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ANNULAR APERTURE DIFFRACTED ENERGY DISTRIBUTION FOR AN EXTENDED SOURCE

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GODDARD SPACE FLIGHT CENTER
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ABSTRACT

The annular aperture diffracted energy within a circular area in the image plane for an extended circular incoherent source is obtained by adding the diffraction contributions from all the elemental areas making up the source surface. Each of the source elemental areas is treated as a distant point source so that the formula for annular aperture Fraunhofer diffraction intensity may be used. The necessary integration for both the point and extended source cases were carried out numerically by means of the IBM 360 computer and the results are listed for source angular sizes up to five times the angular size of the Airy disk. Assuming that the angular diameter of the circular area in the image plane containing 83.8% of the diffracted energy from the extended source is the effective angular resolution of a scanning optical instrument it is shown that for a central obstruction diameter 40% of the aperture diameter the effective angular resolution is degraded by less than 30% when the instantaneous field of view is increased from 1 to 2 (angular) Airy diameters.

ANNULAR APERTURE DIFFRACTED ENERGY DISTRIBUTION FOR AN EXTENDED SOURCE

I. INTRODUCTION

The diffracted intensity distribution in the image plane for a clear circular aperture and distant point source (Fraunhofer diffraction) was first derived by Airy. Lord Rayleigh integrated Airy's intensity distribution function to find the energy lying within a circular area in the image plane. When a circular obstruction, such as that due to the secondary mirror in a folded optical system, is introduced to form an annular aperture the diffracted intensity distribution is altered and the energy distribution (the integral of the intensity distribution) must be evaluated numerically. For extended sources the integration is even less tractable and a modern high speed computer was used to obtain the results reported here.

Two angularly close point sources such as a double star can be resolved even when the Airy disks (central bright area of the diffraction pattern) associated with the point sources overlap. While there is no precise point at which a double star is just discernible Rayleigh formulated a quantitative resolution criterion that is very simple to apply. Using the assumption that two point sources are just resolved when the central maximum of the diffraction pattern of one point falls at the first minimum of the diffraction pattern of the other point, Rayleigh showed that[1]

$$\sin \theta = \frac{1.22\,\lambda}{D} \qquad (1)$$

where θ = smallest angular separation that can be resolved

λ = radiation wavelength

D = aperture diameter.

When λ/D is very small $\sin \theta \approx \theta$ and Eq. (1) becomes

$$\theta = \frac{1.22\,\lambda}{D} \qquad (2)$$

where θ is measured in radians. This is the famous Rayleigh resolution criterion that is valid under the following assumptions:

1. Clear circular aperture
2. Two distant equally intense point sources
3. Perfect optical system (negligible aberrations).

In order to make practical use of the angular resolution given by Eq. (2) other assumptions such as the following must be satisfied:

(a) Sufficient intensity to detect the Airy disks

(b) Sufficient magnification or resolution in the displayed image to resolve the Airy disks

(c) Negligible blurring by the medium between the source and optical system.

For example, the 200-inch Hale Telescope at Mount Palomar has a measured resolution which is an order of magnitude poorer than that given by Rayleigh's criterion.[2]

Because of the overlapping diffraction patterns at the Rayleigh limit, a measurement of the total energy from one of the point sources (of, say a double star) will include a portion of the energy from the other point source. Since most of the diffracted energy is contained within the Airy disk (approximately 84%) two point sources are sometimes said to be completely resolved when their Airy disks do not overlap.[3] For this condition the angular resolution is

$$\theta_d = \frac{2.44\,\lambda}{D} \tag{3}$$

where θ_d is the Airy disk angular diameter.

Unfortunately Eq. (2) or (3) is sometimes used for any "diffraction limited" system and the above restrictions are simply ignored, especially assumptions 1 and 2. The effect of using an annular aperture for observing extended sources will be described in some detail in section III.

II. REVIEW OF CIRCULAR APERTURE FRAUNHOFER DIFFRACTION

a. Clear Aperture

For a clear aperture the normalized diffracted intensity (I_n) in the image plane from a distant point source (plane wave entering the aperture is

$$I_n = \left[\frac{2J_1(k\,a\,\sin\omega)}{k\,a\,\sin\omega}\right]^2 \tag{4a}$$

where J_1 = Bessel function of first order

$k = 2\pi/\lambda$

a = radius of circular aperture

ω = angular distance in the image plane (see Fig. 1)

Eq. (4a) was derived by Airy in 1835.

Setting $q = k\, a \sin \omega$,

$$I = \left[\frac{2J_1(q)}{q}\right]^2 . \tag{4b}$$

The diffraction pattern consists of concentric circular light and dark rings with a maximum at the center. The first minimum occurs at $q \approx 3.832$. The angular radius of the Airy disk ω_A is therefore given by

$$k\, a \sin \omega_A = 3.832 \tag{5}$$

For small angles $\sin \omega_A \approx \omega_A$ and

$$\omega_A = \frac{1.22\,\lambda}{D} \tag{6}$$

where $D = 2a$ = aperture diameter. The length of the Airy radius r_a is

$$r_A = f\omega_A = \frac{1.22\,\lambda f}{D} \tag{7}$$

where f = effective focal length.

If we let $L(\omega_0)$ be the fraction of the total diffracted energy contained within an angular radius ω_0 in the image plane then

$$L(\omega_0) = \frac{\int_0^{2\pi}\int_0^{\omega_0} I\,\omega\, d\omega\, d\psi}{\int_0^{2\pi}\int_0^{\infty} I\,\omega\, d\omega\, d\psi} \tag{8}$$

where ψ is the polar angle in the image plane. Since I is independent of ψ and $q = k\,a\,\omega$ (for ω small)

$$L(q_0) = \frac{\int_0^{q_0} J_1^2(q)\,\frac{dq}{q}}{\int_0^{\infty} J_1^2(q)\,\frac{dq}{q}} \tag{9}$$

now[4]

$$\int_0^{q_0} J_1^2(q)\,\frac{dq}{q} = \frac{1}{2}\left[1 - J_0^2(q_0) - J_1^2(q_0)\right] \tag{10}$$

and since

$$J_0(\infty) = J_1(\infty) = 0$$

$$L(q_0) = 1 - J_0^2(q_0) - J_1^2(q_0) \tag{11}$$

Eq. (11) was derived by Rayleigh in 1881. For $q_0 = 3.832$, $L(3.832) = .838$, so that approximately 84% of the diffracted energy is contained within the Airy disk. About 7.2% of the diffracted energy is in the second maximum ring (first bright ring surrounding the Airy disk). The next two bright rings contain 2.8% and 1.5% of the diffracted energy, respectively.

b. Annular Aperture

For an annular aperture the diffraction intensity distribution is[5]

$$I = \frac{4\,I_0}{(1-\epsilon^2)^2}\left[\frac{J_1(q) - \epsilon J_1(\epsilon q)}{q}\right]^2 \tag{12}$$

where I_0 = intensity at the geometrical image of the source

ϵ = ratio of the radius of the inner circle (central obstruction) to that of the outer circle (aperture).

