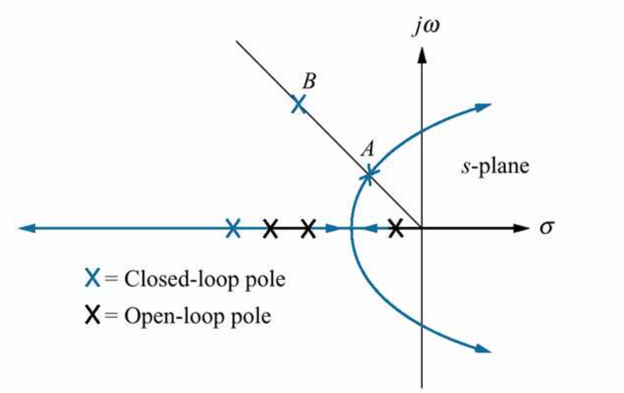
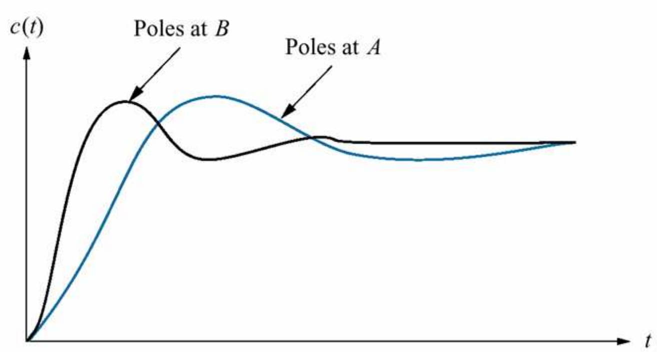
Lecture 1

Design Via Root Locus

Motivation: Consider the example below



(a)



(b)

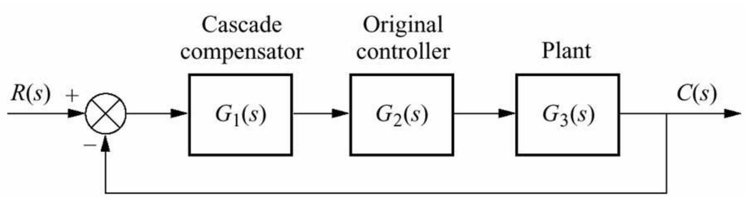
- B is the desired root: we can't access by changing K!
- What can we do?
  1. Change parameters of the original system: Impossible or Expensive
  2. Add a *Compensation System*!.

A Compensation System?

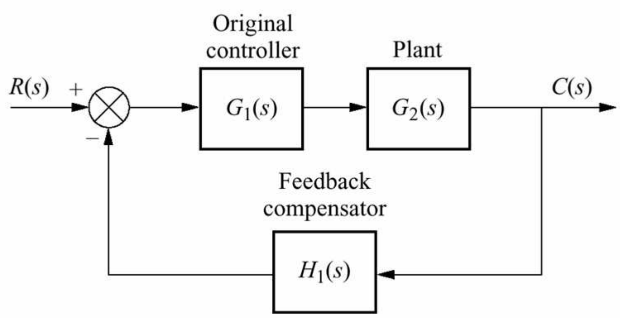
Simple controller system with two purposes:

- Improving the transient response by changing pole locations. (Differentiator Based)
- Improving the steady-state performance. (Integrator Based)

There are 2 types of compensators (Depending on where you place the compensator system): Cascade(a) or Feedback(b)



(a)



(b)

Improving Steady State Performance

Goal: Improve steady state performance without affecting transient response.
Basic Strategy: Add integrators to increase the type of the system
Two Common Techniques:

- Ideal Integrator (a pole on origin): $G_1(s) = K(a + \frac{1}{s})$.
  - Increases the system type, can make steady-state error zero. (Excellent!)
  - Requires use of active elements(i.e., elements requiring power supply)(Expensive!)
- Non-ideal Integrator with a pole near origin. $G_1(s) = \frac{s-z_c}{s-p_c}$
  - Can not the increase system type, but can significantly improve steady state error performance. (Nice!)
  - Requires passive elements only, so it is cheap. (Very Nice!)

Note that both approaches have a zero in addition to the pole. We will see why very soon...

Compensator Naming Convention (for ideal compensators)

- **Proportional Controller:** feed scaled error to the plant.
$$G_1(s) = K. \qquad (1)$$

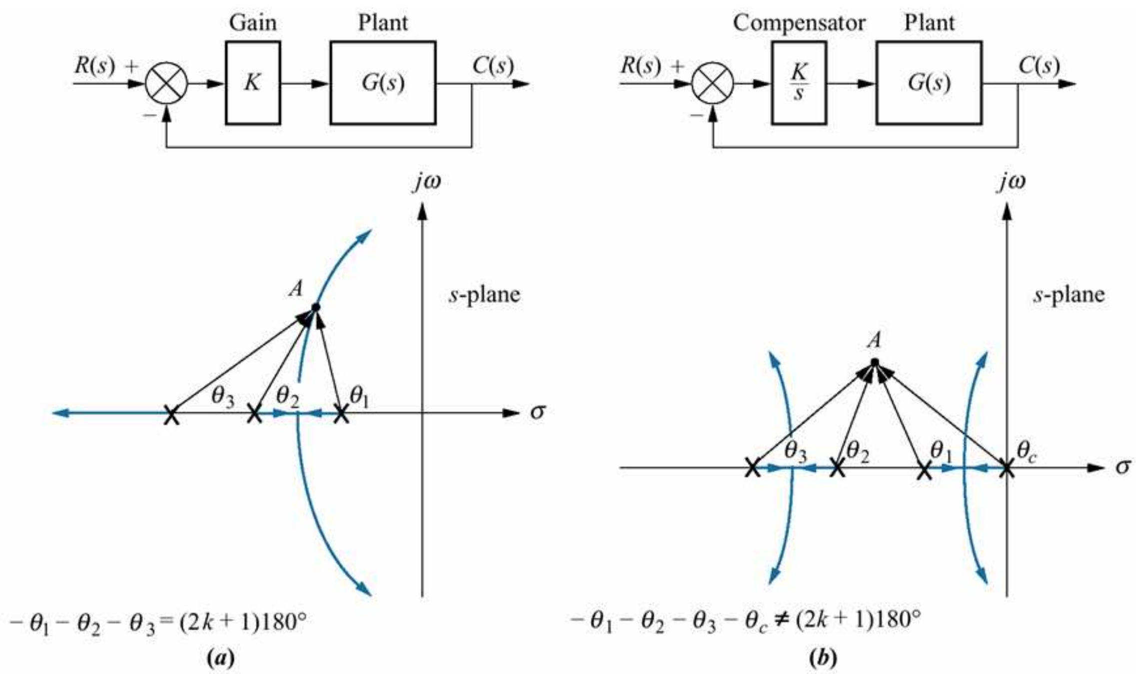
- **Integral Controller:** feed integrated error to the plant.
$$G_1(s) = \frac{K}{s} \qquad (2)$$
.

- **Derivative Controller:** feed differentiated error to the plant.
$$G_1(s) = Ks. \qquad (3)$$

- **Proportional-plus-Integrator (PI)**: feed scaled+integrated error to the plant:
$$G_1(s) = K(a + \frac{1}{s}). \qquad (4)$$

Ideal Integral (PI) Compensator

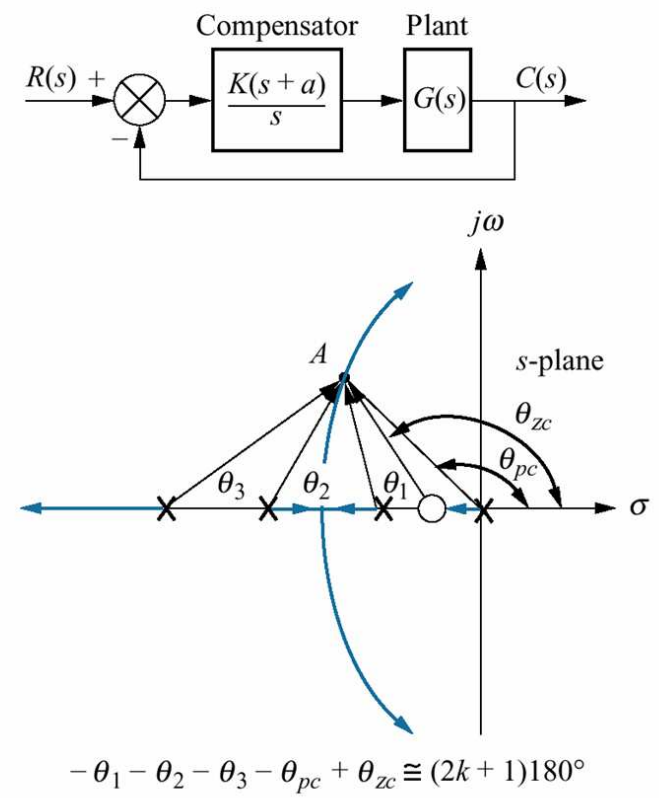
Consider the following Example



- (a) No Compensation
- (b) Only Integrator:
  - Steady-state performance improved.
  - However, the transient response in (a) can not be achieved!

