Digital Design

CS/EEE/ECE/INSTR F215

Lecture 4: Boolean algebra

Boolean Algebra

Postulates of Boolean Algebra

1. $x + 0 = x$

2. $x + x' = 1$

3. $x + y = y + x$

4. $x(y + z) = xy + xz$

Duality Principle

$+\ (OR) \rightarrow .\ (AND)$

$.\ (AND) \rightarrow +\ (OR)$

$1 \rightarrow 0$

$0 \rightarrow 1$

Boolean Algebra

Applying Duality

1. $x + 0 = x$ $\qquad$ $x.1 = x$

2. $x + x' = 1$ $\qquad$ $x.x' = 0$

3. $x + y = y + x$ $\qquad$ $x.y = y.x$

4. $x(y + z) = xy + xz$ $\qquad$ $x + (y.z) = (x + y).(x + z)$

Boolean Algebra

Some important Theorems

1. $x + x = x$ $\quad\quad$ $x.x = x$

2. $(x')' = x$

3. $x + (y + z) = (x + y) + z$ $\quad\quad$ $x.(y.z) = (x.y).z$

4. $(x + y)' = x'y'$ $\quad$ DeMorgan's Th. $\quad$ $(x.y)' = x' + y'$

4. $x + xy = x$ $\quad\quad$ $x.(x + y) = x$

Boolean Algebra

Theorems can be proved in two ways

-using Postulates

$$x + xy = x$$

$$\begin{aligned} LHS &= x.1 + xy \\ &= x.(1 + y) \\ &= x.1 \\ &= x \end{aligned}$$

Boolean Algebra

Theorems can be proved in two ways

-using Truth tables

$$x + xy = x$$

x	y	xy	x+xy		x
0	0	0	0	=	0
0	1	0	0		0
1	0	0	1		1
1	1	1	1		1

$$LHS = RHS$$

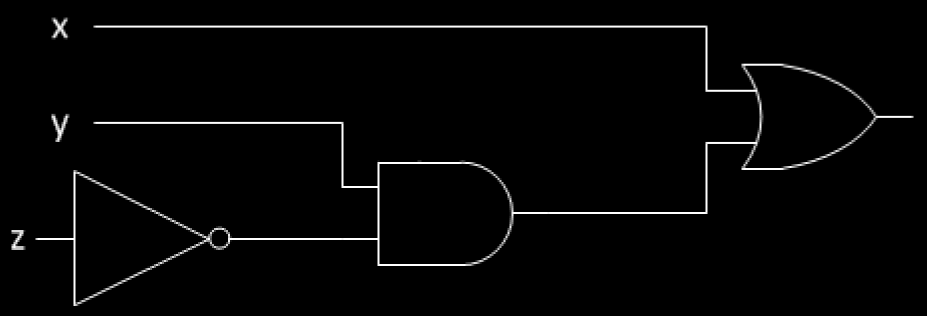
Boolean Functions

Consists of binary variables, constants and Logic symbols

$$F1 = x + yz' \qquad F2 = (x + yz')'$$

Single variable in normal or complemented form - literal

Can be transformed into circuit



$F1$ $\quad$ $F2??$

Boolean Functions

Boolean function can be represented by truth table in only one way

x	*y*	*z*	*F1*	*F2*
0	0	0	0	1
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

$$F1 = x + yz'$$

$$F2 = (x + yz')'$$

Boolean Functions

Boolean function can be represented in many ways in algebraic form

$$F2 = (x + yz')'$$

Both represent same function ?

$$F2 = x'yz' + x'yz + xz'$$

Implementation of both same ? Try it out

Need for simplification

Boolean Functions

Ways of simplification

-Algebraic manipulation

-K Maps

-QM(Quine-McCluskey) Method

Boolean Functions

Algebraic manipulation done using postulates and theorems

For example: $F1 = x(x' + y)$ 1 Not, 1 OR, 1 AND

$F1 = xx' + xy$

$F1 = xy$ 1 AND

Boolean Functions

Canonical Forms-Type1

x	y	F1
0	0	0
0	1	0
1	0	0
1	1	1

$$F1 = xy \quad + \quad F2 = x'y'$$

x	y	F2
0	0	1
0	1	0
1	0	0
1	1	0

x	y	F3
0	0	1
0	1	0
1	0	0
1	1	1

$$F3 = ??$$

$$F3 = xy + x'y'$$

Boolean Functions

Each input entry will be represented by a term

x	y		Designation
0	0	$x'y'$	m_0
0	1	$x'y$	m_1
1	0	xy'	m_2
1	1	xy	m_3

Minterms

Standard Product

F3
1
0
1
1

$$F3 = x'y' + xy' + xy$$

$$F3 = m_0 + m_2 + m_3$$

$$F3 = \sum(0,2,3)$$

Sum of Products

(Canonical form-Type1)

