MATH SCRIBER

ALGEBRA

Inequalities & Quadratic Equations

RAJ WIJESINGHE

Did you hear about the rose that grew
from a crack in the concrete?
Proving nature's law is wrong it
learned to walk without having feet.

Funny it seems, but by keeping its dreams,
it learned to breathe fresh air.

Long live the rose that grew from concrete
when no one else ever cared.

[redacted]

5. [redacted].

6. $\frac{4x^2+3x-1}{(3x-1)(x-2)(4x-1)}$ convert to partial fractions.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

38. Find the partial fractions of $\frac{x+4}{(x+7)(2x-1)}$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted] [redacted]

[redacted] [redacted]

[redacted] [redacted]

[redacted] [redacted]

[redacted] [redacted]

[redacted] [redacted]

[redacted] [redacted]

[redacted] [redacted]

[redacted] [redacted]

[redacted] [redacted]

[redacted] [redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

100. Show $\frac{5x+7}{(x+1)^2(x+2)}$ in the form of partial fractions.

[redacted]

[redacted]

[redacted]

117. The quadratic equations, $ax^2 + bx + c = 0$ and $a'x^2 + b'x + c' = 0$ have the same roots. Show that $\frac{b}{a} = \frac{b'}{a'}$ and $\frac{c}{a} = \frac{c'}{a'}$

[redacted]

119. The quadratic equation $x^2 - x + 1 = 0$ has roots α, β. For positive integers n, let $A_n = \alpha^n + \beta^n$. Without solving the equation show that $A_1 = 1, A_2 = -1$ and $A_{n+2} = A_{n+1} - A_n$ for $n \geq 1$.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

126. [redacted]

(a) [redacted].

(b) Express f(x) in the form $A + \frac{B}{x-4} + \frac{C}{x-3}$, where A, B and C are constant.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

147. (a) $f(x) = px^4 + qx^3 + rx^2 + sx + t$. [redacted].

[redacted]

148. (a) $f(x) = x^4 - bx^3 - 11x^2 + 4(b+1)x + a$. [redacted].

[redacted]

REMAINDER THEOREM & FACTOR THEOREM

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

185. [redacted].

[redacted]

[redacted]

(4) If $p(x) = x^3 + 3x^2 - 5$, $q(x) = 2x^3 + 2x^2 - x - 1$, $r(x) = 3x^4 + x^3 - 2x^2 + 5x - 4$;

[redacted]

c)

[redacted]

(2) If $p(x) = 3x^4 + 2x^3 - 5$, $q(x) = 2x^3 + x^2 - x - 1$; find $p(x)\ q(x)$.

[redacted]

187. 1. $x^2 - x - 4 \div (x - 1)$ 2. $x^3 + x - 1 \div (x + 1)$

3. $x^3 - x^2 + x - 1 \div (x - 2)$ 4. $2x^2 - x - 1 \div (x - 3)$

5. $x^4 + 2x^3 + x^2 \div x - 2 \div (x - 1)$ 6. $2x^4 + 3x - 1 \div (2x - 1)$

7. $x^3 - 2x^2 - x + 5 \div (3x + 1)$ 8. $2x^4 - 7x^3 + 12x^2 - 2x - 5 \div (2x + 1)$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

$$\frac{f(a) - f(b)}{(a - b)}x + \frac{(af(b) - bf(a))}{(a - b)}$$

[redacted]

195. (1) Find m and n as the remainder is 5x - 2 when the polynomial $f(x) \equiv x^4 - mx^2 + n$ is divided by $(x + 1)^2$.

(2) If $(x^2 + 1)$ is a factor of the polynomial $f(x) \equiv x^4 + px^3 + 3x + q$, find p and q. For the values obtained for p and q, find real roots of the equation $x^4 + px^3 + x^2 + 3x + q + 1 = 0$.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

(2) $x^3 - 7x - 6$ [redacted]

[redacted]

209. Express the polynomial $x^3 - 3x^2 + 10x - 5$ in the form $Ax(x - 1)(x + 2) + Bx(x - 1) + (x + 1)$. Here, A, B, C and D are constants that are needed to be found.

[redacted]

[redacted]

[redacted]

214. (1) The polynomial $x^8 + 2x^7 + ax^2 + bx + c$ is divided completely by $x^2 + x - 2$, and the remainder is -8 when the polynomial is divided by $(x + 1)$. Find a, b and c.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

231. If $x^3 + ax^2 + b$ and $ax^3 + bx^2 + x - a$ have a common factor, show that it is a factor of $(b - a^2)\, x^2 + x - a\,(1 + b)$ too.

