Differentiation Past Papers Unit 1 Outcome 3

1. Differentiate $2\sqrt[3]{x}$ with respect to x.

   A. $6\sqrt{x}$

   B. $\frac{3}{2}\sqrt[3]{x}^4$

   C. $-\dfrac{4}{3\sqrt[3]{x}^2}$

   D. $\dfrac{2}{3\sqrt[3]{x}^2}$ **2**

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	1.3	C	0.83	0.38	NC	C2, C3	HSN 091

$2\sqrt[3]{x} = 2x^{1/3}$

$\frac{d}{dx}\left(2x^{1/3}\right) = \frac{1}{3} \times 2x^{-2/3} = \dfrac{2}{3\sqrt[3]{x}^2}$ Option $\boxed{D}$

[SQA] 2. Given $f(x) = 3x^2(2x-1)$, find $f'(-1)$. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
.	3	1.3	3						1.3.4		Source: 1999 P1 qu.5

- •¹ $6x^3 - 3x^2$
- •² $18x^2 - 6x$
- •³ 24

[SQA] 3. Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of $45°$ with the positive direction of the x-axis. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	G2, C4	$(2, 4)$	2002 P1 Q4

•¹ sp: know to diff., and differentiate	•¹ $\frac{dy}{dx} = 4x - 7$
•² pd: process gradient from angle	•² $m_{\text{tang}} = \tan 45° = 1$
•³ ss: equate equivalent expressions	•³ $4x - 7 = 1$
•⁴ pd: solve and complete	•⁴ $(2, 4)$

[SQA] 4. If $y = x^2 - x$, show that $\dfrac{dy}{dx} = 1 + \dfrac{2y}{x}$. **3**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.3
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
.	3	1.3	1	2					1.3.4	0.1	1989 P1 qu.12

- •[1] $\frac{dy}{dx} = 2x - 1$
- •[2] $RHS = 1 + \frac{2\left(x^2 - x\right)}{x}$
- •[3] $1 + 2(x-1)$ and complete

[SQA] 5. Find $f'(4)$ where $f(x) = \dfrac{x-1}{\sqrt{x}}$. **5**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.3
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
.	5	1.3	5						1.3.4		1996 P1 qu.9

- •[1] $\frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}$ *or* $x \times x^{-\frac{1}{2}} - 1 \times x^{-\frac{1}{2}}$
- •[2] $x^{\frac{1}{2}} - x^{-\frac{1}{2}}$
- •[3] $\frac{1}{2}x^{-\frac{1}{2}}$
- •[4] $\frac{1}{2}x^{-\frac{3}{2}}$
- •[5] $\frac{5}{16}$

[SQA] 6. Find $\dfrac{dy}{dx}$ where $y = \dfrac{4}{x^2} + x\sqrt{x}$. **4**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.3
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
.	4	1.3	4						1.3.4		1995 P1 qu.7

- •[1] $4x^{-2}$ stated or implied by •[3]
- •[2] $+x^{\frac{3}{2}}$ stated or implied by •[4]
- •[3] $-8x^{-3}$
- •[4] $+\frac{3}{2}x^{\frac{1}{2}}$

[SQA] 7. If $f(x) = kx^3 + 5x - 1$ and $f'(1) = 14$, find the value of k. **3**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.3
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
.	3	1.3	3						1.3.4	0.1	1994 P1 qu.2

•1 $f'(x) = 3kx^2 + 5$
•2 $f'(1) = 3k + 5$
•3 $k = 3$

[SQA] 8. Find the x-coordinate of each of the points on the curve $y = 2x^3 - 3x^2 - 12x + 20$ at which the tangent is parallel to the x-axis. **4**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.3
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
.	4	1.3	4						1.3.10		1993 P1 qu.4

•1 $\frac{dy}{dx} = \ldots\ldots$
•2 $6x^2 - 6x - 12$
•3 $\ldots\ldots = 0$
•4 $x = -1, 2$

[SQA] 9. Calculate, to the nearest degree, the angle between the x-axis and the tangent to the curve with equation $y = x^3 - 4x - 5$ at the point where $x = 2$. **4**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.3
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
.	4	1.3	4						1.3.9	1.1.3	1989 P1 qu.13

•1 $\frac{dy}{dx} = 3x^2 - 4$
•2 $\frac{dy}{dx}_{x=2} = 8$
•3 $\tan\theta = 8$
•4 $83°$

[SQA] 10. The point $P(-1,7)$ lies on the curve with equation $y = 5x^2 + 2$. Find the equation of the tangent to the curve at P. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	1.3
.	4	1.3	4						1.3.9	1.1.7	Source: 1999 P1 qu.9

- $\bullet^1$ $\frac{dy}{dx} = \ldots\ldots$
- $\bullet^2$ $10x$
- $\bullet^3$ -10
- $\bullet^4$ $y - 7 = -10(x - (-1))$

[SQA] 11. Find the equation of the tangent to the curve $y = 4x^3 - 2$ at the point where $x = -1$. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	1.3
.	4	1.3	4						1.3.9	1.1.7	Source: 1990 P1 qu.2

- $\bullet^1$ *strat:* $\frac{dy}{dx} = \ldots\ldots$
- $\bullet^2$ $\frac{dy}{dx} = 12x^2$
- $\bullet^3$ $m = 12$
- $\bullet^4$ $y - (-6) = 12(x - (-1))$

[SQA] 12. Find the equation of the tangent to the curve with equation $y = 5x^3 - 6x^2$ at the point where $x = 1$. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	1.3
.	4	1.3	4						1.3.9	1.1.7	Source: 1992 P1 qu.1

- $\bullet^1$ $y' = 15x^2 - 12x$
- $\bullet^2$ $y'(1) = 3$
- $\bullet^3$ $y(1) = -1$
- $\bullet^4$ $y - (-1) = 3(x - 1)$

[SQA] 13. Find the equation of the tangent to the curve $y = 3x^3 + 2$ at the point where $x = 1$. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3 Source
.	4	1.3	4						1.3.9	1.1.7	1991 P1 qu.5

- $\bullet^1$ *strat:* $\frac{dy}{dx} = \ldots\ldots$
- $\bullet^2$ $f'(1) = 6$
- $\bullet^3$ $f(1) = 5$
- $\bullet^4$ $y - 5 = 6(x - 1)$

[SQA] 14. The point $\mathrm{P}(-2, b)$ lies on the graph of the function $f(x) = 3x^3 - x^2 - 7x + 4$.

