Development and Implementation of Algorithms for Electronic Warfare Signal Processing

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Abstract: ***Electronic warfare (EW) deals with advances in military technology to master and control the electro-magnetic spectrum to achieve offensive and defensive operational objectives against the adversary radars. EW system generally includes transmit-system like Radar, receiver-system like Radar Warning Receivers (RWRs) and receiver transmit-system like Radar Warning Jammers (RWJ). EW system may be ground-based, ship-borne or air-borne. In the present context, case of air-borne EW is considered from the perspective of a host airborne RWR/RWJ acting against multitude radar in time, frequency, and spatial domains. In a typical airborne-EW scenario, host aircraft encounters various threat radar with wide range of functionalities like search, surveillance, track, gun-fire control etc. Each radar is designed with specific signal and modulation characteristics coupled with allied radiation profiles. Host aircraft deploys ESM/RWR techniques/system for detection, estimation and classification and ECM/RWJ techniques/system to evade/deceive/destroy the threat radar. The objective here is to receive and identify the threat signal, manipulate the signal characteristics and enter the reception path of the threat radar with the jamming signal. The constraints and challenges for the EW system in this process are multi-octave frequency range of operation, wide instantaneous bandwidths, broad spatial coverage, presence of multiple threat radar with different signatures, real-time processing difficulties, large data handling, lack of apriori information on signal and noise statistics, high-level of noise and interference etc.***

Keywords: Radar, Warfare, Signal Processing, Air-borne, Radar warning Jammers, Radar warning Receivers

1. Introduction

Electronic Warfare (EW) is not strictly 'electronic', i.e., it is not conducted using electrons; rather it is electromagnetic, and uses the entire range of the electromagnetic spectrum. Because of this, some people also call it Electromagnetic Warfare. During World War II, Sir Winston Churchill coined the words 'wizard war' and 'battle of beams'. However, the most accepted term for this field of applied science is 'Electronic Warfare'. Electronic circuits are, of course, used in EW equipment.

Electronic Warfare (EW) deals with advances in military technology to master and control the electro-magnetic spectrum to achieve offensive and defensive operational objectives against the adversaries. Radar warning receiver (RWR) systems detect the radio emissions of radar systems. Their primary purpose is to issue a warning when a radar signal that might be a threat (such as a police speed detection radar) is detected. The warning can then be used, manually or automatically, to evade the detected threat. RWR systems can be installed in all kind of airborne, sea-based, and ground-based assets (such as aircraft, ships, automobiles, military bases).

EW is principally divided into three domains viz. Electronic Support Measures (ESM), Electronic Counter Measures (ECM) and Electronic Protection (EP). Candidate EW systems generally includes transmit-systems like Radars, receive-systems like Radar Warning Receivers (RWRs) and receive-transmit-systems like Radar Warning Jammers (RWJ). EW systems may be ground-based, ship-borne or air-borne. In the present context, case of air-borne EW is considered from the perspective of a host airborne RWR/RWJ acting against multitude of adversary radars in time, frequency and spatial domains. The goal of electronic warfare is to control the electromagnetic spectrum. It is generally considered to consist of:

- **Electronic attack**, such as jamming enemy communications or radar, and disrupting enemy equipment using high-power microwaves.
- **Electronic protection**, which ranges from designing systems resistant to jamming, through hardening equipment to resist high-power microwave attack, to the destruction of enemy jammers using anti-radiation missiles.
- **Electronic support** which supplies the necessary intelligence and threat recognition to allow effective attack and protection. It allows commanders to search for, identify and locate sources of intentional and unintentional electromagnetic energy.

2. Objectives of EW

Electronic Warfare, whether it is radar-based or communications- based, employs the electronic devices and techniques for the following purposes.

- Determining the existence and 'placements of the enemy's electronic aids to warfare.
- Destroying or degrading the effectiveness of the enemy's electronic aids to warfare.
- Denying the destruction or degradation of the effectiveness of friendly electronic aids to the warfare.
- EW tries to achieve the above purposes by adopting the following procedures.
- Make full use of electromagnetic emissions released either intentionally or accidentally by the enemy
- Interfere with the enemy's use of electromagnetic spectrum in such a way as to render its use either ineffective, degrading or even dangerous for him
- Defend one's own friendly use of the electromagnetic spectrum.

In this project main objective is to Implement and show performance demonstration of assigned EWSP Algorithms.

Signal Processing in Electronic Warfare

As we know generally filters are used in signal processing to give importance to the required frequency band. By performing filtering operation we can extract information in the band of interest. In our application we use filter bank at receiver side to estimate the exact frequency range where the radar signal lies.

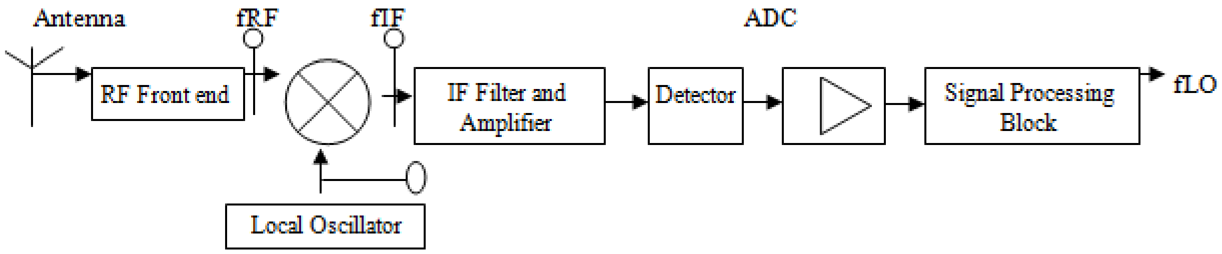

Figure 1: Block diagram of superheterodyne receiver

Signal Processing

This is the main block in any receiver to estimate the signal characteristics or parameters. It mainly includes peak detector and threshold circuitry. This description is discussed in this chapter. Signal estimation can be performed in two methods they are-

1) Time domain approach.
2) Frequency domain approach.

3. Time Domain Approach

Throughout this approach we assume following standards of radar signal for simplicity-

RF range – 1-18GHz, IF range – 750-1250MHz, Sampling frequency (Fs) – 1350MHz

Band pass filters with 20MHz pass band width.

According to our standards radar signal must be in the range given by (Fs-IFmax) to (Fs-IFmin), i.e. 100 to 600 MHz lower side bamd, here we omit the base band peak (675MHz) and upper side band. Hence we design band pass filter in the range of 100 to 600 MHz. This is shown in fig 7.2.

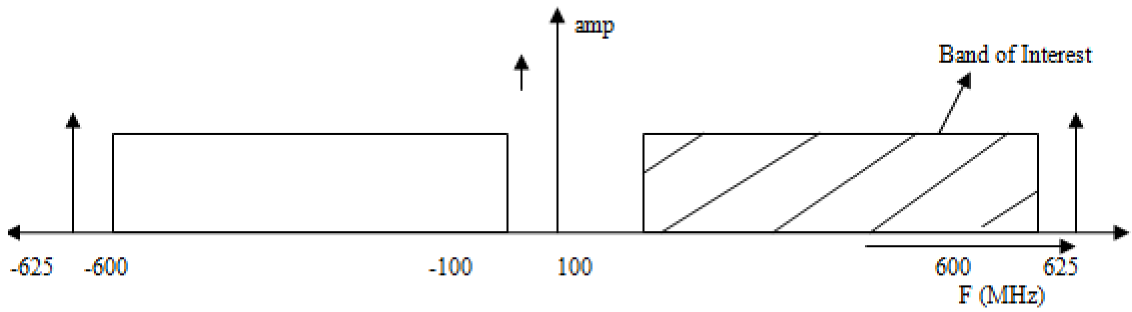

Figure: Frequency band of interest

In time domain approach we follow below steps-

- Passing the received signal to series of band pass filters (BPF) with different pass band frequencies.
- Then output of each filter is correlated with original radar signal in time domain. FFT of output is performed in frequency domain.
- Peak of correlated output is determined; here maximum peak in the plot (filter number v/s amplitude) specifies the BPF number which has pass band equivalent to that of original signal.