Boolean Functions

Canonical Forms-Type2

x	y	F1
0	0	0
0	1	1
1	0	1
1	1	1

$F1 = x + y$. $F2 = x' + y'$

x	y	F2
0	0	1
0	1	1
1	0	1
1	1	0

x	y	F3
0	0	0
0	1	1
1	0	1
1	1	0

$F3 = ??$

$F3 = (x + y).(x' + y')$

Boolean Functions

Each input entry will be represented by a term

x	y		Designation
0	0	$x+y$	M_0
0	1	$x+y'$	M_1
1	0	$x'+y$	M_2
1	1	$x'+y'$	M_3

F3
0
1
0
0

Maxterms

Standard Sums

$$F3 = (x+y).(x'+y).(x'+y')$$

$$F3 = M_0.M_2.M_3$$

$$F3 = \boldsymbol{\pi}(0,2,3)$$

Product of Sums
(Canonical form-Type2)

Boolean Algebra

Some important Theorems

1. $x + x = x$ $\qquad$ $x.x = x$

2. $(x')' = x$

3. $x + (y + z) = (x + y) + z$ $\qquad$ $x.(y.z) = (x.y).z$

4. $(x + y)' = x'y'$ DeMorgan's Th. $(x.y)' = x' + y'$

Boolean Algebra

Theorems can be proved in two ways

-using Truth tables

$$x + xy = x$$

x	*y*	*xy*	*x+xy*
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

=

x
0
0
1
1

$$LHS = RHS$$

Boolean Functions

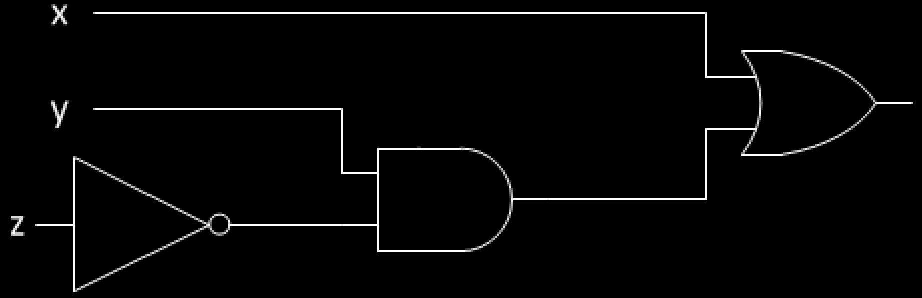
Consists of binary variables, constants and Logic symbols

$$F1 = x + yz'$$

$$F2 = (x + yz')'$$

Single variable in normal or complemented form - literal

Can be transformed into circuit



$F1$

$F2??$

Boolean Functions

Boolean function can be represented by truth table in only one way

x	y	z	F1	F2
0	0	0	0	1
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

$$F1 = x + yz'$$

$$F2 = (x + yz')'$$

Boolean Functions

Boolean function can be represented in many ways in algebraic form

$$F2 = (x + yz')'$$

Both represent same function

$$F2 = x'y'z' + x'y'z + x'z$$

Implementation of both same? Try it out

Need for simplification

Boolean Functions

Algebraic manipulation done using postulates and theorems

For example: $F1 = x(x' + y)$ 1 Not, 1 OR, 1 AND

$F1 = xx' + xy$

$F1 = xy$ 1 AND

Boolean Functions

Before learning

-K Maps

-QM(Quine-McCluskey) Method

Canonical and standard forms of Boolean functions

Boolean Functions

Canonical Forms-Type1

x	y	F1
0	0	0
0	1	0
1	0	0
1	1	1

$$F1 = xy \quad + \quad F2 = x'y'$$

x	y	F2
0	0	1
0	1	0
1	0	0
1	1	0

x	y	F3
0	0	1
0	1	0
1	0	0
1	1	1

$$F3 = ??$$

$$F3 = xy + x'y'$$

Boolean Functions

Each input entry will be represented by a term

x	y		Designation
0	0	$x'y'$	m_0
0	1	$x'y$	m_1
1	0	xy'	m_2
1	1	xy	m_3

Minterms

Standard Product

F3
1
0
1
1

$$F3 = x'y' + xy' + xy$$

$$F3 = m_0 + m_2 + m_3$$

$$F3 = \sum(0,2,3)$$

Sum of Products

(Canonical form-Type1)

