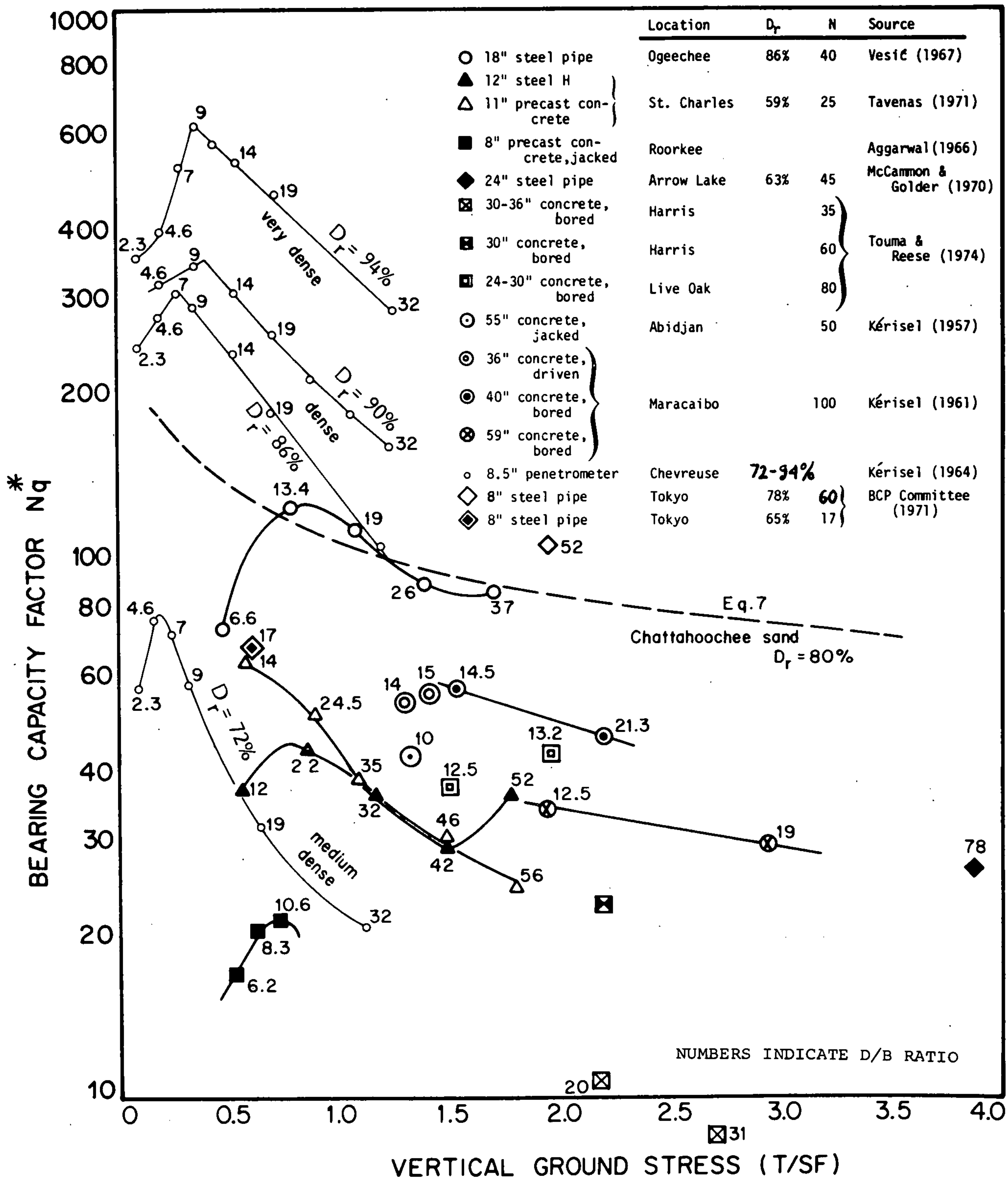


Figure 11. Experimental values of N_q* *in sand from different investigations.*

TABLE 3

EXPERIMENTAL VALUES OF N_q* IN SAND

SAND COMPACTNESS	RELATIVE DENSITY (%)	N_q* DRIVEN PILES	BORED PILES
Very dense	>80	60–200	40–80
Dense	60–80	40–80	20–40
Medium	40–60	25–60	10–30
Loose	<40	20–30	5–15

Source: Figure 11 and other records. Higher values apply to shorter piles.

normally small and for design purposes can be neglected.

The normal stress on the shaft (q_s) is conventionally related to the effective vertical stress at the corresponding level prior to placement of the pile by a coefficient of skin pressure (K_s), defined as q_s/q_v, so that Eq. 9 can be rewritten as

$$f_s = K_s \tan\phi \, q_v \qquad (10)$$

The coefficient K_s depends mainly on the initial ground-stress conditions and the method of placement of the pile; however, it is also affected by pile shape (particularly taper) and length. In bored or jetted piles, K_s is equal to or smaller than the coefficient of earth pressure at rest (K_o). In low-displacement driven piles, such as steel H- or open pipes, it is somewhat larger, rarely exceeding 1.5 even in dense sand. For short, driven, high-displacement piles in sand, K_s can be as high as the coefficient of passive earth pressure, $K_p = \tan^2(45 + \phi/2)$. However its magnitude seems to drop with the increase of depth of penetration, reflecting the fact that the effective stresses in the vicinity of the tips of such piles can be considerably lower than the initial ground stresses at the same level (*39, 40, 41*).

For piles driven into normally consolidated soft-to-firm clays, K_s is equal to or slightly larger than K_o. The skin resistance may be initially low because of the existence of pore pressures set up by pile driving and corresponding reduction in effective overburden stress (q_v). However, as the pore pressures dissipate and q_v approaches its initial value, the skin resistance of many clays may, after sufficient waiting period, become approximately equal to their undrained shear strength (s_u). This long-established fact has led investigators into comparing the skin resistance with the undrained shear strength for all clays. Such comparisons (*42*) show that in general

$$f_s = \alpha s_u \qquad (11)$$

in which α is a coefficient that for different pile types and soil conditions can vary between 0.2 and 1.5. For example, it has been suggested that for soft-to-firm clays ($s_u \leq 0.5$ tons/ft²), α should be equal to 1. For cast-*in-situ* bored piles in London clay, α varies between 0.3 for very short piles to 0.6 for long piles, with an average value of 0.45 (*43*). Figure 12 shows that comparisons of this kind have only limited meaning.

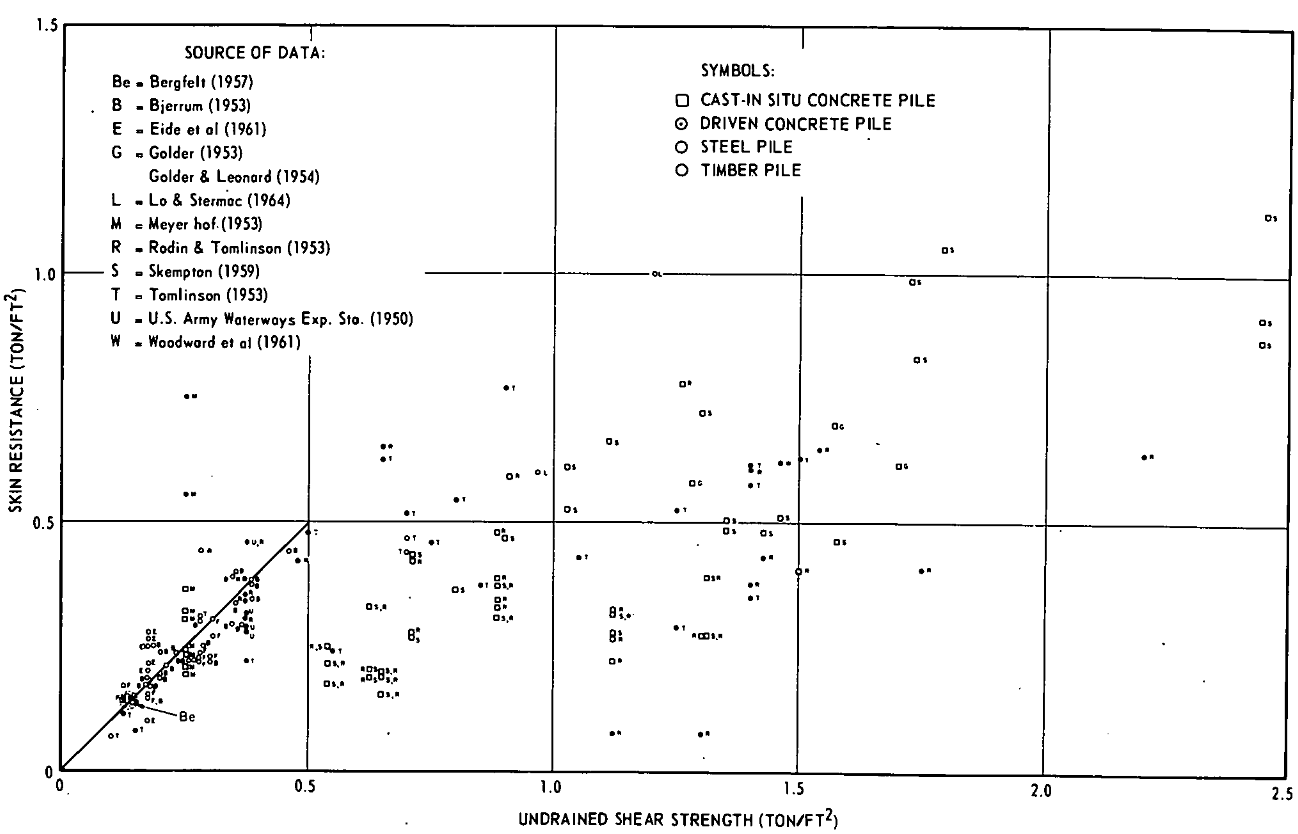


Figure 12. Comparison between skin resistance of piles in clay and undrained strength (44).

There is solid evidence that the skin resistance of piles is governed by the effective stress conditions around the shaft. Early research on behavior of piles in soft clays has established that the increase of skin resistance with time was connected with horizontal migration of pore water caused by excess pore pressures initiated with pile driving (*48, 49, 89*). Soderberg (*50*) has shown that the increase of bearing capacity of friction piles in clay is essentially a phenomenon of radial consolidation of clay, which can be analyzed by the basic equation of the radial diffusion theory. The gain in resistance with time should be controlled by the time factor T_h defined by

$$T_h = \frac{4c_h t}{B^2} \tag{12}$$

in which c_h is the coefficient of radial consolidation of the soil mass; t, elapsed time since pile driving; and B, pile diameter. Thus, the time needed to develop maximum pile capacity should be proportional to the square of pile size. Available field data on the subject are assembled in Figure 13, which shows also a theoretical prediction of increase of bearing capacity with time for two large piles driven into a deep deposit of marine clay (*51, 52*). It is seen that, in comparison with conventional small-diameter piles, the bearing capacity of larger piles continues to increase over a much longer period after driving. This is important to keep in mind when making estimates of waiting period for load testing. Figure 13 (see Horten Quay) also shows that the piles driven in a group exhibit much slower gain in bearing capacity than individual piles. This fact is significant for proper interpretation of group action of piles in soft clay.

On the basis of the aforementioned early research and other observations, it was suggested by Chandler (*45*) and Vesić (*44*) that variations of skin resistance of piles in clay could be better understood if test results were interpreted in

Type	Dia.	Length ft.	Soil type	Location	Source
□ ■ steel H	14	191, 219	silt	Tappan Zee, N.Y.	Yang 1956
△ steel pipe	6	22	soft clay	San Francisco	Seed & Reese, 1957
▲ steel pipe	12	60	soft clay	Michigan	Housel 1958
◎ ⊙ precast concrete	14	40, 56	soft boulder clay	Horten Quay	Bjerrum et al., 1958
● ◑ ○ steel pipe	24	242, 316, 300	soft to stiff clay	Eugene Island	McClelland, 1969
					Stevens, 1974 – – – – (theoretical prediction)

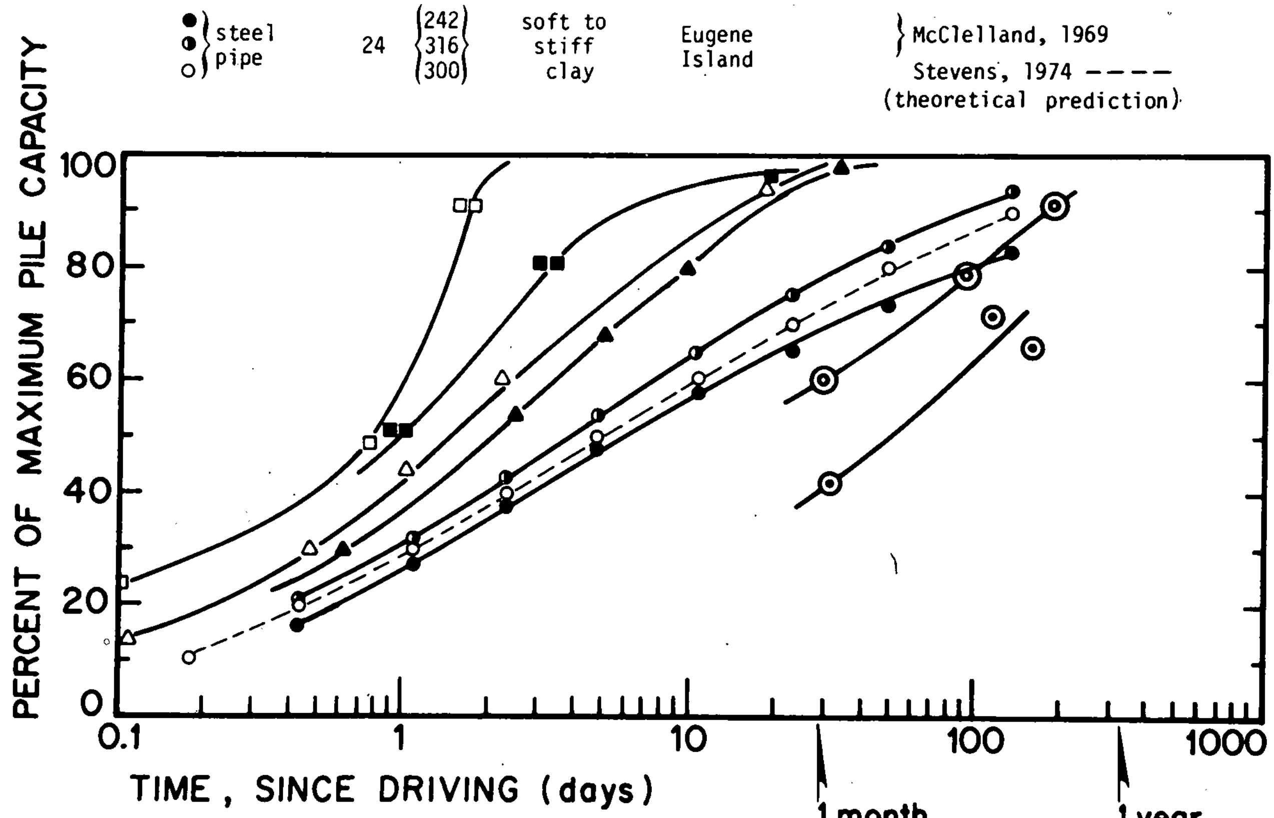


Figure 13. Field data on increase of bearing capacity with time for friction piles in clay.

terms of effective stresses and Eq. 10. Comparisons of this kind (*45-47, 53*) show that shaft friction of bored piles in stiff-to-hard clays, as well as the shaft friction of all piles in soft clays, can be determined from Eq. 10, which, for design purposes, may be written in the following simplified form

$$f_s = N_s q_v \qquad (13)$$

In this equation N_s represents a dimensionless bearing-capacity factor, which apparently varies very little with the angle ϕ. For piles in normally consolidated clays inducing no appreciable change in lateral ground stress conditions, it can be assumed (*46, 53*) that $K_s = K_o = 1\text{-}\sin\phi'$, or that

$$N_s = (1\text{-}\sin\phi')\tan\phi' \qquad (14)$$

where ϕ' represents the angle of shearing resistance of remolded clay in drained conditions. Eq. 14 indicates that, when $15° < \phi' < 30°$, N_s should vary between 0.20 and 0.29.

A different expression for N_s can be derived assuming that the vertical component of ground stress (ρ_z) remains unchanged upon pile driving, although the pile skin becomes a slip surface. In this way one obtains (*63*)

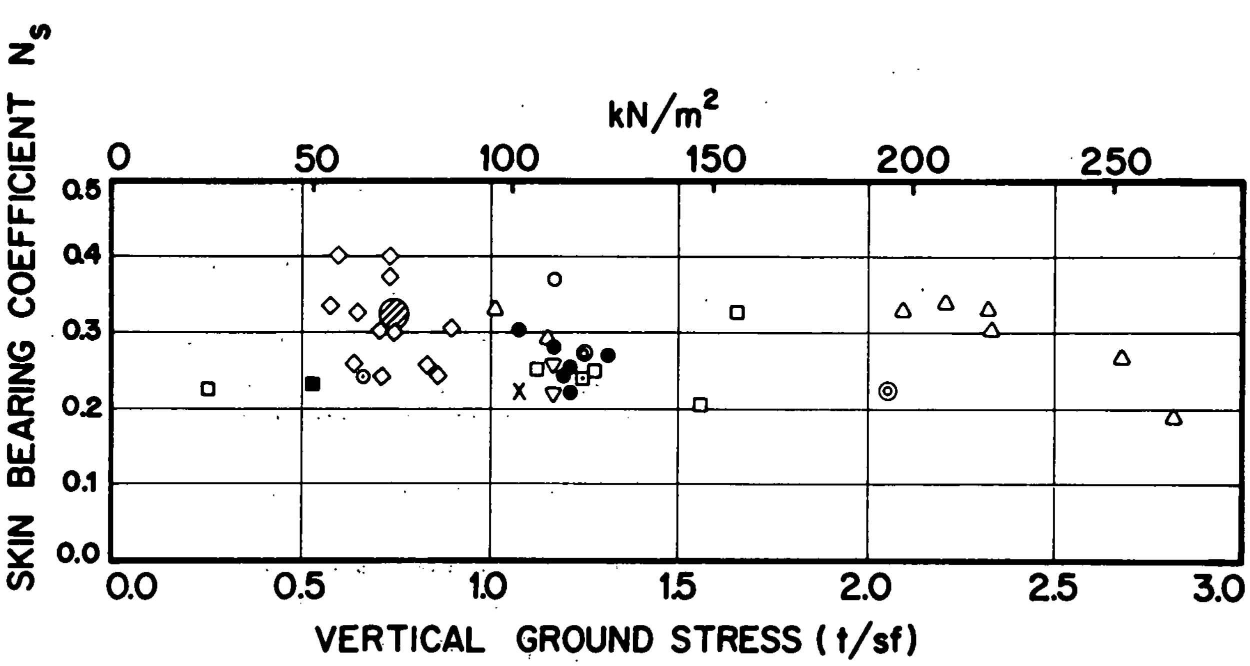
$$N_s = \frac{\sin\phi'\cos\phi'}{1 + \sin^2\phi'} \qquad (15)$$

Eq. 15 yields approximately 20 percent higher N_s-values than Eq. 14. Figure 14 shows the available experimental values of N_s from load tests of piles in a variety of locations. It can be seen that N_s indeed varies very little with soil and pile type and that Eq. 15 or an average value of 0.29 can be proposed for preliminary design. Other comparisons indicate that Eq. 14 or an average value of 0.24 may be more appropriate for tension piles or negative skin friction. There may also be a tendency toward lower N_s-values for very long piles and higher N_s-values for shorter piles, which may derive a greater proportion of their shaft friction from the lightly overconsolidated clay crust. For piles in overconsolidated clay crust, where the at-rest pressure coefficient (K_o) varies with depth, the N_s-value also varies with the pile length (*53*) and can be computed from:

$$N_s = \frac{\tan\phi'}{Dq_{va}} \int_0^D q_v K_o dz \qquad (16)$$

in which q_{va} is the average vertical ground stress over the considered pile length (D). For London clay, this expression gives values varying from about 1.20 at shallow depth to less than 1 at great depth. A comparison with observed values of N_s from a number of tests in London clay is given in Figure 15. It is seen that $N_s = 0.8$ represents a conservative value for preliminary design in London clay.

In the case of piles driven or jacked into stiff, overconsolidated clays, the load tests indicate generally higher N_s-values, resulting from increased lateral stress on the pile



⊙Detroit	Housel, 1950	○ Lemoore	Woodward, 1961
●Morganza	Mansur, 1956	▨ Khorramshahr	Hutchinson & Jensen, 1968
■San Francisco	Seed & Reese, 1957	⊡ Donaldsonville	Darragh, 1969
△Cleveland, Burnside	Peck, 1958	◎ British Columbia	McCammon, 1970
◇Drammen	Eide et al., 1961	X New Orleans	Blessey, 1970
▽Drayton	Peck, 1961	□ Misc. Locations	McClelland Engrs.

Figure 14. Observed values of skin bearing-capacity factor N_s *in normally consolidated clays.*

shaft. Some available data from load tests on such piles are assembled in Figure 16. It is seen that the N_s-values for such piles can be as high as 5 for relatively short piles. However, there is a considerable decrease of N_s with pile length, similar to that in dense sand (shown for comparison in Fig. 16). The explanation for this decrease is analogous to that for piles in sand: apparently the state of stress along the pile shaft does not increase in proportion to pile length or the lateral ground stress (*26, 44*). Other possible factors that have a similar effect include the increase of gap between pile shaft and clay near soil surface (*54*), as well as reduction of K_0 with depth in overconsolidated clay deposits. The combined effect of these factors apparently results in a quasi-constant skin resistance (f_s) beyond a certain pile length. Because of this, it may be easier to relate f_s empirically with the undrained shear strength of clay (Eq. 11), although apparently the coefficient α also decreases with the increase of pile length. However, the explanation of variation of α for different deposits must still be sought by using the effective stress analysis and an expression such as Eq. 10 because there are solid indications that the behavior of piles in stiff clay is frictional in nature and fundamentally similar to that of piles in dense sand (*47*).

Measured values of N_s for driven piles in very dense sand are similar to those for piles in stiff clay, decreasing from about 2 for very short piles to about 0.4 for very long piles (*40, 63*). In loose sand N_s can be as low as 0.1 with no obvious decrease with increasing pile length. Available test data for piles in medium-to-dense sand seem to suggest that after some penetration into the sand stratum, f_s reaches a quasi-constant limit value, which is, for a given sand deposit, a function of the initial sand density, and probably overconsolidation ratio of the deposit only (*29, 39, 63*). Tests for three different sands from Georgia (*39*) indicate for driven piles

$$f_s = (0.08)(10)^{1.5D_r^4} \tag{17}$$

and for bored or jacked piles

$$f_s = (0.025)(10)^{1.5D_r^4} \tag{18}$$

in which f_s is the unit skin resistance in tons/ft² and D_r is the relative density of sand deposit. Later studies (*70*) for a variety of other sand deposits indicate that Eqs. 17 and 18 may represent a lower limit and that the average of all recorded data in different sands indicates values approximately 1.5 times greater (see Fig. 17). Also, values of f_s as much as 1.8 times greater are reported for short, tapered piles (*71*). It remains uncertain, however, to what extent f_s for cylindrical piles still increases with depth, however slowly, and whether N_s for tapered piles decreases with depth as fast as it does for cylindrical piles.

It is significant to note that most recent research on this subject (*30-32*) indicates a steady increase of locally measured f_s with mean normal stress σ_0. Thus, skin bearing-capacity Eq. 13 can be rewritten:

$$f_s = N_s' \sigma_0 \tag{19}$$

in which N_s' represents a dimensionless skin bearing-capacity factor that is apparently independent of K_0. The

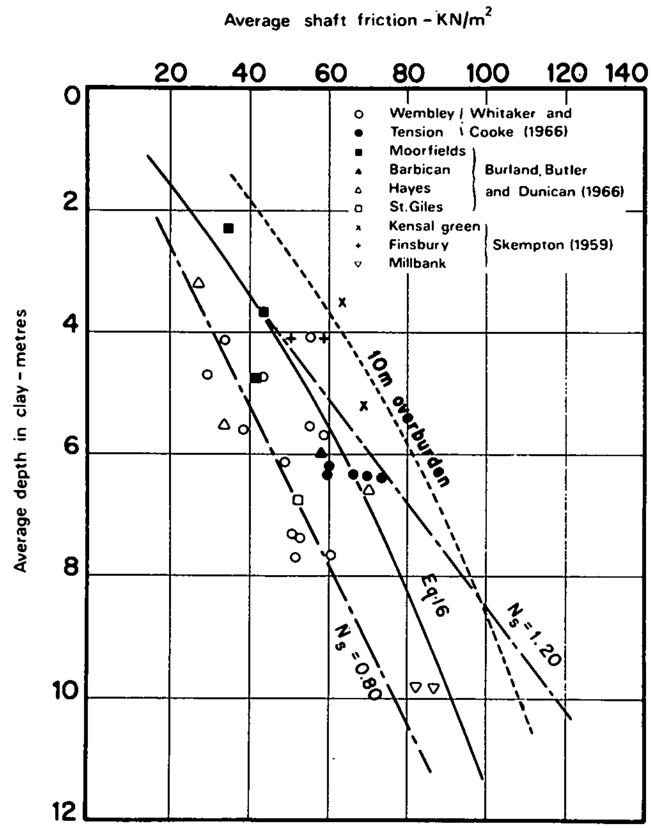


Figure 15. Observed values of N_s *for bored piles in London clay* (53).

experimental data on N_s' are still scarce because no load tests were interpreted in terms of Eq. 19. It is interesting to note, however, that test data on piles in clay given in Figures 14 and 15 define a single range for N_s' between 0.35 and 0.5 for both normally consolidated and overconsolidated clays.

DETERMINATION OF POINT AND SKIN RESISTANCES FROM FIELD TESTS

The determination of point and skin resistances of piles by Eqs. 6 and 19 requires not only detailed knowledge of strength and deformation characteristics of soil strata involved in transmitting pile loads but also knowledge of variation of density and water content within these strata. The cost of taking the necessary number of samples from the soil mass and performing the appropriate laboratory tests to determine soil characteristics needed for design is, however, often prohibitive and cannot be justified except in the case of very important structures in relatively uniform soil situations. In all other circumstances it may be preferable to estimate the unit resistances q_0 and f_s directly from field penetration or expansion tests, using devices such as a static or dynamic penetrometer or a pressuremeter. Most reliable among these is the static-cone-penetration test, performed by means of a classical (Dutch) penetrometer or its modern equivalents (friction-sleeve or electric penetrometer).