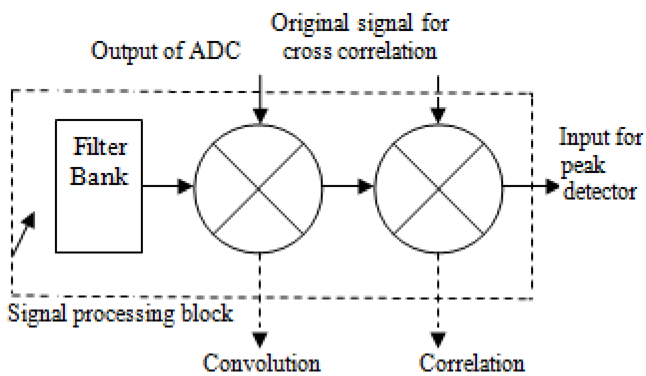

Figure 7.3: Signal Processing Block in Time Domain

Here filter bank is the filter co-efficient which is stored in the form of matrix, where each column refers to individual filters. Hence each column co-efficient are convoluted with signal co-efficient to get filtered output.

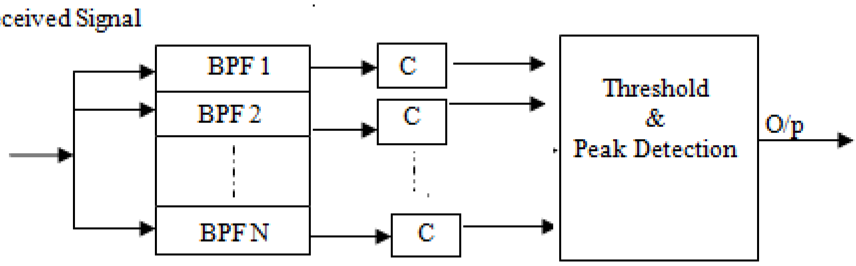

Figure 7.4: Detailed Signal Processing Block

4. Procedure

In this section step by step process of time domain approach is provided

1) Band pass filter design-
According to our standards we want to achieve filtering in frequency band from 100 to 600MHz. If we design BPF of 500MHz band width it is difficult to determine the peak for noisy signal. Hence we use series of band pass filters of 20MHz band width each and collect the result obtained.
2) As there will be large roll off factor for practical filters, we take 5MHz variations in the design, hence final filter design consisting of 25 BPF's is as shown below

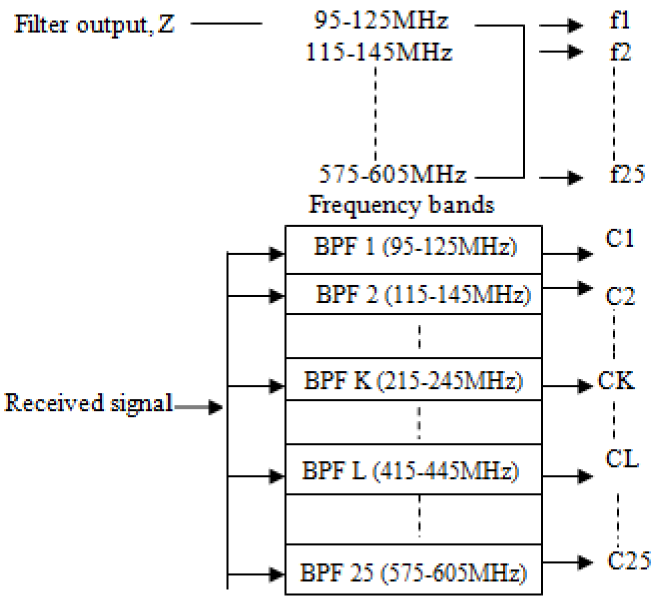

Figure 7.5: BPF Design

3) Here proper FIR filter design technique is used for each filter. Strictly same technique is used for all 25 filters. Then filter coefficients are collected in matrix form of dimension N*25 as shown below-

$$Z = \begin{bmatrix} f1 & f2 & & f25 \\ Z_{1,1} & Z_{1,2} & \text{--------} & Z_{1,25} \\ Z_{2,1} & Z_{2,2} & & Z_{2,25} \\ \vdots & & & \vdots \\ Z_{N,1} & Z_{N,2} & \text{--------} & Z_{N,25} \end{bmatrix}$$

N=Number of coefficients

4) These co-efficient gives the filter response which is convoluted with input coefficients to get filtered signal. Let

X(t) ⟶ received signal
Zn(t) ⟶ filter coefficients
C(t) ⟶ convoluted output

where

$$C(t) = X(t) * Z_n(t) = \int_{-\infty}^{\infty} X(\propto) Z_n(t-\propto) d\propto \quad (7.3)$$

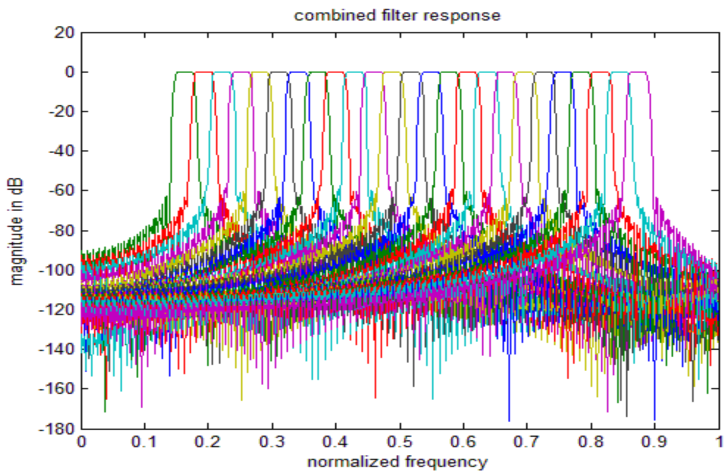

Figure 7.6: Band Pass Filter's Response Designed in MATLAB (25 filters)

5) Then convoluted output is correlated (cross or auto) to determine the signal frequency. In time domain filtering the peak detection is performed using correlation operation. Here there are two methods of determination-
   - First method is performing cross correlation for band filtered output with original radar signal. Here high level of correlated output specifies the particular BPF which filters actual signal frequency.
   - Second method is performing autocorrelation on filtered output. Here low level of correlation specifies the input frequency band.

Cross Correlation Process

In signal processing, cross-correlation is a measure of similarity of two series as a function of the lag of one relative to the other. This is also known as a sliding dot product or sliding inner-product. It is commonly used for searching a long signal for a shorter, known feature. It has applications in pattern_recognition, single particle analysis, electron__tomography, averaging, cryptanalysis, and neurophysiology.

As stated in previous section filtered output (convoluted output) is cross correlated with original radar signal to get proper peak. At the point of signal frequency high correlated output is obtained and at other points scale of correlated output is low. This concept can be understood clearly by considering following cases-

1) First let us consider we are receiving only one signal at given time instant. Let the radar signal frequency be assumed as 500 MHz. Let 'W' be the noise coefficients, 'S' be the signal coefficient. Hence cross correlated output consists of both signal and noise peak components which is represented as- b = S + W.
   Now for different noise conditions we get following results-

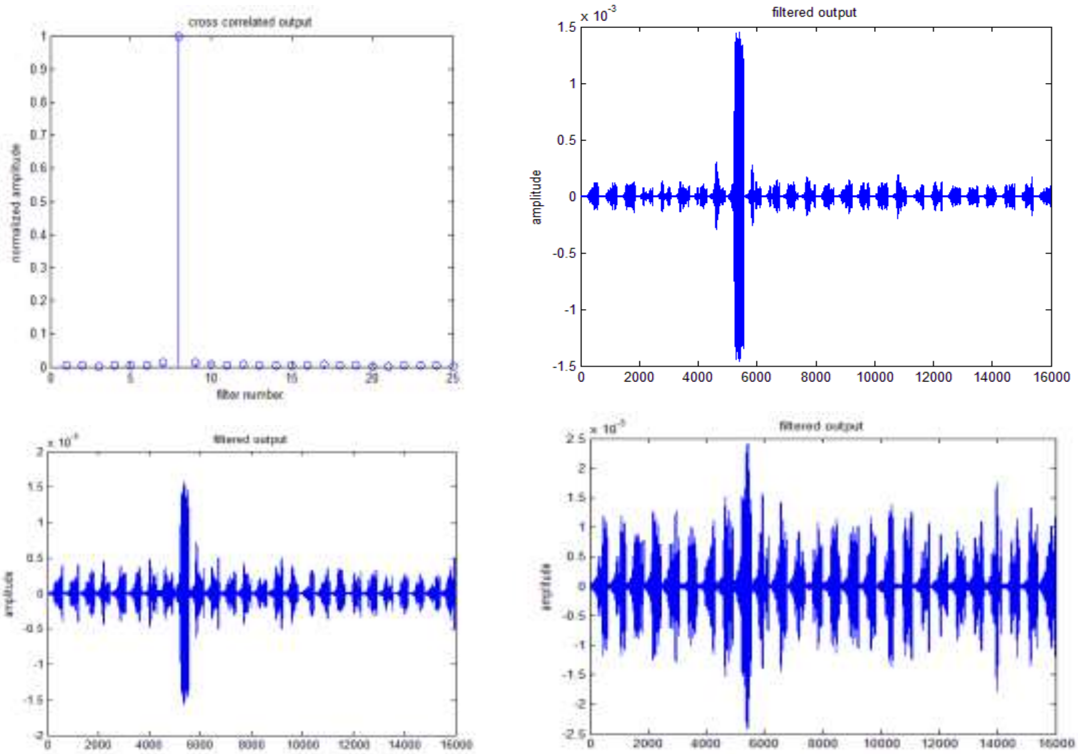

Figure 7.7: Different cases (a) original (b) 10dB noise (c) 0 dB noise (d) -10dB noise for single signal estimation using cross correlation

