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## Designing and Drawing a Sprocket

Visualizing ideas through the creation of CAD solid models is a key engineering skill.
The following text offers the information and procedural steps necessary to engineer a CAD profile of the \#25 pitch, 30 tooth sprocket found in the GEARS-IDS ${ }^{\text {™ }}$ kit of parts.

This process provides a method for generating solid models of any standard sprocket, pitch, number of teeth, or configuration. This exercise also provides students and instructors with an excellent way to incorporate $8^{\text {th }}$ and $9^{\text {th }}$ grade algebra and geometry knowledge with engineering drawing skills to produce the design elements necessary to fully visualize their mechanical creations.


Fig. 1 Sprocket Tooth Geometry
(Adapted from the American Chain Association Chains for Power Transmission and Material Handling handbook.

## Sprocket Tooth Design Formulas

Refer to Fig. 1 Sprocket Tooth Geometry
The tooth form of a sprocket is derived from the geometric path described by the chain roller as it moves through the pitch line, and pitch circle for a given sprocket and chain pitch. The shape of the tooth form is mathematically related to the Chain Pitch (P), the Number of Teeth on the Sprocket (N), and the Diameter of the Roller ( Dr ). The formulas for the seating curve, radius R and the topping curve radius F include the clearances necessary to allow smooth engagement between the chain rollers and sprocket teeth.

The following formulas are taken from the American Chain Association Chains for Power Transmission and Material Handling handbook, and they represent the industry standards for the development of sprocket tooth forms.

| $\begin{array}{ll} \mathbf{P} & =\text { Chain Pitch } \\ \mathbf{N} & =\text { Number of Teeth } \\ \mathbf{D r} & =\text { Roller Diameter ( See Table) } \end{array}$ | $\mathbf{y z}=\operatorname{Dr}\left[1.4 \sin \left(17^{\circ}-\frac{64}{N}\right)-0.8 \sin \left(18^{\circ}-\frac{56^{\circ}}{N}\right)\right]$ |
| :---: | :---: |
| Ds $\quad=($ Seating curve diameter $)=1.0005 \mathrm{Dr}+$ | ab $=1.4 \mathrm{Dr}$ |
| ${ }^{0.003} \mathbf{R}=$ | $\mathbf{W}=1.4 \operatorname{Dr} \cos \frac{180^{\circ}}{N}$ |
| $\begin{aligned} & =\mathrm{Ds} / 2=0.50 \\ & =35^{\circ}+\underline{60^{\circ}} \end{aligned}$ |  |
| A $\quad=35^{\circ}+\frac{}{N}$ | $\mathbf{V}=1.4 \operatorname{Dr} \sin \frac{180^{\circ}}{N}$ |
| $\begin{array}{ll} \text { B } & =18^{\circ}-\frac{J 0}{N} \\ \text { ac } & =0.8 \times \mathrm{Dr} \end{array}$ | $\mathbf{F}=\operatorname{Dr}\left[0.8 \cos \left(18^{\circ}-\frac{56^{\circ}}{\mathrm{N}}\right)+1.4 \cos \left(17^{\circ}-\frac{64^{\circ}}{\mathrm{N}}\right)-1.3025\right]-.0015$ |
| $\mathbf{M} \quad=0.8 \times \operatorname{Dr} \cos \left(35^{\circ}+\frac{60^{\circ}}{N}\right)$ | $\mathbf{H}=\sqrt{F^{2}-\left(1.4 D r-\frac{P}{2}\right)^{2}}$ |
| $\begin{array}{ll} \mathbf{T} & =0.8 \times \operatorname{Dr} \sin \left(35^{\circ}+\frac{60^{\circ}}{N}\right) \\ \mathbf{E} & =1.3025 \mathrm{Dr}+0.0015 \end{array}$ | $\mathbf{S}=\frac{P}{2} \cos \frac{180^{\circ}}{N}+H \sin \frac{180^{\circ}}{N}$ |
| Chordal Length of Arc $\mathbf{x y}=(2.605 \mathrm{Dr}+0.003)$ $\sin \left(9^{\circ}-\frac{28^{\circ}}{N}\right)$ | $\mathbf{P D}=\frac{P}{\sin \left[\frac{180^{\circ}}{N}\right]}$ |

Table 1

## Procedure for Drawing a Sprocket

In this example we will draw the tooth form for the GEARS-IDS 30 tooth Sprocket. Refer to Fig.1, the Sprocket Formulas and the Maximum Roller Diameter Table.

NOTE: This is primarily an algebra and geometry exercise. When drawing a sprocket model for the first time there will likely be problems with arcs that are not exactly tangent to lines, or line lengths that are not exact to 3 or 4 decimal places. This can cause problems if you are using modeling software. Use your drawing tools and common sense in order to create tangent relationships and working line lengths.

With experience, it is possible to draw extremely accurate sprocket models by using the associated spreadsheet and following the directions below.

| Chain Number | Pitch | Max Roller Diameter |
| :---: | :--- | :--- |
| 25 | $1 / 4$ | $0.130^{*}$ |
| 35 | $3 / 8$ | $0.200^{*}$ |
| 41 | $1 / 2$ | 0.306 |
| 40 | $1 / 2$ | $5 / 16$ |