(*a*) Find the value of b. **1**

(*b*) Prove that this function is increasing at P. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3 Source
(a)	1	0.1	1						0.1		1995 P1 qu.10
(b)	3	1.3	3						1.3.11		

- $\bullet^1$ $b = -10$
- $\bullet^2$ know to differentiate and know to show $\frac{dy}{dx}_{x=-2} > 0$
- $\bullet^3$ $9x^2 - 2x - 7$
- $\bullet^4$ show that $\frac{dy}{dx}_{x=-2} > 0$

[SQA] 15. For what values of x is the function $f(x) = \frac{1}{3}x^3 - 2x^2 - 5x - 4$ increasing? **5**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3 Source
.	5	1.3	2	3					1.3.11		1990 P1 qu.16

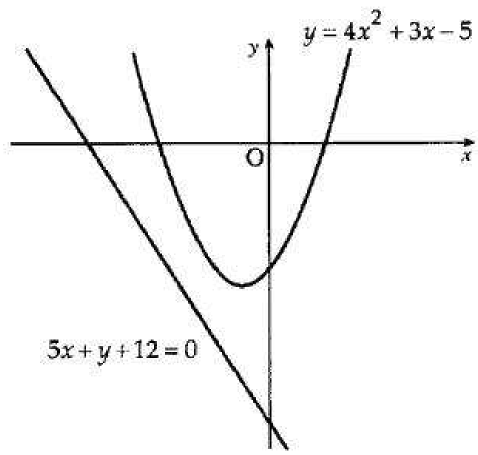
- $\bullet^1$ $f'(x) = x^2 - 4x - 5$
- $\bullet^2$ *use* $f'(x) > 0$
- $\bullet^3$ zeros at $x = 5$ and $x = -1$
- $\bullet^4$ *strat.* e.g. for $-1 < x < 5$ test $x = 0$
- $\bullet^5$ $x < -1,\ x > 5$

[SQA] 16. Given that $y = 2x^2 + x$, find $\frac{dy}{dx}$ and hence show that $x\left(1 + \frac{dy}{dx}\right) = 2y$. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3 Source
.	3	1.3	3						1.3.4		1997 P1 qu.8

- •[1] $\frac{dy}{dx} = 4x + 1$
- •[2] $LHS = x(1 + 4x + 1)$ *or* $RHS = 2(2x^2 + x)$
- •[3] completes proof

[SQA] 17. The diagram below shows a parabola with equation $y = 4x^2 + 3x - 5$ and a straight line with equation $5x + y + 12 = 0$.
A tangent to the parabola is drawn parallel to the given straight line.
Find the x-coordinate of the point of contact of this tangent. 5



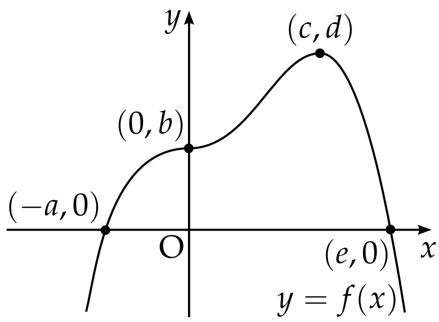
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3 Source
.	5	1.3	5						1.1.8	1.3.7 1.3.1	1997 P1 qu.6

- •[1] equate gradients
- •[2] $m = -5$
- •[3] $\frac{dy}{dx} = \ldots$
- •[4] $\frac{dy}{dx} = 8x + 3$
- •[5] $x = -1$

[SQA] 18. The graph of a function f intersects the x-axis at $(-a, 0)$ and $(e, 0)$ as shown.

There is a point of inflexion at $(0, b)$ and a maximum turning point at (c, d).

Sketch the graph of the derived function f'. **3**

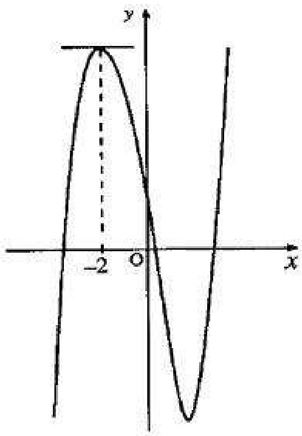


Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	3	C	CN	A3, C11	sketch	2002 P1 Q6

•[1] ic: interpret stationary points •[2] ic: interpret main body of f •[3] ic: interpret tails of f	•[1] roots at 0 and c (*accept a statement to this effect*) •[2] min. at LH root, max. between roots •[3] both 'tails' correct

[SQA] 19. The diagram shows a sketch of the curve $y = x^3 + kx^2 - 8x + 3$. The tangent to the curve at $x = -2$ is parallel to the x-axis.

Find the value of k. **4**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.3 Source
.	4	1.3	4						1.3.4	1.3.7	1998 P1 qu.11

•[1] $\frac{dy}{dx} = \ldots\ldots$ •[2] $3x^2 + 2kx - 8$ •[3] $3x^2 + 2kx - 8 = 0$ when $x = -2$ •[4] $k = 1$

[SQA] 20. A function f is defined by the formula $f(x) = (x-1)^2(x+2)$ where $x \in \mathbb{R}$.

(*a*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **3**

(*b*) Find the stationary points of this curve $y = f(x)$ and determine their nature. **7**

(*c*) Sketch the curve $y = f(x)$. **2**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.3
(*a*)	3	1.2	3						1.2.9		Source
(*b*)	7	1.3	7						1.3.15		1990 Paper 2
(*c*)	2	1.3	2						1.3.13		Qu. 1

(*a*)
- •[1] $x = 1, -2$
- •[2] $(1,0)$ **and** $(-2,0)$
- •[3] $(0,2)$

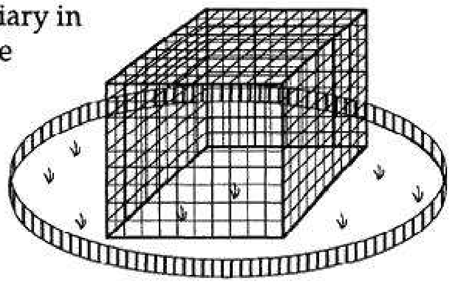
(*b*)
- •[4] $f(x) = x^3 - 3x + 2$
- •[5] $f'(x) = 3x^2 - 3$
- •[6] $f'(x) = 0$ **stated explicitly**
- •[7] $x = 1$ **and** -1
- •[8]

x	-1^-	-1	-1^+	1^-	1	1^+
$f'(x)$	+	0	−	−	0	+

- •[9] max at $(-1,4)$
- •[10] min at $(1,0)$

(*c*)
- •[11] correct shape of sketch
- •[12] correct annotation of sketch(max, min, 2 axes intersections)

[SQA] 21. The owners of a zoo intend to build a new aviary in the shape of a cuboid with a square floor. The volume of the aviary will be 500 m^3.



