DESIGN PROCEDURE FOR DRILLED CONCRETE PIERS IN EXPANSIVE SOIL

by
The Structural Committee

of
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PREFACE

The mission of the Foundation Performance Association (FPA) may be found at www.foundationperformance.org. To help accomplish the "documents" portion of the mission, the Structural Committee was formed in 1999 for the purpose of assembling the information available in the industry on a selected subject, and compiling it into a document, which is then made available to the public.

This document was written by the Structural Committee's FPA-SC-16-0 ad hoc subcommittee and was submitted to peer review by the Foundation Performance Association's (FPA's) entire membership and other selected professionals in the industry who are known to have expertise in the subject. This document is published as FPA-SC-16 Revision 0 (i.e., FPA-SC-16-0) on 17 November 2017 and is made freely available to the public at www.foundationperformance.org so all may have access to the information. To help ensure this document remains as current as possible, it may be periodically updated under the same document number but with higher revision numbers such at 1, 2, etc.

The Structural Committee is a standing committee of the Foundation Performance Association. When this document was written the Structural Committee was chaired by Ron Kelm, P.E., and 50 to 60 members were active on the committee. The Structural Committee sanctioned this project on 28 March 2012, formed an ad hoc subcommittee to write this document with Nicole Wylie, P.E., as chair and provided oversight reviews of the subcommittee throughout this document's development, peer review and publication. The subcommittee's chair and members are listed on the cover sheet of this document and are considered this document's co-authors.

Future suggestions for improvement of this document should be directed to the current chair of the Structural Committee. If comments sufficient to warrant a revision are received, the Structural Committee may form a new subcommittee to revise this document. If the revised document passes the Structural Committee's oversight review and the FPA's peer review, it will be published on the FPA website, superseding the previous revision.

The subcommittee also authored a Microsoft Excel spreadsheet to accompany this document. This spreadsheet software was developed to help the subcommittee vet this design procedure; it was not subjected to the FPA's peer review procedure; a software draft was made available to peer reviewers to augment this document's FPA Peer Review and attain debugging of the software. The software contains the same example calculations made by calculator and contained in this document, though some software results may be slightly different than the calculator results due to rounding of the greater precision solutions offered by the software. The software is provided at no cost as a courtesy to FPA members at www.foundationperformance.org with no guarantee of its accuracy.

> NOTE: In the event of a conflict between this document and the software, this document takes precedence. The software has not been subjected to the FPA peer review process. If "bugs" are encountered in the software, please provide that information to the current Structural Committee chair. The Structural Committee may

opt to revise the software and upload a new version with a later revision date without the need for a subsequent ad hoc subcommittee or FPA peer review.

This document is based on experience gathered by consultants working primarily in the southeast Texas area. The intended audiences for the use of this document are geotechnical engineers, foundation design engineers and other engineers involved in the design or analysis of drilled concrete pier foundations located in areas of the United States with expansive soil.

Special thanks from his fellow subcommittee members and co-authors go to Robert L. Lytton, PhD, PE who spent countless personal hours developing the expansive soil portion of this procedure and commuting between College Station TX and Houston TX to meet with the subcommittee. After multiple iterations, Dr. Lytton developed what the subcommittee believes to be a reasonably accurate suction-based procedure for designing drilled concrete piers in expansive soil without the need for actual suction, hydrometer or swell geotechnical test data.

This document was created with generously donated time in an effort to advance the knowledge, performance, and standards of engineering, construction, and repairs related to foundations, soils, and structures. The text in this document represents the opinions of a majority of the subcommittee members and may not necessarily reflect the opinions of every subcommittee member, Structural Committee member or FPA member at the time of, or since, this document's publication. The FPA and its members make no warranty regarding the accuracy of the information contained herein and will not be liable for any damages, including consequential damages resulting from the use of this document. Each project should be investigated for its individual characteristics in order to determine the appropriate application of the information contained herein.

Please refer to the FPA's website at www.foundationperformance.org for other information pertaining to this publication and other FPA publications.

NOMENCLATURE

Symbol	Units	Description
A	in^2	pier shaft cross section area divided by n_r
A_{bar}	in^2	area of one steel reinforcing bar [Eq. 38]
A_c	in^2	pier shaft cross section area; gross concrete area
AG_{max}	in	maximum specified aggregate size in concrete; the bar clearance between longitudinal reinforcing bars must be less than $3*AG_{max}$
A_{stACT}	in^2	actual area of steel reinforcing, per bar or per pier [Eq. 37]
A_{stREQ}	in^2	required area of steel reinforcing [Eq. 36]
B		base of pier shaft, used as a subscript
cover	in	Thickness of concrete between pier perimeter and tie rebar (see Figure 2-12). A minimum 3 inch cover is recommended for drilled pier shafts. If a cover less than 3 inches is used with a permanent casing, the alpha and beta methods used (see Section 2.4.2) in this procedure may no longer apply.
c_w	in	crack width [Eqs. 27 & 28]
D	in	bell diameter, equal to the pier shaft diameter, d, if there is no bell
d	in	pier shaft diameter
d_{bar}	in	longitudinal steel reinforcing bar diameter [Eqs. 29 & 39]
d_c	in	radial distance from centroid of longitudinal steel reinforcing bar to exterior face of concrete [Eq. 29]
d_{tie}	in	diameter of tie steel reinforcing around longitudinal steel [Eq. 29]
f_s	ksi	steel reinforcing allowable stress [Eq. 29]
f_y	ksi	steel reinforcing yield stress
grade		soil elevation at the time the geotechnical soil testing was performed
GWT	ft	depth from grade to groundwater table
GWT_{high}	ft	depth from grade to highest groundwater table
GWT_{low}	ft	depth from grade to lowest groundwater table
K_o		coefficient of lateral earth pressure at rest [Eq. 4-A]
K_p		coefficient of horizontal soil stress [Eq. 17]
L	ft	length of pier (below grade)
L_A	ft	length of anchor zone
LL	%	Liquid Limit
m		correlation factor for cohesionless soil [Eq. 18]
n		exponent used in computation of y [Eq. 4-B]

Symbol	Units	Description
N_C		bearing capacity factor, function of s_u [Eq. 9]
n_r		number of steel reinforcing bars used [Eq. 32]
n_{r-max}		maximum number of steel reinforcing bars permitted based on AG_{max} [Eq. 34]
n_{r-min}		minimum number of steel reinforcing bars required [Eq. 33]
N_{60}		cohesionless soil blow count for an efficiency of 60%
pF		unit of suction; $\log_{10}$(|suction in cm of water|) [Eq. 3]
pF_{dry}	pF	dry boundary condition of suction [Eq. 2-B]
pF_{eq}	pF	equilibrium condition of suction [Eq. 1]
pF_{wet}	pF	wet boundary condition of suction [Eq. 2-A]
PI		Plasticity Index: Liquid Limit (LL) minus Plastic Limit (PL)
PL	%	Plastic Limit
q	psf, ksf, tsf	denotes a unit load in the direction of pier movement; a subscript indicates the source of the load
q_B	psf, ksf, tsf	unit base load in the direction of pier movement
q_S	psf, ksf	unit side load in the direction of pier movement
Q	lbs, kips	denotes a load in the direction of pier movement; a subscript indicates the source of the load
Q_B	lbs, kips	base load in the direction of pier movement
Q_S	lbs, kips	side load in the direction of pier movement
Q_T	lbs, kips	top of pier load in the direction of pier movement
Q_W	lbs, kips	pier weight for the downward movement case; a load
r	psf, ksf	denotes a unit resistance opposite the direction of pier movement; a subscript indicates the source of the resistance
r_B	psf, ksf	unit base resistance opposite the direction of pier movement
r_{bell}		ratio of bell diameter to shaft diameter, D/d
r_S	psf, ksf	unit side resistance opposite the direction of pier movement
R	lbs, kips	denotes a resistance opposite the direction of pier movement; a subscript indicates the source of the resistance
R_B	lbs, kips	base resistance opposite the direction of pier movement
R_S	lbs, kips	side resistance opposite the direction of pier movement
R_T	lbs, kips	top of pier load in the opposite direction of pier movement, a resistance
R_W	lbs, kips	pier weight for the upward movement case; a resistance

Symbol	Units	Description
RF_S		side resistance factor used to adjust the side resistance/load in cases such as the use of slurry or a permanent casing installation; default is 1.0; a value <1.0 reduces the soil side resistance/load [Eqs. 14 & 21]
S		side of pier shaft, used as a subscript
S.F.		safety factor applied to the resistance or load, see Section 2.5
$S.F._B$		safety factor applied to the base resistance, see Section 2.5
$S.F._S$		safety factor applied to the side resistance or load, see Section 2.5
SPT		Standard Penetration Test
s_u	psf, tsf	average undrained shear strength of a cohesive soil; if actual test pressure values are reported instead of shear strengths, common conversions to obtain s_u are one-half (1/2) the reported unconfined compression test value and one-third (1/3) the reported hand penetrometer test value. For other types of shear tests, contact the geotechnical engineer for a conversion factor.
T	kips	pier tension used in reinforcing calculation; computed from upward case at ZML [Eq. 35]
TMI		Thornthwaite Moisture Index, see Figures 2-4 and 2-5
U	pF	a measure of suction, see Section 2.3
$U_{wet}(y)$	pF	suction; wet boundary condition at depth y [Eq. 5]
$U_{wet\text{-}differential}$	pF	suction differential between the wet boundary condition and equilibrium [Eq. 2-A]
$U_{dry}(y)$	pF	suction; dry boundary condition at depth y [Eq. 6]
$U_{dry\text{-}differential}$	pF	suction differential between the dry boundary condition and equilibrium [Eq. 2-B]
w	%	gravimetric moisture (water) content
y	m	depth below grade at which to establish wet and dry suction boundary conditions [Eq. 4]
z	ft	soil layer penetration depth, used in the summation in Section 2.4.2 with subscripts “i” and “n”, i.e. the length from grade to bottom of soil layer [Eq. 21]
Z_a	ft	depth of movement active zone
Z_i	m	depth, measured from top of grade, typically 0.8m (2.6 ft)
Z_m	ft	depth of moisture active zone [Eq. 7]
$Z_{m\text{-}max}$	ft	maximum user specified depth limit of moisture active zone per criteria specified in Section 2.1.1
$Z_{m\text{-}min}$	ft	minimum user specified depth limit of moisture active zone per criteria specified in Section 2.1.1
ZML		zero movement line; dividing line at the depth between the movement active zone and the anchor zone, see Figure 2-1

Symbol	Units	Description
Greek Letters and Other Symbols:		
#		reinforcing bar size, represents eighths of an inch [Eq. 40]
α_{diff}	$\frac{cm^2}{sec}$	variable in $U_{wet}(y)$ and $U_{dry}(y)$, 0.003 $\frac{cm^2}{sec}$ or 0.015 $\frac{cm^2}{sec}$ [Eq. 6]
α		coefficient from the alpha method relating unit side resistance to undrained shear strength in cohesive soil calculations [Eq. 15]
β		side resistance coefficient from the beta method, used in cohesionless soil calculations [Eq. 20]
γ_d	pcf	dry unit weight of soil
γ_t	pcf	total unit weight of soil [Eq. 21]
γ_t'	pcf	effective unit weight of soil below water table [Eq. 21]
λ		variable; a function of Liquid Limit and used in computing depth, y [Eq. 4]
ρ	%	100 x total pier longitudinal reinforcing steel cross section area divided by pier shaft cross section area [Eq. 41]
σ_p'	psf	effective vertical preconsolidation stress [Eqs. 18 & 19]
σ_{soil}'	psf	effective soil pressure [Eq. 21]
$\sigma_{surcharge}$	psf	surcharge pressure, used to represent unit weight above the pier top such as a foundation, a non-modeled soil layer or other permanent dead load pressure [Eq. 21]
σ_v'	psf	average vertical effective stress [Eq. 21]
ϕ'	deg	soil friction angle, can be computed as a function of PI [Eq. 4-B] for cohesive soil and of N_{60} [Eq. 16] for cohesionless soil

1.0 INTRODUCTION

Local foundation design engineers seldom receive geotechnical investigation reports that follow the minimum requirements of FPA-SC-04-0[1], *Recommended Practice for Geotechnical Explorations and Reports*. The data obtained when following FPA-SC-04-0 is needed to accurately anchor concrete piers in expansive soil against subsidence and heave due to shrinkage and expansion of soil in the active zone. This FPA-SC-16-0 document presents an alternate procedure for both foundation design engineers and geotechnical engineers to use in designing depths of drilled concrete piers in expansive soil on projects where the geotechnical investigation report does not contain suction and hydrometer testing and other FPA-SC-04-0 recommended data.

1.1 INPUT DATA NEEDED TO USE THIS DESIGN PROCEDURE

Data from the geotechnical investigation report required to most accurately use this procedure include:

- Atterberg limits, LL and PL
- soil undrained shear strength, s_u
- moisture (water) content, w
- soil dry unit weight, γ_d
- knowledge of tree and other large vegetation growth on site, past and present, including maximum root depth
- water table (minimum and maximum) depths, GWT
- approximate depth of the moisture active zone, Z_m
- minimum bell ratio to avoid sloughing, r_{bell}

Data required to use this procedure from sources other than the geotechnical investigation report include:

- Thornthwaite Moisture Index, TMI
- local wet and dry suction boundaries, pF_{wet} and pF_{dry}
- maximum vertical loads at pier top for upward and downward cases, Q_T or R_T

If there are cohesionless layers within the depth of the pier, the following input data are required to use this procedure in addition to the above:

- expected surcharge at top of pier, $\sigma_{surcharge}$
- cohesionless soil SPT blow count for an efficiency of 60%, N_{60}
- correlation factor for cohesionless soil, m

[1] Foundation Performance Association. *Recommended Practice for Geotechnical Explorations and Reports*. Document No. FPA-SC-04-0. 2011.

If the engineer is also using this procedure to design the pier reinforcing steel, the following additional input data are required:

- longitudinal steel reinforcing bar yield stress, f_y
- longitudinal steel reinforcing bar allowable stress, f_s
- maximum specified aggregate size in concrete, AG_{max}
- longitudinal steel reinforcing bar diameter, d_{bar}
- diameter of steel reinforcing tie around longitudinal steel, d_{tie}

1.2 RECOMMENDATIONS FOR USING THIS DESIGN PROCEDURE

While the procedure is straightforward, the equations are complex. Therefore, in addition to this document, there is corresponding software to facilitate achieving an optimum pier design. This software has been published in a protected form with the intent that it is to be used by FPA members only.

The subcommittee developed this procedure to be used to design lightly loaded piers with shaft diameter (d) of 30 inches or less that have the potential to heave or subside in active soil. The user may adapt the procedure for use with inactive soil or inactive soil layers and benefit from its side and base design resistance calculations. In the case of piers subjected to net uplift loads, the user may adapt the procedure for use with inactive soil or inactive soil layers and benefit from its pier reinforcing design calculations. This procedure may not fit all scenarios; engineering judgment and prudence is required for its use.

This procedure goes a step beyond the typical methods of pier design since it accounts for the difference between the soil moisture contents at the time the pier is installed and the time the soil investigation was performed. As an example, if a soil investigation is made prior to a drought, the pier parameters given in the geotechnical investigation report may not be accurate for the site conditions if the pier is later installed during the drought. Therefore this procedure accounts for the environmental site conditions at the time of pier installation. This was accomplished by using a common boundary condition for suction at the ground surface for maximum dry and wet conditions. The surface suction boundary conditions recommended in this document can be used in nearly all of the United States. This is discussed in more detail in Sect. 2.3, Step 2.

The depth of the movement active zone (Z_a) computed using this procedure was compared to the moisture active zone (Z_m) determined from the depth of constant suction found in 13 Southeast Texas geotechnical reports that contained suction test data. Good correlation was found between the reported Z_m and computed Z_a for these cases. See Section 4.1 for more details.

Pier depths computed using this procedure were compared to pier depths recommended in 23 recent geotechnical reports for sites across Texas with expansive soil. Each geotechnical report was by a unique geotechnical firm. When the “No Tree” case was considered, this procedure’s computed pier depth was similar to the reports’ recommended pier depth and was on average 1.6 feet deeper than reported. For the “Tree” case, as expected, the calculated pier depth was considerably deeper than reported since most local geotechnical engineers do not currently properly account for the presence of trees and other large vegetation, past or future, when specifying pier depths in expansive soil.

Forensic investigations have shown that some homeowners/builders who added piers to a new slab on ground foundation developed abnormal heave problems where adjacent homes with similar foundations without piers had none. For these problem cases the piers were founded in or just below the bottom of an active stratum and provided a direct heave load path to the slab on ground foundation whereas the adjacent foundation without piers did not experience abnormal heave. For this condition, the foundation design engineer may consider omitting the piers and instead add more stiffness to the slab on ground foundation. If the client wants piers regardless, the engineer should ensure that the piers penetrate sufficiently below the deep active strata to properly anchor them against heave.

Forensic investigations have shown that some homeowners and building owners who structurally isolated their foundations suffered performance failures when the supporting piers heaved along with the soil surface. The subcommittee believes the main use of this procedure will be to design drilled concrete piers in shallow fat clays, such that the lightly loaded slab and grade beams are isolated from the heaving surface clays.

Piers founded in the active zone, or not sufficiently embedded in the anchor zone can allow foundation heave. If piers are to be used, determine the depth of the active zone using suction, swell testing or the procedure presented in this paper, then design the piers to a sufficient depth below the active zone to anchor against the upward side loads in the active zone.

The subcommittee found in developing and using this procedure that bells offer little resistance in the upward cases and should not be used unless the downward cases require bells for added bearing capacity. The user cannot rely on both base resistance and side resistance at the same time in lightly loaded foundations that are prone to heave. Base resistance becomes fully engaged at 5 to 10 times the vertical movement that fully engages the side resistance, i.e. if a vertical movement of 0.5” fully engages the side resistance, the base resistance is not fully engaged until 2.5” to 5” of vertical movement has occurred (see Section 2.1). It does not take much vertical movement to cause objectionable superstructure distress. For this and other reasons, the subcommittee found that piers supporting lightly loaded isolated foundations in expansive soil would not typically require bells.

Finally, the subcommittee found that because drilled concrete piers in expansive soil rely primarily on side resistance, the engineer should find that designing the smallest shaft diameter possible typically provides the greater economy in pier cost. Even though the pier depth may slightly increase when reducing the pier shaft diameter, the cost savings due to using a smaller diameter pier may offset the installation cost increase of the additional depth. This is because side resistance increases linearly with the shaft diameter whereas pier concrete and steel costs increase with the square of the shaft diameter.

1.3 NOTES FOR READING THIS DOCUMENT

- The theory for the design of the piers is found in Section 2.0. Sample Calculations are found in Section 3.0. Comparison of the procedure to the actual geotechnical investigation report's design recommendations is found in Section 4.0.

- Text contained in a bracket indicates units such as feet, e.g. [ft], or information such as the procedure's equation numbers, e.g. [Eq. 7].

- Text after an equation contained in parentheses indicates the section or equation number in the footnoted text from which the equation was duplicated or adapted, e.g. (Sect. 13.3.5).

- Metric, SI and Imperial units are used in the constants and computations, and the units are noted for each, as applicable. The derivations of the equations used in this procedure come from a variety of sources, and in an attempt to more easily reference the equations back to their original sources, the subcommittee elected to maintain original units. Any questions that arise from this combination of units should be answered by reviewing the sample calculations in Section 3.0.

