Differentiation (Part I) Past Paper Questions:

Section 1: To find the derivative using the First Principles.

1. 2012-2013 V2 (2 marks)
   Q3
   Find, from the first principles, the derivative of $3x$.

2. 2014-2015 V1
   #2
   $$\lim_{h\to 0} \frac{f(x+h)-f(x)}{3h} =$$

   - ☐ $3f'(x)$
   - ☐ $\frac{1}{3}f'(x)$
   - ☐ $f'(3)$
   - ☐ $f'\left(\frac{1}{3}\right)$

Section 2: To find the derivative using the Short Cut.

3. 2013-2014 V1
   #1
   If $y = 7x + 9$, then $\frac{dy}{dx} =$

   - ○ $7x$
   - ○ $x + 9$
   - ○ 7
   - ○ 9

4. 2013-2014 V2
   #1
   If $y = x^5$, then $\frac{dy}{dx} =$

   - ☐ x^4
   - ☐ x^5
   - ☐ $5x^4$
   - ☐ $5x^5$

5. 2015-2016 Mock
#2

If $f(x) = 6x^2 + 10$, then $f'(x) =$

a) $6x$ b) $12x$

c) $3x$ d) $12x + 10$

Section 3: To find the derivative of a function at a point.

6. 2013-2014 V1 (2 marks)
Q3ai

Given that $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 2x^2 - 1$, calculate $f'(5)$.

7. 2013-2014 V2 (2 marks)
#19

Given that $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = x + 1$, calculate the gradient of the curve $f(x)$ when $x = 2$.

8. 2012-2013 V1
#1

$\lim_{h \to 0} \frac{f(-2) - f(-2+h)}{h} =$

- ☐ $f'(2)$
- ☐ $-f'(2)$
- ☐ $f'(-2)$
- ☐ $-f'(-2)$

9. 2012-2013 V2
#1

$$\lim_{h \to 0} \frac{f(7) - f(7+h)}{2h} =$$

- ☐ $-\frac{1}{2}f'(7)$
- ☐ $\frac{1}{2}f'(7)$
- ☐ $f'(7)$
- ☐ $-f'(7)$

10. 2015-2016 Mock
#17

Given that $\lim_{h \to 0} \frac{f(x+h)-f(x)}{2h} = \frac{x^2}{3} + 1$, find the gradient of the curve $f(x)$ when $x = 3$ (4 marks)

Section 4: To find points where the derivative equals zero (or any other value).

11. 2014-2015 V1
#3

If $y = ax^2$ has a gradient of 12 at point (2, 12), then a equals:

- ☐ $\frac{1}{6}$
- ☐ $\frac{1}{3}$
- ☐ 3
- ☐ 6

12. 2012-2013 V2
#2 If $f(x) = ax^2 + 3x$ and $f'(2) = -5$, then the value of a is:

- ☐ -2
- ☐ $-\frac{1}{2}$
- ☐ $\frac{1}{2}$
- ☐ 2

13. 2012-2013 V1 (2 marks)
Q3aii Given that $y = 2x^3 + x$ has gradient equal 7 at the point (a, b), find possible values for a and b.

Section 5: To expand or simplify in order to make differentiation easier.

14. 2013-2014 V1 (3 marks)
Q2b Differentiate $f(x) = (x+3)^2 + 2x - 1$ with respect to x.

15. 2013-2014 V2 (3 marks)
#17

Differentiate $f(x) = \frac{x^4 + 2x}{x^2}$ with respect to x.

Section 6: To find the second (or third, etc) derivative.

16. 2014-2015 V1
#16

Find $\frac{d^2y}{dx^2}$ if $y = x^{\frac{3}{2}} + x + 3$ [4 marks]

17. 2012-2013 V1 (2 marks)
Q3ai

If $f(x) = x^{\frac{1}{4}}$, find $f''(x)$

18. 2012-2013 V1
#2

If $y = a\,x^2+5$ and $\frac{d^2y}{dx^2} = 6$ at $x = -1$, then $a=$

- ☐ -3
- ☐ -1
- ☐ 1
- ☐ 3

Section 7: To find the equation of the tangent (or normal) to a curve at a given point.

19. 2013-2014 V1
#2

The equation of the tangent to curve $f(x)=3x^2-2x+4$ *at* $x = 0$ *is :*

- ○ $y = -2x+4$
- ○ $y = \frac{-1}{6}x-2$
- ○ $y = \frac{1}{2}x+4$
- ○ $y = 6x-2$

20. 2012-2013 V2 (3 marks)
Q3aii

Show that there is only one point on the curve $y = \frac{8}{3}x^3 + 4x^2 + 2x$, where the tangent is parallel to $y = 4$.

21. 2012-2013 V1 (3 marks)

Q2b Find the equation of the tangent to $y = x^2 + x$ at $x = 1$

22. 2013-2014 V2 (2 marks)

#20

If $y = 3ax - 16$ is the equation of the tangent of $f(x) = 5x^3 - 3x + c$ at $x = 1$, then calculate the value of a.