A detailed description of different types of static pene-

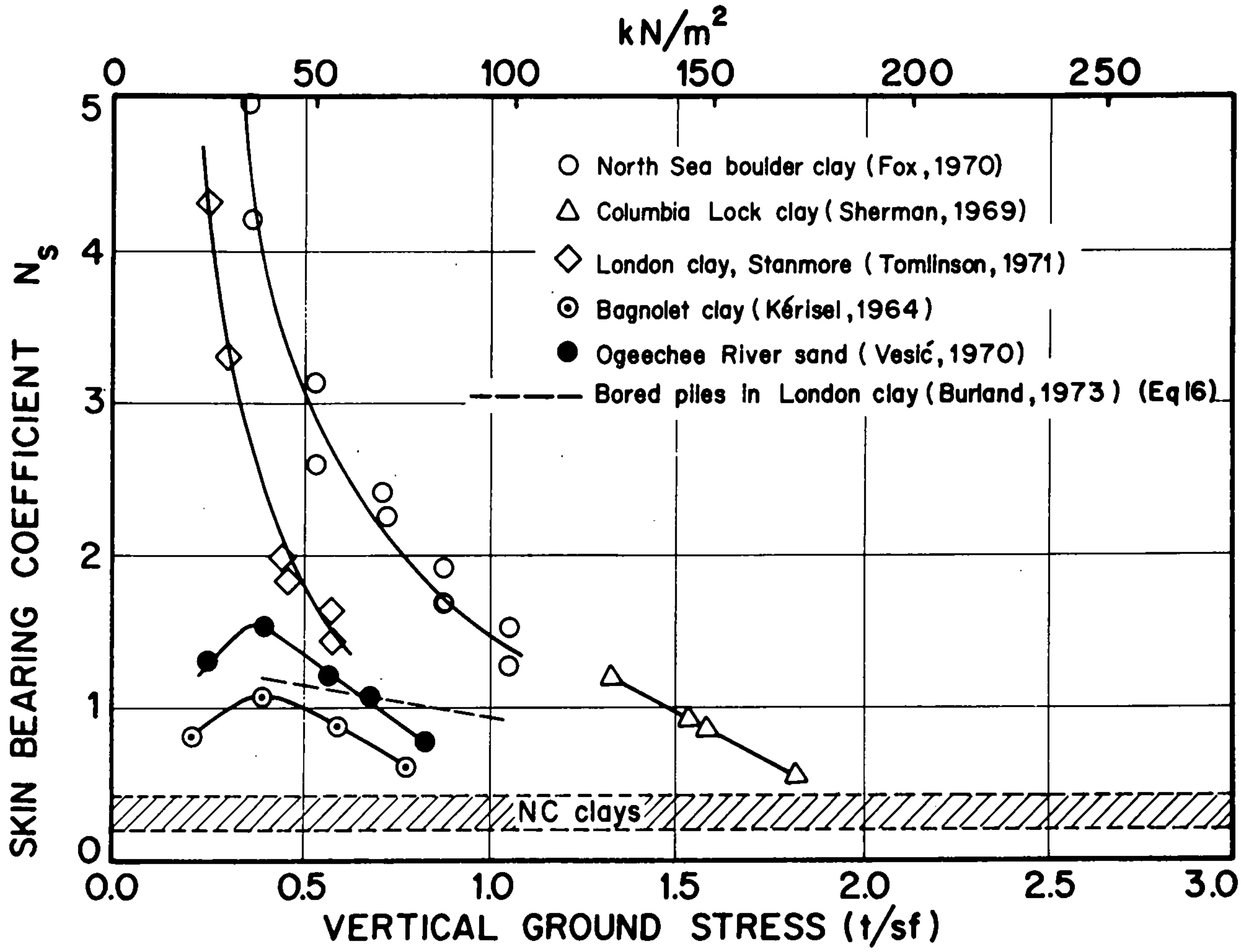


Figure 16. Observed values of N$_s$ *for driven piles in stiff, overconsolidated clays.*

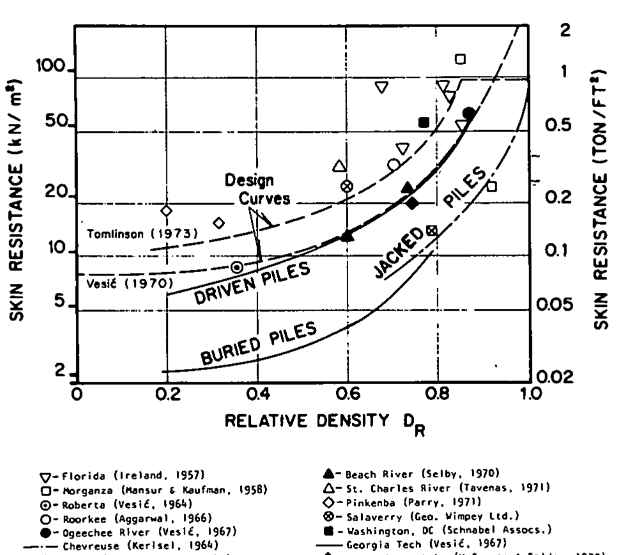


Figure 17. Variation of skin resistance of piles in sand with relative density (39, 53).

trometers can be found in Reference *55*. They all are, in essence, small, 1.4-in. (35-mm)-diameter, jacked piles with a 60° cone at the tip. Extensive experience with these devices, first introduced in Holland and Belgium and later used all over the world, shows that, with due consideration of scale effects, the cone point resistances (q_c) are equal to pile point resistances (q_o)

$$q_c \approx q_o \qquad (20)$$

The scale effects in static-penetration resistance are complex but well understood (*29, 56*). A consistent method for accurate evaluation of resistance of a large pile on the basis of penetration curve of a small penetrometer has been developed by DeBeer (*57*). An example for such evaluation, based on experience at Ogeechee River test site (*39*), is given in Figure 18.

Experience with static penetrations shows also that, with exception of highly sensitive clays, shaft resistance of the friction-sleeve or electric penetrometer (f_c) equals that of the driven pile, whereas the average shaft friction of a classical Dutch cone can be as low as one-half of the friction of a driven pile. In view of uncertainties involved in field measurements of shaft friction, many engineers pre-

$Q_u = q_o \cdot A_p + f_s \cdot A_s$

fer to estimate the shaft friction of piles from measured-cone point resistances, using the relationship

$$f_s = \rho q_o \quad (21)$$

in which ρ is a dimensionless number, often expressed as a percentage, called the friction ratio. This number varies with soil type and mechanical characteristics, particularly ϕ and I_{rr}. For example, tests on driven piles in a variety of soils indicate the following approximate relationship with the angle of shearing resistance (ϕ) (expressed in terms of total stresses) (*39*)

$$\rho = (0.11)(10)^{-1.3\tan\phi} \quad (22)$$

Similarly, penetration tests in several sands at pressures as much as 70 psi (500 kPa) seem to suggest the following simple direct correlation with the rigidity index I_{rr} (*32*)

$$\rho = 3/I_{rr} \quad (23)$$

Considerably lower values (about one-third of these indicated by Eqs. 22 or 23) can be expected in bored piles in dense sand.

Although the existence of correlations such as Eqs. 22 and 23 can be explained on the basis of theoretical expressions for q_o and f_s, they should be used with caution and preferably only after verification in any new condition.

In spite of its great reliability, the static-cone-penetration test is not widely used in North America, partially because of difficulties in using such a test in soil profiles containing stiff soil strata or soft rock. The primary reason is that the standard penetration test, using a split sampling spoon, has a long-established reputation of allowing a safe, though crude and sometimes uneconomical, design. The standard penetration test measures the number of blows (N) needed to advance the standard spoon one foot (300 mm) by driving with a 140-lb (64-kg) hammer and a stroke of 30 in. (760 mm). Experience shows that the point resistance of driven piles, q_o, in ton/ft² can be related to N by the expression

$$q_o = \beta \bar{N} \quad (24)$$

in which $\bar{N} = N$ when $N \leq 15$ and $\bar{N} = 15 + 1/2(N - 15)$ when $N > 15$. The coefficient β varies with soil type, stress level, and possibly some other factors. Values $\beta = 2$ for saturated clays and $\beta = 4$ for sands have been suggested by Meyerhof (*58*). Later research (*59-63*) has shown that β varies with soil characteristics and mean normal ground-stress level. Still, with the exception of piles in loose fine sand, mentioned values probably represent safe minimum values for pile design. For soils of intermediate type, such as micaceous silt, values between 2 and 3 have been reported (*60, 64*). Direct relationships between skin resistance (f_s) and N have also been proposed. It is better, however, to use Eq. 21 with an estimated value of ρ.

The standard penetration test is often criticized for its crudeness and imprecision. It is known that variations in test procedure and lack of attention of test operators in regulating the free fall of the hammer can produce significantly different results in the same soil (*62, 65, 223*). In soft, compressible strata where soils may be liquefied by dynamic impulses, or in granular soils where particles may

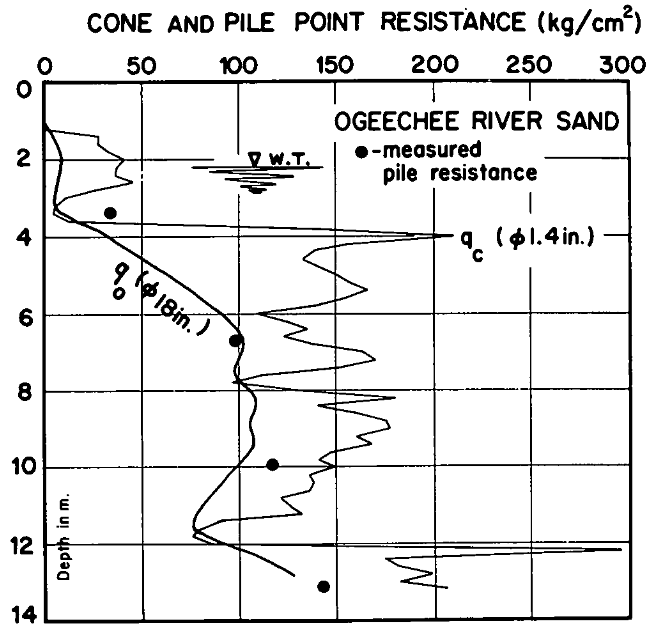


Figure 18. Evaluation of point resistance of 18-in. (450-mm)-diameter pile from results of static-penetration test (57).

be crushed by impact of the spoon, the blow count is frequently not indicative of actual bearing capacity and stiffness of the soil under static loading (*66, 67*). In spite of the great popularity of this test in the United States for use in design, it is increasingly accepted today that the primary purpose of this test should be to correlate experiences of a single organization in familiar geotechnical profiles. The modern view is that it is necessary to have mechanical-release mechanisms to regulate free fall of the test hammer in order to use the results of this test as a design tool (*62, 65*).

The pressuremeter test, which is also widely used in certain areas, measures the ultimate pressure (p_u) needed to expand laterally a vertical cylindrical measuring cell inside a borehole. Considering the failure pattern that typically develops under the pile point (Fig. 7), it is not difficult to conceive that the pile point resistance (q_o) should be related to p_u by:

$$q_o = \lambda p_u \quad (25)$$

where λ is a stress-transfer factor. Theoretical values of λ can be obtained from Eq. 25 by taking q_o according to Eqs. 6 and 7 and assuming that p_u is equal to the ultimate pressure of a cylindrical cavity (*36*). The computations show that when $\phi = 0$ (undrained conditions), λ should be approximately equal to 2. When $c = 0$ (cohesionless soils), λ should increase with ϕ and I_{rr} as shown in Figure 19. These values are substantially in agreement with observations in the field. Corresponding values of the skin resistance can best be found by using Eq. 21 with an estimated value of ρ.

The primary advantage of the pressuremeter test lies in the clarity of stress conditions that it produces in the ground, which allows direct determination of stress-strain

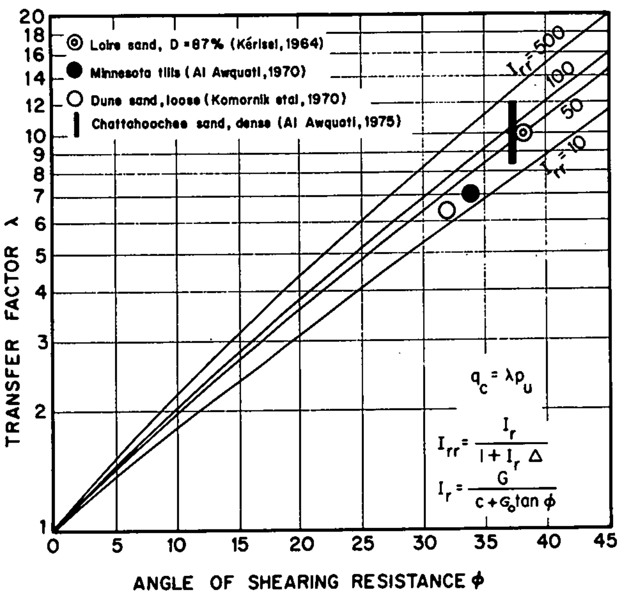


Figure 19. Transfer factor $\lambda = q_o/p_u$ *for cohesionless soils.*

and strength characteristics of the soil in a manner not possible by any other field test. The primary disadvantage of performing the test on a disturbed soil is being eliminated by the introduction of self-boring probes (*68, 69*), allowing the determination of lateral ground stresses, which are needed for complete geotechnical analysis of almost any problem. The relationship (*25*) for determination of pile capacity is less direct than that used with exploration by means of the static cone (Eq. 20). However, the scale effects are taken into consideration more directly because the soil zone stressed by a pressuremeter test is considerably larger than that stressed in a static-cone test.

SUMMARY OF RECOMMENDATIONS FOR DESIGN

In reasonably regular soil profiles, where strength and deformation characteristics of the bearing strata can be reliably determined, the ultimate point resistance of a pile can be computed from Eqs. 6 or 3 and the ultimate skin resistance from Eq. 13. For driven piles in normally consolidated clays, the bearing-capacity factor (N_s) can be found from Eq. 15. For bored or driven piles in overconsolidated clays one can use as a guide Figure 16 or Eq. 11 with $\alpha = 0.45$. For piles in sand, the skin resistance may be related to relative density using as a guide Figure 17. With good information on relative density one can also use Figure 11 or Table 3 as a guide for $N_q{}^*$ in sand.

In most real situations it may be preferable to determine pile point and skin resistances directly from field tests, such as the static (Dutch) cone test, standard penetration test, and the pressuremeter test. With due consideration of scale effects, the static cone measures the point and skin resistances directly. The standard penetration blow count is related to pile point and skin resistances by empirical relationships such as Eq. 24. The pressuremeter limit pressure is related to pile point resistance by a semi-empirical relationship (Eq. 25).

CHAPTER FOUR

PILE-SOIL INTERACTION

LOAD TRANSFER

The use of Eq. 2 for analysis of the bearing capacity of a pile contains a tacit assumption that both the pile point and all points of the pile shaft have moved sufficiently with respect to adjacent soil to develop simultaneously the ultimate point and skin resistance of the pile. As mentioned earlier, the displacement needed to mobilize skin resistance is small, not exceeding 0.4 in. (10 mm), regardless of soil and pile type and pile dimensions. By contrast, the displacement needed to mobilize point resistance may be relatively large, particularly for very large piles, because it amounts to about 8 percent of pile-point diameter for driven piles and as much as 30 percent of pile-point diameter for bored piles. Thus, even in the case of very rigid piles, where the pile-point displacement is not much less than the displacement of the pile head, the ultimate skin resistance is mobilized much sooner than the point resistance, and the fraction of pile total load carried by the pile point is much less at working loads than at the ultimate load. This is illustrated in Figures 20 and 21, which show typical variation of load transfer between the pile shaft and pile point as the total load progresses to failure. Figure 20 shows the mobilization of point and skin loads as a function of pile displacement, measured in a series of tests on 30-in. (760-mm)-diameter bored piles in stiff clay (*72*). Figure 21, based on measurements in a series of load tests on 18-in. (460-mm)-diameter driven steel-pipe piles in dense sand (*39*), illustrates the fact that the percentage of load carried by pile point is much less at working loads than at failure, even in dense sand.

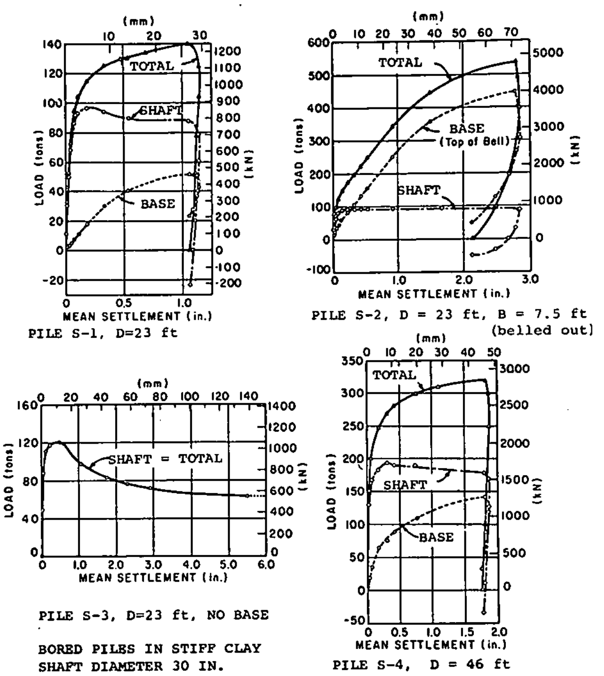


Figure 20. Mobilization of base and shaft resistance as a function of pile displacement (72).

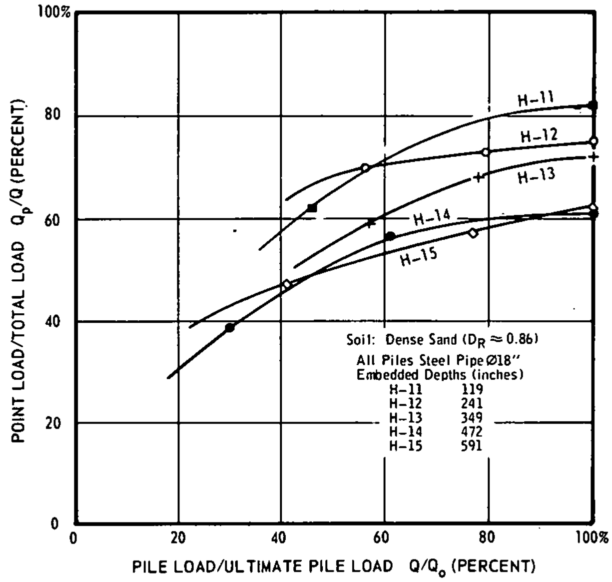


Figure 21. Relative magnitude of point loads at various stages of loading of closed-end pipe piles in dense sand (44).

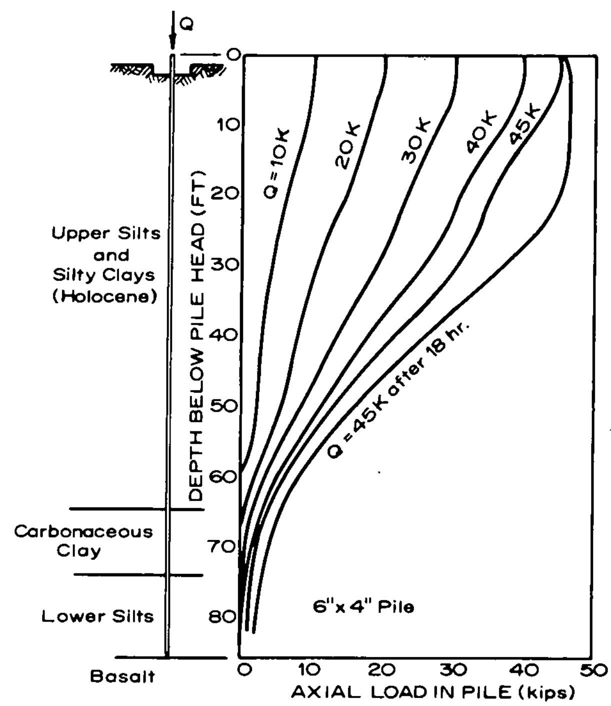


Figure 22. Load transfer from a steel pile driven through compressible silt to rock (73).

The picture of load transfer is more complicated in the case of relatively deformable piles, in which the displacement of the pile head can be considerably greater than that of pile tip and in which the skin resistances in the upper portion of the pile can be mobilized much earlier than those in the lower portion of the pile. In such a case, a substantial load can be carried exclusively by the pile shaft, particularly if the pile is relatively long; no load can obviously be transferred by the pile point without a relative displacement between the point and adjacent soil. This is illustrated in Figure 22, which shows the load transfer from a small-diameter steel-tube pile driven through highly compressible silt to rock. In the early stages of loading, the entire load was taken by the shaft; it took 40 kips (180 kN) of load in skin friction before the load finally reached the point. The same figure shows how, as the soft soil around the pile shaft compresses and creeps under newly applied stresses, downward movement of the soil with respect to the pile causes load redistribution. Thus, in just 18 hours of sustained load of 45 kips (200 kN) on the pile, 10 kips (44 kN) of load initially taken by the upper 20 ft (6 m) of the pile was transferred to lower, stiffer strata, some of it going all the way to the point.

There is another load transfer phenomenon, discovered relatively recently, that is quite significant for understanding of pile response to load, particularly in the case of relatively deformable piles. When the head of a pile is unloaded, after being subjected to a compressive load, the pile shaft tends to return to its initial length. In doing so, the upper portion of the shaft normally moves sufficiently upward against the adjacent soil to develop negative skin friction, which is counterbalanced by residual skin friction in the lower portion of the shaft and also, if the applied compressive load was high enough, by residual point load on the pile (Fig. 6c). In the aforementioned case of bored piles in stiff clay (Fig. 20), a single loading to 140 tons (1 200 kN), of which 50 tons (440 kN) were transferred to pile point, produced an unloading residual point load of more than 20 tons (180 kN). In another case reported in the literature, a tubular-steel pile 430 x 580 mm (17 x 23 in.) jacked 5 metres into stiff clay by a force of

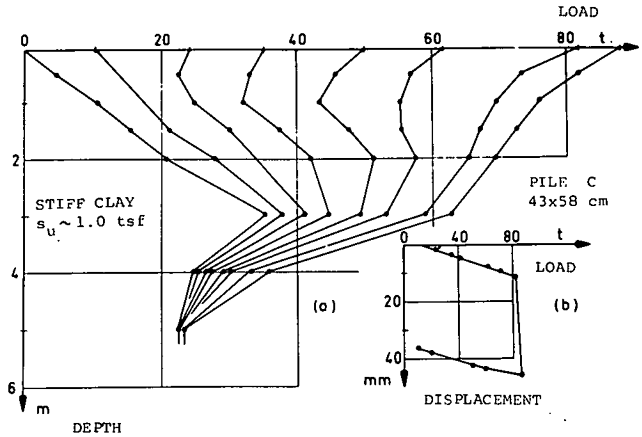


Figure 23. Load transfer from tubular-steel piles in stiff clay (74).

88 metric tons (780 kN) (Fig. 23) showed on unloading a residual point load of 22 metric tons (200 kN), only slightly less than the original point load. Because the impact driving process consists of periodical loading and unloading of pile head by dynamic impulses, driven piles always contain substantial residual loads, the existence of which has an effect on load-settlement response of the pile. This was first evidenced quantitatively in field studies at the Arkansas River Project (*75*) where residual loads up to 50 tons (45 metric tons) were recorded in 53-ft (16-m)-long steel pipe and H-piles. As shown in Figure 24, the existence of residual loads causes an apparent concentration of skin friction in the upper part of the shaft if the pile is loaded in compression. Such concentrations were reported in one of the earliest studies on this subject but could not have been rationally explained before (see Fig. 25 and Ref. *76*). For piles loaded in tension after being loaded in compression or immediately after driving, the apparent load distribution shows a tensile load at pile tip equal in magnitude to the residual point load.

(1) Measured compression load distribution assuming no stress in pile at start of test.
(2) Measured compression load distribution after compression test assuming no stress in pile at start of test.
(3) Measured tension load distribution assuming no stress in pile at start of test.
(4) Measured tension load distribution after tension test assuming no stress in pile at start of test.
(5) Tension load distribution adjusted by subtracting Curve 4 from Curve 3.
(6) Compression load distribution adjusted by adding Curve 4 to and subtracting Curve 2 from Curve 1.

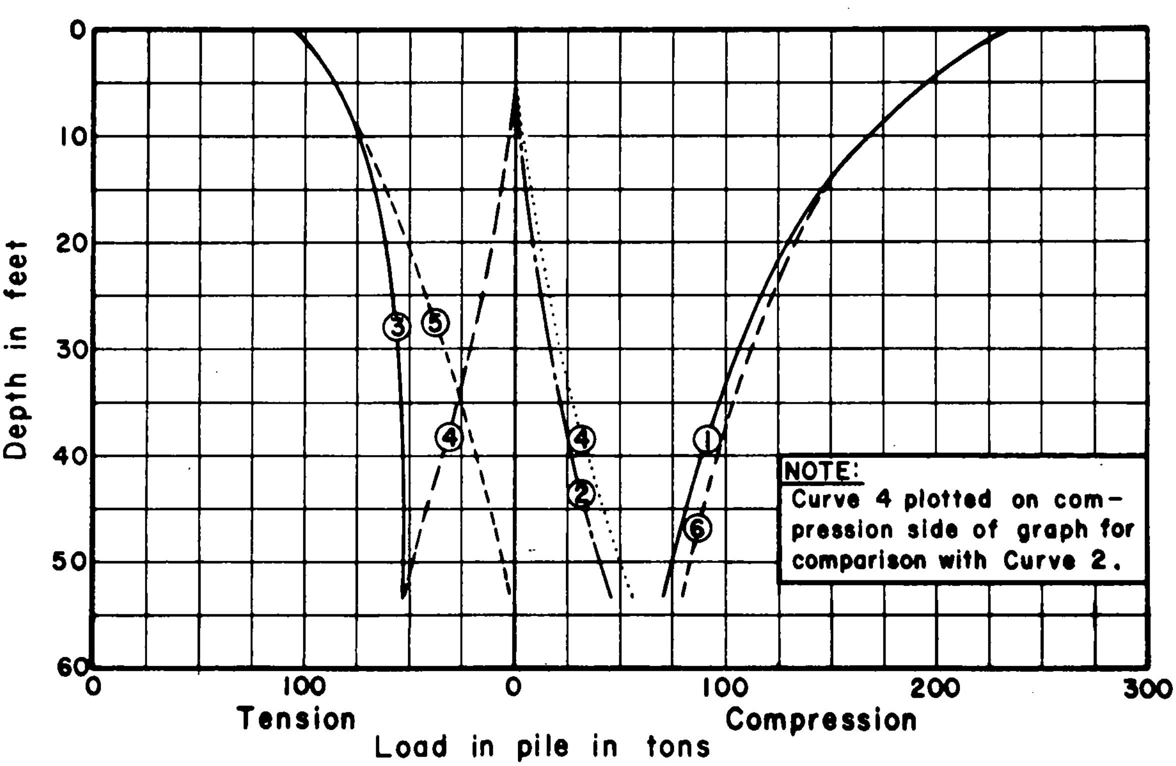


Figure 24. Effect of residual loads on load distribution in driven piles in sand at Arkansas River (134).

NEGATIVE SKIN FRICTION

The preceding discussions mainly concern load transfer between the pile and surrounding soil in normal situations where the soil movement with respect to the pile is caused by stresses transmitted by the pile itself. Of equal practical interest are the situations where the surrounding soil exhibits a downward movement with respect to the pile shaft because of other surface loads or any other cause such as pumping of water from one of the aquifer strata in the profile, which produces an increase of effective stress and settlement of adjacent soil. In such situations, the soil exerts a negative friction (downdrag) against the pile shaft, which has to be taken by the pile as an additional axial force and transmitted to deeper strata. This can cause excessive settlements of the piles with severe damage or even collapse of the structure supported by the piles. In extreme cases, if the axial load in the pile exceeds the strength of the pile shaft, structural collapse of the shaft can occur and undermine the stability of the entire structure. Observaions indicate that a relative movement of 0.6 in. (15 mm) of the soil with respect to the pile may be sufficient to mobilize full negative skin friction.

