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DIAGNOSTIC AND REMEDIAL PROCEDURES IN COMPUTATIONAL SKILLS AND REASONING PROCESSES IN ARITHMETIC

By

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TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

THE PROBLEM

A. A Changing Viewpoint

Ever since Pestalozzi discarded the Eighteenth Century "ciphering system" for a more concrete treatment of numbers, educators have been on the scent for a satisfactory method of teaching arithmetic. Many plans have been advocated, but the method that will combine the adequate teaching of speed, accuracy, and correct reasoning has yet to be discovered.

Perhaps it is the inadequacy of the methods that has caused the focal point of interest to shift from the method of teaching to the reason for teaching and has concentrated the attention of the modern educator on the needs and ability of the child.

This shift of interest became evident near the turn of the century, when students in education began to free themselves from the traditional methods and to view the subject from a more scientific standpoint. The research resulting from this new approach made plain the ineffectiveness of the then current arithmetical teaching practices.

B. Need of a New Procedure

Investigators showed that not only were there more failures in arithmetic than in any other elementary subject, but that these failures occured too often among pupils with a high I. Q. who were successful in every other field. There was no recorded scientific analysis of the pupils' problems and consequently no concerted action to solve them. Such investigation revealed the need of a new procedure, if pupils were to be helped to overcome their difficulties.

C. Scientific Investigations

This need has given rise to the development of a scientific procedure which exposes the arithmetical disabilities of the child and prescribes the needed remedial measures. Diagnostic investigations have provided a standardized system by which teachers are enabled to determine the causes of the arithmetical difficulties of their pupils.

Remedial measures must now be found, as diagnosis exposes the causes of error, but does not offer a method of correction. Much patient investigation with small groups of children is needed to discover the effective means of counteracting the wrong habits established by poor teaching or incomplete learning. A comparison of studies on children's arithmetical errors shows that type errors are widespread and constant. For this reason, remedial measures that help one child with a certain difficulty will be of value in aiding other children with similar weaknesses. Lists of specific remedial measures to correspond with the tabulated charts of typical errors have been requested by leading investigators.

D. The Present Study

These lists of remedial measures will be made possible only by many well-planned, carefully recorded investigations, especially those of the case-study type, where the individual reaction to corrective teaching can be accurately observed. It was for the purpose of contributing, however slightly, to the needed research that the present study was undertaken.

The purpose of this study was to investigate the arithmetical disability of twenty-two children, chosen from grades five to eight, who were found to be retarded a year or more in arithmetic.

The following questions were formulated to direct the investigation:

1. What attitudes are observable in children failing in arithmetic?

2. What difficulties do these children evidence in problem-solving situations?

3. Is there a noticeable relation between reading ability and problem-solving ability?

4. What means are effective in overcoming the arithmetical difficulties of these children?

5. In what ways do children of superior intelligence and

children of low intelligence differ in their response to arithmetic instruction?

6. After a period of remedial instruction in computation and arithmetic reasoning, is there any relation between the improvement in the one and the other?

CHAPTER II

REVIEW OF LITERATURE

A. Nineteenth Century Trends

Although the greatest progress in scientific research in arithmetic has been made within the last twenty-five years, attempts to improve the content and method of arithmetical teaching began over a hundred years ago. Objectives, techniques, and curriculum planning have been the matter of earnest study on the part of educators since 1830. One or other phase of the subject has almost monopolized attention at different periods; various theories have been adopted, pushed to extremes, and then discarded as the result of a marked reaction in the opposite direction. These systems may be roughly grouped under the headings of (1) formal culture, (2) observation and exercise, (3) concentric circle or exhaustive study, and (4) experimental method. Of these, the first three belong to the nineteenth century schools; the last is an outgrowth of the newer educational tendencies.

1. Formal Culture

The great Swiss educator, Pestalozzi, (52,65) made formal or mental culture the aim of his arithmetic teaching, as he made perception its foundation. In his school the children in the primary classes were given a knowledge of number by considering objects as units. Not until the pupils had learned all the elementary operations included in the tables through the tens were Hindu-Arabic numerals introduced as a means of calculation. In the same way fractions were taught; first the concept was formed by objective presentation, then exercise in operation was given, after which the written characters were introduced.

The followers of Pestalozzi, especially Turk and Kawerau, failing to achieve the breadth of vision of their master, narrowed his method by neglecting the training of the perceptive faculty and concentrating on the exercises in thinking, the formal culture aspect of his system. Their extreme views provoked a reaction.

2. Observation and Exercise

The three German educators who led the reaction against over-formalism were Krankes, Denzel, and Grube. Krankes taught his pupils to use the experiences of daily life to gain objective understanding of numbers, and thus avoided the abstract problems of the Pestalozzian school. It was he who first suggested the concentric circle plan, teaching the four operations in all possible combinations from one to ten, then from one to one hundred, one to one thousand, and one to ten thousand.

Denzel sympathized with Krankes in his attempt to establish moderate and systematic methods of arithmetic instruction. His three aims in the teaching of primary numbers are noteworthy:

1. To exercise the thought, perception and memory.
2. To lead the children to the essence and the simple relations of number.
3. To give the children readiness in applying this knowledge to the concrete problems of daily life. (56:88)

3. The Concentric Circle or Exhaustive Method

Grube used the theories of Krankes and Denzel, but unduly emphasized some of their ideas. In his application of the principle of the concentric circle method, Grube said that to teach all the number combinations from one to ten thoroughly, at least a year was necessary, while three years was not too long to spend in teaching the combinations from one to one hundred. Such thoroughness, if used with prudence, had much to commend it, but Grube carried it to an extreme where it became mechanistic. He ignored the number knowledge that the children possessed, and treated the four fundamental processes as if they were of equal difficulty, which they are not.

In the reaction against Grube, a modern pre-Pestalozzian method reestablished counting as the natural approach to the study of number, since counting is fundamental and spontaneous, "free from sensible observation and from the strain of reason." (56:94)

B. Early Twentieth Century Studies

"The advance in the modern teaching of arithmetic is due much more to the recognition of the definite aim than to the discovery of improved methods." (56:110)

The definite aim is clearly defined in "The Teaching of Elementary Mathematics", by D. A. Smith. Appearing in 1902, Dr. Smith's book gave in succinct form an historical sketch of the development of modern arithmetic, and at the same time lifted the subject out of the realm of futile argument by proposing the questions, "Why is arithmetic taught? How is it taught? What should be taught to accomplish the stated aims?"

There are two main aims, the utilitarian and the cultural. For the first, competence in the four fundamental processes, with a little knowledge of fractions, decimals, and denominate numbers will almost suffice. But the broader cultural aim demands much more and gives a much greater return: (56:26)

> . . . arithmetic may train the mind of the child logically to attack the every day problems of life. If he has been taught to <u>think</u> in solving his school problems, he will think in solving the broader ones which he must thereafter meet. The same forms of logic, the same attention to detail, the same patience, and the same care in checking results exercised in solving a problem in greatest common divisor may show itself years later . . . Hence, arithmetic, when taught with this in mind, gives to the pupil not a knowledge of facts alone, but that which transcends such knowledge, namely, power.

Dr. Smith's views had a great influence on educators, as is shown in the acknowledgements made by those who followed him. A. S. Edwards, (24) writing twenty years later, refers to the value that habits of precise thinking and exact calculation formed in arithmetic classes have for a student in the study of languages. He suggests that the idea of accuracy may extend its influence to the moral virtue of honesty.

C. First Scientific Investigations

A first attempt at scientific testing in arithmetic was made in 1902 by J.M. Rice.(54) The present day techniques were lacking and details of administering the tests and interpreting the results were not fully given. The aim of the study was to measure and evaluate the teaching of arithmetic, to determine what results should be accomplished, what length of time should be given to the subject, and the reason for the success or non-success of the teaching. Six thousand children of grades four to eight participated in the test, which consisted of a series of eight problems. Questionaires on the length of the arithmetic period, home work assignments, size of classes, and the methods of teaching were answered by the teachers.

Dr. Rice found that:

1. There is no direct relation between the time given to arithmetic in the school and the results obtained, while the amount of home work is no criterion of the results expected in the class room.

2. Methods in teaching arithmetic are not the controlling

element in the accomplishment of results.

3. Differences in attainment are not explained by difference in the size of classes.

4. Variation in ability is common to all grades, but is greatest in the seventh and the eighth grades.

5. Results in arithmetic correlate with maturity, that is, averages improve from grade to grade.

The improvement that took place in a few years in planning and administering tests is realized after a study of the investigation conducted in 1908 by C. W. Stone. (59)

The purpose of this experiment was to find the nature of the product of the first six years of arithmetical work and to discover the relation between distinctive procedures and the resulting abilities.

Twenty-six school systems of the East and the Middle West were chosen, in which two series of tests were given to the children of the high sixth grade. One series contained examples in the four fundamental processes, the other, arithmetic problems. The tests were timed; in every instance they were administered by the investigator, who also corrected them.

When the correction was completed and the scores were recorded, Dr. Stone formulated the conclusions to be drawn from the study. Among the most important ones are the following:

1. The method of research used in the study is one by which hypotheses may be tested and opinions verified.

2. Freedom and initiative in educational method have in

some instances been the cause of waste, as is evidenced by the variability of the scores.

3. The greatest need shown by the study is a uniform standard of achievement.

4. No one factor is sufficient to produce arithmetic ability; the course of study, time allotment, supervision, and adequate measuring tests are essentials for successful teaching.

5. Among the different school systems tested, there was a high correlation between the fundamental processes; that is, skill in one would seem to promise skill in the others.

6. There is a much lower correlation between arithmetic reasoning and computational skill, the coefficient of correlation being 0.32.

Another advance in the technique of scientific investigation was the use of a control group to check the results of an experiment. Such an investigation was reported by J. C. Brown. (6)

For this research a preliminary survey test was given to both the experimental and the control group. This was followed by a period of thirty days during which the experimental group received daily drill in computational processes while the control group was given ordinary class room instruction with no drill. A final test showed a notable gain made by the class receiving drill. After a summer vacation of twelve weeks, the children were tested again; the results indicated that the period of maturation served to increase the speed of the experimental group.

Dr. Brown also summerized the investigations carried on during the first thirteen years of the century; he pointed out some desired reforms, among them the need of an objective marking system, a better understanding of drill, and the urgent necessity for the reform of the content of texts to provide better grade placement of subject matter and choice of problems.

D. Later Investigations

Later investigations cover numerous aspects of arithmetical theory and practice. For convenience, these may be grouped under certain headings, although a number of the studies deal with two or more phases of the subject.

1. Psychological Aspects

The Psychology of Arithmetic by A.L. Thorndike (63) was a critical study of prevailing methods of arithmetic presentation in the light of his "bond theory". It was one of the first attempts to make educators and publishers of textbooks realize their responsibility of considering the needs of the child.

Dr. Thorndike speaks of the advance made in the psychology of education, and of the knowledge it gives concerning conditions of learning; he then applies this to the teaching of arithmetic. He protests against the persistance of the conventionalized systems which place certain subject matter in specified years, regardless of psychological reasons in favor of other grading. He also gives suggestions for a more logical arrangement of topics.

In discussing the functions of arithmetic, Dr. Thorndike raises the question of the abilities necessary to master the elementary processes of number work. He asks for some student who will analyze arithmetic learning into the unitary abilities which compose it, as a first step in planning the careful teaching that will take into account the hierarchy of psychological connections involved in computational processes. Then, he says, instead of composing problems to fit the instruction given in the school, we can organize the instruction to meet the problems of the child.

Another psychological study dealing with diagnosis and method was made by W. J. Osburn. (50) The study was undertaken to discover means for obtaining better results in the teaching of arithmetic.

The problem was approached by a comparison of the errors made by pupils working the Woody-Theisen Parallel Tests in Arithmetic. Analysis of the results seemed to Dr. Osburn to prove that there are many definite and persistent type errors, caused by difficult combinations, zero combinations and long division. He asserts that if mistakes in these operations were removed, at least forty-five percent of all error would be eliminated.

The underlying cause of these errors is the failure of teachers to realize that there is a minimum of transfer in the learning of arithmetic combinations. Dr. Osburn lists over 1600 number facts or combinations and 100 fraction facts which must be taught separately as a prerequisite for successful work.

Any review of this excellent work would be incomplete if the author's "principles of economy in teaching" were omitted:

> 1. The amount of drill should be apportioned to the difficulty of the task.
> 2. Teach first and most completely that which is most used in life.
> 3. Teach what the pupils do not know. Do not teach what they already know.
> 4. Do not try to teach things which the child could not learn with the best teacher who ever lived.
> 5. Always do your best to make the pupils want to learn what they should learn. (50:I,128)

Although Dr. Osburn based many of his theories upon the information he gained by inspection of the pupils' written exercises, he understood the value of individual analysis; he recommended that the teacher ask the child to work aloud faulty computations, to discover the wrong processes causing the difficulties.

The careful detail of this study as well as its completeness makes it invaluable to anyone interested in the teaching of arithmetic.

In The Psychology and Teaching of Arithmetic, H. G. Wheat (68) traced the development of the number system from its prehistoric beginnings to the present, "in order to describe the peculiarities and characteristics of arithmetic, and to distinguish between its appearances and its actualities." (68:iv) The author's argument is that the idea of number developed from the idea of the group.

> The only importance of the objects in the group, insofar as the idea of number may be concerned, is that they make the group what it is. The idea of number is not concerned with objects; it deals only with their arrangement. If there is one thing that stands out more clearly than another in a review of the gropings of the primitive mind after the idea of number, and of the subsequent development of number, it is that objects do not of themselves furnish the idea, but that it grows only with the systematic study of groups. (68:161)

The tendency to break up the subject of arithmetic into sections or functions was criticized by Wheat, who stated that the distinguishing function of arithmetic was the psychological one that introduced the child to the "meaning and use of the number system." (68:562)

2. Contributions to Method

One outcome of the scientific investigation into the question of arithmetic teaching and learning has been a reopening of the consideration of proper method. Survey tests of school systems exposed the fact that many of the pupils were lacking in the fundamental skills of arithmetic, that they could not handle fractions or decimals, and that they were unable to solve problems. Not only mentally slow children showed these disabilities, pupils of average and even high intelligence appeared similarly handicapped.

This state of affairs has caused much criticism regarding the methods used in the teaching of arithmetic, while experimental studies have been made to determine the best ways of presenting specific processes. Men of national reputation as educators have led in this movement for better arithmetic teaching.

W. A. Brownell, whose investigations are extensive, speaks strongly on the subject:

> The record of arithmetic in the school is an unenviable one. The position taken in this chapter is that the fault lies in the type of instruction generally given. Arithmetic instruction has for a number of years inclined much too far in the direction of the drill theory of instruction. The trend now seems to be in the direction of the incidental theory of instruction. While this change in instructional theory represents distinct improvement, it does not, for reasons given in the foregoing pages, promise the kind and amount of reform needed. . . . The basic tenet in the proposed instructional reorganization is to make arithmetic less a challenge to the pupil's memory and more a challenge to his intelligence.(7:31)

F. B. Knight, (32) who has contributed works of great value on the theory and practice of arithmetic, is insistent upon an effective method that will accomplish its purpose of teaching arithmetic to the child. Such instruction will include careful presentation, formation of good work habits, distributed drill, testing, and the socialization of problems. In a syllabus prepared under his direction, (33) lessons have been planned to show the working out of this theory.

Another prominent figure in educational research, C.H. Judd, (30) asserts that the methods of teaching arithmetic in our schools are so inadequate that many pupils fail in that subject, This is partly due to the fact that numbers and number combinations are not well taught. Dr. Judd criticizes the prevalent methods that introduce pure number combinations by a few exercises in objective counting, but ignore the rich field of experimental number situations. He also questions the wisdom of a method that ignores the need of systematic drill and regularity of sequence in arithmetic learning, especially in the mastering of combinations. He claims that the pupils should have the idea of number presented to them as an orderly, coherent whole. It should be an aid to the memory and to clear thinking, given to help the child gain an understanding of the general, schematic system of number.

In discussing the tendency to use arithmetic only in reference to concrete experience, Dr. Judd states his opinion that this is to lose the general idea of precise thinking which is responsible for our modern progress. A relaxation of training in precision to give more attention to concrete situations will not give the child an insight into the true significance of the number system:

> The conclusions to which the study reported in this monograph lead are diametrically opposed to the doctrine that arithmetic should be reduced to a few exercises in practical calculation. . . . the general ideas which are developed through contact with numbers can be cultivated in the individual only through a broad acquaintance with the properties of a highly perfected number system. To eliminate number instruction from the schools or to give it only a minor place would be to suppress one of the most significant general ideas that the race has evolved. To reduce arithmetic to a few

> practical applications would be to neglect the general idea of precise thinking on which our mechanical and scientific civilization rests. (30:116)

In addition to the constructive criticism offered by experienced thinkers, a number of important investigations have been made to determine the best method of teaching special skills or processes. Among these, that of Dr. Eva Luse, (35) is noteworthy. The object of the study was to discover the amount of transfer that takes place within narrow mental functions. Six hundred fifth grade pupils were chosen and placed in two equated groups, and for fifty consecutive days were given specially prepared drill in arithmetic computation. In the case of one group, the drill was most carefully constructed as to the distribution of practice in the four fundamental processes. Both the easier and the harder combinations were included and all the details of the work, such as the additions of the partial products in multiplication and the subtractions in long division, were chosen for their drill value. To the second group was given similar material, slightly in excess as to amount, but less distributed in character.

At the conclusion of the fifty drill periods, a series of tests was administered, containing matter similar to that given in the daily drills. A study of the resulting scores showed that:

Both groups had made decided gains in skill, speed, and accuracy.

The gains were 13.3 to 60.8 for the group receiving non-distributed drill, and 31.1 to 84.8 for the group receiving distributed drill.

For the particular processes, the gain made by the distributed-drill group over the other was-- addition, 17.7, subtraction, 18.8, multiplication, 35.0, and division, 23.9.

The same relative gains were shown for individuals on the different levels of intelligence and ability as for the whole group.

The residuum of skill after the summer vacation was greater for the group receiving distributed drill.

It was initial learning, not transfer, that caused the difference in the gains made by the two groups.

Another study in transfer by the English educator, W.H. Winch, (73) caused him to deny its effectiveness in arithmetic learning. Mr. Winch, after failing to find any transfer from general practice in computational exercise to work in problem solving, recommended that practice on computations that are to be used in an assignment of problems should be given in a study period. This practice will reduce the amount of inaccuracy due to inability to perform the necessary operations.

There seems to be but little concensus of opinion on the best way to teach subtraction and division. Some educators have advocated the method of presenting the subtraction combinations at the same time that the complementary addition combinations are given, and the division combinations with the multiplication.

The supporters of this opinion claim that it is psychologically sound. In a study reported by Winch a slight superiority was found in the "equal additions" method over the "decomposition" method. (72) Mead and Sears (38), on the other hand, found that keeping subtraction distinctly separate from addition gave better results. In two classes observed by them, the one using the decomposition method made a slightly higher score than the class using the equal addition method. Morton (40) found that children who have been taught the additive method prefer the decomposition way when permitted to use it. He stated that a great majority of the people in this country use the latter way and that it would be folly to attempt to change to the other method.

The difficulties arising in the teaching of percentage were the subject of an investigation by A. Edwards (23). He obtained his data from the study of 215 test papers, received from seventh grade pupils who were given the Compass Diagnostic Test number XIV. He found that the initial teaching of percentage was inadequate. The pupils were not taught how to avoid errors or how to correct them, and they were not given a "sense of the problem" which would enable them to use their judgment in the solution of percentage examples. In a classification of the difficulties in percentage, the following types were prevalent:

Failure to recognize quantities over 100 per cent of a given quantity.

Inability to solve problems using Case II and Case III.

Tendency to express per cent answers in terms of common fractions or mixed numbers.

Habit of forming a wrong attachment, as, $\frac{7}{8}$ is 62½ per cent.