(*a*) If x metres is the length of one edge of the floor, show that the area A square metres of netting required is given by

$$A = x^2 + \frac{2000}{x}.$$ **(4)**

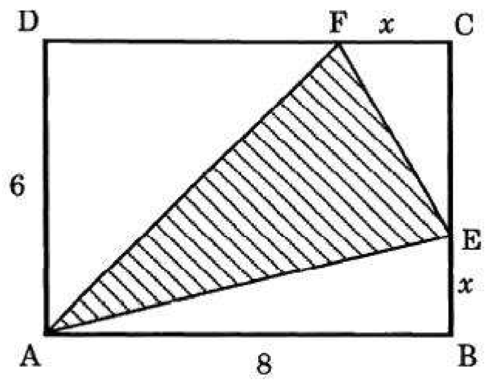
(*b*) Find the dimensions of the aviary to ensure that the cost of netting is minimised. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(*a*)	4	0.1	2	2					0.1		Source
(*b*)	6	1.3	4	2					1.3.15		1992 Paper 2 Qu.5

(*a*)
- •[1] introduce height specific to this cuboid
- •[2] $h = \frac{500}{x^2}$
- •[3] $A = x^2 + 4xh$
- •[4] $A = x^2 + 4x.\frac{500}{x^2}$ explicitly stated

(*b*)
- •[5] $A'(x) = \ldots\ldots$
- •[6] $2x - 2000x^{-2}$
- •[7] $A'(x) = 0$ specifically stated
- •[8] $x = 10$
- •[9] justify minimum e.g. with table
- •[10] dimensions of 10 by 10 by 5

[SQA] 22. An yacht club is designing its new flag. The flag is to consist of a red triangle on a yellow rectangular background. In the yellow rectangle ABCD, AB measures 8 units and AD is 6 units. E and F lie on BC and CD, x units from B and C as shown in the diagram.



(a) Show that the area, H square units, of the red triangle AEF is given by $H(x) = 24 - 4x + \frac{1}{2}x^2$. **(4)**

(b) Hence find the greatest and least possible values of the area of triangle AEF. **(8)**

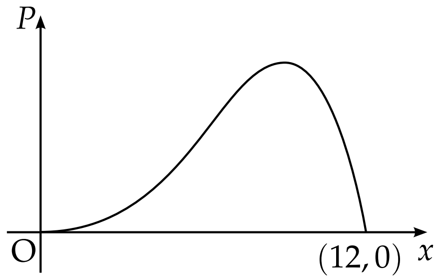
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	1.3 Source
(a)	4	0.1	4						0.1	1994 Paper 2
(b)	8	1.3	3	5					1.3.15	Qu.7

(a)
- •1 rectangle minus 3 triangles
- •2 area of Δ's ADF and ABE
- •3 area of Δ FCE
- •4 3 triangles : $24 + 4x - \frac{1}{2}x^2$ or $48 - 4x - 3x + \frac{1}{2}x^2 - 24 + 3x$

(b)
- •5 $H'(x) = \ldots\ldots$
- •6 $x - 4$
- •7 put $H'(x) = 0$ stated explicitly
- •8 $x = 4$ and $H = 16$
- •9 justify minimum
- •10 consider $x = 0$ and $x = 6$
- •11 $H(0) = 24$, and $H(6) = 18$
- •12 communication re greatest and least.

[SQA] 23. A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model , illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \le x \le 12$.

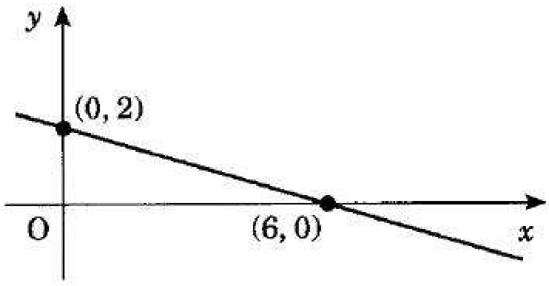
Find the value of x which gives the maximum profit. **5**



Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	5	C	NC	C11	$x = 9$	2001 P1 Q6

•[1] ss: start diff. process	•[1] $\frac{dP}{dx} = 36x^2 \ldots$ **or** $\frac{dP}{dx} = \ldots - 4x^3$
•[2] pd: process	•[2] $\frac{dP}{dx} = 36x^2 - 4x^3$
•[3] ss: set derivative to zero	•[3] $\frac{dP}{dx} = 0$
•[4] pd: process	•[4] $x = 0$ **and** $x = 9$
•[5] ic: interpret solutions	•[5] nature table about $x = 0$ **and** $x = 9$

[SQA] 24. The straight line shown in the diagram has equation $y = f(x)$.
Determine $f'(x)$. 2



part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.3
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
.	2	1.3	2						1.3.7		1995 P1 qu.14

•[1] gradient $= -\frac{1}{3}$
•[2] $f'(x) =$ gradient

[SQA] 25. A ball is thrown vertically upwards. The height h metres of the ball t seconds after it is thrown, is given by the formula $h = 20t - 5t^2$.

(*a*) Find the speed of the ball when it is thrown (i.e. the rate of change of height with respect to time of the ball when it is thrown). **3**

(*b*) Find the speed of the ball after 2 seconds.

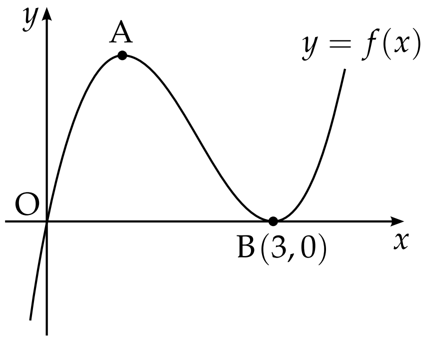
Explain your answer in terms of the movement of the ball. **2**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(a)	3	1.3	1	2					1.3.6		Source
(b)	2	1.2		2					1.2.9		1995 P1 qu.21

$\bullet^1$	knows to differentiate
$\bullet^2$	$20 - 10t$
$\bullet^3$	20
$\bullet^4$	speed = 0
$\bullet^5$	ball stationary at top of flight

[SQA] 26. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below.

The graph has a maximum at A and a minimum at B$(3, 0)$.



(*a*) Find the coordinates of the turning point at A. **4**

(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.

Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(*c*) Write down the range of values of k for which $g(x) = k$ has 3 real roots. **1**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	4	C	NC	C8	A$(1, 4)$	2000 P1 Q2
(*b*)	2	C	NC	A3	sketch (translate 4 up, 2 left)	
(*c*)	1	A/B	NC	A2	$4 < k < 8$	

•1 ss: know to differentiate	•1 $\frac{dy}{dx} = \ldots$
•2 pd: differentiate correctly	•2 $\frac{dy}{dx} = 3x^2 - 12x + 9$
•3 ss: know gradient = 0	•3 $3x^2 - 12x + 9 = 0$
•4 pd: process	•4 $\text{A} = (1, 4)$
	translate $f(x)$ 4 units up, 2 units left
•5 ic: interpret transformation	•5 sketch with coord. of $\text{A}'(-1, 8)$
•6 ic: interpret transformation	•6 sketch with coord. of $\text{B}'(1, 4)$
•7 ic: interpret sketch	•7 $4 < k < 8$ (accept $4 \leq k \leq 8$)

[SQA] 27. A curve has equation $y = x^4 - 4x^3 + 3$.

(*a*) Find algebraically the coordinates of the stationary points. **6**

(*b*) Determine the nature of the stationary points. **2**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(*a*)	6	1.3	6						1.3.12		Source
(*b*)	2	1.3	2						1.3.12		1996 Paper 2 Qu.1

(a)
- •[1] $\frac{dy}{dx} =$
- •[2] $4x^3 - 12x^2$
- •[3] $= 0$ stated explicitly
- •[4] e.g. $4x^2(x-3)$
- •[5] $x = 0,\ 3$
- •[6] $y = 3,\ -24$

(b)
- •[7]

x	0^-	0	0^+	3	3^+
$\frac{dy}{dx}$	$-$	0		0	$+$

- •[8] pt of inflection at $x = 0$ minimum at $x = 3$

[SQA] 28. A curve has equation $y = 2x^3 + 3x^2 + 4x - 5$.

Prove that this curve has no stationary points. **5**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
.	5	1.3	2	3					1.3.12	1.3.11	Source 1999 P1 qu.16

- •[1] $\frac{dy}{dx} = \ldots\ldots$
- •[2] $6x^2 + 6x + 4$
- •[3] e.g. "$b^2 - 4ac$" $= \ldots\ldots$
- •[4] -60 or -15 (from $3x^2 + 3x + 2$)
- •[5] Δ negative so no st. points

OR

- •[1] $\frac{dy}{dx} = \ldots\ldots$
- •[2] $6x^2 + 6x + 4$
- •[3] e.g. complete square……
- •[4] $S = 6\left(x + \frac{1}{2}\right)^2 + 2\frac{1}{2}$
- •[5] $S \geq 2\frac{1}{2}$ so no st. points

[SQA] 29. Find the values of x for which the function $f(x) = 2x^3 - 3x^2 - 36x$ is increasing. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
.	4	2.1	2	2					1.3.11	2.1.9	Source: 1996 P1 qu.16

•1 know to consider $f'(x) > 0$ stated or implied by the evidence for •4.

•2 $\frac{dy}{dx} = 6x^2 - 6x - 36$

•3 $6(x-3)(x+2) > 0$ or by formula or completing the square

•4 $x < -2,\ x > 3$

[SQA] 30. A curve has equation $y = x - \dfrac{16}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at the point where $x = 4$. **6**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	6	C	CN	C4, C5	$y = 2x - 12$	2001 P2 Q2

•1 ic: find corresponding y-coord.	•1 $(4, -4)$ *stated or implied by* •6
•2 ss: express in standard form	•2 $-16x^{-\frac{1}{2}}$
•3 ss: start to differentiate	•3 $\frac{dy}{dx} = 1\ldots$
•4 pd: diff. fractional negative power	•4 $\ldots + 8x^{-\frac{3}{2}}$
•5 ss: find gradient of tangent	•5 $m_{x=4} = 2$
•6 ic: write down equ. of tangent	•6 $y - (-4) = 2(x - 4)$

[SQA] 31. A ball is thrown vertically upwards.

After t seconds its height is h metres, where $h = 1{\cdot}2 + 19{\cdot}6t - 4{\cdot}9t^2$.

(*a*) Find the speed of the ball after 1 second. **3**

(*b*) For how many seconds is the ball travelling upwards? **2**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(a)	3	1.3					1	2	1.3.5	1.3.6	Source
(b)	2	1.3						2	1.3.5	1.3.6	1998 P1 qu.17

•1 $\frac{dh}{dt} = \ldots\ldots$

•2 $19.6 - 9.8t$

•3 9.8

•4 $\frac{dh}{dt} = 0$

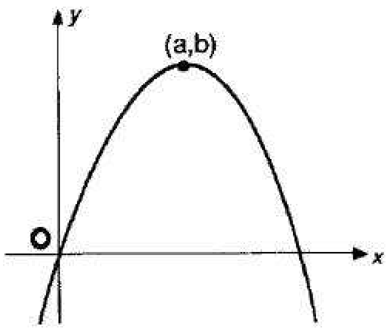
•5 $t = 2$

Alternative

•4 $h(t)$ is a parabola which is **symmetric** about its **maximum**

•5 (*e.g.*) $h(1) = 15{\cdot}9,\ h(2) = 20.8,\ h(3) = 15{\cdot}9$ *so* $t = 2$

[SQA] 32. The line with equation $y = x$ is a tangent at the origin to the parabola with equation $y = f(x)$. The parabola has a maximum turning point at (a, b).
Sketch the graph of $y = f'(x)$. **4**



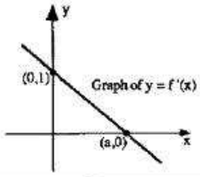
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
.	4	1.3					1	3	1.3.7	1.2.4	Source 1992 P1 qu.19

•[1] $f'(a) = 0$

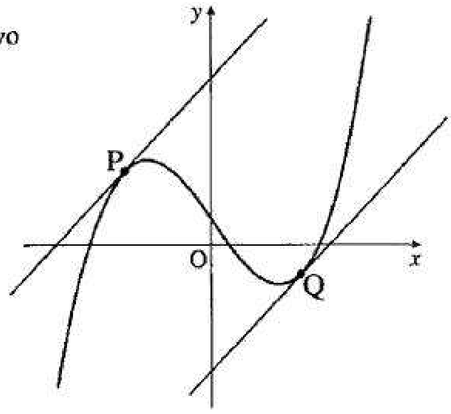
•[2] m_{tgt} at $(0,0) = 1$

•[3] $f'(0) = 1$

•[4] for the sketch



[SQA] 33. The diagram shows a sketch of the graph of $y = x^3 - 9x + 4$ and two parallel tangents drawn at P and Q.