In the figure's shown above we can easily describe performance of correlators at different noise conditions,
- At noiseless condition (SNR=10dB) signal power is very much greater than noise power as shown in fig 7.7(a), hence signal amplitude is higher compared to noise amplitude. So signal can be easily distinguished.
- At SNR=0dB signal and noise power is almost equal as shown in fig 7.7(b), but while correlation process noise power is reduced compared to that of signal.
- At noisy condition (SNR=-10dB) signal power is less than that of noise power as shown in fig 7.7(c). Hence cross correlation with original signal cannot determine proper peak because both correlated output will be almost equal.

2) Cross correlation based peak detection holds good for only single signal again for less noise condition. This statement can be justified using plots shown below, here we are considering 3 signals at 250 MHz, 330 MHz and 450 MHz. Hence correlated output consists of 4 components S1, S2, S3 and W (noise).

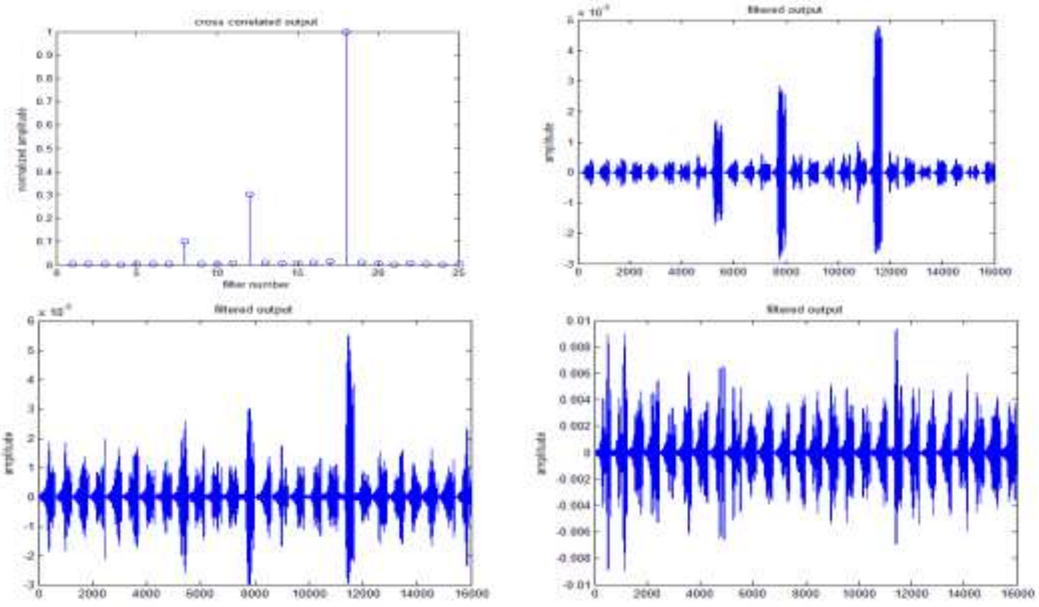

Figure 7.8: Different cases (a) original (b) 10dB noise (c) 0 dB noise (d) -10dB noise for multiple signal estimation using cross correlation

Hence in multiple signal estimation lower power level signal is suppressed by higher power level signal. So lower noise can affect the system very easily.

Again main drawback cross correlation based estimation is setting a threshold value; it is very difficult in noisy conditions. Where there will be high probability of loss of signal. Hence by using auto correlation based approach reduces this error to certain extant.

Auto Correlation Process

Autocorrelation, also known as serial correlation or cross-autocorrelation, is the cross-correlation of a signal with itself at different points in time (that is what the cross stands for). Informally, it is the similarity between observations as a function of the time lag between them. It is a mathematical tool for finding repeating patterns, such as the presence of a periodic signal obscured by noise, or identifying the missing fundamental frequency in a signal implied by its harmonic frequencies. It is often used in signal processing for analyzing functions or series of values, such as time domain signals.

The autocorrelation function tells us the time interval over which a correlation in the noise exists. If the noise is made entirely of waves, and the waves move through the plasma (or other medium) without decaying as they travel, the autocorrelation will be large for all time. In many applications the output is measured with a fixed value of τ that minimizes noise and maximizes the signal. Auto-correlation can remove huge amounts of noise and is particularly useful for frequencies too high for any common laboratory measuring device.

Thus we prefer this method rather than cross correlation, where auto correlation of noise is zero. So peak detection becomes easier.

Frequency Domain Approach

In electronics, control systems engineering, and statistics, the frequency domain refers to the analysis of mathematical functions or signals with respect to frequency, rather than time. Put simply, a time-domain graph shows how a signal changes over time, whereas a frequency-domain graph shows how much of the signal lies within each given frequency band over a range of frequencies. A frequency-domain representation can also include information on the phase shift that must be applied to each sinusoid in order to be able to recombine the frequency components to recover the original time signal.

A given function or signal can be converted between the time and frequency domains with a pair of mathematical operators called a transform. An example is the Fourier transform, which converts the time function into a sum of sine waves of different frequencies, each of which represents a frequency component. The 'spectrum' of frequency components is the frequency domain representation of the signal. The inverse Fourier transform converts the frequency domain function back to a time function. A spectrum analyzer is the tool commonly used to visualize real-world signals in the frequency domain.

Below table shows computation cost for different point FFT

Table 7.1: Computation cost for different point FFT

N-Point	Real signal requirements		Complex signal requirements	
	Number of complex multiplications	Number of complex additions	Number of complex multiplications	Number of complex additions
128	2*128	3*128	7*128	3*128
256	2*256	4*256	8*256	4*256
512	2*512	4*512	9*512	4*512
1024	3*1024	5*1024	10*1024	5*1024
2048	6*2048	6*2048	11*2048	6*2048
4096	6*4096	6*4096	12*4096	6*4096

5. Analysis of N-POINT FFT

In this section let us consider different point FFT for 500 MHz signal. Let us assume signal considered is real. Since signal is at 500 MHz our frequency band of interest will be from 450 to 550 MHz.