[redacted]

[redacted]

234. Equation $3x^4 + 2x^3 - 6x^2 - 6x + p = 0$ have two common roots. Find the value of p.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

247. (i) If the remainders are equal when the function $f(x) \equiv 2x^2 + px^2 - qx + r$ is divided by x-1 and x – 2, show that 3p -q + 21 = 0.

[redacted]

[redacted]

[redacted]

250. [redacted].

(ii) Divide the function $P(x) = 2x^3 + 4x^2 + 3x + 3$ by $x + 2$.

[redacted]

[redacted]

252. [redacted] $x^8 + 2x^7 - 3x^3 + px + q$ [redacted].

[redacted]

255. (i) Here a is a non-zero integer and $f(x) = 2x^3 + 3x^2 - 3x + a$. Show that $x - a$ is a factor of $f(x)$.

[redacted]

[redacted]

[redacted]

[redacted]

260. [redacted].

[redacted]

(ii) Using $x + \frac{1}{x} = t$ or some other way, find all the factors of the equation

$x^4 - 5x^3 + 8x^2 - 5x + 1 = 0.$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

268. When the polynomial function f (x) is divided by $a \neq 0$, $x^2 - a^2$, the remainder is $Ax + B$.

Show that $A = \frac{1}{2a}[f(a) - f(-a)], B = \frac{1}{2}[f(a) - f(-a)]$.

Deduce the remainder when f(x) is divided by, $x^2 - 3^2$. If $f(x) = x^3 + x - 3$ deduce the remainder.

[redacted]

[redacted]

[redacted]

270. $f(x)$ is a polynomial function of second degree. $f(x)$ is divisible exactly by $x+3$. When divided by $(x-2)$ and $(x+1)$, remainders are respectively 5 and -4. Find $f(x)$. Here $f(x) = (px + q - 1)\, f(x)$. p and q are constants. If the remainder is $2x-2$, when the function $f(x)$ is divided by $(x+1)(x+2)$, find p and q.

[redacted]

[redacted]

[redacted]

273. Given that $f(x) = 4x^3 + 3x^2 + 2x + 1,\ g(x) = ax^2 + bx^3 + d$ find the values that would satisfy the condition $f(x) = g(x)$

274. Given that $4x^3 + 2x^2 + 3x + 17 \equiv Ax^2(x-1) + B(x-1)^2(x-2) + C(x+1)$ find the values of A,B,C that satisfy the identity.

[redacted]

276. Given that $f(x) = 4x^4 + 2x^3 + x^2 - 3x + 1$ is the dividend and $x^2 + 2x + 3$ is the divisor, find the answer (quotient) and the remainder.

[redacted]

[redacted]

279. Divide the function $3x^3 + 2x^2 + 4x - 1$ by $3x + 1$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

295. f(x) is a polynomial of squared or higher degree of x. Given $a \neq b$, show that when f(x)is divided by $(x-a)(x-b)$remainder is $f(a)\frac{(x-b)}{1-b}x + f(b)\frac{(x-a)}{b-a}$

296. If, $f(x) = x^{10} + 2x^9 + ax^2 + bx + c$ divide by $x + 1$, remainder is -9 and if same function is divided by $x^2 + x - 2$,remainder is $4x - 1$ find the values of constants a, b and c.

[redacted]

300. show that $(x-a)$is a factor of the polynomial $f(x) = x^3 + ax^2 - a^2x - a^3$ and find the other factors.

301. when polynomial $f(x) = x^{11} + 2x^{10} + px^2 + 2q + r$ is divided by x – 1 [redacted]

302. Find the remainder when $f(x) = 2x^6 + 4x^5 + 3x^2 - 2x + 3$ is divided by (x - 1) (x + 1) (x + 2). Find the quotient when the first function is divided by (x - 1) (x + 1) (x + 2)

303. if x-3 is a common root of $f(x) = 2x^3 - x^2 + 3px + 2q$ and $g(x) = 2qx^3 + 5qx^2 + 4x - 3$ find the values of p, q and r

304. if n is odd positive integers, show that when $x^n + 1$ is divided by $x^2 - 1$, remainder is equal to $x + 1$. Find the solution function (quotient) when it is divided by $x^2 - 1$

305. Divide the polynomial $f(x) = 2x^5 - px^2 + 3x$ by (x - 1) (x - 2) (x - 3). If the remainder does not consist of x^2 find the value of p.

306. When the polynomial f(x) which has higher degree than 2 is $k \neq 0$, show that dividing by $x^2 - k^2$ gives a remainder of $\frac{1}{2k}[f(k) - f(-k)]x + \frac{1}{2}[f(k) + f(-k)]$

[redacted]

[redacted]

[redacted]

310. When f(x) which is a polynomial, is divided by (x -1) (x+1) (x + 2) remainder is equal to A(x+1) (x +2) +B (x-1)(x+2)+ C(x-1) (x+1) . Find the values of A,B and C using f(-1), f(1) f(-2). If $f(x) = 2x^5 + 4x^4 + x^2 - 2x + 1$ deduce the remainder

[redacted]

312. [redacted].