The positions of the first four zeros (minima) of I for various values of ϵ are shown in Table 1. Although the size of the Airy disk decreases as ϵ increases, so does the energy contained in the Airy disk. As ϵ increases from 0 to .4 nearly all of the energy lost by the Airy disk goes into the next bright ring. At $\epsilon = .4$, 30% of the diffracted energy is in the second maximum ring, while only 58% remains within the Airy disk. Fig. 2 shows how the energy in each of the first four bright rings varies with ϵ.

The normalized diffraction energy L is

$$L = \frac{\int_0^{q_0} [J_1(q) - \epsilon J_1(\epsilon q)]^2 \frac{dq}{q}}{\int_0^{\infty} [J_1(q) - \epsilon J_1(\epsilon q)]^2 \frac{dq}{q}} \qquad (13)$$

As shown in the appendix, the denominator of Eq. (13) is equal to $(1 - \epsilon^2)/2$. The numerator was evaluated numerically. The results are given in Table 2.

III. EXTENDED SOURCE AND ANNULAR APERTURE

We shall assume a uniform circular incoherent source with angular radius ρ and center on the optic axis. The intensity distribution in the image plane due to an elemental area of solid angle $d\Omega$ in the extended source is[5]

$$I \, d\Omega = \frac{4 I_0}{(1 - \epsilon^2)^2} \left[\frac{J_1(k a \omega) - \epsilon J_1(\epsilon k a \omega)}{k a \omega} \right]^2 d\Omega \qquad (14)$$

where $I_0 d\Omega$ = intensity at the geometrical image of the source element $d\Omega$

ω = angular radial distance in the image plane from the geometric "point source" image (see Fig. 3).

The diffracted energy dP_B falling on the circular area of angular radius θ due to the source element at B (see Fig. 3) is

$$dP_B = 2 \int_0^{\theta + \phi} \int_0^{\psi} I\ \omega\ d\omega\ d\psi \tag{15}$$

$$= 2 \int_0^{\theta + \phi} \psi\ I\ \omega\ d\omega \tag{16}$$

where ϕ is the angular distance of point B from the optic axis. The relationship between ψ and the other parameters in Eq. (16) is given by (see Fig. 3)

$$\psi = \cos^{-1} \left[\frac{\omega^2 + \phi^2 - \theta^2}{2\theta\omega} \right], \quad 0 \leq \psi \leq \pi \ . \tag{17}$$

The energy P_θ on the circular area of angular radius θ due to all the elements in the source area is

$$P_\theta = 2\pi \int_0^{\rho} \phi\ dP_B\ d\phi \tag{18}$$

where ρ = angular radius of the geometric source image. Using Eqs. (16) and (14), P_θ can be expressed as

$$P_\theta = \frac{16\pi\ I_0}{(1 - \epsilon^2)^2} \int_0^{\rho} \phi d\phi \int_0^{\theta+\phi} \frac{\psi}{k^2\ a^2\ \omega} \left[J_1(k\ a\ \omega) - \epsilon J_1(\epsilon\ k\ a\ \omega) \right]^2 d\omega \tag{19}$$

For computational purposes it is convenient to let

$$q = k\,a\,\omega,\ \theta' = k\,a\,\theta,\ \phi' = k\,a\,\phi,\ \rho' = k\,a\,\rho \tag{20}$$

ψ is now expressed as

$$\psi = \cos^{-1}\left[\frac{q^2 + \phi'^2 - \theta'^2}{2\,\phi'\,q}\right],\ 0 \leq \psi \leq \pi \tag{21}$$

and Eq. (19) becomes

$$P_{\theta'} = \frac{16\pi\, I_0}{(k\,a)^4\,(1 - \epsilon^2)^2} \int_0^{\rho'} \phi'\, d\phi' \int_0^{\theta'+\phi'} \frac{\psi}{q}\,[J_1(q) - \epsilon J_1(\epsilon q)]^2\, dq\,. \tag{22}$$

The <u>total</u> diffracted energy P_T due to all elements in the source area ($\psi = \pi$ for this case) is

$$P_T = \frac{16\pi^2\, I_0}{(k\,a)^4\,(1 - \epsilon^2)^2} \int_0^{\rho'} \phi'\, d\phi' \int_0^{\infty} [J_1(q) - \epsilon J_1(\epsilon q)]^2 \frac{dq}{q} \tag{23a}$$

$$= \frac{16\,\pi^2\, I_0}{(k\,a)^4\,(1 - \epsilon^2)^2} \left(\frac{\rho'^2}{2}\right) \frac{(1 - \epsilon^2)}{2}$$

$$= \frac{4\pi^2\,\rho'^2\, I_0}{(k\,a)^4\,(1 - \epsilon^2)}\,. \tag{23b}$$

Since $L(\theta') = P_{\theta'}/P_T$ = fraction of the total diffracted energy contained within a circle of angular radius θ' in the image plane,

$$L(\theta') = \frac{4}{\pi\rho'^2(1 - \epsilon^2)} \int_0^{\rho'} \phi'\, d\phi' \int_0^{\theta'+\phi'} \frac{\psi}{q}\,[J_1(q) - \epsilon J_1(\epsilon q)]^2\, dq \tag{24}$$

in order to evaluate this integral numerically, three cases must be distinguished.

