3-6 Multiplying Matrices

Determine whether each matrix product is defined. If so, state the dimensions of the product.

1. $A_{2\times 4}\cdot B_{4\times 3}$

ANSWER:

2×3

2. $C_{5\times 4}\cdot D_{5\times 4}$

ANSWER:

undefined

3. $E_{8\times 6}\cdot F_{6\times 10}$

ANSWER:

8×10

Find each product, if possible.

4. $\begin{bmatrix} 2 & 1 \\ 7 & -5 \end{bmatrix}\cdot\begin{bmatrix} -6 & 3 \\ -2 & -4 \end{bmatrix}$

ANSWER:

$\begin{bmatrix} -14 & 2 \\ -32 & 41 \end{bmatrix}$

5. $\begin{bmatrix} 10 & -2 \\ -7 & 3 \end{bmatrix}\cdot\begin{bmatrix} 1 & 4 \\ 5 & -2 \end{bmatrix}$

ANSWER:

$\begin{bmatrix} 0 & 44 \\ 8 & -34 \end{bmatrix}$

6. $\begin{bmatrix} 9 & -2 \end{bmatrix}\cdot\begin{bmatrix} -2 & 4 \\ 6 & -7 \end{bmatrix}$

ANSWER:

$\begin{bmatrix} -30 & 50 \end{bmatrix}$

7. $\begin{bmatrix} -9 \\ 6 \end{bmatrix}\cdot\begin{bmatrix} -1 & -10 & 1 \end{bmatrix}$

ANSWER:

$\begin{bmatrix} 9 & 90 & -9 \\ -6 & -60 & 6 \end{bmatrix}$

8. $\begin{bmatrix} -8 & 7 & 4 \\ -5 & -3 & 8 \end{bmatrix}\cdot\begin{bmatrix} 10 & 6 \\ 8 & 4 \end{bmatrix}$

ANSWER:

Undefined

9. $\begin{bmatrix} 2 & 8 \\ 3 & -1 \end{bmatrix}\cdot\begin{bmatrix} 6 \\ -7 \end{bmatrix}$

ANSWER:

$\begin{bmatrix} -44 \\ 25 \end{bmatrix}$

10. $\begin{bmatrix} -4 & 3 & 2 \\ -1 & -5 & 4 \end{bmatrix}\cdot\begin{bmatrix} 2 & 1 & 6 \\ 8 & 4 & -1 \\ 5 & 3 & -2 \end{bmatrix}$

ANSWER:

$\begin{bmatrix} 26 & 14 & -31 \\ -22 & -9 & -9 \end{bmatrix}$

11. $\begin{bmatrix} 2 & 5 & 3 & -1 \\ -3 & 1 & 8 & -3 \end{bmatrix}\cdot\begin{bmatrix} 6 & -3 \\ -7 & 1 \\ 2 & 0 \\ -1 & 0 \end{bmatrix}$

ANSWER:

$\begin{bmatrix} -16 & -1 \\ -6 & 10 \end{bmatrix}$

12. **CCSS SENSE-MAKING** The table shows the number of people registered for aerobics for the first quarter.
Quinn's Gym charges the following registration fees: class-by-class, \$165; 11-class pass, \$110; unlimited pass, \$239.

Quinn's Gym		
Payment	Aerobics	Step Aerobics
class-by-class	35	28
11-class pass	32	17
unlimited pass	18	12

a. Write a matrix for the registration fees and a matrix for the number of students.
b. Find the total amount of money the gym received from aerobics and step aerobic registrations.