Ideal Integral (PI) Compensator: Continued

Now consider the following compensation:



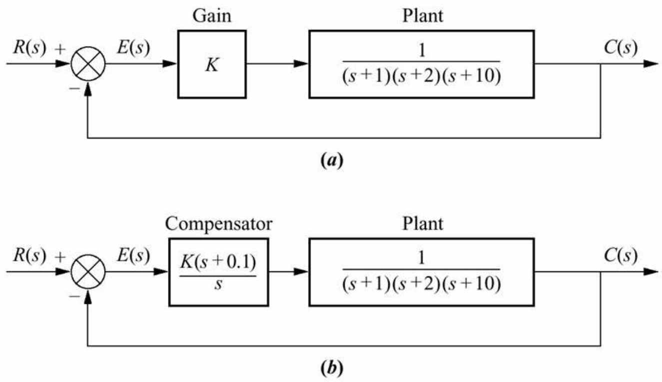
(c)

- (c) Proportional+Integrator:
  - Transient Response almost unaffected!.
  - Steady State Improved.

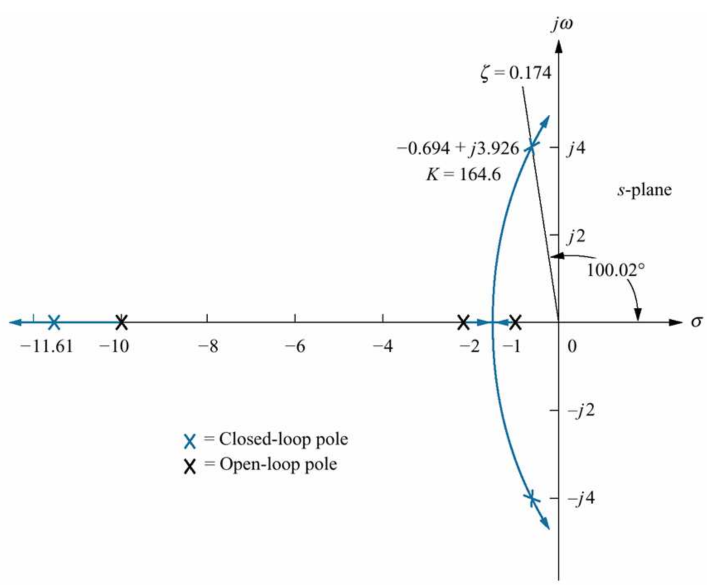
So the choice of $G_1(s) = K(a + \frac{1}{s})$ over $\frac{K}{s}$ should be clear now!: *the inclusion of the proportional part (and therefore the zero) avoids the effect on the transient response*

Ideal Integral (PI) Compensator: Example

Consider the following example:



The Root-Locus for Uncompensated System

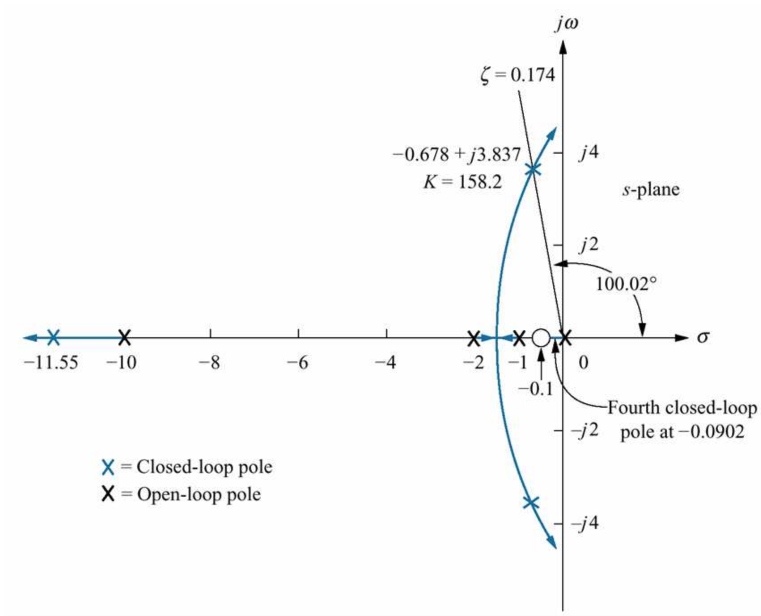


K=164.6 provides:

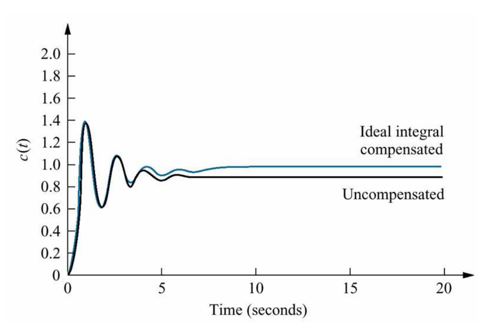
- Damping Ratio: $\zeta = 0.174$.
- Steady State Error: $e(\infty) = \frac{1}{1+Kp} = 0.108$.

Ideal Integral (PI) Compensator: Example Continued

Now with an ideal integrator (PI) controller Root Locus is very similar: For this case



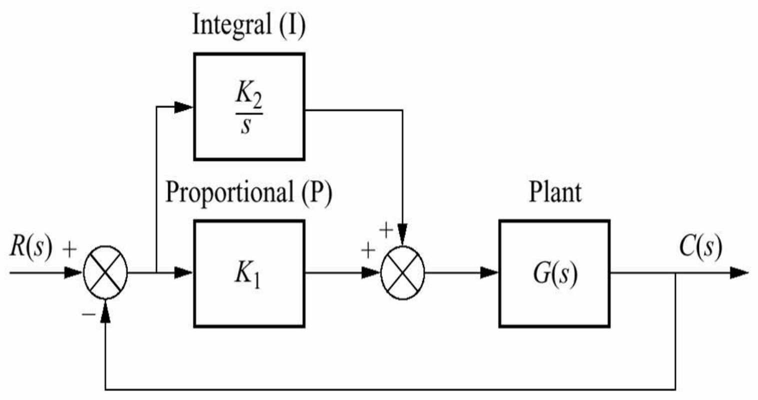
- Damping Ratio unchanged (with $K = 158.2$).
- Steady State Error is ZERO!.



How to Implement PI Controller?

$$G_c(s) = K_1 + \frac{K_2}{s} = K_1 \frac{(s + \frac{K_2}{K_1})}{s} \tag{1}$$

Simple!, use the following:

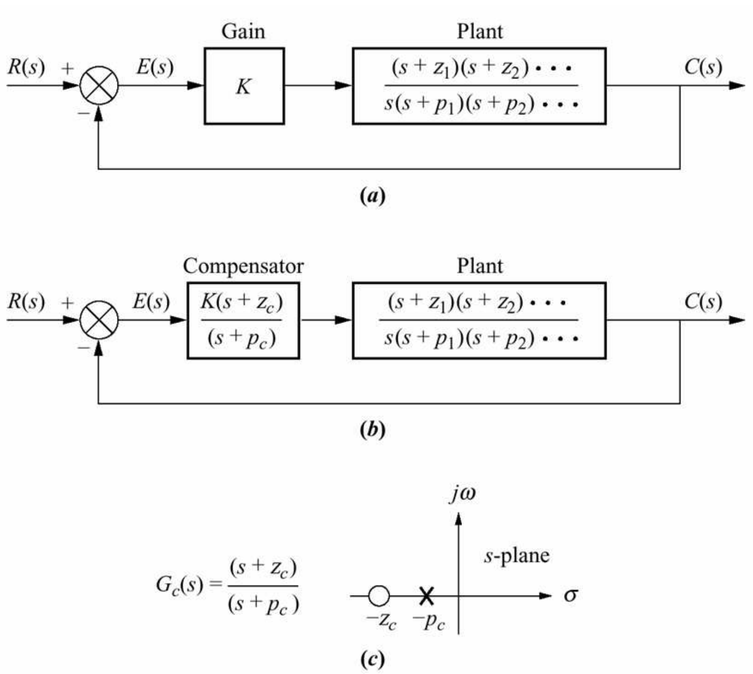


- Made steady-state error zero!.
- However, it is expensive to implement as the integrator requires active elements.
- We may want to use the solution presented next: *Lag Compensation*.