Dr. Edwards said that the great number of wrong attachments, repeated errors, and impossible answers show a lack of the understanding of the relation of numerical quantities. He concluded by stating that "in general, the accomplishments in percentage are disappointing." (23:640)

3. Grade Placement and the Content of Texts

With the investigation on improvement of methods have been associated those dealing with the proper grade placement of topics and the content of textbooks in arithmetic. A great diversity of opinion is shown concerning these subjects. G. M. Wilson (71) wishes to include in the arithmetic curriculum only the subjects that are needed in daily life experiences. B. R. Buckingham (14) declares that such selection will stultify and narrow the ability of pupils, by depriving them of training in quantitative thinking. It will lessen the understanding of number concepts and the power of generalization. Social arithmetic should help interpret the environment; it should not be so limited as to teach facts in isolation rather than in rich association.

The following criteria for determining which of the topics available or in use in the present courses of study should be retained has been suggested:

> 1. Is the material of practical use in life?
> 2. Does the material coincide with the practices of the business world?
> 3. Can the material be understood by the children and used, therefore for a broadening of experience?
> 4. Are there certain phases of the topic which should be taught for informational value as opposed to skill value?
> 5. Does the topic contribute in a real way to development of general quantitative concepts?
> 6. Is the material desirable as a foundation for later mathematical work?
> 7. Will the topic be of interest to superior pupils even if of little or no value to others?
> 8. Does the daily work in arithmetic contribute its full share of situations calculated to build habits of self-reliance and independence in the pupil? (67: 82-83)

L. J. Brueckner (10: 681-709) stressed the need for an enriched arithmetic program and approved "the rejection of the reductionist point of view in curriculum making". He emphasized the fact that a selection of topics chosen solely from children's immediate interests would omit many important phases of learning. In this position he is supported by B. R. Buckingham (16:342:343) who affirms that pupils, especially in the beginning grades, are much more ready for arithmetic than is suspected. Both writers agree that "systematic, regular, provided-for" arithmetic should be taught from the time that the child enters school, not in the simple counting that he already knows, nor in the abstract computation that too quickly succeeds it, but in "the learning of the rich field of concrete arithmetic which lies between". (16:343)

A study as to the proper grade placement of problems has been made by L. N. Neulen. (47) This consists of an analysis of a number of courses of study in use in the larger cities and an investigation of the work of children in problem solving.

The purpose of his work was to ascertain the usual grade placement of problems with regard to the number of steps involved, to discover what ability beginning pupils in grades three to seven have in solving written problems involving one, two, three, or four steps, and to determine what relationships exist between problem solving skill, computational skill, and intelligence.

In the examination of twenty-four courses of study, a wide variation in the placement of problems was found, not only between grades, but also in a single grade. There appeared to be no agreement among the different school systems as to the proper placement of problems of one or more steps, and no consideration of psychological factors or reasoning power had influenced the seemingly arbitrary choice. To obtain a more objective authority for the proper placement of arithmetic problems, Mr. Neulen conducted an experiment in which two thousand children participated.

The children were first given reading and intelligence tests. A series of specially prepared arithmetic tests was then administered, the three units of which consisted in a preliminary test, a computational inventory composed of examples taken from the preliminary test, and finally, a repetition of the first test, given after five weeks of drill in computation. After tabulating the scores, each grade was divided into three groups according to

I. Q. ratings and the statistical tables were drawn up.

The results showed a constant decrease in ability to solve problems from the highest I.Q. group to the lowest, and a consistent decrease in ability to solve problems as the number of steps increased. A comparison of the results achieved in problem solving to those gained in paragraph comprehension gave a correlation of .3513; Comparison of the results achieved in problem solving with those of the intelligence test scores showed no consistent correlation.

When the scores were examined to determine proper grade placement of problems, it was found that the third, fourth, and fifth grade pupils were able to attain a sixty per cent mastery or better only on two-step problems. The sixth and seventh grade pupils gained sixty per cent mastery or better on four-step problems of fifth grade computational difficulty.

As a result of his study, Mr. Neulen stated that it is a mistake to decrease the time given to computational work in the classroom, but that the deliberate teaching of problem solving and of the reasoning processes necessary to it must not be neglected.

4. Diagnostic and Remedial Investigations

In 1926, G. T. Buswell and L. John (18) conducted a scientific study to discover the factors that contribute to failure in arithmetic. They investigated the materials of arithmetic, textbooks, and classroom methods to get a knowledge of the procedure to which children are subjected in the study of numbers. To ascertain the effects of this procedure on the pupils, the investigators made an elaborate study of the children's mental processes in working with the four fundamentals and of the overt movements manifested during the operations.

From the results of the observation of two hundred fifty children, a classified tabulation of work habits was assembled. This experimental table was used in an extention of the investigation, where three hundred fifty-two case studies were made. The advantage of working with the individual child and observing his reactions to the arithmetic situation was made strikingly evident by this research, which enabled the examiner to note the general excitement of some of the children. Twitching and jerking of the limbs, facial contortions, and shaking of the head and body betrayed uncertain methods of procedure. Mature attack by pupils of ability was evident in the regularity and system with which they worked; faulty and inexact operations of those having arithmetic difficulty exposed their confusion. Dr. Buswell remarked, "In the case of the latter, the school has asked for results but has failed to teach adequate methods of securing results." (18:46)

Factors contributing to failures in arithmetic which it is the duty of the school to eliminate are poor material, badly compiled textbooks, and faulty methods. Dr. Buswell criticized the texts for their failure to teach the child how to proceed, as explanations were left to the instructor, who did not always realize this need or who, because of the felt necessity for drill and practice, gave too little time to explain a new process.

Three diagnostic case studies are the subject of an article by W. A. Brownell. (8 :100-107) After a complete diagnosis of the cases, each child's difficulties were listed and special remedial instruction was planned. Every type of material used and the specific examples taught were fully described, while a daily record of the children's responses was kept. These records were invaluable in accounting for the outcome of the remedial classes, which extended over a period of six weeks. After a study of the results of the remedial work, Dr. Brownell claimed that the bond theory did not hold in teaching number concepts, that a child who learned that 2 x 5 are 10 on a card would not know it elsewhere without specific teaching, and that there is but slight transfer of learning in arithmetic, at least in the abstract facts.

C. E. Greene and G. T. Buswell (26:269-316) published, in 1930, a splendid study on the subject of diagnosis. In it they stated the importance of diagnostic investigation, saying that it was difficult to over-emphasize the importance of knowing the mental processes by which a pupil gets his answers. When children show uncertainty, the only real solution is a detailed analysis of their work, followed by instruction to correct faulty methods. The responsibility for the school to provide proper initial teaching was plainly stated:

> The present emphasis of remedial work is

> a reflection on the lack of good teaching in arithmetic. The necessity for a large amount of remedying indicates previous faulty processes which have produced difficulties. While it is probable that no scheme of teaching will ever entirely eliminate remedial work, it is certainly to be expected that the amount of such treatment should grow notably less. The end toward which the school should work is prevention rather than remedy. A school should pride itself on the lack of necessity for remedial work rather than on the elaboratensss of this work. (26:308)

Diagnostic and Remedial Teaching in Arithmetic (12) by L. J. Brueckner is well known and is invaluable to anyone interested in aiding children who have arithmetical difficulties. The same author (11:269-302) has summarized the various aspects of diagnosis. He grouped arithmetic instruction under four heads or functions -- the computational, the informational, the sociological, and the psychological. These, he stated, show that arithmetic is not a tool subject, but a social study, and that from the point of view of diagnosis it is necessary to consider this fact.

An effective program of arithmetic instruction should contain all that will aid the pupil's growth, and nothing that will hinder it. A rich experimental background in the primary grades is needed as a preparation for the social units that the upper grades should offer. Computational instruction, explanation of number relationship, and development of neat, orderly work habits are essential to a complete program of arithmetic teaching.

5. Investigations on Difficulties in Problem Solving

The difficulties met in problem solving are an important field of research, possibly because the results of former teaching have been shown to be inadequate.

In a study by L. N. Neulen (47) quoted above, it was stated that the children were greatly retarded in the solving of problems. Osburn (50) and Judd (30) remarked such a confusion in the wording of problems that the meaning was often obscure. Morton (40) considered that all arithmetic instruction should center about problems, as life presents arithmetic in that form.

G. O. Banting (3) has tabulated the causes that he discovered in problem solving. He found the failure to comprehend the problem, caused by lack of skill in reading or ignorance of the arithmetic vocabulary, to be a great handicap. Inability to identify and perform the proper computational operations caused many errors. Lack of the power to do reflective thinking caused some pupils to depend on verbal signs or cues which often misled them. Mr. Banting found that the pupils could be guided through analysis to accurate habits of problem solving.

Similar results were reported by P. R. Stevenson (57,58), after he had conducted an experiment in problem solving. He arranged a twelve weeks' program of remedial instruction. The first three weeks were devoted to an analysis of textbook problems, the second three weeks were spent in the solution of problems taken from daily life situations. For the third period of three weeks, problems without numbers were studied and for the last period a vocabulary study was made of difficult words found in the arithmetic textbooks. As a result of his observation of the children's work habits, Stevenson claimed that many pupils are influenced more by the form in which a problem is stated than by the meaning of the situation. Children depend upon verbal cues, not upon analysis or reflective thinking, when choosing a computational process to use in the solution of problems.

That children do reflective thinking while solving problems was questioned by E. J. Bradford. (5) He gave a test to nearly four hundred pupils of twelve or thirteen years of age, taking care not to suggest that the five problems offered to them were insoluble. Ninety per cent of 1990 possible attempts at a solution were made. Following this experiment, other tests were given with both real and insoluble problems. The suggestion of impossibility made in this case increased invalid reasoning from two to twenty per cent, reduced correct answers from eighty to sixty per cent, and reduced attempts to work the insoluble problems from ninety to sixty per cent.

From such results, Bradford concluded that many right answers in ordinary classroom arithmetic are not the result of critical thought, but of suggestion. Many children, evidently, do not use their judgment in the schoolroom environment.

Hydle and Clapp (28) found that unfamiliar situations and technical terminology were serious handicaps to pupils in the solution of problems, while ability to read with comprehension and to visualize problem situations were valuable aids to the pupils.

C. W. Stone (60) has written a series of tests to aid pupils in their reasoning difficulties. A survey test measures ability to reason in arithmetic problems, a diagnostic test aids the pupil to "think into and through his difficulty", and practice tests give experience in "rethinking the reasoning involved in his difficulty". Dr. Stone used these tests with an experimental group of children. He found that a comparison of gains over those of a group receiving the regular class instruction placed the experimental group above the other in ability to reason in arithmetic problems. The gain in arithmetic reasoning ability made by the pupils who used the tests transferred to other problem situations.

A great diversity of opinion regarding the proper means to remedy the problem-solving difficulty is evident. From the findings of the studies reviewed above it is obvious that neither computational drills nor training in reasoning gives consistent positive improvement in comprehension and interpretation of problem situations. Perhaps the movement to enrich the arithmetic curriculum and to provide a socialized background for such interpretation will give the needed approach to a solution.

CHAPTER III

METHOD OF STUDY

The investigation reported in this thesis was conducted according to the case study method. The technique involved four distinct steps:

1. Preliminary diagnosis, aimed to determine the need for remedial work and to give a history of the child that would be of aid in interpreting the attitude he evinced toward arithmetic.

2. Detailed diagnosis that would reveal the causes of error, by means of observation of the child's processes in solving prepared examples.

3. Remedial instruction planned to overcome the difficulties thus revealed.

4. Retesting after a period of remedial instruction to determine the progress made.

A. Preliminary Diagnosis

All the children in grades five, six, seven, and eight of a private school in Chicago were given the New Stanford Achievement Tests in Computation and Arithmetic Reasoning. The resulting scores were transmuted into age and grade scores by means of the table accompanying the tests. Twenty-five pupils who showed a retardation of a year or more below their chronological ages were chosen for further observation. They were given an intelligence test and a reading test. Home conditions were investigated. Physical history, school history, interests, and leisure activities were taken into consideration.

The intelligence quotient and mental age were determined by means of the Stanford Revision of the Binet-Simon Test, which was administered to each pupil by the examiner or her assistent, both trained workers in the field. A knowledge of home conditions was obtained in interviews which the examiner held with the parents, while interviews with the children gave information about their interests and out-of-school activities. The examiner obtained the physical history of each child from parents and school records. As the health of all the subjects was carefully supervised by family physicians, no physical tests were given by the examiner except two showing hand and eye preference. The school history was obtained from the individual reports kept on file in the school offices.

The Gates Silent Reading Test was included in the diagnostic program so that a comparison of arithmetic and reading ability might be obtained and so that a possible reading handicap in the study of the arithmetic text and of problems might be discovered.

Repeated absence caused one of the twenty-five subjects to be dropped from the group. Two others withdrew temporarily from school, leaving twenty-two children who, from the data obtained from the investigation, seemed to need remedial instruction in arithmetic.

B. Detailed Diagnosis

A detailed diagnosis of each child's work habits and difficulties in arithmetic was then made. The high rating given the validity and reliability of the Brueckner Diagnostic Tests in whole numbers, common fractions, and decimals caused them to be chosen as the instruments to reveal the nature of the difficulties responsible for the retardation of the children. Each pupil took the test under the observation of the examiner or a competent assistant, who noted on prepared forms the errors and wrong procedures observable during the computational operations of each child.

After this careful observation of the children at work upon the tests, the latter were minutely studied in private. The errors and work habits were compared to lists given in the test manual. Errors, responses, and attitude of each subject were tabulated in a book prepared for that purpose. From these data, remedial measures were planned. Tables XXIII to XXIX present the results of the diagnosis in tabulated form.

C. Remedial Instruction

Remedial instruction was prepared in accord with the needs of the children. For this teaching there were two units, group drill on the fundamental processes and individual instruction for problems of attitude or psychological maladjustment. Arithmetical reasoning and training in problem solving were also included in the individual aid periods. The computational drill was adapted to the group receiving it and varied little after it had been constructed; the individual instruction was suited to the needs of the particular child, was planned one or two lessons at a time, and was very flexible.

1. Group Drill

A program was prepared by the examiner for each class group. It was given by the assistants in periods of ten minutes daily, for twelve weeks. For the sixth, seventh, and eighth grades, the plan explained below was used; the fifth class procedure was similar except that the children did no work in decimals, other than that needed to solve examples involving the use of dollars and cents.

The first two weeks were devoted to work with whole numbers. Reference to Table XXV, page 155, will show the processes that were stressed in these lessons. The object of the drill was explained to the children and a friendly rivalry, stimulated by individual progress charts, gave interest to the work. For the beginning lessons the number wheel, ladder, and flash cards proved helpful, as they seemed to give the pupils a feeling of exactness and quick response which is sometimes lost in the working of long, intricate problems. These devices also enabled the children to correct any faulty work habits they had acquired.

The examples used in the drill were chosen carefully to supply needed exercise on the harder combinations, zero combinations, carrying, and borrowing. They were taken from Osburn, Brueckner, Thorndike and Courtis, and grouped to give distributed drill. No drill was allowed until a clear explanation of the method of procedure had been comprehended by each child. For example, to instill proper habits of column addition, the children were taught to watch themselves as they added columns of figures, and to check the places where they repeated, lost the place, or grouped non-consecutive numbers. As they became conscious of these bad habits, they were helped to add straight up a column. At first the upward movement of a pointer guided their eye movements. Both speed and accuracy increased noticeably, and the success reacted favorably upon other remedial work.

Usually a simple, clear explanation of a process was sufficient to make the pupils conscious of their incorrect operations. Several children who had failed to set over the partial products in multiplication did not repeat the mistake after one explanation of the proper procedure. Many errors in division were caused by ignorance of the right method. Easy examples, whose answers were obvious to the children were used to show the correct operations, step by step, then increasingly difficult division was given, as the pupils became familiar with the process.

The second two-week period was spent on work with fractions, but a daily three minute drill was devoted to whole numbers.

Examples selected as for the former drills were prepared. They were kept very simple, having denominators of the multiples of two and three up to eighteen. Practice was given in reduction, common denominator, improper fractions, and in the process used in subtracting a fraction from a whole or mixed number. Table XXVI, page 156, shows the errors to be corrected in this instruction. The first care of the instructor was that the operation be understood by the child; then drill to establish the process was given. In a number of cases, where the trouble was deep-seated, children were taught individually with specially prepared objective devices. These instances will be found in the Case Studies, chapter Four.

The next two weeks were used for drill on decimals, with a daily review of former work. Table XXVIII, page 159, shows the phases of this subject that gave trouble to the children. In this instruction, a great deal of explanatory work was approached by means of examples employing dollars and cents. Reading and writing decimals were taught by showing that the relations between tenths, hundredths, and thousandths is the same as the relations between tens, hundreds, and thousands. In order to keep the columns straight, zeros were placed to the right of the shorter decimals in addition and subtraction. In the multiplication and division of decimals the children had shown a great lack of understanding of values. To help them in this difficulty, some time was spent in writing the decimal as a common fraction and approximating an answer that would be reasonable. After they had learned to consider the decimal as a certain part of a whole they were allowed to approximate the answers when dealing directly with decimal fractions, and then verify their estimate by working the example. The method of moving the decimal point to make the divisor a whole number was used as it simplified the placing of the point in the answer and also the adding of zeros in the dividend.

At the end of each two-week period, a written test was given to each group. This consisted of the twenty examples most frequently missed in the Brueckner Diagnostic Test of the process being studied. If a child showed by errors on this test that any process was not fully understood, he was given individual aid.

The seventh week was devoted to further work on whole numbers, especially long division; the eighth and ninth weeks were spent on fractions, during which time subtraction and cancellation were stressed; the tenth and eleventh weeks were given to decimals, and for the twelfth week a general review and final test were prepared.

2. Individual Instruction

Two clearly defined objectives motivated the individual instruction arranged by the examiner: to help each child to an understanding of the purposes of arithmetic, and to open his mind to the reasonableness of the subject. In other words, the ultural side of arithmetic was to be developed to give a broader view and the logical aspect was to be emphasized to obtain a more correct interpretation of the meaning of number.

Such topics as the evolution of our modern figures, the daily uses of arithmetic, the meaning of statistical tables, and the different standards of measurement created an observant interest in arithmetic. Traditional arithmetic puzzles gave zest, while they indirectly showed the need of accuracy. Problems dealing with the daily life of the child were most valuable in providing a sense of logical approach to arithmetic situations, others from such books as Richard Haliburton's volumes of travel offered exercise in computing distances, finding the value of foreign currency, and gaining an understanding of longitude and time.

The individual instruction periods were invaluable in the opportunities they afforded of clearing up long-standing difficulties. All children who showed complete lack of comprehension of a process were given aid in these classes. Here much objective arithmetic was taught by means of real measures, weights, coins, and geometrical figures. An understanding of fractions was made easier by actually cutting objects into parts. Charts, graphs, illustrations, and maps were in common use. The utility of these means was demonstrated in a number of instances, when in the midst of some activity a child would exclaim, "O, so that is what it is about. I never could see it before."

The procedures mentioned above are described in greater detail in the case studies, where their use is made evident in the discussions of the progress of the individual child. These explanations will be found in Chapter IV.

CHAPTER IV

CASE STUDIES

Records of the progress made by the children who were given special remedial aid have been summarized in the following reports. It was impossible to include a detailed account of the daily progress of every child. An attempt has been made, however, to give, for each child, the incident that marked the awakening of interest in arithmetic. This interest was invariably aroused through the discussion of number situations in the child's own life.

Due to the nature of the subject, the material used for the remedial instruction was similar for all the pupils, but it was adapted to the needs of each child and presented in a variety of ways.

The table following each case study shows the progress made by the child. This progress is expressed in terms of the per cent of improvement as determined from his scores on the two forms of the New Stanford Achievement Tests in Computation and Arithmetic Reasoning. Table XXXIII, page 167, contains a summary of the same record of progress for the twenty-two children included in the case studies.

Case I, Dorothy T.

I. Introductory Statement.

Dorothy, an only child, lived with her parents in an apartment hotel during the school year. In the summers she traveled with her mother or went to a girls' camp on the Atlantic coast.

Frequent headaches and a susceptibility to colds had been common with her since childhood. She had had whooping-cough and measles during her fourth year, but no other children's diseases. She was of an extremely nervous temperament, with the high-pitched voice and quick, impatient movements characteristic of it.