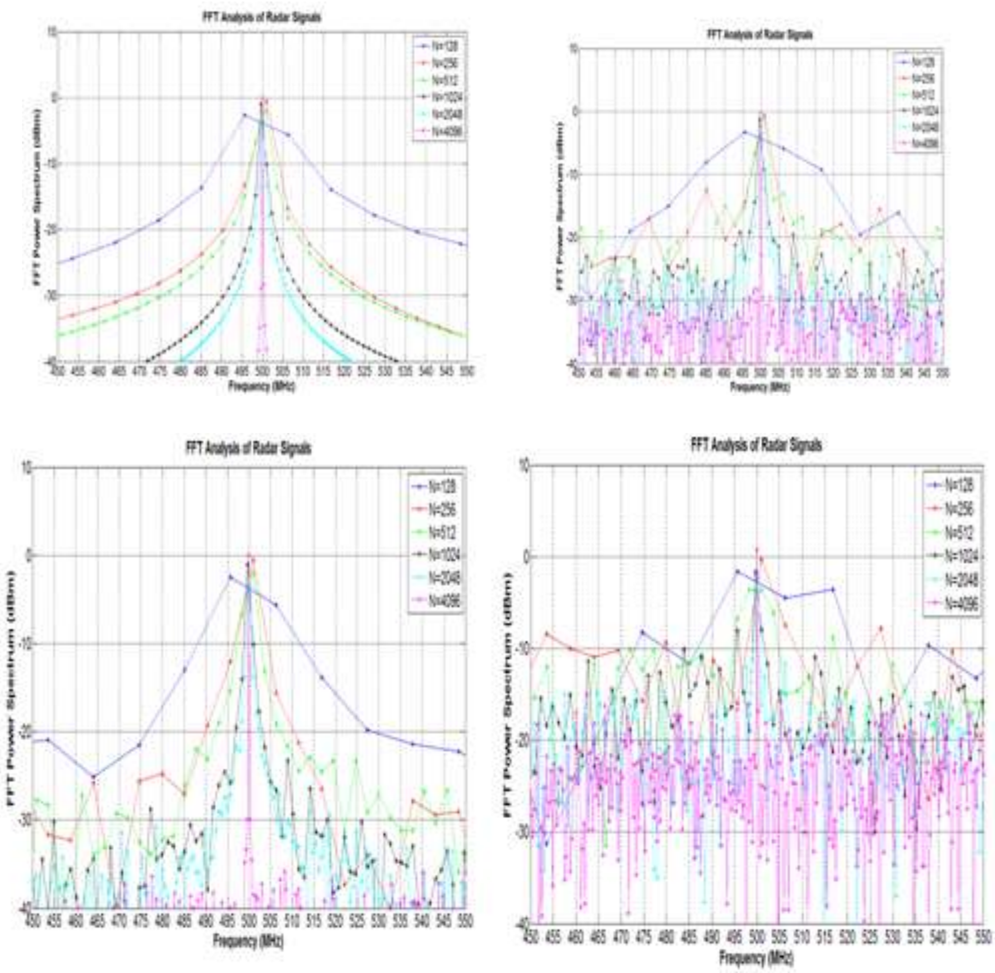

Figure 7.11: FFT plot a) POI region b) 0 dB (c) 10 dB (d) -10 dB different noise levels

Here we can observe that for lower value of N (128) perfect peak is not obtained and frequency band is spread over, but it includes less number of computation at band of interest. By implementing this in the receiver computation cost can be reduced but it is less reliable.

If we choose higher value of N (4096) then perfect peak is obtained at 500 MHz, but it includes large number of computations at the band of interest. Normally our receiver cannot handle these many samples at a time, hence to achieve this we have to design complex receiver, hence it is not preferable.

So we can infer that choosing N in between 512 to 1028 we can get accurate results with reasonable computation cost.

Development of algorithm for signal processing

Present work focuses on **Threat Cognition and Identification** of special class of Radar Signals known as "**Pulse Compressed Radars**". Pulse compression is usually carried out by intra pulse frequency modulation or intra pulse phase modulation. Received signal is considered to be a linear combination of signals of interest embedded in noise. For the sake of establishing basic EWSP concepts, only non time co-existent signal case is presently considered. Handling multiple signals is based on the same concepts with additional processing overheads and further extension of algorithms.

6. Proposed Architecture

In the present work, EWSP for pulse compressed radar signals is proposed to be carried out with the development and MATLAB implementation of two Algorithms viz. Frequency Modulation on Pulse **(FMOP)** Analysis Algorithm and Phase Modulation on Pulse **(PMOP)** Analysis Algorithm. Input signal is assumed to be complex comprising signal of interest in additive, white, gaussian noise of known mean and variance.

Input signal is initially subjected to FMOP Algorithm to detect presence of FM structure within the pulse. IF FMOP is detected, FMOP Analysis is performed to detect linear or non-linear FM, estimation of signal and modulation parameters and reconstruction of IF vector. If FMOP is not detected, analysis proceeds to implement PMOP Algorithm.

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PMOP Algorithm performs projection based analysis and detects the possible intra-pulse phase structure. If PMOP is detected, analysis proceeds to distinguish between bi-phase or poly-phase profiles. Presently Barker Code and Frank Code are considered representing bi-phase and poly-phase classes respectively. All the phase change instances are captured and projection magnitude vector is formed.

If PMOP is not detected, SOI is declared to be unmodulated and the associated parameters are computed and the corresponding Instantaneous frequency vector and projection magnitude vector are presented.

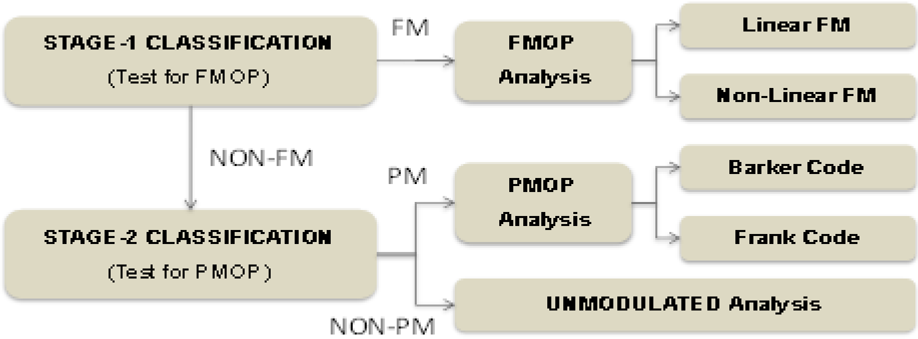


Figure Top Level Architecture of the proposed Pulse Compression Analysis

Signal processing environment definition and test set up

Proposed analysis is a specialized step in the EW signal processing of the received threat radar SOIs involving detailed intra-pulse analysis of the potently complex threats. Input for the present analysis is the SOI which has already undergone various detection and estimation processes in the receiver. It may be noted that signal detection is not performed in the present analysis. The received vector is assumed to contain signal of interest. Both FMOP and PMOP Algorithms are custom designed for the following signal processing environment and receiver architecture. Following table presents the overall parametric bounds, spectral limits, sampling and processing architectures under which the algorithms are designed to perform, noise definition etc.

Table 8.1: Test Setup

	Parameters	*Value/Range/Attribute*
1	Intermediate Frequency Range	750 – 1250 MHz
2	Sampling Scheme	Band Pass Sampling
3	Sampling Frequency	1350 MHz
4	Post ADC Band Selected	Lower Side Band
5	Digital Signal Processing Spectrum	100- 600 MHz
6	Processing Scheme	Complex I and Q based
7	Pulse Compression Classes considered	a. Frequency Modulation b. Phase Modulation
8	FM Sub Classes	a. Linear FM (Up/Down Slopes) b. Non-Linear (Up/Down Slopes)
9	PM Sub Classes	a. Bi-Phase Modulation (Barker Code: 2,3,4,5,7,11 and 13 Bit Codes) b. Poly-Phase Modulation Frank Code (3x3,4x4,6x6 and 12x12 Matrix Cases)
10	Frequency Modulation Bandwidth Range	5 – 500 MHz
11	Noise	Additive, White, Gaussian
12	Signal To Noise Ratio	6 dB
13	Frequency Range for Unmodulated Signals	100 – 600 MHz
14	Amplitude for all cases	Unit Norm

Frequency Modulation On Pulse (Fmop) Analysis Algorithm Objetive

- To detect Frequency Modulation on Pulse in the given radar signal of interest (SOI)
- To Estimate Signal Parameters, if SOI is unmodulated
- To Estimate Signal & Modulation Parameters, if SOI is frequency modulated belonging to Linear, Non-Linear & Arbitrary classes. Both positive & negative slopes are considered.
- Classify SOI based on modulation structure.
- Reconstruct the SOI

Theoretical Concept Derivation

An Unmodulated Radar Signal is expressed as,

$$s[n] = \alpha e^{j2\pi[Fn + \varphi]} + w[n] \; \forall \, w[n]: \mathrm{AWGN}, n \in [0 \; N] \quad (8.1)$$

Instantaneous Frequency vector of the above signal is obtained by differentiating the phase argument vector with respect to 'n' and is given by

$$f[n] = F + v[n] \; \forall \, v[n]: \mathrm{Perturbations}, n \in [0 \; N-1]$$

$$f[n] = p_0 + v[n] \; \forall \, v[n]: \mathrm{Perturbations}, n \in [0 \; N-1] \quad (8.2)$$

It is to be noted that receiver noise added to the signal is additive, white, gaussian, while the noise associated with instantaneous frequency is NOT gaussian and not necessarily white. Present processing architecture does not require modelling the noise associated with IF.