1. $\theta' \geq \rho'$

$$L_1(\theta') = \frac{4}{\pi \rho'^2(1-\epsilon^2)} \int_0^{\rho'} \phi' \, d\phi' \left\{ \pi \int_0^{\theta'-\phi'} [J_1(q) - \epsilon J_1(\epsilon q)]^2 \frac{dq}{q} \right.$$

$$\left. + \int_{\theta'-\phi'}^{\theta'+\phi'} \frac{\psi}{q} [J_1(q) - \epsilon J_1(\epsilon q)]^2 \, dq \right\}$$

2. $\theta' \leq \rho'$, $\phi' \leq \theta'$

$$L_2(\theta') = \frac{4}{\pi \rho'^2(1-\epsilon^2)} \int_0^{\theta'} \phi' \, d\phi' \left\{ \pi \int_0^{\theta'-\phi'} [J_1(q) - \epsilon J_1(\epsilon q)]^2 \frac{dq}{q} \right.$$

$$\left. + \int_{\theta'-\phi'}^{\theta'+\phi'} \frac{\psi}{q} [J_1(q) - \epsilon J_1(\epsilon q)]^2 \, dq \right\}$$

3. $\theta' \leq \rho'$, $\phi' \geq \theta'$

$$L_3(\theta') = \frac{4}{\pi \rho'^2(1-\epsilon^2)} \int_0^{\rho'} \phi' \, d\phi' \int_{\phi'-\theta'}^{\phi'+\theta'} \frac{\psi}{q} [J_1(q) - \epsilon J_1(\epsilon q)]^2 \, dq \quad .$$

L_1 and $L_2 + L_3$ were numerically integrated by means of the IBM 360 computer and the results are given in Tables 3, 4 and 5. A plot of L vs θ' for $\epsilon = .4$ is shown in Fig. 4.

Discussions of the diffracted intensity or irradiance distribution for $\epsilon = 0$ are given in references 7 and 8.

By analogy to the point source, clear aperture case the size of the circular area in the image plane that contains 83.8% of the diffracted energy can be used as a criterion for the effective resolution of an optical system for an extended source.

For illustrative purposes we shall assume that the angular resolution of the optical system is measured by its instantaneous field of view (IFOV). This is generally the case for a scanner containing a "point" detector. For such an instrument the angular source size corresponds to the geometric IFOV (when the source completely fills the IFOV). We also assume that the angular diameter of the circular area in the image plane containing 83.8% of the diffracted energy from the source (of angular diameter 2ρ) is the effective angular resolution θ_E. Under the above assumptions θ_E is completely determined by the four parameters λ, D, ρ and ϵ (in practice, for $\epsilon > 0$, other obscurations such as the spider which supports the secondary mirror must be considered). The values of θ_E can be obtained from Tables 3, 4 and 5. For example, to determine θ_E for $\epsilon = .2$ and $\rho = 1$ Airy unit, look at the Table 4 section headed by $\epsilon = 0.2$ and under the $\rho = 1$ column find (by interpolation) the .838 energy point and the corresponding θ value of 1.61 Airy units. The value of θ_E is twice this value (θ is the radius while θ_E is the diameter of the circle containing 83.8% of the diffracted energy) or 3.22 Airy units.

Therefore

$$\theta_E(.2,\ 1) = \frac{3.9\lambda}{D}$$

where the numbers inside the parenthesis are the values of ϵ and ρ (Airy units) respectively.

Values of θ_E for $\epsilon = 0$ and $\epsilon = 0.4$ are given below.

1. <u>$\epsilon = 0$ (Clear Aperture)</u>

$$\theta_E(0,\ 1) = \frac{3.4\lambda}{D}$$

$$\theta_E(0,\ 2) = \frac{5.2\lambda}{D}$$

$$\theta_E(0,\ 4) = \frac{9.4\lambda}{D}$$

2. <u>$\epsilon = 0.4$</u>

$$\theta_E(.4,\ 1) = \frac{4.9\lambda}{D}$$

$$\theta_E(.4,\ 2) = \frac{6.3\lambda}{D}$$

$$\theta_E(.4,\ 4) = \frac{10\lambda}{D}$$

It is interesting to note that in the region of small source sizes as ρ increases, θ_E does not increase proportionately. This is of particular importance in the measurement of weak signals with instruments for which the signal-to-noise ratio (S/N) varies as ρ^2. For example, with $\epsilon = 0.4$, as ρ increases from 1 to 2 Airy units, the S/N in such an instrument increases by a factor of 4 while the effective angular resolution is degraded by less than 30%. In the region of large values of ρ, θ_E increases as fast as ρ.

APPENDIX

Evaluation of $$I_\infty = \int_0^\infty [J_1(q) - \epsilon J_1(\epsilon q)]^2 \frac{dq}{q}$$

$$I_\infty = \int_0^\infty J_1^2(q) \frac{dq}{q} + \epsilon^2 \int_0^\infty J_1^2(\epsilon q) \frac{d(\epsilon q)}{\epsilon q} - 2\epsilon \int_0^\infty J_1(q) J_1(\epsilon q) \frac{dq}{q}$$

$$= (1 + \epsilon^2) \int_0^\infty J_1^2(q) \frac{dq}{q} - 2\epsilon \int_0^\infty J_1(q) J_1(\epsilon q) \frac{dq}{q} \quad .$$

It is shown in books containing integrals of Bessel functions[6] that

$$\int_0^\infty J_n(at)\, J_n(bt) \frac{dt}{t} = \begin{cases} \dfrac{(b/a)^n}{2n} & , \; b \leq a \\ \dfrac{(a/b)^n}{2n} & , \; b \geq a \end{cases}$$

$$\therefore \int_0^\infty J_1(q)\, J_1(\epsilon q) \frac{dq}{q} = \frac{\epsilon}{2}$$

Also, $$\int_0^\infty J_1^2(q) \frac{dq}{q} = \frac{1}{2}$$

$$\therefore \quad I_\infty = (1 + \epsilon^2)\left(\frac{1}{2}\right) - 2\epsilon\left(\frac{\epsilon}{2}\right) = \frac{1 - \epsilon^2}{2}$$