II. School History

Before coming to her present school, Dorothy had been for four years in a non-sectarian private school where the children were allowed much freedom of choice regarding their studies. Frequent absences retarded her progress, so when she came to the new school her parents agreed with the authorities that a repetition of the fourth year would be desirable. This decision made the child very unhappy for a time. She was not interested in study, but as she became acquainted with her new teachers and class mates, she began to wish to conform to their standards of good work. Soon her changed attitude was noticed, but discouragingly frequent absences due to colds hindered her progress. Her best work was done in reading, as special remedial instruction was given her in that subject. In the eighth grade she was recommended for the arithmetic remedial class.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
14-4	13-6	90

Derived scores on the Gates Silent Reading Tests and New Stanford Achievement Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
13-0	7.3	12-7	6.8

Dorothy showed extreme nervousness during the test; her frequent corrections or contradictions of previous statements, due to lack of self-confidence, lowered her scores appreciably.

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four fundamental processes--
combinations, borrowing, carrying, choice of trial quotients.

In fractions -- computation, comprehension of process.

In decimals -- computation, use of decimal point, comprehension of values.

In problems -- computation, interpretation, choice of process.

Observation of the work of this pupil showed that she worked under a great strain, constantly repeating operations, counting by means of lips or fingers, skipping, and guessing.

Her work was very untidy. She forced herself to a high rate of speed, making no estimates or checks, and using any available figures indiscriminately. When working with a group, she copied from her neighbors.

V. Remedial Instruction.

Remedial instruction was planned (a) to relieve the strain under which this child was laboring, (b) to establish good work habits, (c) to give a mastery of the needed skills and so increase self-confidence, and (d) to aid in the formation of habits of intelligent interpretation of number situations.

To put Dorothy more at her ease in arithmetic situations, many expedients were used to help her realize the daily-life needs of number. For instance, she was given an assignment to bring to school a list of the times she had used numbers over the week-end. The following Monday a brief list was submitted, which was substantially lengthened after a discussion of Sunday activities. A permanent assignment was given to bring in at least one new use for arithmetic every day.

It was, of course, impossible to completely relieve the high tension, but every effort was made to have Dorothy realize that the examiner's aim was to help her, that those working with her liked her and took it for granted that she would do her best. No suggestion of a necessity for speed was made; in fact, her work was deliberately slowed down. Slow oral dictation of examples, verbal problems that required no computations, estimates, and logical analysis were used to give habits of careful thought. Computational accuracy was made the goal for all examples, regardless of the time necessary to get the correct answer.

The establishment of proper work habits was carried out in the drill periods, where Dorothy was one of a group of four. There she was given training in habits of attention and exactitude.

Dorothy had but little visual imagination, which made interpretation of problems very difficult. To remedy this, she was asked to formulate problems about life situations. Some that proved most interesting were centered around a trip she was eager to take. The route was planned, distances, overnight stops, amounts of gas and oil, expenses, all were worked out, with an incidental growth of understanding and interest in such computations. With the improving ability came greater interest in the text book material, which was used for formal exercise in problem analysis. To insure correct answers, Dorothy was given training in rewording, restating, and estimating. She was asked to explain her estimates and to criticize the reasoning by which she made them. There was less variety of subject matter used in the remedial work with Dorothy than with most of the other children, as each new field occasioned a renewal of the nervous strain

For this reason, an effort was made to provide experience in real life situations with which Dorothy was familiar. The furnishing and maintainance of an apartment, planning a tea, and figuring the expenses of a matinee party gave practical problems to solve. Another time, Dorothy had to find how to keep a family's expenditures within a certain income. This type of situation gave her an understanding of the needs of arithmetic in her life, and also instilled a feeling of competence to meet such problems when they arose.

VI. Results.

A comparison of scores made on the two Stanford Achievement Tests shows that Dorothy had made satisfactory progress, although she was still just short of the proper grade level in arithmetic:

	Arithmetic Age	Arithmetic Grade
March 1	12-7	6.8
June 7	14-8	8,7

The improvement in accuracy and the gain in poise and surety with which she approached the final test were most gratifying as they revealed how far Dorothy had advanced from her former attitude of fear and dislike of arithmetic.

TABLE I

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case I, Grade VIII

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	40	45	31	24
Number correct	29	36	17	23
Per cent of number correct to number attempted	62.5	77.7	55	96
Per cent of number correct to entire test	48.3	58.3	42.5	57.5
Per cent of increase in accuracy		7.1	74.5	74.5
Per cent of increase in speed		20.8		35

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. form was given March 1, 1937; the second form was given June 7, 1937.

Case II, Jack T.

I. Introductory Statement.

Jack was a dreamy boy of fourteen, stooped from much reading, awkward in athletics, ill at ease in a group, although able to talk in an interesting manner to one or two companions.

He was subject to frequent colds and was underweight, although his health was carefully watched by physicians. He refused to wear glasses to correct a myöpic condition.

II. School History.

Jack had been doing ailing work in the public school which he had attended from the first grade. The records showed a lack of interest in his studies. For the eighth grade he was placed in a private school where he might receive individual attention. There it was discovered that he enjoyed recreational reading, but failed to get the thought content of the lessons in the social subjects; his spelling and writing were immature and his arithmetic ability was below grade level. He was recommended for special instruction in reading and arithmetic.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
14-2	12-1	85

Derived scores on the Gates Silent Reading Tests and the New Stanford Achievement Tests in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
11-0	5.1	13-0	7.3

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four fundamental processes --
combinations, carrying, borrowing, choice of process, division.

In fractions -- lack of necessary skills.

In decimals -- failure to distinguish values, use of the decimal point, computations.

In problems -- computation, especially with fractions.

Observation of the work of this child showed that he used round-about methods, counted on his fingers, thought aloud, made no estimate or check, and worked very carelessly. He was uninterested in arithmetic as a subject of study, but enjoyed arithmetical puzzles and problems. If his computations had been accurate, he would have received a much higher score in problem-solving, as his method was usually correct.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to give a mastery of the needed skills, (b) to give an appreciation of the need for well organized, accurate work.

In the drill group, Jack found himself with boys whose arithmetic ability was on a par with his own, and he was able to compete with them on equal terms. His interest at once improved and he made good progress in computational skills.

Jack's liking for puzzles was utilized in the remedial periods. He was introduced to a number of the historical arithmetic puzzle problems, such as 12,345,679 multiplied by what number makes 444,444,444? He begged the cue and when it was given, made up numbers of examples to prove its infallibility. He enjoyed completing such tables as 9 x 9 plus 7 is 88
98 x 9 plus 6 is 888, etc.
Thes exercises gave a much needed training in accuracy.

For remedial help in problem solving, Jack was placed with Case X; the method used is described in that study. Toward the end of the period, Jack was absent for two weeks and missed the final reviews. He took the second form of the New Stanford Test, but worked at it in a dilatory fashion and handed in his paper before the allotted time had expired.

VI. Results.

Jack's final record was as follows:

	Arithmetic Age	Arithmetic Grade
March 1	13-0	7.3
June 7	12-9	7.0

The loss in arithmetic age was caused by Jack's failure to work for the allotted time. His interest in arithmetic had noticeably improved, his papers were neater, and the gain in accuracy was encouraging. It was recommended that a repetition of the eighth year, with continued individual assistance, be made.

TABLE II

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case II, Grade VIII

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	38	30	28	23
Number correct	29	25	21	21
Per cent of number corret to number attempted	76.3	85. 3	75	91.2
Per cent of number correct to entire test	48.3	41.6	52.5	52.5
Per cent of increase in accuracy		9.6		21.6
Per cent of increase in speed		-13.8		0

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given Marchl, 1937; the second form was given June 7, 1937.

Case III, Cecil J.

I. Introductory Statement.

Cecil was a nervous, retiring girl of thirteen. The youngest of a large family, her nearest sister was seven years her senior. She had formed the habit of going to some quiet place to read or play dolls by herself, as she had no companions of her own age. A frail constitution kept her from the active games that might have corrected this withdrawing tendency.

II. School History

Cecil did not enter school until she was seven and a half years old; repeated illness caused frequent absence. She made fair progress and was promoted each year. At the beginning of the sixth grade she was transferred to the private school she now attends. During the present year, her seventh, she was recommended for special instruction in arithmetic.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
14-1	12-8	89

Derived scores on the Gates Silent Reading Tests and the New Stanford Achievement Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
13-5	7.7	12-0	6.2

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four fundamental processes -- combinations, use of z ero, process of long division.

In fractions -- very faulty throughout.

In decimals -- computation, comprehension of values, use of the decimal point.

In problems -- comprehension of the problem situation, choice of process.

Observation of this child's work showed that she thought aloud, counted on her fingers, skipped, omitted digits in multiplication, repeated operations; there was no attempt to estimate or check. Cecil had an apprehension of arithmetic and showed great reluctance to do special work. It was suspected that the fundamental processes had never been mastered, so that with the introduction of the more advanced work in the upper grades, Cecil was completely at sea.

When the examiner asked for the processes necessary to solve some of the easier problems she had missed in the Stanford test, Cecil replied that she had forgotten how to work fractions and that she thought the hard problems (geometric figures) assigned for class work were easier to get.

Further discussion of her methods caused her to say, "I know I sometimes start at the wrong end to do my problems. I don't know for sure which end is first." When she was asked if in such an example as "ten minus six hundredths is how much?"

she ever thought of changing the numbers to ten dollars minus six cents, she answered, "I sometimes do that way, because I can understand that."

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to give a mastery of the necessary processes, (b) to give proper habits of work, (c) to give an understanding or problem situations that would obviate absurd answers.

For remedial instruction, Cecil was placed with Case VI; an account of the procedure is given in the study of that child.

VI. Results.

After Cecil had been given the second form of the New Stanford Achievement Test her record showed:

	Arithmetic Age	Arithmetic Grade
March 1	12-0	6.2
June 7	12-10	7.1

Due to the nervousness that Cecil felt in any test situation, her scores do not show the progress that the improvement in her daily work led one to expect. The final test caused a return of some faulty habits which had been overcome in the drill class. If this child could continue the remedial instruction it would be beneficial for her to do so.

TABLE III

Scores Made on the Two Forms of the New Stanford Achievement
Test in Computation and Arithmetic Reasoning
with Percentages of Improvement
Case III, Grade VII

	Computation		Reasoning	
	Form 1	Form2	Form 1	Form 2
Number attempted	43	48	30	25
Number correct	21	31	18	19
Per cent of number correct to number attempted	48.8	64.5	60.0	76.0
Per cent of number correct to entire test	35.0	51.6	30.0	47.5
Per cent of increase in accuracy		31.6		26.6
Per cent of increase in speed		47.4		5.5

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case IV, Peggy R.

I. Introductory Statement.

Peggy was a sturdy little girl of thirteen who lived with her parents and two younger brothers in a large house near the Lake. In the summers, she spent at least two months in a Girl Scout camp.

Rather blunt and outspoken, Peggy had a temperament that precipitated her into many difficulties, but she was an affectionate child, especially kind to younger children. Her favorite pastimes were sports or games with much action; she did not care for reading or imaginative games.

II. School History.

For the first seven grades, Peggy had been in a boarding school where her excellent health and high spirits enabled her to lead in extra-curricular activities. In her studies, however, she did only fair work at best. On entering her present school for the eighth grade she appeared discontented and was slow to adapt herself to the new environment.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
14-0	12-6	89

Derived scores on the Gates Silent Reading Tests and the New Stanford Achievement Test in Computation and Arithmetic

Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
13-9	8.0	12-4	6.6

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four fundamentals -- multiplication, partial products, use of zero, choice of trial quotients.

In fractions -- computation, completion, reduction, comprehension of common denominator, multiplication, division.

In decimals -- comprehension of values, use of decimal point, i.e., 20 - 3. 825 is 3. 805.

In problems -- comprehension of situations, shown by absurd answers.

Observation of the work of this pupil showed that she counted on her fingers and by lip movement, nodded or shook her head, grouped and skipped in adding, began to make her computations before she understood the statement, worked at too high a rate of speed. She showed impatience and worry; her discouragement was expressed in the remark made when she was called for special instruction, "So I'm one of the dumb ones."

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to improve the attitude toward arithmetic, (b) to velop the skills necessary to do satisfactory work, and (c) to give an understanding of the daily life uses of arithmetic.

In the remedial group, Peggy's knowledge of addition and subtraction combinations was praised; it was suggested that as she knew them so well, she could easily do the fastest work in the group if she would add up the columns without skipping about. Eager to excel, Peggy accepted the hint and improved her habits in addition very quickly. This little success made her receive other aid more graciously. She realized that she could do better and set about it in earnest.

In the individual lesson periods, Peggy was first given aid for her difficulties in fractions. She did not know how to multiply twenty by two and a half, but gave forty and a half as the answer. A road map of country with which she was familiar was obtained, and exercise in computing distances between places she had visited was given. Then, using a scale of twenty miles to one inch, she drew lines showing distances. When the question was asked, "How many miles apart are two towns that are two and a half inches apart on your map," she was able to tell the correct distance. The process of the multiplication was then explained to her and other examples of the same type given.

Similar procedure was used to develop other processes, as Peggy's lack of visual imagination was a handicap to her comprehension of problem situations. Objective work was given with coins, weights, and measures. A list of arithmetic experiences in her every-day life was made, from which original problems were constructed. Advertisements, maps, statistical tables and graphs, and baseball scores were used to get real life situations.

Peggy was given much aid in problem analysis. She was taught to read the statement carefully, restate it, answer fact questions about it, and draw a graph or diagram if possible. When the situation was understood, she gave an estimate of the probable answer and decided on the operations necessary for solving the problem. A check was made for computational errors and correct interpretation before the work was considered finished.

VI. Results.

After Peggy had been given the second form of the Stanford Achievement Test, her scores showed:

	Arithmetic Age	Arithmetic Grade
March 1	12-4	6.6
June 7	13-11	8.1

This meant that Peggy was still four months below her chronological age level and nearly a year below her grade level, although she had gained over a year during the remedial period. As she improved in ability, her attitude showed a marked change; she even acquired a liking for arithmetic. Careless errors in the final test, such as writing $\frac{7}{3}$ for $\frac{7}{30}$, suggest that more training might be given in careful checking.

TABLE IV

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case IV, Grade VIII

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
umber attempted	38	48	25	22
umber correct	26	34	18	20
er cent of num- er correct to umber attempted	68.4	70.8	72.0	90.9
er cent of num- er correct to ntire test	43.3	56.6	45.0	50.0
er cent of in- rease in ccuracy		3.5		26.5
er cent of in- rease in speed		30.7		11.1

NOTE: Each form of the computation test contained 60 ex-
ples; each form of the reasoning test contained 40 problems.
e first form was given March 1, 1937; the second form was
iven June 7, 1937.

Case V, Walter G.

Introductory Statement.

Walter was the only son of devoted parents, who gave him everything he wanted. He was spoilt, but very likeable, full of fun and generous, with a love for outdoor sports and a corresponding dislike for study. His only illnesses were an occasional cold, but he sometimes showed fatigue due to over-exertion at games. Myopia made glasses necessary.

II. School History.

For the first five years of his school life, Walter had attended a large parochial school. He was very slow in learning to read and for this reason repeated the fourth grade. At the beginning of the next year he entered his present school. During the fifth and sixth years, Walter received remedial instruction in reading. His best work was done in spelling, grammar, and letter writing; arithmetic was very difficult for him.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
13-6	11-11	88

Derived scores on the Gates Silent Reading Tests and New Stanford Achievement Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
12-0	6.2	11-4	5.5

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four fundamental processes -- combinations, zero, long division.

In fractions -- lack of comprehension of process.

In decimals -- use of the decimal point, comprehension of values.

In problems -- reading, interpretation of statement, choice of correct process.

Observation of the work of this child showed that his work habits were extremely poor, with no estimating or checking. His papers were untidy. He betrayed a feeling of discouragement at his inability. He had never understood some processes, and had devised systems of his own to work the examples. For instance, in long division he used the following unique plan:

$$
\begin{array}{rrr}
 & 5 \text{ R } & 16 \\ \cline{2-3}
27/ & 1465 & \\
 & \underline{135} & 27 \\
 & 11 & -\underline{11} \\
 & & 16
\end{array}
$$

His operations with fractions were equally odd; he said:

1/8 plus 3/8 plus 7/8 plus 1/8 are 12/32,

3/8 divided by 5 is 15/8,

1/4 - 1/3 is 1/1, and

5/6 - 1/9 is 4/2.

In solving problems, he took any available numbers and performed one of the fundamental processes, preferably multiplication. From the information gained in interviews with the parents, the examiner found that Walter had little incentive to improve his scholastic standing.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to stimulate the boy to do the best work of which he was capable, (b) to give him proper habits of work, (c) to teach the skills in which he was lacking, and (d) to establish a reasoned approach to problem solving.

Many methods of motivation had been found ineffective in Walter's case and he seemed to look upon the remedial class as a chance to get out of regular work. A plan frequently used by higher institutions of learning was adopted to ensure real effort. No athletics and no place on the team would be permitted unless satisfactory lessons were done. This proved a splendid stimulant, and the boy did try hard. Years of study without comprehension had established such poor methods that it took much patient explanation and real courage to begin the formation of correct ways.

In long division, Walter was insistent that his way was right, even though it would not check. The device was used of giving simple examples in multiplication, then checking them by division. For several lessons these examples were kept so easy that there would be no remainder in multiplying the first trial quotient, so the habit of bringing down was acquired. Then the example 75 x 15 was given; when the answer, 1125, was to be checked, the necessity of bringing down the five did not admit of argument. Other examples followed until the proper procedure became automatic.

The usual objective work with measures, coins and paper was used to give a grasp of the meaning of fractional parts. The word "part" was substituted for the denominators of the fractions to correct Walter's habit of adding or subtracting both terms. For one example, two paper squares were folded and cut into eighths, although the fractional name was not given to the divisions. Walter was asked, "How many parts would 1/part plus 3/parts plus 7/parts plus 1/part be?" He gave the correct response and asked, "Could you call them twelve eighths?" The examiner requested him to explain how that would be the right answer, which he did.

In examples where it was necessary to find a common denominator, groups of fractions such as halves, thirds, and sixths; halves, fourths, and eighths; and halves, fourths, and twelfths were used. Here also, paper parts were used to make the operations clear. Eleven lessons were spent on this simple work in addition and subtraction of fractions, at the end of which time Walter could change to a common denominator, borrow correctly in subtraction, reduce, and change mixed numbers to improper fractions and improper fractions to mixed numbers.

Multiplication and division of fractions gave less trouble, once the process was explained, as reduction was understood.

For aid in the solution of problems, the first device was that of having problems with numbers missing. Reasonable numbers were to be supplied by the child. One of these was "My pony eats ___ bale of hay a day. A bale of hay costs ___. It costs ___ a day to feed my pony. It costs ___ a week to feed him." Walter had obtained permission to visit a building of his father's which was under construction. He spent several hours there and acquired valuable information about materials, costs, accuracy of measurement, and many other phases of the work. He made several visits to a bank, went to some machine shops, inspected a chemical plant. He was able to formulate some good problems after these visits; he was also more alert and interested in finding other information concerning industry.

Textbook problems were never easy for him. An actual experience seemed to be necessary to make a situation clear to him. One means he was able to adopt to aid in the solution of problems was the use of charts or graphs. He made progress in analyzing by use of the questions, "What am I told? What am I asked? How can I find out?"

VI. Result.

Walter's improvement was very slow, but when he had mastered a process he rarely reverted to his old incorrect ways. He gained greatly in accuracy and neatness; the sense of discouragement was gone and a real interest was shown in arithmetic work. The second form of the Stanford test showed a growth of five months during the remedial period:

	Arithmetic Age	Arithmetic Grade
March 1	11-4	5.5
June 7	11-11	6.0

It was recommended that Walter repeat the seventh grade and that continued individual aid be given to him.

TABLE V

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case V, GradeVII

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	44	32	23	19
Number correct	22	25	12	15
Per cent of number correct to number attempted	50.0	78.1	52.1	78.9
Per cent of number correct to entire test	36.6	41.6	30.0	37.7
Per cent of increase in accuracy		56.0		51.7
Per cent of increase in speed		13.8		25.0

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case VI, Elizabeth F.