ANSWER:

a. $\begin{bmatrix} 165 & 110 & 239 \end{bmatrix} \begin{bmatrix} 35 & 28 \\ 32 & 17 \\ 18 & 12 \end{bmatrix}$

b. \$22,955

Use $X = \begin{bmatrix} -10 & -3 \\ 2 & -8 \end{bmatrix}, Y = \begin{bmatrix} -5 & 6 \\ -1 & 9 \end{bmatrix}, \text{and } Z = \begin{bmatrix} -5 & -1 \\ -8 & -4 \end{bmatrix}$ **to determine whether the following equations are true for the given matrices.**

13. $XY = YX$

ANSWER:

No; $\begin{bmatrix} 53 & -87 \\ -2 & -60 \end{bmatrix} \neq \begin{bmatrix} 62 & -33 \\ 28 & -69 \end{bmatrix}$.

14. $X(YZ) = (XY)Z$

ANSWER:

Yes; $X(YZ) = \begin{bmatrix} 431 & 295 \\ 490 & 242 \end{bmatrix}$ and

$(XY)Z = \begin{bmatrix} 431 & 295 \\ 490 & 242 \end{bmatrix}$.

Determine whether each matrix product is defined. If so, state the dimensions of the product.

15. $P_{2 \times 3} \cdot Q_{3 \times 4}$

ANSWER:

2×4

16. $A_{5 \times 5} \cdot B_{5 \times 5}$

ANSWER:

5×5

17. $M_{3 \times 1} \cdot N_{2 \times 3}$

ANSWER:

undefined

18. $X_{2 \times 6} \cdot Y_{6 \times 3}$

ANSWER:

2×3

19. $J_{2 \times 1} \cdot K_{2 \times 1}$

ANSWER:

undefined

20. $S_{5 \times 2} \cdot T_{2 \times 4}$

ANSWER:

5×4

Find each product, if possible.

21. $\begin{bmatrix} 1 & 6 \end{bmatrix} \cdot \begin{bmatrix} -10 \\ 6 \end{bmatrix}$

ANSWER:

[26]

22. $\begin{bmatrix} 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 2 & -7 \end{bmatrix}$

ANSWER:

$\begin{bmatrix} 12 & -42 \\ -6 & 21 \end{bmatrix}$

23. $\begin{bmatrix} -3 & -7 \\ -2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 4 \\ 9 & -3 \end{bmatrix}$

ANSWER:

$\begin{bmatrix} -75 & 9 \\ -17 & -5 \end{bmatrix}$

24. $\begin{bmatrix} -1 & 0 \\ 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 & -3 \\ 7 & -2 \end{bmatrix}$

ANSWER:

$\begin{bmatrix} -6 & 3 \\ 44 & -19 \end{bmatrix}$

25. $\begin{bmatrix} -1 & 0 & 6 \\ -4 & -10 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 & -7 \\ -2 & -9 \end{bmatrix}$

ANSWER:

Undefined

26. $\begin{bmatrix} -6 & 4 & -9 \\ 2 & 8 & 7 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 2 \\ 4 \end{bmatrix}$

ANSWER:

$\begin{bmatrix} -70 \\ 58 \end{bmatrix}$

27. $\begin{bmatrix} 2 & 9 & -3 \\ 4 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 4 & 2 \\ -6 & 7 \\ -2 & 1 \end{bmatrix}$

ANSWER:

$\begin{bmatrix} -40 & 64 \\ 22 & 1 \end{bmatrix}$

28. $\begin{bmatrix} -4 \\ 8 \end{bmatrix} \cdot \begin{bmatrix} -3 & -1 \end{bmatrix}$

ANSWER:

$\begin{bmatrix} 12 & 4 \\ -24 & -8 \end{bmatrix}$

29. **TRAVEL** The Wolf family owns three bed and breakfasts in a vacation spot. A room with a single bed is $220 a night, a room with two beds is $250 a night, and a suite is $360.

Available Rooms at a Wolf Bed and Breakfast

B & B	Single	Double	Suite
1	3	2	2
2	2	3	1
3	4	3	0

a. Write a matrix for the number of each type of room at each bed and breakfast. Then write a room-cost matrix.
b. Write a matrix for total daily income, assuming that all the rooms are rented.
c. What is the total daily income from all three bed and breakfasts, assuming that all the rooms are rented?