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Table 1

Positions of Minima for Annular Aperture Fraunhofer

Diffraction for Various Values of $\epsilon \cdot q = \pi D \omega / \lambda$

ϵ	q			
	First Minimum	Second Minimum	Third Minimum	Fourth Minimum
0	3.832	7.016	10.173	13.324
.1	3.786	7.128	9.998	13.573
.2	3.665	7.405	9.703	13.732
.3	3.501	7.616	9.725	13.247
.4	3.323	7.501	10.370	12.683
.5	3.144	7.183	10.965	12.953
.6	2.974	6.818	10.646	14.211
.7	2.814	6.458	10.118	13.766
.8	2.667	6.120	9.594	13.071
.9	2.530	5.808	9.105	12.406

Table 2

Annular Aperture Diffraction from Point Source. One Airy Unit = $1.22\lambda/D$. ϵ is the Ratio of the Inner to Outer Radii (of the Annular Aperture).

$$L = \frac{2}{1-\epsilon^2} \int_0^{q_0} [J_1(q) - \epsilon J_1(\epsilon q)]^2 \frac{dq}{q}, \quad q = \left(\frac{\pi D}{\lambda}\right)\omega$$

ω (in Airy units)	L						
	$\epsilon = 0$	$\epsilon = .2$	$\epsilon = .3$	$\epsilon = .4$	$\epsilon = .5$	$\epsilon = .7$	$\epsilon = .9$
0.1	.036	.035	.033	.030	.027	.018	.007
0.2	.136	.131	.123	.113	.100	.067	.024
0.3	.281	.268	.251	.229	.202	.132	.047
0.4	.440	.418	.390	.353	.306	.194	.066
0.5	.588	.554	.513	.459	.393	.240	.078
0.6	.705	.659	.604	.533	.449	.263	.082
0.7	.782	.724	.657	.571	.474	.269	.082
0.8	.822	.755	.679	.583	.479	.269	.085
0.9	.836	.763	.682	.584	.480	.279	.095
1.0	.838	.764	.684	.590	.493	.307	.113
1.1	.839	.769	.695	.611	.526	.355	.136
1.2	.846	.783	.720	.650	.580	.414	.158
1.3	.860	.807	.757	.703	.646	.473	.175
1.4	.877	.836	.799	.760	.712	.520	.182
1.5	.893	.862	.838	.811	.766	.548	.184
1.6	.904	.883	.868	.849	.803	.559	.184
1.7	.909	.894	.887	.872	.822	.560	.190
1.8	.910	.899	.896	.882	.828	.563	.203
1.9	.910	.900	.899	.885	.828	.576	.224
2.0	.912	.900	.899	.885	.830	.603	.247

Table 2 (continued)

ω (in Airy units)	L						
	$\epsilon = 0$	$\epsilon = .2$	$\epsilon = .3$	$\epsilon = .4$	$\epsilon = .5$	$\epsilon = .7$	$\epsilon = .9$
2.2	.922	.903	.901	.889	.849	.684	.279
2.4	.934	.907	.904	.898	.879	.746	.284
2.6	.937	.907	.904	.902	.897	.761	.294
2.8	.938	.912	.907	.903	.901	.765	.330
3.0	.943	.924	.917	.904	.902	.794	.368
3.2	.950	.939	.926	.906	.903	.837	.379
3.4	.952	.946	.929	.906	.903	.861	.383
3.6	.952	.947	.930	.910	.904	.865	.410
3.8	.955	.948	.935	.921	.907	.868	.449
4.0	.959	.950	.943	.933	.909	.881	.467
4.5	.962	.952	.949	.939	.916	.899	.503
5.0	.967	.959	.950	.948	.937	.902	.549
5.5	.970	.962	.956	.950	.941	.902	.603
6.0	.972	.966	.960	.950	.949	.903	.625
6.5	.975	.967	.964	.957	.950	.906	.681
7.0	.975	.968	.966	.959	.950	.911	.699
7.5	.978	.974	.966	.965	.952	.918	.739
8.0	.979	.974	.970	.966	.957	.928	.764
8.5	.980	.975	.972	.966	.961	.934	.785
9.0	.982	.977	.974	.968	.964	.942	.814
9.5	.982	.977	.974	.969	.966	.945	.823
10.0	.983	.979	.975	.973	.966	.948	.848

Table 3

Extended Source Diffraction for $\epsilon = 0, .1$. ρ and θ are in Airy Units (one Airy Unit = $1.22\ \lambda/D$).

Relative Diffracted Energy (L)

$\epsilon = 0$

θ \ ρ	.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
.25	.135	.052	.024	.014	.009	.006	.005	.004	.003	.002
.50	.435	.204	.098	.057	.037	.026	.019	.015	.012	.010
.75	.693	.425	.219	.128	.084	.059	.044	.034	.027	.022
1.00	.818	.644	.384	.227	.149	.105	.078	.060	.048	.039
1.25	.859	.794	.570	.352	.232	.163	.121	.093	.074	.060
1.50	.882	.864	.734	.500	.332	.235	.174	.134	.107	.087
1.75	.902	.891	.840	.655	.449	.318	.237	.183	.145	.118
2.00	.915	.907	.890	.785	.580	.414	.309	.239	.190	.154
2.25	.924	.920	.909	.868	.711	.521	.390	.301	.240	.195
2.50	.933	.930	.921	.906	.819	.637	.479	.371	.296	.241
2.75	.939	.936	.932	.921	.886	.751	.576	.448	.357	.291
3.00	.944	.942	.939	.931	.917	.843	.680	.531	.424	.346
3.25	.949	.947	.944	.939	.930	.899	.780	.620	.497	.405
3.50	.952	.951	.949	.945	.939	.926	.860	.714	.574	.469
3.75	.955	.955	.953	.950	.945	.937	.910	.803	.656	.538
4.00	.958	.958	.956	.954	.951	.944	.933	.874	.741	.610
4.25	.961	.960	.959	.957	.954	.950	.942	.918	.822	.685
4.50	.963	.963	.962	.960	.958	.955	.949	.938	.886	.763
4.75	.965	.965	.964	.963	.961	.958	.954	.947	.924	.837
5.00	.967	.966	.966	.965	.963	.961	.958	.952	.942	.895
5.25	.968	.968	.968	.967	.965	.964	.961	.957	.950	.930
5.50	.970	.970	.969	.968	.967	.966	.964	.961	.956	.946
5.75	.971	.971	.970	.970	.969	.968	.966	.963	.960	.953
6.00	.972	.972	.972	.971	.970	.969	.968	.966	.963	.958