I. Introductory Statement.

Elizabeth was a shy, retiring girl, the only child of parents with outstanding artistic ability whose apartment was the rendez-vous of literary and musical celebrities. Elizabeth loved to watch and listen to the guests from some quiet corner, but had little chance to express herself.

Her life was very sheltered; school and music classes, with an occasional matinee or shopping tour, made up her days. She enjoyed reading, her favorite authors being Andrew Lang and Mrs. Burnett.

II. School History.

Elizabeth entered the public school at the age of six. She learned to read slowly. She repeated the fifth class as all her work was below grade level. For the seventh year she was enrolled in a private school where special aid in arithmetic and reading was given to her.

III. Intelligence and Achievement Record.

The Stanford Revision of the Binet-Simon Test gave the following results:

Chronological Age	Mental Age	Intelligence Quotient
13-5	11-7	86

Derived scores on the Gates Silent Reading Tests and New Stanford Achievement Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
12-6	6.8	11-6	5.7

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four fundamental processes -- combinations, carrying, borrowing, use of zero, long division.

In fractions --no idea of the processes.

In decimals -- computation, use of decimal point, comprehension of values.

In problems -- comprehension of arithmetic situations, no reasoned attack upon the problem.

Observation of the work of this pupil showed that she counted on her fingers, skipped, grouped, repeated operations, and depended upon others. She did not estimate or check her answers and appeared to lack a sense of number. On the Stanford Arithmetic Reasoning Test, thirteen out of twenty-five answers were obvious guesses. In her computations she did not even get the same wrong answer twice. For instance, for the example $6\frac{4}{5} \times \frac{3}{7}$ her first answer was $\frac{42}{35}$, her second answer was $6\frac{43}{35}$.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to instill a sense of number, (b) to give necessary computational skills, (c) to give an idea of the social uses of number, and (d) to give ability to make a reasoned attack upon problems.

It was decided that Elizabeth might be helped by working with a companion who was similarily handicapped by lack of number experience. Each could supplement the other's efforts, while a little rivalry might stimulate more interest than would be awakened in individual work periods.

The first experiences were planned with the help of the parents, who gave the girls opportunities to use money, to assist in planning menus, in marketing, and in depositing money in the bank. One of the mothers undertook to give them cooking lessons to show the use of measures and proportions.

In the remedial class, the two girls were helped to understand the importance of digits' places by a study of the house numbering system. They found how the calendar was made, kept a temperature chart, learned to interpret the graphs and statistical tables in their geographies, and figured distances to familiar places. A visit to the Planetarium gave them opportunity to work with large numbers, while the subject of microscopic investigations was introduced to give meaning to decimal fractions of several places.

Work with problems accompanied these investigations. One form of question was, "How would you find ___if you knew ___?" Restating, choosing the necessary statements in a wordy problem, and answering fact questions were other exercises. Problems obtained from the household experiences were a source of interesting comparison, as were recipes and budgets.

In taking problems from the textbook, those were chosen that contained experiences familiar to the girls, or that had elements common to the individual problems they offered. In solving these, training was given in formal analysis, which was made orally: read the problem, restate, giving the essential facts, estimate a probable answer, choose the proper processes, solve, check.

VI. Results.

After Elizabeth had been given the second form of the Stanford test, her record showed:

	Arithmetic Age	Arithmetic Grade
March 1	11-6	5.7
June 7	12-2	6.3

Elizabeth had gained eight months in arithmetic age, but was still more than a year below her grade level. It is hoped that she will repeat the seventh grade and that she will continue the remedial classes. Individual attention and enriched experience are still needed in this case.

TABLE VI

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement Case VI, Grade VII

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	42	35	25	18
Number correct	25	26	12	17
Per cent of number correct to number attempted	59.5	74.2	48.0	94.4
Per cent of number correct to entire test	41.8	43.3	30.0	42.6
Per cent of increase in accuracy		24.6		96.6
Per cent of increase in speed		4.0		41.6

NOTE: Eacn form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first test was given March 1, 1937; the second form was given June 7, 1937.

Case VII, Blanche A.

I. Introductory Statement.

Blanche, an only child, came from a home of culture and refinement. She had inherited a talent for drawing from her mother, and the furnishings and surroundings of the home had cultivated her love of beauty.

Having twice been ill with pneumonia, Blanche seemed rather frail, although she had no physical disabilities. She enjoyed riding, canoeing, swimming, and children's games, as well as reading and drawing. Her happy, contented disposition made her liked by all who knew her.

II. School History.

Because of her illnesses, Blanche had been taught at home for several years. At the age of ten she entered the fourth grade of a public school, but repeated the grade the next year as she was retarded in reading and arithmetic. For the fifth class, she was enrolled in a private school where she was given remedial instruction in reading for several months. This aid brought her up to grade level in reading and, better still, gave her a real liking for it. Her arithmetic work was very poor.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
12-3	11-0	89

The Gates Silent Reading Tests and the New Stanford Achievement Tests in Computation and Arithmetic Reasoning gave scores as follows:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
11-6	5.6	10-9	4.8

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four fundamental processes --	combinations, carrying, borrowing, use of zero, choice of trial quotients.
In fractions --	lack of comprehension of processes.
In problems --	interpretation of problem situations, computation.

Observation of the work of this pupil showed that she had poor, irregular work habits, lack of skill in the fundamental processes, and inability to estimate or check the answers. Her classroom teacher reported that her written exercises were rarely completed. One day she gave in an exercise paper covered with decorative scroll and flower designs which almost hid two little examples. At the bottom was written, "I'm sorry there is so little."

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to give a mastery of the needed skills, (b) to give motivation for better work, and (c) to inculcate habits of attention and accuracy.

It was evident that lack of knowledge was partly responsible for much of Blanche's difficulty, as her governess had given most of the time to English and French study. Using Brueckner's Diagnostic Tests as a guide in the remedial class, the child worked aloud, at first counting on her fingers, repeating operations, or losing her place. To correct these bad habits, a list was made of the combinations she did not know, and special practice drill cards containing these combinations were given to her for special study. Many examples gave additional exercise to fix the proper response; here it was found that a number of short examples seemed more valuable than long columns of addition, or multiplication with more than two figures in the multiplier. Blanche enjoyed learning the "whole story" about a combination after she had used it -- adding, subtracting, multiplying, and dividing. This method did not confuse her as it would some children.

To teach habits of attention in reading problems, several different ways of stating the question were employed. Multiple choice furnished problems such as "In one pound there are (12, 3, 16, 10) ounces." True-false and completion forms were also useful, as well as questions demanding the knowledge of an unstated fact -- "I must make 12 athletic badges, each 4 inches long. How many yards of ribbon shall I need?" Some problems were followed by thought questions, as for instance, "I bought four handkerchiefs at 60 cents each and gave the clerk a five dollar bill. Was that enough to pay for them? What pieces of money could I receive in change?"

Arithmetic puzzles interested Blanche. Once she spent her free time for two days working on the sequence beginning with 1 x 8 plus 1 is 9. 12 x 8 plus 2 is 98, etc. Original problems, the use of advertisements, maps, graphs, and charts, illustrations, and objective measures were used to develop a sense of number. Blanche was required to estimate the answer to the problem on which she was working, then check herself by computation.

VI. Results.

After Blanche had been given the second form of the Stanford test her scores showed:

	Arithmetic Age	Arithmetic Grade
March 1	10-9	4.8
June 7	11-3	5.4

In this test, Blanche's work showed marked improvement in accuracy and neatness. It was felt that she had in a great measure overcome the habit of inattention. Although not up to her grade level, a good start had been made. Checking up on her work of the following year, it was found that a repetition of the Stanford Achievement Test gave her a standing of seventh grade ability.

TABLE VII

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case VII, Grade V

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	26	28	17	14
Number correct	21	24	8	12
Per cent of number correct to number attempted	80.7	85.7	47.0	85.7
Per cent of number correct to entire test	35.0	40.0	20.0	30.0
Per cent of increase in accuracy		6.1		82.3
Per cent of in-crease in speed		14.2		50.0

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case VIII, Bertha F.

I. Introductory Statement.

Bertha was the youngest of seven children. Her parents were in comfortable circumstances and were able to give their children every care.

When she was two years old, Bertha had had the measles; at seven her tonsils and adenoids were removed. She had no physical defects. Her disposition was happy and she was very fond of other children.

II. School History.

Bertha entered her present school at the age of six. She was not a good student, as reading caused her much trouble. With special tutoring she advanced to the second and third grades, but repeated the latter because of poor reading. In the fourth grade she was again given remedial assistance in reading and was recommended for special help in arithmetic. In all her work she was slow, but diligent, and it was observed that practical experience in life situations was necessary to enable her to interpret the subject matter of her school books.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
11-8	10-0	86

Derived scores on the Gates Silent Reading Test and the New Stanford Achievement Test in Computation and Arithmetic Reasonng were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
9-4	3.8	9-11	4.0

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four fundamental processes -- carrying, borrowing, use of zero, some combinations, subtraction, comprehension of processes of multiplication and division.

In fractions -- no comprehension of process.

In problems -- lack of interpretation of problems.

Observation of the work of this pupil showed that she thought aloud, counted on her fingers, copied from others, was untidy and impatient, and did no estimating or checking. She had a feeling of discouragement at her inability to understand; although she lacked a comprehension of abstract number, she had a practical sense of concrete situations and a good grasp of the combinations, with a few exceptions.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to give Bertha an ability to transfer her understanding of practical number situations to new computations and problems, (b) to correct her faulty work habits, and (c) to teach her the proper computational processes, especially in the four fundamental processes.

Although Bertha's I.Q. was low, she had an understanding of the problems encountered in life situations that she lacked in her school work. It was noticed that a correlation of arithmetic to life problems gave her a grasp of the situation that she could not get with abstract numbers. The plan was tried of giving her puzzle problems, for instance, "Five men are walking down the street and one half go each way. How many go each way?" She saw the absurdity at once. Other problems were presented, some soluable, some absurd. This kept her alert and interested.

As many of Bertha's computations showed a complete lack of interpretation, objective measures such as coins, liquid measures, and yardsticks were used. Two subtraction examples show her need for this method:

$$\begin{array}{r} 89 \\ -89 \\ \hline 89 \end{array} \qquad \begin{array}{r} 414 \\ -7 \\ \hline 413 \end{array}$$

When the same type of example was presented as a problem in traveling, she got the correct answer at once. She was asked to get the distance from home to school by observing the speedometer, to keep a record of the number of miles she traveled by motor, to reckon change from bills given in payment for gas or other purchases. These and other experiences gave the material for many problems, then similar number situations were found in the text. The abstract examples in the arithmetic book were used to make up other problems. Special attention was given to the vocabulary needed for this work. All unfamiliar words were explained and drill was given on arithmetic terms.

In adding fractions, Bertha worked as follows:

$1\frac{2}{5}$ plus $1\frac{1}{2}$ is $2\frac{3}{5}$ In subtraction she had a different method: $\frac{2}{3} - \frac{1}{3}$ equals $\frac{1}{2}$. This answer was obtained by subtracting the bottom number from the top, then the two middle numbers. In the remedial class, all written work in fractions was discontinued for a time. Paper figures of fruits or geometrical shapes were divided into fractional parts, then cut. Instead of using the words "half" or "third", the fractional pieces were simply called "parts", and the fraction was written $\frac{1}{\text{part}}$. Thus, $\frac{2}{\text{parts}} - \frac{1}{\text{part}}$ is $\frac{1}{\text{part}}$, or, as the child advanced in understanding, $1\frac{3}{\text{parts}}$ plus $\frac{2}{\text{parts}}$ is $2\frac{1}{\text{part}}$. It was explained how the number of parts gave them their name--eighths, etc., and that big parts could be divided. When the denominators were introduced, Bertha showed a much better understanding of their meaning. It was thought advisable not to try any work in the multiplication or division of fractions during the remedial period, for fear of confusion of process before the learning or addition and subtraction was fully mastered.

VI. Results.

For two days before the final test, Bertha had been kept at home on account of an indisposition. If it had been possible, the test would have been postponed. The scores, it is felt, do not do justice to the real gain the child had made in ability to handle arithmetic situations. The final test scores were:

	Arithmetic Age	Arithmetic Grade
March 1	9-11	4.0
June 7	9-11	4.0

With Bertha, the greatest improvement had been shown in a growing ability to interpret problem situations, in a more happy attitude toward arithmetic, and in the beginning of a better method of attack. It was felt that with continued guidance the next year she might attain a degree of independence and surety in her arithmetic work.

TABLE VIII

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case VIII, Grade V

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	31	27	12	15
Number correct	13	13	7	7
Per cent of number correct to number attempted	41.9	48.1	58.3	46.6
Per cent of number correct to entire test	21.6	21.6	17.5	17.5
Per cent of increase in accuracy		14.7		-20
Per cent of increase in speed		0		0

NOTE: Each form of the computation test contains 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case IX, Agnes M.

Introductory Statement.

Agnes was a rather immature girl of fourteen whose parents kept her from contact with other children by making her their constant companion. Their home was in an apartment hotel, where they spent the entire year except for an occasional motor trip. Agnes had no physical defects except an astigmatism that made glasses necessary. She did not care for sports, but enjoyed reading and the movies.

II. School History.

For her entire school life, Agnes had attended a private school for girls. She was not an especially good student and showed little interest in her work, but she was promoted each year. At the end of the eighth grade, when she was thirteen years, nine months old, the school authorities persuaded the parents that another term in that grade would strengthen her grasp of the subject matter and make her more ready for high school.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
14-6	15-11	110

Derived scores on the Gates Silent Reading Test and the New Stanford Achievement Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
14-5	8.7	13.5	7.6

. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four fundamental processes --
combinations, use of zero, choice of trial quotients, remainders, incomplete work.
In fractions -- comprehension of process in multiplication and division. (She inverted the multiplier)
In decimals -- Use of the decimal point, computations.
In problems -- Interpretation of situations.

Observation of the work of this child showed that she had poor work habits, with no attempt to estimate or check. She worked too hastily, lacked interest and attention, and made many errors through lack of orderly arrangement. It appeared that Agnes had failed to acquire the skills necessary to do eighth grade arithmetic or to receive proper motivation.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to supply needed motivation, (b) to give proper work habits, and (c) to inculcate an interest in arithmetic that would overcome faults due to lack of skill.

The group of four with whom Agnes was placed for the daily drills planned a racing tournament. Each chose the name of some famous horse and tried to keep him ahead of the others. Daily scores were posted and the girl with the best arithmetic record saw her favorite take the lead. This stimulus sufficed to awaken nterest; it was not one to make a lasting impression, but it ncouraged effort and encouraged confidence by small successes.

In the periods given to private instruction, Agnes said she had never been interested in buying or selling, farm problems, or averages. On being asked what sort of problems she would put into a book, she could not say. The task was given her to bring to school real problems that she had had to solve at home or in town. When these showed but little variety, she was encouraged to use the knowledge she had acquired in her travels to add to them. One project that Agnes enjoyed was to imagine herself as living in different parts of the country and finding what a variety of arithmetic experiences she would have.

This activity gave Agnes a sense of the use of numbers that she had lacked. Her mother helped broaden her experience by asking her aid in planning the meals, shopping, and budgetting.

When a noticeable amount of progress had been made, Agnes was shown her first Stanford Achievement test in reasoning and was asked how she got her answers. On reading a simple one she had missed, she exclaimed, "How did I ever miss that?" She quickly corrected several others and seemed ashamed of the absurd errors. She developed for herself a technique of analysis for problem solving; in it she would write in brief statements the conditions of the problem, using abbreviations of the terms. When symbols were suggested she welcomed them, and made use of equations. As her skill improved, she developed a real liking for arithmetic.

VI. Results.

After Agnes had been given the second form of the Stanford test, her record showed:

	Arithmetic Age	Arithmetic Grade
March 1	13-5	7.6
June 7	15-11	10.0

Agnes' arithmetic age exceeded her chronological age by fourteen months; she showed marked growth both in computational skills and in reasoning. Her greatest gain, however, was felt to be in an awakening interest in her surroundings and in a maturing outlook upon life.

Case X, Dan B.

. Introduction Statement.

The elder of two sons, Dan was a well-built, athletic boy of fourteen. The father, a successful merchant, had acquired a comfortable fortune by his ability and energy; the mother was entirely devoted to her family.

Dan had had no serious illnesses and had no physical defects. He excelled in sports and enjoyed all outdoor pursuits.

II. School History.

Dan had entered his present school in October, when his parents had taken him from the public school he had previously attended. The change was made in hopes of improving his attitude in regard to study. His records showed an uneveness of accomplishment, although there was no failure.

III. Intelligence and Achievement Record.

The Stanford Revision of the Binet-Simon Test gave the following results:

Chronological Age	Mental Age	Intelligence Quotient
14-5	16-5	114

The Gates Silent Reading Test and the New Stanford Achievement Test in Computation and Arithmetic Reasoning showed:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
15-5	9.6	13-1	7.4

IV. Diagnostic Test Results.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four Fundamentals --

combinations, borrowing, carrying, division.

In fractions -- reduction, inversion, cancellation.

In decimals -- use of the decimal point, computation, knowledge of values.

In problems -- computations, interpretation of the statement.

Observation of the work of this pupil showed that he made unnecessary bodily movements, added by skipping and grouping, worked too hastily, lacked accuracy and neatness, made no estimate or check, was impatient and lacked interest. He disliked having to take the Stanford test, and when he became tired of working, wrote, "I don't know," across his paper. Although Dan had had some practical experience with numbers in managing a small bank account and helping in his father's office, he made no use of this knowledge in his arithmetic work. It was felt that he possessed the necessary ability to do good work, but failed because of his attitude.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to secure Dan's interest and cooperation in the attempt to give him the necessary aid, (b) to replace the bad work habits by efficient ones, and (c) to coordinate arithmetic with daily life and experience.

Dan was placed with four other boys for the daily drill periods. There the work was stimulated by a "horse race". Each oy chose his favorite racer, for whom he won points by speed and ccuracy in the drills. Here Dan did good work in checking his wn faults; he showed a steady gain in accuracy by correcting his rong processes after listening to explanations given to other oys. He tried to avoid any supervision of his own work, seeming to feel that it caused a loss of prestige.

In spite of Dan's practical experience with numbers, he showed no ability to attack problems in a logical way. "I don't see why this way won't work," was the oft-repeated reply when an absurd answer showed that his reasoning was at fault. The reading test gave a cue to the source of the difficulty, as there he had manifested weakness in picking out significant details. Problems were prepared in which a false statement had been made, in others, missing numbers were to be supplied. Wordy problems were restated and abstract examples used to construct original problems These devices seemed to help in the interpretation of problem situations.

It was found that Dan enjoyed the class more when he was allowed to work with a friend of his who had difficulties similar to his own. Both boys asked if the special instruction might be on the subject matter of the impending final examination. Their request was granted, although it meant a narrowing of the field of number experience. It is possibly for this reason that less progress was made by these two cases than by the others.

The boys were given work in completion of tables of measure, lists of measures to match properly, true-false statements such as "Interest is the same as amount," and formulae to complete, as "The area of a square is ____ (A is s^2)

Problem analysis was taught with the form, "What am I told? What am I asked? How can I find out?" Listing the necessary steps in the correct order, estimating, and checking were insisted upon. Graphs, charts, and illustrations helped make difficult statements clear. For problems of geometric measurement, such as computing the height of a building by means of shadows, the boys took the necessary instruments outside and made real measurements.

With these two pupils, accuracy was insisted upon, but no attempt was made to develop rapid calculations. On the contrary, emphasis was put upon neat, well-organized solutions.

VI. Results.

After Dan had been given the second form of the Stanford test, his record showed:

	Arithmetic Age	Arithmetic Grade
March 1	13-1	7.4
June 7	12-6	6.7

In this test only one error was made in the computation form, two errors and two failures to complete in the problem form. Dan did not wish to take the test, and stopped before the expiration of the allotted time.