(a) Find the equations of the tangents to the curve $y = x^3 - 9x + 4$ which have gradient 3. **6**

(b) Show that the shortest distance between the tangents is $\frac{16\sqrt{10}}{5}$. **6**

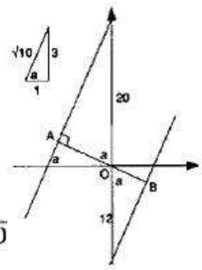
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	1.3
(a)	6	1.3					6		1.3.9	1.1.7	Source 1999 Paper 2 Qu. 11
(b)	6	1.1						6	1.1.10		

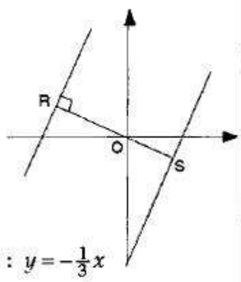
(a)
- •¹ strategy: $\frac{dy}{dx} = \ldots\ldots = 3$
- •² $3x^2 - 9$
- •³ $x = 2,\ -2$
- •⁴ $y = -6,\ 14$
- •⁵ $y + 6 = 3(x - 2)$
- •⁶ $y - 14 = 3(x + 2)$

OR

- •³ $x = 2,\ y = -6$
- •⁴ $x = -2,\ y = 14$

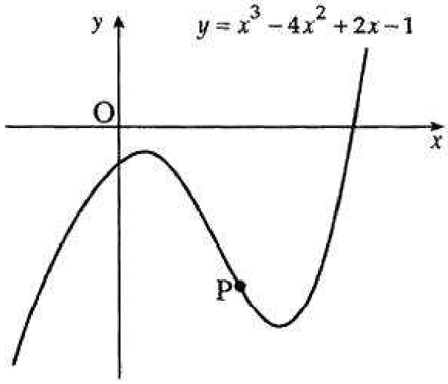
(b)
- •⁷ diagram with $y = -\frac{1}{3}x$
- •⁸ for 20 and 12
- •⁹ $AB = AO + OB$
- •¹⁰ $AB = 20\cos a + 12\cos a$
- •¹¹ using $\tan a = \frac{3}{1}$
- •¹² $AB = 32 \times \frac{1}{\sqrt{10}} = 32 \times \frac{\sqrt{10}}{10} = \frac{32}{5}\sqrt{10}$



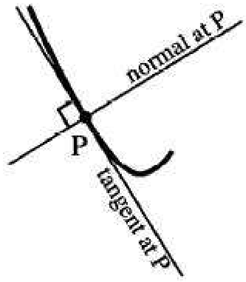


- •⁷ $m_{RS} = -\frac{1}{3}$
- •⁸ equ of RS : $y = -\frac{1}{3}x$
- •⁹ $-\frac{1}{3}x = 3x - 12$ & $-\frac{1}{3}x = 3x + 20$
- •¹⁰ $R(-6, 2)$ and $S\left(\frac{18}{5}, -\frac{6}{5}\right)$
- •¹¹ $d^2 = \left(-6 - \left(\frac{18}{5}\right)\right)^2 + \left(2 - \left(-\frac{6}{5}\right)\right)^2$
- •¹² $d^2 = \frac{48^2}{25} + \frac{16^2}{25}$ and completes proof

[SQA] 34. (*a*) The diagram shows an incomplete sketch of the curve with equation $y = x^3 - 4x^2 + 2x - 1$. Find the equation of the tangent to the curve at the point P where $x = 2$. **(5)**



(*b*) The normal to the curve at P is defined as the straight line through P which is perpendicular to the tangent to the curve at P.
Find the angle which the normal at P makes with the positive direction of the x-axis. **(2)**

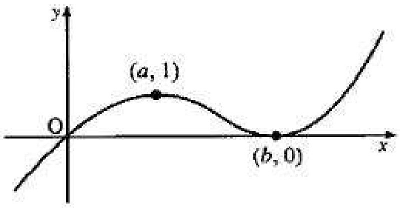


part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	1.3
(*a*)	5	1.3					5		1.1.7, 1.3.9, 1.1.6	Source **1998 Paper 2**
(*b*)	2	1.1					2		1.1.3, 1.1.9	**Qu. 3**

(*a*) •1 $\frac{dy}{dx} = \ldots$
•2 $3x^2 - 8x + 2$
•3 gradient $= -2$ (calculated from $\frac{dy}{dx}$)
•4 $y_A = -5$
•5 $y + 5 = -2(x - 2)$

(*b*) •6 $m_{\text{normal}} = \frac{1}{2}$
•7 angle $= \tan^{-1}\frac{1}{2}$

[SQA] 35. A sketch of the graph of the cubic function f is shown. It passes through the origin, has a maximum turning point at $(a, 1)$ and a minimum turning point at $(b, 0)$.



(a) Make a copy of this diagram and on it sketch the graph of $y = 2 - f(x)$, indicating the coordinates of the turning points. 3

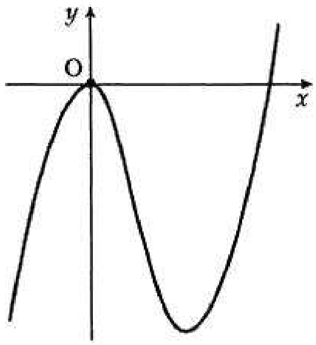
(b) On a separate diagram sketch the graph of $y = f'(x)$. 2

(c) The tangent to $y = f(x)$ at the origin has equation $y = \frac{1}{2}x$. Use this information to write down the coordinates of a point on the graph of $y = f'(x)$. 1

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.2
(a)	3	1.2					3		1.2.4		Source
(b)	2	1.2						2	1.2.4		1998 P1 qu.13
(c)	1	1.2						1	1.3.8		

•1	clear evidence of reflection in $y = 0$	•4	roots at $x = a$ and $x = b$
•2	clear evidence of translation $\begin{pmatrix}0\\2\end{pmatrix}$ subsequent to a reflection	•5	parabolic shape with min. turning point between the roots and no other turning points
•3	indication of passing through $(a,1)$ and $(b,2)$	•6	$\left(0, \frac{1}{2}\right)$

[SQA] 36. (*a*) The diagram shows a part of the curve with equation $y = 2x^2(x-3)$. Find the coordinates of the stationary points on the graph and determine their nature. **(5)**



(*b*) State the range of values of k for which $y = k$ intersects the graph in three distinct points. **(2)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.3
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
(*a*)	5	1.3					5		1.3.12		1991 Paper 2
(*b*)	2	1.2					2		1.2.1		Qu. 1

(*a*) $\bullet^1$ $\frac{dy}{dx} = 6x^2 - 12x$

$\bullet^2$ $\frac{dy}{dx} = 0$

$\bullet^3$ $x = 0,\ x = 2$

$\bullet^4$

x	0^-	0	0^+	2^-	2	2^+
$\frac{dy}{dx}$	+	−	+	+	−	+

$\bullet^5$ max. at $(0,0)$ min at $(2,-8)$

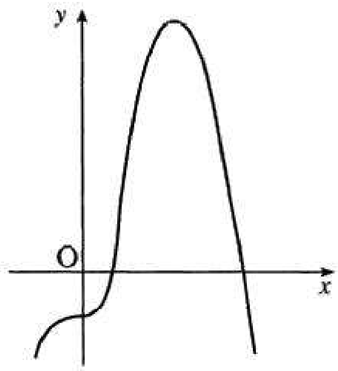
(*b*) $\bullet^6$ $k < 0$

$\bullet^7$ $k > -8$

[SQA] 37. A curve has equation $y = -x^4 + 4x^3 - 2$. An incomplete sketch of the graph is shown in the diagram.