- Refer to the foregoing Nomenclature section for definitions and units of the variables used in this document. Where possible, the subcommittee attempted to retain the same variables used in the referenced literature. In most cases, the subcommittee did not attempt to define the variables where they were discussed in the document. For some of the variables, the Nomenclature section also contains helpful information on use of the variable that is not available in the body of the document.

2.0 DRILLED PIER DESIGN

The minimum required depth of the pier is a function of the drilled shaft diameter, the load at the top of the drilled shaft and factors related to soil conditions such as the computed depth of the moisture active zone. Following is a description of a procedure for designing the depth of a drilled concrete pier when the input data from Section 1.0 are available.

The required depth of the concrete pier is calculated by the sum of the movement active zone depth and the pier anchor zone depth (see Figure 2-1). Pier anchor zone depth is determined by finding the depth in which the resistance on the pier below the movement active zone equals or exceeds the loads in the movement active zone and/or at the pier top with appropriate safety factors applied.

$$\sum Q \leq \sum \frac{R}{S.F.}$$

That is, the sum of the loads in the direction of the movement, Q, must be less than or equal to the sum of the resistances opposite the direction of movement, R, divided by the appropriate safety factors, S.F. See Section 2.5 for more detail.

The equations presented are stand-alone with respect to each pier. In other words, the equations are developed with the assumption that the foundation or first floor is isolated from the top of the soil, such that there is no contribution of loading from the soil expanding against or shrinking away from the bottom of the foundation. However, the engineer could estimate these loads, if present, and include them in the calculations if the piers are coupled with a slab-on-ground foundation.

2.1 SOIL ZONE DEFINITIONS

This section describes the various soil zones referenced in this procedure and shown in
Figure 2-1:

- the moisture active zone (Z_m),
- the movement active zone (Z_a), and
- the anchor zone (L_A).

The depth of the moisture active zone (Z_m) is an input parameter to the procedure presented in this paper and is used to calculate the depth of the movement active zone (Z_a) and anchor zone (L_A). The length of the anchor zone, L_A, is equal to the total pier length, L, minus the depth of the movement active zone, Z_a. A procedure for determining the moisture active zone is presented below.

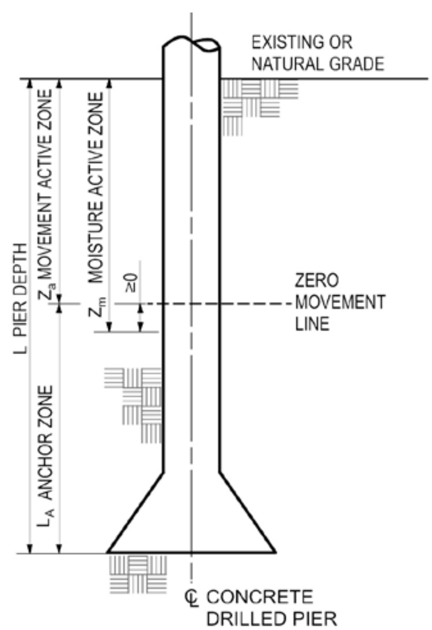

Figure 2-1

2.1.1 Moisture Active Zone

The moisture active zone is that depth of an active soil, measured from the grade downward, wherein moisture fluctuations occur.

The depth of the moisture active zone is best determined by suction measurements. If suction measurements are available, the moisture active zone is the depth of constant suction. The depth of constant suction is defined as the elevation (depth) where suction variance is less than or equal to 0.03 pF per foot of depth.

However, suction data at regular depths may not be available and is not required to use the procedure presented here. Less accurate methods to estimate the depth of the moisture active zone include:

- Root Depth - The moisture active zone will be *at least two feet deeper* than the deepest (dead or living) vegetation root observed in the cohesive soil samples.
- Ground Water Table - The moisture active zone will be *no deeper* than the lowest elevation of the ground water table.
    - Be aware that various geologic conditions may produce perched water zones that are not continuous, but may create significant wet and low suction zones within highly expansive unsaturated clay soils. These expansive clay zones can produce substantial moisture gradient anomalies that will produce soil deformations at the base of the pier and bell as well as potentially higher stress and force along the pier/pile shaft above the base or tip.
- Hard Layer – The moisture active zone will be *no deeper* than the upper elevation of a hardpan or cemented soil layer, a rock layer, or another stratum that vegetation roots cannot penetrate.
- Cohesionless Layer – The moisture active zone will be *no deeper* than the upper elevation of a relatively thick cohesionless stratum located below an active soil.

In addition, there may be other observable data used to correlate with the above methods. One indicator is the deepest depth of fissures and animal/insect-burrowed tunnels sufficient to allow rapid diffusion of moisture in the active soil. Another indicator that can be used to correlate with the above methods is the observation of the deepest depth of slickensides, which indicate passive earth failure of an active soil due to moisture fluctuations causing large horizontal stresses. Although these indicators alone cannot be used to quantify the depth of the moisture active zone, it is possible that the deepest depth of either of these indicators could approximately equal the depth of the moisture active zone.

The placement of concrete at grade may or may not affect the moisture active zone. Though pavements and foundations may limit vertical moisture penetration, water may still reach the covered soil.

The moisture active zone may include water migration in deeper desiccated layers where moisture content changes are influenced by higher suction soil and/or clay-shale with lower suction wet zones, including perched water zones. A comprehensive geotechnical and geophysical exploration program may be used to identify subsurface anomalies that affect the actual moisture equilibration process at depth.

2.1.2 Movement Active Zone

The movement active zone is that depth of the active soil measured from the grade downward where soil movement can occur due to volumetric moisture changes. The bottom of this zone is defined as the zero movement line (ZML).

In cases where the active soil is gaining moisture, it attempts to expand in all directions, particularly upward because the other directions are usually more confined at lower depths. However, since the overburden pressure of the soil increases with depth, it will eventually resist and prevent the upward movement at some depth. The depth below grade where vertical upward movement due to expanding soil can no longer occur will almost always be above and never below the bottom elevation of the moisture active zone (see Figure 2-1). Examples of where the bottom of the moisture active zone and the movement active zone could be equal include shallow active clay strata underlain by cemented or cohesionless strata.

As the soil tries to expand laterally, the pressure it applies to a pier shaft facilitates its grip of the pier and attempts to pull it vertically upward. In order to prevent pier movement due to soil heave, the load from this upward soil movement must be resisted. This load is resisted along the portion of the pier founded below the zero movement line (i.e., in the “anchor zone”, see Figure 2-2) and is modified by the net vertical load acting on the pier.

In the case where the active soil is losing moisture, it will shrink in any direction possible, usually downward and horizontally, with the latter creating vertical fissures. The loss of moisture can occur over the entire moisture active zone (see Figure 2-1) but the shrinkage will occur only within the movement active zone. The soil next to the portion of the pier shaft in the movement active zone will lose its side friction when a gap occurs. In this case, the vertical loads transmitted along the pier centerline must be resisted by the base resistance and/or side resistance within the anchor zone, discussed below.

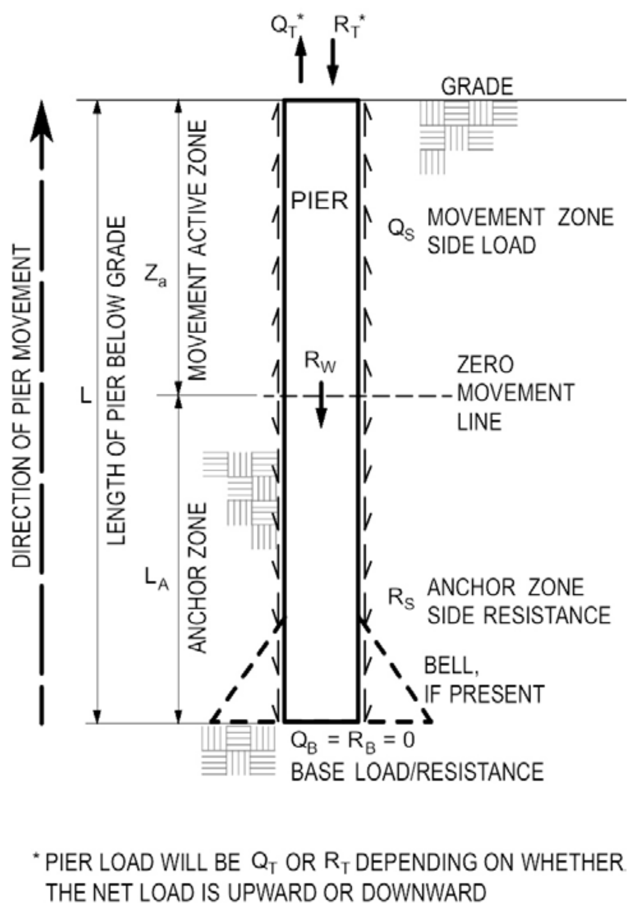

Figure 2-2

2.1.3 Anchor Zone

The anchor zone is that depth of soil measured from the zero movement line downward to the base of the pier. The anchor zone should be sufficient in depth to resist the vertical loads. Examples of these loads are the vertical components of gravity, wind, seismic, and snow load, and by movement of the soil mass in the active zone for cases of either gaining or losing moisture. Four cases were considered for active soils: pier movement upward with heaving soil, pier movement downward with heaving soil, pier movement upward with subsiding soil and pier movement downward with subsiding soil. However only two cases control: pier movement upward with heaving soil and pier movement downward with subsiding soil, discussed below.

Pier movement upward with heaving soil: To prevent upward pier movement in the case where the active soil is gaining moisture, the upward side load applied along the pier shaft in the movement active zone, Q_S, and any net upward pier top load, Q_T, must be resisted by side resistance, R_S, along the portion of the pier shaft embedded in the anchor zone plus the effective weight of the pier, R_W, and any net downward pier top load R_T. See Figure 2-2.

If the pier is configured with a bell at the bottom, while the bell's additional effective weight can be used to resist the upward side loading from the expanding soil in the movement active zone, resistance due to the top of the bell should not be included. This is because side resistance will be fully mobilized after moving only a fraction of an inch, whereas bearing resistance on the top of the bell is not fully realized until the pier has moved upwards on the order of 5 percent of the bell diameter[2], or about 2 inches for a 36-inch to 42-inch bell diameter. Designing for this magnitude of bell movement will mean side resistance on/above the bell's cone cannot be relied upon.

Furthermore, despite the depiction of bell-bottomed piers as cone-shaped, the subcommittee's experience and its discussions with other engineers revealed that a bulb-shaped bell is the norm and a cone-shaped bell is the exception. Because of the typical as-constructed geometry of a belled pier and the magnitude of movement required to engage the bell top for bearing, for the purposes of the uplift calculations in this procedure the subcommittee elected to neglect all bearing resistance at the top of the bell. This is of particular importance since designing for 2" or more of upward movement may be unacceptable to owners that choose to pay the additional cost for foundations that are isolated from grade in order to reduce their risk of foundation movement. The cone of the bell in such a large upward movement condition will offer some side resistance however. See further discussion of this below and the resistance arrows depicted in Figure 2-2.

Pier movement downward with subsiding soil: When the active soil starts losing moisture, the soil shrinks away from the concrete pier and the side load in the movement active zone is zero wherever the soil has pulled away. For a pier design that includes a load case with a net

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[2] Reese, L.C. and O'Neill, M.W. *Drilled Shafts: Construction and Design.* FHWA. Publication No. HI-88-042. 1988.

downward load that includes the pier's effective weight, the side resistance in the anchor zone and the bearing resistance at the base of the pier must resist the maximum design downward load on the pier plus the entire effective pier weight. In this case the bell, if present, will provide limited side resistance along the cone that can be approximated as a cylinder whose diameter is the same as the pier shaft's diameter (see Figure 2-3).

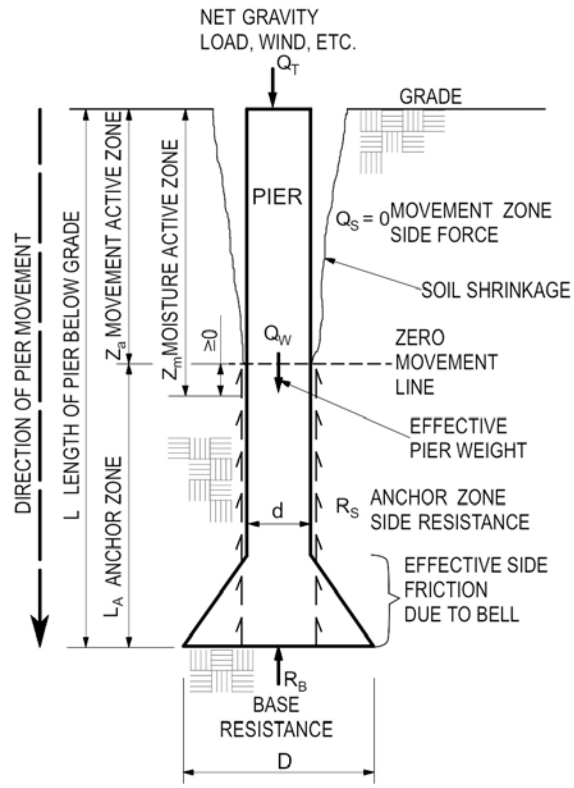


Figure 2-3

2.2 ANCHOR ZONE LENGTH CALCULATION

The anchor zone is the portion of soil below the zero movement line that does not experience volumetric changes due to moisture variations. The anchor zone depth must be determined to accommodate both soil active zone moisture change scenarios: the active soil gaining moisture and expanding (heaving), and the active soil losing moisture and shrinking (subsiding).

The anchor zone length is determined by calculating the depth at which the resistances on the pier below the movement active zone balance the loads in those upper zones using appropriate safety factors. Resistances and loads are based on soil shear strength, external forces and pier geometry. The pier must be sufficiently embedded into the anchor zone so that no additional side resistance is required to resist the net upward loads.

2.3 DEPTH OF THE MOISTURE ACTIVE ZONE (Z_M) CALCULATION

Matric suction is defined as the suction exerted by the soil matrix, which induces water to flow in unsaturated soil. It is a negative pressure that results from the combined effects of adsorption and capillary action due to the soil matrix. Water flows from soil with low matric suction (wet soil) to soil with high matric suction (dry soil). Matric suction is the difference between the pore water pressure and pore air pressure in a soil mass.

When matric suction data is not available from the geotechnical report, the depth of the moisture active zone can be estimated to predict the lower boundary of the depth of the movement active zone in an expansive soil. This procedure uses the computed moisture active zone to approximate the movement active zone. It requires knowledge of the following parameters.

Data from the geotechnical investigation report:

- Atterberg limits, LL and PL
- soil undrained shear strength, s_u
- moisture (water) content, w

- soil dry unit weight, γ_d
- knowledge of tree and other large vegetation growth on site, past and present, including maximum root depth
- water table (minimum and maximum) depths, GWT
- approximate depth of the moisture active zone, Z_m
- minimum bell ratio to avoid sloughing, r_{bell}

Data from sources other than the geotechnical investigation report:

- Thornthwaite Moisture Index, TMI
- local wet and dry suction boundaries, pF_{wet} and pF_{dry}
- maximum vertical loads at pier top for upward and downward cases, Q_T or R_T

If there are cohesionless layers within the depth of the pier, the following data is also required:

- expected surcharge at top of pier, $\sigma_{surcharge}$
- cohesionless soil SPT blow count for an efficiency of 60%, N_{60}
- correlation factor for cohesionless soil, m

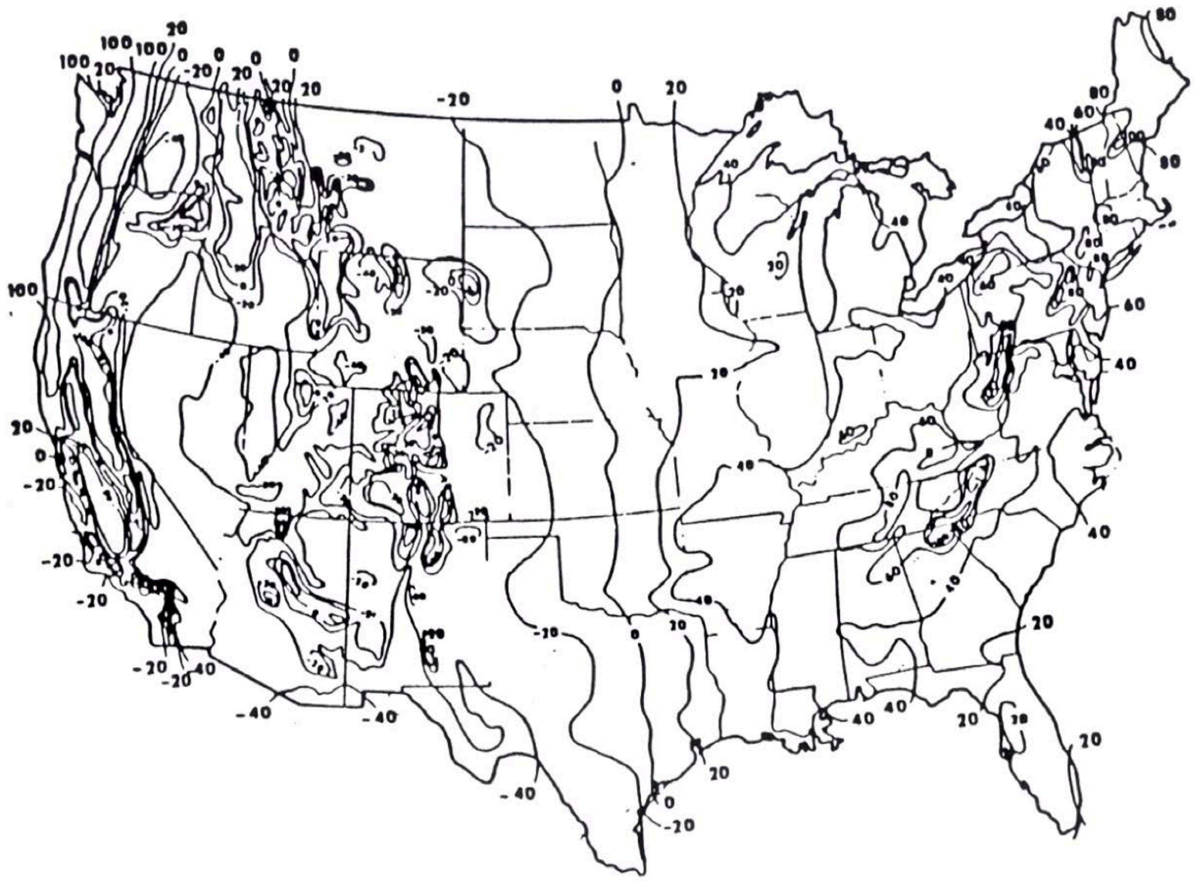
The procedure to compute Z_m is as follows:

Step 1: Calculate the Equilibrium Suction, pF_{eq}, as a function of the Thornthwaite Moisture Index, TMI:

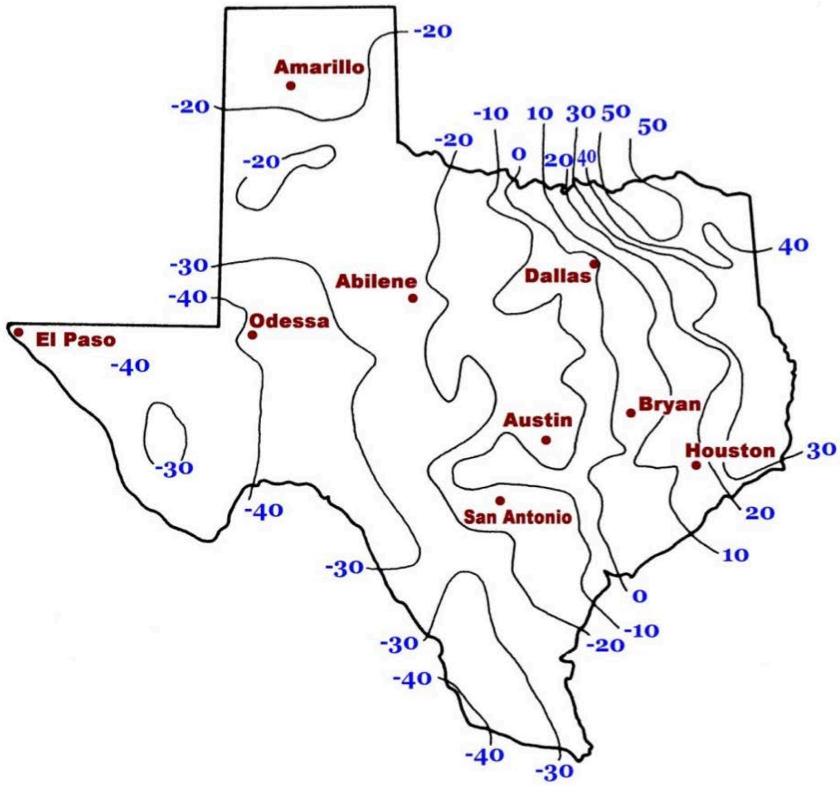
[Eq. 1] $$pF_{eq} = 3.659\ e^{(-0.0033 * TMI)}$$

Typical pF values are listed below Figure 2-5. TMI, if unknown, can be extrapolated from the following United States or Texas maps (Figures 2-4 and 2-5)[3]:

[3] *Design of Post-Tensioned Slabs-on-Ground*. Post-Tensioning Institute. Third Edition.