TABLE X

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case X, Grade VIII

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	46	30	28	21
Number correct	37	28	18	17
Per cent of number correct to number attempted	80.4	93.3	64.3	80.9
Per cent of number correct to entire test	61.6	46.6	45	42.5
Per cent of increase in accuracy		16		25.8
Per cent of increase in speed		-24.3		-5.5

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case XI, Mary W.

I. Introductory Statement

Mary was quiet, old-fashioned child. Her parents were middle-aged and her only sister was several years her senior. She did not make friends easily, showed little interest in the happenings around her, and spent most of her time reading or drawing.

Although May had never had a serious illness, she was subject to frequent colds that sometimes affected the sinus. Near-sightedness made glasses necessary.

II. School History.

Before coming to her present school, which she entered for the seventh grade, Mary had attended a public school. Her reports showed that she had done fair work in arithmetic, spelling, and geography, and very good work in reading and history. In the eighth grade her arithmetic work became very poor.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
14-0	15-8	112

Derived scores on the Gates Silent Reading Test and the New Stanford Achievement Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	ARithmetic Age	Arithmetic Grade
15-1	9.2	13-1	7.4

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four fundamental operations -- carrying, borrowing, use of zero, partial payments, choice of trial quotient, remainders in division.

In fractions -- no evident comprehension of process; she wrote 40 for 4 $\frac{0}{4}$,
6 $\frac{7}{8}$ equals $\frac{42}{24}$

In decimals -- comprehension of values, use of decimal point.

In problems -- wrong processes, computation.

Observation of the work of this pupil showed an irregular procedure, too hasty and untidy for accuracy to be expected. Her attitude toward arithmetic was one of diffidence and uncertainty. She said she had forgotten how to do the "easy work" when she began to study the hard seventh and eighth class arithmetic. Lack of interest in the subject also impeded progress.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to develop a sense of arithmetic values, (b) to establish habits of clear thinking, and (c) to correct faulty techniques.

The last named aim was taken up especially in the drill group in which Mary was placed. Her persistent fault of not setting over the second partial product in multiplication was corrected by a clear explanation of the meaning of the digits' columns. As with many of the other children who were receiving remedial instruction, Mary seemed to have forgotten the explanations given in the early years of arithmetic study. After a careful exposition of these processes, there was rarely any recurrence of the error. Distributed practice was used to fix the new habits, once the principle was understood.

Although Mary had a reading age of fifteen years, the arithmetic vocabulary was particularly difficult for her; she could not visualize the situations. Problems of the same types as those that caused trouble were made for her, with daily life incidents to replace those in the text. Advertisements were used for the same purpose.

Often articles in the newspapers gave statistical information that was used in connection with geography tables for purposes of comparison. Thus the estimated wheat and cotton crops for 1937 were likened to the amount produced in other years, and the percent of increase, probable value, percent to be exported, etc., were worked out.

When problems in the text were taken up, the first step was to connect them with similar situations in the problems Mary had formulated; thus a real meaning was given to them. To solve them, Mary stated the terms briefly, making, if possible, a rough graph or illustration; then she estimated the answer, planned the steps in the solution, and performed the operations. All work was checked.

Mary became interested in the development of modern number. She practiced using the Roman numerals and discovered their limitations. The story of the introduction of the zero, its importance, and the effect it had on our number system were discussed. Mary was able to appreciate the relation of the decimal system to our modern business development. The logic of number appealed to her; she was able to see the use of formulae and literal terms, and was eager to take up the study of algebra and geometry. Once she realized the concrete applications, the abstractions of number became of great interest to her.

VI. Result.

Mary's ability to appreciate the higher possibilities of mathematics was her greatest gain. In the final test, she worked slowly, but confidently. She made four errors in forty-seven examples, but missed twelve problems either because of computational errors or inexact copying of figures.

After Mary had been given the second form of the Stanford test, her record showed:

	Arithmetic Age	Arithmetic Grade
March 1	13-1	7.4
June 7	15-4	9.3

TABLE XI

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case XI, Grade VIII

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	46	45	35	34
Number correct	34	43	17	22
Per cent of number correct to number attempted	74.0	95.4	48.5	64.7
Per cent of number correct to entire test	56.6	71.6	42.5	55.0
Per cent of increase in accuracy		28.9		33.3
Per cent of increase in speed		26.5		29.4

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case XII, Adele N.

I. Introductory Statement.

Adele was a tall, lovely girl of thirteen, the only child of rather elderly parents. She lived in a beautiful home on the Lake, and spent the summers traveling with her father and mother in this country or in Europe.

Her only illnesses had been light attacks of measles and chickenpox. Here eye-sight was excellent and she had no physical defects. Her good health enabled her to enjoy all sorts of sports, while a quiet sense of humor and generous disposition made her a favorite among her companions.

II. School History.

Adele entered her present school three years ago, being placed in the fifth grade. Before that time, she had been taught by a governess. She did outstanding work in the English subjects, especially in composition, and showed promise of talent in art and music. In arithmetic, however, she was noticeably retarded. In the seventh grade she had received special tutoring, but without any planned remedial program. In the eighth class a need for special work was evident.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
13-9	16-1	117

Derived scores on the Gates Silent Reading Test and the New

Stanford Achievement Tests in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
14-9	9.0	12-7	6.8

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of computation and errors in the following operations:

In the four fundamental processes --

combinations, borrowing, carrying, choice of trial quotient, use of zero.

In fractions -- lack of skill and knowledge in all the processes.

In decimals -- comprehension of values, use of the decimal point.

In problems -- inexactness in reasoning and computation.

Observation of the work of this pupil showed that she was nervous and discouraged. She had no well-established, orderly habits; she did not estimate or check her work, nor did she attempt any analysis of problems, but seized upon the first process that came to her mind, worked in a maze of uncertainty, and did extraordinary things with decimal points to make the answer "seem right". Lack of self-confidence caused her to use the work of other pupils.

V. Remedial Instruction.

Remedial instruction was planned to accomplish the following purposes: (a) to establish proper work habits, (b) to give a mastery of the fundamental skills, (c) to make an intelligent attack upon problems, and (d) to give an idea of the relation of number to ordinary life situations.

It was suspected that somewhere in the lower grades, Adele had failed to gain a knowledge of the essential abilities and for this reason had fallen into habits of trial and error. Therefore, as was explained to her, the first necessity was to find the "bad spots". She was grateful to be helped and willingly performed aloud the operation of examples given her. She was quick to catch herself when an error was made. Merely calling her attention to faulty and wasteful habits of work enabled her to improve her procedure. It was plain that not her intelligence, but her attention had been at fault. In the drill classes her progress was rapid.

Adele became interested in the evolution of number; her artist's eye appreciated the symbolism of the old Hindu and Arabic forms. When she learned of the introduction of the zero and its importance, the rhythm of the decimal system was opened to her. With this awakening, numbers seemed to fall into their proper place.

Analysis of problems was the other aid that gave Adele ability to deal with number situations. She had never thought of correlating the experiences of travel, such as foreign exchange, distance, mileage, time tables, or latitude and longitude with formal arithmetic situations, but was quick to see their application when it was suggested. This made the interpretation of problems readily understood. Practice was given in rewording, estimating answers, and choosing the proper processes. Adele's maturity of mind gave her a critical sense of logical analysis, so that after four weeks of individual instruction she was able to return to her regular class for problem work. However, it was thought best to have her continue the special drill class in computation, that the correct work habits she had acquired there might be made permanent.

VI. Results.

After Adele had been given the second form of the Stanford Achievement test, her record showed:

	Arithmetic Age	Arithmetic Grade
March 1	12-7	6.8
June 7	16-3	Adult Standing

Adele continued to be interested in the theory of number, in fact, the final test showed more improvement in work with abstract numbers than in problems. Adele is looking forward with zest to high school mathematics.

TABLE XII

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case XII, Grade VIII

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	48	48	40	33
Number correct	26	44	20	24
Per cent of number correct to number attempted	54.1	91.6	50.0	72.7
Per cent of number correct to entire test	43.3	73.3	50.0	60.0
Per cent of increase in accuracy		69.3		45.4
Per cent of increase in speed		69.2		20.0

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case XIII, Sally J.

I. Introductory Statement.

Sally was a leader among her friends, full of fun, generous, and kindly. She was fond of swimming, riding, and tennis, and as she had excellent health, she was able to devote herself to these sports. She also showed talent in music and drawing.

II. School History.

Sally had entered her present school at the age of six. For the first five years she had received excellent grades in all her studies. In the sixth class she had some difficulty with the arithmetic work, which became more pronounced in the seventh.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
13-0	14-3	110

Derived scores on the Gates Silent Reading Test and the New Stanford Achievement Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
13-9	8.0	12-0	6.2

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four fundamentals --

carrying, borrowing, partial products,

choice of trial quotients, combinations, In fractions -- reduction, improper fractions, common denominator.

In decimals -- inexact computations, use of the decimal point.

In problems -- computation and comprehension of process.

Observation of the work of this child showed that she grouped, skipped, repeated operations, and worked too rapidly. There were persistent faults of inaccuracy due to inattention, although the work was neat. She had a good spirit and a desire to improve. The feeling of failure caused a noticeable strain during arithmetic periods.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to reestablish confidence, (b) to give a mastery of the needed skills, (c) to teach better work habits, and (d) to give ability in the every day uses of number.

As the first difficulties in Sally's work in arithmetic had been remarked shortly after the introduction of fractions, it was thought possible that some uncertainty there might be at the root of the trouble. The fifth grade fractions were reviewed and it was found that Sally had failed to grasp the development of the work in common denominator. S he quickly learned the process, and with it the reason for the reduction of fractions. Having had but little practical experience with number situations Sally felt this lack. Her parents were asked to give her opportunities to acquire such knowledge, which they willingly did.

she was taken to the stock exchange, the Planetarium, on shopping tours and other excursions. She was given her allowance by check and was taken to the bank to obtain the money; she also began to keep an account of her expenditures.

Sally brought a variety of problems to school as a result of these experiences. From the practice of estimating costs, she was able to judge more accurately in the textbook problems. Correlation with life problems aided in the interpretation of the statements in the book. At the same time, training in analysis, estimating, and checking corrected the careless, illogical choice of process that had hindered her advance in problem solving.

VI. Results.

The second form of the New Stanford Achievement Test showed the final standing to be:

	Arithmetic Age	Arithmetic Grade
March 1	12-0	6.2
June 7	14-6	8.5

The signs of strain disappeared as Sally gained skill in arithmetic. Her final test contained errors that indicated that she still worked too rapidly, although there was a marked gain in accuracy. Careful guidance during the coming year is recommended.

TABLE XIII

Scores Made on the Two Forms of the New Standard Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case XIII, Grade VII

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	36	45	28	24
Number correct	20	36	15	21
Per cent of number correct to number attempted	55.5	80.0	53.5	87.5
Per cent of number correct to entire test	33.3	60.0	37.5	52.5
Per cent of increase in accuracy		44.1		63.5
Per cent of increase in speed		80.1		40.0

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case XIV, Ned K.

I. Introductory Statement.

Ned was a tall, extremely nervous boy of twelve. His health had always been frail, twice necessitating long periods of rest in bed.

When able to play, the boy was excellent at sports. He enjoyed camping, hiking, and fishing, as well as working in his shop where he made airplanes, boats, radio ear-sets, and other objects popular with boys of his age.

II. School History.

Ned had attended a public school for the first three years. A long illness kept him home the following winter, but at his insistence, he was tutored to keep up in his school work. On his recovery he entered his present school, where he was allowed to try the fifth grade studies, although special aid was necessary in reading. Passing to the sixth grade at the end of the year, remedial instruction in arithmetic was found advisable.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
12-6	13-9	110

Derived scores on the Gates Silent Reading Test and New Stanford Achievement Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
12-2	6.4	11.5	5.6

IV. Diagnostic Test Record

The Brueckner Diagnostic Test revealed difficulties of comprehension and errors in the following operations:

In the four fundamental processes --

borrowing, carrying, use of zero, choice of trial divisor.

In fractions -- comprehension of process.

In decimals -- computation, use of decimal point.

In problems -- computation, inexact reading of problems.

Observation of the work of this pupil showed that he was handicapped by nervous habits such as stopping to repeat an operation, losing his place, counting vocally, and erasing a half-finished example to begin again. He was over-anxious, self-distrustful, and discouraged.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to give habits of deliberation in computation, (b) to give a sense of self-confidence by a mastery of the needed skills, and (c) to give a critical approach to problem solving.

As fractions were a source of much difficulty, it was decided to begin the remedial work with them. An objective approach was made by studying the parts of a gallon, a dollar, and a yard. Geometric figures were cut into fractional parts. To explain improper fractions, paper "pies" were used. They were divided into sixths, then Ned put aside pieces to serve two people, then four, six, seven, etc. Ned said that he had never understood fractions, but that this plan gave him an idea of their meaning. It did seem as if this simple procedure had supplied the necessary connection between concrete and abstract work with fractions, as the boy increased rapidly in skill. The explanations on methods given in the drill class and the supervised practice helped him form proper work habits.

In spite of his work shop, Ned had never connected arithmetic study with real life situations. He was quick to see the relationship when questioned about it, and he made a long list of the uses of arithmetic with problems to illustrate each item.

In problem solving, an attempt was made to correct Ned's overcareful manner. He was allowed to read the problem only once, then asked to repeat it or answer fact questions about it. Sometimes an insoluable problem was placed among the others. At first this was not noticed and he would have great difficulty in trying to find a way to solve it. When he did realize what had been done, he challenged the instructor to catch him again. He also enjoyed puzzle problems. With growing skill in careful reading, comprehension increased, a habit of visualization developed, and problem analysis became a logical process, different from the former trial and error system.

VI. Results.

After Ned had been given the second form of the Stanford Test, his record showed:

	Arithmetic Age	Arithmetic Grade
March 1	11-5	5.6
June 7	12-2	6.3

Ned was recommended for further individual guidance the following year, although it was expected that continued improvement would enable him to take up the regular class work in arithmetic within a few months.

TABLE XIV

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case XIV, Grade VI

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	38	36	20	19
Number correct	17	25	14	17
Per cent of number correct to number attempted	44.7	69.4	70.0	89.4
Per cent of number correct to entire test	28.3	41.6	35.0	42.5
Per cent of increase in accuracy		55.2		27.7
Per cent of increase in speed		47.0		21.4

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case XV, Rose H.

I. Introductory Statement.

Rose was a charming girl, full of fun, thoughtful, and unselfish. Though not robust, she had no serious illnesses or physical defects, but sometimes suffered from nervous fatigue caused by over-anxiety about her duties or school work.

II. School History.

Rose had entered her present school for the first grade. Her progress was steady; her work in reading and the social subjects was especially good. In the seventh grade she ranked among the highest pupils in all subjects except arithmetic. Here she seemed completely at a loss and was recommended for special instruction.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
12-5	13-8	110

Derived scores on the Gates Silent Reading Test and New Stanford Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
15-2	9.3	11-3	5.4

IV, Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four fundamental processes --

use of zero, carrying, choice of trial

quotients.

In fractions -- lack of comprehension of processes of reduction, subtraction, division.

In decimals -- lack of comprehension of value, omission of decimal point, wrong placement of decimal point.

In problems -- confusion of processes, lack of comprehension of problem situations.

Observation of the work of this pupil show that she counted by lips and tapping, thought aloud, worked too hastily with no estimate or check. She was over-eager to do well and was worried and self-conscious because of her failure.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to build up a feeling of self-confidence by giving her a mastery of the needed skills, (b) to decrease the rate of speed and increase accuracy, and (c) to give a knowledge of critical approach to problem solving that would obviate absurd answers.

Rose entered the remedial class gladly, saying she would do anything to improve her arithmetic work. In the daily drill group she was quick to grasp the value of regular habits of work and by attentive practice corrected her old habits of skipping and counting. Her attitude of interest made it easy for her to understand the careful explanations given on the different processes of division and fraction computations.

For individual aid, Rose was given objective work with measures, coins, and weights; she even studied cooking at home to gain experience in using measures. A list of arithmetic situations was kept, from which problems were formulated.

As Rose had trouble in estimating a reasonable answer, she did much work in problem analysis. Among the devices used were reading the problem, then answering fact questions, making graphs or charts of the problem, choosing a reasonable answer out of several suggestions, and giving the reasons for the choice, and criticizing badly stated or faulty problems.

As skill in analysis grew, Rose developed a method of procedure as follows:

1. Read the problem carefully.
2. Form a clear mental picture of the situation.
3. Criticize the problem as to possibility.
4. Estimate the answer.
5. Decide on the necessary processes for solution.
6. Work the problem.
7. Check thinking and computations.

VI. Results.

After Rose had been given the second form of the Stanford Achievement test, her record showed:

	Arithmetic Age	Arithmetic Grade
March 1	11-3	5.4
June 7	13-1	7.4

Rose had lost the tenseness caused by trying to work at a speed beyond her control. she had gained a sense of deliberation and control. Her gains in both accuracy and speed were among the highest of all in the remedial classes.

TABLE XV

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case XV, Grade VII

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	48	38	22	19
Number correct	21	31	12	19
Per cent of number correct to number attempted	43.7	81.5	54.5	100
Per cent of number correct to entire test	35.0	51.6	30.0	47.5
Per cent of increase in accuracy		86.5		83.5
Per cent of increase in speed		47.4		58.3

NOTE: Each form of the computation test contained 60 examples; each form on the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case XVI, Louise S.

I. Introductory Statement.

Louise was a sturdy little girl of eleven, the youngest of five daughters and the favorite of all. The family lived quietly in a large house with ample grounds which made permissible a collection of pets that were the special interest of Louise's life.

The child had a little garden of her own where she could grow whatever she wished. She spent all the time possible out of doors with her hobbies.

II. School History.

Louise had been in attendance at a public school for kindergarten and the first four grades. When the family moved to Chicago, she was enrolled in a private school. She had never been interested in school, although her work was reported as fair or good. Very early in the fifth class, a weakness in arithmetic was remarked.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
11-9	13-4	113

Derived scores on the Gates Silent Reading Test and New Stanford Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
11-6	5.7	10-7	4.6

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four fundamental processes --
combinations, borrowing, use of zero,
entire process of division.

In fractions -- no knowledge of the process; Louise wrote on her paper, "I cannot do this."

In problems -- interpretation of arithmetic situations, choice of process.

Observation of the work of this child showed that she lacked initial skills, made no estimate or check, failed to discern absurd answers, and did not understand problem analysis.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to give proper work habits, (b) to give an understanding of the utility of number in daily life, (c) to give an understanding of the quantitative relations, and (d) to give ability in analysis of problems.

In talking about her difficulties, Louise did not show discouragement, but asked to be helped. Because of her habit of inexactness, a game of "solitaire" was planned where she could play against herself by checking the work for errors. This proved to be an excellent device, as she disliked being beaten in anything. When she began her game, she discovered errors uch as $\begin{array}{r}307\\-192\\\hline 205\end{array}$ and $\begin{array}{r}20\\20\overline{)40}\end{array}$. Although checking disclosed the error, Louise did not know how to rectify it. An explanation of digits' place made this clear; when the processes were understood, the error was not repeated.

Objective materials were used to give a sense of the meaning of fractions. After measures, coins, weights, and geometric figures had been utilized, a real project was developed out of Louise's gardening activities. She planned an old-fashioned, "geometric" garden on the basis of sixteen squares. This was done, for the most part, at home, where she plotted the garden on the lawn, then transferred her ideas to paper. The dimensions of each unit were worked out, the necessary number of plants calculated, and the cost of plants and labor ascertained.

Other life situations gave material for problems. Opportunities to estimate the cost of food for her pets, the amount of seed necessary for the lawn, and other experiences in estimation and checking provided a natural approach to the analysis of problems. Textbook situations that the child could correlate with her own knowledge were chosen for study. Devices to aid in the analysis included restating the problem, making graphs, supplying missing numbers, estimating answers.

VI. Results.

After Louise had been given the second form of the Stanford Achievement, her record showed:

	Arithmetic Age	Arithmetic Grade
March 1	10-7	4.6
June 7	11-8	5.8

A gain of thirteen months had been made and all Louise's teachers remarked a more alert interest in her classes. It was felt that the realization that school and home had many points of contact, that school really helped a child to work out her plans, was the most valuable lesson that Louise had received in the remedial period.