Lag Compensation: A Cheaper Solution

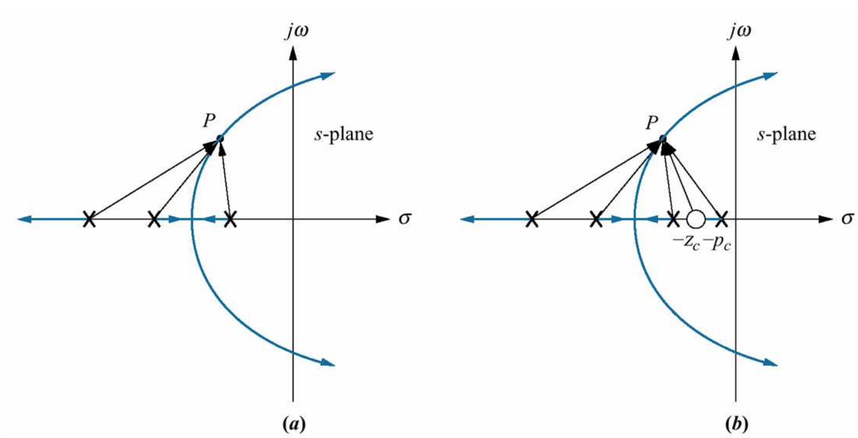
Similar to the Ideal Integrator, however it has a pole not on origin but close to the origin.

$$G_1(s) = \frac{s+z_c}{s+p_c} \tag{1}$$



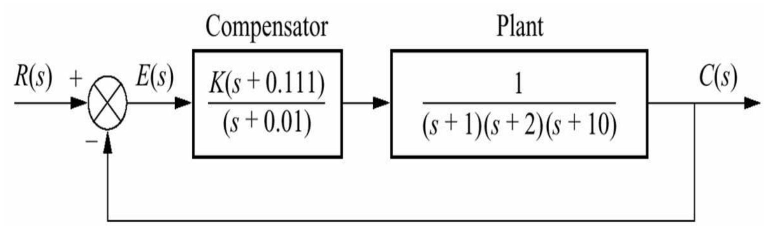
Lag Compensation: Continued

- Steady State Improvement:
  - Before compensation:
    $K_{v_0} = \lim_{s \to 0} G(s) = K \frac{z_1 z_2 \ldots}{p_1 p_2 \ldots}$.
  - After compensation: $K_{v_{new}} = \frac{z_c}{p_c} \underbrace{K \frac{z_1 z_2 \ldots}{p_1 p_2 \ldots}}_{K_{v_0}}$
- the effect on the transient response is negligible:

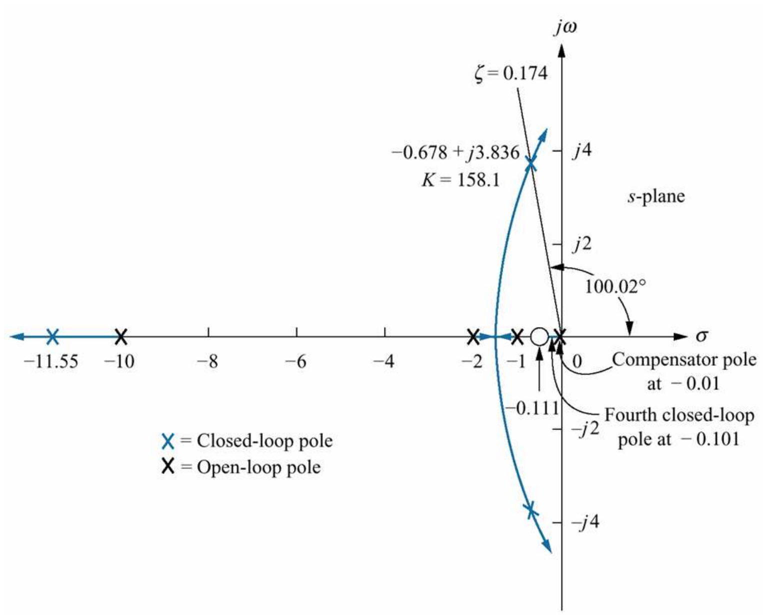


Lag Compensation: Example Revisited

Consider the following lag compensation for the previous example:



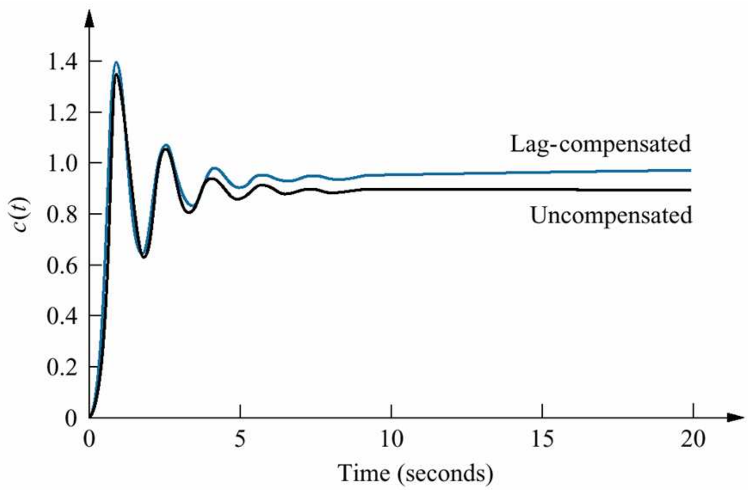
- The Root Locus: almost unchanged



Lag Compensation: Example Revisited

- New Steady State Error:

$$e(\infty) = \frac{1}{1+K_p} = 0.0108 \qquad (1)$$



- Comparison of the Lag-Compensated and the Uncompensated Systems

Parameter	**Uncompensated**	**Lag-compensated**
Plant and compensator	$\frac{K}{(s+1)(s+2)(s+10)}$	$\frac{K(s+0.111)}{(s+1)(s+2)(s+10)(s+0.01)}$
K	164.6	158.1
K_p	8.23	87.75
$e(\infty)$	0.108	0.011
Dominant second-order poles	$-0.694 \pm j3.926$	$-0.678 \pm j3.836$
Third pole	-11.61	-11.55
Fourth pole	None	-0.101
Zero	None	-0.111

Improving Steady State Response with Cascade Compensation: Summary

- Include Integrators or integrator-type systems to improve steady state performance
- Ideal Integral(Proportional-plus Integrator): $G(s) = K\frac{(s+a)}{s}$.
  - Can create zero steady state error.
  - Zero $-a$ is to avoid change in the transient response.
  - Expensive due to the ideal integrator.
- Lag Compensation: $G(s) = K\frac{(s+z_C)}{s+p_C}$.
  - Can be considered as the cheaper approximation of PI.
  - Steady-state error is not zero but can be made small.

Up to this point we dealt with improving steady-state response without affecting the transient response. Next subject is improving the transient response!

Improving Transient Response with Cascade Compensation

If the closed loop root locus doesn't go through the desired point, it needs to be reshaped.
Two approaches

- Ideal Derivative (Proportional-plus-Derivative (PD)):

$$G_1(s) = s + z_c \tag{1}$$

  - Can provide better performance than the other alternative. :)
  - Requires active elements for implementation. : (
  - Can amplify the high frequency noise. : (

- Lead Compensation:

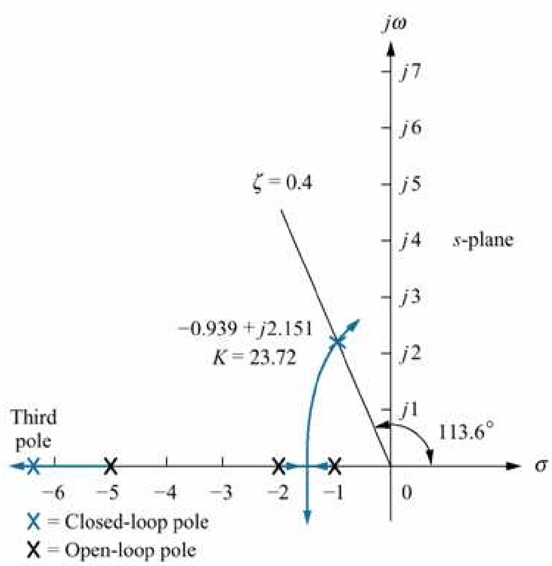
$$G_1(s) = K\frac{s + z_c}{s + p_c} \tag{2}$$

  where p_c is a distant pole in this case.

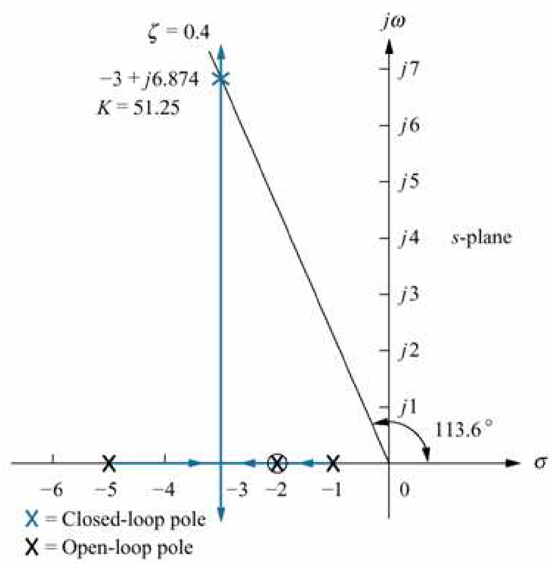
  - Can provide reasonable performance. :)
  - Requires passive elements only. :)
  - Less sensitive to high frequency noise. :)

Ideal Derivative Compensation (PD)

– $G_1(s) = s + z_c$: Introduction of a new zero. Lets see how it affects by an example:(a) uncompensated (b) zero at -2