Measurements of magnitude and distribution of negative skin friction performed in recent years show its dependence on the effective ground stress similar to that expressed by Eq. 13 for positive skin friction (*77-79*). For ordinary, uncoated piles the N_s-values reported are equal to or slightly less than those for positive skin friction (*53, 81*). However, the negative skin friction develops only along the portion of the pile shaft where the soil settlement exceeds the downward displacement of the pile shaft. Below the "neutral point," where there is no relative movement between the pile and surrounding soil, friction remains positive and the load transmitted to pile point may be significantly less than the maximum axial load in the pile. This is illustrated in Figure 26, taken from the very thorough study of Endo et al. (*78*). This figure shows the measured development of axial force in a 24-in. (610 mm)-diameter steel-pipe pile driven into a 140 ft (43 m) deep deposit of silt, which settles around 6 in. (150 mm) per year because of pumping of groundwater from the underlying sand stratum. It also shows the measured development of axial load in four such pipes tested on this site, one of which was open-ended. There is clearly a neutral point in all four piles at approximately 75 percent of pile length. A similar position is found in at least two other recent investigations of a similar nature (*79, 82*). However, other investigations (*83, 84*) show that the neutral point can be located higher or lower. Its position coincides with the point of no relative movement between the pile and the adjacent soil (*78*). It is influenced by factors such as relative compressibility of the pile shaft and underlying soil with respect to surrounding soil, relative magnitude of axial load in the pile with respect to the effective stress change that causes settlement of surrounding soil, as well as the position of the most compressible stratum in the over-all soil profile (*83, 85*).

Negative skin friction can become particularly damaging in the case of foundations supported by battered piles. Although detailed measurements of developed axial forces in

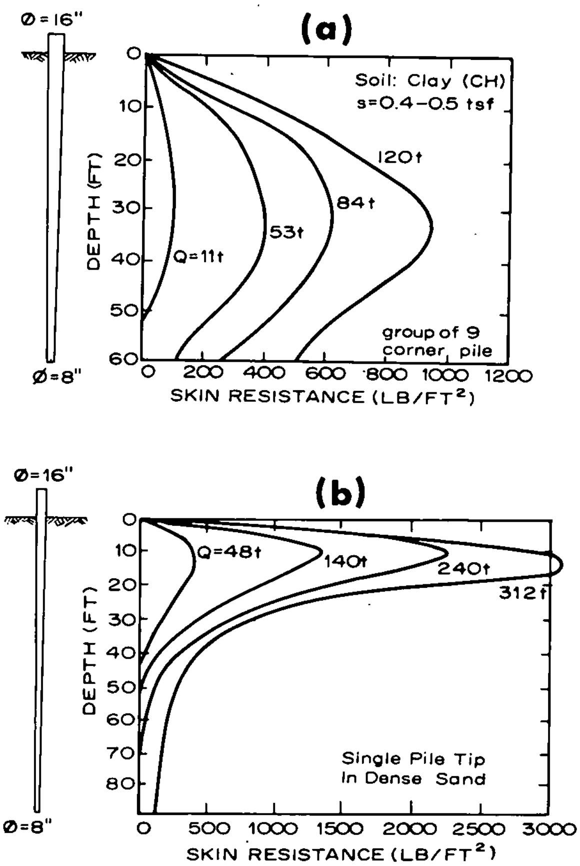


Figure 25. Measured distributions of skin resistance (76).

such cases are scarce, there are many documented records of serious damage to the foundations. Apparently the negative friction force against the outer side of battered piles is considerably larger than that against a vertical pile in a similar situation. At the same time the movement of the soil away from the piles on their inner side causes an imbalance of lateral forces and may induce substantial pile bending for which the piles have normally not been designed. Thus, battered piles should be avoided in all cases where significant negative friction is expected to develop.

Should the negative skin friction be excessive (normally if it exceeds the magnitude of live load to be supported by piles) it may require extra piles just to carry the parasite downdrag loads and, thus, may have a detrimental effect on the economy of the project. Consequently, there is great interest in practical methods of reducing the negative skin friction. This can be done by installing a casing around the pile shaft to prevent direct contact with the settling soil. Alternatively, the pile can be placed in a pre-drilled hole of larger diameter than the pile shaft, filling the gap with bentonite slurry, which, in view of its low friction characteristics, limits the negative friction to a relatively low value. One of the newest and most effective friction-reducing methods consists of applying on the pile shaft a slip layer of bitumen of appropriate consistency. Experience shows that it is possible in this way to reduce downdrag to as little as 10 percent of the magnitude to be expected for uncoated piles in the same soil (*82, 86-88*).

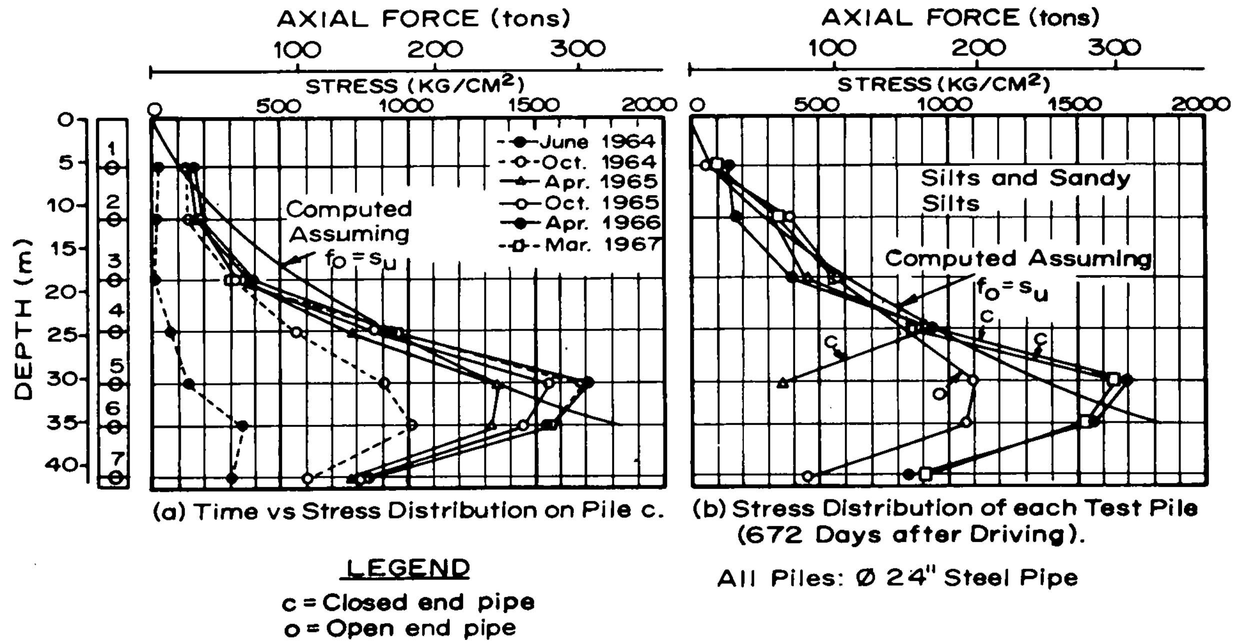


Figure 26. Axial load developed by negative skin friction in open-end and closed-end pipes in silt, ending in dense sand (78).

This method is gaining in popularity, although it can not be used effectively if the pile has to penetrate a rigid stratum, which can damage the bitumen coating.

In summary, negative skin friction can be expected above the point of the pile where the relative downward displacement of the soil with respect to the pile exceeds 0.6 in. (15 mm). As indicated by Eq. 13, the intensity of the downdrag is proportional to effective vertical ground stress. The skin resistance factor (N_s) for ordinary, uncoated fills in soft compressible strata of clay and silt falls in the 0.15-to-0.30 range, generally increasing with ϕ' and over-all soil stiffness. With the use of suitable coating, such as bitumen or bentonite, the N_s-values can be reduced to the 0.01-to-0.05 range. In loose sand, N_s is substantially greater, varying generally between 0.30 and 0.80. Still greater values can be expected in dense sands and stiff clays. Group action should be considered as in the case of positive skin friction. As long as the downdrag force is less than the live load, its negative effect on pile foundation performance is minimal.

ANALYSIS OF LOAD TRANSFER

The preceding examples are given to illustrate the fact that the mechanics of load transfer between the pile and the adjacent soil represent a relatively complex phenomenon affected by stress-strain-time and failure characteristics of all elements of the pile-soil system, including peculiarities stemming from the procedures used in placing the pile in the particular location in the ground. Some parameters affecting this load transfer are often difficult, if not impossible, to express in numerical terms. Yet numerical assessment of load transfer characteristics of a pile-soil system is essential for settlement computations and for rational design of pile foundations. The following presents the essential elements of analytical approaches currently used for that purpose.

Consider the case of a single pile of diameter B placed in soil to depth D and loaded by a central, vertical load Q (Fig. 5). The simplest way to obtain an idea about the load transfer along the pile is to install strain-gage bridges at different depths (z) along the pile axis. If the measured axial force in the pile is plotted against depth z (Fig. 27) the function Q (z) indicates the load transfer along the pile shaft. The ordinate of this curve at $z = D$ represents the pile point load (Q_p), whereas the difference $Q - Q_p = Q_s$ represents the pile skin load. The slope of the function Q (z) divided by pile perimeter length P yields the distribution of skin resistance (f_o) along the shaft

$$f_o = -\frac{1}{P}\frac{dQ}{dz} \tag{26}$$

Note that f_o remains positive (as shown in Fig. 27) as long as Q (z) decreases with depth z. A few examples showing the shape of curves Q (z) and f_o (z) for some simple distributions of skin resistance are shown in Figure 28. Example E in this figure represents the case of negative skin resistance.

From the experimentally established curve Q (z) it is also possible to find the vertical displacement (w_z) of any pile section at a depth (z) below the ground surface, provided the cross-sectional area (A) and the modulus of deformation (E_p) of the pile shaft are known and the vertical displacement of pile top (w_o) is measured during the loading test. Assuming that both A and E_p are constant along the shaft and using known strength-of-materials formulae, it can be shown that

$$w_z = w_o - \frac{1}{AE_p}\int_0^z Q(z)\,dz \tag{27}$$

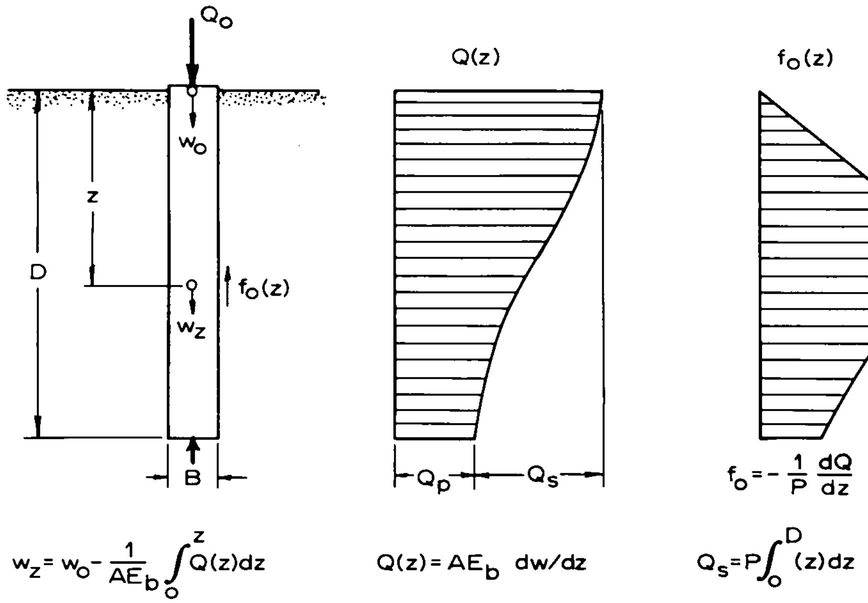


Figure 27. Load transfer from a single pile.

Another way to determine the displacements of different points along the pile is to use the so-called strain rods. These are simple steel rods, attached to different points of the pile, extending all the way to the ground surface where their movements can be observed by dial gages and leveling instruments. If the measured displacements are plotted versus depth as a function $w(z)$, it follows from Eq. 27 that

$$Q(z) = AE_p\, dw/dz \qquad (28)$$

The distribution of skin resistance f_0 can then be found by using Eq. 26. All analytical methods proposed for analysis of load transfer use, in essence, one of two possible approaches to this problem, either the transfer-function approach or the elastic-solid approach.

In the transfer-function approach, illustrated in Figure 29b, the pile is divided into n elements, which are considered as compressible short columns of length $\Delta D = D/n$, acted upon by axial forces (Q_i) and skin resistances (f_i). The skin resistances can be computed if the axial forces are known from simple relationships of statics:

$$f_i = \frac{Q_i - Q_{i-1}}{P\,\Delta D} = \frac{\Delta Q_i}{P \Delta D} \qquad (29)$$

The relative vertical displacements of centroids of the elements can also be computed as follows from simple relationships of the strength of materials, provided the cross-sectional area (A) and the modulus of deformation (E_p) of the pile shaft are known.

$$\Delta w_i = w_{i+1} - w_i = \frac{Q_i\,\Delta D}{E_p A} \qquad (30)$$

With two analogous expressions containing the displacements of the pile top (w_o) and pile point (w_p), there are $n + 1$ expressions of this kind for $n + 2$ unknown displacements. Thus, if the axial forces are known, all displacements (w) can be computed, provided one of them is known or assumed.

The forces (Q) are computed from the so-called trans-

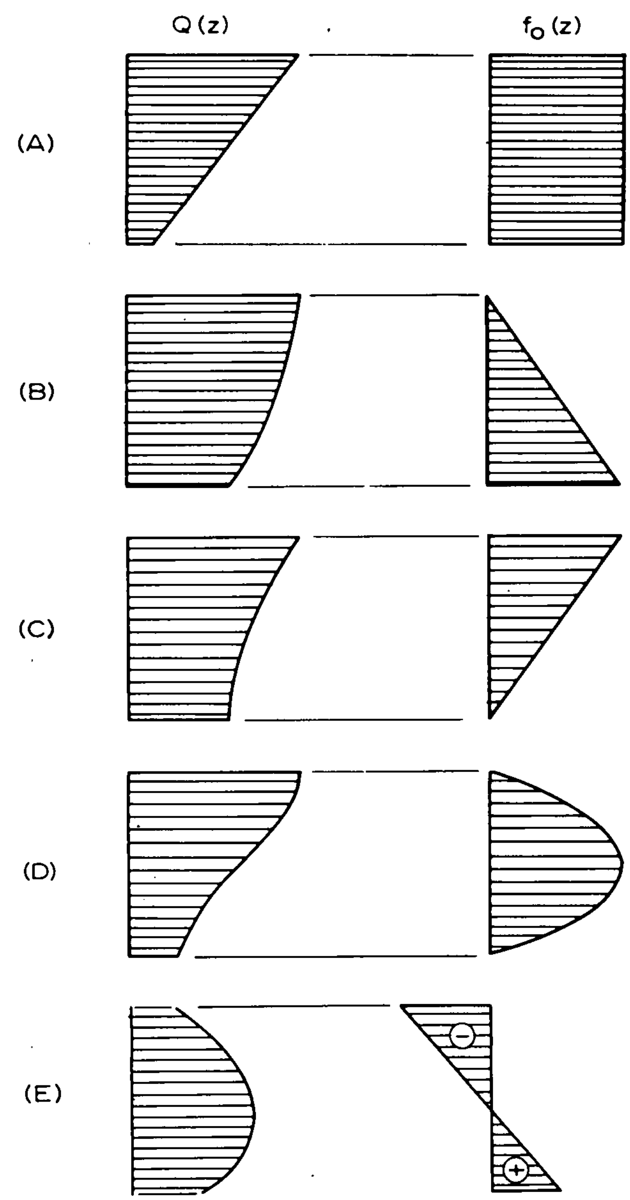


Figure 28. Typical simple distributions of skin resistance.

fer function, which is an empirical or analytical relationship of the form:

$$\Delta Q_i = Q_i - Q_{i-1} = f(w_i) \tag{31}$$

Some transfer functions are given in Table 4, which lists the principal methods of analysis based on this approach.

The transfer function provides a unique relationship between the load transferred from an element and the displacement of that element. It contains also a hidden assumption that the displacements along any element are not affected by skin loads (ΔQ) transferred by other elements, except through the pile itself. In other terms, the soil surrounding the pile is replaced by a set of nonlinear springs, supporting the pile at mid-points of each element and entirely independent of each other. Because any loads (Q) transmitted to the soil must affect the points below, as well as the points above to some distance from the considered element, the concept of a unique transfer function is in obvious contradiction with reality.

In an attempt to circumvent this inconsistency, the elastic-solid approach (Fig. 29a) considers the effects of transmitted shaft loads (ΔQ) on the points above and below. It also allows the possibility that the displacement of pile element (${}_{p}w_i$) may be different from the displacement of the adjacent soil (${}_{s}w_i$). Fundamental to this approach is the assumption that the soil transmits loads (ΔQ) as a homogeneous, elastic, isotropic (Hookean) solid defined by two deformation characteristics: namely, modulus of deformation (E_s) and Poissons' ratio (ν_s).

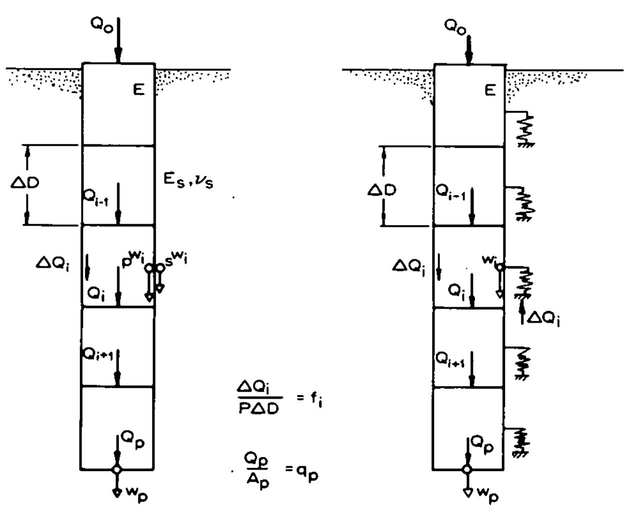


Figure 29. Load-transfer analysis: (a) elastic-solid approach and (b) transfer-function approach.

With this assumption, the transfer-function relationship (Eq. 31) is replaced by n equations of the form

$${}_{s}w_i = \frac{B}{E_s}\sum_{j=1}^{n} I_{ij}f_j + I_{ip}q_p \tag{32}$$

plus one equation of the form

$$w_p = \frac{B}{E_s}\sum_{j=1}^{n} I_{pj}f_j + I_{pp}q_p \tag{33}$$

In these equations, B represents the pile diameter and I_{ij} are influence factors for settlement of elements i caused by forces ΔQ_j. These factors are evaluated by using the Mindlin solution for stresses and displacements in a semi-infinite solid under the action of point loads in the interior of the solid (*113*).

In addition to Eqs. 32 and 33, assuming that there is no slip between the pile and the soil (${}_{s}w_i = {}_{p}w_i$), there are n equations of the form

$${}_{p}w_{i+1} - {}_{p}w_i = Q\,\Delta D / E_p A \tag{34}$$

Should slip occur, the equations (Eq. 34) are replaced by

$$\Delta Q_j = f_s P\,\Delta D \tag{35}$$

in which f_s is the maximum skin resistance for the considered point. In any event there are $2n + 1$ equations for n unknown forces (Q) and $n + 1$ unknown displacements (w). In most recent research, these are written in the matrix form and evaluated by means of a digital computer. Different methods proposed in the literature, all using this approach, are listed in Table 5.

The elastic-solid approach is, in principle, usable for

TABLE 4

METHODS OF LOAD TRANSFER ANALYSIS BY TRANSFER FUNCTION APPROACH

Author(s)	Transfer function
Seed & Reese (1955)	experimental
Kezdi (1957)	$f = K_o \gamma z \tan\phi \left\{1 - \exp\left[-kw/(w_c - w)\right]\right\}$
Reese (1964)	experimental
Coyle & Reese (1966)	experimental
Reese et al (1969)	$f = K\left[\,2\sqrt{w/w_c} - w/w_c\right]$
Holloway et al (1975)	$f = K\gamma_w(\sigma/p)^n \;w\left[1 - (R_f/\sigma\tan\delta)\right]^2$

TABLE 5

METHOD OF LOAD TRANSFER ANALYSIS BY ELASTIC SOLID APPROACH

Author(s)	Characteristics of the method
D'Appolonia & Romualdi (1963)	w_p = 0, special case, no slip, E_s = const.
Thurman & D'Appolonia (1965)	special case, E_s = const.
Salas & Belzunce (1965)	$E_p = \infty$, E_s = const., no slip
Pichumani & D'Appolonia (1967)	groups, special case, E_s = const.
Poulos & Davis (1968)	$E_p = \infty$, general case, E_s = const.
Poulos (1968)	groups ($E_p = \infty$), general case, E_s = const.
Mattes & Poulos (1969)	general case, E_s = const.
Poulos & Mattes (1969)	w_p = 0, general case, E_s = const.
Poulos & Mattes (1971)	groups, general case, E_s = const.
Poulos & Mattes (1971,1974)	groups, general case, layered soil

analysis of negative friction as well. Some of the methods listed in Table 5 (Salas and Belzunce, *96*; and Poulos and Davis, *98*) include considerations of this kind of problem of load transfer. A number of attempts along similar lines have been undertaken in recent years (*105, 106, 112*) with limited success, however. An alternate method of De Beer-Zeevaert (*107, 108*), using some concepts of silo-analysis, has often produced more reliable estimates of maximum pile loads in this situation.

The elastic-solid approach offers the advantage of considering the adjacent soil as a continuum, defined by material characteristics of definite physical meaning. However, the approach still possesses a number of serious shortcomings.

The assumption that the soil response to loading can be adequately described with only two deformation characteristics (E_s and ν_s) may be oversimplifying to a greater degree than in a comparable situation involving shallow foundations. Most soils show stress-, stress-history-, and time-dependent response to loading. Typically, a mass of cohesionless sand will have *E*-values increasing with the square root of mean normal stress and different in loading from those in unloading. The pile behavior in such a mass is greatly affected by the possible prestress in lateral direction caused by pile driving.

In addition, the elastic-solid approach assumes the same *E* in tension as in compression, whereas most soils can take little stress, if any, in tension. It should be remembered that the load transfer according to Mindlin's solution induces sometimes significant tensile stresses beyond the point of load application. Unless these tensile stresses are compensated by compressive stresses coming from loads applied above the considered point, they are not transmitted by the soil and the stress distribution can be considerably different from that assumed by the blind use of Mindlin's solution. This is illustrated in Figure 30, which shows the distribution of vertical stresses (σ_z) due to the pile point load (Q_p) (on the right) as well as due to skin load (Q_s), assumed to be uniformly distributed along the pile shaft (on the left) (*109*). A study of the charts in this

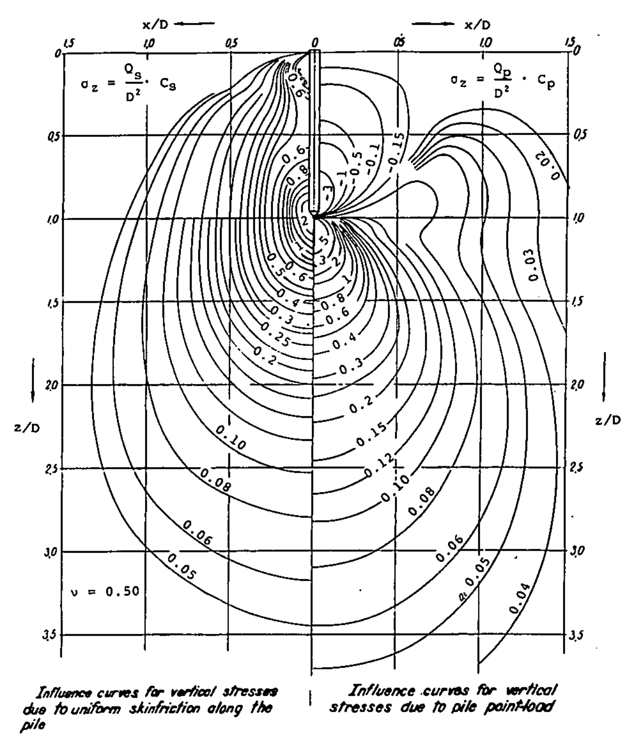
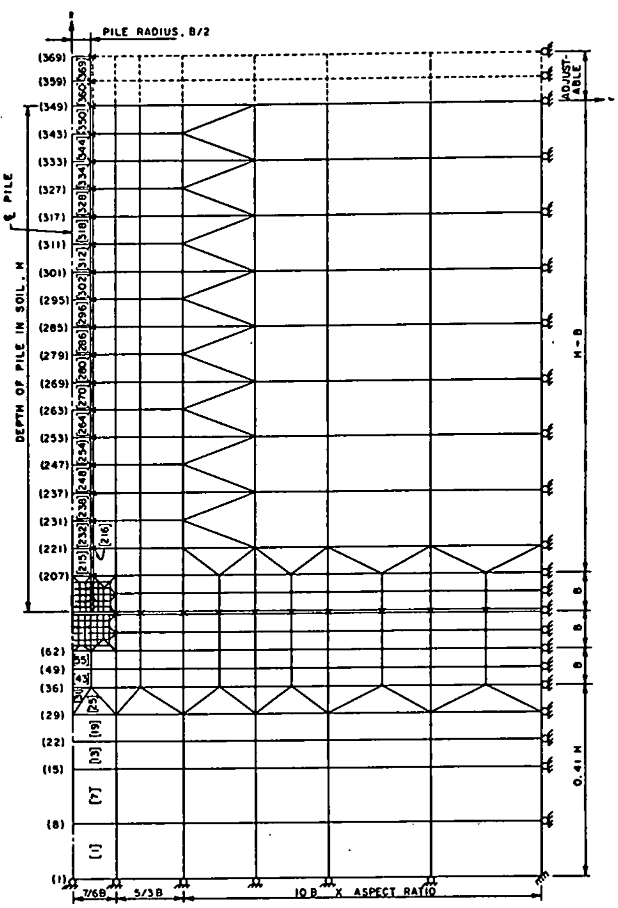


Figure 30. Distribution of vertical stresses around a pile in elastic solid (109).



LEGEND
X SPRING ELEMENTS
[377] ELEMENT NUMBER
(379) NODE NUMBER

Figure 31. Finite-element analysis of load transfer (110).

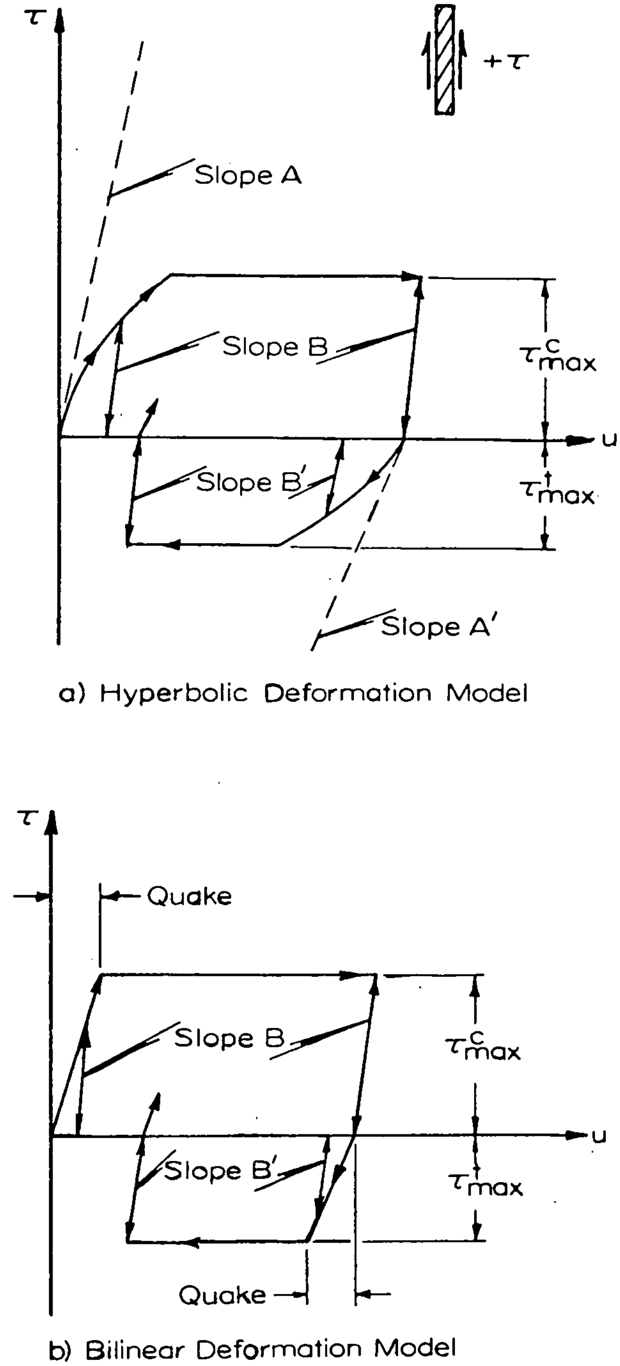


Figure 32. Deformation models used in load-transfer analysis (93).

figure reveals that, to avoid an extensive tension zone above the pile point, the skin load (Q_s) must be at least twice as large as the point load (Q_p). An examination of available load-transfer data shows that this condition can more easily be satisfied in cohesive than in cohesionless soils.