TABLE XVI

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case XVI, Grade V

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	20	27	24	18
Number correct	11	22	13	16
Per cent of number correct to number attempted	55.0	81.5	54.0	88.8
Per cent of number correct to entire test	18.3	36.6	32.5	40.0
Per cent of increase in accuracy		48.1		64.4
Per cent of increase in speed		100		23.0

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case XVII, Janet B.

I. Introductory Statement.

Janet was extremely shy and reserved. From her rather colorless exterior, one would never gues the vivid imagination and alert mind within. Due to her father's profession, the family had no settled home, but moved from city to city as necessary. This circumstance kept Janet from forming real friendships with other children.

A slightly imperfect muscular coordination was noticeable, although Janet's health was good; astigmatism made glasses necessary.

II. School History.

Attendance at school had been irregular because of the frequent changes of residence. Janet was too shy to recite well, but her written exercises and letters were phrased in a quaint, pleasing style that showed the influence of good reading. When Janet entered her present school at the beginning of the sixth year, she was able to do excellent work in all her studies, arithmetic excepted. Here she seemed to have no skill whatever.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
11-8	13-0	111

Derived scores on the Gates Silent Reading Test and New Stanford Achievement Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
16-9	10.9	10-8	4.7

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four fundamental processes --

use of zero, choice of trial quotients, All the work was extremely uneven; one example of a type would be solved correctly, others incorrectly.

In fractions -- no evident comprehension of process.

In decimals -- no evident comprehension of process.

In problems -- meaningless numbers inserted in spaces provided for the answers.

Observation of the work of this pupil showed that she thought aloud, had irregular procedures, made no estimate or check. The entire test was done in such a fantastic way that it was difficult to analyze. All kinds of methods were used to avoid the test -- attempts at conversation, broken pencils, and pleas of inability. In spite of this attitude, the examiner suspected that Janet could do the work in whole numbers if she would; in fractions and decimals the ignorance seemed more sincere, as skills and a sense of values were plainly lacking.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to bring about a better attitude toward arithmetic by correlating it with subjects in which Janet was interested, (b) to supply needed skills, and (c) to give good habits of work.

The child disliked the remedial class, that is, the portion of it that was spent on arithmetic. She loved to talk about her travels, to discuss books, or tell about her pets, but resisted every effort to interest her in numbers. Finally she was told that each day she would be given a certain task within her ability, and that it must be done. The first assignment was in column addition. Janet began by bridging the tens and grouping; losing her place caused her to count on her fingers. The instructor had her try adding aloud, going directly up the column. She was able to do this and seemed to know the combinations quite well. This exercise was timed. The following day it was repeated, Janet being challenged to better her former record. For a time the instructor used a pencil to lead her eyes up the column, then Janet offered to keep the place with her own pencil.

Realizing that the work must be done, the child applied herself to do it as quickly as possible. One day the instructor asked her to make up a story about the numbers in an example. Immediately she said, "It is 1004 miles to Brownsville and back. When Daddy had driven 938 miles he ran out of gas. How far had he to walk?"

In learning any process, Janet was helped by having the number facts presented in concrete form; 975 divided by 25 meant nothing to her, but a problem made from the abstract numbers attracted her attention at once. After fractions had been introduced by means of objective measures such as pints and quarts, feet and yards, and parts of a dollar, she seemed to attain an understanding of the relation of parts to a whole.

One day Janet confessed that she had always disliked arithmetic because she could see no sense in it, but that now that she understood it, she liked it. From that day her improvement in abstract drill was constant.

Practice in analyzing problems became another pleasure. Her extensive reading had given the child a knowledge of many of the situations in the text book, while the use of statistical tables, maps, foreign currency, and newspaper items containing arithmetical information widened the problem field. Janet was interested in the story of the evolution of number; an account of the telescope and its uses thrilled her, and the minute measurements made possible through the magnifying glass opened new aspects of number to her. Before the remedial period was over, she had acquired a growing interest in the science of number, and also an appreciation of the enjoyment that work with number gives.

VI. Results.

After Janet had been given the second form of the Stanford Achievement Test in Computation and Arithmetic Reasoning, her record showed the following improvement:

	Arithmetic Age	Arithmetic Grade
March 1	10-8	4.7
June 7	12-8	6.9

Janet had gained two years in arithmetic age and was able to enter the seventh grade with no arithmetic handicap. Unfortunately, the family moved to the East during the summer and it was impossible to follow Janet's further progress.

TABLE XVII

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case XVII, Grade VI

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	33	41	25	20
Number correct	20	30	9	18
Per cent of number correct to number attempted	60.6	73.1	36.0	90.0
Per cent of number correct to entire test	33.3	50.0	36.0	90.0
Per cent of increase in accuracy		20.6		150
Per cent of increase in speed		50.0		100

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case XVIII, Edith D.

I. Introductory Statement.

Edith was a frail child of eleven. Her health had been weakened by a severe attack of pneumonia when she was three; she had a susceptibility to colds and throat infection. Due to her continued illnesses, she had received much care, which left her with a feeling of dependence and inability to do things for herself. She had a quick, nervous temperament, relieved by a strong sense of humor.

II. School History.

As Edith was taken to Florida during the winter of each year, her school life was very broken. She entered her present school at the age of eight and was placed in the third grade; continued absences caused a repetition of this class. In the fourth grade, Edith was given special remedial instruction in reading. In the fifth she was recommended for the arithmetic remedial class.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
11-8	12-10	110

Derived scores on the Gates Silent Reading Test and New Stanford Achievement Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
11-8	5.9	10-7	4.6

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties in computation and errors in the following operations:

In the four fundamental processes --

combinations, carrying, borrowing, zero,
lack of fundamental skills, incomplete work.

In fractions -- common denominator, reduction, subtraction.

In problems -- computational errors.

Observation of the work of this child showed that she counted on her fingers, repeated operations, skipped, worked hastily and inexactly. She was discouraged and wept over her inability to do well.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to give a detailed guidance in the learning of the fundamental processes, (b) to give confidence through the mastery of needed skills, and (c) to teach habits of careful, deliberate attack upon problems.

During the first remedial lesson, Edith tearfully told the instructor that her father said she could not learn arithmetic, and that she had never been able to do so. The reply was that the tests she had taken showed that she not only could, but had learned quite an amount of arithmetic. She was shown her Brueckner Test paper in whole numbers and allowed to correct all the errors that she could. When one was found that she could not solve, it was put on a list to be checked off when she had developed the necessary skill. A long list faced her at the end of the period. It was explained that the cause of many errors was a zero, and that every day for a week she would learn what to do with zero. Followed by a concise explanation of the meaning and use of this symbol, Edith was given drill examples to give skill in each process when it occurred. She learned so quickly that after the fourth day she was placed in a group of four to continue study and drill.

In problem analysis, Edith showed an unusual maturity of judgment. She had no difficulty in interpreting two or even three-step problems. Training was given in estimating and checking, to overcome the errors caused by hasty, thoughtless attack; a record of reasonable estimates verified by calculation was kept. After four weeks individual work, she was able to return to her regular class, but remained in the special drill group for twelve weeks, to establish proper work habits.

VI. Results.

It was a pleasure to notice the change in Edith's attitude toward arithmetic and her increase in self-confidence. The ability she had acquired seemed to give her greater zest in attacking all her work, as she was reported as showing more interest in other classes.

The second form of the Stanford Achievement recorded her improvement:

	Arithmetic Age	Arithmetic Grade
March 1	10-7	4.6
June 7	12-0	6.2

TABLE XVIII

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case XVIII, Grade V

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	30	24	11	15
Number correct	18	28	10	15
Per cent of number correct to number attempted	60.0	82.3	90.9	100
Per cent of number correct to entire test	30.0	46.6	25.0	37.5
Per cent of increase in accuracy		37.1		9.9
Per cent of increase in speed		55.3		50.0

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case XIX, Kate N.

I. Introductory Statement.

Kate was a quiet, serious little girl of eleven, the only child of cultured, well-to-do parents. In the winters, the family lived in an apartment; in the summers, they usually traveled through the East.

The child had no disease history except minor childish ailments and no physical defects. She enjoyed reading, especially Stevenson and Dickens. She studied the piano and went to the Saturday classes at the Art Institute. Her unselfish kindness to her schoolmates endeared her to them.

II. School History.

Entering her present school three years ago, Kate was placed in the third grade. Her work was very good in the social studies and passable in arithmetic until in the fifth class, when a growing difficulty in her arithmetic exercises became noticeable.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
11-7	12-10	110

Derived scores on the Gates Silent Reading Test and New Stanford Achievement Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
14-5	8.7	10-7	4.6

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties of comprehension and errors in the following operations:

In the four fundamental processes --

borrowing, carrying, use of zero.

In fractions -- lack of comprehension of process.

In problems -- lack of ability to analyze.

Observation of the work of this pupil showed that she made many errors through poor work habits. Her mind was alertly eager to grasp the meaning of arithmetic and the proper methods of operation, but lack of success discouraged her.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to give a mastery of the necessary skills, (b) to give an appreciation of number situations in daily life, and (c) to give an understanding of problem analysis.

A high degree of accuracy in the easier combinations and the exquisite neatness of her papers made it possible to convince Kate that hers was not the hopeless case she thought it. Daily explanations and practice drills showed their value in correcting the wrong processes in zero combinations and fractions.

The problem solving skills were not acquired so easily. Her mature tastes in reading tempted the instructor to give her a glimpse of the wider meaning of arithmetic as a helpful background for the interpretation of number situations. The story of the development of our modern number system was sketched, during which Kate was very interested in the evolution of the forms of the Arabic numerals. When she was asked to use the Roman numerals to solve a problem, she was much amused, and remarked on the trouble bankers would have with such a system.

Kate enjoyed keeping a record of her daily number experiences. She figured distances, compared routes, kept an account of her expenditures, and planned the cost of a trip to Florida. In solving the original problems, there was far less tendency to give absurd answers than in working those in the textbook. When the book situations came, care was taken to show the similarity to some actual experience.

Among the devices used for problem solving were: problems with missing numbers, verbal problems, and problems with absurd or impossible statements. Answers were always estimated and checked.

VI. Results.

After Kate had been given the second form of the Stanford test, her record showed:

	Arithmetic Age	Arithmetic Grade
March 1	10-7	4.6
June 7	12-4	6.5

A gain of two years had been made, there was much more confidence shown, and a real interest in arithmetic was evident.

TABLE XIX

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case XIX, Grade V

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	30	32	20	25
Number correct	20	25	9	29
Per cent of number correct to number attempted	66.6	78.1	45.0	76.0
Per cent of number correct to entire test	33.3	41.6	22.5	47.5
Per cent of increase in accuracy		17.1		68.8
Per cent of increase in speed		24.9		111

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case XX, Dora H.

I. Introductory Statement.

Dora was a jolly child of eleven. The youngest of several children, she managed the brothers and sisters near her own age with a firm hand, thus relieving her mother, who was not strong.

In the winter, the family lived in a large apartment; in the summer, they went to their home on the Lake where they passed the time swimming, playing tennis, taking hikes, and entertaining the many friends for whom they kept open house.

II. School History.

Dora had entered her present school for the first grade. She had learned to read easily and seemed to have no difficulty in any of her studies until the fifth grade, when she was recommended for special instruction in arithmetic.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were:

Chronological Age	Mental Age	Intelligence Quotient
11-7	13-10	120

Derived scores on the Gates Silent Reading Test and New Stanford Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
11-8	5.9	10-6	4.5

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties in computation and errors in the following operations:

In the four fundamental processes -- combinations, carrying, borrowing, wrong process in division.

In fractions -- no comprehension of process.

In problems -- a nervous, uncertain procedure with no analysis.

Observation of the work of this pupil showed that she counted with lips and fingers, worked aloud, skipped and repeated operations, She was inaccurate and dependent upon those near her for cues. Her manner was tense and hurried.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to correct faulty processes, (b) to develop good work habits, and (c) to give an understanding of the uses of arithmetic.

Dora had worked a division example thus: $\begin{array}{r} 5 \\ 5\overline{)\,254} \end{array}$

and a fraction example calling for subtraction thus:

$$\begin{array}{l} 2/3 = 1/3 \\ \underline{1/3 = 0/3} \\ \qquad\quad /3 \end{array}$$

When asked to correct the first example, she said, "I know better than that", and solved it without error; for the fraction, she had no idea what to do. She was allowed to correct her mistakes on the Brueckner Test in whole numbers, as an object lesson on the value of accuracy. In the drill group where she was placed, careful explanations and directed drill soon eliminated much of her inexact, over-hasty manner of computation. She was given special aid in the fraction processes, when objective measures and coins were used to teach fractional parts.

In problem solving, Dora's idea seemed to be to put down a number in the space left for the answer, then go to the next problem and do the same. A variety of devices was employed to teach her how to interpret the problem situation. She read the problem aloud, answered questions, and estimated answers. After the estimate was made, Dora was faced with the task of choosing the process necessary to check it. At first, any process was named at random, as she did not take time to think what the outcome would be. To correct this lack of consideration Dora learned to say, "I am asked to find how many times as much gas is needed for 135 miles than for 15 miles. The answer will be smaller than 135. I would not subtract because that would not give a sensible answer. I shall divide 135 by 15. . . Seven times as much is needed. Seven times 15 are 135."

As with the other remedial cases, much work in life problems was given, interesting situations were developed, and geography graphs and road maps utilized.

VI. Results.

After Dora had been given the second form of the Stanford test, her record showed:

	Arithmetic Age	Arithmetic Grade
March 1	10-6	4.5
June 7	11-8	5.8

The test showed that further aid in fractional processes was desirable. This was given the following year, at the end of which Dora achieved an arithmetic age of thirteen years, six months. A recent check on her progress shows that she continues to gain interest and skill.

TABLE XX

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case XX, Grade V

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	44	32	29	21
Number correct	17	23	9	15
Per cent of number correct to number attempted	38.6	71.9	31.0	71.4
Per cent of number correct to entire test	28.3	38.3	22.5	37.5
Per cent of increase in accuracy		86.2		130
Per cent of increase in speed		35.3		66.0

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case XXI, Elinor R.

I. Introductory Statement.

Elinor was ten years old, tall for her age, with a very serious manner that reminded one of the girls of our grandmother's childhood.

She came from a cultured home, where she and two younger sisters received the best of care. She had no disease history, but wore glasses because of myopia.

II. School History.

After having spent the first four grades in a public school, Elinor had been placed for her fifth year in a private school. Her reports showed that she had done consistently good work. A marked weakness in arithmetic was noticed before she had been long in the fifth grade.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
10-10	11-11	110

Derived scores on the Gates Silent Reading Test and New Stanford Achievement Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
11-8	5.9	9-7	3.8

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties in comprehension and errors in the following operations:

In the four fundamental processes --
combinations, carrying, borrowing, partial products, trial divisor, remainder, use of zero.

In fractions -- lack of comprehension of the process.

In problems -- computational skills.

Observation of the work of this child showed that she was slow but exact, always doing her best; she was eager to succeed, but felt the lack of ability. She showed strain by an over-anxious manner in the tests. In problem solving, she had ability to think clearly, but failed to check or estimate. Computational difficulties prevented correct solution of problems.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to relieve the strain by giving a mastery of the fundamental skills, (b) to give a clear understanding of the fractional processes.

Elinor was eager for the promised help and interested in number processes. She said she had always wanted to know how to get them right because she liked arithmetic.

One of the first difficulties attacked was subtracting with zeros in the minuend. The instructor told a simple story about a child who needed three cents, but had only one, so she borrowed a dime from her mother, which gave her eleven cents in all. She could then spend three cents and have some money left over. Elinor enjoyed the story, which was applied to dollars also, to enable her to grasp the value of tens' and hundreds' places. A similar story was used for division errors such as $\begin{array}{r} 22 \\ 22\overline{)44} \end{array}$. Elinor was asked how many cakes each of twenty-two girls could take at a party where forty-four cakes were served. Checking the answers by multiplication helped her understand the multiplication necessary in division examples.

In Elinor's work, it was found that the use of problems aided in teaching abstract computations, so a great deal of objective material was given her; some of the projects used were planning a party, making shopping lists, figuring distances, keeping a bird calendar, making graphs, and keeping a comparative record of class scores.

VI. Results.

After the second form of the Stanford test had been given to Elinor, her record showed:

	Arithmetic Age	Arithmetic Grade
March 1	9-7	3.8
June 7	11-0	5.1

A gain of seventeen months in arithmetic age had been made, although Elinor was still below her grade level. She had improved in arithmetical skills and in speed, but she still betrayed an over-anxiety that seemed to indicate a lack of confidence in her computational operations. It is planned to continue the remedial instruction during the coming year.

TABLE XXI

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	36	32	14	19
Number correct	10	19	7	12
Per cent of number correct to number attempted	27.7	59.4	50.0	63.1
Per cent of number correct to entire test	16.6	31.6	17.5	30.0
Per cent of increase in accuracy		114		26.0
Per cent of increase in speed		90.3		71.4

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

Case XXII, Ann H.

I. Introductory Statement.

Ann was a clever, self-contained little girl, but thoughtful and kindly. Her parents had a pleasant home where they lived quietly, preferring books, music, and simple hospitality to a more active social life.

Although Ann had no serious disease history, she had never been robust. She was her mother's companion and did not play much with other children. Her main interests were reading, sewing, and keeping house.

II. School History.

After attending a school near her home for the first four grades, Ann entered a private school for girls at the beginning of her fifth year. She was ambitious to get good reports in her studies and her work in the English classes was excellent, but it was soon evident that she was seriously handicapped in arithmetic.

III. Intelligence and Achievement Record.

The results of the Stanford Revision of the Binet-Simon Test were as follows:

Chronological Age	Mental Age	Intelligence Quotient
10-10	12-0	111

Derived scores on the Gates Silent Reading Test and New Stanford Achievement Test in Computation and Arithmetic Reasoning were:

Reading Age	Reading Grade	Arithmetic Age	Arithmetic Grade
11-8	5.9	9-10	4.0

IV. Diagnostic Test Record.

The Brueckner Diagnostic Tests revealed difficulties in comprehension and errors in the following operations:

In the four fundamental processes -- combinations, borrowing, position of partial product, choice of trial quotients, use of zero.

In fractions -- comprehension of the processes of reduction, subtraction, and division.

In problems -- interpretation of statement of the problem, comprehension of process.

Observation of the work of this pupil showed that she had no established work habits, but used irregular procedures such as counting, skipping, repeating operations, and guessing. There was no attempt to estimate answers or check work. The child lacked the skills necessary to perform the operations of multiplication and division. She was discouraged and tearful.

V. Remedial Instruction.

Remedial instruction was planned with the following aims in view: (a) to give confidence to the child by showing her the correct procedures that would ensure accuracy, (b) to give a mastery of the needed skills, and (c) to develop an interest in arithmetic by pointing out the every-day uses of number.

Ann was not put into the drill class for a few days; instead, the processes of subtraction with zero in the minuend and of multiplication by more than one digit were explained to her. As she had been taught subtraction by the equal additions method, which had proved very confusing, it was decided to use the decomposition plan in the remedial work. A story was used to explain the borrowing process, and at first all the examples were stated in terms of dollars and cents. In this way it was easy to "borrow a dime" or "borrow a dollar". Later the decimal point was omitted and Ann was taught how to borrow tens and hundreds.

Ann habitually forgot to set the second partial product over, even though she realized that she was multiplying by tens. To help her remember, vertical lines were drawn, separating the units, tens, and hundreds. In two digit division, it was also necessary to use a device. Ann's method was unique -- $\begin{array}{r} 42 \\ 12\overline{)48} \end{array}$. She divided the 4 by the 1 and the 8 by the 4. To correct this, she was taught to say, "Twelve into four won't go," and to place a zero over the 4. Then she would divide the 48 by the 12 and obtain the correct answer.

When Ann was put into the drill group, she enjoyed the work and soon gained first place. She had little trouble in improving her score in computations with fractions and rarely made a mistake once a process had been carefully explained.

Five weeks of individual help were given for the study of number situations and the uses of number. Maps, objective measures, graphs, and advertisements were among the materials used for this. At the end of twenty-five lessons, Ann's improvement was so marked that the individual aid was discontinued; the corrective drill work was kept up for twelve weeks.