Table 3 (continued)

$\epsilon = 0.1$

θ \ ρ	.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
.25	.133	.051	.024	.014	.009	.006	.005	.004	.003	.002
.50	.428	.200	.097	.057	.037	.026	.019	.015	.012	.010
.75	.680	.417	.216	.127	.083	.058	.043	.033	.027	.022
1.00	.802	.634	.379	.224	.147	.104	.077	.059	.047	.038
1.25	.844	.782	.562	.348	.230	.162	.120	.093	.074	.060
1.50	.872	.853	.724	.494	.329	.233	.172	.134	.106	.086
1.75	.895	.880	.829	.647	.444	.316	.235	.182	.145	.118
2.00	.909	.897	.879	.776	.573	.410	.306	.237	.189	.154
2.25	.916	.912	.899	.858	.703	.516	.387	.299	.238	.194
2.50	.922	.921	.913	.896	.810	.631	.475	.369	.294	.240
2.75	.928	.928	.924	.912	.877	.743	.572	.445	.355	.290
3.00	.935	.934	.931	.923	.908	.834	.674	.527	.422	.344
3.25	.942	.940	.936	.932	.922	.891	.773	.616	.494	.403
3.50	.948	.944	.941	.938	.931	.918	.853	.708	.570	.467
3.75	.950	.948	.946	.943	.939	.929	.902	.797	.651	.535
4.00	.952	.952	.950	.947	.944	.937	.925	.867	.736	.606
4.25	.954	.955	.954	.951	.948	.944	.935	.911	.816	.681
4.50	.956	.957	.957	.954	.952	.949	.943	.931	.879	.758
4.75	.960	.960	.959	.958	.955	.952	.948	.940	.918	.831
5.00	.963	.962	.961	.960	.958	.956	.953	.947	.936	.889
5.25	.966	.964	.963	.962	.961	.959	.956	.952	.945	.924
5.50	.966	.966	.965	.964	.963	.961	.959	.956	.950	.940
5.75	.967	.968	.967	.966	.965	.964	.962	.959	.955	.948
6.00	.968	.969	.969	.968	.967	.966	.964	.961	.959	.953

Table 4

Extended Source Diffraction for ϵ = .2, .3. ρ and θ are in Airy Units.

Relative Diffracted Energy

$\epsilon = 0.2$

θ \ ρ	.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
.25	.127	.047	.024	.014	.009	.006	.005	.004	.003	.002
.50	.406	.189	.094	.056	.036	.026	.019	.015	.012	.010
.75	.642	.397	.209	.125	.082	.058	.043	.033	.026	.022
1.00	.756	.606	.366	.220	.146	.103	.077	.059	.047	.038
1.25	.805	.752	.543	.339	.227	.160	.119	.092	.074	.060
1.50	.846	.823	.700	.481	.323	.231	.172	.133	.106	.086
1.75	.881	.856	.804	.630	.436	.313	.233	.181	.144	.117
2.00	.899	.879	.855	.756	.562	.405	.304	.235	.188	.153
2.25	.904	.898	.880	.838	.688	.508	.383	.297	.237	.193
2.50	.908	.910	.899	.878	.793	.620	.470	.366	.292	.238
2.75	.915	.918	.914	.897	.860	.730	.564	.442	.353	.288
3.00	.925	.925	.923	.912	.893	.820	.664	.522	.419	.342
3.25	.937	.932	.929	.924	.909	.876	.761	.609	.490	.401
3.50	.945	.939	.935	.931	.922	.904	.840	.699	.565	.464
3.75	.948	.945	.940	.937	.931	.918	.889	.786	.645	.531
4.00	.950	.949	.945	.941	.938	.929	.913	.855	.727	.601
4.25	.951	.952	.950	.946	.942	.937	.925	.899	.805	.675
4.50	.953	.954	.953	.950	.947	.943	.935	.920	.868	.750
4.75	.956	.956	.956	.954	.950	.947	.942	.931	.907	.822
5.00	.958	.958	.957	.957	.954	.951	.947	.940	.926	.878
5.25	.960	.960	.959	.959	.957	.954	.951	.946	.936	.914
5.50	.962	.962	.961	.960	.960	.957	.954	.951	.944	.931
5.75	.964	.964	.963	.962	.961	.960	.957	.954	.950	.940
6.00	.966	.965	.964	.964	.963	.962	.960	.957	.954	.947

Table 4 (continued)

$\epsilon = 0.3$

θ \ ρ	.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
.25	.118	.043	.023	.014	.009	.006	.005	.004	.003	.002
.50	.373	.172	.090	.056	.036	.025	.019	.015	.012	.009
.75	.584	.367	.200	.124	.081	.057	.043	.033	.026	.021
1.00	.690	.566	.349	.216	.145	.102	.076	.059	.047	.038
1.25	.750	.708	.518	.330	.224	.159	.118	.092	.073	.059
1.50	.814	.785	.670	.466	.318	.229	.170	.132	.105	.086
1.75	.868	.828	.773	.608	.427	.310	.231	.179	.143	.116
2.00	.895	.863	.828	.731	.547	.400	.302	.233	.186	.152
2.25	.902	.890	.860	.812	.669	.499	.380	.295	.235	.192
2.50	.904	.905	.885	.855	.771	.607	.465	.364	.290	.237
2.75	.909	.911	.904	.880	.838	.713	.556	.438	.350	.286
3.00	.917	.916	.916	.900	.873	.800	.652	.517	.416	.340
3.25	.924	.922	.922	.915	.894	.857	.745	.600	.486	.398
3.50	.930	.929	.927	.924	.911	.887	.822	.687	.560	.461
3.75	.935	.935	.932	.929	.923	.905	.871	.771	.637	.527
4.00	.941	.940	.936	.934	.931	.919	.898	.839	.716	.596
4.25	.947	.943	.941	.938	.936	.930	.913	.883	.792	.667
4.50	.949	.947	.945	.942	.939	.936	.926	.906	.853	.740
4.75	.950	.950	.948	.946	.943	.941	.935	.920	.892	.809
5.00	.951	.952	.951	.950	.947	.944	.941	.931	.913	.865
5.25	.953	.954	.954	.952	.950	.947	.945	.940	.926	.900
5.50	.956	.956	.956	.955	.953	.951	.948	.945	.936	.919
5.75	.959	.958	.958	.957	.956	.953	.951	.948	.943	.931
6.00	.961	.960	.959	.959	.958	.956	.954	.951	.948	.940

Table 5

Extended Source Diffraction for ϵ = .4, .5. ρ and θ are in Airy Units.