* This refers to the bushing diameter since the chain pitch is small and roller-less.
1.) Determine the values for $\mathrm{P}, \mathrm{N}$ and Dr for a 30 T sprocket
2.) Calculate the remaining values using the formulas on the preceding page. Enter the data in the table below.
Note: this can be accomplished with a calculator or by using the GEARS-IDS ${ }^{\text {TM }}$ spreadsheet (sprocketequations.xls)

| $\mathrm{P}=1 / 4 "$ (For the example Sprocket) | $\mathrm{A}=$ | $\mathrm{W}=$ |
| :--- | :--- | :--- |
| $\mathrm{N}=30$ (For the example Sprocket) | $\mathrm{B}=$ | $\mathrm{V}=$ |
| $\mathrm{Dr}=0.130$ " (See table above) | $\mathrm{ac}=$ |  |
| $\mathrm{Ds}=$ | $\mathrm{M}=$ | $\mathrm{F}=$ |
| $\mathrm{R}=$ | $\mathrm{T}=$ | $\mathrm{H}=$ |
|  | $\mathrm{E}=$ | $\mathrm{S}=$ |
|  | $\mathrm{Yz}=$ | $\mathrm{PD}=$ |
|  | $\mathrm{ab}=$ |  |

## Pitch Circle


location of point a.

6.) Draw a circle with radius $=\mathrm{R}$ at the intersection of lines 1,2 and 3,4 .
5.) Draw a horizontal line ( 3,4 ) through point a, (the top quadrant,) of the pitch circle. Make this line tangent to the pitch circle.


Radius $=\mathbf{R}$ from calculations or spreadsheet $=0.0668$ "

7.) Offset lines 1,2 and 3,4 distances M and T respectively. These lines will intersect at point c

Note: Throughout this exercise it is important to continually save your work and to continually verify the accuracy of every dimension.
8.) Draw line cx at angle
A. Line cx must extend beyond the circle.

9.) Draw line cy. Line cy $=$ length E ( $E=.1708$ for this example)
and is drawn at an angle B from cx.


## 10.) Draw arc $x y$ (Shown in blue) with radius E. ( $E=.1708$ for this example)

Use the 3 point arc
command to set the arc from point $x$ to point $y$. After drawing the arc dimension the radius to length $E$ )

Note: The center of arc xy appears to be at or near point c. Do not make the assumption that this is always the case. With sprockets of differing numbers of teeth, the center of arc xy will likely NOT be at point $c$.

The image on the right illustrates the arc dimension for the 30 tooth \#25 pitch sprocket used in this example.


11.) Draw line segment yz (Shown in green)
perpendicular to line cy. This can be drawn initially to any point on line cy. After drawing the line segment yz perpendicular to cy, move the lower end of the line segment to point y as shown below.


In the example on the left, line yz (shown in green) has been moved to the endpoint of line cy (point y). It remains perpendicular to line cy.

Constrain this line by fixing it in position. Line zy is fully constrained when it appears in black.

12.) Locate point b by offsetting lines 1,2 and 3,4 distances W and V respectively. Offset lines are shown in green.

Note: Some of the geometry is shown in blue as a means for making the visual information more clear.

13.) With point $b$ as the center, draw a circle with radius $F$.
(Radius $F=.1050$ in this example ) The circle is shown in green.

Note: The circle should end up tangent to line zy. It may be necessary to force a tangent relationship in the event that rounding errors prevent that from happening.


These 2 lines meet at the center of the ditch circle
14.) Locate the tooth tip by drawing a line (Shown in green) from the center of the pitch circle at an angle of $\frac{180^{\circ}}{N}$. In the example above, a 30 tooth sprocket yields an angle of $180 / 30=6$ degrees. The tooth tip is the intersection of the circle (center at b) and the line drawn from the center of the pitch circle, 6 degrees left of line 1,2.
15.) The tooth form NL, is outlined in red below. This represents $1 / 2$ of the tooth form. The tooth form is comprised of 4 sections.


These 2 lines meet at the center of the ditch circle

The images below illustrate the beginning of the trimming process necessary to reveal the tooth form.


16.) Continue the trimming process until only the tooth form and the (mirror axis) line 1,2 are remaining.


Make line 1,2 a construction line. Make the pitch circle a construction line. Refer to the illustration below.

17.) Mirror the tooth form about line 1,2 . This represents an entire tooth form.

18.) Array this tooth form about the center of the pitch circle. Since there are 30 teeth on this example sprocket, array 30 instances of the tooth form, evenly spaced around the center of the pitch circle.

19.) Complete the sprocket drawing by extruding the sprocket to a measured thickness equal to that of the GEARS-IDS 30 tooth sprocket. Note: Industry standard thickness $=0.93 \mathrm{~W}-0.006$. The 0.006" represents a "clearance" number.

20.) Create a 1 " diameter hub with a 0.3 " projection.

1" dia.


21.) Cut a $7 / 16$ " hex bore through the hub and sprocket. The hex is specified as the distance across two opposite flats. Create the hex by circumscribing it around a circle with a $7 / 16$ " diameter.

12.) Nylon or delrin sprockets need extra support to remain stiff and non deforming under load. The GEARS 30 tooth sprocket is cast with webbing as shown in figures 1 and 2 below. For added realism, measure the webbing on the GEARS-IDS sprocket with a dial caliper and create the sprocket geometry shown in the examples below.

Note: For added realism, bevel the sprocket teeth as shown in figures 3 and 4 below. Other options include adding keyways and set screw or pin bores. For set screws refer to the GEARS drill and tapping lessons to learn how to specify hole sizes and thread pitch for various size screws.


Fig.2.) CAD Solid Model


Fig.4.) 10 Tooth Solid Model w/ Bevel Tooth


Fig.3.) Rendered Solid Model


Fig.5.) 20 Tooth Solid Model w/ Bevel Tooth and 3/32" Keyway

Note: Use the GEARS-IDS spread sheet entitled Equations for the Design of Standard Sprocket Teeth to check the calculated values found using the equations listed in Table 1.