The known methods of load-transfer analysis using the elastic-solid approach all make a tacit assumption that the presence of piles in the soil does not affect the stress distribution in the soil mass. In other terms the stress distribution assumed is that of an ideal homogeneous soil mass without piles. The "reinforcing" effect of piles in a vertical direction has been only partially explored.

It appears that many of the aforementioned shortcomings of the elastic-solid approach can be overcome, at least in principle by the use of finite-element analysis. For example Ellison (*110*) formulated and programmed for the computer a general analysis of load transfer for an arbitrary pile in a soil mass, the stress-strain response of which is bilinear and stress dependent. The general scheme of the model used, containing 377 elements, is shown in Figure 31. Holloway et al. (*93*) introduced a similar analysis with nonlinear (hyperbolic) stress-dependent stress-strain response, using different characteristics in compression and tension (Fig. 32). This allows a complete analysis of pile response to load-unload cycles in both compression and tension. Some typical results of this analysis are shown in Figure 33. Analyses of this kind allow the introduction of stress and displacement conditions imposed by the method of pile placing and consider the fact that the pile presence in the soil mass affects the stress distribution. In addition, the finite-element method by its nature allows the introduction of arbitrary inhomogeneities in the soil mass, such as layers and lenses of different characteristics. The results obtained so far appear quite encouraging.

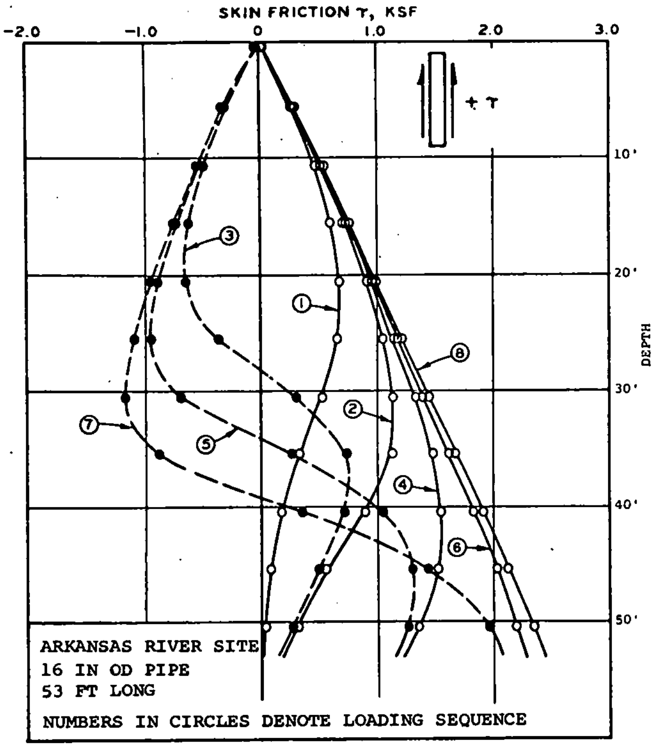


Figure 33. Computed distribution of skin friction for Arkansas River site piles loaded in cyclic compression (93).

CHAPTER FIVE

SETTLEMENT ANALYSIS

The settlement analysis of pile foundations bears some similarity to the settlement analysis of shallow foundations, in that both are based on the same principles. There are, however, some distinct differences. These come, in part, from previously discussed disturbances of the adjacent soil and changes in its state of stress caused by the operation of pile placement in the ground. Because of these disturbances, the soil surrounding a pile, even if it was initially homogeneous, can exhibit sharp variations of stiffness in both vertical and horizontal directions at least in the highly stressed zones around the pile. If the pile is driven, it can retain large residual stresses which, in turn, significantly influence the pile response to load and its load-settlement characteristics. The exact position of load transfer from the pile to the soil is unknown and usually varies with load intensity. As in the case of the parent bearing-capacity problem, the situation is further complicated by the effects of placing of adjacent piles and possible group action, making the rational formulation of this problem extremely difficult. Thus, at the present state of knowledge, only approximate solutions of this problem are available and their limitations must be kept in mind in all applications.

For design purposes, it is convenient to separate the pile-head settlement (w_0) into its three components:

(a) settlement due to axial deformation of the pile shaft (w_s),

(b) settlement of pile point caused by load transmitted at the point (w_{pp}), and

(c) settlement of pile point caused by load transmitted along the pile shaft (w_{ps}).

Thus, it can be written that, in general

$$w_o = w_s + w_{pp} + w_{ps} \tag{36}$$

in which the three components are determined separately (*29, 39*).

The settlement due to the deformation of the pile shaft (w_s) can be determined easily if the magnitude and distribution of the skin friction is known or assumed. The well-known strength-of-materials formula for axial deformation of a bar yields in this case

$$w_s = (Q_p + \alpha_s Q_s)\frac{L}{A\,E_p} \qquad (37)$$

in which Q_p and Q_s represent, respectively, the actual point and skin loads transmitted by the pile in the working stress range; L is the length; A, cross-sectional area; and E_p, modulus of elasticity of the foundation shaft. The coefficient α_s is a number that depends on distribution of skin friction along the pile shaft. For uniform and parabolic distributions (Figs. 28A and 28D), $\alpha_s = 0.5$, whereas for two extreme cases of linear distribution (Figs. 28B and 28C), α_s can be as much as 0.67 (Fig. 28B) and as little as 0.33 (Fig. 28C). Lesser values are observed in driven piles because of residual stresses. A typical value for hard-driven slender piles in sand, corresponding to distribution shown for peak load in Figure 25b, may be around 0.1. Lesser values are also observed in the case of long friction piles where, under working loads, only a fraction of shaft length effectively transmits the loads (see Figs. 22 and 25b).

The components of settlement of the pile point (w_{pp} and w_{ps}) can, in principle, be found by assuming that the soil surrounding the pile behaves as an elastic, isotropic (Hookean) solid, defined by a modulus of deformation (E_s) and a Poisson's ratio (ν_s). Computations of this kind lead to solutions that can be presented in the following form:

$$w_{pp} = \frac{q_p B}{E^*_s} I_{pp} \qquad (38)$$

$$w_{ps} = \frac{\bar{f}_s B}{E^*_s} I_{ps} \qquad (39)$$

in which q_p represents the net (excess) pressure on the pile point; $\bar{f}_s$, the average unit shear transmitted across the pile shaft. Both are in stress units. $E^*_s = E_s/(1 - \nu_s^2)$ is the plane-strain modulus of deformation of the soil underlying the pile point. I_{pp} and I_{ps} are dimensionless influence factors, analogous to settlement factors of shallow foundations, which are often computed by integration of Mindlin solution for displacement in a solid loaded by point load in its interior (*2, 113*). In spite of known limitations of that approach, discussed briefly in the preceding section, such computations can give reasonable answers, at least for I_{ps}. The I_{pp}-values so obtained are normally too low (*114*). To find reasonably realistic values of these influence factors it is necessary to consider the pile as a rigid or elastic body within the soil mass. A finite-element analysis of this problem (*63*) gives for $\nu = 0.25$ and $D/B > 5$ a I_{pp}-value of 0.54. The corresponding I_{ps}-values can be approximated in the range $0 > D/B > 50$ by the expression

$$I_{ps} = 2 + 0.35\sqrt{D/B} \qquad (40)$$

A consistent theory of prediction of settlements of piles that considers piles as compressible columns in an elastic soil mass and makes a systematic use of the Mindlin solution has been proposed by Poulos (*115*). This theory assumes that the distribution of load between the pile shaft and pile point is governed by the elastic properties of the pile and the soil and disregards the existence of residual stresses in the pile. In view of its assumptions, this theory is potentially applicable to bored piles in clay.

In order to use Eqs. 38 and 39 or the Poulos theory, it is necessary to determine the modulus of deformation (E_s). With known limitations this can be, in principle, done by triaxial or consolidation tests in the laboratory, knowing that the compression modulus from the consolidation test $M_v = 1/m_v$ is related to E_s and $E_s{}^*$ by:

$$\frac{E_s}{1 - \nu_s^2} = E_s{}^* = \frac{1 - 2\nu_s}{(1 - \nu_s)^2} M_v \qquad (41)$$

In the absence of other data it may be assumed that $\nu_s = 0.3$. However, in view of the fact that most soil moduli are stress-level and stress-path dependent and that placement of driven piles causes substantial changes in structure and state of stress of the soil mass, the successful selection of a representative E_s-value for this kind of computation involves a strong element of art. Also, as mentioned earlier in bearing-capacity discussions, the cost of taking and testing the necessary number of samples to make such an analysis meaningful is often prohibitive. For these and other reasons it is normally preferred to determine deformation moduli from empirical relationships based on field penetration or expansion tests. Examples of such correlations can be found for example, in References *39* and *55*.

On the basis of Eqs. 38 and 39 and available empirical correlations between E_s and the ultimate point resistance (q_o) for a number of construction sites, the following general expressions can be proposed (*39, 63*):

$$w_{pp} = \frac{C_p Q_p}{B\,q_o} \qquad (42)$$

$$w_{ps} = \frac{C_s Q_s}{D\,q_o} \qquad (43)$$

in which Q_p denotes the net point load and Q_s the shaft load of the pile (both in force units), mobilized under working conditions of the pile. C_p and C_s are empirical coefficients that depend on soil type and the method of construction of the pile. Some typical values of C_p are given in Table 6. C_s is related to C_p by the expression

$$C_s = (0.93 + 0.16\sqrt{D/B})\,C_p \qquad (44)$$

It should be emphasized that q_o in Eqs. 42 and 43 represents the ultimate point resistance of the particular pile for which the settlement analysis is made. As explained earlier in bearing-capacity discussions, this quantity is not necessarily the same for different foundation types in the same soil and is often affected by the foundation size. In this way, the scale effects other than those explicit in Eqs. 42 and 43 are introduced, making the coefficients C_p and C_s practically independent of pile dimensions.

The values of coefficients C_p given in Table 6 give total, long-term settlements of the pile in conditions where the bearing stratum under the pile tip extends at least 10 pile

TABLE 6

TYPICAL VALUES OF COEFFICIENT C_p

Soil type	Driven piles	Bored piles
sand (dense to loose)	0.02 to 0.04	0.09 to 0.18
clay (stiff to soft)	0.02 to 0.03	0.03 to 0.06
silt (dense to loose)	0.03 to 0.05	0.09 to 0.12

diameters below its point and where the soil below is of comparable or higher stiffness. These values are slightly lower if a firm stratum (rock) exists nearer to the pile point.* As established both by full-scale sustained load tests (*96*) and by theoretical analysis (*78*), a predominant portion of these settlements in the working load range are immediate in nature. Unless a highly compressible stratum of low permeability exists somewhere below the pile tips, the consolidation settlement should not be significant and normally does not exceed 15 percent of the total settlement. However, should such a compressible layer be present in the zone influenced by the pile-transmitted load, a consolidation-settlement analysis similar to that conventionally performed for shallow footings is needed.

* It can be shown that the reduction in settlement depends on the ratio of depth of compressible stratum (z) under pile tip to pile diameter (B). If this ratio drops to 5, the settlement is 88 percent of value obtained by Eq. 42. When the ratio drops to 1, the settlement is still about 51 percent of that value.

CHAPTER SIX

PILE GROUPS

The previous chapters of this synthesis refer primarily to bearing capacity or settlement of an individual pile. However, pile foundations are normally constructed as groups of closely spaced piles joined together by a cap, a crossbeam, or a truss-like frame (Fig. 34). The cap is usually a reinforced concrete block that must be properly designed to distribute loads equally (as much as possible) among individual piles. The crossbeam performs a similar function without contact with the soil and as a part of a structural frame that includes the piles as columns protruding from the soil. The truss-like frame, usually made of steel members with piles as columns, is designed to assure joint action of all piles in the group under lateral loads.

The spacing of piles is determined from considerations of stability and economy, combined with a number of practical considerations. Ideally, piles should be spaced so that the bearing capacity of the group is not less than the sum of the bearing capacities of individual piles. This could be translated into a requirement that the outer perimeter of the group (shown by dashed line in Fig. 34) should be greater than or equal to the sum of the perimeters of individual piles. A simple computation shows that this requirement is satisfied if the relative spacing of the piles (s/B) is greater than $\sqrt{n} + 1$ for square piles and 0.785 ($\sqrt{n} + 1$) for circular piles, in which n is the number of piles in the group. However, for groups containing more than nine piles, this may lead to spacings of more than four pile diameters, which is normally avoided as uneconomical. On the other hand, in order to reduce the extent of soil disturbance and provide an allowance for errors in position and alignment of piles, the lower limit of spacing must be kept at 2.5 pile diameters. The optimum spacing is usually selected in the range of 3 to 3.5 *d*.

Once the problem of pile spacing is settled, the principal questions faced in the design of pile groups remains the same as those for individual foundations, namely: what is the ultimate load of the group ($\overline{Q}_o$) and how can the settlement of the group ($\overline{w}$) under a working load ($\overline{Q}$) be predicted. It is well known that the ultimate load of the group is generally different from the sum of ultimate loads of individual piles ΣQ_o. The factor,

$$\eta_g = \frac{\overline{Q}_o}{\Sigma Q_o} \tag{45}$$

called group efficiency, depends on parameters such as soil type, size and shape of the group, spacing and relative length of piles, as well as the construction procedures. It is also known that the settlement of the group ($\overline{w}$) is normally greater than the settlement of a single pile at comparable working load. In general

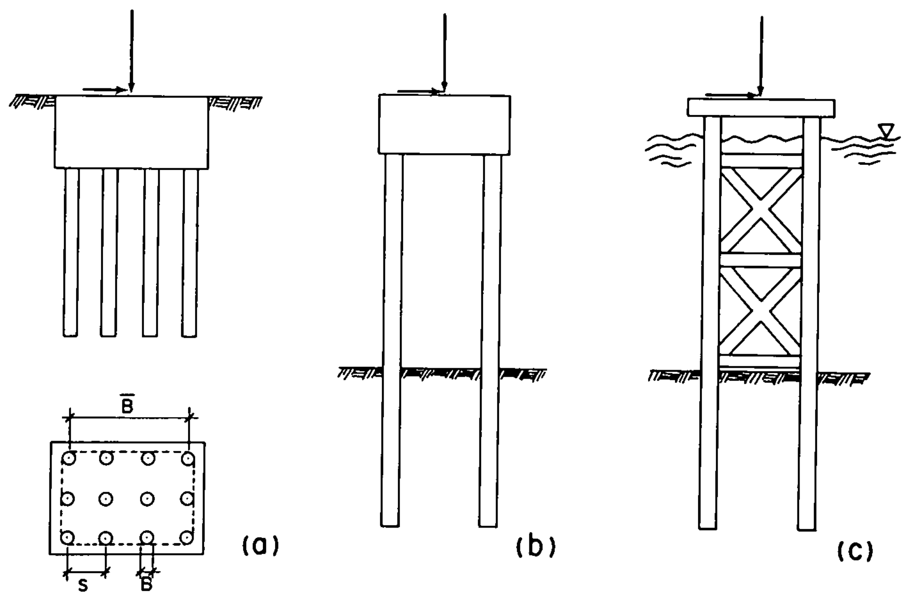
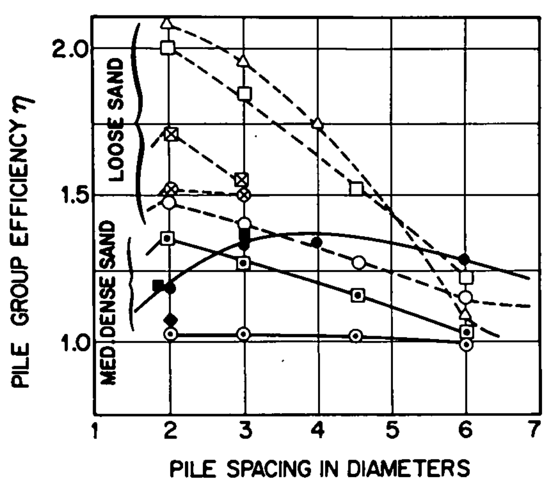


Figure 34. Typical configurations of pile groups.



SYMBOL	NO. OF PILES IN GROUP	PILE DIAMETER	PENETRATION IN BEARING STRATUM(DIA)	SOIL TYPE	SOURCE
△	4	4"	20	LOOSE SAND	KÉZDI (1957)
● ■	4 9	4"	15	MED. DENSE SAND	VESIĆ (1968)
◆	4	4"	3	DENSE SAND OVERLAIN BY LOOSE SAND	VESIĆ (1968)
○ □	4 9	1.4"	15	LOOSE SAND	TEJCHMAN (1973)
⊙ ▣	4 9	1.4"	15	MED. DENSE SAND	TEJCHMAN (1973)
⊗ ⊠	4 9	12"	20	LOOSE SAND	KISHIDA (1967)

Figure 35. Observed efficiencies of square pile groups in sand.

$$\overline{w} = \zeta_g w \qquad (46)$$

in which ζ_g is the group settlement factor. This factor also depends on a number of parameters including soil type and over-all soil profile, size and shape of the group, and the method of construction.

There is, at present, no acceptable rational theory of bearing capacity of pile groups. A number of empirical "efficiency formulae," such as the Feld or Converse-Labarre rule, have been proposed and used in the past. A comprehensive review of these formulae, all of which give efficiencies less than unity, can be found in Reference *19*. (See also Ref. *117*.)

Terzaghi and Peck (*118*) suggested that the bearing capacity of a pile group can not be greater than that of a block foundation defined by the exterior perimeter of the group. This idea was generalized into a suggestion that the bearing capacity of any pile group could be computed by adding the skin resistance along the outer perimeter of the group and the resistance of an imaginary base defined by that perimeter (*117, 119*). In this manner, efficiencies greater than unity are obtained for groups in sand.

The well-documented information on actual bearing capacities of full-size pile groups is extremely scarce. There are known to be only five reported full-scale tests with pile groups in clay (*76, 120, 135, 136, 222*); all indicate efficiencies of approximately unity. Only six investigations with full-size piles in sand can be found in the literature (*121-125, 227*). As shown in Figure 35, they all indicate efficiencies greater than unity. Model studies using small [0.5 to 1.5-in. (12 to 38-mm)]-diameter piles in dense and loose sand show a similar trend (*126*), whereas tests with small-scale-model piles in clay (*127-129*) indicate efficiencies somewhat less than unity. However, the value of quantitative information from small-scale-model tests remains questionable (*124*). Summarizing all available data, the following basic findings relative to group bearing capacity appear to be valid, at least as long as pile spacings exceed 2.5 pile diameters:

• The ultimate point load of a pile group can be taken to be equal to the sum of the ultimate point loads of individual piles.

• The ultimate shaft load of a group of piles in cohesive soil can not be greater than the sum of the ultimate shaft loads of individual piles multiplied by the ratio of the outer perimeter of the group (dashed line in Fig. 34) to the sum of the perimeters of individual piles. However, in comparing shaft loads of piles and pile groups in cohesive soils, it is important to keep in mind that the dissipation of pore pressures set by pile driving is slower for groups than for individual piles (cf. Fig. 13).

• The ultimate shaft load of a group of piles in cohesionless soil may be greater than the sum of individual shaft loads. This is explained by increased compaction and lateral compression caused by driving a greater number of piles within a relatively small area. The sequence of driving is important in this regard, and there is experimental proof that later driven piles have a higher capacity than those driven earlier (Fig. 36). If the design is not controlled by settlement considerations, it may be possible to take advantage of this increase in group capacity at least for piles totally embedded in loose sand. However, in view of scarce information on this subject, group load tests are highly recommended in the latter case.

• Pile caps contribute to the over-all bearing capacity of the group to the extent that they are supported by competent soil outside the outer perimeter of the group (*124*). However, in view of the potential danger of erosion or loss of support by settlement of the soil surrounding the piles, the use of this increased capacity in design is discouraged.

The allowable design load for larger groups is often determined from settlement considerations. Although knowledge of the general mechanism of load transfer in pile groups has increased considerably over the last several years, it is still not easy to explain all observations and provide a theory of group settlement that can be recommended without reservations. Empirical or semiempirical expressions for ζ_g, proposed in the literature for pile groups in sand (*130, 131, 134*), obviously have a limited value and can yield unrealistic results if applied in conditions quite different from those existing in tests on which they were based (*132, 133*). The simplest of these expressions (*124*) suggests that the group settlement factor should be found from

$$\zeta_g = \sqrt{\frac{\overline{B}}{B}} \tag{47}$$

in which $\overline{B}$ is the width of the pile group and B, as before, the diameter of the individual pile. The following points should be kept in mind if expressions of this kind are to be used in design:

• Driving or jacking adjacent piles reduces the residual load in the previously driven piles. With the resulting change in distribution of skin friction, the coefficient α_s from Eq. 37 may increase as much as five times for large groups. In spite of this, there is only a minimal group effect on that part of settlement coming from elastic deformation of the pile shaft if the pile is predominantly point bearing. There is no such effect when dealing with bored piles.

• The group settlement effect is generally greater in predominantly shaft-bearing (friction) piles. Because Eq. 46 is based on tests with piles penetrating 15 diameters in a uniform sand mass, it may overestimate the group effect for piles penetrating less than 15*B* into the bearing stratum and, conversely, may underestimate the group effect in long, slender piles.

Considering all circumstances, equations for settlement factor (such as Eq. 47) represent gross approximations and should be used with greater caution. A consistent theoretical approach for determining ζ_g can be found in the work of Poulos (*99, 102*). However, in view of the shortcomings of basic assumptions of this approach, mentioned

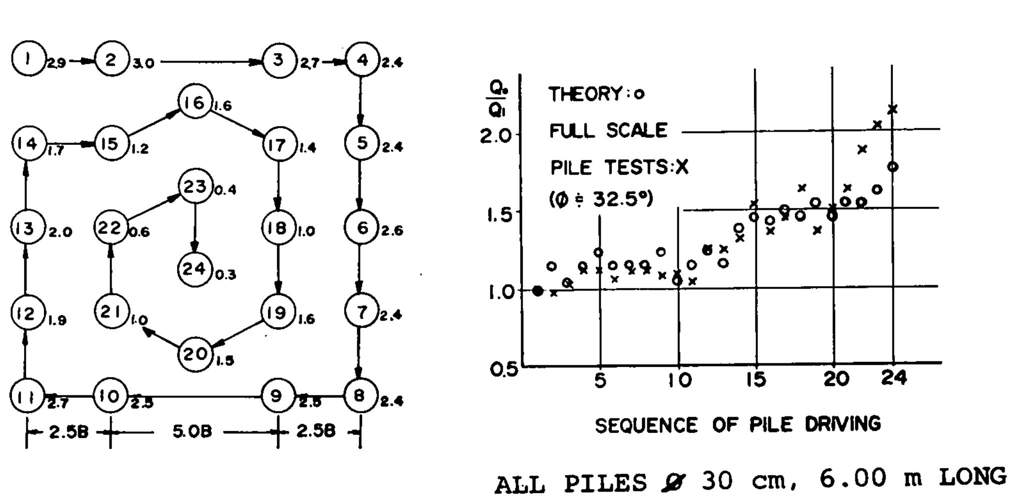


Figure 36. Effect of driving sequence on efficiency of piles in loose sand (125).

earlier in discussing load transfer and settlement analysis of a single pile, the information from this work remains somewhat questionable.

Knowledge about a variety of factors affecting bearing capacity and settlement of pile groups remains limited. A considerable amount of research, including well-instrumented observations on full-size pile groups is needed for safer and more economical design in the future. In the meantime, this aspect of pile foundation design continues to contain a substantial element of art.

CHAPTER SEVEN

LATERAL LOADING

SINGLE PILE UNDER LATERAL LOADS

In addition to axial loads, piles are often expected to transmit lateral loads into the ground. In so doing, they are subjected to bending moments and shearing forces that need to be evaluated in order to assess the safety against structural failure of the pile. At the same time, lateral deflections and slopes of the pile axis are needed for determination of lateral displacement and tilt of the supported structure.

The basic problem of a pile under the action of lateral loads can be formulated as follows (Fig. 37). A single, vertical pile of diameter B and structural stiffness E_pI, is placed to depth D in a soil mass of known characteristics. At the ground level, the pile is acted upon by a transverse load (P) and a moment (M). To be determined are the lateral deflection (u) and the slope of the pile axis (θ) at the ground level, as well as the position and magnitude of maximum bending moment in the pile.

Because pile diameter is usually small compared to pile length, the statical influences along the pile can be determined by considering the pile as a beam and using the well-known differential equation of bending:

$$E_pI \frac{d^4u}{dz^4} = -p(z) \qquad (48)$$

in which $p(z)$ represents the reactive lateral pressure of the soil against the pile, expressed in force-per-length units such as lb/in. or kN/m. The complex distribution of this pressure (p_z) to deflection (u) at any point of the pile, stress-strain characteristics of the adjacent soil are known or assumed.

Relatively simple solutions of this problem can be obtained by introducing an assumption similar to that used in theory of beams and slabs resting on soil that the ratio of pressure (p_z) to deflection (u) at any point of the pile, called soil reaction modulus, is known. For example,

$$\frac{p_z}{u} = K_h = const. \qquad (49)$$

The assumption of constant modulus (K_h), called Winkler's hypothesis, leads to closed-form solutions of differential Eq. 48 that are well known in the literature (*137*) and that can easily be adapted to the considered problem. However, observations on laterally loaded piles indicate that it is more plausible, at least for free-standing, uncapped piles, to assume a modulus (K_h) linearly increasing with depth according to

$$\frac{p_z}{u} = K_h = n_h z \qquad (50)$$

in which n_h represents an empirical quantity called coefficient of soil reaction.