Relative Diffracted Energy

$\epsilon = 0.4$

θ \ ρ	.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
.25	.105	.037	.022	.014	.009	.006	.005	.004	.003	.002
.50	.329	.152	.086	.055	.036	.025	.019	.014	.012	.009
.75	.512	.330	.188	.122	.081	.056	.042	.033	.026	.021
1.00	.611	.517	.327	.210	.143	.100	.075	.058	.046	.038
1.25	.687	.655	.487	.318	.221	.157	.117	.090	.072	.059
1.50	.775	.737	.632	.446	.312	.226	.168	.130	.104	.085
1.75	.848	.792	.734	.581	.414	.305	.229	.177	.141	.115
2.00	.882	.841	.793	.698	.528	.393	.298	.231	.184	.150
2.25	.893	.877	.833	.778	.644	.487	.375	.292	.233	.190
2.50	.898	.894	.866	.826	.743	.589	.457	.360	.287	.234
2.75	.903	.900	.890	.857	.809	.690	.544	.433	.347	.283
3.00	.905	.904	.903	.883	.848	.775	.636	.509	.412	.337
3.25	.907	.910	.910	.903	.875	.832	.725	.589	.481	.395
3.50	.911	.915	.915	.913	.896	.865	.799	.672	.553	.457
3.75	.920	.920	.920	.919	.912	.887	.849	.752	.626	.522
4.00	.929	.926	.925	.924	.921	.906	.878	.819	.702	.589
4.25	.934	.933	.930	.928	.926	.919	.897	.863	.775	.657
4.50	.941	.938	.935	.932	.930	.927	.913	.888	.834	.726
4.75	.944	.943	.939	.936	.934	.931	.925	.906	.874	.793
5.00	.947	.946	.943	.940	.937	.935	.932	.920	.897	.847
5.25	.949	.948	.947	.944	.941	.939	.936	.930	.912	.883
5.50	.950	.950	.949	.947	.945	.942	.940	.936	.925	.904
5.75	.950	.951	.951	.950	.948	.945	.943	.940	.934	.918
6.00	.952	.953	.953	.952	.951	.948	.946	.943	.940	.930

Table 5 (continued)

$\epsilon = 0.5$

θ \ ρ	.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
.25	.089	.031	.021	.013	.009	.006	.005	.004	.003	.002
.50	.278	.131	.080	.052	.035	.025	.018	.014	.011	.009
.75	.432	.289	.172	.116	.079	.056	.041	.032	.026	.021
1.00	.525	.457	.298	.200	.139	.099	.074	.057	.045	.037
1.25	.616	.586	.444	.301	.214	.154	.115	.089	.071	.058
1.50	.719	.671	.581	.419	.301	.221	.166	.128	.102	.084
1.75	.797	.738	.681	.545	.398	.297	.224	.175	.139	.114
2.00	.836	.800	.746	.656	.503	.381	.292	.228	.182	.149
2.25	.859	.845	.794	.736	.611	.470	.366	.287	.230	.188
2.50	.880	.870	.837	.787	.705	.565	.445	.353	.283	.232
2.75	.896	.883	.867	.825	.772	.660	.528	.424	.342	.280
3.00	.902	.894	.883	.857	.814	.742	.614	.498	.405	.332
3.25	.903	.901	.893	.881	.846	.799	.698	.574	.472	.389
3.50	.904	.905	.901	.894	.873	.835	.769	.652	.541	.450
3.75	.906	.907	.908	.903	.892	.862	.819	.727	.611	.513
4.00	.909	.911	.912	.910	.904	.885	.851	.791	.682	.578
4.25	.913	.915	.916	.916	.911	.902	.875	.835	.751	.643
4.50	.919	.920	.921	.921	.918	.911	.895	.864	.809	.708
4.75	.927	.925	.925	.924	.923	.918	.909	.885	.849	.771
5.00	.934	.931	.930	.928	.927	.924	.918	.902	.864	.823
5.25	.938	.937	.934	.932	.930	.928	.923	.915	.893	.860
5.50	.942	.941	.938	.935	.933	.932	.929	.923	.909	.883
5.75	.946	.944	.942	.939	.936	.935	.933	.928	.921	.900
6.00	.949	.947	.945	.942	.940	.938	.936	.933	.928	.915