Thornthwaite Moisture Index Map for United States
Figure 2-4



Thornthwaite Moisture Index Map for Texas
Figure 2-5

Typical suction levels, in pF and atmospheres (sea level pressure is 1 atm = 14.7 psi), are:

Air Dry[3]	6.0 pF	970 atm
Wilting in Grass and Tree Root Zones[3]	4.5 pF	30 atm
Plastic Limit in Fat Clays[3]	3.5 pF	3.1 atm
Natural Water Content in Clays	3.2 – 3.7 pF	1.5 – 4.8 atm
Clay Wet Limit[3]	2.5 pF	0.31 atm
Clay Liquid Limit[3]	1.0 pF	0.01 atm

where: $\text{suction in pF} = \log_{10}(|\text{suction in cm of water}|) =$

$$\log_{10}\left(\left|\text{suction in psi} * 70.37\ \frac{\text{cm of water}}{\text{psi}}\right|\right)$$

Step 2: Given the Wet Suction Boundary (pF_{wet}) and Dry Suction Boundary (pF_{dry}) for the site, compute the wet envelope $U_{\text{wet-differential}}$ and dry envelope $U_{\text{dry-differential}}$:

[Eq. 2-A] $U_{\text{wet-differential}} = pF_{wet} - pF_{eq}$ (a negative number)

[Eq. 2-B] $U_{\text{dry-differential}} = pF_{dry} - pF_{eq}$ (a positive number)

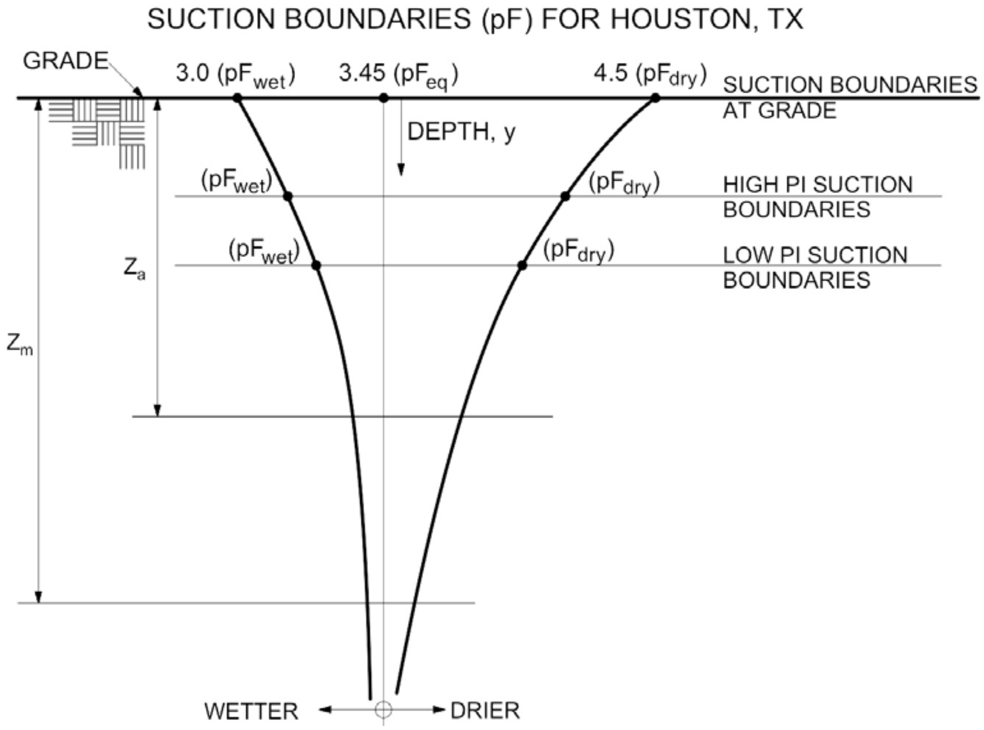


Figure 2-6

In most of the United States it is appropriate to use 4.5 for pF_{dry} and 3.0 for pF_{wet}. In areas of bare sunbaked ground (i.e. desert) the dry suction boundary can approach 6.0 pF[3].

Using the full range (at grade level) of the Wet Suction Boundary to the Dry Suction Boundary to compute the Moisture Active Zone is unrealistically conservative. This procedure uses a more realistic computation by moving down the suction envelope curve to a depth, y, which is calculated in [Eq. 4], and using the corresponding Wet Suction, at pF_{wet}, and Dry Suction, at pF_{dry}, at that depth on the profile (Figure 2-6).

The user may check the moisture content relative to the liquid limit (LL) before continuing, particularly for low-LL soil. An expansive soil at moisture equilibrium is commonly near its PL. A soil sample with a moisture content that is much higher than its PL may result in an overly deep pier design. Conversely, a soil with a moisture content that is much lower than its PL may result in an undersized pier design. The check is easily done by first plotting moisture content on the x-axis, pF on the y-axis, then drawing a line connecting (0, 5.6 pF) to (LL, 1.0 pF). The equation of this line is

[Eq. 3] $pF = -(4.6/LL) * w + 5.6$.

Then, use [Eq. 3] to calculate pF for the reported moisture content, w. If the result is lower than pF_{wet}, typically 3.0, or higher than pF_{dry}, typically 4.5, then *this procedure may not be applicable*.

Table 2-1 shows the boundary condition moisture contents at pF_{wet} = 3.0 (maximum w) and pF_{dry} = 4.5 (minimum w) for select liquid limits.

Example boundary moisture content, w vs. LL for pF_{wet} = 3.0 and pF_{dry} = 4.5 pF(w) = -(4.6/LL) * w + 5.6		
LL	**minimum w**	**maximum w**
20	5	11
30	8	16
40	10	22
50	12	28
60	15	33
70	17	39
80	20	45

Table 2-1

Step 3: Calculate the depth, y, with the following equation:

[Eq. 4] $$y[m] = \left(\frac{Z_i[m] * \lambda}{2K_0+1}\right)\left(\frac{\phi'}{35°}\right)^n$$

where: Z_i is typically[4] 0.8m, and is considered a constant in this procedure

λ = 3.75 for Liquid Limit (LL) > 60
4.19 for 50 ≤ LL ≤ 60
4.63 for LL < 50

[Eq. 4-A] $$K_0 = 1 - \sin\phi'$$

[Eq. 4-B] $$\phi' = 0.0016\,(PI)^2 - 0.3021\,(PI) + 36.208,$$ limited by:
$30 \leq \phi' \leq 34$ for LL < 50
$23 \leq \phi' \leq 27$ for LL ≥ 50

n = 0.2π for the "No Tree" case as defined below
0.35π for the "Tree" case as defined below and LL < 50
0.625π for the "Tree" case as defined below and LL ≥ 50

Tree vs. No Tree Option

The case designation of "Tree" or "No Tree" *significantly* affects the depth of the required pier. If the site meets the criteria for the "No Tree" case, the user should use "No Tree", otherwise the designed pier depth will be deeper than necessary. This designation affects the variables n and α_{diff}.

The "Tree" case applies for projects where clay soils are present plus *any* of the following occur:

- Site contains mature broadleaf or conifer trees now or historic aerials show it contained them in the past, even if they existed many decades prior and are no longer present. Some broadleaf trees require significantly more water than conifers

[4] *Class notes from Texas A&M University Course CVEN 646*. 3 April 2002.

but depending on several factors the opposite can also be true. Trees have been documented to have a major influence on soil moisture content within the root zone. The effect of trees on soil moisture can vary based on tree size, age, type and specie, and other factors such as soil type, seasonal variations, exposure, etc. It is left to the design engineer to determine the overall effect of the soil moisture variations from trees for each site considered.

- Site is covered with brush-type vegetation (likely 4-5 ft root depth).
- Roots are found at greater than 3 ft depth.

The "No Tree" case applies for projects where *none* of the items listed in the "Tree" case occur. This is true even if any or all of the following occur:

- Site contains or did contain only grass or field grass (likely 2-3 ft root depth)
- Slickensides exist at any depth. Slickensides indicate large horizontal stresses in expansive soil that were sufficient to cause passive earth failure in the past. The moisture fluctuations that caused the horizontal stresses may or may not have been due to trees.
- Calcareous nodules exist at any depth

Step 4: Calculate the wet and dry suction boundary conditions at depth y as follows:

[Eq. 5] $$U_{wet}(y)=pF_{eq}+U_{wet\text{-}differential}\, e^{-\left(\frac{3.171*10^{-8}*\pi}{\alpha_{diff}}\right)^{0.5} * y[m]\left(\frac{100\text{ cm}}{1\text{ m}}\right)}$$ (5.1[5], modified)

[Eq. 6] $$U_{dry}(y)=pF_{eq}+U_{dry\text{-}differential}\, e^{-\left(\frac{3.171*10^{-8}*\pi}{\alpha_{diff}}\right)^{0.5} * y[m]\left(\frac{100\text{ cm}}{1\text{ m}}\right)}$$ (5.1[5], modified)

where: $\alpha_{diff} = 0.003\,\frac{cm^2}{sec}$ if "No Trees" and $0.015\,\frac{cm^2}{sec}$ if "Trees", as defined in Step 3

3.171E-8 is the frequency with units $\frac{1}{sec}$

$pF_{wet} < pF_{dry}$

$U_{dry}(y) < pF_{dry}$

Step 5: Compute the depth of the moisture active zone, Z_m, as follows:

[5] Mitchell, Peter W. *The Structural Analysis of Footings on Expansive Soil*. Newton, South Australia, 1980. Kenneth W. G. Smith & Associates Research Report No. 1, Second Edition. The original equation has been modified to omit the time portion of the equation.

[Eq. 7] (below)

$$Z_m[\text{ft}] = Z_i[\text{m}] * \left(\frac{3.2808\ \text{ft}}{\text{m}}\right) * \left(10^{U_{wet}(y) - \frac{1}{2}\left(pF_{eq} + U_{dry}(y)\right)}\right)^{-\left(1 + \frac{0.4343}{0.5\left(\frac{pF_{eq}+U_{dry}(y)}{2} + U_{wet}(y)\right) - 6.032}\right)}$$

The movement active zone, Z_a, is almost always more shallow than the moisture active zone, Z_m, i.e. $Z_a \leq Z_m$. The movement active zone cannot be accurately computed without suction tests, so the [Eq. 7] computed value of Z_m is used to determine the pier depth required to resist the movement in the active zone. However, the computed Z_m may be overridden by the user-determined zero movement line in accordance with Section 2.1.1.

For this procedure Z_a, the portion of the pier in the movement active zone, is set equal to the calculated Z_m, minus the equivalent soil thickness of any surcharge present, such as pad buildup. This applies to piers supporting both isolated and slab-on-ground foundations with piers. If the foundation type is a slab-on-ground with piers, the pad buildup surcharge may also include the dead load of the slab and grade beams as well as dead loads of the superstructure. If the foundation type is an isolated (structural) slab/grade beam system that is designed to span to the piers, then instead of a foundation/superstructure surcharge, additional sustained gravitational pier top loads would be applied in the design.

> COMMENTARY: The movement active zone, Z_a, may be equal to the moisture active zone, Z_m, in the case of a hardpan layer beneath a relatively shallow, expansive clay layer. Where this geography occurs with trees, past or present, roots will penetrate the entire depth of the expansive clay layer. An increase in the applied surcharge will tend to reduce the required pier length in cohesive and cohesionless soil.

2.4 BASE RESISTANCE AND SIDE LOADS AND RESISTANCES IN COHESIVE AND COHESIONLESS SOIL

This section contains the procedure for computing pier base resistance, side loads and side resistances in both cohesionless soil and cohesive soil. A procedure for using the calculated loads and resistances in the equilibrium equations to determine pier depths follows in Section 2.5.

2.4.1 Base Resistance

Soil resistance at the base of drilled shafts, whether the shaft is straight or belled, is computed for cohesive and cohesionless soil as follows:

a) Cohesive Soil Base Resistance

For cohesive soil, base soil resistance is computed as a function of the undrained shear strength of the cohesive soil at the base. Unless the pier is shallow, the equation for computing bearing resistance of the soil at the base is:

[Eq. 8] $q_B = N_c s_u$ for $\frac{L}{D * \frac{1}{12}} \geq 3$ (Eq. 13-16[6])

where D = d for straight shafts and the bearing capacity factor N_c = 9.0 in most cases for bearing at the base of drilled shafts, and is dependent upon s_u.

s_u is the average undrained shear strength of a cohesive soil. If actual test pressure values are reported instead of shear strengths, common conversions to obtain s_u are one-half (1/2) the reported unconfined compression test value and one-third (1/3) the reported hand penetrometer test value. For other types of shear tests, such as torvane, contact the geotechnical engineer for a conversion factor.

In softer cohesive soil, when s_u < 2000 psf, N_c can be reduced to as much as 6.5 (per Table 13-2[6]).

Curve fitting the data given in Table 13-2[6]:

[Eq. 9] $N_c = 10.25 - \left(\frac{2812.5}{S_u+250}\right)$ $6.5 \leq N_c \leq 9.0$

For shallow piers, base resistance computed in [Eq. 8] is further reduced for a small pier length to base diameter ratios as follows:

[Eq. 10] $q_B = \frac{2}{3}\left[1+ \frac{1}{6}\left(\frac{L}{D* \frac{1}{12}}\right)\right][N_c\, s_u]$ for $\frac{L}{D* \frac{1}{12}} < 3$ (Eq. 13-17[6])

b) Cohesionless Soil Base Resistance

Base resistance in cohesionless soil is proportional to the number of blows per foot with an efficiency of 60% (N_{60}) measured in the field Standard Penetration Test (SPT):

[Eq. 11] $q_B[\text{tsf}] = 0.6\, N_{60} \leq 30$ (Eq. 13-14[6])

Converting to other units:

[Eq. 12] $q_B[\text{ksf}] = 1.2\, N_{60} \leq 60$
[Eq. 13] $q_B[\text{psf}] = 1200\, N_{60} \leq 60{,}000$

When the number of blows per foot (N_{60}) is not known, Table 2-2 below can be used to estimate N_{60} based on a description of the relative density of the soil. If the relative density is unknown, assume the sand is loose. Note Table 2-2 provides N-values and these should be reduced to 60% to use in the equations above, or the provided N_{60}-value may be used.

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[6] Brown, Dan A., John P. Turner and Raymond J. Castelli. *Drilled Shafts: Construction Procedures and LRFD Design Methods.* Report No. FHWA NHI-10-016. National Highway Institute, U. S. Department of Transportation. May 2010.

Penetration resistance and soil properties based on the SPT[7]		
Number of blows per foot, N*	**Relative Density**	**N_{60}**
0-4	Very loose	2
5-10	Loose	4
11-30	Medium	12
31-50	Dense	24
Over 50	Very dense	30+

Table 2-2

* Note that $N = N_{60}$ when the SPT equipment used in the field has an efficiency rating of 60%, typical for rope and cathead drop hammer systems.

2.4.2 Side Resistance/Load

Side resistance of drilled shafts is derived for cohesive and cohesionless soil as follows:

a) Cohesive Soil Side Resistance/Load
For cohesive soil, side resistance (or, in active zones – side load) is computed using the Alpha Method[6]:

[Eq. 14] $q_S = \alpha * s_u * RF_S$ (Eq. 13-15[6], modified with RF_S)
where: RF_S is the Side Resistance Factor, default is 1.0

$\propto\ = 0$ over the length of the movement active zone, Z_a (downward case only)

$\propto\ = 0.55$ $s_u \le 1.5\ (2116) \cong 3200$ psf (Sect. 13.3.5.2[6])

$\propto = 0.55 - 0.1\left[\left(\frac{s_u}{2116}\right) - 1.5\right]$ $3200 \text{ psf} < s_u \le 5300 \text{ psf}$ (Sect. 13.3.5.2[6])

or simply:

[Eq. 15] $\propto\ = \left[0.55 - 0.1\left[\left(\frac{s_u}{2116}\right) - 1.5\right]\right]$ $0.45 \le \propto \le 0.55$

b) Cohesionless Soil Side Resistance
For cohesionless soil, the beta (β) method is used to compute side resistance as follows.

Given γ_t, water table depth and N_{60}

[Eq. 16] $\phi' = 27.5 + 9.2 \log_{10}(N_{60})$ (Eq. 3-8[6])

[Eq. 17] $K_p = \tan^2\left(45^\circ + \frac{\phi'}{2}\right)$ (Eq. 13-10[6])

[Eq. 18] $\sigma'_p = 990\ (N_{60})^m$ (Eq. 13-11[6])
where: m = 0.6 for clean sand

[7] Peck, Ralph B. *Foundation Engineering*. Second Edition. John Wiley & Sons. 1973.

m = 0.8 for silty sands, clayey sands, sandy silts, etc.[6]

Note that [Eq. 18] is for non-gravelly cohesionless soil. For gravelly sands use [Eq. 19].

[Eq. 19] $\sigma_p' = 320 N_{60}$ (Eq. 13-12[6])

[Eq. 20] $\beta = (1 - \sin\phi')\left(\frac{\sigma_p'}{\sigma_v'}\right)^{\sin\phi'} * \tan\phi' \leq K_p \tan\phi'$ (Eq. 13-13[6])

[Eq. 21] $q_S = \sigma_{v_n}' \beta * RF_S$ (Eq. 13-7[6] modified with RF_S)

where: $\sigma_{v_n}' = \sigma_{soil_n}' + \sigma_{surcharge}$ = effective vertical overburden pressure of soil computed at middle of layer (n) in question plus surcharge, as depicted in Figure 2-8,

$$\sigma_{soil_n}' = \sum_{i=1}^{n-1} \gamma_{t_i}' \Delta z_i + \frac{\gamma_{t_n}' \Delta z_n}{2}$$

where: $\gamma_{t_i}' = \gamma_{t_i} - 62.4$ (below ground water table)

$\gamma_{t_i}' = \gamma_{t_i}$ (above ground water table)

$\sigma_{surcharge}$ = surcharge pressure of grade beams, slab, superstructure, fill, etc., acting on soil around the pier at the elevation of pier top.

