

## CHAPTER 2

## Digital Image Fundamentals

## Part 2



## 1. Theoretical

## Basic Relationships Between Pixels

- Neighborhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries


## * Neighbors of a Pixel

$\mathbf{x}$ The 4- neighbors of pixel p are: $\mathbf{N}_{\mathbf{4}}(\mathbf{p})$
Any pixel $\mathrm{p}(\mathrm{x}, \mathrm{y})$ has two vertical and two horizontal neighbors, given by:

$$
(\mathrm{x}+1, \mathrm{y}),(\mathrm{x}-1, \mathrm{y}),(\mathrm{x}, \mathrm{y}+1),(\mathrm{x}, \mathrm{y}-1)
$$


$\mathbf{x}$ The 4- diagonal neighbors are: $\mathbf{N}_{\mathbf{D}}(\mathbf{p})$ given by:

$$
(x+1, y+1),(x+1, y-1),(x-1, y+1),(x-1, y-1)
$$


$\mathbf{x}$ The 8-neighbors are : $\mathbf{N}_{\mathbf{8}}(\mathbf{p})$
8 -neighbors of a pixel pare its vertical, horizontal and 4 diagonal neighbors denoted by $\mathrm{N}_{8}(\mathrm{p})$


## Connectivity

Two pixels are said to be connected if they are adjacent in some sense.

- They are neighbors $\left(\mathrm{N}_{4}, \mathrm{~N}_{\mathrm{D}}, \mathrm{N}_{8}\right)$ and
- Their intensity values (gray levels) are similar.


## Adjacency

Let V be the set of intensity used to define adjacency; e.g. $\mathrm{V}=\{1\}$ in a binary image or $\mathrm{V}=\{100,101,102, \ldots, 120\}$ inn a gray-scale image.
We consider three types of adjacency:

## - 4-adjacency:

Two pixels $p$ and $q$ with values from $V$ are 4 -adjacent if $q$ is in the set $\mathrm{N}_{4}(\mathrm{p})$.


## - 8-adjacency:

Two pixels p and q with values from V are 8 -adjacent if q is in the set $\mathrm{N}_{8}(\mathrm{p})$.


- m-adjacency (mixed adjacency):

Two pixels $p$ and $q$ with values from $V$ are $m$-adjacent if:
(i) $q$ is in $N_{4}(p)$,or
(ii) q is in $\mathrm{N}_{\mathrm{D}}(\mathrm{p})$ and $\mathrm{N}_{4}(\mathrm{p}) \cap \mathrm{N}_{4}(\mathrm{q})$ is empty (has no pixels whose values are from $V$ )


Two image subsets S1 and S2 are adjacent if some pixel in S1 is adjacent to some pixel in S2.

## Question1:

Consider the two image subsets, $S 1$ and $S 2$, shown in the following figure. For $\mathrm{V}=\{1\}$, determine whether these two subsets are (a) 4-adjacent, (b) 8-adjacent, or (c) m-adjacent.

|  | $S_{1}$ |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |

## Solution:

Let $p$ and $q$ be as shown in Fig. Then:
(a) $S 1$ and $S 2$ are not 4 -connected because $q$ is not in the set $N_{4}(p)$;
(b) $S 1$ and $S 2$ are 8 -connected because $q$ is in the set $N_{8}(p)$;
(c) $S 1$ and $S 2$ are $\boldsymbol{m}$-connected because
(i) $q$ is in $N_{D}(p)$, and
(ii) the set $N_{4}(p) \cap N_{4}(q)$ is empty.


## Paths

A (digital) path (or curve) from pixel p with coordinates ( $\mathrm{x}, \mathrm{y}$ ) to pixel q with coordinates ( $\mathrm{s}, \mathrm{t}$ ) is a sequence of distinct pixels with coordinates

$$
\left(x_{0}, y_{0}\right),\left(x_{1}, y_{1}\right), \ldots \ldots,\left(x_{n}, y_{n}\right)
$$

- where $\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)=(\mathrm{x}, \mathrm{y}),\left(\mathrm{x}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}}\right)=(\mathrm{s}, \mathrm{t})$,
- and pixels $\left(\mathrm{x}_{\mathrm{i}}, \mathrm{y}_{\mathrm{i}}\right)$ and $\left(\mathrm{x}_{\mathrm{i}-1}, \mathrm{y}_{\mathrm{i}-1}\right)$ are adjacent for $1 \leq \mathrm{i} \leq \mathrm{n}$.
- In this case, $n$ is the length of the path.
- If $\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)=\left(\mathrm{x}_{\mathrm{n}}, \mathrm{y}_{\mathrm{n}}\right)$ the path is a closed path.
- The path can be defined $4-, 8-$,m-paths depending on adjacency type.

Let $S$ be a subset of pixels in an image. Two pixels $p$ and $q$ are said to be connected in $S$ if there exists a path between them consisting entirely of pixels in $S$

- For any pixel $p$ in $S$, the set of pixels that are connected to it in $S$ is called a connected component of $S$.
- If it only has one connected component, then set S is called a connected set.