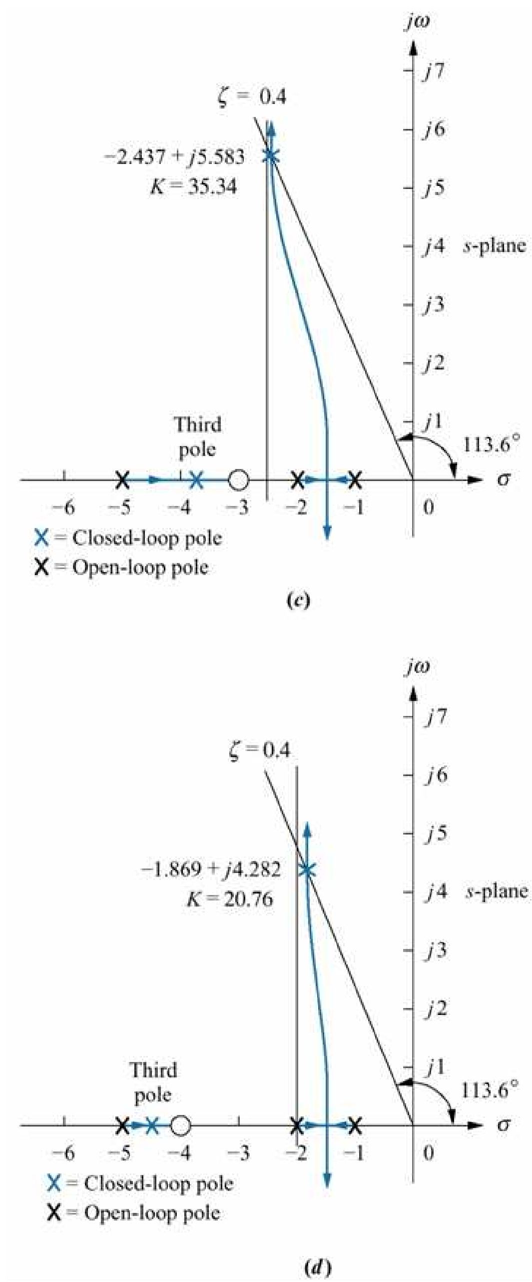
(a)



(b)

Ideal Derivative Compensation (PD)

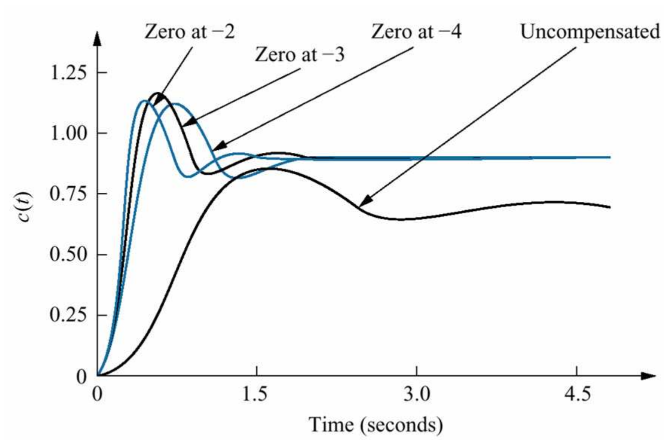
(c) zero at -3 (d) zero at -4



Ideal Derivative Compensation (PD)

Observations and facts:

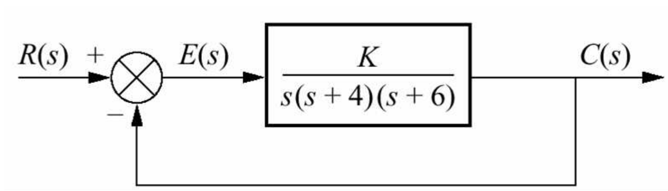
- In each case gain K is chosen such that percent overshoot is same.
- Compensated poles have more negative real and imaginary parts: smaller settling and peak times.



- Farther the zero from the dominant poles, closer the the dominant pole to the origin.

Ideal Derivative Compensation (PD): Example

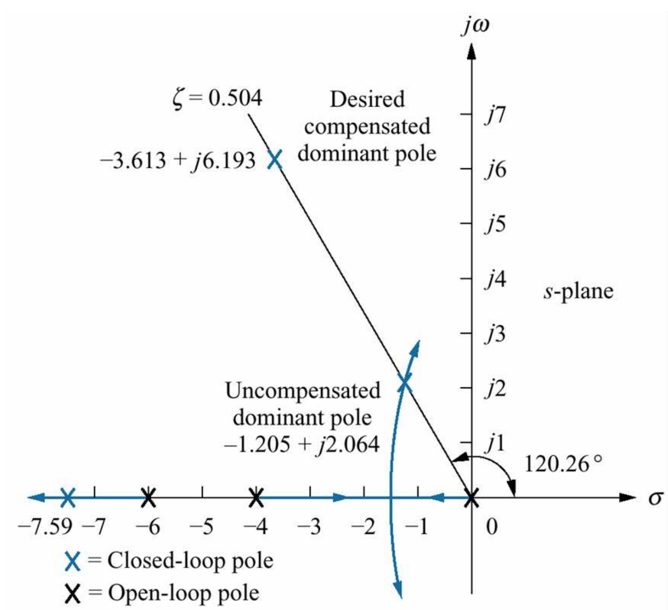
Given



Design an ideal derivative compensator to yield, $\%16$ overshoot with threefold reduction in settling time.

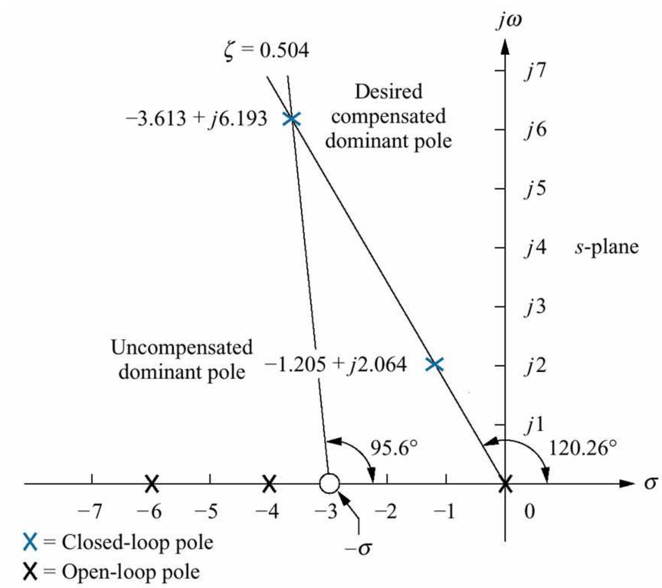
Solution:

Root-Locus and desired pole location:



Ideal Derivative Compensation (PD): Example Continued

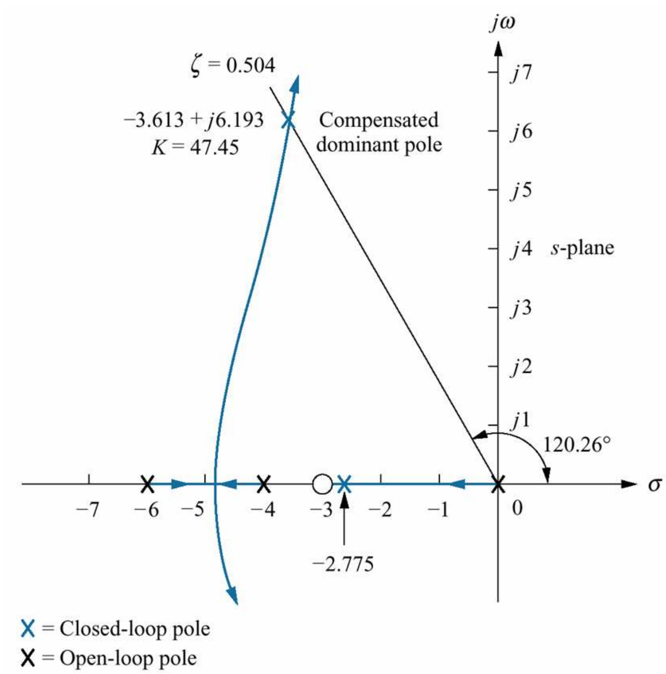
Determining the location of the zero:



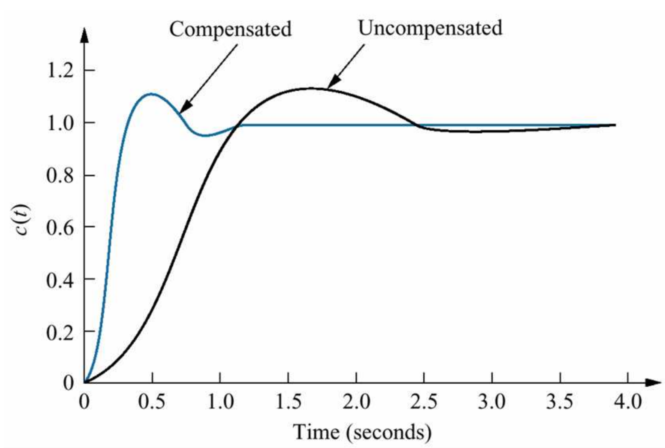
- The angle contribution of poles for the desired pole location: -275.6
- In order to achieve -180 the angle contribution of the placed zero should be 95.6.
- From the figure: $\frac{6.193}{3.613-\sigma} = tan(180 - 95.6)$ which yields $\sigma = 3.006$.