(a) Find the coordinates of the stationary points. **(6)**

(b) Determine the nature of the stationary points. **(2)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(a)	6	1.3					6		1.3.12		Source 1998 Paper 2 Qu. 2
(b)	2	1.3					2		1.3.12		

(a)
- •1 $\frac{dy}{dx} = \ldots\ldots$ *stated or implied* by •2
- •2 $-4x^3 + 12x^2$
- •3 $-4x^3 + 12x^2 = 0$ *or* $\frac{dy}{dx} = 0$ *explicitly stated*
- •4 $-4x^2(x-3)$ (*accept* $x^2(-4x+12)$)
- •5 $x = 0$ *and* 3
- •6 $y = -2$ *and* 25

(b)
- •7

x	0^-	0	0^+	3	3^+
$\frac{dy}{dx}$					

- •8 $+ \quad 0 \quad + \quad 0 \quad -$

  PI at $x = 0$, max *at* $x = 3$

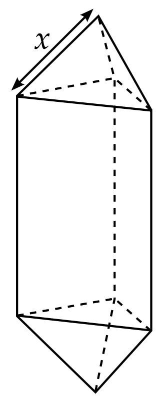
[SQA] 38. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.

The surface area, A, of the solid is given by

$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

where x is the length of each edge of the tetrahedron.

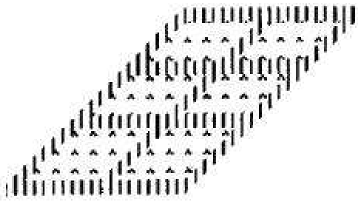
Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid. **6**

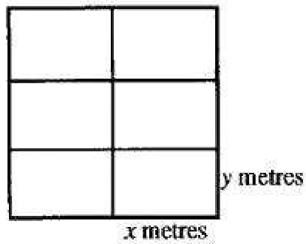


Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	6	A/B	CN	C11	$x = 2$	2000 P2 Q6

•1 ss: know to differentiate •2 pd: process •3 ss: know to set $f'(x) = 0$ •4 pd: deal with x^{-2} •5 pd: process •6 ic: check for minimum	•1 $A'(x) = \ldots$ •2 $\frac{3\sqrt{3}}{2}(2x - 16x^{-2})$ **or** $3\sqrt{3}x - 24\sqrt{3}x^{-2}$ •3 $A'(x) = 0$ •4 $-\frac{16}{x^2}$ **or** $-\frac{24\sqrt{3}}{x^2}$ •5 $x = 2$ •6 $\begin{array}{c\|ccc} x & 2^- & 2 & 2^+ \\ \hline A'(x) & -ve & 0 & +ve \end{array}$ so $x = 2$ is min.

[SQA] 39. A zookeeper wants to fence off six individual animal pens.





Each pen is a rectangle measuring x metres by y metres, as shown in the diagram.

(*a*) (i) Express the total length of fencing in terms of x and y.

(ii) Given that the total length of fencing is 360m, show that the total area, A m^2, of the six pens is given by $A(x) = 240x - \frac{16}{3}x^2$. **4**

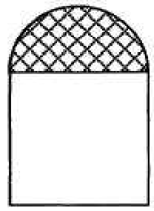
(*b*) Find the values of x and y which give the maximum area and write down this maximum area. **6**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(*a*)	4	0.1					2	2	0.1		Source
(*b*)	6	1.3					6		1.3.15		1999 Paper 2 Qu. 5

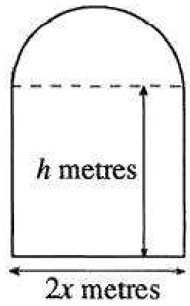
(*a*)	•1	$9y + 8x$
	•2	$A = 3y \times 2x$
	•3	$9y = (360 - 8x)$
	•4	$2x.3.\frac{1}{9}(360 - 8x)$ and complete proof
(*b*)	•5	$A'(x) = \ldots\ldots$
	•6	$240 - \frac{32}{3}x$
	•7	$A'(x) = 0$ *or* $240 - \frac{32}{3}x = 0$
	•8	$x = 22\frac{1}{2}$, $y = 20$
	•9	see table below
	•10	2700

x	$22\frac{1}{2}^-$	$22\frac{1}{2}$	$22\frac{1}{2}^+$
$A'(x)$	+	0	–
		maximum	

[SQA] 40. A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light.
The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.



The rectangle measures $2x$ metres by h metres.



(*a*) (i) If the perimeter of the whole window is 10 metres, express h in terms of x. **(2)**

(ii) Hence show that the amount of light, L, let in by the window is given by $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$. **(2)**

(*b*) Find the values of x and h that must be used to allow this design to let in the maximum amount of light. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	1.3
(*a*)	4	0.1					1	3	0.1		Source
(*b*)	5	1.3					2	3	1.3.15		**1996 Paper 2 Qu.11**

(*a*)
•1 eg $2h + 2x + \text{semicircle} = 10$
•2 $h = \frac{1}{2}(10 - \pi x - 2x)$
•3 $L = 2 \times 2xh + \frac{1}{2}\pi x^2$
•4 $L = 4x \times \frac{1}{2}(10 - \pi x - 2x) + \frac{1}{2}\pi x^2$
$L = 20x - 4x^2 - \frac{3}{2}\pi x^2$

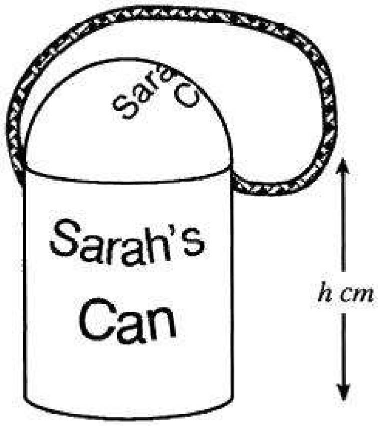
(*b*)
•5 $L' = 20 - 8x - 3\pi x$
•6 $L' = 0$
•7 $x = \dfrac{20}{3\pi + 8} = x_0$ $(= 1.148)$
•8

x	x_0^-	x_0	x_0^+
L'	+	0	−

maximum at x_0

•9 $h = \dfrac{5\pi + 20}{3\pi + 8}$ $(= 2.049)$

[SQA] 41. A child's drinking beaker is in the shape of a cylinder with a hemispherical lid and a circular flat base. The radius of the cylinder is r cm and the height is h cm. The volume of the cylinder is 400 cm^3.