RF_S = Side Resistance Factor, commonly 1.0.

Research shows (FHWA NHI-10-016[6] Section 7.5.2) that using a polymer slurry effectively increases the diameter of the pier for the purposes of calculating the side load/resistance. A mineral slurry, such as bentonite and other clays, may degrade the side load/resistance because the mineral slurry's side friction angle, ϕ', can be lower than the in situ clay's side friction angle. Permanent casings will also degrade side friction. For these and other reasons, the user may choose to increase or degrade the side load/resistance with a Side Resistance Factor, RF_S.

IBC[8] 1810.3.3.1.4 "Allowable frictional resistance" states the assumed frictional resistance developed by any uncased cast-in-place deep foundation element shall not exceed one-sixth of the bearing value of the soil material at minimum depth as set forth in Table 1806.2, up to a maximum of 500 psf (24 kPa), unless a greater value is allowed by the *building official* on the basis of a geotechnical investigation as specified in Section 1803 or a greater value is substantiated by a load test in accordance with Section 1810.3.3.1.2. This procedure requires a geotechnical investigation; therefore q_B is *not* limited to 500 psf in this procedure.

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[8] *IBC 2015*. International Building Code. International Code Council. Washington, DC.

Geotechnical reports often provide data for multiple strata in a given boring. Figures 2-7 and 2-8 depict layers in the soil. When layers are used, calculations are made at each layer and then the forces and resistances are summed in the equilibrium equations. The examples in Section 3 illustrate this. Note that in Figures 2-7 and 2-8 the pier is drawn deeper than the sum of the movement active zone, Z_a, and the anchor zone, L_A. The anchor zone length, L_A, is the required length to resist the opposing forces, but the engineer may have a reason for designing the pier deeper than required. Throughout the rest of this document it is assumed that $Z_a + L_A = L$.

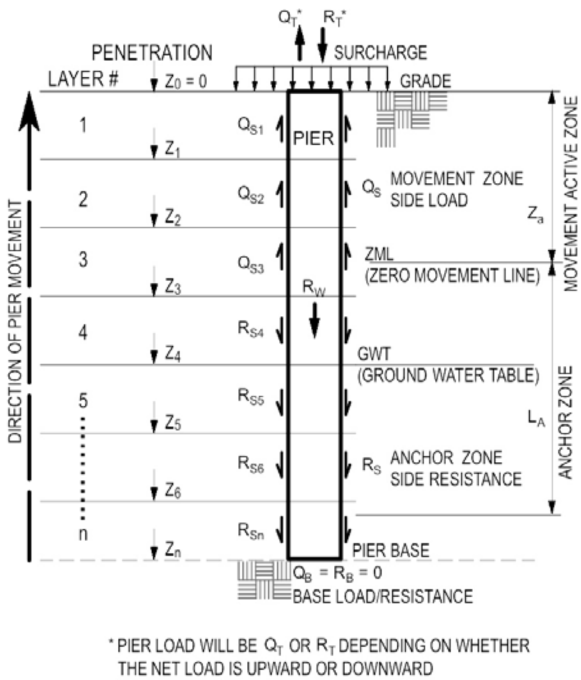


Figure 2-7

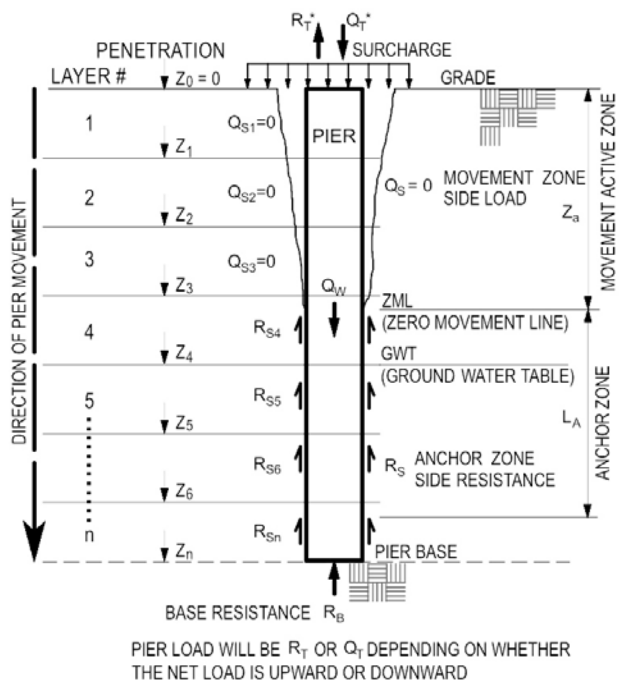


Figure 2-8

2.5 DEPTH OF THE PIER

The following equation is used to check the pier anchor zone depth by balancing loads and resistances:

[Eq. 22-A] $$\sum Q \leq \sum \frac{R}{S.F.}$$

where Q denotes loads in the direction of the pier movement and R denotes resistances.

The above equation must satisfy the unity check:

[Eq. 22-B] $$U.C. = \frac{\Sigma Q}{\Sigma \frac{R}{S.F.}} \leq 1.0$$

The subcommittee recommends the safety factors (S.F.) in Table 2-3 be used for the general "suction-range" design premise. When the soil moisture at the time of installation is unknown, this premise is needed.

Recommended Safety Factors for Soil Resistance			
Pier Movement Direction	**Soil Layer Type**	**Safety Factor (S.F.)**	
		Base (S.F.$_B$)[a]	**Side (S.F.$_S$)**
In cases with an active zone ($Z_a > 0$ ft)			
Up[b]	Cohesive	-	$Q_T/3Q_S + 1 <= 2.0$
	Cohesionless	-	$Q_T/3Q_S + 1.1 <= 2.2$
Down	Cohesive	3 r_{bell}	2.0
	Cohesionless	10.0	2.2
In cases with no active zone ($Z_a = 0$ ft)[c]			
Up	Cohesive	-	2.0
	Cohesionless	-	2.2
Down	Cohesive	3 r_{bell}	2.0
	Cohesionless	10.0	2.2

Table 2-3

Recommended Safety Factor Table Notes:

a. Base resistance safety factors are higher than commonly used because it takes considerably less settlement to fail side resistance than it does to fail base resistance. Where shallow piers are used to support pier top loads by base resistance only (i.e., no side load or side resistance), these safety factors can be reduced to S.F.$_B$ = 2 or 3. For more detail, see FHWA 1999[9] Appendix C, Figures C.1 to C.4, which show settlement versus side load transfer and settlement versus end bearing for cohesive and cohesionless soils.

b. The use of the suction envelope in this procedure yields conservative values. Because of these conservative values, the safety factors on Q_S can be low for the upward movement case when the primary uplift load is from expanding clay in the movement active zone.

c. $Z_a = 0$ for the case of downward moving pier condition where the active clay has shrunk away from the side of pier.

COMMENTARY: The reader will observe a varying safety factor that approaches 1.0 in the above table for cohesive soil resistance in the upward moving case, when the load is predominantly due to side load rather than top load, i.e., $Q_S >> Q_T$. This may initially seem low because IBC[8] 1810.3.3.1.5 requires that the uplift capacity of a single deep foundation element be determined by an approved method of analysis based on a minimum factor of safety of three or by a load test. However, this procedure is conservative in that it was developed to apply to piers installed any day of any year, whether in an abnormally wet or drought condition, and independent of when the

[9] Reese, L.C. and O'Neill, M.W. 1999. *Drilled Shafts: Construction and Design.* FHWA-IF-99-025.

geotechnical investigation occurred. In other words, this procedure has a "built-in" factor of safety. Had the subcommittee applied the IBC's factor of safety of three to the soil's heave, it would have been necessary to apply the same factor of safety to the soil's resistance in the anchor zone. Thus for simplicity, in the above table the subcommittee recommends a factor of safety closer to 1.0 than 2.0 in the case when the load is primarily due to heave of the soil in the active zone. The same sliding scale is applied to the resistance of anchor zone cohesionless soils in the upward moving case, within a range of 1.1 to 2.2. The recommended cohesionless safety factors are 10 percent higher than cohesive safety factors due to the increased variability commonly found in granular strata.

To use [Eq. 22-A] an initial pier length is selected, loads and resistances on the pier are calculated in the upward and downward pier movement cases and, if the loads and resistances (with applied safety factors from Table 2-3) balance in both load cases, then the pier length is sufficient.

In Figures 2-9 and 2-10, Q denotes loads in the direction of the pier movement and R denotes resistances opposite the direction of the pier movement.

For this procedure Z_a, the portion of the pier in the movement active zone, is set equal to the Z_m calculated in [Eq. 7] minus any soil surcharge present. Z_m is in feet and the soil surcharge, $\sigma_{surcharge}$, is in psf. The soil surcharge must be converted into an equivalent soil layer in feet so that it can be used to reduce the active zone. The procedure for this step is to divide the soil surcharge ($\sigma_{surcharge}$ [psf]) by the soil density (γ_t [pcf]). I.e. $Z_a = Z_m - (\sigma_{surcharge} / \gamma_t)$.

For the upward pier movement case (see Figure 2-9), the soil side loading in the movement active zone, Z_a, acts upwards along the pier shaft and the soil in the anchor zone, L_A, acts downwards, resisting the upward loads. There is no base resistance in the upward pier movement case ($Q_B = R_B = 0$). Additionally, if the load at the pier top acts upward, it is denoted Q_T. If it acts downward it is denoted R_T, because it is a resistance. As shown in Figure 2-9, the weight of the pier, R_W, resists the upward movement of the pier. The equation for calculating R_W of a belled pier is provided in Section 3.3.1. For the purpose of calculating side resistance, the pier is assumed to be a straight shaft pier even if the actual geometry is belled (see Section 2.1.3).

For the downward pier movement case (see Figure 2-10), we assume the soil in the entire movement active zone provides no resistance because it is assumed to have shrunk away from the pier. This is a conservative assumption. There is base resistance, R_B, and side resistance in the anchor zone along the length of the pier, R_S. If the load at the pier top acts upward it is a resistance, R_T, and if it acts downward it is a load, Q_T. As shown in Figure 2-10, the weight of the pier, Q_W, is a load in the direction of the pier movement.

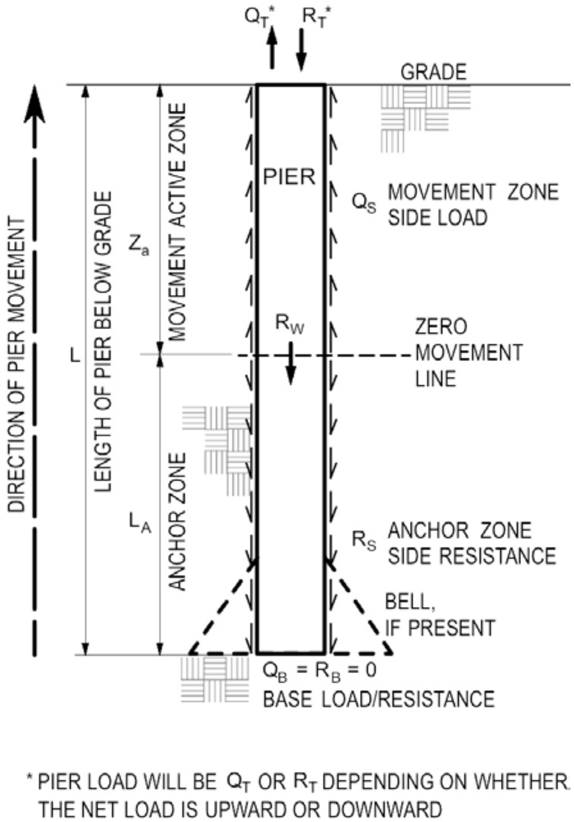


Figure 2-9

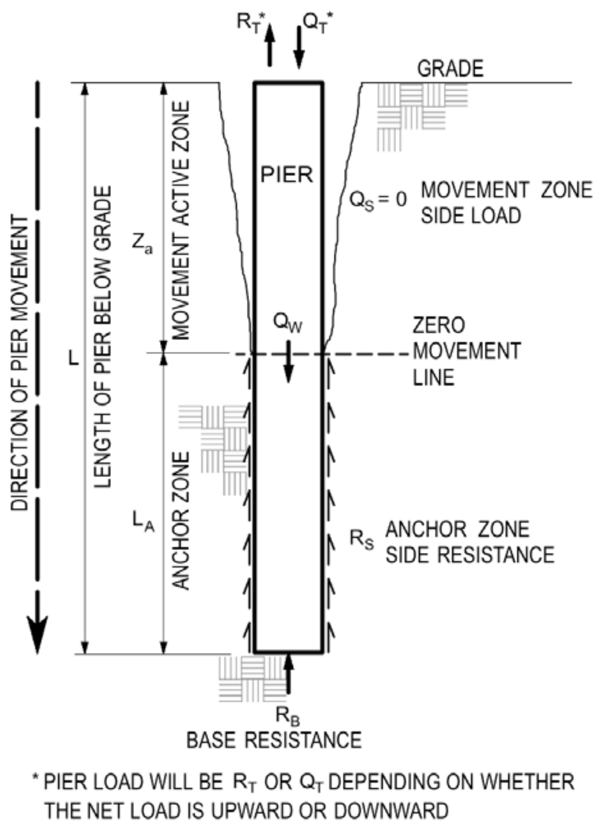


Figure 2-10

For the upward and downward pier movement cases, the primary equilibrium equation is [Eq. 22-A]:

[Eq. 22-A] $$\sum \mathrm{Q} \leq \sum \frac{\mathrm{R}}{\mathrm{S.F.}}$$

Q denotes loads in the direction of the pier movement, R denotes resistances opposite the direction of the pier movement, and S.F. is the safety factor chosen by the engineer or from Table 2-3.

For the upward movement case shown in Figure 2-9, if the top load is in the upward direction, the primary equilibrium equation becomes:

[Eq. 23] $$\mathrm{Q_T} + \mathrm{Q_S} \leq \mathrm{R_W} + \frac{\mathrm{R_S}}{\mathrm{S.F._S}}$$

or, for the same upward movement case, if the top load is in the downward direction, the primary equilibrium equation becomes:

[Eq. 24] $$\mathrm{Q_S} \leq \mathrm{R_T} + \mathrm{R_W} + \frac{\mathrm{R_S}}{\mathrm{S.F._S}}$$

For the downward movement case shown in Figure 2-10, if the top load is in the downward direction, the primary equilibrium equation becomes:

$$[\text{Eq. } 25] \qquad Q_T + Q_W + Q_S \leq \frac{R_S}{S.F._S} + \frac{R_B}{S.F._B}$$

or, for the same downward movement case, if the top load is in the upward direction, the primary equilibrium equation becomes:

$$[\text{Eq. } 26] \qquad Q_W + Q_S \leq R_T + \frac{R_S}{S.F._S} + \frac{R_B}{S.F._B}$$

The subcommittee did not find an example where [Eq. 26] controlled the pier design.

2.6 PIER REINFORCING

Under certain design conditions, building codes may not require drilled concrete piers to be reinforced. On sites containing clay, in order to account for unforeseen conditions such as recent removal of trees, lateral pier loading, additional uplift loadings, etc., this subcommittee recommends that a minimum 0.50% longitudinal steel reinforcing be used along the length of concrete piers. In other words, as a minimum the ratio of the cross sectional area of the longitudinal steel reinforcing divided by the gross cross sectional area of the pier shaft should be $\rho = 0.0050 = 0.50\%$, regardless of the pier load conditions.

Concrete piers that are known to be subjected to uplift, such as those penetrating an active movement zone may require additional reinforcing ($\rho > 0.50\%$,) to prevent the shaft concrete from failing in tension. If there is a design bending moment in the pier shaft, it will be necessary to account for the additive compression and or tension in the design of the longitudinal steel reinforcing due to bending when designing for the maximum downward or upward movement cases.

For cases where the net loading on the pier tends to cause the pier to move downward, pier shafts are considered laterally braced by the anchor zone soil against Euler type buckling under compression, even in weak soil. In reviewing cases of small diameter shafts with the largest possible bells ($r_{bell} = 3.0$) bearing in very stiff clays, the subcommittee did not find a case where it would be necessary for the engineer to consider Euler type buckling of the shaft above the ZML for the condition where the upper clay has shrunk away from the shaft (see Figure 2-10).

The derivation included in this section is intended for the design of steel reinforcing in drilled concrete piers under pure tension, a common load case for piers designed for the upward movement case in expansive soil, i.e., the case where the pier tends to move upward due to the net loads acting on the pier. Above the ZML, the axial forces on the pier due to uplift from soil heave will increase with depth, with some reduction for the pier weight. Below the ZML, the axial forces on the pier will decrease with depth due to downward acting side resistance in the anchor zone. Therefore, the maximum tensile load (T) in the pier will occur somewhere between the pier top and the ZML (see Figure 2-11).

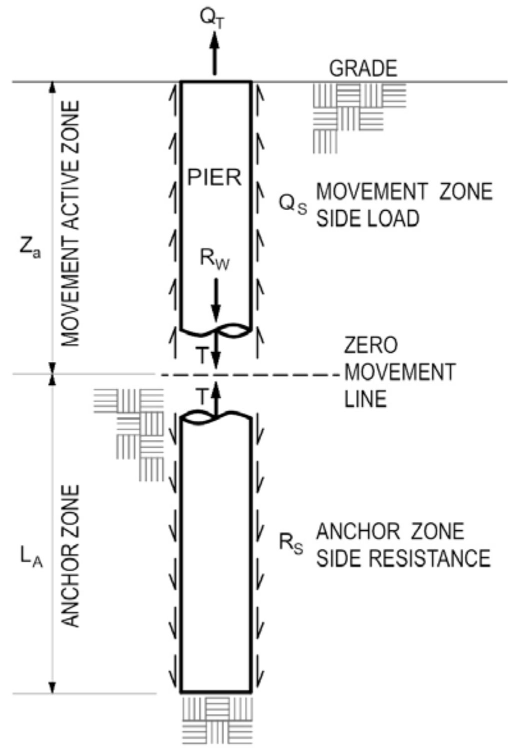


Figure 2-11

Although the tension will vary along the depth of the pier shaft, the longitudinal steel area in drilled concrete piers in expansive soil is typically designed to be constant from the pier's top to its base. The longitudinal bars should be uniformly spaced in a circle near the shaft perimeter using the ACI minimum cover for earth-formed concrete.

The longitudinal and horizontal bars need to be sufficiently spaced to allow concrete to flow to the outside of the steel cage. ACI-336.3R[10] Section 4.4.3 recommends, "Minimum concrete cover of 3 in. (76 mm) should be maintained and the minimum clear spacing between reinforcing steel should be 4.0 in. (102 mm) between horizontal reinforcement and 3 times (5 times preferable) the size of the maximum coarse aggregate between longitudinal steel (a minimum of 3 in. [76 mm])." ACI-336.3R[10] Section 4.4.4b states that when slurry is used, "Maximum aggregate size should be ¾ in." ACI-336R[10] Section 1.3-Limitations cautions, "This report is generally limited to piers of 30 in. or larger diameter". Since the design procedure in this (FPA-SC-16) document is limited to piers of diameter 30 inches or *smaller*, the subcommittee recommends the maximum aggregate size be less than or equal to the ¾ in. specified by ACI-336R[10] if slurry is used.

In this procedure the subcommittee recommends 3" cover be used outside the ties to set the maximum longitudinal bar centerline circle diameter. If the engineer designs shaft diameters greater than 30", cover could increase beyond 3" per some codes. If the engineer attempts to reduce cover from 3" using a permanent (disposal) casing to avoid the concrete being cast directly against soil, this procedure may not apply due to the alpha and beta method assumptions made in Section 2.4.2 for side resistance and side loading of drilled cast-in-place concrete piers.