23. 2014-2015 V1

#17a

Find the equation of the tangent to the curve $y = 16 - x^2$ for $x \geq 0$ at the point of intersection of curve y with the x-axis. [3 marks]

24. 2015-2016 Mock

#18

Find the equation of the tangent and normal to the curve $y = 3x^2 + 2x + 4$ at the point of intersection of curve y with the $y-axis$ (3 marks)

25. 2015-2016 Mock

#3 Though this question is fairly clear, it should have said, "an equation for the normal to the curve $y = x^2 + 1$ at (2,5) is:

Given that $f(2) = 5$, and $f'(2) = 4$ an equation for the normal to curve$y = x^2 + 1$ is:

a) $y = 4x + 13$

b) $y = 4x - 3$

c) $y = -\frac{1}{4}x + \frac{11}{2}$

d) $y = -\frac{1}{4}x - \frac{9}{2}$

26. 2013-2014 V2
#2

The equation of the normal to curve $f(x) = x^2 + 3x$ at $x = 0$ is $y =$

- ☐ $-\frac{1}{2}x + 3$
- ☐ $-\frac{1}{3}x$
- ☐ $\frac{1}{3}x$
- ☐ $2x + 3$

27. 2013-2014 V1 (2 marks)
Q3aii

Prove that $6y + x - 7 = 0$ is the equation for the normal to curve $f(x) = 2x^3 - 1$ at the point $(1,1)$.

28. 2012-2013 V2 (3 marks)
Q2b

Find the equation of the normal to $y = x^2$ at the point where $x = 1$.

Answers to Differentiation Part I:

1. 2012-2013 V2

   $$\lim_{h\to 0}\frac{3(x+h)-3x}{h}$$

   Q3 $$\lim_{h\to 0}\frac{3x+3h-3x}{h}$$

   $$\lim_{h\to 0}\frac{3h}{h}=3$$

2. 2014-2015 V1

   #2 $\frac{1}{3}f'(x)$

3. 2013-2014 V1

   #1 7

4. 2013-2014 V2

   #1 $5x^4$

5. 2015-2016 Mock

   #2 12x

6. 2013-2014 V1

   Q3ai 49

7. 2013-2014 V2

   #19 3

8. 2012-2013 V1

   #1 $-f'(-2)$

9. 2012-2013 V2

   #1 $-\frac{1}{2}f'(7)$

10. 2015-2016 Mock

    #17 8

11. 2014-2015 V1

    #3 3

12. 2012-2013 V2

    #2 -2

13. 2012-2013 V1

    Q3aii Two answers:

    a = 1 and b = 3

    also, a = -1 and b = -3

14. 2013-2014 V1

    Q2b 2x + 8

15. 2013-2014 V2

    $2x-2x^{-2}$

    #17 or $2x-\frac{2}{x^2}$

16. 2014-2015 V1

    #16

    $$y'=\frac{3}{2}x^{1/2}+1$$

    $$y''=\frac{3}{4}x^{-1/2}$$

17. 2012-2013 V1

    Q3ai $\frac{-3}{16}x^{-7/4}$

18. 2012-2013 V1

    #2 3

19. 2013-2014 V1

    #2 y = -2x + 4

    or y – 4 = -2(x – 0)

20. 2012-2013 V2

    Q3aii

    $y' = \frac{8}{3}(3x^2) + 4(2x) + 2$

    $y' = \frac{8}{3}(3x^2) + 4(2x) + 2 = 0$

    $8x^2 + 8x + 2 = 0$

    $2(4x^2 + 4x + 1) = 0$

    the gradient is zero $\rightarrow (2x+1)^2 = 0$

    $x = \frac{-1}{2}$

21. 2012-2013 V1

    Q2b y – 2 = 3(x – 1)

    y = 3x – 1

22. 2013-2014 V2

#20 4

23. 2014-2015 V1

#17a

17)a)To find the intersection points of y with $x-axis$ $\frac{1}{2}$

Putting $y=0$

$16-x^2=0$

$(4-x)(4+x)=0$ $\frac{1}{2}+\frac{1}{2}$

$x=4, \quad or \quad x=-4$ rejected

So the point of intersection is $(4,0)$ $\frac{1}{2}$

$y'=-2x$

Gradient of tangent at $(x=4) = -2\times 4=-8$ $\frac{1}{2}$

$equation \quad y-y_1=m(x-x_1)$

$y-0=-8(x-4)$ $\frac{1}{2}$

$y=-8x+32$

24. 2015-2016 Mock

#18 Tan:

$$y=2x+4$$

or: $y-4=2(x-0)$

Nor: $y=-\frac{1}{2}x+4$

or: $y-4=-\frac{1}{2}(x-0)$

25. 2015-2016 Mock

#3 $y-5=-\frac{1}{4}(x-2)$

which becomes $y=-\frac{1}{4}x+\frac{11}{2}$

26. 2013-2014 V2

#2 $-\frac{1}{3}x$

27. 2013-2014 V1

Q3aii

$f'(x)=6x^2$

$f'(1)=6$

The gradient of the tangent =6

The gradient of the normal $=\frac{-1}{6}$

The equation of the normal

$y-1=\frac{-1}{6}(x-1)$

$y=\frac{-1}{6}x+\frac{7}{6} \quad \times 6$

$6y=-x+7$

$6y+x-7=0$

28. 2012-2013 V2

Q2b

$$y-1=-\frac{1}{2}(x-1)$$

$$y=-\frac{1}{2}x+\frac{3}{2}$$