(a) Show that the surface area of plastic, $A(r)$, needed to make the beaker is given by $A(r) = 3\pi r^2 + \dfrac{800}{r}$. **(3)**

Note: The curved surface area of a hemisphere of radius r is $2\pi r^2$.

(b) Find the value of r which ensures that the surface area of plastic is minimised. **(6)**

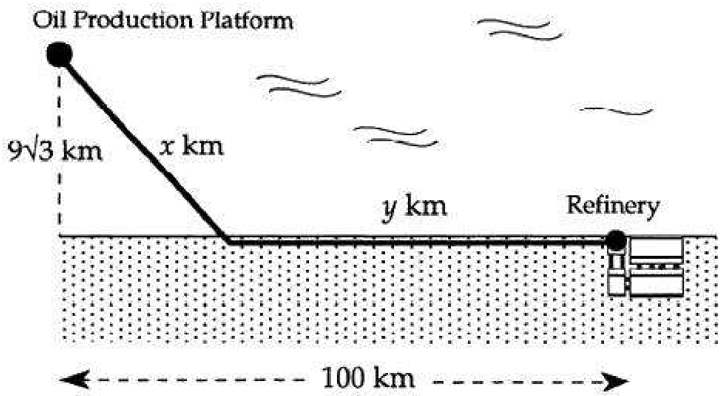
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(a)	3	0.1				3			0.1		Source
(b)	6	1.3			3	3			1.3.15		**1998 Paper 2 Qu. 10**

(a)
- •¹ $\pi r^2 + 2\pi rh + 2\pi r^2$
- •² $h = \frac{400}{\pi r^2}$ or equivalent (*e.g.* $\pi rh = \frac{400}{r}$)
- •³ $2\pi r \frac{400}{\pi r^2} + 3\pi r^2$ and completes proof

(b)
- •⁴ $\frac{dA}{dr} = \ldots$
- •⁵ $800r^{-1}$
- •⁶ $6\pi r - 800r^{-2}$
- •⁷ e.g. $6\pi r - \frac{800}{r^2} = 0$
- •⁸ 3.5
- •⁹

r	3.5^-	3.5	3.5^+
$\frac{dA}{dr}$	$-ve$	0	$+ve$

[SQA] 42. An oil production platform, $9\sqrt{3}$ km offshore, is to be connected by a pipeline to a refinery on shore, 100 km down the coast from the platform as shown in the diagram.



The length of underwater pipeline is x km and the length of pipeline on land is y km. It costs £2 million to lay each kilometre of pipeline underwater and £1 million to lay each kilometre of pipeline on land.

(*a*) Show that the total cost of this pipeline is £$C(x)$ million where

$$C(x) = 2x + 100 - \left(x^2 - 243\right)^{\frac{1}{2}}.$$ **(3)**

(*b*) Show that $x = 18$ gives a minimum cost for this pipeline.
Find this minimum cost and the corresponding total length of the pipeline. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.2
(*a*)	3	0.1	1	2					0.1		Source 1993 Paper 2 Qu.11
(*b*)	7	1.3	1	6					1.3.15,	3.2.2	

(*a*) •1 $C = 2x + y$

•2 $\sqrt{x^2 - (9\sqrt{3})^2}$

•3 for completing proof

(*b*) •4 knowing to differentiate

•5 $\frac{1}{2}\left(x^2 - 243\right)^{-\frac{1}{2}}$

•6 $\times\ 2x$

•7 $C'(18) = 0$

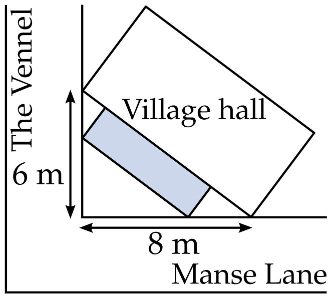
•8 justification of minimum e.g. nature table

•9 $C = 127$

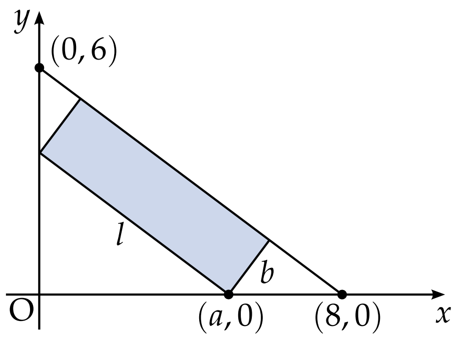
•10 $x + y = 109$

	18^-	18	18^+
$C'(x)$	$-$	0	$+$
	\	—	/
		minimum	

[SQA] 43. The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a, 0)$.



(*a*) (i) Show that $l = \frac{5}{4}a$.

(ii) Express b in terms of a and hence deduce that the area, A m^2, of the extension is given by $A = \frac{3}{4}a(8-a)$. **3**

(*b*) Find the value of a which produces the largest area of the extension. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	3	A/B	CN	0.1	proof	2002 P2 Q10
(*b*)	4	A/B	CN	C11	$a = 4$	

•1 ss: select strategy and carry through	•1 proof of $l = \frac{5}{4}a$
•2 ss: select strategy and carry through	•2 $b = \frac{3}{5}(8-a)$
•3 ic: complete proof	•3 complete proof leading to $A = \ldots$
•4 ss: know to set derivative to zero	•4 $\frac{dA}{da} = \ldots = 0$
•5 pd: differentiate	•5 $6 - \frac{3}{2}a$
•6 pd: solve equation	•6 $a = 4$
•7 ic: justify maximum, e.g. nature table	•7 e.g. nature table, comp. the square

[SQA] **44.** Diagram 1 is an artist's impression of a new warehouse based on the architect's plans. The warehouse is in the shape of a cuboid and is supported by three identical parabolic girders spaced 30 metres apart.
With coordinate axes as shown in Diagram 2, the shape of each girder can be described by the equation $y = 9 - \frac{1}{4}x^2$.

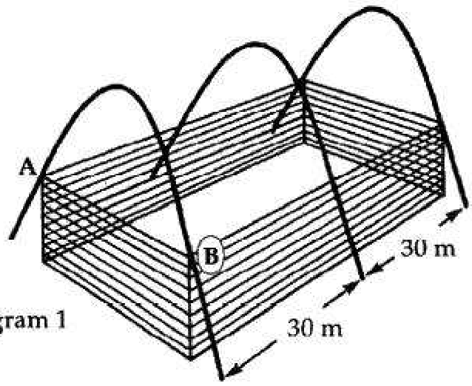


Diagram 1

(*a*) Given that AB is $2x$ metres long, show that the shaded area in Diagram 2 is $\left(18x - \frac{1}{2}x^2\right)$ square metres.