[10] American Concrete Institute. ACI 336.3R-93 (Reapproved 2006). *Design and Construction of Drilled Piers*.

In this procedure, the minimum longitudinal bar centerline spacing is based on the use of a maximum concrete aggregate size of one-third the clear spacing between the longitudinal bars, measured along the longitudinal bar centerline circle. If uniform spacing of the longitudinal steel is not sufficient to meet ACI regarding aggregate size, the shaft diameter should be enlarged or the aggregate size (AG_{max}) should be decreased. AG_{max} could be considered 0" in the design equations if the concrete installer is able to directly distribute part of the concrete evenly outside the rebar cage. If this assumption is made, the subcommittee recommends an AG_{max} of no more than 1" be specified.

In order to inhibit moisture intrusion sufficient to cause corrosion of reinforcing steel, ACI 224R-01[11] dictates a maximum crack width design for sizing longitudinal reinforcing in drilled concrete piers. The maximum allowed crack width specified is 0.012" for soil or moist air exposure and 0.016" for dry air or protective membrane (Table 4.1)[11].

The derivation for the pier shaft reinforcing equations follows (see Figure 2-12). In order to limit the concrete shaft crack widths in tension:

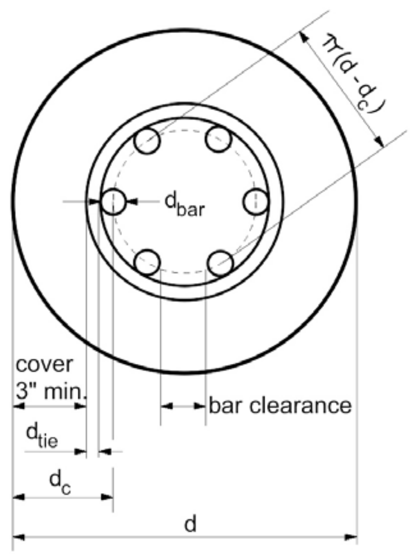


Figure 2-12

[Eq. 27] $c_w = 0.012"$ moist air/soil, $= 0.016"$ dry air (Table 4.1)[11]

[Eq. 28] $c_w = 0.10\, f_s (d_c A)^{1/3} \times 10^{-3}$ [in.] (Eq. 4-21)[11]

where:

c_w = crack width [in.]

[Eq. 29] $d_c = \frac{d_{bar}}{2} + \text{cover} + d_{tie}$ [in.]

cover = 3 [in.], minimum

f_s = allowable stress in reinforcement [ksi]

$A = \frac{\text{shaft area}}{n_r} = \frac{\pi d^2}{4 n_r}$ $[\text{in}^2]$

and n_r = number of rebar, limited by a function of $3 * AG_{max}$

The yield stress of the reinforcing steel, f_y, is the Grade in ksi., e.g., Gr. 60 rebar has $f_y = 60$ ksi.

[11] American Concrete Institute. ACI 224R-01. *Control of Cracking in Concrete Structures.* 2001

COMMENTARY: This procedure was developed using Allowable Stress Design, even though current ACI codes follow Load Resistance Factor Design. However older ACI codes still included their original Allowable Stress Design as late as 1989 through its ACI 318-89[12]. In that edition, ACI limited Grade 60 rebar tension $f_s \le 0.40\ f_y$ (ACI-318-89[12] Appendix A Para. 3.2). In some steel grades, higher allowables were permitted. AISC currently allows $f_s \le 0.60\ f_y$ for solid steel bars in tension. For this procedure it was chosen to use the more conservative ACI-318-89 value for allowable tensile stress in rebar, i.e. $f_s \le 0.40\ f_y$. The election of $0.40\ f_y$ complies with IBC 2015[8] Table 1810.3.26 for Grade 60 rebar which states maximum allowable stress of nonprestressed reinforcement in tension is limited to $f_s = 0.40 * 60$ ksi $= 24$ksi.

Then, from [Eq. 28],

$$[\text{Eq. }30] \qquad c_w = \frac{(0.10)(0.40)f_y}{1000}\left((d_c)\frac{\pi d^2}{4n_r}\right)^{1/3} \ [\text{in.}]$$

Rearranging to solve for the minimum number of bars, n_r, required for crack control,

$$[\text{Eq. }31] \qquad \left(\frac{4n_r}{\pi d^2}\right)^{1/3} = \frac{f_y}{25000c_w}(d_c)^{1/3}$$

$$[\text{Eq. }32] \qquad (n_r)^{1/3} = \frac{f_y d^{2/3}}{27100c_w}(d_c)^{1/3}$$

Observe from [Eq. 32] that the minimum number of rebar (n_r) needed to maintain a maximum design crack width increases as the design crack width c_w decreases. For the purposes of this procedure, piers will be in soil so from [Eq. 27] the subcommittee recommends $c_w = 0.012$" and [Eq. 32] becomes:

$$[\text{Eq. }33] \qquad n_{r\text{-min}} = \left(\frac{f_y}{325}\right)^3 d^2(d_c) \qquad [\text{-}], \text{ integer, round up}$$

$$[\text{Eq. }34] \qquad n_{r\text{-max}} = \frac{\pi\ (d - 2d_c)}{3\ AG_{max} + d_{bar}} \qquad [\text{-}], \text{ integer, round down}$$

Note that $n_{r\text{-max}}$ is limited by the maximum aggregate size, AG_{max}, as discussed above.

Compute the maximum pier tension, T.

$$[\text{Eq. }35] \qquad T = \sum Q \ \text{ for the upward case} \qquad [\text{kips}]$$

[12] American Concrete Institute. ACI 318-89 Appendix A. *Alternate Design Method.*

The required steel area is calculated according to the following equation:

[Eq. 36] $$A_{st_{REQ}} = \frac{T}{f_s}$$ $[in^2]$

The actual steel area is calculated according to the following equation:

[Eq. 37] $$A_{st_{ACT}} = \frac{\pi}{4} n_r d_{bar}^2$$ $[in^2]$

[Eq. 38] $$A_{bar} = \frac{A_{st_{REQ}}}{n_r}$$
$[in^2]$, minimum bar area

[Eq. 39] $$d_{bar} = \left(\frac{4}{\pi} A_{bar} \right)^{1/2} = 1.13 \left(\frac{A_{st_{REQ}}}{n_r} \right)^{1/2}$$ [in], minimum bar diameter

Calculate the rebar size as follows:

[Eq. 40] $$\# = 8\, d_{bar} = 9.04 \left(\frac{A_{st_{REQ}}}{n_r} \right)^{1/2}$$ [-], bar size, integer, round up

Calculate the steel to pier area ratio as follows:

[Eq. 41] $$\rho = \frac{A_{st_{ACT}}}{A_c} = \frac{4A_{st_{ACT}}}{\pi d^2} = \frac{4}{\pi}\ \frac{\pi}{4}\ \frac{n_r d_{bar}^2}{d^2} = n_r \left(\frac{d_{bar}}{d}\right)^2$$ [-], actual

or, in percent,

[Eq. 42] $$\rho = 100\ n_r \left(\frac{d_{bar}}{d}\right)^2 = 100\ n_r \left(\frac{\frac{\#}{8}}{d}\right)^2 = 100\ n_r \left(\frac{\#}{8d}\right)^2$$ [%]

Simplify and check that ρ is greater than or equal to 0.50%:

[Eq. 43] $$\rho = 1.56\ n_r \left(\frac{\#}{d}\right)^2$$ [%] ; ≥ 0.50%

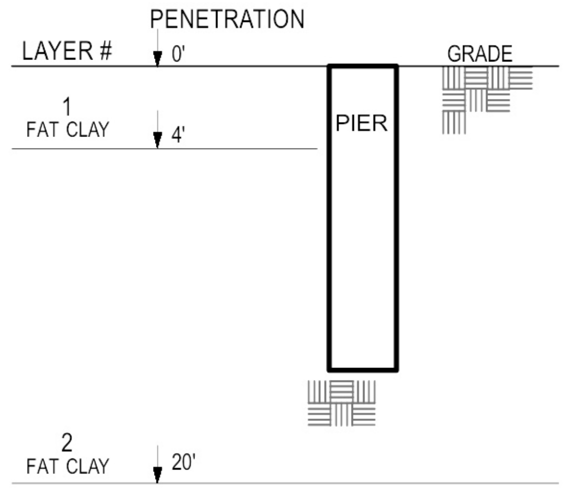
Iterate $n_{r\text{-min}} \leq n_r \leq n_{r\text{-max}}$ as needed to achieve an optimal design.

3.0 SAMPLE CALCULATIONS

3.1 SAMPLE CALCULATION 1 – ALL CLAY IN HOUSTON, NO TREES

Sample Calculation 1 is for a clay soil in Houston with the parameters listed in the table below, on a lot without trees, with pier loading of Q_T = 3 kips per pier upward in the upward case and Q_T = 30 kips per pier downward in the downward case. Assume a 14' pier.

Layer Number	[1]	[2]
Soil Description	Fat clay	Fat clay
Layer Penetration, z (ft)	4	20
Liquid Limit, LL (%)	64	78
Plastic Limit, PL (%)	21	23
s_u (tsf)	0.9	1.8
Moisture Content, w (%)	32	32
Dry Unit Weight of Soil γ_d (pcf)	102	102
RF_S	1.0	1.0



TMI = 18 for Houston, extrapolated from Thornthwaite Moisture Index map (Section 2.3)
Surcharge = 150 psf
n = 0.2π ("no tree" case per Section 2.3)
α_{diff} = 0.003 ("no tree" case per Section 2.3)
Z_i = 0.8m (typical value per Section 2.3)
λ = 3.75 (LL > 60 per Section 2.3)

Wet Suction Boundary Condition at Grade = 3.0 pF (standard for Houston)
Dry Suction Boundary Condition at Grade = 4.5 pF (standard for Houston)

Check that the moisture contents do not exceed the wet or dry suction boundary conditions at grade as discussed in Step 2. For pF_{wet} = 3.0 and pF_{dry} = 4.5 the boundary equation is: pF = -(4.6/LL) * w + 5.6

$pF_{[1]} = -(4.6/64) * 32 + 5.6 = 3.30$; $pF_{wet} < 3.30 < pF_{dry}$ [Eq. 3]
$pF_{[2]} = -(4.6/78) * 32 + 5.6 = 3.71$; $pF_{wet} < 3.71 < pF_{dry}$ [Eq. 3]

For both layers, the suction as a function of the liquid limit and moisture content is within the boundary conditions.

Calculate ϕ':
$\phi' = 0.0016\,(PI)^2 - 0.3021\,(PI) + 36.208$, limited by: [Eq. 4-B]
$30 < \phi' < 34$ for LL < 50

$$23 < \phi' < 27 \text{ for LL} \geq 50$$

$$\phi'[1] = 0.0016\,(64-21)^2 - 0.3021\,(64-21) + 36.208 = \mathbf{26.2°}$$
$$\phi'[2] = 0.0016\,(78-23)^2 - 0.3021\,(78-23) + 36.208 = \mathbf{24.4°}$$

In both layers, the calculated soil friction angle is between the limits for LL ≥ 50 (per Section 2.3).

Calculate K_0 .

$$K_0 = 1 - \sin\phi' \qquad \text{[Eq. 4-A]}$$
$$K_0[1] = 1 - \sin\phi'[1] = 1 - \sin(26.2°) = \mathbf{0.558}$$
$$K_0[2] = 1 - \sin\phi'[2] = 1 - \sin(24.4°) = \mathbf{0.587}$$

Using the suction boundaries at the surface to compute the depth of the movement active zone is conservative, as explained in Section 2.3. Calculate the depth, y, at which to impose the suction boundaries.

$$y = \left(\frac{Z_i * \lambda}{2K_0+1}\right)\left(\frac{\phi'}{35°}\right)^n \qquad \text{[Eq. 4]}$$

$$y[1] = \left(\frac{0.8[\text{m}] * 3.75}{2 * 0.558 + 1}\right)\left(\frac{26.2°}{35°}\right)^{0.2\pi} = \mathbf{1.18\ m}$$

$$y[2] = \left(\frac{0.8[\text{m}] * 3.75}{2 * 0.587 + 1}\right)\left(\frac{24.4°}{35°}\right)^{0.2\pi} = \mathbf{1.10\ m}$$

Calculate the Equilibrium Suction, $U_{\text{wet-differential}}$, $U_{\text{dry-differential}}$, U_{wet} at depth y, U_{dry} at depth y, and then the depth of the movement active zone Z_m

$$pF_{eq} = 3.659\, e^{(-0.0033 * \text{TMI})} = 3.659\, e^{(-0.0033 * 18)} = \mathbf{3.45\ pF} \qquad \text{[Eq. 1]}$$

$$U_{\text{wet-differential}}[1,2] = pF_{wet} - pF_{eq} = 3.0 - 3.45 = \mathbf{-0.45\ pF} \qquad \text{[Eq. 2-A]}$$
$$U_{\text{dry-differential}}[1,2] = pF_{dry} - pF_{eq} = 4.5 - 3.45 = \mathbf{1.05\ pF} \qquad \text{[Eq. 2-B]}$$

$$U_{wet}(y[1]) = pF_{eq} + U_{\text{wet-differential}}[1]\, e^{-\left(\frac{3.171*10^{-8} * \pi}{\alpha_{diff}}\right)^{0.5} * \, y[1][\text{m}]\left(\frac{100\text{ cm}}{1\text{ m}}\right)} \qquad \text{[Eq. 5]}$$
$$= 3.45 + (-0.45) * e^{-\left(\frac{3.171*10^{-8} * \pi}{0.003}\right)^{0.5} * \, 1.18[\text{m}]\left(\frac{100\text{ cm}}{1\text{ m}}\right)} = \mathbf{3.22\ pF}$$

$$U_{dry}(y[1]) = pF_{eq} + U_{\text{dry-differential}}\, e^{-\left(\frac{3.171*10^{-8} * \pi}{\alpha_{diff}}\right)^{0.5} * \, y[\text{m}]\left(\frac{100\text{ cm}}{1\text{ m}}\right)} \qquad \text{[Eq. 6]}$$
$$= 3.45 + 1.05 * e^{-\left(\frac{3.171*10^{-8} * \pi}{0.003}\right)^{0.5} * \, 1.18[\text{m}]\left(\frac{100\text{ cm}}{1\text{ m}}\right)} = \mathbf{3.98\ pF}$$

$$U_{wet}(y[2]) = pF_{eq} + U_{wet\text{-}differential}[1]\ e^{-\left(\frac{3.171*10^{-8}*\pi}{\alpha_{diff}}\right)^{0.5} *\ y[1][m]\left(\frac{100\ cm}{1\ m}\right)}$$

$$= 3.45 + (-0.45) *\ e^{-\left(\frac{3.171*10^{-8}*\pi}{0.003}\right)^{0.5} *\ 1.10[m]\left(\frac{100\ cm}{1\ m}\right)} = \mathbf{3.21\ pF}$$

$$U_{dry}(y[2]) = pF_{eq} + U_{dry\text{-}differential}\ e^{-\left(\frac{3.171*10^{-8}*\pi}{\alpha_{diff}}\right)^{0.5} *\ y[2][m]\left(\frac{100\ cm}{1\ m}\right)}$$

$$= 3.45 + 1.05 *\ e^{-\left(\frac{3.171*10^{-8}*\pi}{0.003}\right)^{0.5} *\ 1.10[m]\left(\frac{100\ cm}{1\ m}\right)} = \mathbf{4.01\ pF}$$

$$Z_m[1][ft] = Z_i[m]*\left(\frac{3.2808\ ft}{m}\right) * \left(10^{U_{wet}(y[1]) - \frac{1}{2}\left(pF_{eq}+U_{dry}(y[1])\right)}\right)^{-\left(1+\frac{0.4343}{0.5\left(\frac{pF_{eq}+U_{dry}(y[1])}{2}+U_{wet}(y[1])\right)-6.032}\right)} \text{[Eq. 7]}$$

$$= 0.8\ [m]*\left(\frac{3.2808\ ft}{m}\right) * 10^{\left(3.22 - \frac{1}{2}(3.45+3.98)\right)^{-\left(1+\frac{0.4343}{0.5\left(\frac{3.45+3.98}{2}+3.22\right)-6.032}\right)}} = \mathbf{6.7\ ft}$$

$$Z_m[2][ft] = Z_i[m]*\left(\frac{3.2808\ ft}{m}\right)$$

$$* \left(10^{U_{wet}(y[2]) - \frac{1}{2}\left(pF_{eq}+U_{dry}(y[2])\right)}\right)^{-\left(1+\frac{0.4343}{0.5\left(\frac{pF_{eq}+U_{dry}(y[2])}{2}+U_{wet}(y[2])\right)-6.032}\right)}$$

$$= 0.8\ [m]*\left(\frac{3.2808\ ft}{m}\right) * 10^{\left(3.21 - \frac{1}{2}(3.45+4.01)\right)^{-\left(1+\frac{0.4343}{0.5\left(\frac{3.45+4.01}{2}+3.21\right)-6.032}\right)}} = \mathbf{7.0\ ft}$$

The upper layer of clay, 4 feet thick, has a Z_m of 6.7 feet. The second layer of clay, beginning below 4 feet, has a Z_m of 7.0 feet. The weighted average of Z_m across the soil profile is:

$$Z_{m\ weighted} = \frac{(6.7 * 4\ ft\ [Layer\ 1]) + (7.0 * 10\ ft\ [Layer\ 2])}{(4\ ft + 10\ ft)} = 6.9\ ft, \text{ round up to } \mathbf{7\ ft}$$

The depth of the movement active zone, Z_a, is assumed to be equal to Z_m, though in reality it may be shallower.

$$Z_a = Z_{m\ weighted} - \frac{\sigma_{surcharge}}{\gamma_d\left(1+\frac{w}{100}\right)} = 7\ ft - \frac{150\ psf}{102\ pcf\left(1+\frac{32}{100}\right)} = \mathbf{5.9\ ft}$$

The following is calculated:

$$\gamma_t[1]=\gamma_d[1]\left(1+\frac{w[1]}{100}\right)=102\text{pcf}*\left(1+\frac{32}{100}\right)=\mathbf{134.6\ pcf}=\boldsymbol{\gamma_t[2]}$$

$$\alpha = 0.55 - 0.1\left(\left(\frac{S_u}{2116}\right) - 1.5\right) \qquad 0.45 \le \propto \le 0.55 \qquad \text{[Eq. 15]}$$

$$\alpha[1] = 0.55 - 0.1\left(\left(\frac{S_u}{2116}\right) - 1.5\right) = 0.55 - 0.1\left(\left(\frac{0.9\ \text{tsf} * \frac{2000\ \text{psf}}{1\ \text{tsf}}}{2116}\right) - 1.5\right)$$

$$= \mathbf{0.61},\ \textbf{use}\ \mathbf{0.55}.$$

$$\alpha[2] = 0.55 - 0.1\left(\left(\frac{1.8\ \text{tsf} * \frac{2000\ \text{psf}}{1\ \text{tsf}}}{2116}\right) - 1.5\right) = \mathbf{0.53}$$

$$N_c = 10.25 - \left(\frac{2812.5}{S_u+250}\right) \qquad 6.5 \le N_c \le 9.0 \qquad \text{[Eq. 9]}$$

$$N_c[1] = 10.25 - \left(\frac{2812.5}{0.9\ \text{tsf} * \frac{2000\ \text{psf}}{1\ \text{tsf}} + 250}\right) = \mathbf{8.88}$$

$$N_c[2] = 10.25 - \left(\frac{2812.5}{1.8\ \text{tsf} * \frac{2000\ \text{psf}}{1\ \text{tsf}} + 250}\right) = \mathbf{9.52,\ use\ 9.0}.$$