A LFM signal and its IF are expressed as,

$$s[n] = \alpha e^{j2\pi\left[\left(F \mp \frac{B}{2}\right)n \pm \frac{B}{2T}n^2 + \varphi\right]} \; \forall \, n \in [0 \; N]$$

$$f[n] = \left(F \mp \frac{B}{2}\right) \pm \left(\frac{B}{T}\right)n + v[n] \; \forall \, n \in [0 \; N-1]$$

$$f[n] = p_0 + p_1 n + v[n] \; \forall \, n \in [0 \; N-1]$$

Similarly, a quadratic FM signal and its IF are given by,

$$s[n] = \alpha e^{j2\pi\left[\left(F \mp \frac{B}{2}\right)n \pm \frac{B}{3T^2}n^3 + \varphi\right]} \; \forall \, n \in [0 \; N]$$

$$f[n] = \left(F \mp \frac{B}{2}\right) \pm \left(\frac{B}{T^2}\right)n^2 + v[n] \; \forall \, n \in [0 \; N-1]$$

$$f[n] = p_0 + p_2 n^2 + v[n] \ \forall \ n \in [0 \ N-1]$$

In general, IF of any continuous time FM can be expressed in the Taylors' series form as,

$$f[n] = p_0 + p_1 n + p_2 n^2 \cdots + p_q n^q + v[n] \ \forall \ n \in [0 \ N-1] \quad (8.3)$$

F: Carrier Frequency/Centre Frequency, B: Modulation Bandwidth, T: Modulation Period

α: Signal Magnitude, φ: Initial Phase Offset, n: Time Vector, N: Processing Vector Length

p_i : i^{th} Coefficient , q : Model Order, AWGN: Additive White Gaussian Noise

Analysis of entire class of FM signal is possible by computing the coefficients of the model representing the underlying instantaneous frequency (IF) vector. IF of an unmodulated signal has only a constant, 0^{th} order coefficient. IF of LFM has 0^{th} and 1^{st} order coefficient, while IF of quadratic FM has only 0^{th} and 2^{nd} order coefficients in the expression. Any other class can be approximated by higher order coefficients or a judicious combination of many coefficients. As the order of expression increases, closeness of approximation also improves. However, the cost paid is in terms of computation and algorithmic implementation complexity.

Fmop Algorithm Concepts

Consider the Taylor's series expression for the generic FM class.

$$f[n] = p_0 + p_1 n + p_2 n^2 \cdots + p_q n^q + v[n] \ \forall \ n \in [0 \ N-1]$$

Time vector is known apriori with the knowledge of sampling frequency. Proposed Algorithm synthesizes the coefficients based on 'LEAST SQUARES ERROR MINIMIZATION' criterion. Corresponding cost function is expressed as,

$$\arg\min_{\hat{p}_0, \hat{p}_1 \cdots \hat{p}_q} \sum_{n=0}^{N-1} [f[n] - (\hat{p}_0 + \hat{p}_1 n + \hat{p}_2 n^2 \cdots + \hat{p}_q n^q$$

(8.4)

Differentiating the above cost function w.r.t coefficients one by one, 'q' times and equating the corresponding gradient vector to zero results in 'q+1' equations in 'q+1' unknowns.

Differentiating w.r.t 0^{th} order,

$$\sum_{n=0}^{N-1} 2[f[n] - (\hat{p}_0 + \hat{p}_1 n + \hat{p}_2 n^2 \cdots + \hat{p}_q n^q)](-1) = 0$$

$$\sum_{n=0}^{N-1} [\hat{p}_0 + \hat{p}_1 n + \hat{p}_2 n^2 \cdots + \hat{p}_q n^q] = \sum_{n=0}^{N-1} f[n]$$

$$\hat{p}_0 N + \hat{p}_1 \sum_{n=0}^{N-1} n + \hat{p}_2 \sum_{n=0}^{N-1} n^2 \cdots + \hat{p}_q \sum_{n=0}^{N-1} n^q = \sum_{n=0}^{N-1} f[n] \quad (8.5)$$

Differentiating w.r.t 1^{st} order,

$$\sum_{n=0}^{N-1} 2[f[n] - (\hat{p}_0 + \hat{p}_1 n + \hat{p}_2 n^2 \cdots + \hat{p}_q n^q)](-n) = 0$$

$$\sum_{n=0}^{N-1} [\hat{p}_0 + \hat{p}_1 n + \hat{p}_2 n^2 \cdots + \hat{p}_q n^q] n = \sum_{n=0}^{N-1} n f[n]$$

$$\hat{p}_0 \sum_{n=0}^{N-1} n + \hat{p}_1 \sum_{n=0}^{N-1} n^2 + \hat{p}_2 \sum_{n=0}^{N-1} n^3 \cdots + \hat{p}_q \sum_{n=0}^{N-1} n^{q+1} = \sum_{n=0}^{N-1} n f[n] \quad (8.6)$$

Differentiating w.r.t q^{th} order,

$$\sum_{n=0}^{N-1} 2[f[n] - (\hat{p}_0 + \hat{p}_1 n + \hat{p}_2 n^2 \cdots + \hat{p}_q n^q)](-n^q) = 0$$

$$\sum_{n=0}^{N-1} [\hat{p}_0 + \hat{p}_1 n + \hat{p}_2 n^2 \cdots + \hat{p}_q n^q] n^q = \sum_{n=0}^{N-1} n^q f[n]$$

$$\hat{p}_0 \sum_{n=0}^{N-1} n^q + \hat{p}_1 \sum_{n=0}^{N-1} n^{q+1} + \hat{p}_2 \sum_{n=0}^{N-1} n^{q+2} \cdots + \hat{p}_q \sum_{n=0}^{N-1} n^{2q} = \sum_{n=0}^{N-1} n^q f[n] \quad (8.7)$$

Rearranging the above equations and expressing in matrix form, we get,

$$\begin{bmatrix} N & \sum_{n=0}^{N-1} n & \cdots & \sum_{n=0}^{N-1} n^q \\ \sum_{n=0}^{N-1} n & \sum_{n=0}^{N-1} n^2 & & \sum_{n=0}^{N-1} n^{q+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{n=0}^{N-1} n^{q+1} & \sum_{n=0}^{N-1} n^{q+2} & \cdots & \sum_{n=0}^{N-1} n^{2q} \end{bmatrix} \begin{bmatrix} \hat{p}_0 \\ \hat{p}_1 \\ \hat{p}_2 \\ \vdots \\ \hat{p}_q \end{bmatrix} = \begin{bmatrix} \sum_{n=0}^{N-1} f[n] \\ \sum_{n=0}^{N-1} n f[n] \\ \vdots \\ \sum_{n=0}^{N-1} n^q f[n] \end{bmatrix} \quad (8.8)$$

Coefficient vector minimizing the cost function in Least-Squares Sense is obtained by computing the inverse of the LHS square matrix and finding the vector product

$$\begin{bmatrix} \hat{p}_0 \\ \hat{p}_1 \\ \hat{p}_2 \\ \vdots \\ \hat{p}_q \end{bmatrix} = \begin{bmatrix} N & \sum_{n=0}^{N-1} n & \cdots & \sum_{n=0}^{N-1} n^q \\ \sum_{n=0}^{N-1} n & \sum_{n=0}^{N-1} n^2 & & \sum_{n=0}^{N-1} n^{q+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{n=0}^{N-1} n^{q+1} & \sum_{n=0}^{N-1} n^{q+2} \cdots & & \sum_{n=0}^{N-1} n^{2q} \end{bmatrix}^{-1} \begin{bmatrix} \sum_{n=0}^{N-1} f[n] \\ \sum_{n=0}^{N-1} n f[n] \\ \vdots \\ \sum_{n=0}^{N-1} n^q f[n] \end{bmatrix} \quad (8.9)$$

For LFM case, only 0^{th} and 1^{st} order coefficients exist and the above solution takes the following form.

$$\begin{bmatrix} \hat{p}_0 \\ \hat{p}_1 \end{bmatrix} = \begin{bmatrix} N & \sum_{n=0}^{N-1} n \\ \sum_{n=0}^{N-1} n & \sum_{n=0}^{N-1} n^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{n=0}^{N-1} f[n] \\ \sum_{n=0}^{N-1} n f[n] \end{bmatrix}$$