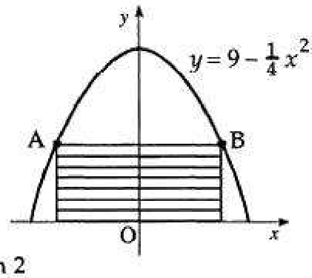


Diagram 2

(2)

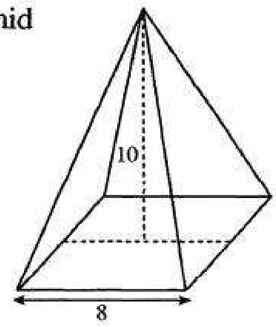
(*b*) The architect wished to fit into the girders the cuboidal warehouse which had the maximum volume. Find the value of this maximum volume. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	1.3
(*a*)	2	0.1					2		0.1		Source
(*b*)	6	1.3					3	3	1.3.15		**1989 Paper 2 Qu. 7**

(*a*)
- •[1] $B = (x, y)$ where $y = 9 - \frac{1}{4}x^2$
- •[2] area $= 2x\left(9 - \frac{1}{4}x^2\right)$

(*b*)
- •[3] $V = 1080x - 30x^3$
- •[4] $\frac{dV}{dx} = 1080 - 90x^2$
- •[5] $\frac{dV}{dx} = 0$ **stated explicitly**
- •[6] $x = 2\sqrt{3}$
- •[7] $\begin{array}{cccc} x & 2\sqrt{3}^- & 2\sqrt{3} & 2\sqrt{3}^+ \\ \frac{dV}{dx} & + & 0 & - \end{array}$
- •[8] max at $x = 2\sqrt{3}$ of $1440\sqrt{3}$

[SQA] 45. A cuboid is to be cut out of a right square-based pyramid. The pyramid has a square base of side 8 cm. and a vertical height of 10cm.

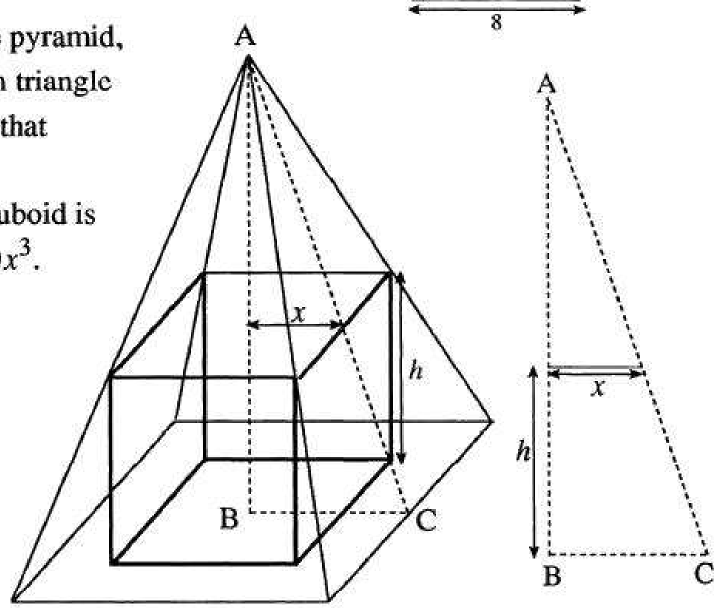


(*a*) The cuboid has a square base of side $2x$ cm and a height of h cm.

If the cuboid is to fit into the pyramid, use the information shown in triangle ABC, or otherwise, to show that

(i) $h = 10 - \frac{5}{2}x$. **(3)**

(ii) the volume, V, of the cuboid is given by $V = 40x^2 - 10x^3$. **(1)**



(*b*) Hence find the dimensions of the square-based cuboid with the greatest volume which can be cut from the pyramid. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	1.3
(*a*)	4	0.1					1	3	0.1		Source
(*b*)	6	1.3					3	3	1.3.15		**1997 Paper 2 Qu.10**

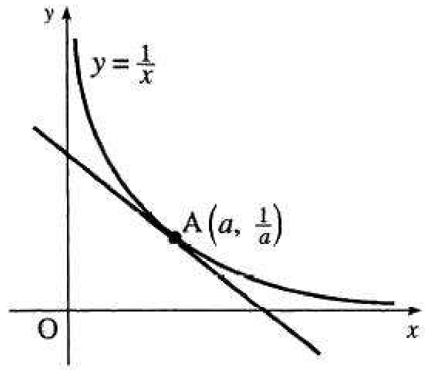
(*a*)
- •1 strategy: e.g. equate ratios from similar triangles
- •2 $\frac{10}{4} = \frac{10-h}{x}$ or equivalent
- •3 complete proof
- •4 $V = 40x^2 - 10x^3$

(*b*)
- •5 $\frac{dV}{dx} =$
- •6 $80x - 30x^2$
- •7 $\frac{dV}{dx} = 0$ for stationary points
- •8 $0, \frac{8}{3}$
- •9

x	…	$\frac{8}{3}$	…
$\frac{dV}{dx}$	+	0	−
		max	

- •10 $\frac{16}{3}$ and $\frac{10}{3}$

[SQA] 46. (*a*) A sketch of part of the graph of $y = \frac{1}{x}$ is shown in the diagram. The tangent at A $\left(a, \frac{1}{a}\right)$ has been drawn.
Find the gradient of this tangent. **(4)**



(*b*) Hence show that the equation of this tangent is $x + a^2y = 2a$. **(2)**

(*c*) This tangent cuts the *y*-axis at B and the *x*-axis at C.

(i) Calculate the area of triangle OBC **(3)**

(ii) Comment on your answer to *c*(i). **(1)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	1.3
(*a*)	4	1.3					4		1.3.7		Source
(*b*)	2	1.1					1	1	1.1.7		1997 Paper 2
(*c*)	4	0.1						4	0.1		Qu.6

(*a*)
•[1] $\frac{1}{x} = x^{-1}$
•[2] $\frac{dy}{dx} = \ldots\ldots$
•[3] $\frac{dy}{dx} = -x^{-2}$
•[4] gradient $= -a^{-2}$

(*b*)
•[5] use $y - \frac{1}{a} = -\frac{1}{a^2}(x-a)$
•[6] $a^2y - a = -(x-a)$ and completes proof

(*c*)
•[7] $y_B = \frac{2a}{a^2}$
•[8] $x_A = 2a$
•[9] 2
•[10] independent of *a*

[END OF QUESTIONS]