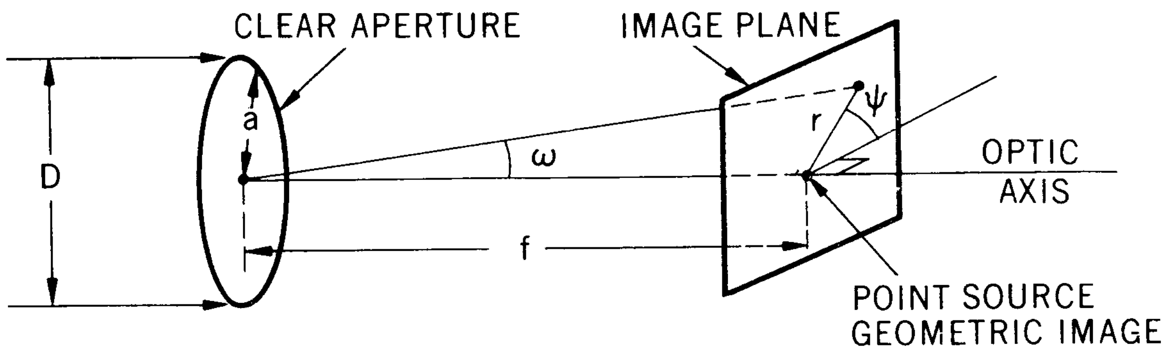

Figure 1–Diagram illustrcting Fraunhofer diffraction parameters

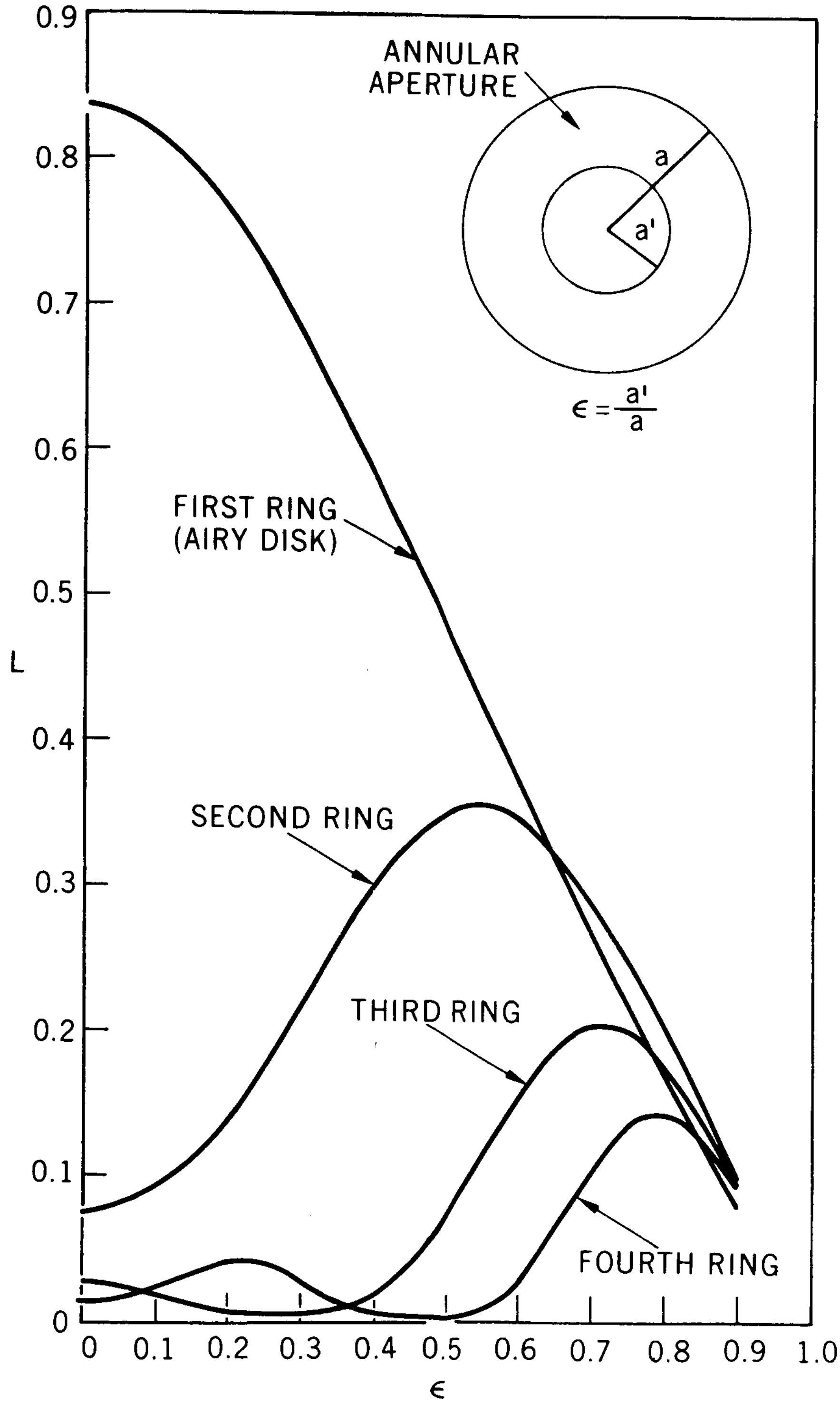

Figure 2–Relative energy in the first four diffraction rings due to an annular aperture