ANSWER:

a. $I = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 1 \\ 4 & 3 & 0 \end{bmatrix}, C = \begin{bmatrix} 220 \\ 250 \\ 360 \end{bmatrix}$

b. $\begin{bmatrix} \$1880 \\ \$1550 \\ \$1630 \end{bmatrix}$

c. $5060

Use $P = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}, Q = \begin{bmatrix} 6 & 4 \\ -2 & -5 \end{bmatrix}, R = \begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix}$, and $k = 2$ to determine whether the following equations are true for the given matrices.

30. $k(PQ) = P(kQ)$

ANSWER:

Yes; $k(PQ) = \begin{bmatrix} 52 & 42 \\ 4 & -12 \end{bmatrix}$ and $P(kQ) = \begin{bmatrix} 52 & 42 \\ 4 & -12 \end{bmatrix}$

31. $PQR = RQP$

ANSWER:

No; $PQR = \begin{bmatrix} -22 & 240 \\ 44 & -12 \end{bmatrix}$ and $RQP = \begin{bmatrix} 34 & -40 \\ -220 & -44 \end{bmatrix}$.

32. $PR + QR = (P + Q)R$

ANSWER:

Yes; $PR + QR = \begin{bmatrix} 22 & 72 \\ 14 & -18 \end{bmatrix}$ and $(P + Q)R = \begin{bmatrix} 22 & 72 \\ 14 & -18 \end{bmatrix}$.

33. $R(P + Q) = PR + QR$

ANSWER:

No; $R(P + Q) = \begin{bmatrix} 34 & -6 \\ -64 & -30 \end{bmatrix}$ and $PR + QR = \begin{bmatrix} 22 & 72 \\ 14 & -18 \end{bmatrix}$.

34. **CCSS SENSE-MAKING** Student Council is selling flowers for Mother's Day. They bought 200 roses, 150 daffodils, and 100 orchids for the purchase prices shown. They sold all of the flowers for the sales prices shown.

Flower	Purchase Price	Sales Price
rose	$1.67	$3.00
daffodil	$1.03	$2.25
orchid	$2.59	$4.50

a. Organize the data in two matrices, and use matrix multiplication to find the total amount that was spent on the flowers.
b. Write two matrices, and use matrix multiplication to find the total amount the student council received for the flower sale.
c. Use matrix operations to find how much money the student council made on their project.

ANSWER:

a. $747.50
b. $1387.50
c. $640

35. **AUTO SALES** A car lot has four sales associates. At the end of the year, each sales associate gets a bonus of $1000 for every new car they have sold and $500 for every used car they have sold.

Cars Sold by Each Associate		
Sales Associate	New Cars	Used Cars
Mason	27	49
Westin	35	36
Gallagher	9	56
Stadler	15	62

a. Use a matrix to determine which sales associate earned the most money.
b. What is the total amount of money the car lot spent on bonuses for the sales associates this year?

ANSWER:

a. $\begin{bmatrix} 27 & 49 \\ 35 & 36 \\ 9 & 56 \\ 15 & 62 \end{bmatrix} \cdot \begin{bmatrix} 1000 \\ 500 \end{bmatrix} = \begin{bmatrix} 51,500 \\ 53,000 \\ 37,000 \\ 46,000 \end{bmatrix}$;
b. $187,500

Use matrices $X = \begin{bmatrix} 2 & -6 \\ 3y & -4.5 \end{bmatrix}$, $Y = \begin{bmatrix} -5 & -1.5 \\ x+2 & y \\ 13 & 1.2 \end{bmatrix}$, and $Z = \begin{bmatrix} -3 \\ x+y \end{bmatrix}$ **to find each of the following. If the matrix does not exist, write undefined.**

36. XY

ANSWER:

undefined

37. YX

ANSWER:

$\begin{bmatrix} -10 - 4.5y & 36.75 \\ 2x + 4 + 3y^2 & -6x - 4.5y - 12 \\ 3.6y + 26 & -83.4 \end{bmatrix}$

38. ZY

ANSWER:

undefined

39. *YZ*

ANSWER:

$$\begin{bmatrix} -1.5x-1.5y+15 \\ y^2+xy-3x-6 \\ 1.2x+1.2y-39 \end{bmatrix}$$

40. *(YX)Z*

ANSWER:

$$\begin{bmatrix} 36.75x+50.25y+30 \\ -6x^2-18x-10.5xy-13.5y^2-12y-12 \\ -83.4x-94.2y-78 \end{bmatrix}$$

41. *(XZ)X*

ANSWER:

undefined

42. *X(ZZ)*

ANSWER:

undefined

43. *(XX)Z*

ANSWER:

$$\begin{bmatrix} 15x+69y-12 \\ -18y^2+42.75y-18xy+20.25x \end{bmatrix}$$

44. **CAMERAS** Prices of digital cameras depend on features like optical zoom, digital zoom, and megapixels.

Optical Zoom	6 MP	7 MP	10 MP
3 to 4	$189.99	$249.99	$349.99
5 to 6	$199.99	$289.99	$399.99
10 to 12	$299.99	$399.99	$499.99

a. The 10-mp cameras are on sale for 20% off, and the other models are 10% off. Write a new matrix for these changes.

b. Write a new matrix allowing for a 6.25% sales tax on the discounted prices.

c. Describe what the differences in these two matrices represent.

ANSWER:

a. $\begin{bmatrix} \$170.99 & \$224.99 & \$279.99 \\ \$179.99 & \$260.99 & \$319.99 \\ \$269.99 & \$359.99 & \$399.99 \end{bmatrix}$

b. $\begin{bmatrix} \$181.68 & \$239.05 & \$297.49 \\ \$191.24 & \$277.30 & \$339.99 \\ \$286.86 & \$382.49 & \$424.99 \end{bmatrix}$

c. sales tax

3-6 Multiplying Matrices

45. **BUSINESS** The Kangy Studio has packages available for senior portraits.

Size (price)	Packages			
	A	B	C	D
4 × 5 ($7)	10	10	8	0
5 × 7 ($10)	4	4	4	4
8 × 10 ($14)	2	2	2	2
11 × 14 ($45)	1	1	0	0
16 × 20 ($95)	1	0	0	0
Wallets (8 for $13)	88	56	16	0

a. Use matrices to determine the total cost of each package.
b. The studio offers an early bird discount of 15% off any package. Find the early bird price for each package.

ANSWER:

a. A: $421; B: $274; C: $150; D: $68
b. A: $357.85; B: $232.90; C: $127.50; D: $57.80

46. **REASONING** If the product matrix AB has dimensions 5×8, and A has dimensions 5×6, what are the dimensions of matrix B?

ANSWER:

6×8

47. **CCSS ARGUMENTS** Show that each property of matrices.
a. Scalar Distributive Property
b. Matrix Distributive Property
c. Associative Property of Multiplication
d. Associative Property of Scalar Multiplication

ANSWER:

a.

$c(A+B)$

$= c\left(\begin{bmatrix} a & b \\ d & e \end{bmatrix} + \begin{bmatrix} w & x \\ y & z \end{bmatrix}\right)$ Substitution

$= c\begin{bmatrix} a+w & b+x \\ d+y & e+z \end{bmatrix}$ Definition of matrix additon

$= \begin{bmatrix} ca+cw & cb+cx \\ cd+cy & ce+cz \end{bmatrix}$ Definiton of scalar multiplication

$= \begin{bmatrix} ca & cb \\ cd & ce \end{bmatrix} + \begin{bmatrix} cw & cx \\ ce & cz \end{bmatrix}$ Definition of matrixaddition

$= CA + CB$ Substitution

b.