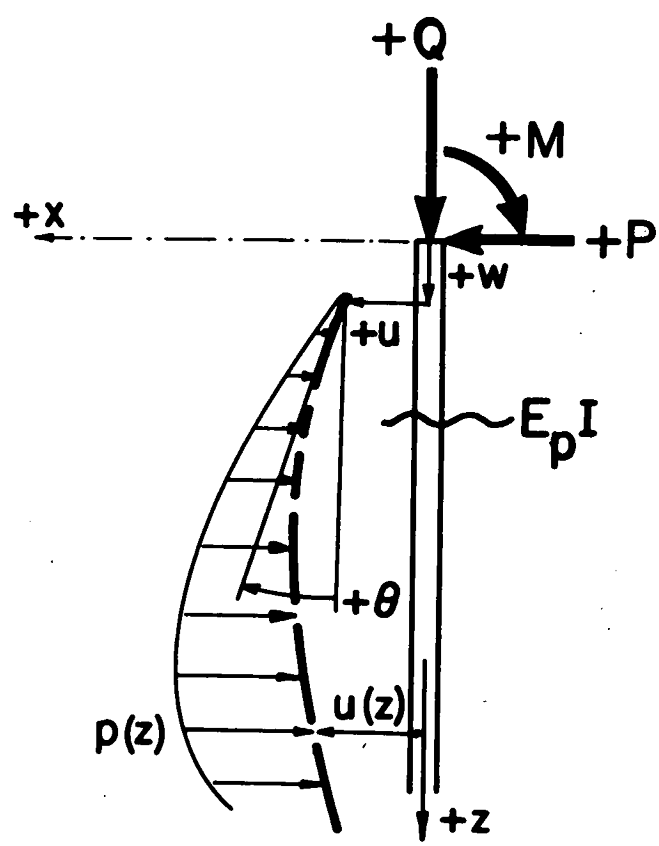


Figure 37. Single pile under the action of lateral loads.

The resulting differential equation has a solution in the form of an infinite series (*138*); however, it also can be easily integrated by using the method of finite differences (*139*). Computations of this kind (*140, 141*) show that, for a sufficiently long pile, the deflection (u) and the slope (θ) at the ground line ($z = 0$) can be obtained from:

$$u = \frac{2.43}{n_h T^2} P - \frac{1.62}{n_h T^3} M \tag{51}$$

$$\theta = -\frac{1.62}{n_h T^3} P + \frac{1.75}{n_h T^4} M \tag{52}$$

in which T represents the characteristic length of the pile-soil system defined by:

$$T = \sqrt[5]{\frac{E_p I}{n_h}} \tag{53}$$

The maximum bending moment in the pile can be found with an error less than 4 percent from the approximate expression (*142*):

$$M_{max} = 0.80\,(M + PT) \tag{54}$$

which is valid as long as M/PT is algebraically greater than -0.47. The location of this maximum moment varies with the ratio M/PT; for M/PT equal to or greater than unity, it is approximately at depth T below ground line increasing to $1.6T$ for $M/PT = -0.47$. For M/PT less than -0.47, the critical bending moment is at ground line, negative in sign, and equal to M. It should be noted that M and P in Eq. 54 have to be introduced with proper sign convention as shown in Figure 37.

The preceding analysis is sufficiently accurate as long as the pile length exceeds four characteristic lengths T defined by Eq. 53. For short, semi-rigid piles, the deflection and slope coefficients, as well as the maximum moments, can be found from appropriate graphs and tables. (See Refs. *140* and *141*.) For very rigid piles having lengths shorter than two characteristic lengths T, the deflections and slopes at the ground line can be computed from expressions (*140*):

$$u = \frac{18}{n_h D^2} P - \frac{24}{n_h D^3} M \tag{55}$$

$$\theta = -\frac{24}{n_h D^3} P + \frac{36}{n_h D^4} M \tag{56}$$

A detailed review of literature on behavior of rigid piles can be found in Reference *192*. Such piles should also be checked for safety against overturning (*190*).

In the case of nonhomogeneous soil conditions and abrupt changes in soil stiffness with depth, it is important to note that the lateral resistance of a pile is governed by the properties of the soil in the immediate vicinity of the pile head. Studies of the lateral-load behavior of piles in a layered soil (*143*) show that the lateral resistance is little affected, if any, by the soil extending deeper than $0.5T$, or just a few pile diameters below the pile head. Thus, investigations for this purpose should concentrate on the surface layer. Also, significant improvements of lateral-load resistance can be achieved by improvement or densification of that layer. Finally, the fact that this layer is commonly exposed to seasonal variations in soil moisture and stiffness can be important for proper interpretation of lateral load tests. Large seasonal variations of lateral stiffness of piles should be expected in many situations.

The coefficient of subgrade reaction (n_h) appearing in the previously described theory of lateral bending of piles can best be obtained from measured deflections and slopes in a lateral-load test. Terzaghi (*144*) found that n_h in sands should be directly related to the rate of increase of overburden stress with depth or to the effective unit weight of the sand, γ. He suggested that n_h could be determined from:

$$n_h = N_h \gamma \tag{57}$$

in which N_h is a dimensionless constant which, for 1-ft (0.3-m)-wide piles in sand, increases from 75 for very loose to 1,500 for very dense sand. Subsequent detailed observations on instrumented piles (*145-151*) have shown that these values are usually conservative. However, the same observations also show that n_h or N_h are not constants but that they generally vary with the lateral deflection of the pile (u). Similar nonlinear behavior has been reported for piles in clay (*150-154*). The variation of the coefficient of subgrade reaction with deflection observed in different pile load tests reported in the literature is illustrated in Figures 38 and 39.

In view of the nonlinearity of the pressure-deflection relationship, it has been suggested (*153*) that the basic equation of pile bending (Eq. 48) be used in the following nonlinear form:

$$E_p I \frac{d^4 u}{dz^4} = -p(z,u) \tag{58}$$

in which, for piles with variable cross section, moment of inertia I can also be a function of z rather than a constant. The function $p(z,u)$ can be formulated in the following hyperbolic form:

$$\frac{p}{u} = K_h = \frac{1}{\dfrac{1}{K_i} + \dfrac{u}{p_o B}} \tag{59}$$

in which K_i represents the soil reaction modulus at the origin (for zero deflection) and p_o, the ultimate horizontal pressure of the soil for a loaded area of diameter B, which is evaluated by the appropriate bearing-capacity theory. (Both K_i and p_o normally vary with depth z.) This type of analysis has been programmed for a computer and can produce nonlinear load-displacement curves that agree with observations over a considerable range of horizontal deflections (*153, 154*). However, this apparently refined analysis suffers from the same shortcomings as the elementary, linear subgrade-reaction theory, namely:

(a) It deals with empirical constants that cannot be related to other known soil and pile characteristics.

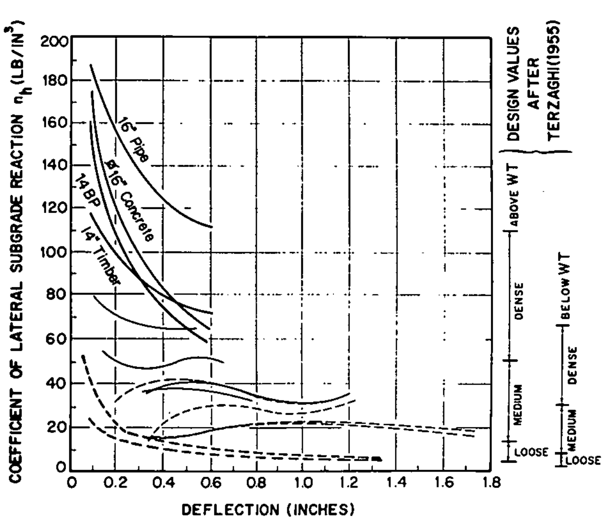
(b) It does not allow a rational analysis of interference of adjacent piles.

These shortcomings can, in principle, be corrected by considering the laterally loaded pile as a bar in a solid of defined properties. Early analyses of this kind (*155-158*) assume that the surrounding soil acts as an elastic-isotropic solid defined by a constant modulus of deformation (E_s) and a Poisson's ratio (ν_s). This assumption represents a serious departure from reality because the uppermost elements of the soil adjacent to the pile are highly stressed and generally deform in the plastic range. In addition, the soil around the pile can not transmit large horizontal tensile stresses perpendicular to the plane of pile bending, as assumed in three of these analyses based on the Mindlin solution (*156-158*). Consequently, possible advantages of these theories may be overshadowed by the lack of reality of some of their basic assumptions. The theories can be improved by introducing a "yield pressure," thus simulating the plastic behavior of soil close to the pile head in a manner similar to that used in nonlinear subgrade-reaction theory (*159*). However, the advantages of this slightly more rigorous approach remain questionable, except for analysis of interference of adjacent piles in a group. As pointed out earlier, such an analysis can not be made by the subgrade-reaction method. A real improvement, from the theoretical point of view, can be achieved only if the problem is solved by using a three-dimensional finite-element analysis for a nonlinear elastic-plastic solid. However, such an analysis is not meaningful before more is learned about the constitutive relationships of soils in three-dimensional stress conditions and about the stress conditions around a pile after driving.

Most of the previous discussion has been concerned with pile deflections and bending moments under short-term, first-loading conditions. Sustained loading generally increases the deflections of laterally loaded piles in clay and, to a certain extent, in sand. The increase is, however, not significant, at least when only dealing with clays of reasonably low liquidity index. Both theoretical and experimental studies on this subject (*158, 160, 163*) indicate that the increase in deflection caused by sustained loads should rarely exceed 25 percent. Considerably larger increases should be expected under cyclic loading. For example, cyclic load tests on two single piles in sand at an Arkansas River site (*146*) showed a sharp increase of deflections, stabilizing after about 100 load applications at almost 100 percent of the first-loading value (see Fig. 40). The corresponding increase in bending moments is much smaller, as might be expected from an analysis according to Eqs. 51 through 54. Similar conclusions were reached earlier from laboratory-model tests on piles in sand (*145, 161*). Limited information available (*162, 163*) shows that analogous cyclic-loading effects exist in piles in clay. Little is known, however, in a general sense about the relative magnitude of these effects in different clay types.

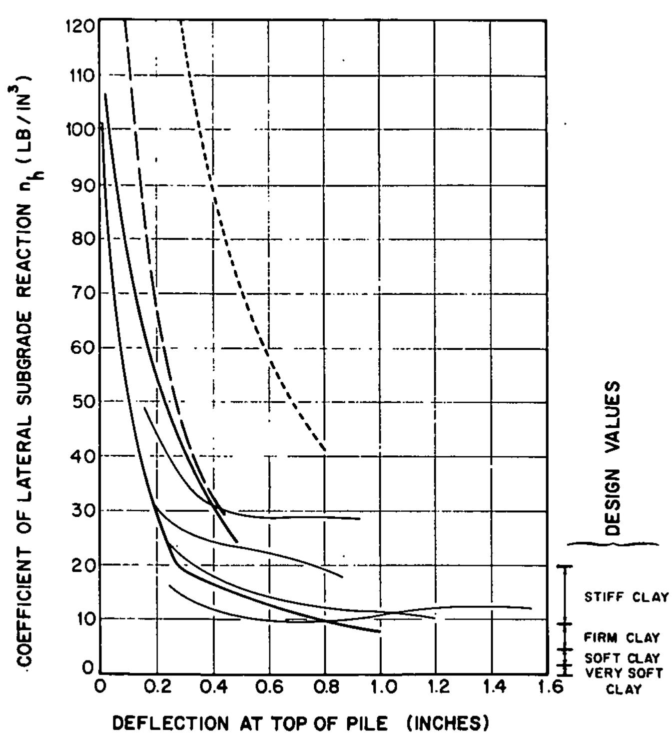
PILES SUBJECTED TO LATERAL SOIL MOVEMENT

A particularly dangerous type of lateral loading on piles, not covered by preceding considerations, is that of pile loading along the shaft by horizontal movements of the ground. A relatively common situation of this kind is shown in Figure 41a. Because of a vertical surcharge, such as an embankment, the underlying soil, particularly if it



Site	Soil	Pile Size	Source
Arkansas River	medium sand	14"-16"	Alizadeh & Davisson (1970)
Danube Bank	silty sand	35" pier	Botea et al. (1970)
Occidental Drive	silty sand	10 ¾"	Paduana & Yee (1974)
Linda Creek	fine silty sand	H 8" x 8"	Paduana & Yee (1974)

Figure 38. Variation of coefficient of subgrade reaction (n_h) for piles in sand.



Soil	Pile Type	Source
silty-sandy clay	10BP42 (group of 6)	Kim et al (1973)
sandy clay and sand	48" pier	Davisson & Salley (1968)
stiff sandy clay	10 ¾" pipe concrete filled	Paduana & Yee (1974)
sandy clay	40" pier	Botea et al (1973)

Figure 39. Variation of coefficient of subgrade reaction (n_h) piles in clay.

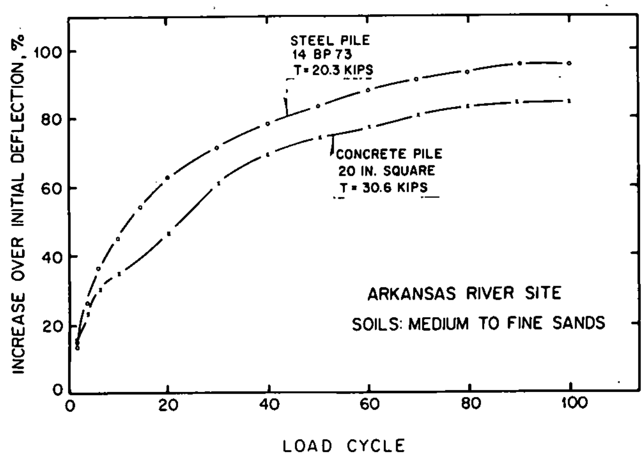


Figure 40. Increase of deflection of laterally loaded piles under cyclic loading (146).

is soft, undergoes sometimes substantial horizontal movement. If a pile is placed in the ground close enough to the surcharge before the horizontal movements of the soil are completed, the soil exerts horizontal pressures on the pile. In extreme cases, such as that shown in Figure 41b, the soil along a portion of pile shaft can be involved in a general shear-type movement along a slip surface or a narrow slip zone in the ground. Reported measurements and observations in the field (*164-168*) show that the resulting forces acting on the pile may be large enough to break the pile in bending or shear. Thus, piles subject to horizontal movements must be designed to guard against this type of failure.

The basic problem of a pile subjected to lateral soil movement can be formulated as follows: A pile has a diameter of B and structural stiffness of E_pI and is embedded in a soil of known properties (Fig. 42). The pile can be free-standing or connected to a superstructure, which may, to a given degree, restrain the free movement of its head. Because of a surcharge or an excavation in the vicinity, the soil surrounding the pile undergoes a lateral movement represented by a function $u_s(z)$. The deflected shape of the pile, $u_p(z)$; the bending moments, $M(z)$; and shears along the pile axis, $P(z)$, are to be determined.

To solve the problem, the differential equation of bending, Eq. 48, and an assumed relationship between the reactive pressure of the soil (p_z) and the relative displacement between the soil and the pile ($u_p - u_s$) are used, as follows:

$$p_z = K_h(u_p - u_s) \qquad (60)$$

The modulus of subgrade reaction (K_h) can, in general, be a function of z and u. Convenient solutions can be obtained by dividing the pile into small elements and using a difference-equation equivalent of Eq. 48 (*169*). Comparisons with measured bending moments in piles under investigated conditions show a good agreement, provided a judicious choice of the subgrade modulus and a realistic assumption of boundary conditions of pile head can be made.

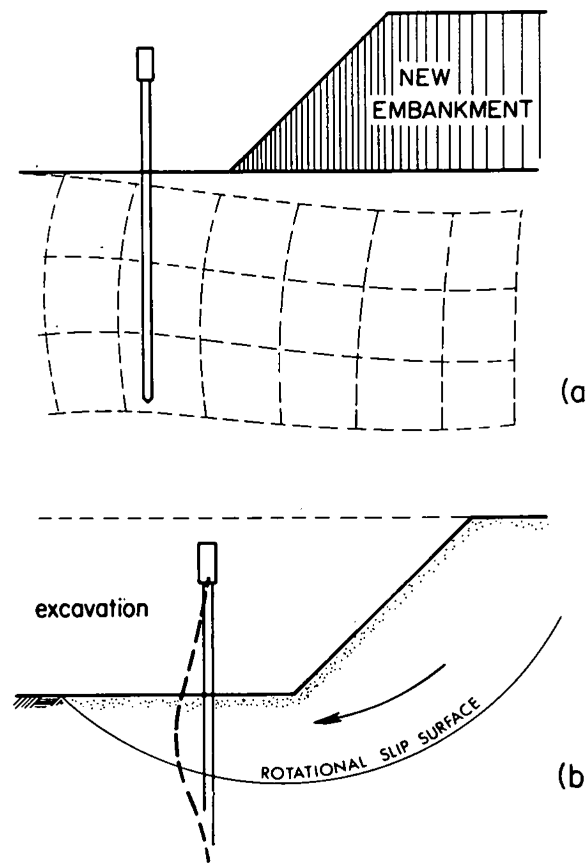


Figure 41. Piles subjected to lateral movement of soil.

An alternate method, proposed by Poulos (*170*), replaces the subgrade-reaction relationship (Eq. 60) by a series of load-transfer functions obtained under the assumption that the soil behaves as an elastic solid. This method also assumes that there is an upper limit of pressure (p_o) that the soil can transfer to the pile. Poulos (*170*) presents some ready solutions for a few simple conditions that can be used in practice. It should be noted that relatively simple solutions of the problem can also be found by assuming that the relative movement between the pile and the soil is large enough everywhere to produce punching failure in the soil and mobilize its ultimate lateral resistance (p_o) (*171*). Another type of ultimate design analysis can be used in situations where an entire soil mass engaged in general shear along a slip surface or a narrow slip zone is moving against the pile or a row of piles (*168, 172*).

AXIAL CAPACITY OF INITIALLY BENT PILES

Another type of potentially dangerous pile bending may occur under axial loads if the pile axis deviates from the intended straight line. Such deviations are not uncommon in the case of relatively slender steel H- or pipe piles, particularly if the soil contains obstructions in the form of small boulders or, more generally, if there are randomly distributed soft or stiff pockets or both in the profile. They also occur with cast-in-place piles installed in long lengths with a casing but without a solid mandrel. The departure of a pile axis from a straight line causes parasite bending moments and thus, additional, bending stresses in the piles, which may reduce their axial capacity.

Once it is established that one or several piles on a site deviate from a straight-line axis, the foundation engineer is faced with the decision of accepting or rejecting these piles. In some instances such decisions are reached by rules-of-thumb such as that requiring that the total offset of the pile from the theoretical straight-line axis should not exceed some percentage of the pile length (usually 2 to 3 percent). Such rules are, however, arbitrary in nature and tend to be overly conservative. A more precise evaluation of possible reduction in pile capacity from a bent-pile axis can be made by analyzing bending moments and shears in the bent piles. At least two general methods for such an analysis with appropriate examples can be found in the literature (*224, 225*).

PILE BUCKLING

Experience shows that buckling of fully embedded piles is extremely rare, even in soft soils, as long as they are capable of supporting a pile in friction. This fact was explained in early studies of this problem (*173, 174*), which showed that a minimum of lateral support can prevent a fully embedded, straight, centrally loaded pile from buckling. Subsequent model and full-scale tests (*175-178*) gave some indications about the order of magnitude of this "minimum support" and provided valuable insight into the nature of the buckling phenomenon in piles.

According to Davisson (*179*) the critical load (Q_{crit}) of a pile of constant stiffness (E_pI), fully embedded in a soil with reaction modulus proportional to depth can be found from the expression

$$Q_{crit} = 0.78\, n_h T^3 \qquad (61)$$

in which T is the characteristic length given by Eq. 53. This expression is valid for piles with free head, transmitting a constant axial load all the way to the pile tip, and is computed under the assumption that the pile is at least $4T$ long. If the pile head is fixed against rotation, the critical load is approximately 13 percent higher; if the pile head is pinned (i.e., prevented from translation but free to rotate), the critical load is 62 percent greater. The effect of reduction of axial load with depth due to shaft friction is minimal as long as the pile head is free; it can be substantial if the pile head is fixed (*180*). The effect of pile taper is, similarly, more pronounced if the pile head is fixed (*181*).

If the piles are only partially embedded in soil, forming part of a structural frame (Fig. 44a), the buckling load can be determined as if the piles were free-standing columns fixed at their base at a depth (L_f) below the ground surface (Fig. 43). Assuming a soil-reaction modulus proportional to depth, the depth of fixity (L_f) is found to be approximately $1.8T$, when T is, as before, the characteristic length given by Eq. 53 (*182*). The same depth of fixity (L_f) can be assumed for complete structural analysis of bending moment and shears in the upper frame (*212*). Following this, the maximum bending moment in the pile can be found from ground-line shear (P) and moment (M) by using Eq. 54.

PILE GROUPS UNDER ECCENTRIC AND INCLINED LOADS

Earlier considerations of pile group action were limited to groups of vertical piles subjected to central, vertical loads. However, in view of the existence of horizontal loads (such as wind; braking or earthquake forces; and earth, ice, or wave pressures), the design reactions in many situations include eccentric and inclined loads. As long as the horizontal components of those reactions are relatively small, they may be transmitted by vertical piles in bending. For large horizontal loads and great eccentricities, it may happen, particularly if the adjacent soil is relatively compressible, that the horizontal displacement and tilt of the foundation become excessive or that bending moments in piles become too great. In such instances batter piles may be

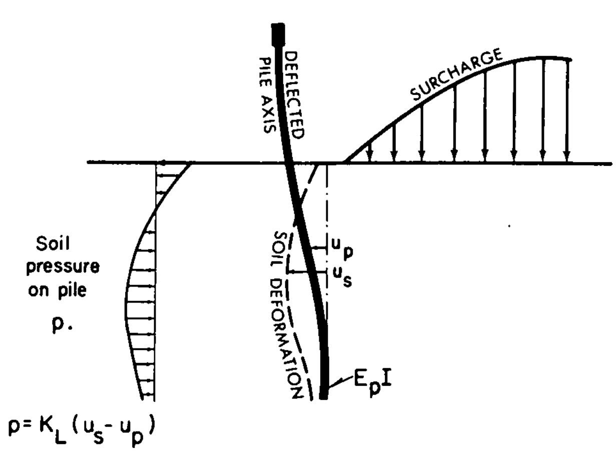


Figure 42. Basic problem of a pile subjected to lateral soil movement.

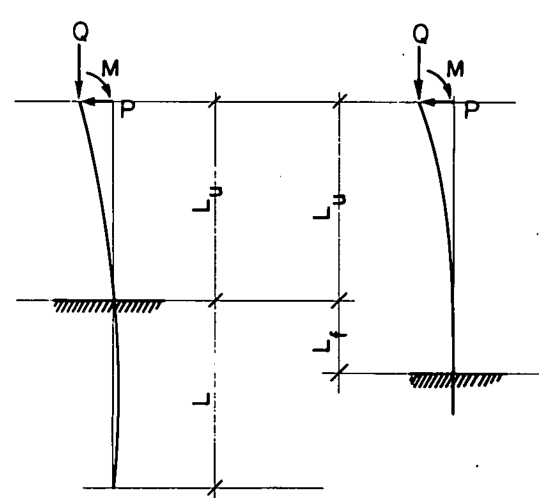


Figure 43. Buckling of partially embedded piles (212).

needed in the group, resulting in pile configurations such as those shown in Figure 44.*

In any case, the presence of eccentric and inclined loads requires a structural analysis of the system. The problem can be formulated as follows: a group of vertical and batter piles is rigidly connected by a cap (Fig. 45). Under the action of the resultant (R) of external loads acting on the system, the foundation is displaced both horizontally and vertically, as well as tilted. The piles resist this displacement by normal forces (Q), shear forces (P), and moments (M) (all positive as shown in the figure). These pile reactions need be determined for computation of bending moments and shears in individual piles. Also needed are components of displacement of the cap or pile heads from which any displacement in the system can be determined.

This problem has been treated in the past by many authors, starting with Gullander (*183*). More recent solutions, based on more realistic assumptions on soil-pile interaction, have been presented by Hrennikoff (*184*), Vesić (*155*), Asplund (*185*), Francis (*186*), Saul (*187*), Reese et al. (*188*) and others. Practically all these solutions are based on the following common assumptions:

(a) The passive pressure and the friction along the sides and on the base of pile cap are neglected.

(b) It is assumed that the spacing of the piles is such that they do not influence each other through the soil mass. In other terms, it is assumed that each pile carries its own reactions independently, as if it were an isolated pile in the soil mass.

(c) It is assumed that the components of displacement (u, w, and θ) of a pile head are linear functions of reactions Q, P, and M of that pile and independent of reactions of other piles.

The first assumption is justified if the cap is embedded in a weak, relatively compressible stratum or in a stratum that can be eroded by scour. Operating under this assumption is always on the safe side; however, there are situations in which it may be too conservative. Information from both model and full-scale tests (*149, 163*) indicates that an embedded cap in sand can take as much as 30 percent of the lateral load in shear on the base alone.

According to Prakash (*161*), the second assumption may be justified if pile spacings are less than 8 pile diameters in the direction of the lateral load and less than 3 pile diameters in the perpendicular direction. His model experiments have shown, however, that the effect of pile interference in the direction of the load can be accounted for by reducing the value of coefficient n_h obtained for a single pile.

As explained in the preceding section, the third assumption is generally not justified because load-displacement relationships for the head of a single pile tend to be nonlinear. However, the effects of nonlinearity can be accounted for by selecting secant values of coefficient n_h (cf. Figs. 38 and 39).

* In selecting the batter pile arrangements, the designer should keep in mind certain facts of practical order. Batter does not greatly affect the cost of driving as long as it is less than 1 to 4. Many designers consider 1 to 3 to be the most efficient batter when lateral stability is critical; this is also the limit for most driving rigs. For free-standing tall bents, large batter should be avoided for aesthetic reasons; in such situations 1 to 10 may be the maximum tolerable.

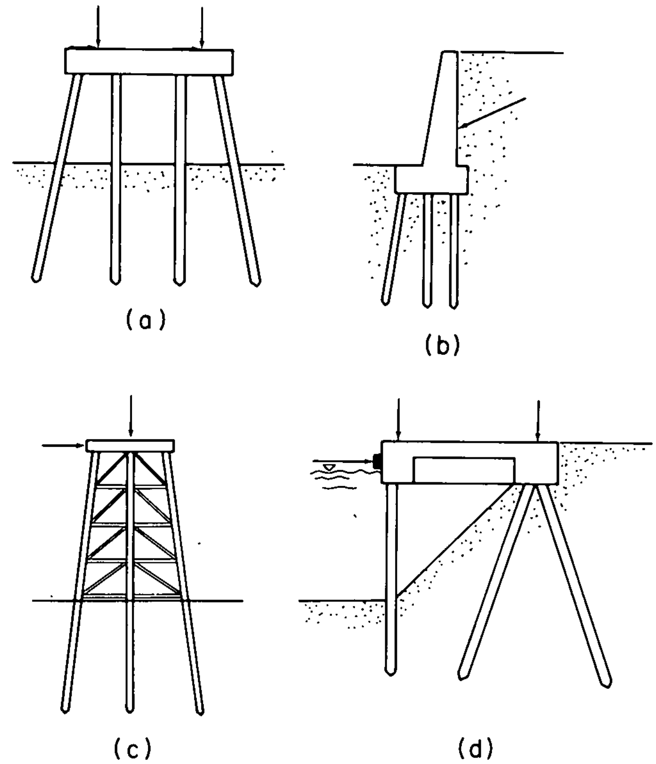


Figure 44. Examples of structural systems with batter piles.

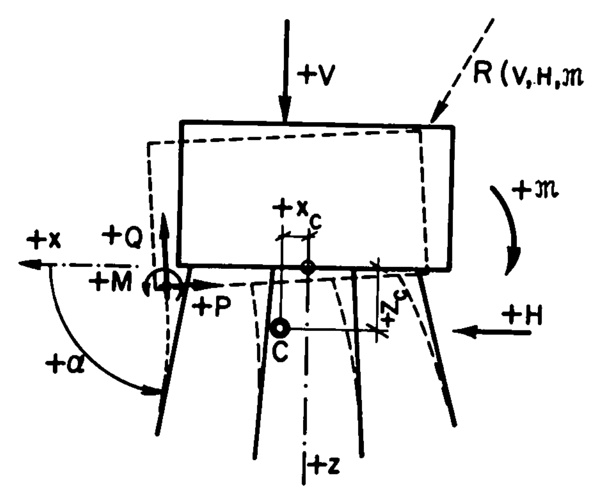


Figure 45. Problem of a pile foundation subjected to eccentric and inclined loads.