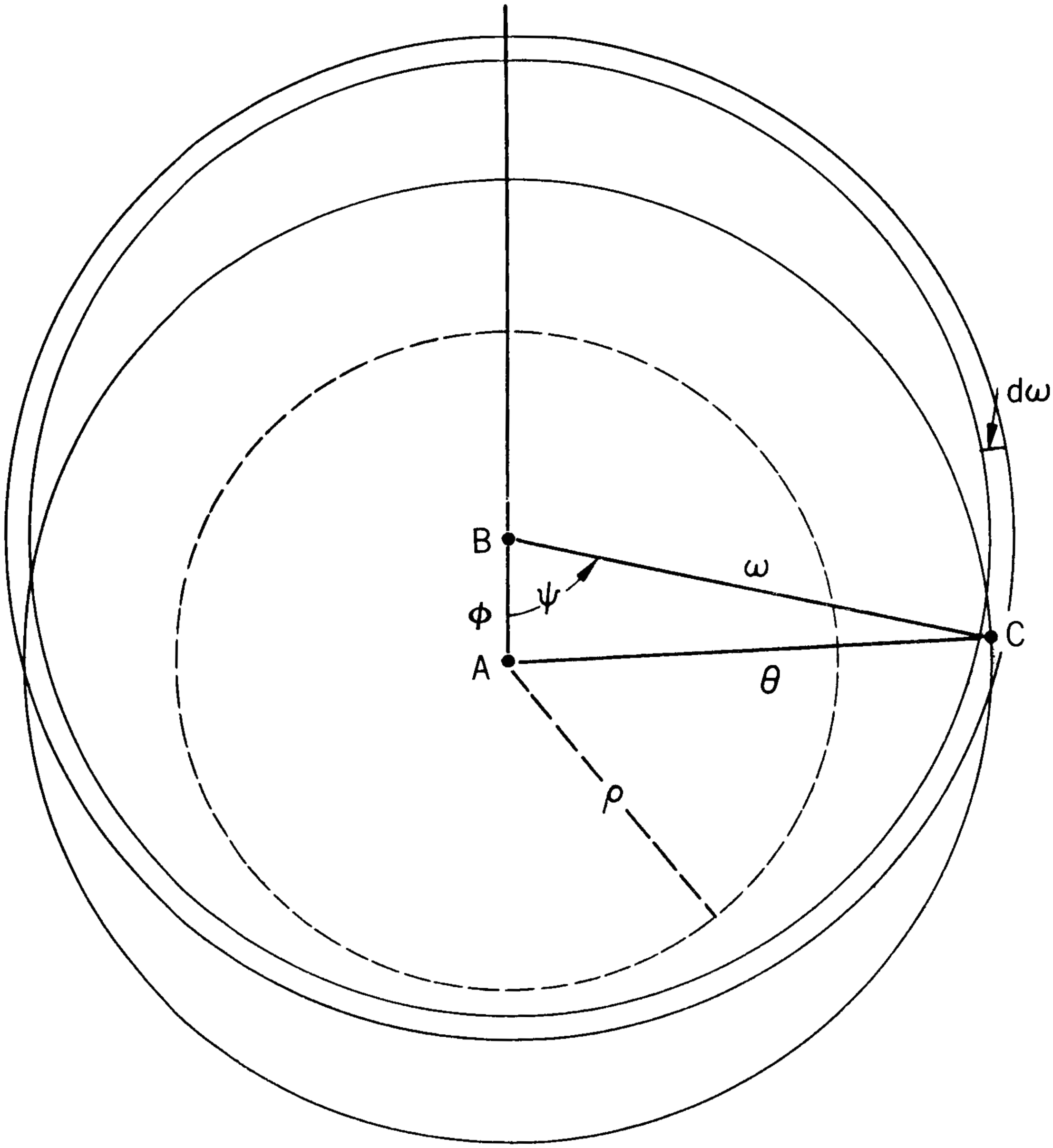

Figure 3—Extended source diffraction geometry in the image plane. Point A is the center of the extended source image and lies on the optic axis. Point B is the image of an elemental source area. Point C is at the center of intersection of an elemental diffraction ring of radius ω with circle (center at A) of radius θ. ρ = radius of circular extended source image, ϕ = distance from source center image to elemental area image. ϕ, ω, θ and ρ are measured in radians (i.e., actual length divided by effective focal length).

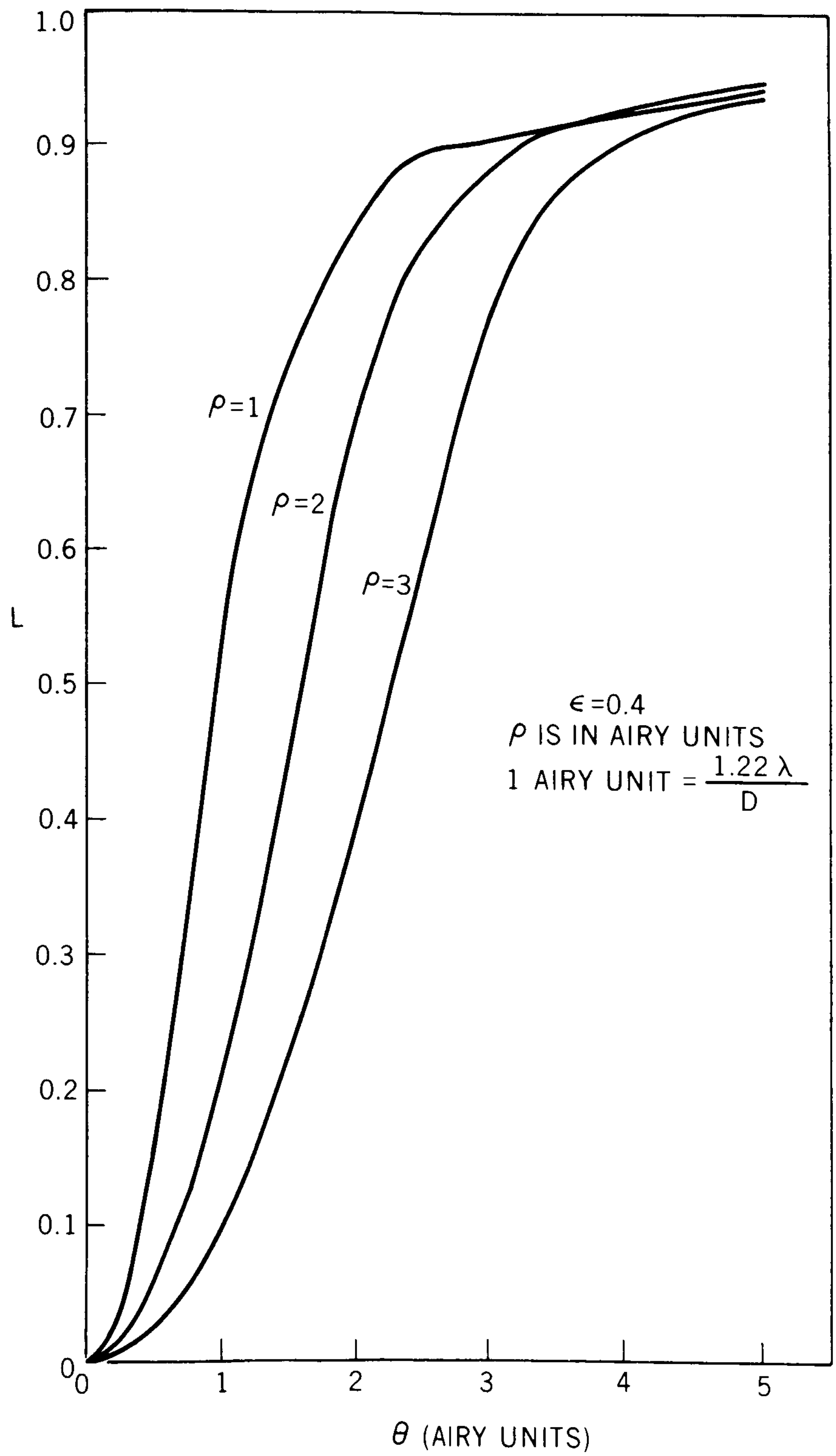

Figure 4–Extended source diffraction for ϵ =.4