313. $f(a,b,c) = a^3(b-c) + b^3(c-a) + c^3(a-b)$ Find factors

[redacted]

325. Given that the remainder of $f(x) = 2x^3 + 4x^2 - 3x + 2$ and $\emptyset(x) = x^3 - 3x^2 + 2x + 5$ functions when divided by x – a is equal, show that $a^3 + 7a^2 - 5a - 3 = 0$ find the integer values that satisfy the equation.

[redacted]

[redacted].

328. (i) Given that $3x^2 + 5xy - 2y^2 + 5x - 4y + k = (lx + my + n)\ (lx + my + n.)$

find the constants l., m., n., k

[redacted].

329. (i) $x^4 - 5x^3 + 4x^2 + 5x + 1 = 0$ solve the equation.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

333. (a) $f(x) \equiv x^4 - bx^3 - 11x^2 + 4(b+1)x + a$. in this function, a and b are constants. Show that

(i) f(x) is a perfect square.

(ii) x + 2 is a factor of f(x). find the a and b. Find the all factors of f(x)

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

(b) f(x) is a polynomial

$f(x) \equiv x^5 + 3x^4 - 2x^3 + 2x^2 - 3x + 1$

(i) Show that $x - 1$ or $x + 1$ are not factors of f(x)

(ii) Find the remainder when f(x) is divided by $x^2 - 1$

339. (i) $x(y^4 - z^4) + y(z^4 - x^4) + z(x^4 - y^4)$ factorize

(ii) assume that $f(x) = x^2 - 2x + 2$ and $g(x) = 6x^2 - 16x + 19$. Find the value of λ when $f(x) + \lambda g(x)$ function is converted to the form of $a(x + b)^2$ where a and b are real constants. Based on that providing values for A,B and C express f(x) in the form of $A(x - 3)^2 + B(x + c)^3$. Also show $g(x) = 10A(x - 3)^2 + 5B(x + c)^2$ find the smallest and largest values of $\frac{f(x)}{g(x)}$

[redacted]

[redacted]

[redacted]

[redacted].

(ii) $x^8 + 2x^7 + ax^2 + bx + c$ is perfectly divisible by $x^2 + x - 2$. When the same equation is divided by $x+1$, there is a remainder of 8. Find the a, b and c.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

---

352. $f(x)$ is a polynomial in x of degree greater than 3. When $f(x)$ is divided by $(x-1)$, $(x-2)$ and $(x-3)$, the remainders are a, b and c respectively. By repeated application of the Remainder Theorem, show that when $f(x)$ is divided by $(x-1)(x-2)(x-3)$, the remainder can be expressed as

$\lambda(x-1)(x-2)+\mu(x-1)+\nu$, where λ and μ are constants.

Find λ, μ and ν in terms of a, b and c.

(2007)

[redacted]

---

354. Prove that if a polynomial $f(x)$ is divided by $(x-\alpha)$ then the remainder is $f(\alpha)$.

When the polynomial $f(x)$ is divided by $(x-\alpha)(x-\beta)(x-\gamma)$, where α, β and γ are unequal real numbers the remainder takes the form $A(x-\beta)(x-\gamma) + B(x-\alpha)(x-\gamma) + C(x-\alpha)(x-\beta)$.

Express the constants A,B and C in terms of $\alpha\ \beta, \gamma, f(\alpha),, f(\beta)$ and $f(\gamma)$.

Hence, find the value of the constant k for which the remainder when $x^5 - kx$ is divided by $(x+1)\ (x-1)(x-2)$ contains no term in x.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

---

361. Sketch the graphs of $y = |x| + 1$ and $y = 2|x-1|$ in the same diagram. Hence or otherwise, find all real values of x satisfying the inequality $|x| + 1 > 2|x-1|$

(2016)

[redacted] Let $p(x) = x^3 + 2x^2 + 3x - 1$ and $q(x) = x^2 + 3x + 6$. Using the remainder [redacted]

[redacted]

[redacted]

[redacted]

---

364. Let c$(\neq O)$ and d be real numbers, and let $f(x) = x^3 + 4x^2 + cx + d$. [redacted]

[redacted]

What's Next ?

ALGEBRA 2

ALGEBRA
Inequalities & Quadratic Equations

Maths Simplified !

mathscriber .com
0772461477

RAJ WIJESINGHE
B.Sc (Math Sp.) M.Sc