Ideal Derivative Compensation (PD): Example Continued

Root-Locus After Compensation

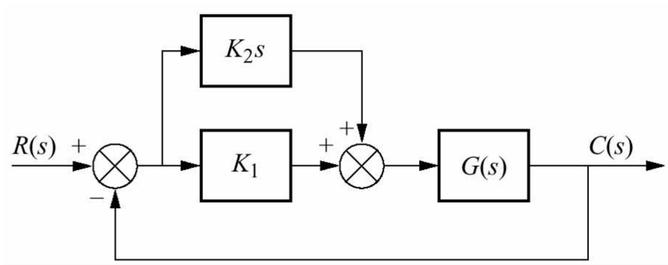


Improvement in the transient response



Ideal Derivative Compensation (PD): Implementation

$G_c(s) = K_2 s + K_1 = K_2(s + \frac{K_1}{K_2})$.
A trivial implementation:

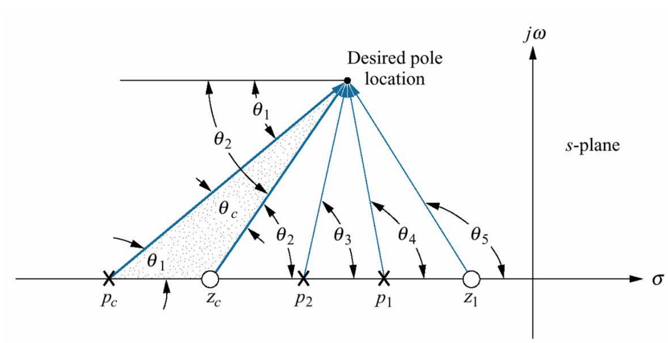


Implementation of ideal differentiator is expensive. So we may use the next technique: *Lead Compensation*

Lead Compensation

- Passive element approximation of PD.
- it has an additional pole far away on the real axis.
- Advantage 1: Cheaper
- Advantage 2: Less noise amplification
- Disadvantage: Doesn't reduce the number of branches.

  Basic Idea: Angular contribution of the lead

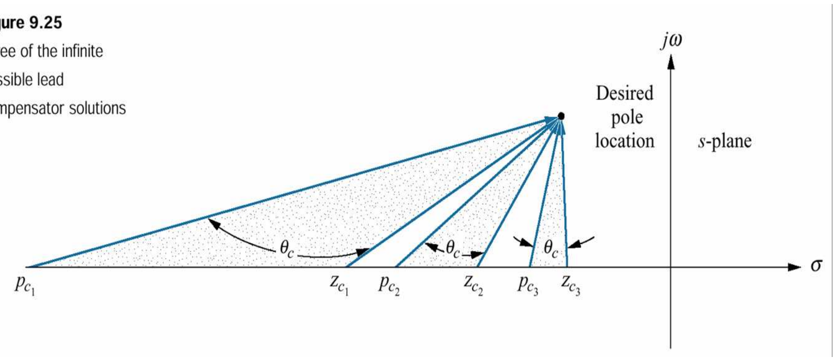


compensator is $\Theta_2 - \Theta_1$.

Lead Compensation: Continued

There are infinitely many choices of z_c, p_c providing same $\Theta_c = \Theta_2 - \Theta_1$.

Figure 9.25
Three of the infinite possible lead compensator solutions

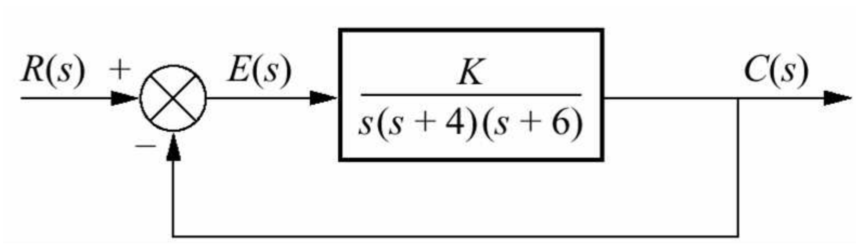


The choice from infinite possibilities affects:

- Static Error Constants.
- Required gain to reach the design point.
- Justification of the second order assumption.

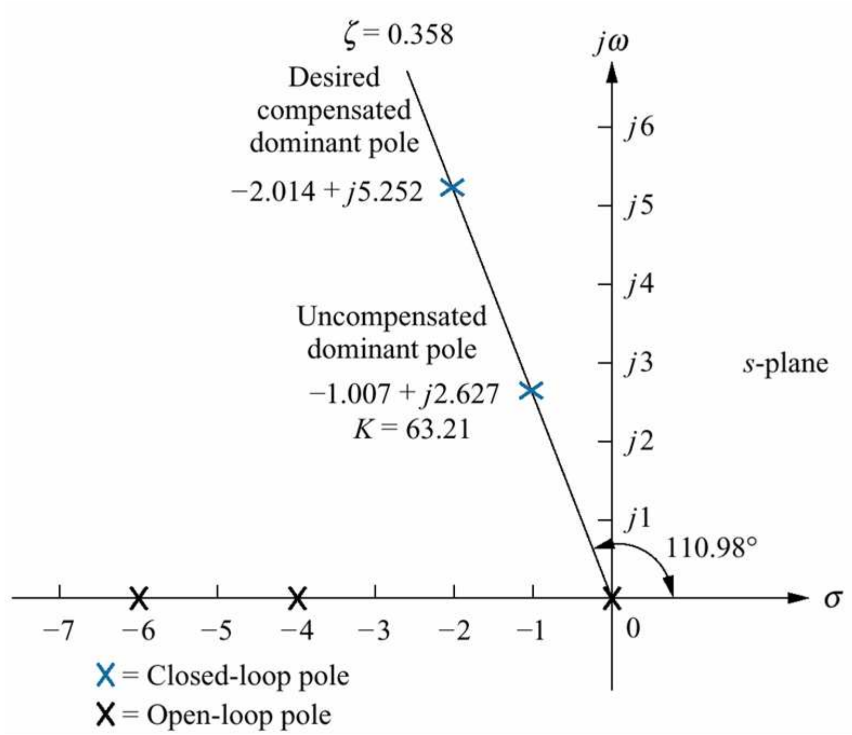
Lead Compensation: Example

Design three lead compensators for the system to reduce the settling factor by a factor of 2 while maintaining %30 overshoot for the system



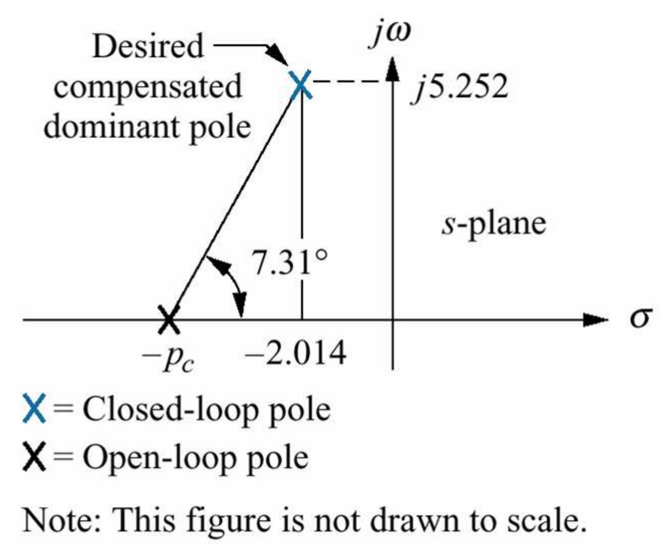
Solution:

Root-Locus and the desired pole location

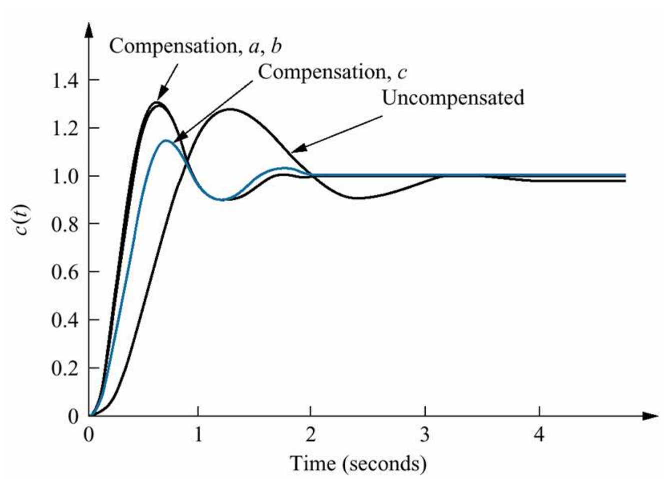


Lead Compensation: Example

Place the zero on -5 arbitrarily. Figure out the required p_c



From this figure, $p_c = 42.96$. We also obtain p_c for $z_c = 4$(Case b) and $z_c = 2$ (Case c). The transient responses are shown in Figure below Second order



approximation is not valid for case C!

Improving Steady-State Error and Transient Response

Suggested Method:

- Improve the transient response first.(PD or lead compensation)
- Then improve the steady-state response. (PI or lag compensation).