## Question2:

Consider the image segment shown.
Let $\mathrm{V}=\{0,1\}$ and compute the lengths of the shortest $4-, 8$-, and m-path between p and q . If a particular path does not exist between these two points, explain why.

| 3 | 1 | 2 | $1(q)$ |
| ---: | :--- | :--- | :--- |
| 2 | 2 | 0 | 2 |
| 1 | 2 | 1 | 1 |
| $(p) 1$ | 0 | 1 | 2 |

## Solution:

- When $V=\{0,1\}, 4$-path does not exist between $p$ and $q$ because it is impossible to get from $p$ to $q$ by traveling along points that are both 4 -adjacent and also have values from $V$. Fig. $a$ shows this condition; it is not possible to get to $q$.
- The shortest 8 -path is shown in Fig. $b$ its length is $\mathbf{4}$.
- The length of the shortest $m$ - path (shown dashed) is $\mathbf{5}$.
- Both of these shortest paths are unique in this case.

(a)

(b)


## Regions and boundaries

- Let $R$ be a subset of pixels in an image. We call $R$ a region of the image if $R$ is a connected set.
- The boundary (also called border or contour) of a region R is the set of pixels in the region that have one or more neighbors that are not in R .


## Distance Measures

Given pixels $\mathrm{p}, \mathrm{q}$ and z with coordinates ( $\mathrm{x}, \mathrm{y}$ ), ( $\mathrm{s}, \mathrm{t}$ ), ( $\mathrm{u}, \mathrm{v}$ ) respectively, the distance function D has following properties:

- $D(p, q) \geq 0[D(p, q)=0$, iff $p=q]$
- $\mathrm{D}(\mathrm{p}, \mathrm{q})=\mathrm{D}(\mathrm{q}, \mathrm{p})$
- $D(p, z) \leq D(p, q)+D(q, z)$
- The following are the different Distance measures:
a. Euclidean Distance:

$$
\mathbf{D}_{\mathrm{e}}(\mathbf{p}, \mathbf{q})=\left[(\mathbf{x}-\mathrm{s})^{2}+(\mathrm{y}-\mathrm{t})^{2}\right]^{1 / 2}
$$

- The pixels having a distance less than or equal to some value $r$ from $(x, y)$ are the points contained in a disk of radius $r$ centered at ( $x, y$ ).
b. City Block Distance:

$$
D_{4}(p, q)=|x-s|+|y-t|
$$

- The pixels having a $D_{4}$ distance from ( $\mathrm{x}, \mathrm{y}$ ) less than or equal to some value r form a diamond centered at ( $x, y$ ). For example, the pixels with $D_{4}$ distance $\leq 2$ from ( $\mathrm{x}, \mathrm{y}$ ) (the center point) form the following contours of constant distance:

|  |  | 2 |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | 2 | 1 | 2 |  |
| 2 | 1 | 0 | 1 | 2 |
|  | 2 | 1 | 2 |  |
|  |  | 2 |  |  |

- The pixels with $D_{4}=1$ are the 4-neighbors of (x,y).


## c. Chess Board Distance:

$$
D_{8}(p, q)=\max (|x-s|,|y-t|)
$$

- The pixels with $D_{8}$ distance from ( $x, y$ ) less than or equal to some value $r$ form a square centered at (x, y). For example, the pixels with $D_{8}$ distance $\leq 2$ from ( $\mathrm{x}, \mathrm{y}$ ) (the center point) form the following contours of constant distance:

| 2 | 2 | 2 | 2 | 2 |
| :--- | :--- | :--- | :--- | :--- |
| 2 | 1 | 1 | 1 | 2 |
| 2 | 1 | 0 | 1 | 2 |
| 2 | 1 | 1 | 1 | 2 |
| 2 | 2 | 2 | 2 | 2 |

- The pixels with $D_{8}=1$ are the 8 -neighbors of (x,y).


## 2. Practical

## Example1: Image rotate

```
%ex1.m
clear all;
I = imread('1.jpg');
J = imrotate(I,35); %rotates an image 35' counterclockwise
subplot(121) ,imshow(I) ,title('Original');
subplot(122),imshow(J),title('Rotatated by 35');
```


## Output:

Original


Rotatated by 35


Example2: Cropping an Image

```
%ex2 .m
clear all;
I = imread('1.jpg');
J = imrotate (I, 35);
J2= imcrop(J) ;
subplot(131),imshow(I),title('Original');
subplot(132),imshow(J),title('Rotatated by 35');
subplot(133),imshow(J2),title('Rotatated by 35 croped');
```

Output:


Original


Rotatated by 35


Rotatated by 35 croped


## 3. Homework:

1. Consider the image segment shown.

Let $\mathrm{V}=\{1,2\}$ and compute the lengths of the shortest $4-, 8$-, and m-path between p and q . If a particular path does not exist between these two points, explain why.

| 3 | 1 | 2 | $1(q)$ |
| ---: | :--- | :--- | :--- |
| 2 | 2 | 0 | 2 |
| 1 | 2 | 1 | 1 |
| $(p) 1$ | 0 | 1 | 2 |

2. Write a Matlab code to rotate an image by $180^{\circ}$ (do not use the imrotate function, you should implement it)
Hint (Use end function)
3. Given the two images below, perform an enhancement operation to get Fig (1). Hint (Use a logical operation)


Fig (1)