For Quadratic FM case, the solution is expressed as,

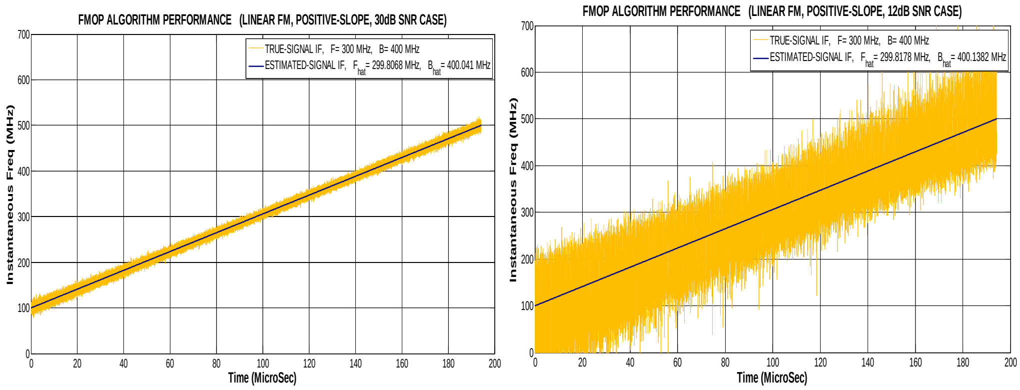
$$\begin{bmatrix} \hat{p}_0 \\ \hat{p}_2 \end{bmatrix} = \begin{bmatrix} N & \sum_{n=0}^{N-1} n^2 \\ \sum_{n=0}^{N-1} n^2 & \sum_{n=0}^{N-1} n^4 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{n=0}^{N-1} f[n] \\ \sum_{n=0}^{N-1} n^2 f[n] \end{bmatrix}$$

With respect to the practically existing radar signals of FM class, the above representation using qth order Taylors' series is an excellent approximation of the underlying IF structure and hence the SOI definition and offers a single-concept solution covering a wide family of frequency modulation types. From the implementation point of view, it can be easily seen that inverse matrices are functions of 'n' and 'N' only and hence are pre-computable for different values of N. This saves run-time computation efforts involved in lengthy mathematical operations of matrix formation and its inverse computation.

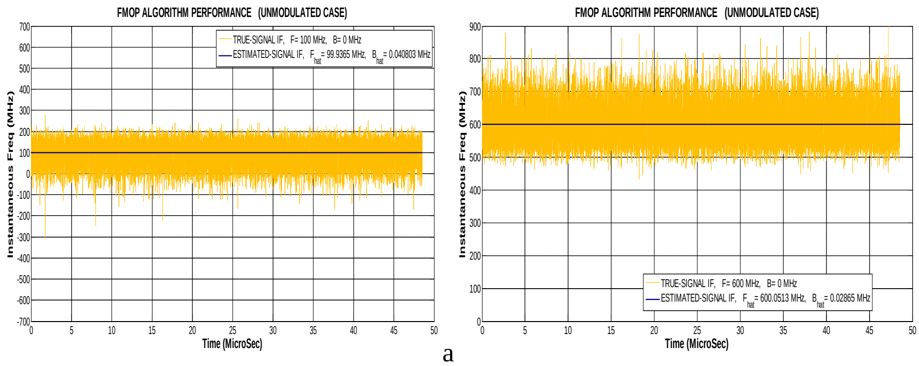
Once the coefficient vector, $[p_0 \quad p_1 \quad p_2 \quad \cdots \quad p_q]$ is computed for the qth order, the next step is analysis of values of obtained for each coefficient for pattern matching. For unmodulated signals, all coefficients except p_0 are actually non-existent and practically below a pre-set threshold. For LFM case, $[p_2 \quad p_3 \quad \cdots \quad p_q]$ are below the pre-set threshold. For non-linear case, IF structure is supported by many higher order coefficients with the possibility of one dominant coefficient. For example, cubic FM has p_3 as the dominant coefficient while other coefficients also support with varying degrees of prominence. Log FM has many coefficients actually contributing to the underlying IF structure. Although it is difficult to classify within non-linear FM class into sub classes like Log FM, quadratic FM and Cubic FM etc, it is much easier to classify between LINEAR and NON-LINEAR simply based on coefficient profile. Present algorithm is designed to just classify between purely linear and purely non-linear classes, while sub class definition is beyond the current scope.

MATLAB implementation of the FMOP Algorithm

- Algorithm models the underlying Instantaneous Frequency (IF) under Least Squares Error Minimization and hence is an optimal solution.
- For 30dB SNR case (almost noise free condition), variation of true IF over theoretical IF curve is very small. As SNR decreases IF spreads more and more spectrally as shown in Case-1 and Case-2. Algorithm shows degraded performance for SNR<10dB (Cut-Off)
- Algorithm is robust against variations in SNR above the cut-off SNR as seen by the parameter estimation performances for 30dB and 12dB SNR cases presented below.
- Algorithm is simple to implement in hardware/firmware or software of a modern Radar Warning Receiver (RWR) and is useful against a large family of FM Radar Signals.
- Warning Receiver (RWR) and is useful against a large family of FM Radar Signals.

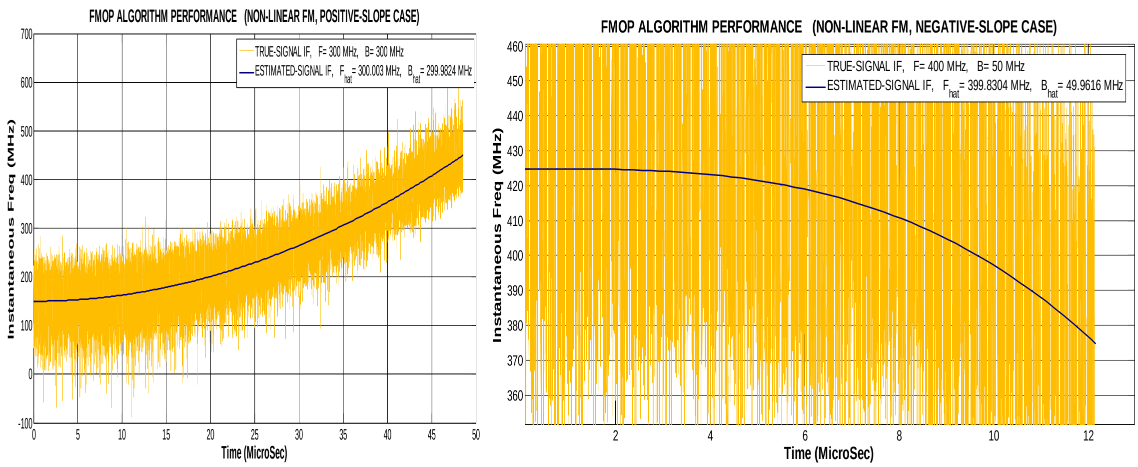


LFM Radar Signal with F=300MHz, B=400MHz at 30dB SNR & 12 dB SNR



a

Unmodulated Radar Signal at lower &Upper Band Edge. F = 100 & 600 MHz resp.



Non-Linear FM Radar Signal with Positive & Negative Slope. @F=300MHz, B = 300 MHz & . F=450MHz, B = 50 MHz

Figure: FMOP Algorithm Performance

PM on Pulse (PMOP) Analysis Algorithm

PMOP Radar Signals are characterised by discrete phase profiles embedded within the pulse. In the present project, following widely used PMOP classes are considered.

a) Bi-Phase Coded Modulation
   Barker Codes (2, 3, 4, 5, 7, 11 and 13 Bits)
b) Poly-Phase Coded Modulation
   Frank Codes (4x4, 7x7 and 12x12)

7. Theoretical Concept Derivation

A phase modulated Radar Signals is expressed as,

$$s[n] = \alpha e^{j2\pi[Fn + \theta[n] + \varphi]} + w[n]\ ;\ \theta[n]:\ \text{PMOP Vector}, w[n]: AWGN, ;\ \forall\ n \in [0\ N] \quad (8.10)$$

Phase vector is a discrete phase change structure with phase change profile dictated by the underlying modulation scheme like bi phase or poly phase and particular class profiles. Following table gives the Phase Structure of Barker coded signals.

Table 8.2: Phase of Barker codes

Barker Type	Phase Profile (in Degrees)
2 – Bit	π, - π
3 – Bit	π, π, - π
4 – Bit	π, π, - π, π
5 – Bit	π, π, π, - π, π
7 – Bit	π, π, π, - π, - π, π, - π
11- Bit	π, π, π, - π, - π, - π, π, - π, - π, π, - π
13 – Bit	π, π, π, π, π, - π, - π, π, π, - π, π, - π, π

MxM Frank Coded Radar signals are customarily expressed in matrix form as,

$$\Theta_{MxM} = \frac{2\pi}{M}\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 2 & \cdots & M-1 \\ 0 & 2 & 4 & \cdots & 2(M-1) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & M-1 & 2(M-1) & \cdots & (M-1)^2 \end{bmatrix}$$

Hence, 4x4 Frank Coded Signal has the following matrix representations (in degrees)

$$\Theta_{4x4} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 90 & 180 & 270 \\ 0 & 180 & 360 & 540 \\ 0 & 270 & 540 & 810 \end{bmatrix}$$

8. PMOP Analysis Algorithm Concepts

In the proposed method, PMOP algorithm is implemented after FMOP stage, where carrier frequency is already estimated. Since FMOP follows Least Squares Error Minimization concept, the estimated frequency is very close to the actual frequency.