Boolean Functions

Canonical Forms-Type2

x	y	F1
0	0	0
0	1	1
1	0	1
1	1	1

$F1 = x + y$. $F2 = x' + y'$

x	y	F2
0	0	1
0	1	1
1	0	1
1	1	0

x	y	F3
0	0	0
0	1	1
1	0	1
1	1	0

$F3 = ??$

$F3 = (x + y).(x' + y')$

Boolean Functions

Each input entry will be represented by a term

x	y	Maxterms	Designation	F3
0	0	$x + y$	M_0	0
0	1	$x + y'$	M_1	1
1	0	$x' + y$	M_2	0
1	1	$x' + y'$	M_3	0

Maxterms

Standard Sums

$$F3 = (x + y).(x' + y).(x' + y')$$

$$F3 = M_0.M_2.M_3$$

$$F3 = \boldsymbol{\pi}(0,2,3)$$

Product of Sums

(Canonical form-Type2)

Boolean Functions

Canonical forms from Truth table

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Sum of Products Representation ?

Product of Sums Representation ?

Boolean Functions

Can we convert from one canonical form to another ?

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Using Truth Table

$$F = m_1 + m_2 + m_4 + m_7$$

$$(Three\ variables - 0,1,2,3,4,5,6,7)$$

$$F = M_0 . M_3 . M_5 . M_6$$

Minterm and Maxterm are Complementary to each other

Boolean Functions

x	y	z	F	F'
0	0	0	0	1
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0

$$F = m_1 + m_2 + m_4 + m_7$$

$$F' = m_0 + m_3 + m_5 + m_6$$

$$F' = x'y'z' + x'yz + xy'z + xyz'$$

$$F = (F')'$$

$$F = (x'y'z' + x'yz + xy'z + xyz')'$$

$$F = (x'y'z')'.(x'yz)'.(xy'z)'.(xyz')'$$

$$F = (x + y + z).(x + y' + z').(x' + y + z').(x' + y' + z)$$

$$F = M_0.M_3.M_5.M_6$$

Boolean Functions

Express in minterms:

Example 1: $F = y + x'yz'$

$$F = y\,(x + x')(z + z') + \ x'yz'$$ Simplify

$$F = xyz + xyz' + x'yz + x'yz' + \ x'yz'$$

F= $m_7 + m_6 + m_3 + m_2$

Boolean Functions

Express in minterms:

Example 1: $F = y + x'yz'$

x	y	z	x'yz'
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

F
0
0
1
1
0
0
1
1

$F = m_2 + m_3 + m_6 + m_7$

$F = M_0.M_1. .M_4.M_5$

Boolean Functions

Standard Form – Terms that form the function may contain one, two or many literals

$$F1 = y' + xy + x'yz'$$ Sum of Products

$$F2 = x(y' + z).(x + y + z')$$ Product of Sums

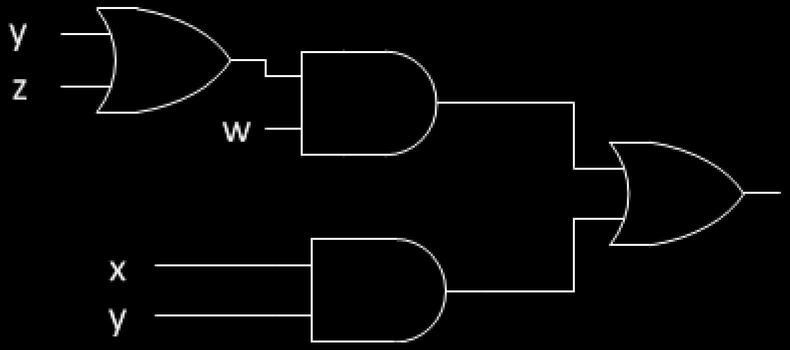
How is it different from canonical form ??

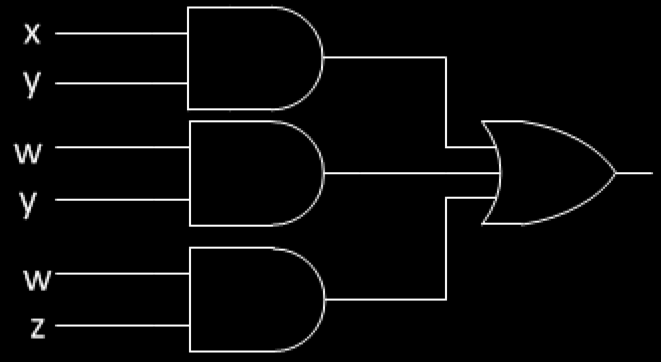
Boolean Functions

Some times standard forms result in better implementation

What form ??

$$F = xy + w(y+z)$$





$$F = xy + wy + wz$$

Standard Form

SPOT THE DIFF ??

Thank You