$C(A+B) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\left(\begin{bmatrix} e & f \\ g & h \end{bmatrix} + \begin{bmatrix} j & k \\ m & n \end{bmatrix}\right)$ Substitution

$= \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e+j & f+k \\ g+m & h+n \end{bmatrix}$

$= \begin{bmatrix} a(e+j)+b(g+m) & a(f+k)+b(h+n) \\ c(e+j)+d(g+m) & c(f+k)+d(h+n) \end{bmatrix}$

$= \begin{bmatrix} ea+ja+gb+mb & fa+ka+hb+nb \\ ec+jc+gd+md & fc+kc+hd+nd \end{bmatrix}$

$= \begin{bmatrix} ea+gb+ja+mb & fa+hb+ka+nb \\ ec+gd+jc+md & fc+hd+kc+nd \end{bmatrix}$

$= \begin{bmatrix} ec+gb & fa+hb \\ ec+gd & fc+md \end{bmatrix} + \begin{bmatrix} ja+mb & ka+md \\ jc+md & kc+nd \end{bmatrix}$

$= CA + CB$ Definition of matrix multiplication

$(A+B)C$

$= \left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \left[\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\right]\right)\left(\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\right)$ Substitution

$= \left(\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix}\right)\left(\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\right)$ Definition of Matrix Addition

$= \begin{bmatrix} (a_{11}+b_{11})c_{11}+(a_{12}+b_{12})c_{21} & (a_{11}+b_{11})c_{12}+(a_{12}+b_{12})c_{22} \\ (a_{21}+b_{21})c_{11}+(a_{22}+b_{22})c_{21} & (a_{21}+b_{21})c_{12}+(a_{22}+b_{22})c_{22} \end{bmatrix}$

$= \begin{bmatrix} a_{11}c_{11}+b_{11}c_{11}+a_{12}c_{21}+b_{12}c_{21} & a_{11}c_{12}+b_{11}c_{12}+a_{12}c_{22}+b_{12}c_{22} \\ a_{21}c_{11}+b_{21}c_{11}+a_{22}c_{21}+b_{22}c_{21} & a_{21}c_{12}+b_{21}c_{12}+a_{22}c_{22}+b_{22}c_{22} \end{bmatrix}$

$= \begin{bmatrix} a_{11}c_{11}+a_{12}c_{21}+b_{11}c_{11}+b_{12}c_{21} & a_{11}c_{12}+a_{12}c_{22}+b_{11}c_{12}+b_{12}c_{22} \\ a_{21}c_{11}+a_{22}c_{21}+b_{21}c_{11}+b_{22}c_{21} & a_{21}c_{12}+a_{22}c_{22}+b_{21}c_{12}+b_{22}c_{22} \end{bmatrix}$

$= \begin{bmatrix} a_{11}c_{11}+a_{12}c_{21} & a_{11}c_{12}+a_{12}c_{22} \\ a_{21}c_{11}+a_{22}c_{21} & a_{21}c_{12}+a_{22}c_{22} \end{bmatrix} + \begin{bmatrix} b_{11}c_{11}+b_{12}c_{21} & b_{11}c_{12}+b_{12}c_{22} \\ b_{21}c_{11}+b_{22}c_{21} & b_{21}c_{12}+b_{22}c_{22} \end{bmatrix}$

$= AC + BC$ Definition of Matrix Multiplication

c.