This estimate of frequency is used PMOP to form the TEMPLATE signal for implementing Projection Analysis. Considering the PMOP signal,

$$s[n] = \alpha e^{j2\pi[Fn + \theta[n] + \varphi]} + w[n]\ ;\ w[n]: AWGN, ;\ \forall\ n \in [0\ N] \quad (8.11)$$

Let $\hat{F}$ be the estimate of the frequency obtained in the earlier stage of analysis. Constructing a noise free, perturbation free, and unit amplitude template signal of dimension Mx1.

$$r[m] = e^{j2\pi\hat{F}m}\ ;\ \forall\ m \in [0\ M-1], \text{such that } M \ll N \quad (8.12)$$

r[m] is a column vector of dimension Mx1, whereas s[n] is a column vector of dimension Nx1 and M<<N. Proposed projection analysis method involves progressively projecting the template vector r[m] onto Mx1 segments of s[n] and computing the scalar dot product for each projection and forming the projection magnitude vector J[k], assuming that k=1,2,... K projections are carried out, where $\boldsymbol{K} = \frac{N}{M}$. The process is mathematically derived as below.

First projection is carried out between the hermitian of r[m] and first M elements of s[n]. The resultant is a complex number of unit dimension, whose magnitude is assigned to J[1].

$$J[1] = \frac{1}{M}\left|\begin{bmatrix} r^*[0] & r^*[1] \cdots r^*[M-1] \end{bmatrix}\begin{bmatrix} s[0] \\ s[1] \\ \vdots \\ s[M-1] \end{bmatrix}\right|$$

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Second Projection is carried out between the hermitian of r[m] and (L+1) to (L+M) elements of s[n], where L<M and is arbitrary. Presently $L = \frac{M}{4}$ is considered.

$$J[2] = \frac{1}{M}\left| \left[r^*[0]\ r^*[1]\ \cdots\ r^*[M-1] \right] \begin{bmatrix} s[L] \\ s[L] \\ \vdots \\ s[L+M-1] \end{bmatrix} \right|$$

Similarly Kth projection is computed using last M elements of SOI and is expressed as,

$$J[K] = \frac{1}{M}\left| \left[r^*[0]\ r^*[1]\ \cdots\ r^*[M-1] \right] \begin{bmatrix} s[N-M-1] \\ s[N-M] \\ \vdots \\ s[N-1] \end{bmatrix} \right|$$

Projection Magnitude Vector J[1], J[2], ... J[K] is normalized to account for the different amplitude profiles of the received PMOP signals. The values so obtained are theoretically between zero to one and may vary marginally due to noise and other perturbations.

Plotting the vector J against projection filter index vector gives the overall projection picture and reveals time instances where loss of correlation has happened. In the present algorithm, two THRESHOLD values are used for PMOP analysis.

Threshold-1 is used to classify between unmodulated signals and PMOP signals. The theory behind this thresholding is that if there is no phase change occurring within the Mx1 vector then the template signal and the Mx1 segment of SOI are close in frequency and maintain continuity of phase throughout M samples and hence the complex projection has a high value, very close to 1. However, if there is a phase change within a given Mx1 projection process, then the two signals differ in phase, resulting in loss of correlation. The same is revealed as low value of projection magnitude and is typically a function of how much phase change has happened. The loss of projection magnitude is worst for 180 degrees phase shift cases and reduces for other values in a predetermined fashion.

Threshold-2 is used to classify between Bi-Phase and Poly-Phase PMOPs. This second threshold is much below the first threshold. If SOI happens to be Bi-Phase, then only phase changes involved are +180 and -180 degrees, which result in heavy loss of correlation and hence projection assumes very low values, close to zero for all those segments.

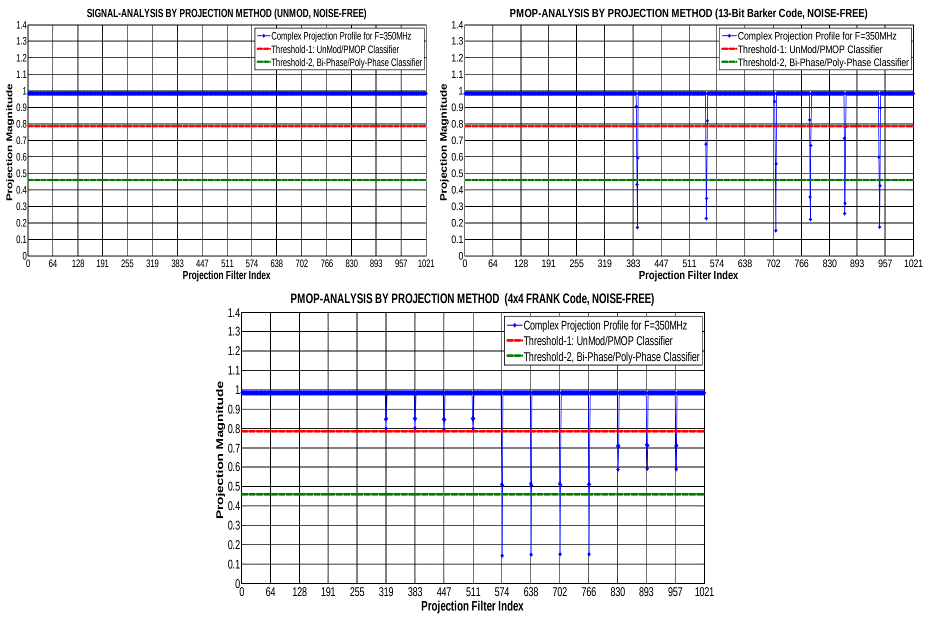
Real Time computation of THRESHOLDS is a very complicated process and depends on many factors such as EW receiver design, AWGN noise structure, noise statistics, parameters of various filters and down converters in the receiver path etc. "THRESHOLD computation is beyond the scope of the present project. Apriori nominal values used in practical receivers have been considered in the analysis.

Following is the decision mechanism in the proposed PMOP algorithm.

- If the SOI is unmodulated, then NO threshold crossing occurs in the entire analysis.
- If SOI contains Barker-coded structure, then both THRESHOLD-1 and THRESHOLD-2 crossings definitely occurs for each of the phase-change instances.
- If SOI contains Frank-coded structure, then THRESHOLD-1 occurs for some (or all) of the phase-change instances and number of THRESHOLD-2 crossings would be definitely less than the number of THRESHOLD-1 crossings.

9. Algorithm Design and Implementation in MATLAB

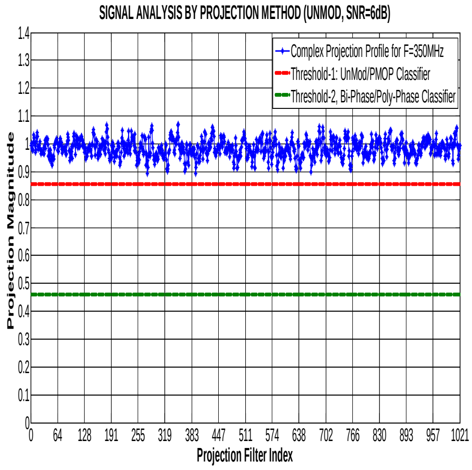
PMOP Algorithm results are presented below for the noise free case as proof of concept and demonstration of various theories discussed so far for a test signal of 350 MHz carrier. It is shown that NO threshold crossings occur for unmodulated case, both threshold crossing occur for Barker-13 Bit case at all instances of phase changes and for 4x4 Frank Case, a mixed crossing profile is observed.