Once that assumption is made, the relationships between the displacement components (u, w, and θ) and pile reactions can be written in the following form (*155*):

$$
\begin{aligned}
Q &= k_n w \\
P &= k_t(u + t\,\theta) \\
M &= k_t t(u + \zeta t\,\theta)
\end{aligned}
\qquad (62)
$$

The meaning of coefficients k_n, k_t, t, and ζ is evident from

Figure 46 and Eq. 62. The first coefficient (k_n) represents simply the ratio of axial load to settlement; the second coefficient (k_t) represents the ratio of lateral load to lateral deflection in condition of pure translation of pile head (fixed-end condition). Both k_n and k_t appear in units such as kip/in. or kN/m; they are usually called coefficient of axial and lateral pile reaction, respectively. The characteristic length t represents the ratio of moment to lateral load in the "fixed-end" condition shown in Figure 46b, whereas the pure number ζ represents the ratio of moment to lateral load in the "pure-rotation" condition shown in Figure 46c divided by the characteristic length t.

The axial coefficient (k_n) can be determined from an axial load test or from computed pile settlement under a working load. The other three coefficients can be determined from lateral-load tests. It should be noted, however, that they can also be derived if the pile and soil properties are defined. Thus, if it is assumed that the pile-soil system is defined by a reaction modulus that increases with depth according to Eq. 50, it can be shown from Eqs. 51, 52 and 62 that:

$$\begin{aligned} k_t &= 1.075 n_h T^2 \\ t &= 0.926T \\ \zeta &= 1.62 \end{aligned} \tag{63}$$

If it is assumed, on the other hand, that the pile is a square bar of diameter B, having a modulus of elasticity E_p, and that the soil behaves as a linear, elastic solid defined by a modulus of deformation (E_s), the following expressions for a solid square or circular pile are obtained (*122, 135*)

$$\begin{aligned} k_t &= 0.60\, B\, E_s \sqrt[5]{\frac{E_p}{E_s}} \\ t &= 0.44\, B \sqrt[4]{\frac{E_p}{E_s}} \\ \zeta &= 1.88 \end{aligned} \tag{64}$$

In reality, Eq. 64 should be used with a reduced modulus (E_s), which can take into account plastic phenomena around the pile head (*159*). These expressions are, nevertheless, quite instructive. For example, they indicate that the lateral stiffness of a pile is directly proportional to its diameter and the soil stiffness, so that in a given soil the lateral stiffness of the pile can be increased most effectively by increasing the pile diameter. The expressions also show that the characteristic length (t), only slightly less than T, may vary in extreme cases from approximately 2 pile diameters for a wooden pile in extremely dense sand to as much as 10 pile diameters for a steel pile in very soft clay.*

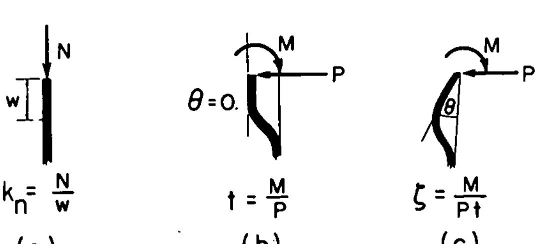


Figure 46. Definition of pile coefficients.

A relatively simple solution to this problem can be obtained for a plane problem and a rigid cap. For such a system, it can be shown (*155*) that there is point (C) (see Fig. 45) called center of the pile group such that the system experiences no rotation if the resultant of external forces (R) passes through it. The position of group center (C) can be found from geometrical and statical considerations.

Take any system with n equal-size piles, each axis of which makes an angle α with the x-axis located in the plane of the pile heads. Assume a coordinate center (O) anywhere on x-axis, with z-axis as shown in the figure. The coordinates of center C can then be found from

$$\begin{aligned} x_c &= \frac{M_2 S_1 - M_1 S_2}{S_1 S_3 - S_2^2} \\ z_c &= \frac{M_2 S_2 - M_1 S_3}{S_1 S_3 - S_2^2} \end{aligned} \tag{65}$$

in which S and M represent pile group constants defined as:

$$\begin{aligned} S_1 &= \Sigma(\cos^2\alpha + \sin^2\alpha) \\ S_2 &= (1 - \lambda)\Sigma\sin\alpha\cos\alpha \\ S_3 &= \Sigma(\sin^2\alpha + \lambda\cos^2\alpha) \\ M_1 &= (1 - \lambda)\Sigma x\sin\alpha\cos\alpha \\ M_2 &= \Sigma x(\sin^2\alpha + \lambda\cos^2\alpha) \end{aligned} \tag{66}$$

in which $\lambda = k_t/k_n$ represents the stiffness ratio of the piles.

Reducing the external load to the center C and denoting the components of that load by V, H, and $\mathcal{M}$ (positive as shown in Fig. 45), the pile reactions P, Q, and M can be computed from:

$$\begin{aligned} Q &= C_{qh}H + C_{qv}V + C_{qm}\mathcal{M} \\ P &= C_{ph}H + C_{pv}V + C_{pm}\mathcal{M} \\ M &= (C_{mh}H + C_{mv}V + C_{mm}\mathcal{M})t \end{aligned} \tag{67}$$

The coefficients in these equations represent the pile-reaction factors defined as:

$$\begin{aligned} C_{qh} &= \frac{S_3\cos\alpha - S_2\sin\alpha}{S_1 S_3 - S_2^2} \\ C_{qv} &= \frac{S_1\sin\alpha - S_2\cos\alpha}{S_1 S_3 - S_2^2} \\ C_{ph} &= \lambda\frac{S_3\sin\alpha + S_2\cos\alpha}{S_1 S_3 - S_2^2} \\ C_{pv} &= -\lambda\frac{S_1\cos\alpha + S_2\sin\alpha}{S_1 S_3 - S_2^2} \\ C_{qm} &= -\frac{k_n p}{\Sigma_m} \end{aligned} \tag{68}$$

* Eqs. 64 have been derived for solid square sections. They can be used for other sections by applying a correction to the pile modulus of elasticity to take care of the difference in moment of inertia. Thus for a circular section, a fictitious pile modulus equal to $0.589E_p$ should be introduced, changing the factors in Eq. 64 to 0.54 and 0.38, respectively. For steel H-piles, the factors are approximately 0.48 and 0.32.

$$C_{pm} = \frac{k_t(q + t)}{\Sigma_m}$$

$$C_{mm} = \frac{k_t(q + \xi t)}{\Sigma_m}$$

in which p represents the shortest distance between pile axis and the center of the group and q the distance between pile head and the point on pile axis closest to the center of the group (both positive as shown in Fig. 47). Σ_m is the sixth foundation constant defined by

$$\Sigma_m = \Sigma[k_n p^2 + k_t(q + t)^2 + k_t(\xi - 1)t^2] \quad (69)$$

The pile-reaction factors (Eq. 68) have been tabulated (*105*) for symmetrical as well as for unsymmetrical groups of vertical and batter piles. The tables available are made for two different batters (3 to 1 and 4 to 1) and for an arbitrary number of equidistant piles. The entire analysis can also easily be programmed for a computer.

Measurements of models and full-size pile groups (*109, 161, 149, 190*) show a reasonable agreement between observations and predictions by this relatively simple analysis. The only major departure observed is that the front pile row in the direction of bending receives a greater share of lateral load than the back row, although this theory predicts that both rows should carry the same lateral load.

In the simpler case of a group containing only vertical piles (Fig. 48), the group center C is on the axis of symmetry of the group at a distance $z_c = t$ from the center of coordinates O. The reactions of individual piles are:

$$Q = \frac{k_n}{\Sigma k_n} V - \frac{k_n x}{\Sigma_m} \mathcal{M}$$

$$P = \frac{k_t}{\Sigma k_t} H \quad (70)$$

$$M = \frac{k_t t}{\Sigma k_t} H + \frac{k_t(\xi - 1)t^2}{\Sigma_m} \mathcal{M}$$

in which x represents the distance from pile axis to pile center (positive as shown in Fig. 48).

The displacement components of the foundation are given by equations:

$$u = \frac{H}{\Sigma k_t} \qquad w = \frac{V}{\Sigma k_n} \qquad \theta = \frac{\mathcal{M}}{\Sigma_m} \quad (71)$$

in which the constant Σ_m (Eq. 69) is reduced to:

$$\Sigma_m = \Sigma[k_n x^2 + k_t(\xi - 1)t^2]$$

It should be noted that u and w need to be corrected for group action, unless this was done through reaction coefficients k_n and k_t.

The analysis of three-dimensional pile groups containing batter piles is based on similar principles (*185, 187, 188, 191*). However, in view of the complexity of the computations, a matrix formulation and solution by computer is preferred.

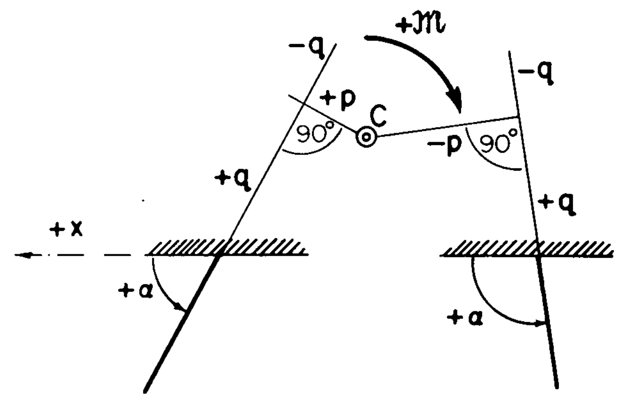


Figure 47. Sign convention for p *and* q.

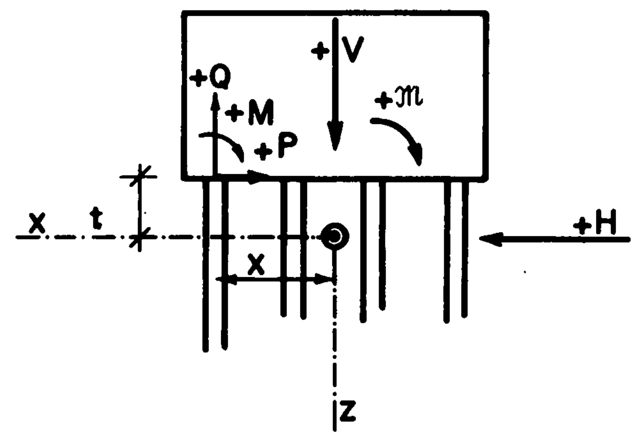


Figure 48. Group of vertical piles subjected to eccentric and inclined loads.

CHAPTER EIGHT

CONSTRUCTION PROBLEMS

DYNAMICS OF PILE DRIVING

Pile driving can be defined as the operation of forcing the pile into ground by dynamic means such as impact or vibration. In the most commonly used impact method, the pile is driven by means of a hammer containing a ram (Fig. 49a), which falls along two parallel guides (leads) on the pile head. If the ram weight (W_r) and the falling distance (stroke) (h) are properly selected, the pile experiences on each blow a permanent downward displacement (set)(s). A sequence of such blows can bring the pile to a desired elevation as long as there is sufficient driving energy to overcome the dynamic resistance (R) of the pile and the pile is not damaged or broken by excessive driving stresses.

The mechanics of operation of principal hammer types is shown in Figure 49. The ram of a drop-hammer (Fig. 49a) is raised by a winch and allowed to drop freely from a prescribed height. The modern versions of this ancient method operate at approximately 10 blows per minute. In the case of single-acting hammers (Fig. 49b), the ram is lifted by steam or air pressure, increasing the frequency of blows to approximately 60 per minute. Free-fall still remains the principal source of driving energy. In the case of double- and differential-acting hammers (Fig. 49c), the steam or air pressure is used also to accelerate the ram downward. In this way, both the impact velocity and the driving frequency are considerably increased. In diesel hammers (Fig. 49d), the ram fall compresses the air inside an enclosed cylinder. A properly timed fuel injection during the ram-anvil impact causes an explosion of the fuel-air mixture, which is used to lift the ram and also to give the pile an additional dynamic impulse. In some models, the cylinder may be closed-ended, in which case a "bounce-chamber" of compressed air on the upper end of the cylinder provides a double-acting effect. Hydraulic hammers, which are still not used extensively, operate on a system similar to steam or air hammers. However, oil at a high pressure of 5,000 psi (34.5 MPa) replaces steam, which operates at usual pressures of slightly more than 100 psi (689 kPa). Finally vibratory drivers (Fig. 49e) consist of a pair of counter-rotating eccentric weights whose horizontal impulses from centrifugal forces cancel, while vertical impulses add in producing a pulsating load of variable frequency. Principal characteristics of commonly used impact hammers are given in Table 7 and those of vibratory drivers in Table 8. An up-to-date discussion of their relative merits can be found in Reference *195*.

To soften the impact and prevent pile and hammer damage, the pile head may be covered by a steel helmet (cap), with a renewable cushion (capblock) made of materials such as wood, fiber, or plastics. As shown in Figure 50, another cushion may be placed between the helmet and the pile head. The presence of the helmet with cushions

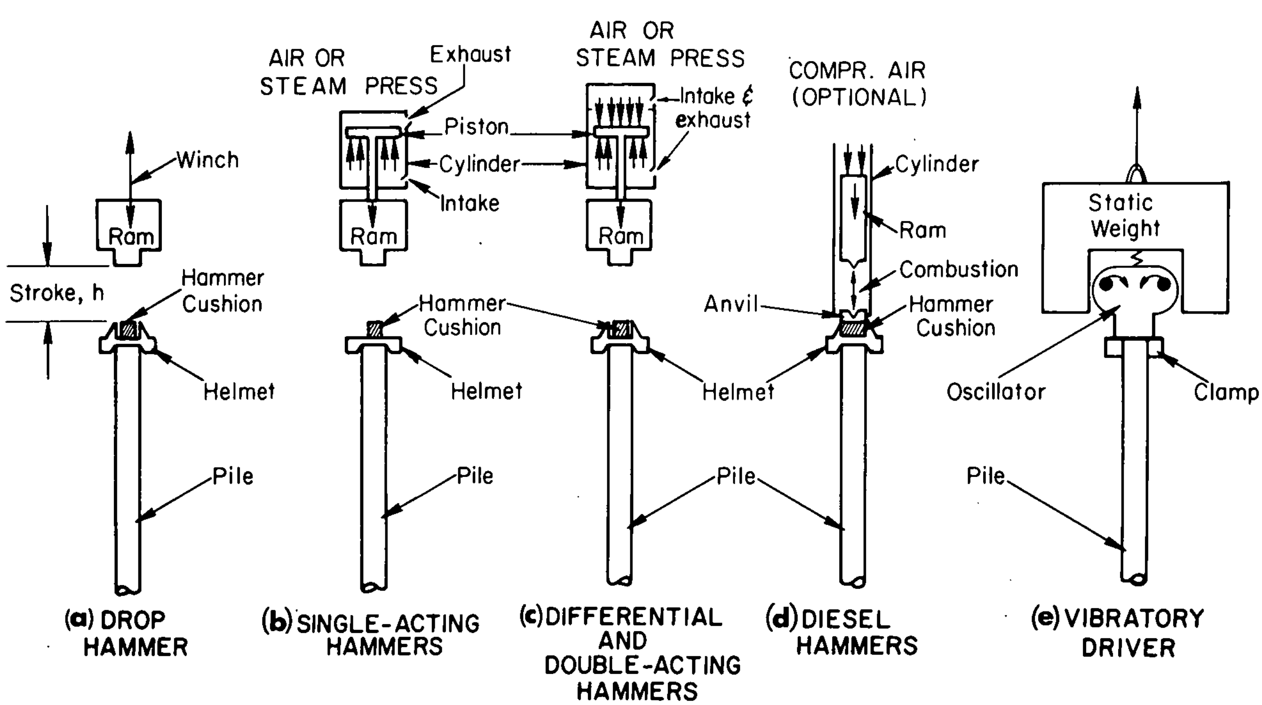


Figure 49. Principle of operation of pile drivers (18, 93).

TABLE 7

IMPACT PILE-DRIVER DATA

Rated Energy Kip-ft	Make of Hammer	Model No.	Type	Blows per min	Stroke at Rated Energy	Weight Striking Parts Kips.	Total Weight Kips.
180.0	Vulcan	060	S-A	62	36	60.0	121.0
130.0	MKT*	S-40	S-A	55	39	40.0	96.0
120.0	Vulcan	040	S-A	60	36	40.0	87.5
113.5	S-Vulcan	400C	Diff.	100	16.5	40.0	83.0
97.5	MKT	S-30	S-A	60	39	30.0	86.0
79.6	Kobe	K42	Dies.	52	98	9.2	22.0
60.0	Vulcan	020	S-A	60	36	20.0	39.0
60.0	MKT	S20	S-A	60	36	20.0	38.6
56.5	Kobe	K32	Dies.	52	98	7.0	15.4
50.2	S-Vulcan	200C	Diff.	98	15.5	20.0	39.0
48.7	Vulcan	016	S-A	60	36	16.2	30.2
48.7	Raymond	0000	S-A	46	39	15.0	23.0
44.5	Kobe	K22	Dies.	52	98	4.8	10.6
42.0	Vulcan	014	S-A	60	36	14.0	27.5
40.6	Raymond	000	S-A	50	39	12.5	21.0
39.8	Delmag	D-22	Dies.	52	n/a	4.8	10.0
37.5	MKT	S14	S-A	60	32	14.0	31.6
36.0	S-Vulcan	140C	Diff.	103	15.5	14.0	27.9
32.5	MKT	S10	S-A	55	39	10.0	22.2
32.5	Vulcan	010	S-A	50	39	10.0	18.7
32.5	Raymond	00	S-A	50	39	10.0	18.5
32.0	MKT	DE-40	Dies.	48	96	4.0	11.2
30.2	Vulcan	OR	S-A	50	39	9.3	16.7
26.3	Link-Belt	520	Dies.	82	43.2	5.0	12.5
26.0	MKT	C-8	D-A	81	20	8.0	18.7
26.0	Vulcan	08	S-A	50	39	8.0	16.7
26.0	MKT	S8	S-A	55	39	8.0	18.1
24.4	S-Vulcan	80C	Diff.	111	16.2	8.0	17.8
24.4	Vulcan	8M	Diff.	111	n/a	8.0	18.4
24.3	Vulcan	0	S-A	50	39	7.5	16.2
24.0	MKT	C-826	D-A	90	18	8.0	17.7
22.6	Delmag	D-12	Dies.	51	n/a	2.7	5.4
22.4	MKT	DE-30	Dies.	48	96	2.8	9.0
24.4	Kobe	K13	Dies.	52	98	2.8	6.4
19.8	Union	K13	D-A	110	24	3.0	14.5
19.8	MKT	11B3	D-A	95	19	5.0	14.5
19.5	Vulcan	06	S-A	60	36	6.5	11.2
19.2	S-Vulcan	65C	Diff.	117	15.5	6.5	14.8
18.2	Link-Belt	440	Dies.	88	36.9	4.0	10.3
16.2	MKT	S5	S-A	60	39	5.0	12.3
16.0	MKT	DE-20	Dies	48	96	2.0	6.3
16.0	MKT	C5	Comp.	110	18	5.0	11.8
15.1	S-Vulcan	50C	Diff.	120	15.5	5.0	11.7
15.1	Vulcan	5M	Diff.	120	15.5	5.0	12.9
15.0	Vulcan	1	S-A	60	36	5.0	10.1
15.0	Link-Belt	312	Dies.	100	30.9	3.8	10.3
13.1	MKT	10B3	D-A	105	19	3.0	10.6
12.7	Union	1	D-A	125	21	1.6	10.0
9.0	Delmag	D5	Dies.	51	n/a	1.1	2.4
9.0	MKT	C-3	D-A	130	16	3.0	8.5
9.0	MKT	S3	S-A	65	36	3.0	8.8
8.8	MKT	DE-10	Dies.	48	96	11.0	3.5
8.7	MKT	9B3	D-A	145	17	1.6	7.0
8.2	Union	1.5A	D-A	135	18	1.5	9.2
8.1	Link-Belt	180	Dies.	92	37.6	1.7	4.5
7.2	Vulcan	2	S-A	70	29.7	3.0	7.1
7.2	S-Vulcan	30C	Diff.	133	12.5	3.0	7.0
7.2	Vulcan	3M	Diff.	133	n/a	3.0	8.4
6.5	Link-Belt	105	Dies.	94	35.2	1.4	3.8
4.9	Vulcan	DGH900	Diff.	238	10	.9	5.0
3.6	Union	3	D-A	160	14	.7	4.7
3.6	MKT	7	D-A	225	9.5	.8	5.0
.4	Union	6	D-A	340	7	.1	.9
.4	Vulcan	DGH100A	Diff.	303	6	.1	.8
.4	MKT	3	D-A	400	5.7	.06	.7
.3	Union	7A	D-A	400	6	.08	.5

*Codes

MKT - McKiernan-Terry
S-Vulcan - Super-Vulcan
S-A - Single-Acting
D-A - Double-Acting
Diff. - Differential
Dies. - Diesel
Comp. - Compound

(After Parola, Ref. 193)

TABLE 8

COMPARISON OF VIBRATORY DRIVERS

Make	Model	Total Weight Kips	Available HP	Frequency range cps	Force*, Kips / Frequency cps
Foster	2-17	6.2	34	18-21	
(France)	2-35	9.1	70	14-19	62/19
	2-50	11.2	100	11-17	101/17
Menck	MVB22-30	4.8	50		48/
(Germany)	MVB65-30	2.0	7.5		14/
	MVB44-30	8.6	100		97/
Muller	MS-26	9.6	72		
(Germany)	MS-26D	16.1	145		
Uraga	VHD-1	8.4	40	16-20	43/20
(Japan)	VHD-2	11.9	80	16-20	86/20
	VHD-3	15.4	120	16-20	129/20
Bodine (USA)	B	22	1000	0-150	63/100-175/100
(Russia)	BT-5	2.9	37	42	48/42
	VPP-2	4.9	54	25	49/25
	100	4.0	37	13	44/13
	VP	11.0	80	6.7	35/7
	VP-4	25.9	208		198/

*Forces given are present maximums. These can usually be raised or lowered by changing weights in the oscillator.

(from Davisson, Ref. 194)

generally reduces driving efficiency; however, it can improve the shape of the dynamic impulse transmitted to the pile, making it more like a push than a sharp rap, and thus aiding the driving process. Typical deformation characteristics of some commonly used cushion materials are given in Table 9; a brief description of their features is given in Reference *195*.

A thorough understanding of the mechanics of pile driving is essential to the foundation engineer facing decisions on a piling site. Experience alone, however important, is not always sufficient to answer a series of questions such as:

(a) Can a given pile be driven to the design depth and capacity with a specific hammer considered for the particular job?

(b) If so, what will be the set in the final blows and what will be the maximum stress experienced by the pile? Should a hard or a soft cushion be used?

(c) If driving cannot be accomplished as specified under (a), what hammer characteristics are needed to perform the job with the pre-selected pile? Or, alternately, what other pile of the same dimensions can, perhaps, be driven with the first considered hammer.

To answer these and other questions that may appear in connection with a particular piling job, a rational analysis, based on the principles of dynamics of the pile-soil system may be useful. In some situations involving unusually large or new pile types in a new location such information might be considered even indispensable. The problem can be formulated as follows (Fig. 50): given a pile of a certain size and material characteristics that may be capped by a helmet with cushions. The pile is being driven into a soil mass of known properties by means of a ram of weight (W_r) falling over a stroke (h) with or without propelling forces other than gravity, so that it achieves an impact

TABLE 9

TYPICAL PILE CUSHION MATERIAL PROPERTIES

MATERIAL	SECANT MODULUS, E (PSI)	COEFFICIENT OF RESTITUTION, e [a]
Micarta plastic	450,000	0.80
Oak (green)	45,000 [b]	0.50
Asbestos discs	45,000	0.50
Plywood, fir	35,000 [b]	0.40
Plywood, pine	25,000 [b]	0.30
Gum	30,000 [b]	0.25

[a] $e = E_{load}/E_{unload}$ (From Ref. 211)
[b] Properties of wood with load applied perpendicular to wood grain.

velocity v_o. The stress-displacement-time relationships for selected points of the system are to be determined. Of special interest are the plastic displacement (set) of the pile head and its correlation with ultimate dynamic resistance of the pile (R), as well as the maximum compressive and tensile stresses in the pile caused by driving.

The following simplifying assumptions are usually made in attempting to solve this complex problem:

(a) The helmet and the pile are assumed to behave as elastic solids, defined by their elasticity moduli E_h and E_p, respectively. The cushions are also assumed to behave as elastic solids, sometimes with bilinear moduli, however.

(b) The soil resistance to pile penetration is simulated by means of interface elements as shown in Figure 51, which include elastic, plastic, and viscous dynamic response of the soil.

(c) The cross-section of the pile (A) is assumed to be small in comparison with its length (L), so that the pile can be considered as an elastic bar. It can be shown (*196, 197*) that the pile displacement (w) for any pile point at the depth z in function of time t is governed by the following second-order partial-differential equation

$$AE_p \frac{\partial^2 w}{\partial z^2} - R(w,z,t) = \rho_p A \frac{\partial^2 w}{\partial t^2} \tag{72}$$

in which ρ_p represents the mass density of the pile, equal to γ_p/g in which g is the acceleration of gravity. R is the mentioned response function of the surrounding soil, which can but does not have to be assumed in the simplified form shown in Figure 51.

Closed-form solutions of this equation can be obtained if $R = 0$, in which case Eq. 72 becomes

$$c^2 \frac{\partial^2 w}{\partial z^2} = \frac{\partial^2 w}{\partial t^2} \tag{73}$$

in which $c = \sqrt{E_p/\rho_p}$ represents the velocity of wave propagation through the bar. This velocity should be distinguished from the particle velocity v, which is related to the stress σ produced by the wave by the expression

$$v = \frac{c\sigma}{E_p} \tag{74}$$

The product ρcA, called pile impedance, can be used as a measure of force that can be transmitted through a pile by a dynamic impulse. The higher the impedance of a particular pile, the greater is its potential of overcoming soil resistance to driving and of developing a high bearing capacity. This is shown in Figure 52, taken from Reference *193*. That study, using an analog computer solution of Eq. 73, found the following optimum range of impedances for any given hammer-cushion combination

$$\rho_p cA = (0.60 \text{ to } 1.10)\sqrt{\frac{W_r K}{g}} \tag{75}$$

in which K represents the cushion stiffness, defined as $A_c E_c/h_c$, in which A_c, E_c, and h_c are, respectively, the cross-sectional area, modulus of elasticity, and thickness of the cushion. A comparison of impedances for some common pile types of about the same exterior dimensions is presented in Table 10.