$(AB)C$

$= \left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\right)\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$ Substitution

$= \begin{bmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \end{bmatrix}\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$ Definition of Matrix Multiplication

$= \begin{bmatrix} (a_{11}b_{11}+a_{12}b_{21})c_{11}+(a_{11}b_{12}+a_{12}b_{22})c_{21} & (a_{11}b_{11}+a_{12}b_{21})c_{12}+(a_{11}b_{12}+a_{12}b_{22})c_{22} \\ (a_{21}b_{11}+a_{22}b_{21})c_{11}+(a_{21}b_{12}+a_{22}b_{22})c_{21} & (a_{21}b_{11}+a_{22}b_{21})c_{12}+(a_{21}b_{12}+a_{22}b_{22})c_{22} \end{bmatrix}$

$= \begin{bmatrix} a_{11}b_{11}c_{11}+a_{12}b_{21}c_{11}+a_{11}b_{12}c_{21}+a_{12}b_{22}c_{21} & a_{11}b_{11}c_{12}+a_{12}b_{21}c_{12}+a_{11}b_{12}c_{22}+a_{12}b_{22}c_{22} \\ a_{21}b_{11}c_{11}+a_{22}b_{21}c_{11}+a_{21}b_{12}c_{21}+a_{22}b_{22}c_{21} & a_{21}b_{11}c_{12}+a_{22}b_{21}c_{12}+a_{21}b_{12}c_{22}+a_{22}b_{22}c_{22} \end{bmatrix}$

$= \begin{bmatrix} a_{11}b_{11}c_{11}+a_{11}b_{12}c_{21}+a_{12}b_{21}c_{11}+a_{12}b_{22}c_{21} & a_{11}b_{11}c_{12}+a_{11}b_{12}c_{22}+a_{12}b_{21}c_{12}+a_{12}b_{22}c_{22} \\ a_{21}b_{11}c_{11}+a_{21}b_{12}c_{21}+a_{22}b_{21}c_{11}+a_{22}b_{22}c_{21} & a_{21}b_{11}c_{12}+a_{21}b_{12}c_{22}+a_{22}b_{21}c_{12}+a_{22}b_{22}c_{22} \end{bmatrix}$

$= \begin{bmatrix} a_{11}(b_{11}c_{11}+b_{12}c_{21})+a_{12}(b_{21}c_{11}+b_{22}c_{21}) & a_{11}(b_{11}c_{12}+b_{12}c_{22})+a_{12}(b_{21}c_{12}+b_{22}c_{22}) \\ a_{21}(b_{11}c_{11}+b_{12}c_{21})+a_{22}(b_{21}c_{11}+b_{22}c_{21}) & a_{21}(b_{11}c_{12}+b_{12}c_{22})+a_{22}(b_{21}c_{12}+b_{22}c_{22}) \end{bmatrix}$

$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11}c_{11}+b_{12}c_{21} & b_{11}c_{12}+b_{12}c_{22} \\ b_{21}c_{11}+b_{22}c_{21} & b_{21}c_{12}+b_{22}c_{22} \end{bmatrix}$ Definition of Matrix Multiplication

$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\left(\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\right)$ Definition of Matrix Multiplication

$= A(BC)$ Substitution

d.

$c(AB) = c\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\right)$ Substi

$= c\begin{bmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \end{bmatrix}$ Defini

$= \begin{bmatrix} c(a_{11}b_{11}+a_{12}b_{21}) & c(a_{11}b_{12}+a_{12}b_{22}) \\ c(a_{21}b_{11}+a_{22}b_{21}) & c(a_{21}b_{12}+a_{22}b_{22}) \end{bmatrix}$ Defini

$= \begin{bmatrix} ca_{11}b_{11}+ca_{12}b_{21} & ca_{11}b_{12}+ca_{12}b_{22} \\ ca_{21}b_{11}+ca_{22}b_{21} & ca_{21}b_{12}+ca_{22}b_{22} \end{bmatrix}$ Distri