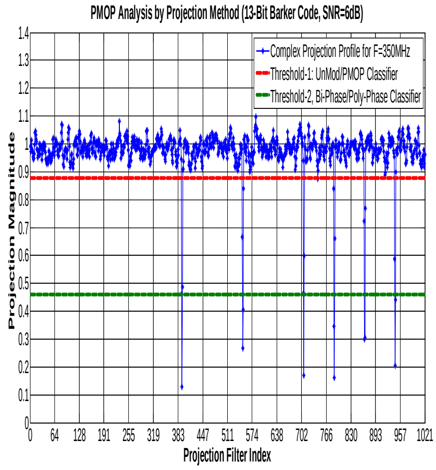


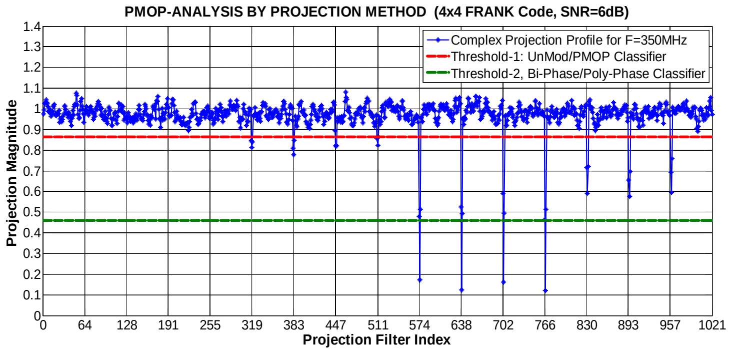
Case-1, 2 & 3: Unmodulated, Barker-13Bit & 4x4 Frank SOIs (NOISEFREE)

Pmop algorithm performance at cut off snr-

It is noted that Projection Analysis performance is a function of SNR also and is valid for SNR above a certain value and depends on length of template vector and initial estimation of frequency used to construct the template vector. Following 3 diagrams show the nature of projection analysis for 6 dB SNR under AWGN assumption.



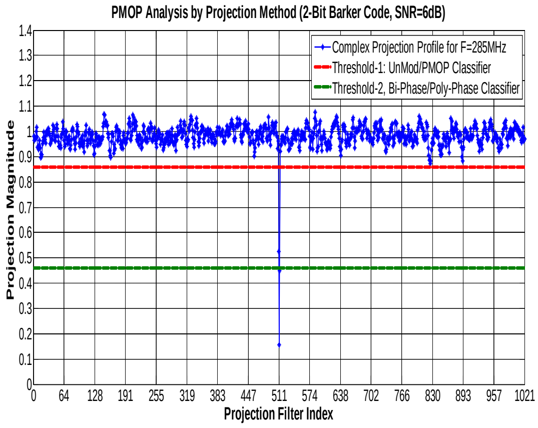


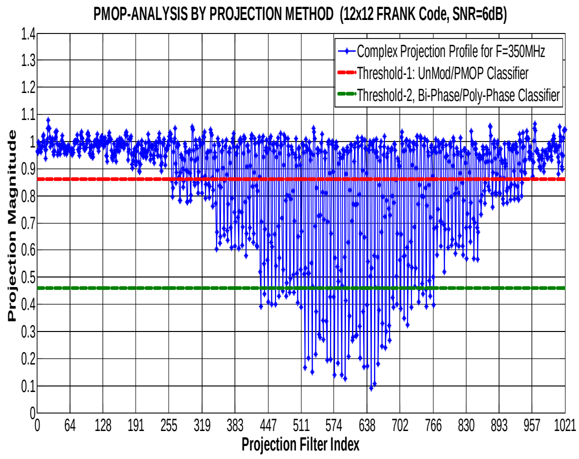


Case-4, 5 & 6: Unmodulated, Barker-13Bit & 4x4 Frank SOIs (CUT-OFF SNR = 6dB)

PMOP Algorithm Performance at below Cut off SNR (Failure Characteristics)-

As SNR decreases, increased loss of correlation between r[m] and s[n] results in threshold crossings even for unmodulated SOIs. Moreover, frequency estimation is less accurate, resulting in Template Signal synthesis away from the spectral zone of the actual SOI. Characterising the failure of the algorithm allows the designed to judiciously design the overall receiver architecture.





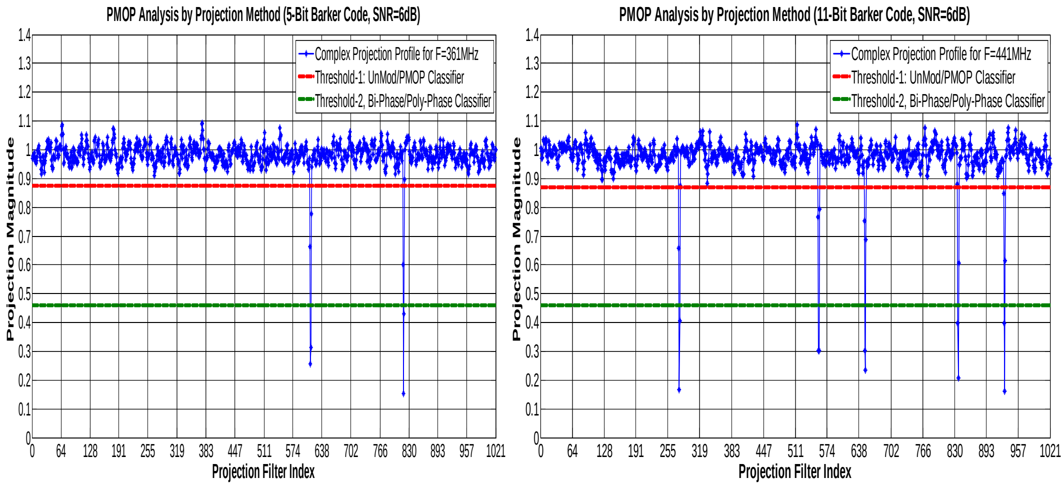


Figure 8.5: Frank code plots

Conclusion of PMOP Algorithm

- Proposed Algorithm performs satisfactorily for all sub classes of Barker and Frank coded PM Radar Signals, with provision for extension to other PM classes.
- Consistent and accurate performance is observed for SNR $\geq$ 6dB. Algorithm completely fails for SNR $\leq$ 3 dB and shows a continuously improving performance over 3dB to 6dB SNR transition.
- Phase transition of less than 15 degrees are not possible to detect with the proposed algorithm, since noise dominates the projection process. The suggested improvement is using a longer template signal which provides SNR improvement.
- Algorithm is computationally fast, since it facilitates parallel projection of SOI segments and is feasible for implementation in hardware or software of a RWR.

10. Conclusion and Future Scope

Our project includes study of basic radar signals and simulating them in MATLAB, then analysing the procedure for signal processing in EWSP and development and implementing algorithm for signal analysis. By the end of this project, using our algorithm we can give the characteristics of any random radar signal given to us and determine the threat of the signal. This is possible because we will know the frequency of transmission, modulation type and power of our base station. Then by getting frequency of received signal we can determine the threat (received signal is from enemy camp) if there is any frequency variation. Suppose if same frequency is used by enemy station then we differentiate it by power level and modulation type.

Proposed FMOP Algorithm addresses the entire family of Continuous Time FM Radar Signals. Analysis Model is mathematically derived and Solution is OPTIMAL in Least Squares Error Minimization Sense. Most of analysis is pre-computable, thus reducing real-time processing time and resources. Algorithm is suitable for implementation in modern day Radar Warning Receivers and it gives most accurate till date.

PMOP algorithm is also suitable for implementing in modern day RWR’s but main drawback is setting of threshold. Because it is very difficult and again it includes some complex algorithm which must be developed to adapt threshold value for signals with lesser power and signal having lesser SNR.

Hence in future PMOP algorithm must be modified to set adaptive threshold and to detect other phase modulation techniques rather than Barker and Frank code.

References

[1] Merrill I Skolnik, “Introduction to RADAR systems”, third edition, Mcgraw-Hill Book Company.
[2] Leon Cohen, “Time frequency analysis”, Prentice Hall PTR, Upper Saddle River, New Jersey 07458.
[3] B P Lathi , “Analog and Digital communications”, third edition, Mcgraw-Hill Book Company.
[4] “Electronic Warfare Fundamentals”, Det 8, ACC TRSS 4349 Duffer Drive, Ste 437, Nellis AFB NV 89191-7007.
[5] “MATLAB Primer” by Math works, Inc.
[6] MATLAB GUI (Graphical User Interface) Tutorial for Beginners By J.S. Park, University of Incheon
[7] Merrill I Skolnik, “RADAR Handbook”, third edition, Mcgraw-Hill Book Company.
[8] S. Salivahanan, A. Vallavaraj And C. Gnanapriya, “Digital Signal Processing”, First edition, Mcgraw-Hill Book Company, 2008.
[9] Alan V. Oppenheim, Alan V. *Willsky*, S.Hamid Nawab; Title: Signals and Systems; Publisher: Prentice-Hall India; Place: New Delhi; Year: 1992.