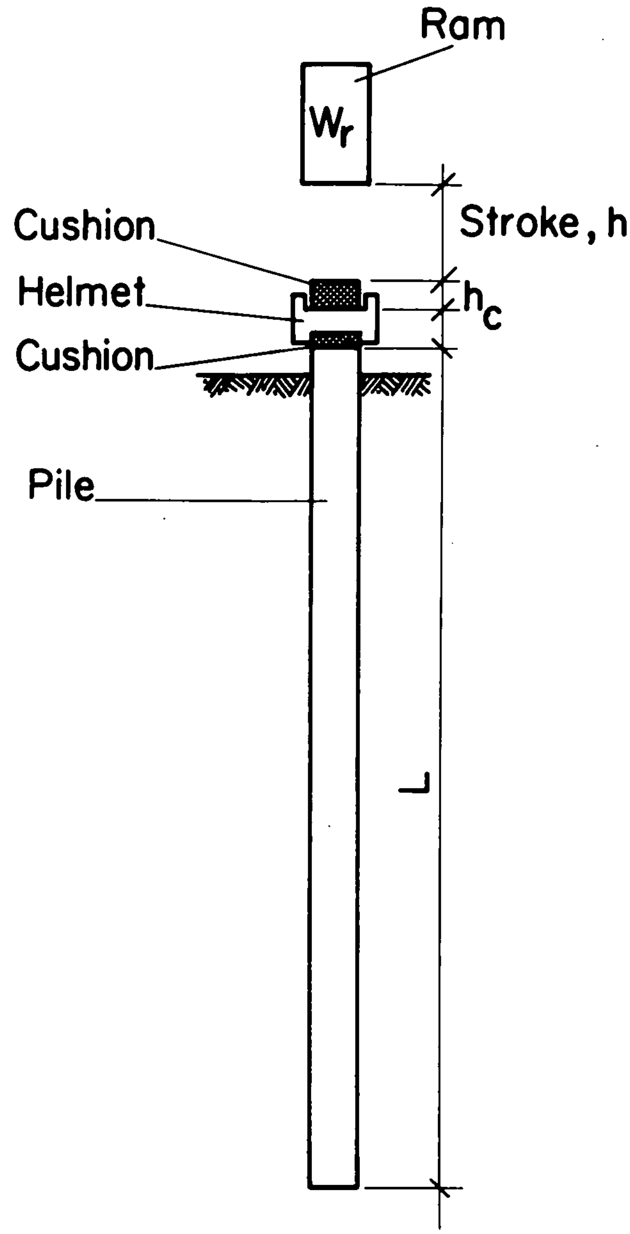


Figure 50. Problem of pile driving.

It is not difficult to show that the maximum stress in a long pile occurs at the impact point. Thus, for an unprotected, long pile, the maximum compressive driving stress should be equal to:

$$\sigma_{max} = \frac{E_p v_o}{c} \tag{76}$$

in which $v_o = \sqrt{2gh}$ represents the ram impact velocity. For a pile protected by a cushion, the maximum compressive stress becomes equal to (*198, 199*):

$$\sigma_{max} = \frac{K_c v_o e^{-nt}}{A\sqrt{p^2 - n^2}} \sin\left(t\sqrt{p^2 - n^2}\right) \tag{77}$$

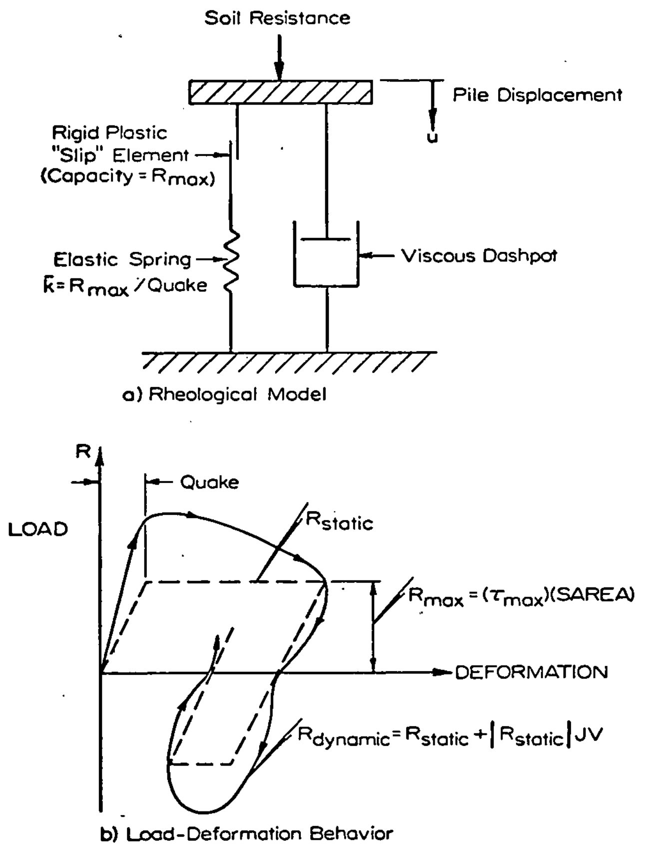


Figure 51. Rheological model of soil resistance at pile-soil interface.

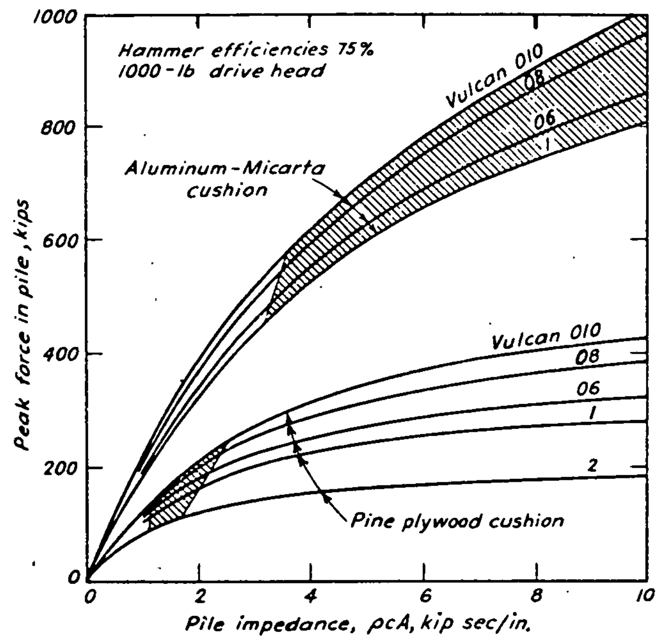


Figure 52. Relationship between peak driving force and pile impedance for Vulcan SA hammers. Shaded area indicates conditions of maximum transmission of driving energy (173).

TABLE 10

STRESS-TRANSMISSION CHARACTERISTICS OF TYPICAL PILES

Pile Type	Unit Weight γ_p (lb/ft^3)	Mass Density $\rho_p = \frac{\gamma_p}{g}$ (lb sec^2/ft^4)	Stress Wave Velocity c (ft/sec)	Area A (in.^{2})	Impedance $\rho_p cA$ (lb sec/in.)
Wood					
10-in. diameter kiln dry	40	1.24	13,600	78.5	768
10-in. diameter treated southern pine	60	1.86	10,600	78.5	898
Concrete	150	4.66	11,100		
10-in. diameter				78.5	2360
20-in. diameter				314.2	9410
Steel	490	15.2	16,900		
HP 10 X 57				16.76	2500
HP 12 X 53				15.58	2430
HP 14 X 117				34.44	5370
10-3/4 X 0.188 pipe				6.24	928
10-3/4 X 0.279 pipe				9.18	1440
10-3/4 X 0.365 pipe				11.91	1770
10-3/4 X 0.188 pipe with steel mandrel 160 lb/ft				53.30	7930
Steel/concrete					
10-3/4 X 0.279 pipe filled with concrete	185	5.76	12,100	87.9	3550

(After Peck, Hanson, Thornburn; Ref. 18) in which A represents, as before, the cross-sectional area of the pile and K_c the cushion stiffness. Quantities p and n are two parameters given by $p = \sqrt{K_c g / W_r}$ and $n = (K_c/2A)\sqrt{g/E_p \gamma_p}$ and t is a characteristic time, to be found from the expression:

$$\tan\,(t\sqrt{p^2 - n^2}) = (\sqrt{p^2 - n^2})/n \tag{78}$$

Equations 77 and 78 are valid as long as $n < p$. For the less frequent case in which $n > p$, they are to be replaced by:

$$\sigma_{max} = \frac{K_c v_0 e^{-nt}}{A\sqrt{n^2 - p^2}} \sinh\,(t\sqrt{n^2 - p^2}) \tag{77a}$$

$$\tanh\,(t\sqrt{n^2 - p^2}) = (\sqrt{n^2 - p^2})/n \tag{78a}$$

It can be further shown (*197*) that, if the pile has a finite length and meets complete refusal, the maximum stresses occur at pile tip and can be, theoretically, twice as great as indicated by Eqs. 76 or 77. If, however, a bar of finite length meets no resistance on its free end, the compressive wave is reflected from that end as a tension wave. The stress conditions in this situation depend on the relative length of the pile (L) with respect to the stress wave length (L_s), which can be computed from

$$L_s = \frac{c\pi}{\sqrt{p^2 - n^2}} \tag{79}$$

The maximum tensile stress, assuming no damping in either the pile or the soil, would be equal to the maximum compressive stress as long as $L/L_s > 0.5$. For shorter piles, $\sigma_{t\,max}$ is given by the approximate expression (*199*):

$$\sigma_{t\,max} = 8\,\sigma_{c\,max}\,(L/L_s)^3 \tag{80}$$

This analysis presents an idea of maximum tensile stresses theoretically possible under conditions of very soft driving, in which the pile tip offers little resistance to penetration. If these stresses are not compensated by concurrent compressive stresses, net tensions may develop and cause cracking in concrete piles. Tensile stresses may also appear in very hard driving if the high-compression wave reflected from the tip hits the pile head after contact with the ram has been lost. (For additional information, see Refs. *197* and *199*.)

The solutions to Eq. 73, although referring to highly idealized conditions, present an insight into the relative importance of principal parameters that affect the stresses in a pile during driving. They show that both compressive and tensile stresses in piles can be effectively reduced by using a heavier ram with lower impact velocity. These stresses can also be reduced by the use of cushions that can transform the short, sharp impulse conveyed by a hammer blow into a more modulated, longer and lower stress-peak impulse. In soft-driving conditions, it may be necessary, at least when driving concrete piles, to reduce the hammer stroke and impact velocity in order to avoid damaging tensile stresses.

All detailed studies point out the importance of the soil resistance term R in Eq. 72 for accurate predictions of pile behavior during driving. The presence of this term in the wave equation, even in the simplest form, makes rigorous solutions to Eq. 72 virtually impossible. However, convenient numerical solutions can always be found by using a discrete-element approach [first proposed by Smith (*196*)] in which the pile is modelled as a series of mass and spring elements (Fig. 53), while the soil support is simulated by a series of interface rheological elements shown in Figure 51. The analysis has been programmed for a digital computer by a group of researchers at Texas A&M University, and the programs are widely used in engineering practice (*200, 201*). A typical solution can furnish the axial force or axial stress caused by a blow at any point of the pile as a function of elapsed time (Fig. 54), as well as a relationship between the pile set (s) (or blow count $1/s$) and dynamic resistance of the pile (R) (Fig. 55). Parametric studies to investigate the effects of different variables on pile capacity or driving ability are also routinely available.

A significant improvement of the original Smith analysis and Texas A&M computer program has been developed recently at Duke University (*93*). The resulting DUKFOR program offers a multiple-blow solution, which allows the consideration of residual stresses left in the pile after each blow in analysis of the subsequent blow. The analysis also allows the introduction of more versatile, nonlinear stress-transfer models at the pile-soil interface (see Fig. 32). In addition to the usual output, the DUKFOR program furnishes the residual stresses in the pile after driving, which, as explained in earlier sections, have a significant effect on load-transfer and pile-settlement computations.

One major uncertainty left in all analyses of this kind is often in the actual energy transmitted by the hammer to the soil-pile system, which is entered into the computer program as an input. Field measurements have shown that the average hammer efficiency, defined as the ratio of actual energy delivered to nominal energy quoted by the manufacturer, is often considerably less than conventionally as-

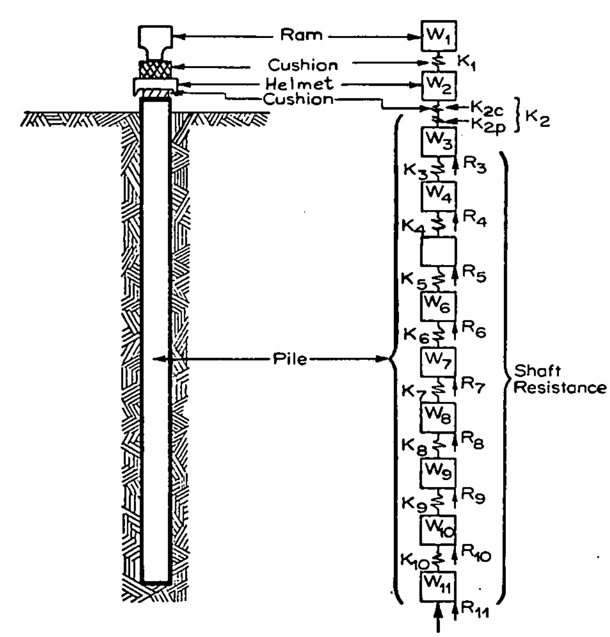


Figure 53. Discrete-element model of the pile-soil system (196).

sumed. In a thorough field study of this problem performed by the Michigan State Highway Commission (*204*), average efficiencies of only 25 to 65 percent for steam hammers and 45 to 65 percent for diesel hammers were reported. A somewhat higher average of 73 percent for diesel hammers has been reported in another situation (*205*). Moreover, considerable delivered energy variation, from blow to blow, has been consistently reported (*204, 206*)—in one case as high as ±70 percent from the average.

This uncertainty can be circumvented by direct measurement of the dynamic force transmitted by the hammer impact. Recent developments in instrumentation have made such measurements possible under field conditions, although they require highly specialized personnel. Such measurements, combined with measurement of acceleration of pile head under impact, form the basis for an alternate method of analysis of dynamic response of piles developed at Case-Western Reserve University (*207-209*). Using pile and soil models similar to those introduced by Smith (*196*) (Figs. 51 and 53), this method computes the pile resistance (R) from measured input force and displacement at the pile head. The measured signals are directly transmitted to a field computer, which displays the result immediately following the measurement. The method has some attractive features, although it requires highly sophisticated electronic instrumentation and specialized personnel on the site.

A common weakness of all known methods of analysis of pile driving by wave equation lies in the manner in which the soil resistance to pile penetration is modelled. In conventional pile-soil models (Fig. 53), the soil reaction is represented by a series of independent rheological interface elements, which offer only a crude approximation of actual soil response. As described in earlier sections of this synthesis, the soil reacts to pile driving as a two- or three-phase mass in which the reactions of different pile-soil-interface elements are interdependent and related to reactions of soil elements located as far as several diameters from the soil-pile interface. There is also complete disregard for the nature of effective shear strength developed at the interface. Thus, phenomena such as increase and decrease of driving resistance caused by driving interruptions (known among practitioners as "freeze" and "relaxation") remain outside the reach of the presently known methods of analysis.

In spite of these shortcomings, the wave-equation approach represents a valuable tool in pile foundation analysis that is far superior to the conventional "pile-formula" approach, which considers the pile as a rigid mass experiencing a motion caused by Newtonian impact of a ram of weight W_r. The energy delivered in a blow is generally expressed as $\eta W_r h$, in which η is the efficiency and h the stroke of the hammer. This energy can be equated with the sum of energy spent in displacing the pile over a distance (set) (s) against the soil resistance (R) and the energy lost in elastic rebound of the pile, soil, helmet, and cushion, as well as in plastic deformations and heat on contact surfaces. The lost energy can be expressed as Rc, in which c is a constant, having the dimension of length. Thus,

$$R = \frac{\eta W_r h}{s + c} \tag{81}$$

This is the well-known, general form of practically all pile formulas, a good summary of which can be found in Reference *19*. For example, when $\eta = 1$ (correct for drop

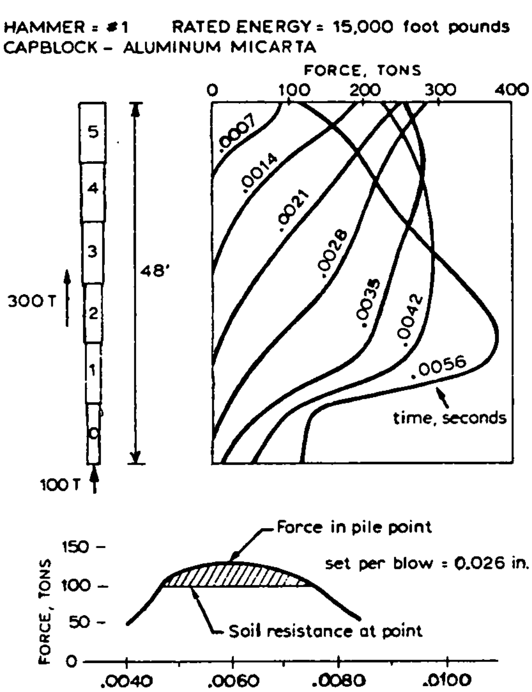


Figure 54. Typical result of wave-equation analysis of driving stresses (202).

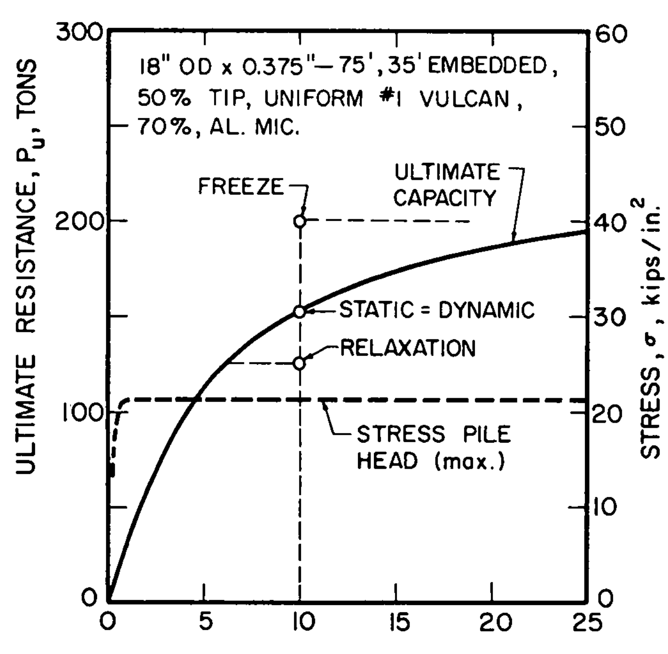


Figure 55. Typical result of wave-equation analysis of pile resistance (203).

hammers) and c = 1 in. (25 mm) (approximately correct for average timber piles in sand), the old *Engineering News* formula is obtained. Replacing c with 0.1 in. (2.5 mm) the revised *Engineering News* formula, which was developed for single-acting steam hammers is obtained. A more recent Danish formula (*210*), which shows relatively good statistical correlation for piles in sand, replaces c with:

$$c = \sqrt{\frac{\eta W_r h L}{2AE_p}} \tag{82}$$

However, these and other formulas of a similar nature suffer from the same fundamental weakness—a disregard of the true nature of dynamic stress transmission on hammer-pile impact. They should not be used as a substitute for static analysis of bearing capacity or dynamic analysis of pile driveability. Nevertheless, observing the shapes of actual blow count–pile resistance relationships (Fig. 55), it should be clear that formulas of the simplest form (Eq. 81) can represent a good approximation of such complex relationships. Indeed, it has been known for a long time that there exists, for many particular sites involving defined soil conditions, driving equipment, and pile type, a single value of c that can provide good correlation between observed sets (s) and pile resistances (R). Once this value is known, the formula works well, as long as no variable in the problem is changed. Thus, the simple Eq. 81 or its variants can continue to be useful in construction control, particularly on smaller jobs.

LOAD TESTING OF PILES

In view of the many uncertainties involved in analysis of pile foundations, it has become customary, and in many cases mandatory, to perform a certain number of full-scale pile load tests at the site of more important projects. The main purpose of these tests is to verify experimentally that the actual pile response to load, as reflected in its load-displacement relationships, corresponds to the response assumed by the designer and that the actual ultimate load of the pile is not less than the computed ultimate load used as a basis for foundation design. In some instances comparative tests are made on several different pile types to select one that best satisfies the requirements for a particular project. Such conventional tests include the axial compressive- and axial tensile-load tests and the lateral load tests.

Typical arrangements for an axial compressive-load test are shown in Figures 56 and 57. In Figure 56, the reaction of the hydraulic jack used to load the pile is transmitted by a reaction beam to a pair of anchor piles. In Figure 57 the hydraulic jack reacts against a dead weight, such as a water-filled tank or ballast of pig iron, rails, bricks, concrete blocks, sand or gravel, stacked on a platform. A combination of the two methods of taking the reaction has also been used (*39*). The load transmitted by the jack should preferably be measured by a load cell inserted between the jack and the reaction beam or platform. Using the measured pressure of the jack fluid (as often was done in the past) as the sole measurement of force is not sufficiently accurate because of ram friction (*213*). The settlements of a pile head are usually measured directly by micrometer dial gages supported by reference beams. An alternate method is to use reference wires under constant tension supported far enough from the pile to be unaffected by soil displacements caused by the test loads. It is considered good practice to always check those measurements by a leveling instrument, such as a conventional level or a laser beam. Settlements of other points on the pile axis, useful in determination of load-transfer characteristics of the pile, can also be measured directly by means of strain rods (tell-tales), which have to be built in the test pile prior to driving. Details of strain-rod arrangements for a steel and a concrete pile are shown in Fig. 58. The information provided by strain-rod measurements can be vital for understanding the performance of a particular pile; it should be routinely included in pile load testing whenever possible, at least for the pile tip.

To minimize the interference with the test pile, the

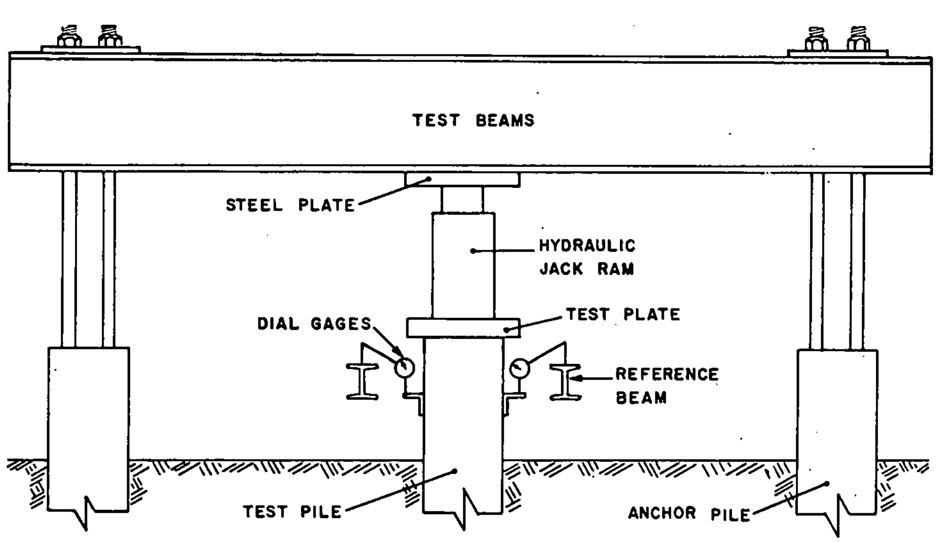


Figure 56. Typical setup for pile load testing in axial compression using anchor piles (**214**).

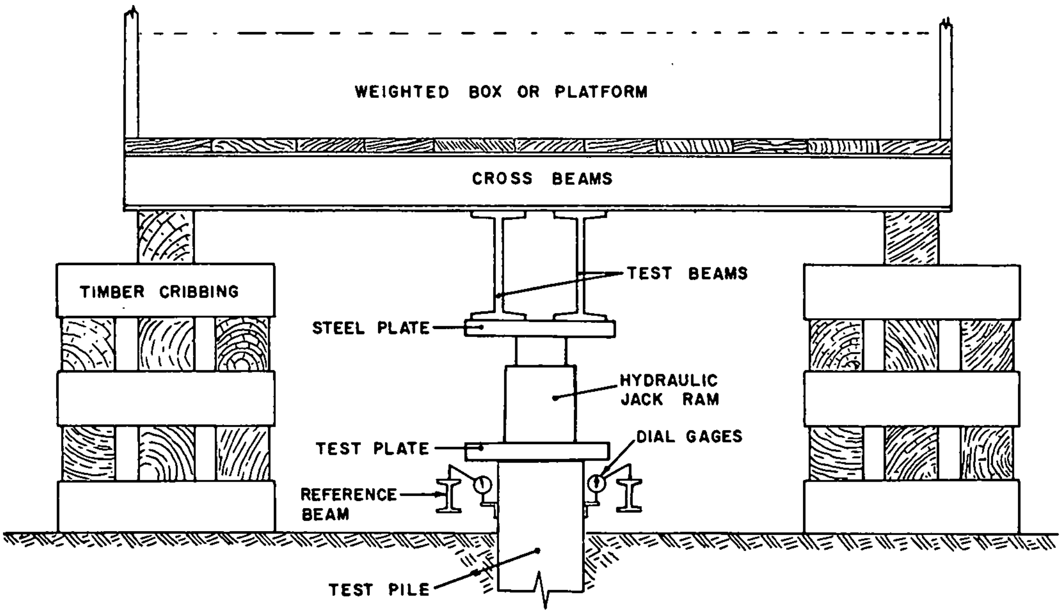


Figure 57. Typical setup for pile load testing in axial compression using a loading platform (214).

reaction piles should be kept at a reasonable distance. ASTM standards (*214*) specify this distance as a minimum of 5 pile diameters; however, theoretical studies (*215*) indicate that this may not always be sufficient and recommend 10 pile diameters as the minimum distance. The latter distance is also found in some more recent testing specifications (*216*). Under otherwise identical conditions, a somewhat smaller distance could be allowed for deeply seated ground anchors or for the supports of a reaction platform. Theoretical estimates of errors that may be introduced by using reaction piles and anchors at shorter distances can be found in Reference *215*.

Typical arrangement for a tension-load test are shown in Figures 59 and 60. The jack reactions in this case can always be transmitted directly to the ground across timber crib mats or concrete blocks, which must be of adequate size to avoid bearing capacity failure of the underlying soil. The distance between the mats and the test pile should be such that the pile remain outside of the zone of significant stress influence of the mats. The mats can be replaced by reaction piles, in which case the minimum-distance rules set for compressive-load tests apply. The force and displacement measuring devices are basically the same as those used in compressive testing.

A typical arrangement for a lateral-load test is shown in Fig. 61. Jacking is usually done against a reaction pile or against the cap of a completed pile group. The use of a load cell for force measurement is strongly recommended. The horizontal displacement and rotation of the pile head are measured by means of micrometer dial gages supported by a reference beam. To measure rotations at different points along the pile axis, inclinometers can be used, at least with steel piles. To minimize the interference with the test pile, the reaction pile should preferably be at a distance of 10 pile diameters.

Once the experimental setup for pile testing is completed, the test is executed according to a prescribed loading procedure. Two principal modes of load application can be used. In the load-controlled (maintained-load) mode, the load is applied in increments of, perhaps, 25 percent of the design load and maintained at each loading stage until the pile settlements cease or reach a specified small rate. In the displacement-controlled mode, small increments of settlement of 1 percent of the pile diameter are imposed on the pile and maintained until the load reaches equilibrium or a specified small rate of decrease with time. A simplified version of this mode in which the settlement increments are imposed in specified time intervals (e.g., 15 or 30 minutes) is called the controlled-rate-of-penetration mode.

If the loading arrangements are such that the pile is set into continuous movement at a constant speed, while the force is measured, the pile is loaded in what is called the constant-rate-of-penetration mode. Typically constant rates between 0.01 and 0.10 in./min (0.25 to 2.5 mm/min) have been recommended for piles in different soil types. Most prescribed loading procedures include one or several unload cycles, which allow the determination of permanent (plastic) deformation under given load. It is recommended that provisions be made to carry the test until the ultimated load of the pile is obtained; however, most common standards require that the test be carried to only twice the design load.