The base resistance of the clay for each layer is:

$$q_B = N_c * s_u \qquad \text{for } \frac{L}{D*\frac{1}{12}} \ge 3 \qquad \text{[Eq. 8]}$$

$$q_B[1] = N_c[1] * s_u[1] = 8.88 * 0.9\text{tsf}\left(\frac{2000\ \text{pounds}}{\text{ton}}\right) = \mathbf{15{,}984\ psf}$$

$$q_B[2] = N_c[2] * s_u[2] = 9.0 * 1.8\text{tsf}\left(\frac{2000\ \text{pounds}}{\text{ton}}\right) = \mathbf{32{,}400\ psf}$$

The side resistance of the pier for each layer is calculated as:

$$q_S = \alpha * s_u * RF_S \qquad \text{[Eq. 14]}$$

$$q_S[1] = \alpha[1] * s_u[1] * RF_S[1] = 0.55 * 0.9\text{tsf}\left(\frac{2000\ \text{pounds}}{\text{ton}}\right) * 1.0 = \mathbf{990\ psf}$$

$$q_S[2] = \alpha[2] * s_u[2] * RF_S[2] = 0.53 * 1.8\text{tsf}\left(\frac{2000\text{ pounds}}{\text{ton}}\right) * 1.0 = \mathbf{1908\ psf}$$

To summarize:

Layer Number	[1]	[2]
Soil Description	Fat clay	Fat clay
Layer Penetration, z (ft)	4	20
q_B	15,984 psf	32,400 psf
q_S	990 psf	1908 psf

3.1.1 Upward Case of Sample Calculation 1

For a pier diameter, d, of 12 inches, a pier depth of 14 feet, no bell, and Z_a of 6 feet (5.9, rounded up), it is assumed that all the soil from grade to 6 feet is acting upward on the pier. Pier loading, Q_T, is upward, as a worst-case scenario. The weight of the pier, R_W, acts downward; the soil below 6 feet is resisting uplift and therefore is a resistance. Base resistance is zero because the pier is moving up. See Figure 2-9 for a generic force diagram.

$$\sum Q \le \sum \frac{R}{\text{S.F.}} \qquad \text{[Eq. 22-A]}$$

$$Q_T + Q_S \le R_W + \frac{R_S}{\text{S.F.}_S} \qquad \text{[Eq. 23]}$$

$Q_T = \mathbf{3\ kips}$ from the problem statement.

$$Q_S = \left[\left(990\text{ psf} * 4\text{ft} * \pi * 12\text{ in} * \frac{1\text{ ft}}{12\text{ in}}\right) + \left(1908\text{ psf} * (6\text{ft} - 4\text{ft}) * \pi * 12\text{ in} * \frac{1\text{ ft}}{12\text{ in}}\right)\right] * \frac{1}{1.0\text{ S.F.}} = 24{,}429\text{ lb} = \mathbf{24.4\ kips}$$

$$R_W = 145\text{ pcf} * 14\text{ft} * \pi * \left(\frac{12\text{ in} * \frac{1\text{ ft}}{12\text{ in}}}{2}\right)^2 = 1594\text{ lb} = \mathbf{1.6\ kips}$$

$$R_S = \left(1908\text{ psf} * (14\text{ft} - 6\text{ ft}) * \pi * 12\text{ in} * \frac{1\text{ ft}}{12\text{ in}}\right) = 47{,}950\text{ lb} = \mathbf{48.0\ kips}$$

Compute the side safety factor, per Table 2-3.

$$\text{S.F.}_S = \frac{Q_T}{3\,Q_S} + 1 = \frac{3\text{ kips}}{3 * 24.4\text{ kips}} + 1 = \mathbf{1.04}$$

Summing the loads and resistances for the upward case, check that the loads are less than the resistances.

$$Q_T + Q_S \leq R_W + \frac{R_S}{S.F._S} \qquad [\text{Eq. } 23]$$

$$3 \text{ kips} + 24.4 \text{ kips} \leq 1.6 \text{ kips} + \frac{48.0 \text{ kips}}{1.04}$$

27.4 kips ≤ 47.7 kips ; TRUE

Unity check:

$$\frac{\text{Loads}}{\text{Resistances}} = \frac{27.4 \text{ kips}}{47.7 \text{ kips}} = \mathbf{0.57} \leq 1.00 \qquad [\text{Eq. 22-B}]$$

The unity check is less than one; the 14' deep 12" diameter pier is adequate for the upward case. The unity check is much less than 1, indicating that a shorter pier might be acceptable. If the user wished to revise the calculations using a shorter pier, one would find that an 11' deep pier is adequate for the upward case. However, a 14' deep pier is required for the downward case.

3.1.2 Downward Case of Sample Calculation 1

For a pier diameter of 12 inches, a pier depth of 14 feet, and Z_a of 6 feet (5.9', rounded up), it is assumed that all the soil from grade to 6 feet has pulled away from the pier due to drying out and shrinkage of the soil. Pier loading, Q_T, is downward, as a worst-case scenario. The weight of the pier, Q_W, acts downward. The soil below Z_a is resisting the downward loads and therefore all side resistance is an upward load. The base is also resisting the downward load and therefore is an upward resistance. See Figure 2-10 for a generic force diagram.

$$\sum Q \leq \sum \frac{R}{S.F.} \qquad [\text{Eq. } 22 - A]$$

$$Q_T + Q_W + Q_S \leq \frac{R_S}{S.F._S} + \frac{R_B}{S.F._B} \qquad [\text{Eq. } 25]$$

$Q_T = \mathbf{30\ kips}$ from the problem statement.

$$Q_W = 145 \text{ pcf} * 14\text{ft} * \pi * \left(\frac{12 \text{ in} * \frac{1 \text{ ft}}{12 \text{ in}}}{2}\right)^2 = 1594 \text{ lb} = \mathbf{1.6\ kips}$$

Note that $Q_S = 0$ because the soil has pulled away from the pier in the movement zone, see Figure 2-3.

$$\frac{R_S}{S.F._S} = \left(1908 \text{ psf} * (14\text{ft} - 6 \text{ ft}) * \pi * 12 \text{ in} * \frac{1 \text{ ft}}{12 \text{ in}}\right) * \frac{1}{2.0 \text{ S.F.}} = 23{,}980 \text{ lb} = \mathbf{24.0 \text{ kips}}$$

$$\frac{R_B}{S.F._B} = \left(32{,}400 \text{ psf} * \pi * \left(\frac{12 \text{ in} * \frac{1 \text{ ft}}{12 \text{ in}}}{2}\right)^2\right) * \frac{1}{3.0 * r_{bell} \text{ [S.F.]}} = 8480 \text{ lb} = \mathbf{8.5 \text{ kips}}$$

$$Q_T + Q_W + Q_S \leq \frac{R_S}{S.F._S} + \frac{R_B}{S.F._B} \qquad \text{[Eq. 25]}$$

$$30 \text{ kips} + 1.6 \text{ kips} + 0 \leq 24.0 \text{ kips} + 8.5 \text{ kips}$$

$\mathbf{31.6 \text{ kips} \leq 32.5 \text{ kips ;}}$ TRUE

Unity check:

$$\frac{\text{Loads}}{\text{Resistances}} = \frac{31.6 \text{ kips}}{32.5 \text{ kips}} = \mathbf{0.97} \leq 1.00 \qquad \text{[Eq. 22-B]}$$

The unity check is less than one; the 14’ deep 12” diameter pier is adequate for the downward case.

3.1.3 Pier Reinforcing

Design the reinforcing per Section 2.6 with the following input from the sample calculation above:

f_y = 60 ksi
diameter of the pier shaft = 12 inches
Z_a, the length of the active zone = 6 feet
Q_T = 3 kips (pier top load, upward)
Q_S = 24.4 kips (side load, upward)
Required ρ, the ratio of the area of the steel to the area of the concrete = 0.50%, min.
AG_{max} = 0.75 inches
cover = 3 inches

$$n_{r\text{-}min} = \left(\frac{f_y}{325}\right)^3 d^2(d_c) \qquad \text{[-], integer, round up} \quad \text{[Eq. 33]}$$

$$\text{where } d_c = \frac{d_{bar}}{2} + \text{cover} + d_{tie} \qquad \text{[in.]} \quad \text{[Eq. 29]}$$

$$d_c = \frac{\left(\frac{4}{8} \text{ in}\right)}{2} + 3 \text{ in} + 0.375 \text{ in} = \mathbf{3.6 \text{ in.}}$$

$n_{r\text{-}min} = \left(\frac{60\text{ ksi}}{325}\right)^3 (12\text{ in})^2(3.6\text{ in}) = \mathbf{3.3, round\ up\ to\ 4}$, meaning that 4 bars are the minimum number to be used in order to limit crack width c_w to 0.012".

Check $n_{r\text{-}max}$ for AG_{max} = 3/4 inch, using #4 longitudinal bars with d_{bar} = 0.5".

$$n_{r\text{-}max} = \frac{\pi\,(d - 2d_c)}{3\,AG_{max} + d_{bar}}$$ [-], integer, round down [Eq. 34]

$n_{r\text{-}max} = \frac{\pi\,(12\text{ in} - 2 * (3.6\text{ in})}{3 * 0.75\text{ in} + 0.5\text{ in}}$ = 5.5, meaning that the maximum number of longitudinal reinforcing bars is 5.

Solving for the maximum pier tension, T, due to soil side load in the movement zone (Q_S) and the pier top load (Q_T)

$$T = \sum Q$$ [kips] [Eq. 35]

$T = Q_T + Q_S = 3\text{ kips} + 24.4\text{ kips} = \mathbf{27.4\ kips}$

Calculate the required steel area:

$$A_{st_{REQ}} = \frac{T}{f_s} = \frac{T}{0.40 f_y} = \frac{27.4\text{ kips}}{0.40(60\text{ ksi})} = \mathbf{1.1\ \ in^2}$$ [Eq. 36]

The actual steel area is calculated according to the following equation:

$$\# = 8\,d_{bar} = 9.04\left(\frac{A_{st_{REQ}}}{n_r}\right)^{1/2}$$ [-], bar size, integer, round up [Eq. 40]

$$\# = 9.04\left(\frac{1.1\text{ inches}^2}{4\text{ bars}}\right)^{\frac{1}{2}} = \mathbf{4.7, round\ up\ to\ 5}.$$

The above calculation shows 4**#5** is adequate.

Check that the minimum steel ratio, ρ = 0.5% is met.

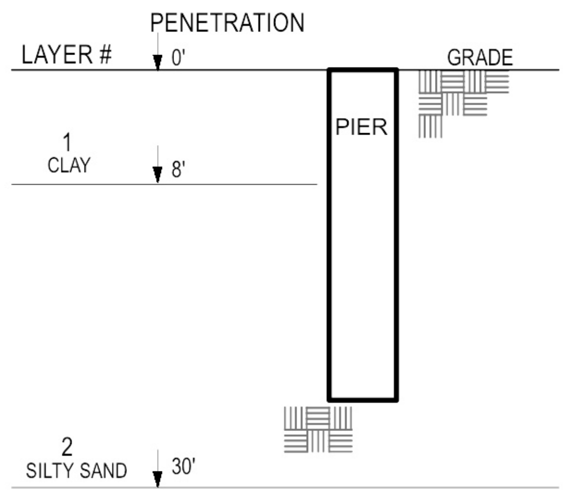
$$\rho = 1.56\,n_r\left(\frac{\#}{d}\right)^2 = 1.56\,(4)\left(\frac{5}{12}\right)^2 = \mathbf{1.08}$$ [%] ; ≥ 0.50%; **TRUE** [Eq. 41]

4#5 reinforcing bars meet the requirements of [Eq. 33], [Eq. 34], and [Eq. 41]. One may iterate to determine whether fewer #5 bars or a quantity of #3 or #4 bars would satisfy the requirements.

3.2 SAMPLE CALCULATION 2 – CLAY AND SAND IN TYLER, WITH TREES

Sample Calculation 2 is for a Tyler, TX soil with clay and sand strata and the parameters in the following table, on a lot with past trees, with pier loading of Q_T = 5 kips per pier upward in the upward case and Q_T = 20 kips per pier downward in the downward case. The water table is at 35 feet. Assume a pier penetration (L) of 20 feet.

Layer Number	[1]	[2]
Soil Description	Clay	Silty Sand
Layer Penetration, z (ft)	8	30
Liquid Limit, LL (%)	35	-
Plastic Limit, PL (%)	16	-
s_u (tsf)	2.25	-
Moisture Content, w (%)	17	17
Dry Unit Weight of Soil γ_d (pcf)	102	102
N_{60}	-	24
m	-	0.8
RF_S	1.0	1.0



n = 0.35π ("tree" case per Section 2.3)
α_{diff} = 0.015 ("tree" case per Section 2.3)
Z_i = 0.8m (typical value per Section 2.3)
λ = 4.63 (LL < 50)
Wet Suction Boundary Condition at Grade = 3.0 pF
Dry Suction Boundary Condition at Grade = 4.5 pF
TMI = 18 for Tyler, Texas, extrapolated from Thornthwaite Moisture Index map (Section 2.3)
Surcharge = 0

Wet Suction Boundary Condition at Grade = 3.0 pF (standard for Tyler)
Dry Suction Boundary Condition at Grade = 4.5 pF (standard for Tyler)

Check that the moisture contents do not exceed the wet or dry suction boundary conditions at grade as discussed in Step 2. For pF_{wet} = 3.0 and pF_{dry} = 4.5 the boundary equation is: pF = -(4.6/LL) * w + 5.6

$pF_{[1]} = -(4.6/35) * 17 + 5.6 = 3.35$; $pF_{wet} < 3.36 < pF_{dry}$ [Eq. 3]

For the cohesive layer, the suction as a function of the liquid limit and moisture content is within the boundary conditions.

For the clay layer, layer [1]:

Calculate ϕ'
$\phi' = 0.0016\,(PI)^2 - 0.3021\,(PI) + 36.208$, limited by: [Eq. 4-B]

$30 < \phi' < 34$ for LL < 50
$23 < \phi' < 27$ for LL ≥ 50

$$\phi'[1] = 0.0016\,(35 - 16)^2 - 0.3021\,(35 - 16) + 36.208 = \mathbf{31.0°}$$

In layer [1], the calculated soil friction angle is between the limits for LL < 50 .

Calculate K_0

$$K_0 = 1 - \sin\phi' \qquad \text{[Eq. 4-A]}$$

$$K_0[1] = 1 - \sin\phi'[1] = 1 - \sin(31.0°) = \mathbf{0.485}$$

Calculate the depth, y, at which to impose the suction boundaries.

$$y = \left(\frac{Z_i * \lambda}{2K_0+1}\right)\left(\frac{\phi'}{35°}\right)^n \qquad \text{[Eq. 4]}$$

$$y[1] = \left(\frac{0.8[\text{m}] \ * \ 4.63}{2 \ * \ 0.485 \ + \ 1}\right)\left(\frac{31°}{35°}\right)^{0.35\pi} = \mathbf{1.65\ m}$$

Calculate the Equilibrium Suction, $U_{\text{wet-differential}}$, $U_{\text{dry-differential}}$, U_{wet} at depth y, U_{dry} at depth y, and then the depth of the movement active zone Z_m

$$pF_{eq} = 3.659\ e^{(-0.0033 * \text{TMI})} = 3.659\ e^{(-0.0033 * 18)} = \mathbf{3.45\ pF} \qquad \text{[Eq. 1]}$$

$$U_{\text{wet-differential}}[1] = pF_{wet} - pF_{eq} = 3.0 - 3.45 = \mathbf{-0.45\ pF} \qquad \text{[Eq. 2-A]}$$

$$U_{\text{dry-differential}}[1] = pF_{dry} - pF_{eq} = 4.5 – 3.45 = \mathbf{1.05\ pF} \qquad \text{[Eq. 2-B]}$$

$$U_{wet}(y[1]) = pF_{eq} + U_{\text{wet-differential}}[1]\ e^{-\left(\frac{3.171*10^{-8} * \pi}{\alpha_{diff}}\right)^{0.5} * \ y[1][\text{m}]\left(\frac{100\text{ cm}}{1\text{ m}}\right)} \qquad \text{[Eq. 5]}$$

$$= 3.45 + (-0.45) * \ e^{-\left(\frac{3.171*10^{-8} * \pi}{0.015}\right)^{0.5} * \ 1.65[\text{m}]\left(\frac{100\text{ cm}}{1\text{ m}}\right)} = \mathbf{3.16\ pF}$$

$$U_{dry}(y[1]) = pF_{eq} + U_{\text{dry-differential}}\ e^{-\left(\frac{3.171*10^{-8} * \pi}{\alpha_{diff}}\right)^{0.5} * \ y[\text{m}]\left(\frac{100\text{ cm}}{1\text{ m}}\right)} \qquad \text{[Eq. 6]}$$

$$= 3.45 + 1.05 * \ e^{-\left(\frac{3.171*10^{-8} * \pi}{0.015}\right)^{0.5} * \ 1.65[\text{m}]\left(\frac{100\text{ cm}}{1\text{ m}}\right)} = \mathbf{4.14\ pF}$$

$$Z_m[1][ft] = Z_i[m] * \left(\frac{3.2808\text{ ft}}{m}\right) * \left(10^{U_{wet}(y[1]) - \frac{1}{2}\left(pF_{eq} + U_{dry}(y[1])\right)}\right)^{-\left(1+\frac{0.4343}{0.5\left(\frac{pF_{eq}+U_{dry}(y[1])}{2}+U_{wet}(y[1])\right)-6.032}\right)} \quad \text{[Eq. 7]}$$

$$= 0.8\ [m] * \left(\frac{3.2808\text{ ft}}{m}\right) * \left(10^{3.16 - \frac{1}{2}(3.45+4.14)}\right)^{-\left(1+\frac{0.4343}{0.5\left(\frac{3.45+4.14}{2}+3.16\right) - 6.032}\right)} = \mathbf{8.9\ ft}$$

The top layer of clay, 8 feet thick, has a Z_m of 8.9 feet. This means that if the clay layer were deeper than 8 feet, and not underlain by a sand layer, Z_m would be 8.9 feet. However, the sand in layer 2 starts at 8 feet penetration, is cohesionless and inactive, and not underlain by an active soil, so we use 8 feet for the maximum Z_m, which is the total depth of the clay layer.

The depth of the movement active zone, Z_a, is assumed to be equal to Z_m, though in reality it may be shallower.

$$Z_a = Z_m - \frac{\sigma_{surcharge}}{\gamma_d\left(1+\frac{w}{100}\right)} = 8\text{ ft} - \frac{0\text{ psf}}{102\text{ pcf}\left(1+\frac{17}{100}\right)} = \mathbf{8\ ft}$$

The following is calculated for the clay layer:

$$\gamma_t[1] = \gamma_d[1]\left(1+\frac{w[1]}{100}\right) = 102\text{pcf} * \left(1+\frac{17}{100}\right) = \mathbf{119.3\ pcf}$$

$$\alpha = 0.55 - 0.1\left(\left(\frac{S_u}{2116}\right) - 1.5\right) \qquad 0.45 \le \propto \le 0.55 \qquad \text{[Eq. 15]}$$

$$\alpha[1] = 0.55 - 0.1\left(\left(\frac{S_u}{2116}\right) - 1.5\right) = 0.55 - 0.1\left(\left(\frac{2.25\text{ tsf} * \frac{2000\text{ psf}}{1\text{ tsf}}}{2116}\right) - 1.5\right) = \mathbf{0.49}$$

$$N_c = 10.25 - \left(\frac{2812.5}{S_u+250}\right) \qquad 6.5 \le N_c \le 9.0 \qquad \text{[Eq. 9]}$$

$$N_c[1] = 10.25 - \left(\frac{2812.5}{2.25\text{ tsf} * \frac{2000\text{ psf}}{1\text{ tsf}} + 250}\right) = \mathbf{9.66},\ \textbf{use 9.0}.$$

The bearing strength of the clay at the base of the pier is not necessary to compute as the pier will be founded in sand.