VI. Results.

After Ann had been given the second form of the Stanford test, her record showed:

	Arithmetic Age	Arithmetic Grade
March 1	9-10	4.0
June 7	11-9	5.9

Ann had gained two years in arithmetic age and had come up to her grade level in arithmetic. Her enjoyment in her improvement more than repaid the special attention given to her. At the end of the following year, a repetition of the Stanford test showed her to have reached the eighth grade level in arithmetic.

TABLE XXII

Scores Made on the Two Forms of the New Stanford Achievement Test in Computation and Arithmetic Reasoning with Percentages of Improvement
Case XXII, Grade V

	Computation		Reasoning	
	Form 1	Form 2	Form 1	Form 2
Number attempted	29	38	16	15
Number correct	15	30	8	12
Per cent of number correct to number attempted	51.7	79.0	50.0	80.0
Per cent of number correct to entire test	51.7	79.0	50.0	80.0
Per cent of number correct to entire test	25.0	50.0	20.0	30.0
Per cent of increase in accuracy		53.0		60.0
Per cent of increase in speed		100		50.0

NOTE: Each form of the computation test contained 60 examples; each form of the reasoning test contained 40 problems. The first form was given March 1, 1937; the second form was given June 7, 1937.

CHAPTER V

FINDINGS

A. Discussion of the Findings

During the course of this study, certain characteristics and attitudes were found to be exhibited by the children under observation as evidences of disturbances caused by arithmetic failure. A sympathetic investigation into the underlying causes of the failure enabled the examiner to formulate certain remedial measures. The first need was to overcome the dislike or even dread of arithmetic expressed by the children. It was also necessary to replace the sense of failure by a more hopeful attitude, and to give better motivation to the study of arithmetic. Remedial exercises in computation and reasoning were needed, to improve the arithmetic ability of the Children. Each of these aspects of the investigation will be discussed in detail.

1. Attitudes Observable in Children Failing in Arithmetic

The expression of attitude differed with each child, but a feeling of discouragement and a lack of self-confidence were observable in all the subjects. This expression varied from a tendency toward resentment in a child who possessed an impatient, aggressive temperament to a feeling of quiet dispair in a child who had taken her failure as an indication of stupidity.

Among the most frequently observed manifestations were:

1. An excitability tending to nervous strain during arithmetic class, accompanied, at times, by a tendency to stutter.

2. A more or less openly expressed dislike of arithmetic, with the accompanying desire to avoid it, or to satisfy the instructor with many computations, regardless of accuracy.

3. A sulkiness at the approach of the arithmetic class.

4. A lack of interest, evidenced by yawning or restlessness.

5. A feeling of discouragement, expressed by such statements as, "I'm one of the dumb ones" ... "I never could get arithmetic" ..."My father says I can't learn arithmetic" .... "I don't know why I make so many mistakes." In some cases, the children wept because of their lack of success.

2. <u>Summary of Diagnostic Findings</u>

a. Evidences of Difficulty

An examination of the scores made by the twenty-two subjects who took the diagnostic tests gives evidence of the seriousness of the difficulties. In Table XXIII are shown the chronological age, the mental age as ascertained by the Stanford Revision of Binet-Simon, and the arithmetic age as shown by the New Stanford Achievement Test. The first eight cases listed are of pupils having a mental age one year or more below their

TABLE XXIII

Chronological, Mental , and Arithmetic Ages of Twenty-two Cases at Time of Diagnosis

Case	Chronological Age, March 1	Mental Age March 1	Arithmetic Age March 1
1	14-4	13-0	12-7
2	14-2	12-0	13-0
3	14-1	12-8	12-0
4	14-0	12-6	12-4
5	13-6	11-11	11-4
6	13-4	11-7	11-6
7	12-3	10-9	10-9
8	11-8	10-0	9-11
9	14-6	15-11	13-5
10	14-5	16-6	13-1
11	14-0	15-8	13-1
12	13-9	16-1	12-7
13	13-0	14-3	12-0
14	12-6	13-7	11-5
15	12-5	13-7	11-3
16	11-9	13-4	10-7
17	11-8	12-10	10-8
18	11-7	12-8	10-7
19	11-7	12-10	10-7
20	11-6	13-10	10-6
21	10-10	11-10	9-7
22	10-10	12-0	9-10

chronological age. It will be seen that in seven of the cases the arithmetic age goes still lower. Case II is an exception, having an arithmetic age one year in advance of his mental age. Among the children whose mental age exceeds their chronological age, the difference between the mental age and the arithmetic age is extremely noticeable. In all cases there is a varience of at least two years, while in several, a difference of three or more years is found.

Table XXIV shows the evidences of difficulty manifested by the children in their regular classroom work in arithmetic. The list was compiled by the arithmetic teachers, then amplified by theexaminer during the diagnostic testing period.

b. Computational Difficulties

In Tables XXV, XXVI, and XXVII will be found the number of children making errors in whole numbers, fractions, and decimals. on the Brueckner Diagnostic Tests. Weakness in the combinations was found to be an ontstanding source of error in operations with whole numbers. In all four processes, the zero caused much difficulty. Mistakes in carrying and borrowing indicate both incomplete learning and inattention. Failure to judge correctly in choosing trial quotients in long division is of frequent occurrence.

The results of the fifth grade test in fractions are not included, as they evidenced a lack of comprehension in all fractional computations. Out of the fourteen children whose work is

TABLE XXIV

Evidences of Difficulty Shown by Twenty-Two Children in Grades V, VI, VII, and VIII During Arithmetic Diagnosis

Evidence of Difficulty	Number of Cases
Thinking aloud	6
Counting by fingers, lip movement, or tapping	15
Using irregular procedure -- grouping, skipping, losing the place, repeating	21
Making unnecessary movements of the arms, legs, or head	8
Working too hastily	12
Working untidily	5
Failing to estimate answer	19
Failing to check work	18
Showing impatience during arithmetic computations	7
Showing worry	10
Showing lack of interest	10
Showing lack of self-confidence	15
Showing discouragement	8
Copying from others	7

TABLE XXV

Number of Children Making Various Errors in the Four Fundamental Processes

Addition		Subtraction	
Weakness in combinations	14	Weakness in combinations	15
Bridging tne tens	4	Zero difficulties	
		In minuend	6
Zero difficulties	6	In subtrahend	4
Breaking up combinations	16	Borrowing difficulties	
		Does not allow for borrowing	15
Carrying difficulties		Gives zero instead	6
Forgets to carry	10		
Adds carried number irregularly	4	Does not reduce minuend after borrowing	6
Carries wrong number	9	Borrows unnecessarily	8
Carries unnecessarily	1	Deducts 2 from minuend	5
Omitting digits	1	Using wrong process	2
Beginning at left-hand column	2	Subtracts minuend from subtrahend	5

Multiplication		Division	
Weakness in combinations	20	Weakness in combinations	17
Zero difficulties	11	Difficulties with remainder	15
Carrying difficulties	11	Zero difficulties	19
Errors in addition	12	Errors in choice of trial quotient	15
Omitting digits	10	Using digits of divisor separately	6
Errors in position of partial product	6	Errors in subtraction	6
Incomplete operation	2	Incomplete operation	10

TABLE XXVI

The Number of Children from a Group of Fourteen in Grades VI, VII, and VIII Making Various Errors in Fractions

Addition		Subtraction	
Comprehension of process	3	Comprehension of Process	8
Adding numerators and denominators	1	Borrowing unnecessarily	5
Failing to change to common denominator	1	Prefixing borrowed number to numerator	5
Failing to change to lowers terms	10	Disregarding fraction in minuend	10
Failing to change improper fraction to mixed number	10	Subtracting minuend from subtrahend	6
Denominator divided by numerator	4	Denominator divided by numerator	4
Computation errors	7	Computation errors	8
Incomplete operation	9	Omitting whole number in answer	4

Multiplication		Division	
Comprehension of process	3	Comprehension of process	7
Computation errors	9	Computation errors	9
Inverting multiplier	6	Wrong process	6
Denominator not expressed in product	3	Multiplying numerators, adding denominators	1
Failing to reduce to lowest terms	10	Failing to reduce to lowest terms	10
Numerator only multiplied	2	Inverting dividend	2
Dividing denominator by numerator	2	Dividing denominator by numerator	2
Error in changing to improper fraction	2	Failure to change to mixed number	9
Cancellation	8	Cancellation	7
Failing to changeto mixed number	9	Error in changing to improper fraction	5

TABLE XXVII

The Number of Children from a Group of Fourteen in Grades VI, VII, and VIII Making Various Errors in Decimals

General Information		Addition		Subtraction	
Comprehension of values	10	Addition errors	8	Subtraction errors	4
Expressing decimals in words	8	Placing decimal point	8	Misplacing decimal point	8
Reading and writing decimals	8	Adding common fraction to decimal	8	Omitting decimal point	4
Writing decimal as common fraction	11	Misplacing whole number	4	Omitting zero	4
Changing common fraction to decimal	8	Showing lack of comprehension	7	Failing to allow for zero not expressed in minuend	8
Lack of fundamental knowledge	12			Showing lack of comprehension	5

Multiplication		Division	
Multiplication errors	6	Division errors	5
Misplacing decimal point	7	Misplacing decimal point	8
Omitting decimal point	6	Omitting decimal point	7
Failing to prefix zero in product	4	Failing to prefix zero in quotient	7
Inability to write answer in form	4	Failing to add zero to dividend	7
Prefixing unnecessary zero in product	2	Unnecessary zero in quotient	4
Inability to multiply common fraction and decimal	8	Remainder not expressed as decimal	8
Showing lack of comprehension	5	Showing lack of comprehension	5

recorded in Table XXVI, ten habitually failed to reduce fractions to lowest terms. Other errors frequently noted are computational mistakes and faulty reductions of mixed numbers; the use of the wrong process is quite rare. While six children inverted the multiplier, only one pupil inverted the dividend; in all four fundamental processes some children divided the denominator by the numerator. A few pupils constantly used individual methods of work that betrayed a lack of comprehension of proper operations.

A general lack of comprehension of values and of the use of the decimal point was evident in the test on decimals, as recorded in Table XXVII. Five children showed an ignorance of all four fundamental processes, seven habitually misplaced the decimal point, while the use of the zero was a source of many errors.

c. Difficulties Evidenced in Problem Solving.

In Table XXVIII will be found the mental, arithmetic, and problem-solving ages of the children. It will be observed that the problem-solving age is higher than the arithmetic age. In eleven cases the difference is only a few months, but in six instances it exceeds a year. It is interesting to observe that among the cases having a low I. Q., there is a much closer approximation to the mental age than in those having a high I. Q. There the arithmetic age is sometimes between three and four years lower than the mental age.

TABLE XXVIII

Mental Age, Arithmetic Age, and Problem-Solving Age of Twenty-Two Children, March 1

Case	Mental Age	Arithmetic Age	Problem-Solving Age
1	13-0	12-7	13-1
2	12-0	13-0	14-8
3	12-8	12-0	13-5
4	12-6	12-4	13-5
5	11-11	11-4	11-7
6	11-7	11-6	11-7
7	10-9	10-9	10-6
8	10-0	9-11	9-11
9	15-11	13-5	13-9
10	16-6	13-1	13-5
11	15-8	13-1	13-1
12	16-1	12-7	14-1
13	14-3	12-0	12-7
14	13-7	11-5	12-3
15	13-7	11-3	11-7
16	13-4	10-7	11-11
17	12-10	10-8	11-11
18	12-8	10-7	11-1
19	12-10	10-7	10-9
20	13-10	10-6	10-9
21	11-10	9-7	9-11
22	12-0	9-10	10-6

In Table XXIX are tabulated the arithmetical causes of wrong answers in the first form of the New Stanford Achievement Test in Arithmetic Reasoning. These items were obtained by asking each subject to explain the method of solution on the problems that he had worked incorrectly. Under the heading "Wrong Process" are grouped all the methods of work showing ignorance of correct procedure, lack of comprehension of the terms of the problem, and guessing. When the process chosen was incorrect, computational errors were not considered.

In problem solving, faulty reasoning or the lack of comprehension of the number situation seemed to be the main source of error; the outstanding difficulties were:

1. Lack of intelligent understanding of the number situation expressed in the problem, shown by absurd answers.

2. Inability to see the relationships expressed in the problem, evidenced by haphazard manipulation of any numbers appearing in the problem.

3. Inability to restate the problem or to answer fact questions about it.

4. Ignorance of quantitative relations.

5. Lack of experience with many of the situations given in the problems, with consequent inability to judge the reasonableness of the answer.

6. Lack of the use of visual imagination to interpret the problem situations.

7. Presence of computational errors.

TABLE XXIX

Arithmetical Causes of Wrong Answers in New Stanford Achievement Test in Arithmetic Reasoning

Problem Number	Computation Error	Comprehension of Computation	Wrong Process	Incomplete
1	1		1	
2				
3	1			
4			1	
5	2		5	
6	2		2	
7			2	
8			5	
9	3		2	
10	4			
11	4		7	
12			9	2
13			6	
14	1			
15		5	4	
16	3		4	
17	2	3	6	
18	13	11	3	
19		1	8	1
20	2	1	11	
21	2		6	
22			11	
23			4	
24			7	3
25			8	
26	2		9	
27			9	
28	1		12	
29	2		7	
30			6	
31			5	4
32			1	
33			3	1
34			1	
35			1	
36			2	
37			1	
38			1	
39			2	
40			1	

8. Untidiness of arrangement.

d. Relation Between Reading Ability and Problem Solving Ability

A comparison of the problem-solving age of the children after the first form of the New Stanford Achievement Test was given, with the reading age as shown by the Gates Silent Reading Tests is shown in Table XXX. There it may be seen that in four cases the problem-solving age was more than one month above the reading age. In two of these cases the reading age was very low. In sixteen cases, the problem-solving age was below the reading age, while in two cases the difference in ages was one month or less. The scores for the second form of the New Stanford Achievement Tests, administered after twelve weeks of remedial instruction, show a much closer approach to the reading age; in fourteen instances, the problem-solving age rose above the reading age.

3. Results of Remedial Instruction

a. Improvement Made by Children of High and of Low I. Q.

Table XXXI gives the mental and arithmetic ages of the subjects. Of these, fourteen had mental ages of a year or more above their chronological ages. All of this group made noticeable gains with the exception of one boy, whose failure was due rather to contributing causes than to inability to do the work. Among the other children in this group, the least gain was made by an eighth grader whose mental age is two years and four months

TABLE XXX

Mental Age, Reading Age, and Problem-Solving Age, March 1, and Problem-Solving Age, June 7, of Twenty-Two Children

Case	Mental Age March 1	Reading Age March 1	Problem-Solving Age, March 1	Problem-Solving Age, June 7
1	13-0	13-8	13-1	15-4
2	12-0	11-0	14-8	14-8
3	12-8	13-5	13-5	13-7
4	12-6	13-9	13-5	14-1
5	11-11	12-0	11-7	12-2
6	11-7	12-6	11-7	12-10
7	10-9	11-6	10-6	11-3
8	10-0	9-4	9-11	9-11
9	15-11	14-5	13-9	15-0
10	16-6	15-5	13-5	12-10
11	15-8	15-1	13-1	15-0
12	16-1	14-9	14-1	15-8
13	14-3	13-9	12-7	14-8
14	13-7	12-2	12-3	12-10
15	13-7	15-2	11-7	13-7
16	13-4	11-6	11-11	12-4
17	12-10	16-9	11-11	12-10
18	12-8	11-8	11-1	13-7
19	12-10	15-5	10-9	13-7
20	13-10	11-8	10-9	12-2
21	11-10	11-8	9-11	11-3
22	12-0	11-8	10-6	11-3

TABLE XXXI

Chronological Age, Mental Age, and Arithmetic Age, March 1, and Arithmetic Age, June 7, of Twenty-Two Children

Case	Chronological Age, March 1	Mental Age March 1	Arithmetic Age, March 1	Arithmetic Age, June7	Gain
1	14-4	13-0	12-7	14-8	2-1
2	14-2	12- 0	13-0	12-9	-0-3
3	14-1	12-8	12-0	12-10	0-10
4	14-0	12-6	12-4	13-11	1-7
5	13-6	11-11	11-4	11-10	0-6
6	13-4	11-7	11-6	12-2	0-8
7	12-3	10-9	10-9	11-3	0-6
8	11-8	10-0	9-11	9-11	0-0
9	14-6	15-11	13-5	15-11	2-6
10	14-5	16-6	13-1	12-6	-0-7
11	14-0	15-8	13-1	15-4	2-3
12	13-9	16-1	12-7	16-3	3-8
13	13-0	14-3	12-0	14-6	2-6
14	12-6	13-7	11-5	12-2	0-9
15	12-5	13-7	11-3	13-1	1-10
16	11-9	13-4	10-7	11-8	1-1
17	11-8	12-10	10-8	12-8	2-0
18	11-7	12-8	10-7	12-0	1-5
19	11-7	12-10	10-7	12-4	1-9
20	11-6	13-10	10-6	11-8	1-2
21	10-10	11-10	9-7	11-0	1-5
22	10-10	12-0	9-10	11-9	1-11

above her chronological age. Her arithmetic age rose nine months during the remedial instruction period. Four pupils of the sixth, seventh, and eighth grades gained two years or more; fifth grade children advanced from one to two years.

The two greatest gains of the group a year or more retarded in mental age were made by two eighth grade girls who raised their arithmetic ages one year seven months and one year eleven months' growth. The child with the lowest I. Q. made no improvement on the final test scores, and one boy showed a retrogression of three months through failure to complete the test.

During the remedial period, it was observed that the children under age mentally needed much guidance in applying experiences gained in life situations to similar situations found in their textbooks. Although a number of these children knew the number combinations, they did not apply their knowledge in the solution of problems. The guidance given in the remedial classes was centered around the interpretation of arithmetic situations and the application of acquired skills to new situations. This training seems to have been effective, as the scores gained on the second form of the problem solving test are markedly higher than those made on the first test. Table XXXII shows the initial and final scores made by the twenty-two subjects on the New Stanford Achievement Test in Arithmetic Reasoning.

b. Comparison of Gains Made in Computation and in Problem Solving

A study of Table XXXIII will show that in most cases pro-

TABLE XXXII

Chronological Age, Mental Age, and Problem-Solving Age, March 1, and Problem-Solving Age, June 7, of Twenty-Two Children

Case	Chronological Age, March 1	Mental Age March 1	Problem-Solving Age, Mar. 1	Problem-Solving Age, June 7	Gain
1	14-4	13-0	13-1	15-4	2-3
2	14-2	12-0	14-8	14-8	0-0
3	14-1	12-8	13-5	13-7	0-2
4	14-0	12-6	13-5	14-1	0-8
5	13-6	11-11	11-7	12-2	0-7
6	13-4	11-7	11-7	12-10	1-3
7	12-3	10-9	10-6	11-3	0-9
8	11-8	10-0	9-11	9-11	0-0
9	14-6	15-11	13-9	15-0	1-3
10	14-5	16-6	13-5	12-10	-0-7
11	14-0	15-8	13-1	15-0	1-11
12	13-9	16-1	14-1	15-8	1-7
13	13- 0	14-3	12-7	14-8	2-1
14	12-6	13-7	12-3	12-10	0-7
15	12-5	13-7	11-7	13-7	2-0
16	11-9	13-4	11-11	12-4	0-5
17	11-8	12-10	11-11	12-10	0-11
18	11-7	12-8	11-1	13-7	2-6
19	11-7	12-10	10-9	13-7	2-10
20	11-6	13-10	10-9	12-2	1-5
21	10-10	11-10	9-11	11-3	1-4
22	10-10	12-0	10-6	11-3	0-9

TABLE XXXIII

Percentage of Improvement in Accuracy and Speed as Shown by the Second Form of the New Stanford Achievement Test

ase	Per cent of Improvement Computation		Per cent of Improvement Reasoning	
	Accuracy	Speed	Accuracy	Speed
1	7.0	20.8	74.5	35.0
2	9.6	-13.8	21.6	0
3	31.6	47.4	26.6	5.5
4	3.5	30.7	26.6	11.1
5	56.0	13.8	51.7	25.0
6	24.6	4.0	96.6	41.6
7	6.1	14.2	82.3	50.0
8	14.7	0	-20	0
9	33.0	30.1	53.2	5.5
10	16.0	-24.3	25.8	-5.5
11	28.9	26.5	33.3	29.4
12	69.3	69.2	45.4	20.0
13	44.1	80.0	63.5	40.0
14	55.2	47.0	27.7	21.4
15	86.5	47.4	83.5	58.3
16	48.1	100	64.4	23.0
17	20.6	50.0	150	100
18	37.1	55.3	9.9	50.0
19	17.1	24.9	68.8	111
20	86.2	35.3	130	66
21	114	90.3	26.0	71.4
22	53.0	100	60.0	50.0

nounced gains were made either in computation or in reasoning. The three children, Cases XVII, XIX, and XX, who improved the most in problem solving failed to make equal progress in computation, while Cases XVI, XXI, and XXII, who improved their scores greatly in computation did not show similar growth of ability in reasoning. The child with the lowest I, Q., Case VIII, failed to raise either of her scores; other pupils who made but slight gains in one skill improved markedly in the other. A wide divergence between the two scores in all the cases would seem to indicate that there is no relation between improvement in computation and in arithmetic reasoning.

c. Appreciation of the Broader Implications of Number

In this study there were a few children whose mature intelligence enabled them to appreciate the wider aspects of number. When these pupils were given a brief introduction into the history of the development of our modern system they were eager to learn more of the orderly arrangement made possible by the use of the Hindu-Arabic notation and the zero. The need of such a system was understood when present-day methods of banking and finance were investigated. The children admired the decimal system that made possible the measurements of modern scientific studies. They were also intrigued by the use of formulae and symbols and looked forward eagerly to the study of algebra and geometry.