Defini

$$= \begin{bmatrix} ca_{11} & ca_{12} \\ ca_{21} & ca_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$ Defini

$$c\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$= (cA)B$ Substi

Subst

$$c(AB) = c\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\right)$$ Defini

$$= c\begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$ Defini

$$= \begin{bmatrix} c(a_{11}b_{11} + a_{12}b_{21}) & c(a_{11}b_{12} + a_{12}b_{22}) \\ c(a_{21}b_{11} + a_{22}b_{21}) & c(a_{21}b_{12} + a_{22}b_{22}) \end{bmatrix}$$ Distri

$$= \begin{bmatrix} ca_{11}b_{11} + ca_{12}b_{21} & ca_{11}b_{12} + ca_{12}b_{22} \\ ca_{21}b_{11} + ca_{22}b_{21} & ca_{21}b_{12} + ca_{22}b_{22} \end{bmatrix}$$ Com

$$= \begin{bmatrix} a_{11}cb_{11} + a_{12}cb_{21} & a_{11}cb_{12} + a_{12}cb_{22} \\ a_{21}cb_{11} + a_{22}cb_{21} & a_{21}cb_{12} + a_{22}cb_{22} \end{bmatrix}$$ Defini

$$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} cb_{11} & cb_{12} \\ cb_{21} & cb_{22} \end{bmatrix}$$

$= A(cB)$ Substi

48. **OPEN ENDED** Write two matrices A and B such that $AB = BA$.

ANSWER:

Sample Answer: $A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$

49. **CHALLENGE** Find the missing values in $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 11 \\ 20 & 29 \end{bmatrix}$

ANSWER:

$a = 2, b = 1, c = 3, d = 4$

50. **WRITING IN MATH** Use the data on Lisa Leslie found at the beginning of the lesson to explain how matrices can be used in sports statistics. Describe a matrix that represents the total number of points she has scored during her career and an example of a sport in which different point values are used in scoring.

ANSWER:

Sports statistics are often listed in columns and matrices. In this case, you can find the total number of points scored by multiplying the point value matrix, which does not change, by the scoring matrix, which changes after each season. The total number of points for her career can be found by multiplying the scoring matrix S by the point matrix P. Basketball and wrestling use different point values in scoring.

51. **GRIDDED RESPONSE** The average (arithmetic mean) of r, w, x, and y is 8, and the average of x and y is 4. What is the average of r and w?

ANSWER:

12

52. Carla, Meiko, and Kayla went shopping to get ready for college. Their purchases and total amounts spent are shown in the table below.

Person	Shirts	Pants	Shoes	Total Spent
Carla	3	4	2	$149.79
Meiko	5	3	3	$183.19
Kayla	6	5	1	$181.14

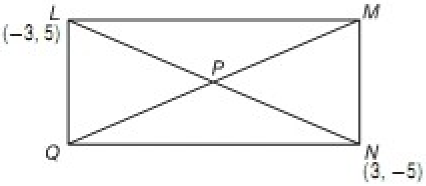
Assume that all of the shirts were the same price, all of the pants were the same price, and all of the shoes were the same price. What was the price of each item?

A shirt, $12.95; pants, $15.99; shoes, $23.49
B shirt, $15.99; pants, $12.95; shoes, $23.49
C shirt, $15.99; pants, $23.49; shoes, $12.95
D shirt, $23.49; pants, $15.99; shoes, $12.95

ANSWER:

A

53. **GEOMETRY** Rectangle *LMNQ* has diagonals that intersect at point P. Which of the following represents point *P*?



F (2, 2)
G (1, 1)
H (0, 0)
J (−1, −1)

ANSWER:
H

54. **SAT/ACT** What are the dimensions of the matrix that results from the multiplication shown?

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 4 \\ 6 \end{bmatrix}$$

A 1 × 4
B 3 × 3
C 3 × 1
D 4 × 1
E 4 × 3

ANSWER:
D

Perform the indicated operations. If the matrix does not exist, write *impossible*.