The side load on the pier for layer [1] is calculated as:

$$q_S = \alpha * s_u * RF_S \qquad \text{[Eq. 14]}$$

$$q_S[1] = \alpha[1] * s_u[1] * RF_S[1] = 0.49 * 2.25\text{tsf} * \left(\frac{2000 \text{ pounds}}{\text{ton}}\right) * 1.0 = \mathbf{2205\ psf}$$

Methodology for calculating the sand's strength comes from USDOT FHWA 2010 *Drilled Shafts: Construction Procedures and LRFD Design Methods*, Chapter 13[6]. Sand calculations are as follows:

$$\phi'[2] = 27.5 + 9.2 \log(N_{60}) = 27.5 + 9.2 \log(24) = \mathbf{40.2°}$$

$$K_p[2] = \tan\left(45° + \frac{\phi'}{2}\right)^2 = \tan\left(45° + \frac{40.2°}{2}\right)^2 = \mathbf{4.6}$$

$$\gamma_t[2] = \gamma_d[2]\left(1 + \frac{w[2]}{100}\right) = 102 \text{ pcf} * \left(1 + \frac{17}{100}\right) = \mathbf{119.3\ pcf}$$

$\gamma_t'[2] = \gamma_t[2] = \mathbf{119.3\ \ pcf}$ as water table is below the layer

$$\sigma_p'[2] = 990 * (N_{60})^m = 990 * (24)^{0.8} = \mathbf{12584\ psf}$$

Note that m = 0.6 clean sand
0.8 silty sand

$$\sigma_v'[2] = \sigma_{soil}' + \sigma_{surcharge}$$

$$= 119.3 \text{ pcf} * 8 \text{ ft} + 119.3 \text{ pcf} * \frac{(22\text{ft} - 8\text{ft})}{2} + 0 \text{ surcharge} = \mathbf{1670\ psf}$$

$$\beta[2] = (1 - \sin\phi[2]')\left(\frac{\sigma_p'}{\sigma_v'}\right)^{\sin\phi[2]'} * \tan\phi[2]' \leq K_p[2]\tan\phi[2]'$$

$$= (1 - \sin 40.2°)\left(\frac{12584}{1670}\right)^{\sin 40.2°} * \tan 40.2° \leq 4.6 \tan 40.2°$$

$$= \mathbf{1.103} \leq 3.89; \text{ use } 1.103$$

$$q_B[2] = 1200 * N60 = 1200 * 24 = \mathbf{28,800\ psf}$$

Note: q_B [2], a sandy soil, must be less than or equal to 60,000

$$q_S[2] = \sigma_v'[2] * \beta * RF_S[2] = 1670 \text{ psf} * 1.103 * 1.0 = \mathbf{1843\ psf}$$

To summarize:

Layer Number	[1]	[2]
Soil Description	Clay	Silty Sand
Layer Penetration (ft)	8	30
q_B	N/A	28,800 psf
q_S	2205 psf	1843 psf

3.2.1 Upward Case of Sample Calculation 2

For a pier diameter of 12 inches, a pier depth of 20 feet, and Z_a of 8.0 feet, it is assumed that all the soil from grade to 8 feet is acting upward on the pier. Pier loading, Q_T, is upward, as a worst-case scenario. The weight of the pier, R_W, acts downward, resisting the upward movement. The soil below 8 feet is resisting uplift and therefore is a resistance. There is no base resistance.

$$\sum Q \leq \sum \frac{R}{\text{S.F.}} \qquad [\text{Eq. } 22 - \text{A}]$$

$$Q_T + Q_S \leq R_W + \frac{R_S}{\text{S.F.}_S} \qquad [\text{Eq. } 23]$$

$Q_T = \mathbf{5\ kips}$ from the problem statement.

$$Q_S = \left(2205 \text{ psf } * 8\text{ft} * \pi * 12 \text{ in} * \frac{1 \text{ ft}}{12 \text{ in}}\right) * \frac{1}{1.0 \text{ S.F.}} = 55{,}418 \text{ lb} = \mathbf{55.4\ kips}$$

$$R_W = 145 \text{ pcf} * 20\text{ft} * \pi * \left(\frac{12 \text{ in} * \frac{1 \text{ ft}}{12 \text{ in}}}{2}\right)^2 = 2280 \text{ lb} = \mathbf{2.3\ kips}$$

Compute the side safety factor, per Table 2-3 for cohesionless soil.

$$\text{S.F.}_S = \frac{Q_T}{3\ Q_S} + 1.1 = \frac{5 \text{ kips}}{3 * 55.1 \text{ kips}} + 1.1 = \mathbf{1.13}$$

$$\frac{R_S}{\text{S.F.}_S} = \left(1843 \text{ psf } * (20\text{ft - 8 ft}) * \pi * 12 \text{ in} * \frac{1 \text{ ft}}{12 \text{ in}}\right) * \frac{1}{1.13 \text{ [S.F.]}} = 61{,}486 \text{ lb} = \mathbf{61.5\ kips}$$

Summing the loads and resistances for the upward case, check that the loads are less than the resistances.

$$Q_T + Q_S \leq R_W + \frac{R_S}{\text{S.F.}_S} \qquad [\text{Eq. } 23]$$

$$5 \text{ kips} + 55.4 \text{ kips} \leq 2.3 \text{ kips} + 61.5 \text{ kips}$$

$\mathbf{60.4\ kips \leq 63.8\ kips}$; TRUE

Unity check:

$$\frac{\text{Loads}}{\text{Resistances}} = \frac{60.4\text{ kips}}{63.8\text{ kips}} = \mathbf{0.95} \le 1.00 \qquad \text{[Eq. 22-B]}$$

The unity check is less than one; the 20’ deep 12” diameter pier is adequate for the upward case.

3.2.2 Downward Case of Sample Calculation 2

For a pier diameter of 12 inches, a pier depth of 20 feet, and Z_a of 8.0 feet, it is assumed that all the soil from grade to 8 feet penetration has pulled away from the pier due to drying out and shrinkage of the soil. Pier loading, Q_T, is downward, as a worst-case scenario. The weight of the pier, Q_W, acts downward. The soil below Z_a is resisting the downward movement of the pier and therefore the side resistance is a resistance. The base is also resisting the downward movement and is a resistance.

$$\sum Q \le \sum \frac{R}{\text{S.F.}} \qquad [\text{Eq. } 22 - \text{A}]$$

$$Q_T + Q_W + Q_S \le \frac{R_S}{\text{S.F.}_S} + \frac{R_B}{\text{S.F.}_B} \qquad [\text{Eq. } 25]$$

$Q_T = \mathbf{20\ kips}$ from the problem statement.

$$Q_W = 145\text{ pcf} * 20\text{ft} * \pi * \left(\frac{12\text{ in} * \frac{1\text{ ft}}{12\text{ in}}}{2}\right)^2 = 2280\text{ lb} = \mathbf{2.3\ kips}$$

Note that $Q_S = \mathbf{0}$ because the soil has pulled away from the pier in the movement zone, see Figure 2-3.

$$\frac{R_S}{\text{S.F.}} = \left(1843\text{ psf} * (20\text{ft} - 8\text{ ft}) * \pi * 12\text{ in} * \frac{1\text{ ft}}{12\text{ in}}\right) * \frac{1}{2.2\text{ S.F.}}$$
$$= 31{,}580\text{ lb} = \mathbf{31.6\ kips}$$

$$\frac{R_B}{\text{S.F.}} = \left(28{,}800\text{ psf} * \pi * \left(\frac{12\text{ in} * \frac{1\text{ ft}}{12\text{ in}}}{2}\right)^2\right) * \frac{1}{10.0\text{ S.F.}} = 2260\text{ lb} = \mathbf{2.3\ kips}$$

$$Q_T + Q_W + Q_S \le \frac{R_S}{\text{S.F.}_S} + \frac{R_B}{\text{S.F.}_B} \qquad [\text{Eq. } 25]$$

$$20\text{ kips} + 2.3\text{ kips} + 0\text{ kips} \le 31.6\text{ kips} + 2.3\text{ kips}$$

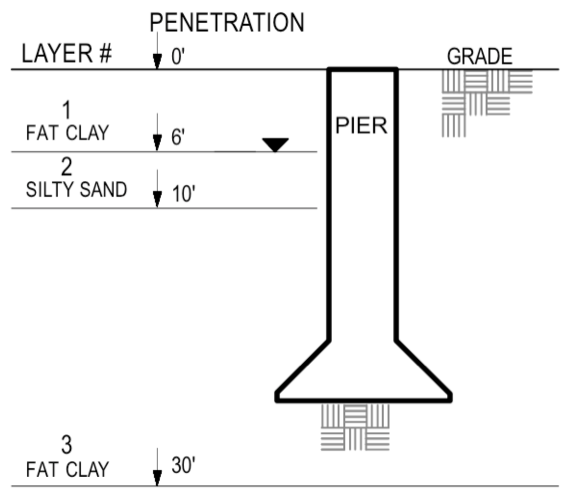
$\mathbf{22.3\ kips \le 33.9\ kips}$; TRUE

Unity check:

$$\frac{\text{Loads}}{\text{Resistances}} = \frac{22.3\text{ kips}}{33.9\text{ kips}} = \mathbf{0.66} \le 1.00 \qquad \text{[Eq. 22-B]}$$

The unity check is less than one, therefore the 20’ deep 12” diameter pier is adequate for the downward case. If the user wished to revise the calculations using a shorter pier, one would find that a 16’ deep pier is adequate for the downward case; however, a 20’ deep pier is required for the upward case.

3.3 SAMPLE CALCULATION 3 – CLAY AND SAND IN TULSA, WITH TREES AND HIGH WATER TABLE



Sample Calculation 3 is for a Tulsa, OK soil with clay and sand strata, the parameters in the following table, on a lot with trees, with pier loading of Q_T = 5 kips per pier upward in the upward case and Q_T = 30 kips per pier downward in the downward case. The water table is constant at 6 feet. Assume a pier depth (L) of 17 feet.

Layer Number	[1]	[2]	[3]
Soil Description	Lean Clay	Clean Sand	Fat Clay
Layer Penetration, z (ft)	6	10	30
Liquid Limit, LL (%)	45	-	80
Plastic Limit, PL (%)	15	-	20
s_u (tsf)	2.25	-	2
Moisture Content, w (%)	17	17	20
Dry Unit Weight of Soil γ_d (pcf)	102	102	102
N_{60}	-	20	-
m	-	0.6	-
RF_S	1.0	0.7	1.0

n[1] $=0.35\pi$ (“tree” case per Section 2.3) and n[3] $= 0.625\pi$

$\alpha_{diff} = 0.015\ \frac{\text{cm}^2}{\text{sec}}$ (“tree” case per Section 2.3)

Z_i = 0.8m (constant per Section 2.3)

$\lambda[1] = 4.63$) and $\lambda[3] = 3.75$ per Section 2.3

pF_{wet} at Grade = 3.0 pF

pF_{dry} at Grade = 4.5 pF
TMI = 18 for Tulsa, OK, extrapolated from Thornthwaite Moisture Index map (Section 2.3)
Surcharge = 0
RF_S = 0.7 for the sand layer because a permanent casing will be used

Wet Suction Boundary Condition at Grade = 3.0 pF (standard for Tulsa)
Dry Suction Boundary Condition at Grade = 4.5 pF (standard for Tulsa)

Check that the moisture contents do not exceed the wet or dry suction boundary conditions at grade as discussed in Step 2. For pF_{wet} = 3.0 and pF_{dry} = 4.5 the boundary equation is: pF = -(4.6/LL) * w + 5.6

$pF_{[1]} = -(4.6/45) * 17 + 5.6 = 3.86$; $pF_{wet} < 3.86 < pF_{dry}$ [Eq. 3]
$pF_{[3]} = -(4.6/80) * 20 + 5.6 = 4.45$; $pF_{wet} < 4.45 < pF_{dry}$ [Eq. 3]

For both cohesive layers, the suction as a function of the liquid limit and moisture content is within the boundary conditions.

For the clay layers, [1] and [3]:

Calculate ϕ'
$\phi' = 0.0016\,(PI)^2 - 0.3021\,(PI) + 36.208$, limited by: [Eq. 4-B]
$30 < \phi' < 34$ for LL < 50
$23 < \phi' < 27$ for LL $\geq$ 50

$$\phi'[1] = 0.0016\,(45-15)^2 - 0.3021\,(45-15) + 36.208 = \mathbf{28.6°}\textbf{, use 30.0°}$$

$$\phi'[3] = 0.0016\,(80-20)^2 - 0.3021\,(80-20) + 36.208 = \mathbf{23.8°}$$

Calculate K_0
$K_0 = 1 - \sin\phi'$ [Eq. 4-A]

$K_0[1] = 1 - \sin\phi'[1] = 1 - \sin(30.0°) = \mathbf{0.500}$
$K_0[3] = 1 - \sin\phi'[3] = 1 - \sin(23.8°) = \mathbf{0.596}$

Calculate the depth, y, at which to impose the suction boundaries.

$$y = \left(\frac{Z_i * \lambda}{2K_0 + 1}\right)\left(\frac{\phi'}{35}\right)^n \quad \text{[Eq. 4]}$$

$$y[1] = \left(\frac{0.8[m] * 4.63}{2 * 0.500 + 1}\right)\left(\frac{30°}{35°}\right)^{0.35\pi} = \mathbf{1.56\ m}$$

$$y[3] = \left(\frac{0.8[m] * 3.75}{2 * 0.596 + 1}\right)\left(\frac{23.8°}{35°}\right)^{0.625\pi} = \mathbf{0.642\ m}$$

Calculate the Equilibrium Suction, $U_{\text{wet-differential}}$, $U_{\text{dry-differential}}$, U_{wet} at depth y, U_{dry} at depth y, and then the depth of the movement active zone Z_m for Layers [1] and [3].

$$pF_{eq} = 3.659\, e^{(-0.0033 * \text{TMI})} = 3.659\, e^{(-0.0033 * 18)} = \mathbf{3.45\ pF} \qquad \text{[Eq. 1]}$$

$$U_{\text{wet-differential}}[1,3] = pF_{\text{wet}} - pF_{eq} = 3.0 - 3.45 = \mathbf{-0.45\ pF} \qquad \text{[Eq. 2-A]}$$

$$U_{\text{dry-differential}}[1,3] = pF_{\text{dry}} - pF_{eq} = 4.5 - 3.45 = \mathbf{1.05\ pF} \qquad \text{[Eq. 2-B]}$$

$$U_{\text{wet}}(y[1]) = pF_{eq} + U_{\text{wet-differential}}[1]\ e^{-\left(\frac{3.171*10^{-8} * \pi}{\alpha_{\text{diff}}}\right)^{0.5} * \ y[1][m]\left(\frac{100\ cm}{1\ m}\right)} \qquad \text{[Eq. 5]}$$

$$= 3.45 + (-0.45) * \ e^{-\left(\frac{3.171*10^{-8} * \pi}{0.015}\right)^{0.5} * \ 1.56[m]\left(\frac{100\ cm}{1\ m}\right)} = \mathbf{3.15\ pF}$$

$$U_{\text{dry}}(y[1]) = pF_{eq} + U_{\text{dry-differential}}\ e^{-\left(\frac{3.171*10^{-8} * \pi}{\alpha_{\text{diff}}}\right)^{0.5} * \ y[1][m]\left(\frac{100\ cm}{1\ m}\right)} \qquad \text{[Eq. 6]}$$

$$= 3.45 + 1.05 * \ e^{-\left(\frac{3.171*10^{-8} * \pi}{0.015}\right)^{0.5} * \ 1.56[m]\left(\frac{100\ cm}{1\ m}\right)} = \mathbf{4.15\ pF}$$

$$U_{\text{wet}}(y[3]) = pF_{eq} + U_{\text{wet-differential}}[3]\ e^{-\left(\frac{3.171*10^{-8} * \pi}{\alpha_{\text{diff}}}\right)^{0.5} * \ y[3][m]\left(\frac{100\ cm}{1\ m}\right)}$$

$$= 3.45 + (-0.45) * \ e^{-\left(\frac{3.171*10^{-8} * \pi}{0.015}\right)^{0.5} * \ 0.644[m]\left(\frac{100\ cm}{1\ m}\right)} = \mathbf{3.07\ pF}$$

$$U_{\text{dry}}(y[3]) = pF_{eq} + U_{\text{dry-differential}}\ e^{-\left(\frac{3.171*10^{-8} * \pi}{\alpha_{\text{diff}}}\right)^{0.5} * \ y[3][m]\left(\frac{100\ cm}{1\ m}\right)}$$

$$= 3.45 + 1.05 * \ e^{-\left(\frac{3.171*10^{-8} * \pi}{0.015}\right)^{0.5} * \ 0.644[m]\left(\frac{100\ cm}{1\ m}\right)} = \mathbf{4.34\ pF}$$

$$Z_m[1][ft] = Z_i[m] * \left(\frac{3.2808\ ft}{m}\right) * \left(10^{U_{\text{wet}}(y[1]) - \frac{1}{2}\left(pF_{eq} + U_{\text{dry}}(y[1])\right)}\right)^{-\left(1 + \frac{0.4343}{0.5\left(\frac{pF_{eq} + U_{\text{dry}}(y[1])}{2} + U_{\text{wet}}(y[1])\right) - 6.032}\right)} \qquad \text{[Eq. 7]}$$

$$= 0.8\ [m] * \left(\frac{3.2808\ ft}{m}\right) * \left(10^{3.15 - \frac{1}{2}(3.45 + 4.15)}\right)^{-\left(1 + \frac{0.4343}{0.5\left(\frac{3.45 + 4.15}{2} + 3.15\right) - 6.032}\right)} = \mathbf{9.1\ ft}$$

$$Z_m[3][ft] = Z_i[m]*\left(\frac{3.2808\ ft}{m}\right)$$
$$*\left(10^{U_{wet}(y[3]) - \frac{1}{2}\left(pF_{eq}+U_{dry}(y[3])\right)}\right)^{-\left(1+\frac{0.4343}{0.5\left(\frac{pF_{eq}+U_{dry}(y[3])}{2}+U_{wet}(y[3])\right)-6.032}\right)}$$
$$= 0.8\ [m] * \left(\frac{3.2808\ ft}{m}\right) * \left(10^{3.07\ -\ \frac{1}{2}(3.45+4.34)}\right)^{-\left(1+\frac{0.4343}{0.5\left(\frac{3.45\ +\ 4.34}{2}+3.07\right)\ -\ 6.032}\right)} = \mathbf{12.7\ ft}$$

The upper layer of clay, 6 feet thick, has a Z_m of 9.1 feet. The sand in layer 2 is cohesionless and inactive. The lower layer of clay, below 10 feet, has a Z_m of 12.7 feet. Normally, one would use 11.3 feet for Z_m, which is a weighted average determined by summing the products of the Z_m for each layer and the length of the pier in each layer, and dividing by the length of the pier in the clay:

$$Z_{m\ weighted} = \frac{(9.1 * 6\ ft\ [Layer\ 1]\ ) + (12.7 * 9\ ft\ [Layer\ 3])}{(6\ ft + 9\ ft)} = \mathbf{11.3\ ft}$$