4. Educational Implications

a. Improvement in Computational Skills

There seems to be a need for deliberate training in attentive and precise application of the information and skill gained in the recitation periods. The written exercises done in unsupervised study periods indicated that the pupils failed to utilize the abilities they had developed in oral arithmetic or in supervised written work.

The gains made by the pupils, as indicated by the final test scores, seem to show that there is quick growth in the comprehension of a process and in accuracy of work when an intelligent attack is made upon the computational operations, followed by attentive exercises and drills.

b. Problem Solving difficulties

If the difficulties of the children observed in this study may be regarded as typical, they would seem to imply that problem solving must be taught by means of well-planned, graded steps, from the simple to the complex. It might be said that where such teaching is not done, there is a lack of ability on the part of the child to choose from his acquired knowledge or experience the processes pertaining to a problem situation.

The inability to interpret problems seems to indicate that:

1. The reasoning demanded in the problems of a certain grade placement are beyond the reasoning powers of the children

of that grade, or that

2. The pupils are not given sufficient preparatory instruction in the lower grades to enable them to make a logical attack upon the problems they encounter.

c. Questions Suggested for Further Study

Among the questions arising for further study, three seem to be especially pertinent:

1. Is there a plateau of learning about the time of the introduction of fractions, usually at the beginning of the fifth year, where retrogression may occur if further formal study of the fundamental processes is discontinued?

2. To what extent are the bad arithmetic work habits of children due to the fact that assignments are made upon operations which the pupils have not mastered, and are solved in unsupervised study periods, where no assistence is given to pupils who are in need of it?

3. After having made notable gains in arithmetic reasoning, is it possible for a child to use this increase of reasoning power to raise his score on an intelligence test of the Binet-Simon type?

CHAPTER VI

SUMMARY AND CONCLUSIONS

A. Summary

This thesis is the report of an investigation into (a) the attitudes of children failing in arithmetic, (b) the difficulties evidenced by the children in their arithmetic work, and (c) the means found to be effective in overcoming these difficulties.

The twenty-two children who participated in the experiment were chosen from the fifth, sixth, seventh, and eighth grades of one school, although twelve of the group had been in the school less than two years, In the investigation the case study method was used according to the following plan:

1. A preliminary test in arithmetic was given to all the children in grades five to eight by means of the New Stanford Achievement Tests in Computation and Arithmetic Reasoning. After the scores of the test had been transmuted into arithmetic age and grade scores, all children having an arithmetic age of one year or more below their chronological age were listed for further observation.

2. The mental ability of the children so segregated was measured by means of the Stanford Revision of the Binet-Simon

tests, in which sixteen subjects showed an I. Q. of 110-120, and nine subjects showed an I. Q. of 90-80. Of these twenty-five children chosen for the experiment, three were later dropped because of prolonged absence.

3. A thorough investigation of possible factors contributing to the failure of the pupils was made through three channels. Interviews with parents provided information regarding the environmental and physical history of the subjects. Records from former classes or schools furnished the pedigogical history. Lastly, interviews with the children themselves gave the examiner an insight into their difficulties.

4. Diagnosis of the arithmetic abilities was made by means of the Brueckner Diagnostic Tests, while possible reading handicaps were discovered by the Gates Silent Reading Tests.

5. The data obtained by means of these interviews and tests were then assembled and tabulated. The various types of error made by each child, his habits of work, and his attitude were noted. In the light of this information, remedial instruction was organized along the following lines.

a. Further interviews with the parents secured their interest and cooperation. The examiner advised them to encourage the children while providing them with opportunities to make a practical use of arithmetic.

b. Groups of three or four children having similar difficulties were organized for daily explanations and drill.

c. Each child received individual aid in the correlation of number work with life situations, in the interpretation of statistical information and news articles containing information expressed in quantitative terms, and in the understanding the uses and importance of number. Problem solving aids were developed along the lines of child experience, while at the same time, interests were broadened by the grasp of the use of number in business, travel, commerce, science, and art.

6. The results of thetests given during the progress of the study were summarized in tables showing the initial standing, progress, and final scores of the subjects. The per cent of improvement in accuracy and speed in computation and in arithmetic reasoning was also tabulated. Lists of errors, bad work habits, and overt movements of pupils having arithmetic difficulties were prepared. A number of remedial measures and devices found helpful in overcoming these difficulties were described.

B. Conclusions

An analysis of the data obtained during the course of this experiment seems to give support to the following conclusions:

1. Instruction in abstract skills is not sufficient preparation for the reasoned solution of problems.

2. Children must be taught how to bridge the gap between the mastery of computational skills and the application of these skills in problems.

3. Daily, directed drill increased the skill of the

pupils on computational operations.

4. The errors made in problem solving were due more often to lack of correct interpretation of the problem situation than to mistakes in computation.

5. Guidance in visualization, analysis, and interpretation of problem situations increased the ability of the children to solve problems.

6. Greater improvement was made by each child in the processes in which he had received the lowest initial scores.

7. Less improvement was manifested by each child in the processes in which he had received higher initial scores.

8. Children with high I. Q.'s who are in need of remedial instruction make rapid progress in computational processes, once they have a grasp of the proper method of operation.

9. Children with high I. Q.'s learn quickly to use their daily life experiences with number to interpret the textbook situations.

10. Children with I. Q.'s below 90 may acquire a good command of the computational processes.

11. Children with I. Q.'s below 90 need much training in the visualization of number situations.

12. Children with I. Q.'s below 90 must be taught how to realize the every-day uses of arithmetic. They are benefitted by employing original problems about their own experiences.

13. All children need regular, directed drill on the ab-

stract processes they have learned, to ensure retention of the skills.

14. The following types of instruction were factors in the improvement in problem solving shown by the subjects of this study:

a. Giving the child an understanding of number.

b. Teaching the child to interpret the problem in terms of his experience.

c. Teaching the child to make a careful analysis of the problem.

d. Teaching the child to estimate the probable answer before making any computations.

e. Teaching the child to think back over his procedure to determine if the attack were logical.

BIBLIOGRAPHY

1. Ballard, P. B., "Norms of Performance in the Fundamental Processes of Arithmetic with Suggestions for Their Improvement." Journal of Experimental Pedagogy 2:386-405, December, 1914, and 3:9-20, March, 1915.

2. ______, Teaching the Essentials of Arithmetic. London: University of London Press, Limited, 1928. 260p.

3. Banting, G. O., "The Elimination of Difficulties in Reasoning." Second Yearbook. Department of Elementary School Principles. Washington, D. C.: National Education Association, 1923. p. 411-421.

4. Bowdren, M. E., Five Case Studies of Arithmetic Failures. Unpublished Master's Thesis, Boston University, Boston, Massachusetts, 1934.

5. Bradford, E. J., "Suggestion, Reasoning, and Arithmetic." Forum of Education 3:3-12, February, 1925

6. Brown, J. C., and Coffman, L. D., How to Teach Arithmetic. New York: Row, Peterson Company, 1914. 391 p.

7. Brownell, W. A., "Psychological Considerations in the Learning and Teaching of Arithmetic." Tenth Yearbook. National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1935. p. 1-31.

8. ______, "Remedial Cases in Arithmetic." Peabody Journal of Education 9:100-107, September 1929.

9. ______ and Chazal, C. B., "The Effect of Premature Drill in Third-Grade Arithmetic." Journal of Educational Research 29:17-28. September, 1935.

10. Brueckner, L. J., "A Critique of the Yearbook." Twenty-Ninth Yearbook. National Society for the Study of Education. Bloomington, Illinois: Public School Publishing Company, 1930. p. 681-709.

11. ______, "Diagnosis in Arithmetic." Thirty-Fourth Yearbook. National Society for the Study of Education. Bloomington, Illinois: Public School Publishing Company, 1935. p. 269-302

12. ______, *Diagnostic and Remedial Teaching in Arithmetic*. Philadelphia: Winston Company, 1930. ix+341 p.

13. Brunner, R. S., *The Relation of Certain Factors to Ability in Arithmetic Reasoning*. Unpublished Master's Thesis, University of Pennsylvania, Philadelphia, Pennsylvania, 1933.

14. Buckingham, B. R., "*The Social Value of Arithmetic*." *Twenty-Ninth Yearbook*. National Society for the Study of Education. Bloomington, Illinois: Public School Publishing Company, 1930. p. 9-62.

15. ______, "Informational Arithmetic." *Tenth Yearbook*. National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1935. p. 51-73.

16. ______, "When to Begin the Teaching of Arithmetic." *Childhood Education*, 11:339-343, May 1935.

17. Buswell, G. T., "Summary of Arithmetic Investigations." *Elementary School Journal* 30:766-775, June, 1930, and 31: 756-766, June, 1931.

18. ______, and John, Lenore, *Diagnostic Studies in Arithmetic*. Supplementary Educational Monograph, No. 30. Chicago: University of Chicago Press, 1926. xiii+212 p.

19. ______, and Judd, C. H., *Summary of Educational Investigations Relating to Arithmetic*. Supplementary Educational Monograph, No. 27. Chicago: University of Chicago Press, 1925. 212 p.

20. Cantril, N. E., *Some Influences of Reading Ability Upon Achievement in Arithmetic*. Unpublished Master's Thesis, University of Colorado, Boulder, Colorado, 1933.

21. Carroll, N. M., *A Study of Two Methods of Teaching Problem-Solving in Arithmetic*. Unpublished Master's Thesis, Loyola University, Chicago, Illinois, 1934.

22. Conant, L. L. *The Number Concept: Its Origin and Development*. New York: The MacMillan Company, 1896. 218 p.

23. Edwards, Arthur, "A Study of Errors in Percentage." *Twenty-Ninth Yearbook*. National Society for the Study of Education. Bloomington, Illinois: Public School Publishing Company, 1930. p. 621-640.

24. Edwards, A. S., *Psychology of Elementary Education.* Boston: Houghton Mifflin Company, 1925. xiii+317 p.

25. Engelhardt, M. D., "The Relative Contribution of Certain Factors to Individual Differences in Arithmetical Problem Solving Difficulties." *Journal of Experimental Education* 1:19-27, September, 1932.

26. Greene, C. E. and Buswell, G. T., "Testing Diagnosis and Remedial Work in Arithmetic." *Twenty-Ninth Yearbook*. National Society for the Study of Education. Bloomington, Illinois: Public School Publishing Company, 1930. p. 269-316.

27. Harap, H. and Mapes, C. E., "The Learning of Decimals in an Activity Program." *Journal of Educational Research* 27: 686-693, May 1936.

28. Hydle, L. L. and Clapp, F. L., *Elements of Difficulty in the Interpretation of Concrete Problems in Arithmetic*. University of Wisconsin Bulletin No. 9. Madison: The University, 1927.

29. Holloway, H. V., *An Experimental Study to Determine the Relative Difficulty of the Elementary Number Combinations in Addition and Multiplication*. Philadelphia: University of Pennsylvania, 1914. 102 p.

30. Judd, C. H., *Psychological Analysis of the Fundamentals of Arithmetic*. Supplementary Educational Monograph No. 32. Chicago: University of Chicago Press, 1927. ix+121 p.

31. Klapper, P., *The Teaching of Arithmetic*. New York: D. Appleton-Century Company, 1934. 525 p.

32. Knight, F. B., Report on Organization of Drill in Arithmetic, *Third Yearbook*. Department of Superintendence. Washington, D. C.: The National Education Association, reprinted 1926. p. 63-91.

33. ______; Ruch, G. M.; and Luse, E., *Problems in the Teaching of Arithmetic: A Syllabus for Discussions on Important Aspects of Elementary School Arithmetic Prepared for Use in Advanced Courses in the Teaching of Arithmetic*. Iowa City, Iowa: Iowa Supply Company, 1924.

34. Lennes, N. L., *The Teaching of Arithmetic*. New York; MacMillan Company, 1923. x+486 p.

35. Luse, Eva M., "*Transfer Within Narrow Mental Functions.*" University of Iowa Monograph in Education, First Series, No. 5. Iowa City: The University.

36. Lutes, O. S., An Evaluation of Three Techniques for Improving Ability to Solve Arithmetic Problems. University of Iowa Monograph in Education, First Series, No. 6. Iowa City: The University, 1926.

37. Mangan, M. C., A Comparative Study of Two Methods of Problem-Solving in Arithmetic. Unpublished Master's Thesis, Loyola University, Chicago, Illinois, 1933.

38. Mead, C. D. and Sears, Isabel, "Additive Subtraction and Multiplicative Division Tested." Journal of Educational Psychology 7:261-270, May 1916.

39. Monroe, Marion, Children Who Cannot Read. Chicago: University of Chicago Press, 1932. xvi+205.

40. Morton, R. L., Teaching Arithmetic in the Elementary School. New York: Silver Burdett Company, 2 vol. 1937, vol. 1, x+410 p. 1938, vol. 2, xii+538 p.

41. Myers, G. C. "Presistence of Errors in Arithmetic." Journal of Educational Research 12:19-28, June, 1924.

42. ______, The Prevention and Correction of Errors in Arithmetic. Chicago: The Plymouth Press, 1925. 75 p.

43. National Council of Teachers of Mathematics, Tenth Yearbook. New York: Bureau of Publications, Teachers College, Columbia University, 1935. 289 p.

44. National Education Association, Department of Superintendence, A Reprint of the Third Yearbook. Washington, D.C.: The Department of Superintendence of the National Education, 1926. 520 p.

45. National Society for the Study of Education, Twenty-Ninth Yearbook. Bloomington, Illinois: Public School Publishing Company, 1930. x+749 p.

46. ______, Thirty-Fourth Yearbook. Bloomington, Illinois: Public School Publishing Company, 1935. x+563 p.

47. Neulen, L. N., Problem Solving in Arithmetic. Contributions to Education, No. 483. New York: Bureau of Publications, Teachers College, Columbia University, 1931, vi+87 p.

48. Newcomb, R. S., "Teaching Pupils How to Solve Problems in Arithmetic." Elementary School Journal 23:183-189, November, 1922.

49. Olander, H. T., "The Need for Diagnostic Testing." *Elementary School Journal* 33:736-745, June 1933.

50. Osburn, W. J., *Corrective Arithmetic*. Boston: Houghton Mifflin Company. 2 vol. 1924, vol. 1, vii+264 p. 1939, vol. 2, iv+279 p.

51. Parker, S. C., *Types of Elementary Teaching and Learning*. Boston: Ginn and Company, 1923. 555 p.

52. Pestalozzi, J. H., *How Gertrude Teaches Her Children*. (Translated by Lucy E. Holland and Francis C. Turner). Syracuse, New York: C. W. Bardeen. n.d. 256 p.

53. Phillips, D. E., "Number and Its Application Pyschologically Considered." *Pedagogical Seminary* 5:221-281, October, 1897.

54. Rice, J. M., "An Evaluation of the Teaching of Arithmetic." *Forum* 34:281-297. October, 1902. 437-452, January, 1903.

55. Ruch, G. M., and Mead, C. D., "A Review of Experiments in Subtraction." *Twenty-Ninth Yearbook*. National Society for the Study of Education. Bloomington, Illinois: Public School Publishing Company, 1930. p. 671-678.

56. Smith, D. E., *The Teaching of Elementary Mathematics*. New York: The Mac Millan Company, 1900. 312 p.

57. Stevenson, P. R., "Difficulties in Problem Solving." *Journal of Educational Research* 11:95-103, February, 1925.

58. ______, "Increasing the Ability of Pupils to Solve Arithmetic Problems." *Educational Research Bulletin*, vol. 3, No. 13: 263-271, November, 1924.

59. Stone, C. W., *Arithmetic Abilities and Some Factors Determining Them*. Contributions to Education, No. 19. New York: Bureau of Publications, Teachers College, Columbia University, 1908. 101 p.

60. ______, *Standardized Reasoning Tests in Arithmetic and How to Utilize Them.* New York: Bureau of Publications, Teachers College, Columbia University, 1929.

61. Stretch, L. B., "The Value and Limitations of Drill in Arithmetic." *Childhood Education* 11:413-416. June, 1925.

62. Thorndike, E. L., *New Methods in Arithmetic*. Chicago: Rand McNally and Company, 1926. viii+260 p.

3. ______, The Psychology of Arithmetic. New York: The MacMillan Company, 1920. xvi+314 p.

4. Tyler, R. W., "Elements of Diagnosis." Thirty-Fourth Yearbook. National Society for the Study of Education. Bloomington, Illinois: Public School Publishing Company, 1935. p. 113-129.

65. Ulich, R., Sequence of Educational Influences. Cambridge, Massachusetts: Harvard Press, 1925. 91 p.

66. Washburne, C. W., and Osburn, R., "Solving Arithmetic Problems." Elementary School Journal 27:219-226 and 293-304. November and December, 1926.

67. West, R. L., Greene, C. E., and Brownell, W. A., "The Arithmetic Curriculum." Twenty-Ninth Yearbook. National Society for the Study of Education. Bloomington, Illinois: Public School Publishing Company, 1930. p. 65-142.

68. Wheat, H. G., Psychology and Teaching of Arithmetic. New York: D. C. Heath and Company, 1937. 591 p.

69. ______, The Relative Merits of Conventional and Imaginative Types of Problems in Arithmetic. Contributions to Education, No. 359. New York: Bureau of Publications, Teachers College, Columbia University, 1929. 123 p.

70. Wilson, E. "Improving the Ability to Read Arithmetic Problems." Elementary School Journal 22:380-386, January, 1922.

71. Wilson, G. M., What Arithmetic Shall We Teach? Boston: Houghton Mifflin Company, 1926. 149 p.

72. Winch, W. H., "Equal Additions' Versus 'Decomposition' in Teaching Subtraction: An Experimental Research." Journal of Experimental Pedagogy 5:207-220 and 261-270. June 5 and December 6, 1920.

73. ______, "Further Work on Numerical Accuracy in School Children." Journal of Educational Psychology 2:262-271.

74. Woody, C., "The Arithmetical Backgrounds of Young Children." Journal of Educational Research 24:188-201, October, 1931.

The thesis, "Diagnostic and Remedial Procedures in Computational Skills and Reasoning Procedures in Arithmetic," written by Mother R. Fullerton, has been accepted by the Graduate School with reference to form, and by the readers whose names appear below, with reference to content. It is, therefore, accepted in partial fulfillment of the requirements for the degree of Master of Arts.

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Rev. Austin G. Schmidt, S.J.	March, 1939