The relative merits of the conventional load-controlled testing mode, as opposed to the relatively newer controlled- or constant-rate-of-penetration mode, have been discussed in the literature. (See, for example, *217* and *218*.) It may be argued that the load-controlled mode better simulates the actual conditions under which most foundation piles approach failure. That mode also allows a better estimate of deformation characteristics of the pile under short-term loading conditions. However, comparative studies (*217*,

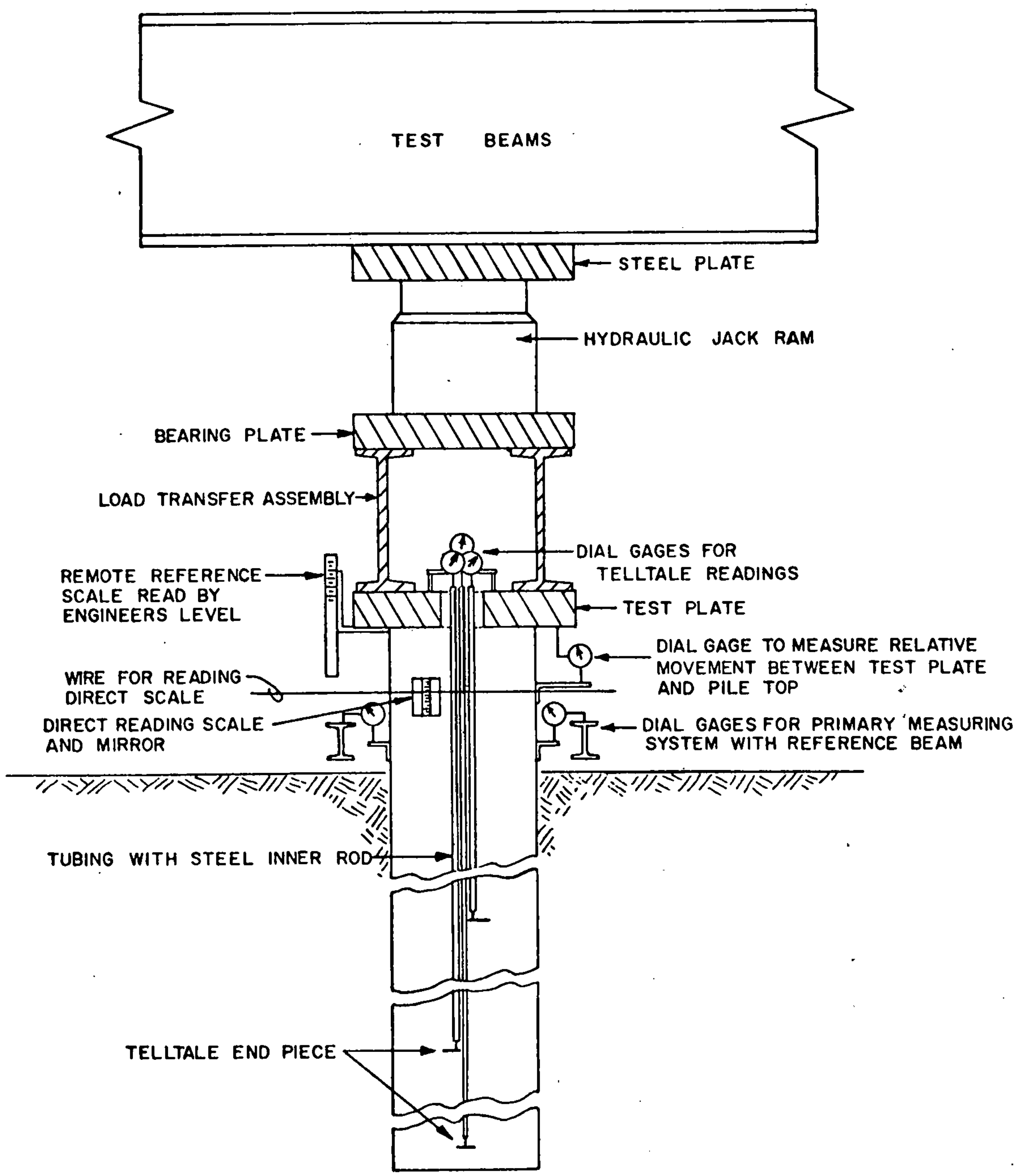


Figure 58. Typical setup for measurement of pile displacements (214).

218) show that the ultimate loads determined by the two methods are practically the same. It is also well established that the conventional load-testing method, however slow and time-consuming, does not give information about long-term settlements of the pile. Thus, the distinct advantages of the controlled-rate-of-penetration mode (simplicity and considerable time saving) make that method more attractive for general use in engineering practice. Its recent introduction in ASTM standards as an acceptable alternative to the load-controlled method will allow its wider application.

It is important to note here that the response of a driven pile in a load test can be greatly affected by the time elapsed between driving and testing. As explained in earlier sections, the gain in pile-bearing capacity with time is governed by the rate of dissipation of pore-water stresses through the surrounding soil mass, which, in turn, depends on time factor given by Eq. 12. Most existing codes prescribe a minimum waiting period between driving and testing not exceeding one month. Although this requirement may be adequate for piles in relatively pervious soils, such as sands and inorganic silts, it is obviously not sufficient for piles in clay, particularly if they are of larger size (cf. Fig. 13). Because it may be impractical to prescribe longer waiting periods, an estimate of additional gains in bearing capacity between pile testing and application of service loads may be in order. Equally important at all times is to keep in mind the differences in over-all behavior and load response of a single pile as compared with that of a pile in a group. Consequently, load testing of piles can never be used as a substitute for an engineering analysis of the pile's anticipated performance under load. Such an analysis should be based on principles discussed in this synthesis and must include an adequate soil exploration and testing program.

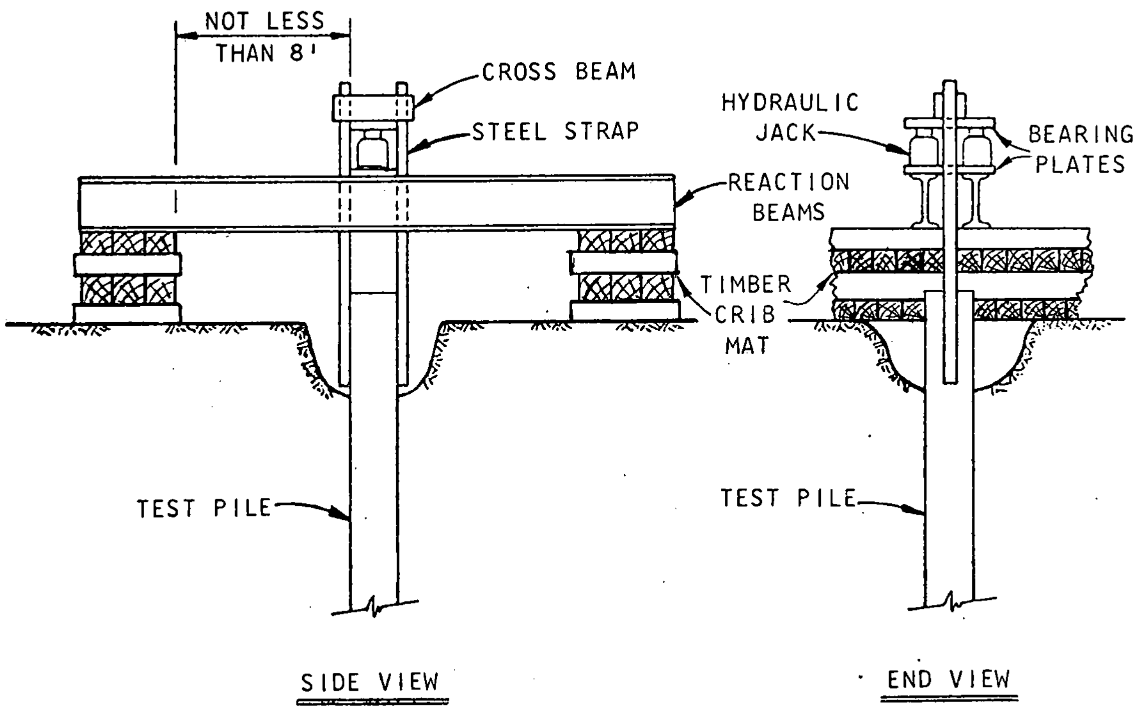


Figure 59. Typical setup for pile load testing in tension using direct jacking with straps (214).

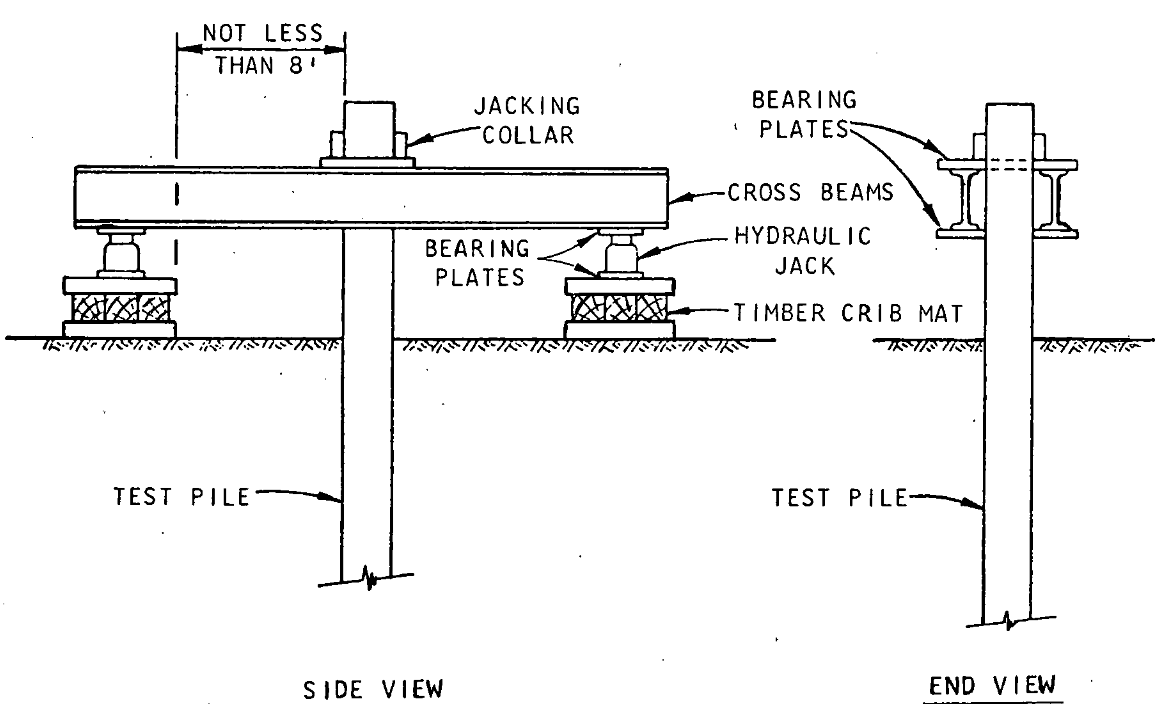


Figure 60. Typical setup for pile load testing in tension using cross-beams (214).

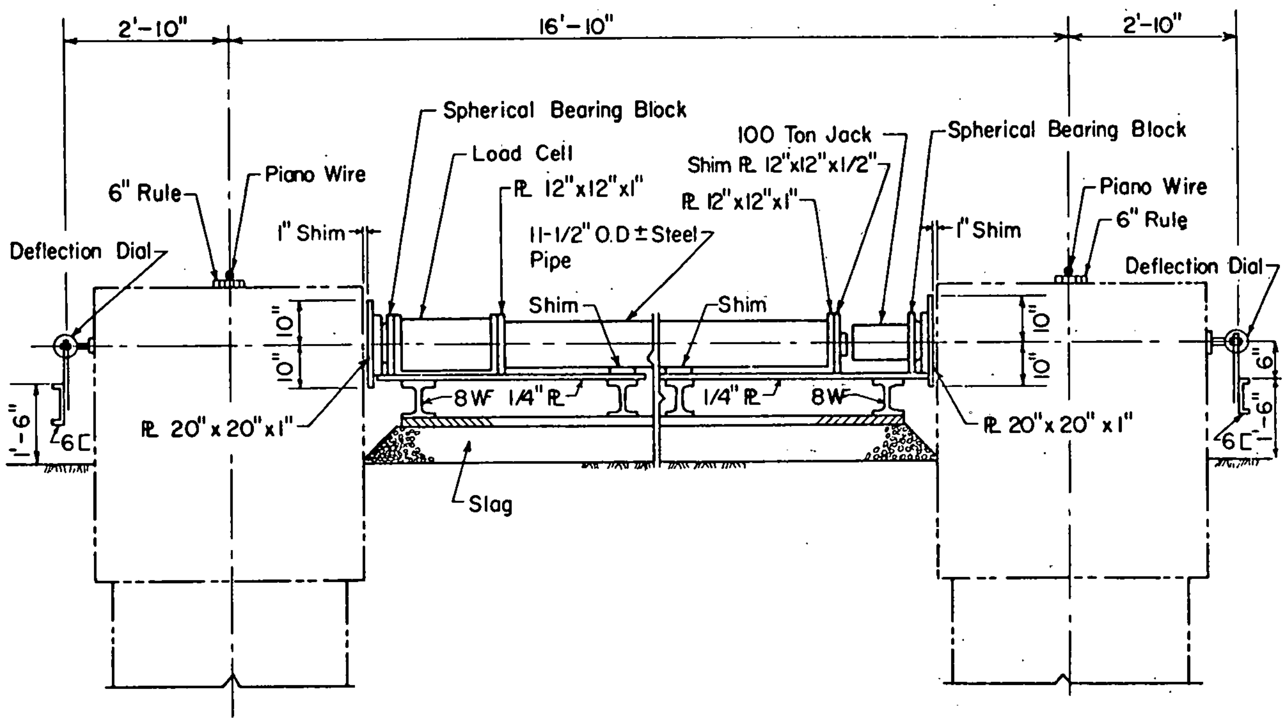


Figure 61. Typical setup for lateral load tests (145).

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AUTHOR INDEX *

* Entries refer to numerical sequence in Reference List.

APPENDIX A

ANALYSIS OF POINT RESISTANCE

Based on geometry and boundary conditions shown in Figure 7 of this synthesis, an approximate value of N_σ can be found if the average normal stress along the ring *BD* is assumed to be equal to the ultimate pressure needed to expand a spherical cavity in an infinite soil mass. That mass may be assumed to behave as an ideal elastic-plastic solid, characterized by already defined strength parameters c, ϕ, deformation parameters E (modulus of deformation) and μ (Poisson's ratio), as well as by a volume change parameter, Δ, representing average volumetric strain in the plastic zone III surrounding the cavity (*36*). Computations based on these assumptions yield for N_σ the expression

$$N_\sigma = \frac{3}{3 - \sin\phi} e^{\left(\frac{\pi}{2}-\phi\right)\tan\phi} \tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right) I_{rr}^{\frac{4\sin\phi}{3(+\sin\phi)}} \quad \text{(A-1)}$$

in which, according to Reference *36*, I_{rr} represents the reduced rigidity index of the soil

$$I_{rr} = \frac{I_r}{1 + I_r\Delta} \quad \text{(A-2)}$$

which in conditions of no volume change (undrained conditions) or little volume change (dense strata) can be taken equal to the rigidity index I_r, given by expression

$$I_r = \frac{E}{2(1+\nu)(c+\bar{q}\tan\phi)} = \frac{G}{c+\bar{q}\tan\phi} \quad \text{(A-3)}$$

The magnitude of N_c^* can be found from Eq. 4 and Eq. A-1. It can be shown that for a frictionless soil ($\phi = 0$), these expressions yield:

$$N_c^* = \frac{4}{3}(\ln I_{rr} + 1) + \frac{\pi}{2} + 1 \quad \text{(A-4)}$$

Numerical values of N_σ and N_c^* after Eqs. A-1 and A-4, respectively, are given in Table A-1. Typical values of rigidity index of some known soils are given in Table A-2.

TABLE A-1

BEARING CAPACITY FACTORS FOR DEEP FOUNDATIONS[b]

ϕ \ I_r	10	20	40	60	80	100	200	300	400	500
0	6.97	7.90	8.82	9.36	9.75	10.04	10.97	11.51	11.89	12.19
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	7.34	8.37	9.42	10.04	10.49	10.83	11.92	12.57	13.03	13.39
	1.13	1.15	1.16	1.18	1.18	1.19	1.21	1.22	1.23	1.23
2	7.72	8.87	10.06	10.77	11.28	11.69	12.96	13.73	14.28	14.71
	1.27	1.31	1.35	1.38	1.39	1.41	1.45	1.48	1.50	1.51
3	8.12	9.40	10.74	11.55	12.14	12.61	14.10	15.00	15.66	16.18
	1.43	1.49	1.56	1.61	1.64	1.66	1.74	1.79	1.82	1.85
4	8.54	9.96	11.47	12.40	13.07	13.61	15.34	16.40	17.18	17.80
	1.60	1.70	1.80	1.87	1.91	1.95	2.07	2.15	2.20	2.24
5	8.99	10.56	12.25	13.30	14.07	14.69	16.69	17.94	18.86	19.59
	1.79	1.92	2.07	2.16	2.23	2.28	2.46	2.57	2.65	2.71
6	9.45	11.19	13.08	14.26	15.14	15.85	18.17	19.62	20.70	21.56
	1.99	2.18	2.37	2.50	2.59	2.67	2.91	3.06	3.18	3.27
7	9.94	11.85	13.96	15.30	16.30	17.10	19.77	12.46	22.71	23.73
	2.22	2.46	2.71	2.88	3.00	3.10	3.43	3.63	3.79	3.91
8	10.45	12.55	14.90	16.41	17.54	18.45	21.51	23.46	24.93	26.11
	2.47	2.76	3.09	3.31	3.46	3.59	4.02	4.30	4.50	4.67
9	10.99	13.29	15.91	17.59	18.87	19.90	23.39	25.64	27.35	28.73
	2.74	3.11	3.52	3.79	3.99	4.15	4.70	5.06	5.33	5.55
10	11.55	14.08	16.97	18.86	20.29	21.46	25.43	28.02	29.99	31.59
	3.04	3.48	3.99	4.32	4.58	4.78	5.48	5.94	6.29	6.57
11	12.14	14.90	18.10	20.20	21.81	23.13	27.64	30.61	32.87	34.73
	3.36	3.90	4.52	4.93	5.24	5.50	6.37	6.95	7.39	7.75
12	12.76	15.77	19.30	21.64	23.44	24.92	30.03	33.41	36.02	38.16
	3.71	4.35	5.10	5.60	5.98	6.30	7.38	8.10	8.66	9.11
13	13.41	16.69	20.57	23.17	25.18	26.84	32.60	36.46	39.44	41.89
	4.09	4.85	5.75	6.35	6.81	7.20	8.53	9.42	10.10	10.67
14	14.08	17.65	21.92	24.80	27.04	28.89	35.38	39.75	43.15	45.96
	4.51	5.40	6.47	7.18	7.74	8.20	9.82	10.91	11.76	12.46
15	14.79	18.66	23.35	26.53	29.02	31.08	38.37	43.32	47.18	50.39
	4.96	6.00	7.26	8.11	8.78	9.33	11.28	12.61	13.64	14.50
16	15.53	19.73	24.86	28.37	31.13	33.43	41.58	47.17	51.55	55.20
	5.45	6.66	8.13	9.14	9.93	10.58	12.92	14.53	15.78	16.83
17	16.30	20.85	26.46	30.33	33.37	35.92	45.04	51.32	56.27	60.42
	5.98	7.37	9.09	10.27	11.20	11.98	14.77	16.69	18.20	19.47
18	17.11	22.03	28.15	32.40	35.76	38.59	48.74	55.80	61.38	66.07
	6.56	8.16	10.15	11.53	12.62	13.54	16.84	19.13	20.94	22.47
19	17.95	23.26	29.93	34.59	38.30	41.42	52.71	60.61	66.89	72.18
	7.18	9.01	11.31	12.91	14.19	15.26	19.15	21.87	24.03	25.85
20	18.83	24.56	31.81	36.92	40.99	44.43	56.97	65.79	72.82	78.78
	7.85	9.94	12.58	14.44	15.92	17.17	21.73	24.94	27.51	29.67
21	19.75	25.92	33.80	39.38	43.85	47.64	61.51	71.34	79.22	85.90
	8.58	10.95	13.97	16.12	17.83	19.29	24.61	28.39	31.41	33.97
22	20.71	27.35	35.89	41.98	46.88	51.04	66.37	77.30	86.09	93.57
	9.37	12.05	15.50	17.96	19.94	21.62	27.82	32.23	35.78	38.81
23	21.71	28.84	38.09	44.73	50.08	54.66	71.56	83.68	93.47	101.83
	10.21	13.24	17.17	19.99	22.26	24.20	31.37	36.52	40.68	44.22
24	22.75	30.41	40.41	47.63	53.48	58.49	77.09	90.51	101.39	110.70
	11.13	14.54	18.99	22.21	24.81	27.04	35.32	41.30	46.14	50.29
25	23.84	32.05	42.85	50.69	57.07	62.54	82.98	97.81	109.88	120.23
	12.12	15.95	20.98	24.64	27.61	30.16	39.70	46.61	52.24	57.06

ϕ \ I_r	10	20	40	60	80	100	200	300	400	500
26	24.98	33.77	45.42	53.93	60.87	66.84	89.25	105.61	118.96	130.44
	13.18	17.47	23.15	27.30	30.69	33.60	44.53	52.51	59.02	64.62
27	26.16	35.57	48.13	57.34	64.88	71.39	95.02	113.92	128.67	141.39
	14.33	19.12	25.52	30.21	34.06	37.37	49.88	59.05	66.56	73.04
28	27.40	37.45	50.96	60.93	69.12	76.20	103.01	122.79	139.04	153.10
	15.57	20.91	28.10	33.40	37.75	41.51	55.77	66.29	74.93	82.40
29	28.69	39.42	53.95	64.71	73.58	81.28	110.54	132.23	150.11	165.61
	16.90	22.85	30.90	36.87	41.79	46.05	62.27	74.30	84.21	92.80
30	30.03	41.49	57.08	68.69	78.30	86.64	118.53	142.27	161.91	178.98
	18.24	24.95	33.95	40.66	46.21	51.02	69.43	83.14	94.48	104.33
31	31.43	43.64	60.37	72.88	83.27	92.31	126.99	152.95	174.49	193.23
	19.88	27.22	37.27	44.79	51.03	56.46	77.31	92.90	105.84	117.11
32	32.89	45.90	63.82	77.29	88.50	98.28	135.96	164.29	187.87	208.43
	21.55	29.68	40.88	49.30	56.30	62.41	85.96	103.66	118.39	131.24
33	34.41	48.26	67.44	81.92	94.01	104.58	145.46	176.33	202.09	224.62
	23.34	32.34	44.80	54.20	62.05	68.92	95.46	115.51	132.24	146.87
34	35.99	50.72	71.24	86.80	99.82	111.22	155.51	189.11	217.21	241.84
	25.28	35.21	49.05	59.54	68.33	76.02	105.90	128.55	147.51	164.12
35	37.65	53.30	75.22	91.91	105.92	118.22	166.14	202.64	233.27	260.15
	27.36	38.32	53.67	65.36	75.17	83.78	117.33	142.89	164.33	183.16
36	39.37	55.99	79.39	97.29	112.34	125.59	177.38	216.98	250.30	279.60
	29.60	41.68	58.68	71.69	82.62	92.24	129.87	158.65	182.85	204.14
37	41.17	58.81	83.77	102.94	119.10	133.34	189.25	232.17	268.36	300.26
	32.02	45.31	64.13	78.57	90.75	101.48	143.61	175.95	203.23	227.26
38	43.04	61.75	88.36	108.86	126.20	141.50	201.78	248.23	287.50	322.17
	34.63	49.24	70.03	86.05	99.60	111.56	158.65	194.94	225.62	252.71
39	44.99	64.83	93.17	115.09	133.66	150.09	215.01	265.23	307.78	345.41
	37.44	53.50	76.45	94.20	109.24	122.54	175.11	215.78	250.23	280.71
40	47.03	68.04	98.21	121.62	141.51	159.13	228.97	283.19	329.24	370.04
	40.47	58.10	83.40	103.05	119.74	134.52	193.13	238.62	277.26	311.50
41	49.16	71.41	103.49	128.48	149.75	168.63	243.69	302.17	351.95	396.12
	43.74	63.07	90.96	112.68	131.18	147.59	212.84	263.67	306.94	345.34
42	51.38	74.92	109.02	135.68	158.41	178.62	259.22	322.22	375.97	423.74
	47.27	68.46	99.16	123.16	143.64	161.83	234.40	291.13	339.52	382.53
43	53.70	78.60	114.82	143.23	167.51	189.13	275.59	343.40	401.36	452.96
	51.08	74.30	108.08	134.56	157.21	177.36	257.99	321.22	375.28	423.39
44	56.13	82.45	120.91	151.16	177.07	200.17	292.85	365.75	428.21	483.88
	55.20	80.62	117.76	146.97	172.00	194.31	283.80	354.20	414.51	468.28
45	58.66	86.48	127.28	159.48	187.12	211.79	311.04	389.35	456.57	516.58
	59.66	87.48	128.28	160.48	188.12	212.79	312.03	390.35	457.57	517.58
46	61.30	90.70	133.97	168.22	197.67	224.00	330.20	414.26	486.54	551.16
	64.48	94.92	139.73	175.20	205.70	232.96	342.94	429.98	504.82	571.74
47	64.07	95.12	140.99	177.40	208.77	236.85	350.41	440.54	518.20	587.72
	69.71	103.00	152.19	191.24	224.88	254.99	376.77	473.42	556.70	631.25
48	66.97	99.75	148.35	187.04	220.43	250.36	371.70	468.28	551.64	626.36
	75.38	111.78	165.76	208.73	245.81	279.06	413.82	521.08	613.65	696.64
49	70.01	104.60	156.09	197.17	232.70	264.58	394.15	497.56	586.96	667.21
	81.54	121.33	180.56	227.82	268.69	305.37	454.42	573.38	676.22	768.53
50	73.19	109.70	164.21	207.83	245.60	279.55	417.82	528.46	624.28	710.39
	88.23	131.73	196.70	248.68	293.70	334.15	498.94	630.80	744.99	847.61

[a] Upper number N_σ^*, lower number N_c.

TABLE A-2

TYPICAL VALUES OF RIGIDITY INDEX, I_r

(a) sands and silts

Soil	Relative density D_r	Mean Normal stress level σ_o (kg/cm^2) (psf)	Rigidity index I_r	Source
Chattahoochee sand	80%	0.1 205 1 2048 10 20480 100 204800	200 118 52 12	Vesić and Clough (1968)
	20%	0.1 1	140 85	
Ottawa sand	82% 21%	0.05 0.05	265 89	Roy (1966)
Piedmont silts		0.70	10-30	Vesić (1972)

(b) clays (undrained conditions)

Soil	Plasticity index I_p	Water content	OC ratio	Effective stress level σ_o (kg/cm^2)	Rigidity index	Source
Weald clay	25	23.1% 22.5%	1 24	2.1 0.35	99 10	Ladanyi (1963)
Drammen clay	19	24.9% 25.1% 27.2%	1	1.5 2.5 4.0	267 259 233	
Lagunillas clay	50	65%*	1	6.5 4.0	390 300	

*prior to consolidation

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