Two Alternatives

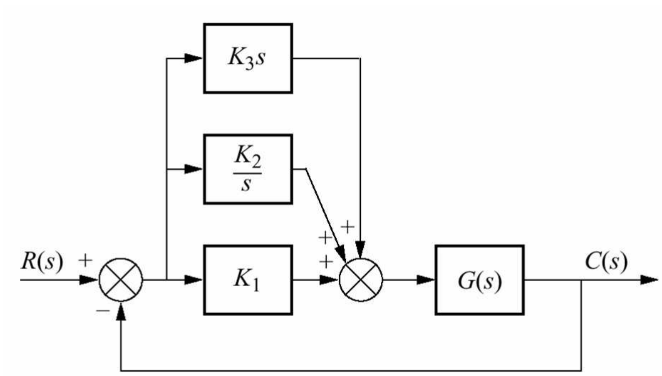
- PID (Proportional-plus-Integral-plus-Derivative) (with Active Elements)
- Lag-Lead Compensator. (with Passive Elements)

PID Controller

- Transfer Function:

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s \tag{1}$$

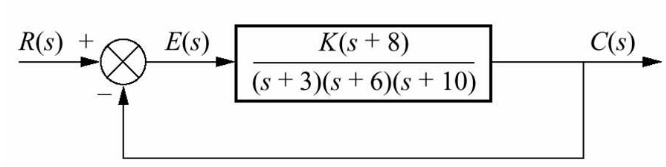
- Implementation



- Design Procedure
  1. From the requirements figure out the desired pole location to meet transient response specifications.
  2. Design the PD controller.
  3. Check validity of the design by simulation.
  4. Design PI controller to yield steady state error performance.
  5. Combine PD and PI to obtain K_1, K_2, K_3.

PID Controller: Example

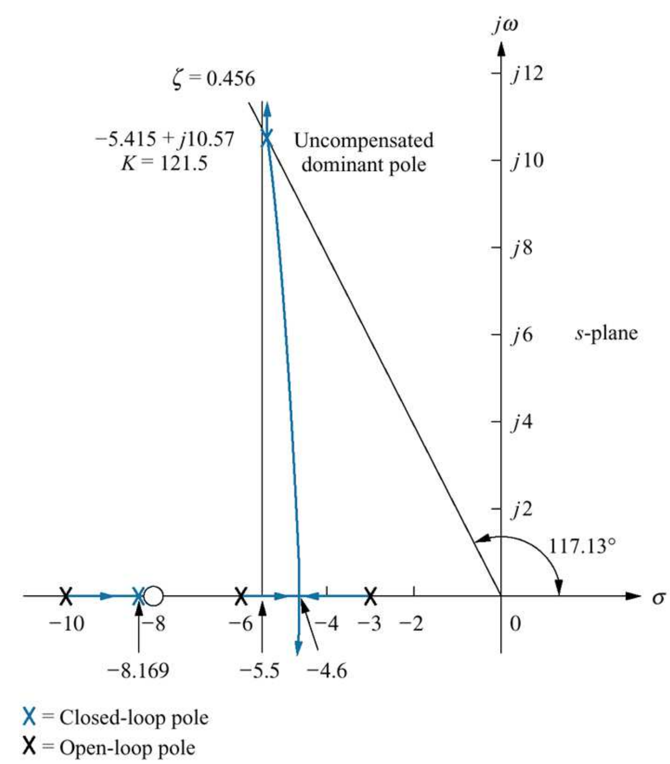
Consider the system below:



Design a PID controller such that

- The peak time is $\frac{2}{3}$ of the uncompensated system with 20%OS.
- Zero steady state error for unit-step input.

The uncompensated system has the following root-locus

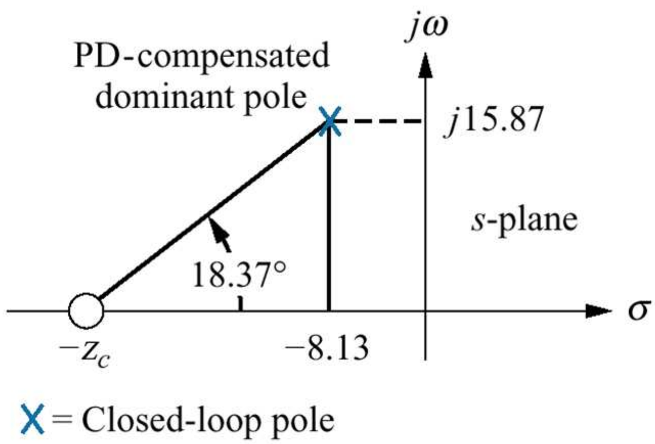


PID Controller: Example Continued

In order to reduce the peak time by $\frac{2}{3}$ the new pole location

$$p_{desired} = \frac{3}{2} \times \underbrace{-5.415 + j10.57}_{\text{uncompensated pole location}} = -8.13 + 15.87 \quad (1)$$

The angle of $G(p_{new})H(p_{new})$ is -198.37. So the desired contribution from the PD zero is $180 - 198.37 = 18.37$.

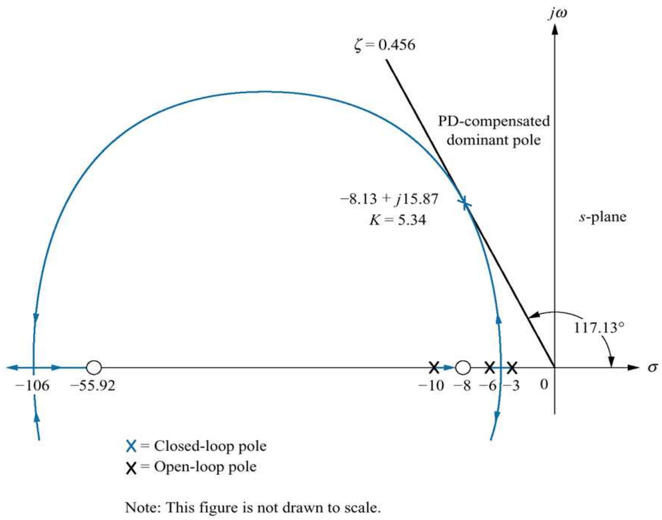


Note: This figure is not drawn to scale.

Controller's zero position:
$\frac{15.87}{z_c - 8.13} = tan(18.37) \Rightarrow z_c = 55.92.$

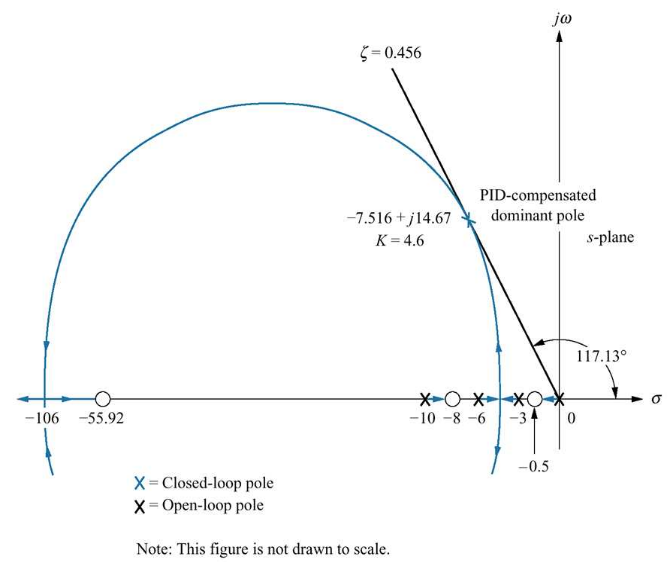
PID Controller: Example Continued

The new root locus with PD



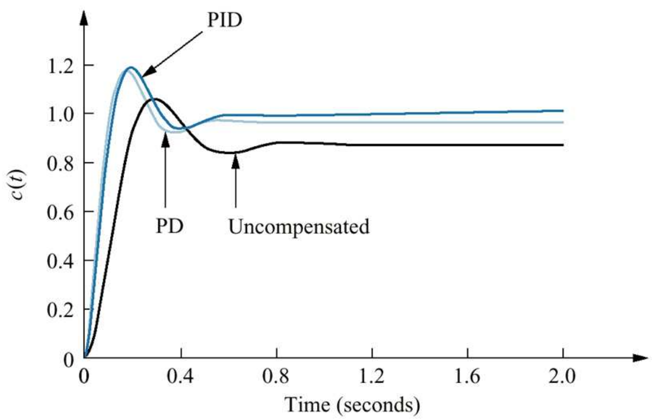
For PI controller use:$G_{PI}(s) = \frac{s+0.5}{s}$

The new root locus with PID



PID Controller: Example Continued

Comparison of step responses



Calculation of the PID parameters:

$$
\begin{aligned}
G_{pid}(s) &= K\frac{(s+55.92)(s+0.5)}{s} = \frac{4.6(s+55.92)(s+0.5)}{s} \\
&= \underbrace{259.5}_{K_1} + \underbrace{128.6}_{K_2}\frac{1}{s} + \underbrace{4.6}_{K_3}s
\end{aligned}
$$

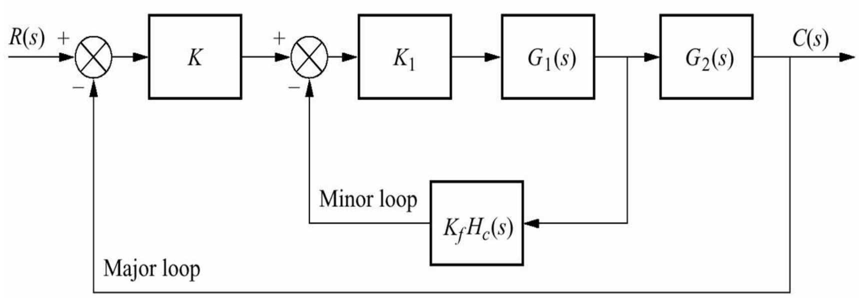
Lag-Lead Compensation: Cheaper solution then PID

Procedure:

1. Determine the desired pole location based on specifications.
2. Design the lead compensator.
3. Evaluate the steady state performance of the lead compensated system to figure out required improvement.
4. Design the lag compensator to satisfy the improvement in steady state performance.