However, the second layer is a thick layer of sand, and the ground water table in this example is constant at 6', which is the penetration depth of the first clay layer. Because the GWT is constant, the moisture content of the lower layer of clay is constant and therefore the soil volume is constant. Thus, the 6' penetration depth of the first clay layer is selected for Z_m. If $Z_{m\ weighted}$ were less than 6 feet, the smaller value would be used.

$$Z_m = \mathbf{6.0\ ft}$$

The depth of the movement active zone, Z_a, is assumed to be equal to Z_m, though in reality it may be shallower. In this case Z_a will be the smaller of 6 ft or 11.3 ft minus any soil surcharge.

$$Z_a = Z_m - \frac{\sigma_{surcharge}}{\gamma_d\left(1+\frac{w}{100}\right)} = 11.3\ ft - \frac{0\ psf}{102\ pcf\left(1+\frac{17}{100}\right)} = \mathbf{11.3\ ft; use\ 6\ ft}$$

The following is calculated for the clay layer:

$$\gamma_t[1] = \gamma_d[1]\left(1+\frac{w[1]}{100}\right) = 102pcf * \left(1+\frac{17}{100}\right) = \mathbf{119.3\ pcf}$$

$$\gamma_t[3] = \gamma_d[3]\left(1+\frac{w[3]}{100}\right) = 102pcf * \left(1+\frac{20}{100}\right) = \mathbf{122.4\ pcf}$$

$$\alpha = 0.55 - 0.1\left(\left(\frac{S_u}{2116}\right) - 1.5\right) \qquad 0.45 \le \alpha \le 0.55 \qquad [Eq.\ 15]$$

$$\alpha[1] = 0.55 - 0.1\left(\left(\frac{s_u}{2116}\right) - 1.5\right) = 0.55 - 0.1\left(\left(\frac{2.25 \text{ tsf } * \frac{2000 \text{ psf}}{1 \text{ tsf}}}{2116}\right) - 1.5\right) = \mathbf{0.49}$$

$$\alpha[3] = 0.55 - 0.1\left(\left(\frac{s_u}{2116}\right) - 1.5\right) = 0.55 - 0.1\left(\left(\frac{2 \text{ tsf } * \frac{2000 \text{ psf}}{1 \text{ tsf}}}{2116}\right) - 1.5\right) = \mathbf{0.51}$$

$$N_c = 10.25 - \left(\frac{2812.5}{S_u + 250}\right) \qquad 6.5 \le N_c \le 9.0 \qquad \text{[Eq. 9]}$$

$$N_c[1] = 10.25 - \left(\frac{2812.5}{2.25 \text{ tsf} * \frac{2000 \text{ psf}}{1 \text{ tsf}} + 250}\right) = \mathbf{9.66, use\ 9.0.}$$

$$N_c[2] = 10.25 - \left(\frac{2812.5}{2 \text{ tsf } * \frac{2000 \text{ psf}}{1 \text{ tsf}} + 250}\right) = \mathbf{9.59, use\ 9.0.}$$

The ultimate base resistance of the clay at the base of the pier is:

$$q_B = N_c * s_u$$

$$q_B[3] = N_c[3] * s_u[3] = 9.0 * 2.0 \text{ tsf} \left(\frac{\mathbf{2000\ pounds}}{\mathbf{ton}}\right) = \mathbf{36{,}000\ psf}$$

The ultimate side resistance on the pier for layer [1] is calculated as:

$$q_S = \alpha * s_u * RF_S \qquad \text{[Eq. 14]}$$

$$q_S[1] = \alpha[1] * s_u[1] * RF_S[1] = 0.49 * 2.25 \text{ tsf} * \left(\frac{2000 \text{ pounds}}{\text{ton}}\right) * 1.0 = \mathbf{2205\ psf}$$

$$q_S[3] = \alpha[3] * s_u[3] = 0.51 * 2.0 \text{ tsf} * \left(\frac{2000 \text{ pounds}}{\text{ton}}\right) * 1.0 = \mathbf{2040\ psf}$$

Calculating the side resistance in the cohesionless Layer 2:

$$\phi'[2] = 27.5 + 9.2 \log(N_{60}) = 27.5 + 9.2 \log(20) = \mathbf{39.5°} \qquad \text{[Eq. 16]}$$

$$K_p[2] = \tan^2\left(45° + \frac{\phi'}{2}\right) = \tan^2\left(45° + \frac{39.5°}{2}\right) = \mathbf{4.5} \qquad \text{[Eq. 17]}$$

$$\gamma_t[2] = \gamma_d[2]\left(1 + \frac{w[2]}{100}\right) = 102\ \text{pcf} * \left(1 + \frac{17}{100}\right) = \mathbf{119.3\ pcf}$$

$\gamma_t'[2] = \gamma_t[2] - 62.4 = \mathbf{56.9\ pcf}$ because the water table is at 6.0 ft. and therefore the entire cohesionless layer is buoyant.

$$\sigma_p'[2] = 990 * (N_{60})^m = 990 * (20)^{0.6} = \mathbf{5973\ psf} \qquad \text{[Eq. 18]}$$

where m = 0.6 clean sand
0.8 silty sand, sandy silt, clayey silt, etc.

$$\sigma_v'[2] = \sigma_{soil}' + \sigma_{surcharge}$$

$$= (119.3\ \text{pcf} * 6\ \text{ft}) + \left(56.9\ \text{pcf} * \frac{(10\text{ft} - 6\text{ft})}{2}\right) + 0\ \text{surcharge} = \mathbf{830\ psf}$$

$$\beta[2] = \left(1 - \sin\phi[2]'\right)\left(\frac{\sigma_{p[2]}'}{\sigma_{v[2]}'}\right)^{\sin\phi[2]'} * \tan\phi[2]' \leq K_p[2]\tan\phi[2]' \qquad \text{[Eq. 20]}$$

$$= (1 - \sin 39.5°)\left(\frac{5973}{830}\right)^{\sin 39.5°} * \tan 39.5° \leq 4.5\tan 39.5°$$

$$= \mathbf{1.05} \leq \mathbf{3.71;\ use\ 1.05}$$

$$q_B[2] = 1200 * N_{60} \leq 60{,}000\ \text{psf} = 1200 * 20 = \mathbf{24{,}000\ psf} \qquad \text{[Eq. 13]}$$

$$q_S[2] = \sigma_v'[2] * \beta[2] * RF_S[2] = 830\ \text{psf} * 1.05 * 0.7 = \mathbf{610\ psf} \qquad \text{[Eq. 21]}$$

To summarize:

Layer Number	[1]	[2]	[3]
Soil Description	Lean Clay	Silty Sand	Fat Clay
Layer Penetration (ft)	6	10	30
q_B	N/A	24,000 psf	36,000 psf
q_S	2205 psf	610 psf	2040 psf

3.3.1 Upward Case of Sample Calculation 3

For a pier diameter of 12 inches, bell diameter of 36 inches, a pier depth of 17 feet, and Z_a of 6 feet, it is assumed that all cohesive soil from grade to 6 feet is acting upward on the pier. The resistance provided by the upper face of the bell is reduced such that, over the length of the bell, only the surface area of the 12 inch diameter pier shaft is considered to resist heave rather than the surface area of the top of the bell (see Figure 2-2). Pier loading, Q_T, is upward, as a worst-case scenario. The weight of the pier, R_W, acts downward therefore it is a resistance. The soil below 10 feet is resisting uplift and therefore it is also a resistance. There is no base resistance.

$$\sum Q \leq \sum \frac{R}{S.F.} \qquad [Eq. 22 - A]$$

$$Q_T + Q_S \leq R_W + \frac{R_S}{S.F._S} \qquad [Eq. 23]$$

$$Q_S = \left(2205 \text{ psf} * 6\text{ft} * \pi * 12 \text{ in} * \frac{1 \text{ ft}}{12 \text{ in}}\right) * \frac{1}{1.0 \text{ S.F.}}$$

$$= 41{,}563 \text{ lb} = \mathbf{41.6 \text{ kips}}$$

Compute the side safety factor, per Table 2-3 for cohesionless soil.

$$S.F._S = \frac{Q_T}{3\, Q_S} + 1.1 = \frac{5 \text{ kips}}{3 * 41.7 \text{ kips}} + 1.1 = \mathbf{1.14}$$

Compute the side safety factor, per Table 2-3 for cohesive soil.

$$S.F._S = \frac{Q_T}{3\, Q_S} + 1.1 = \frac{5 \text{ kips}}{3 * 41.7 \text{ kips}} + 1.0 = \mathbf{1.04}$$

$$\frac{R_S}{S.F._S} = \left(610 \text{ psf} * (10\text{ft} - 6 \text{ ft}) * \pi * 12 \text{ in} * \frac{1 \text{ ft}}{12 \text{ in}}\right) * \frac{1}{1.14 \text{ S.F.}}$$
$$+ \left(2040 \text{ psf} * (17\text{ft} - 10 \text{ ft}) * \pi * 12 \text{ in} * \frac{1 \text{ ft}}{12 \text{ in}}\right) * \frac{1}{1.04 \text{ S.F.}} = 49{,}860 \text{ lb} = \mathbf{49.9 \text{ kips}}$$

The volume of a pier whose bell is a cone with hypotenuse 60 degrees from its base is as follows:

Pier Volume =

$$0.7854\, d^2 L + 0.6082\, d^3 (r_{bell} - 1)^2 \left[\left(\frac{r_{bell} - 1}{3}\right) + 1\right] + 0.3927\, d^2 (r_{bell}^2 - 1)$$

where $r_{bell} = \frac{D}{d} = 3$ for this pier.

$$\text{Pier Volume} = 0.7854 * \left(12 \text{ in} * \frac{1 \text{ ft}}{12 \text{ in}}\right)^2 * 17 \text{ ft} +$$
$$0.6082 \left(12 \text{ in} * \frac{1 \text{ ft}}{12 \text{ in}}\right)^3 (3 - 1)^2 \left[\left(\frac{3-1}{3}\right) + 1\right] + 0.3927 \left(12 \text{ in} * \frac{1 \text{ ft}}{12 \text{ in}}\right)^2 (3^2 - 1)$$
$$= 20.5 \text{ cu. ft. or } 0.76 \text{ cu. yd.}$$

$$R_W = \text{Effective concrete weight } * \text{ Pier Volume}$$

Note that for simplicity in calculating the pier weight R_W, if the water table is above the base of the pier, the effective concrete weight is assumed to apply over the entire pier.

$$\mathrm{R_W} = (145\ \mathrm{pcf} - 62.4\ \mathrm{pcf})\ * 20.5\ \mathrm{cu.ft.} = 1693\ \mathrm{pounds} = \mathbf{1.7\ kips}$$

Summing the loads and resistances for the upward case, check that the loads are less than the resistances.

$$\mathrm{Q_T} + \mathrm{Q_S} \leq \mathrm{R_W} + \frac{\mathrm{R_S}}{\mathrm{S.F._S}} \qquad [\mathrm{Eq.}\ 23]$$

$$5\ \mathrm{kips} + 41.7\ \mathrm{kips} \leq 1.7\ \mathrm{kips} + 49.9\ \mathrm{kips}$$

$\mathbf{46.7\ kips \leq 51.6\ kips}$; TRUE

Unity check:

$$\frac{\mathrm{Loads}}{\mathrm{Resistances}} = \frac{46.7\ \mathrm{kips}}{51.6\ \mathrm{kips}} = \mathbf{0.90} \leq 1.00 \qquad [\mathrm{Eq.\ 22\text{-}B}]$$

The unity check is less than one; the 17’ deep 12”/36” diameter pier is adequate for the upward case.

3.3.2 Downward Case of Sample Calculation 3

For a pier diameter of 12 inches, bell diameter of 3 feet, a pier depth of 17 feet, and Z_a of 6 feet, it is assumed that all the cohesive soil from grade to 6 feet has pulled away from the pier due to drying out and shrinkage of the soil. The 30 kip pier loading, Q_T, is downward as a worst-case scenario. The weight of the pier, Q_W, acts downward. The cohesive soil below Z_a is resisting the downward movement of the pier and therefore is a side resistance. The base is resisting the downward movement of the pier and it is also a resistance.

$$\sum \mathrm{Q} \leq \sum \frac{\mathrm{R}}{\mathrm{S.F.}} \qquad [\mathrm{Eq.}\ 22 - \mathrm{A}]$$

$$\mathrm{Q_T} + \mathrm{Q_W} + \mathrm{Q_S} \leq \frac{\mathrm{R_S}}{\mathrm{S.F._S}} + \frac{\mathrm{R_B}}{\mathrm{S.F._B}} \qquad [\mathrm{Eq.}\ 25]$$

$\mathrm{Q_T} = \mathbf{30\ kips}$ acting downward from the problem statement.

$\mathrm{Q_W} = \mathbf{1.7\ kips}$

Note that $\mathrm{Q_S} = \mathbf{0}$ in this case because the soil has pulled away from the pier in the movement zone, see Figure 2-3.

$$\frac{\mathrm{R_S}}{\mathrm{S.F.}} = \left(610\ \mathrm{psf} * (10\ \mathrm{ft} - 6\ \mathrm{ft}) * \pi * 12\ \mathrm{in} * \frac{1\ \mathrm{ft}}{12\ \mathrm{in}}\right) * \frac{1}{2.2\ \mathrm{S.F.}} + \left(2040\ \mathrm{psf} * (17\ \mathrm{ft} - 10\ \mathrm{ft}) * \pi * 12\ \mathrm{in} * \frac{1\ \mathrm{ft}}{12\ \mathrm{in}}\right) * \frac{1}{2.0\ \mathrm{S.F.}}$$

$$= 25{,}910 \text{ lb} = \mathbf{25.9 \text{ kips}}$$

$$\frac{R_B}{S.F.} = \left(36{,}000 \text{ psf} * \frac{\pi}{4} * \left(36 \text{ in} * \frac{1 \text{ ft}}{12 \text{ in}}\right)^2\right) * \frac{1}{3.0 * r_{bell} \text{ S.F.}} = 28{,}270 \text{ lb} = \mathbf{28.3 \text{ kips}}$$

$$Q_T + Q_W + Q_S \le \frac{R_S}{S.F._S} + \frac{R_B}{S.F._B} \qquad \text{[Eq. 25]}$$

$$30 \text{ kips} + 1.7 \text{ kips} + 0 \text{ kips} \le 25.9 + 28.3 \text{ kips TRUE}$$

$$\mathbf{31.7 \text{ kips} \le 54.2 \text{ kips}} \text{ ; TRUE}$$

Unity check:

$$\frac{\text{Loads}}{\text{Resistances}} = \frac{31.7 \text{ kips}}{54.2 \text{ kips}} = \mathbf{0.58} \le 1.00 \qquad \text{[Eq. 22-B]}$$

The unity check is less than one; the 17’ deep 12”/36” diameter pier is adequate for the downward case. If the user wished to revise the calculations using a shorter pier, one would find that a 10’ deep pier is adequate for the downward case; however, a 17’ deep pier is required for the upward case. Similarly, one would find that the bell is not necessary to design this pier for the downward case. Using a straight 12” shaft would give a unity check of 0.88, which is within the allowable for a 17’ deep pier.

4.0 COMPARISON OF THIS DESIGN PROCEDURE TO CURRENT PRACTICE

This section includes comparisons made between the results of this procedure and local practice. The following subsections include comparison of active zones and pier depths.

4.1 ACTIVE ZONE COMPARISON

Using geotechnical reports that included suction testing at 13 Southeast Texas sites with trees, the subcommittee compared the movement active zone, Z_a, computed using this procedure to the moisture active zone, Z_m, determined from the soil reports. The depth of Z_m should always be greater than or equal to Z_a.

The actual depth of Z_m based on suction testing was determined by the depth where the suction profile variance was less than or equal to 0.03 pF per foot of depth (see Section 2.1.1) at each of the 13 sites. At some sites Z_m was limited by soil stratigraphy such as a hardpan layer.

The subcommittee expected the moisture active zone depth of Z_m (computed via suction testing) to be greater than or equal to the movement active zone depth Z_a (computed via this procedure) and this was the case, with the ratio of Z_a/Z_m varying from 0.71 to 1.00.

The detailed results are tabulated in Table 4-1 below.

	Location	**Active Zone Depth (ft)**		**Ratio**
#	**(all in SE TX)**	**Z_m (From Suction)**	**Z_a (From SC-16)**	**Z_a / Z_m**
1	Bellaire	12	12	1.00
2	Friendswood	14	14	1.00
3	Rosenberg	13	11	0.85
4	Sugar Land	12	12	1.00
5	Bellaire	15	13	0.87
6	Piney Point	14	10	0.71
7	Cypress	14	10	0.71
8	SW Houston	13	11	0.85
9	Spring Branch	12.5	9	0.72
10	SW Houston	12	11	0.92
11	River Oaks	11	11	1.00
12	Brenham	12	11	0.92
13	River Oaks	11	11	1.00
	Average	12.7	11.2	0.89
	Median	12.5	11.0	0.92
	Minimum	11.0	9.0	0.71
	Maximum	15.0	14.0	1.00

Table 4-1

4.2 PIER DEPTH COMPARISON

Using recent geotechnical reports that included pier depth recommendations from 23 different geotechnical firms in southeast Texas, the subcommittee compared the recommended pier depths with this procedure's recommended pier depths for both the "No Tree" case and the "Tree" case (see Section 2.3, Step 3). Of the 23 locations tested, only 8 were confirmed to possibly meet the "No Tree" case by examination of historic aerial photographs. Because the southeast Texas area pier depth recommendations in expansive soil have been gradually increasing over the past two decades, the geotechnical reports in the comparison study were limited to those written within the previous five years of the date of this publication.

The computed pier depths using this procedure for the "No Tree" case were, on average, 1.6 feet deeper than the reports' average recommendations. Therefore, SC-16's "No Tree" case approximately correlates with the recommendations in the geotechnical reports. The pier depths for the "Tree" case were, on average, 10.9 ft deeper, indicating that most of the geotechnical engineers in this study did not consider tree effects in their pier depth recommendations. The detailed study results are tabulated as follows in Table 4-2 below.

Pier Depth Comparison (ft)						
#	**Location**	**Proposed by Geo**	**"No Tree" Case**		**"Tree" Case**	
			SC-16	**Difference**	**SC-16**	**Difference**
1	Houston	12	12	0	25	13
2	Pflugerville	20	23	3	31	11
3	Houston	12	14	2	25	13
4	Memorial	10	12	2	16	6
5	Fulshear	10	12	2	20	10
6	Jersey Village	9	10	1	17	8
7	Bellaire	12	13	1	23	11
8	Sugar Land	7	15	8	22	15
9	Manvel	13	19	6	29	16
10	West University	13	12	-1	28	15
11	Bellaire	14	14	0	24	10
12	Houston	14	13	-1	21	7
13	Cedar Creek	12	13	1	24	12
14	Houston	15	11	-4	21	6
15	Houston	13	13	0	22	9
16	Angleton	6	12	6	22	16
17	Houston	9	10	1	17	8
18	Houston	10	12	2	21	11
19	Malakoff	15	15	0	26	11
20	Houston	9	9	0	19	10
21	Houston	12	15	3	26	14
22	Simonton	15	14	-1	20	5
23	Houston	8	14	6	22	14
	Average	11.7	13.3	1.6	22.7	10.9
	Median	12	13	1	22	11
	Minimum	6	9	-4	16	5
	Maximum	20	23	8	31	16

Table 4-2