55. $4\begin{bmatrix} 8 & -1 \\ -3 & -4 \end{bmatrix} - 5\begin{bmatrix} -2 & 4 \\ 6 & 3 \end{bmatrix}$

ANSWER:
$$\begin{bmatrix} -42 & -24 \\ -42 & -31 \end{bmatrix}$$

56. $5\left(2\begin{bmatrix} -2 & -5 \\ -1 & 3 \end{bmatrix} - 3\begin{bmatrix} -1 & -2 \\ 6 & 4 \end{bmatrix}\right)$

ANSWER:
$$\begin{bmatrix} -5 & -20 \\ -100 & -30 \end{bmatrix}$$

57. $-4\left(\begin{bmatrix} 8 & 9 \\ -5 & 5 \end{bmatrix} - 2\begin{bmatrix} -6 & -1 \\ 6 & 3 \end{bmatrix}\right)$

ANSWER:
$$\begin{bmatrix} -80 & -44 \\ 68 & 4 \end{bmatrix}$$

Solve each system of equations.

58. $$2x - 4y + 3z = -3$$
$$-7x + 5y - 4x = 11$$
$$x - y - 2z = -21$$

ANSWER:
(–2, 5, 7)

59. $$-4x - 2y + 9z = -29$$
$$10x - 12y + 7z = 51$$
$$3x + 5y - 14z = 25$$

ANSWER:
(8, 3, 1)

60. $$-7x + 8y - z = 43$$
$$3x - 2y + 5z = -43$$
$$2x - 4y + 6z = -50$$

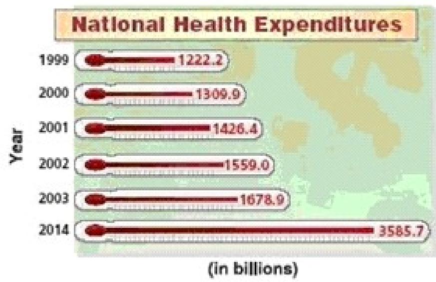
ANSWER:
(–3, 2, –6)

61. **MEDICINE** The graph shows how much Americans spent on doctors' visits in some recent years and a prediction for 2014.

a. Find a regression equation for the data without the predicted value.

b. Use your equation to predict the expenditures for 2014.

c. Compare your prediction to the one given in the graph.



ANSWER:

a. Sample answer: $y = 116.25x - 231,176.97$

b. Sample answer: $2950.53 billion

c. The value predicted by the equation is significantly lower than the one given in the graph.

62. How many different ways can the letters of the word *MATHEMATICS* be arranged?

ANSWER:

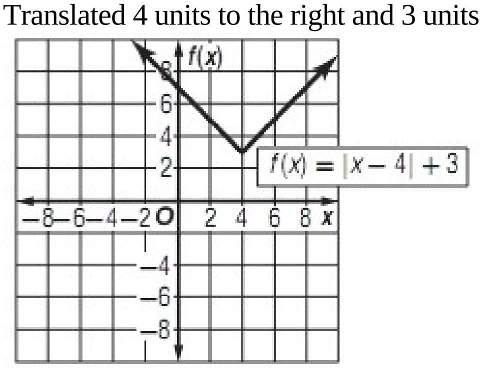
4,989,600

Describe the transformation in each function. Then graph the function.

63. $f(x) = |x - 4| + 3$

ANSWER:

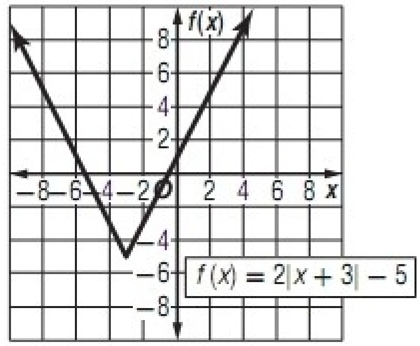
Translated 4 units to the right and 3 units up.



64. $f(x) = 2|x + 3| - 5$

ANSWER:

Translated 3 units to the left and 5 units down and stretched vertically..



65. $f(x) = (x + 2)^2 - 6$

ANSWER:

Translated 2 units to the left and 6 units down.