Feedback Compensation

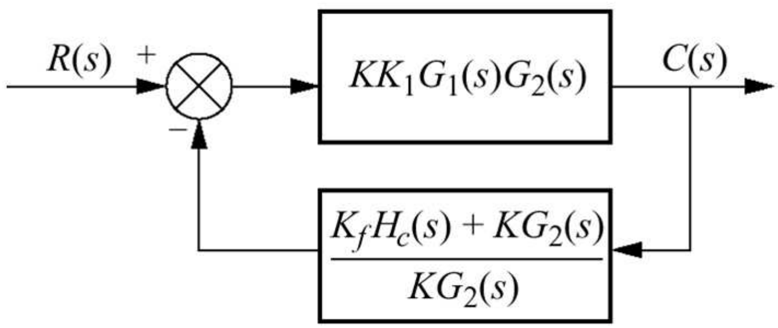
Compensator is at the feedback... (as opposed to the cascade compensators we have seen up to this point..)



- More complicated then cascade.
- Generally provide faster response.
- Can be used in cases where noise is a concern if we use cascade compensators.
- May not require additional gain.

Two Approaches for Feedback Compensation

1. Consider compensation as adding poles and zeros to feedback section for the equivalent system:



2. First design the minor loop then design the major loop.
   - The minor loop is designed to change the open loop poles and open loop transient-response.
   - Loop gain is used to adjust the closed loop performance.

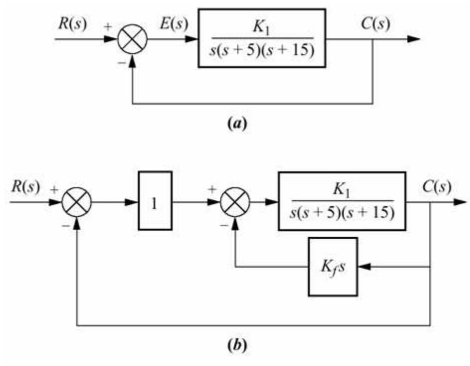
Feedback Compensation: Approach 1

- Does it make a difference whether you place a zero
  - in $G(s)$ by a cascade compensator.
  - or in $H(s)$ by a feedback compensator.
- In terms of root-locus you obtain the same diagram because what matters is the product $G(s)H(s)$!.
- The difference is the following: Since the overall transfer function

$$T(s) = \frac{KN_G(s)D_H(s)}{D_G(s)D_H(s) + KN_G(s)N_H(s)} \tag{1}$$

  - the zeros of $G(s)$ are the zeros of $T(s)$.
    * When a closed loop pole in root locus is close to the zero of $G(s)$ we can (most probably) assume that it will be cancelled,
    * then the second order assumption is better justified.
  - the zeros of $H(s)$ are not the zeros of $T(s)$.
    * Therefore, the closed loop pole close to the zero of $H(s)$ may not be cancelled by a zero of $T(s)$,
    * then we need to be more careful about the second order approximation.

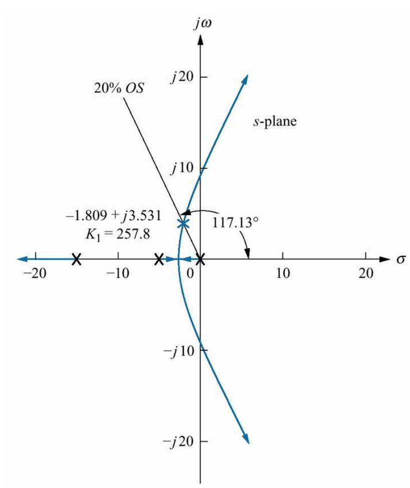
Feedback Compensator Example: Tachometer



Design a feedback compensator to reduce the settling time by a factor of 4 while continuing to operate the system with 20% overshoot.

Solution:

Uncompensated System: The root locus and 20% OS line:



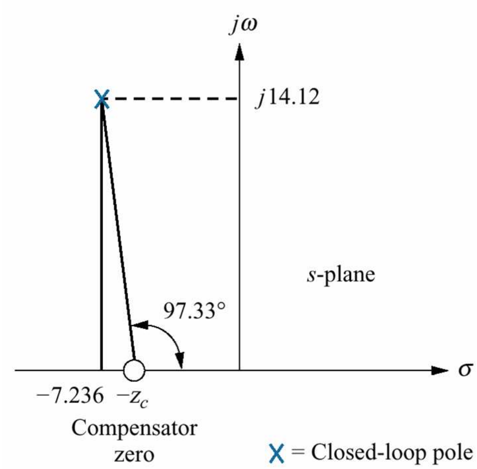
Intersection point: $p = -1.809 \pm j3.531$.

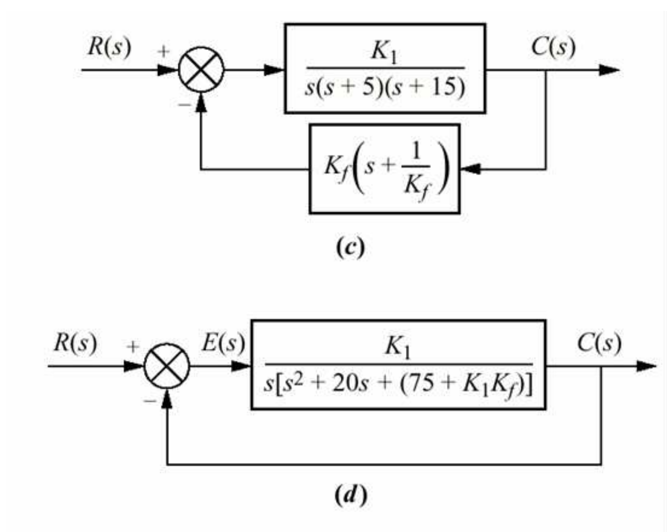
Desired poles: $4 \times p = -7.236 \pm j14.12$.

Feedback Compensator : Tachometer Example Continued

The angle of $G(s)$ at the desired pole point is -277.33^o. Required contribution from the compensator zero is 97.33^o. The zero location

$\frac{14.12}{7.236 - z_c} = tan(180 - 97.33) \Rightarrow z_c = 5.42.$

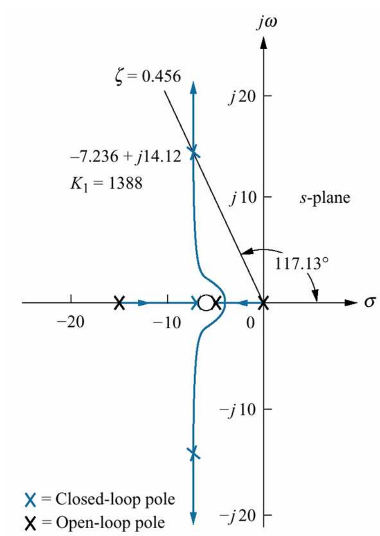




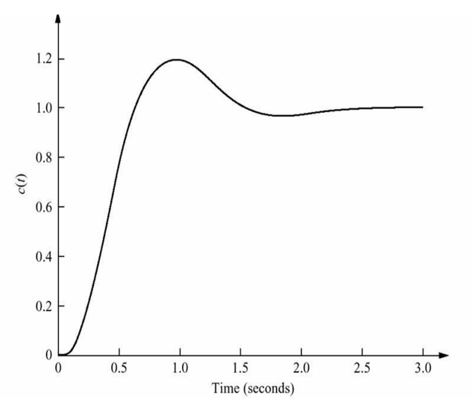
$K_f = \frac{1}{z_c} = 0.185.$

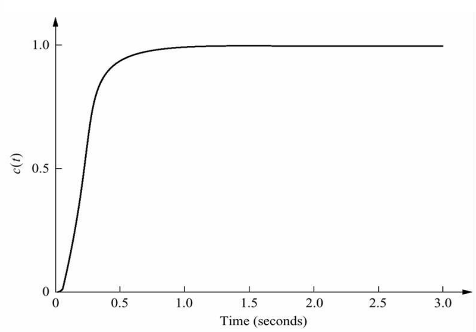
Feedback Compensator : Tachometer Example Continued

Root-Locus of the compensated system:



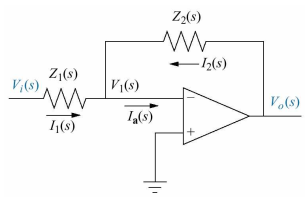
So $K_1 = 1388$. Transient (unit step) response of uncompensated and compensated systems:





Physical Realization of Compensation Systems

Active Systems where $T(s) = -\frac{Z_2(s)}{Z_1(s)}$. Impedances we



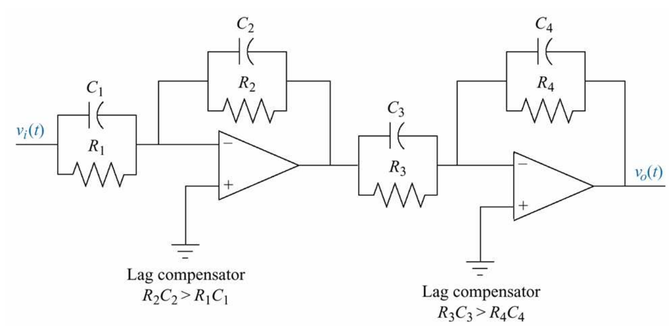
use determine the type of compensator:

Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\frac{Z_2(s)}{Z_1(s)}$
Gain	R_1	R_2	$-\frac{R_2}{R_1}$
Integration	R	C	$-\frac{\frac{1}{RC}}{s}$
Differentiation	C	R	$-RCs$
PI controller	R_1	R_2 C	$-\frac{R_2}{R_1}\frac{\left(s+\frac{1}{R_2C}\right)}{s}$
PD controller	C R_1	R_2	$-R_2C\left(s+\frac{1}{R_1C}\right)$

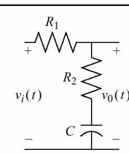
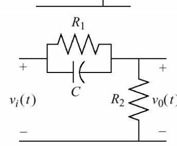
Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\frac{Z_2(s)}{Z_1(s)}$
Lag compensation	C_1 R_1	C_2 R_2	$-\frac{C_1}{C_2}\frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$ where $R_2C_2 > R_1C_1$
Lead compensation	C_1 R_1	C_2 R_2	$-\frac{C_1}{C_2}\frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$ where $R_1C_1 > R_2C_2$

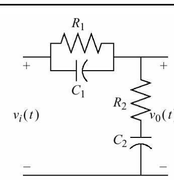
Physical Realization of Compensation Systems: Continued

As an example, active lag-lead compensator



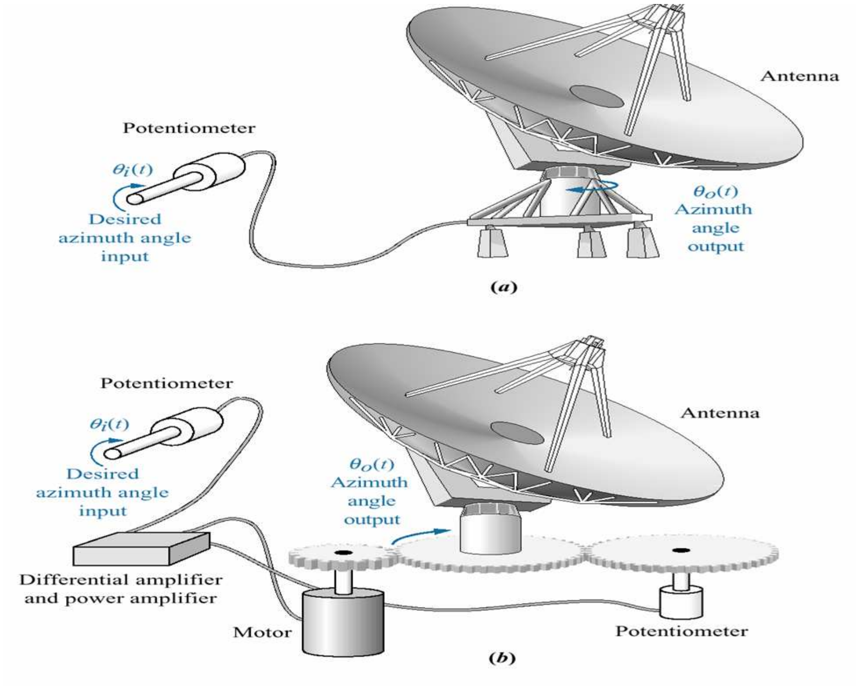
We are actually able to implement lag, lead compensators with passive circuits:

Function	Network	Transfer function, $\frac{V_o(s)}{V_i(s)}$
Lag compensation		$\frac{R_2}{R_1+R_2}\frac{s+\frac{1}{R_2C}}{s+\frac{1}{(R_1+R_2)C}}$
Lead compensation		$\frac{s+\frac{1}{R_1C}}{s+\frac{1}{R_1C}+\frac{1}{R_2C}}$

Function	Network	Transfer function, $\frac{V_o(s)}{V_i(s)}$
Lag-lead compensation		$\frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)s+\frac{1}{R_1R_2C_1C_2}}$

Antenna Control Case Example

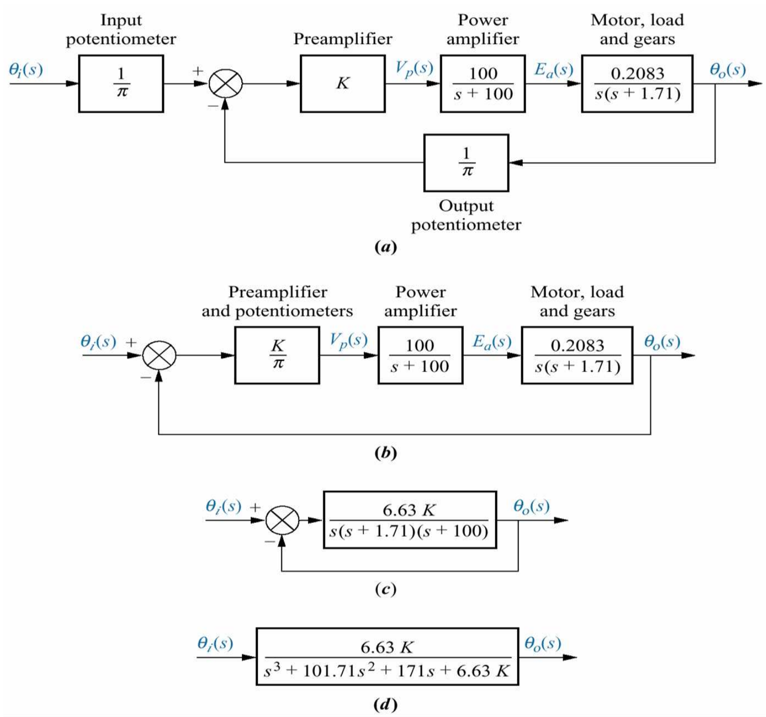
Remember the antenna position control system?



We want to add a cascade compensator for

- 25% OS
- $2-$second settling time
- $K_V = 20$.

Antenna Control Case Example Continued



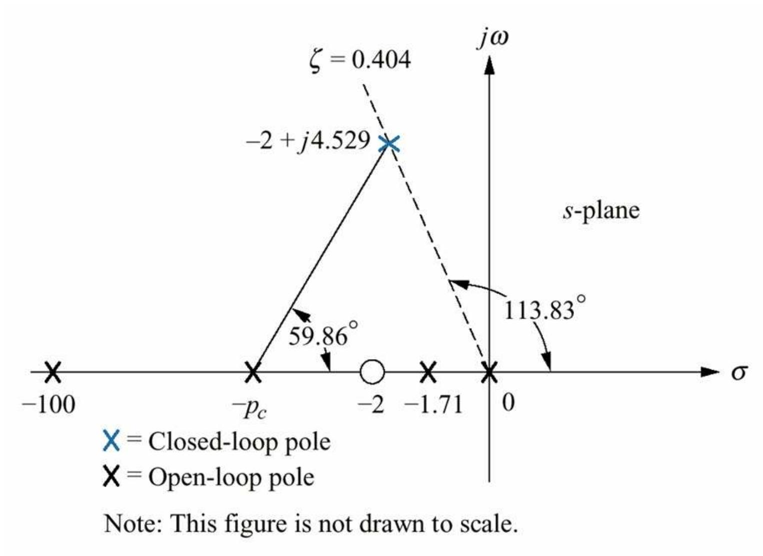
Uncompensated Case:

- 25% OS is achieved with preamplifier gain of 64.21,
- The dominant poles are at $-0.833 \pm j1.888$.
- The settling time $T_s = \frac{4}{0.833} = 4.8$ seconds.
- $K_V = \frac{1.61K}{1.71\times100} = 2.49$

Antenna Control Case Example Continued

Lead Compensation to improve transient

- The desired pole location $\frac{4.8}{2} \times -0.833 \pm j1.888 = -2 \pm j4.529.$
- Assume a compensator zero at -2.
- The poles angular contribution should be -59.86.



- From this figure $p_c = 4.63$.
- The gain $6.63K = 2549$.

Antenna Control Case Example Continued

Lag Compensation to improve steady state

- K_v of the lead compensated system

$$K_v = \frac{2549 \times 2}{1.71 \times 100 \times 4.63} = 6.44 \qquad (1)$$

- Since the desired $K_v = 20$, a factor of $\frac{20}{6.44} = 3.1$ improvement is required.
- choose $p_c = -0.01$ then $z_c = 0.031$.
- Overall lag-lead Compensator

$$G_{LLC}(s) = \frac{6.63K(s+2)(s+0.031)}{s(s+0.01)(s+1.71)(s+4.63)(s+100)} \qquad (2)$$

- The corresponding circuit