DIGITAL SIGNAL PROCESSING

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SIMPLY EASY LEARNING

About the Tutorial

Digital Signal Processing is an important branch of Electronics and Telecommunication engineering that deals with the improvisation of reliability and accuracy of the digital communication by employing multiple techniques. This tutorial explains the basic concepts of digital signal processing in a simple and easy-to-understand manner.

Audience

This tutorial is meant for the students of E&TC, Electrical and Computer Science engineering. In addition, it should be useful for any enthusiastic reader who would like to understand more about various signals, systems, and the methods to process a digital signal.

Prerequisites

Digital signal processing deals with the signal phenomenon. Along with it, in this tutorial, we have shown the filter design using the concept of DSP. This tutorial has a good balance between theory and mathematical rigor. Before proceeding with this tutorial, the readers are expected to have a basic understanding of discrete mathematical structures.

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Part 1 – Introduction to Signals

1. SIGNALS – DEFINITION

Definition

Anything that carries information can be called as signal. It can also be defined as a physical quantity that varies with time, temperature, pressure or with any independent variables such as speech signal or video signal.

The process of operation in which the characteristics of a signal (Amplitude, shape, phase, frequency, etc.) undergoes a change is known as signal processing.

Note - Any unwanted signal interfering with the main signal is termed as noise. So, noise is also a signal but unwanted.

According to their representation and processing, signals can be classified into various categories details of which are discussed below.

Continuous Time Signals

Continuous-time signals are defined along a continuum of time and are thus, represented by a continuous independent variable. Continuous-time signals are often referred to as analog signals.

This type of signal shows continuity both in amplitude and time. These will have values at each instant of time. Sine and cosine functions are the best example of Continuous time signal.

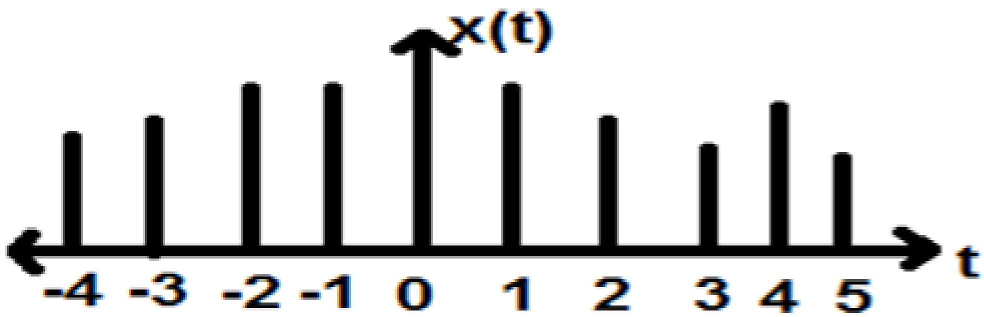
Continuous Time signal

The signal shown above is an example of continuous time signal because we can get value of signal at each instant of time.

Discrete Time signals

The signals, which are defined at discrete times are known as discrete signals. Therefore, every independent variable has distinct value. Thus, they are represented as sequence of numbers.

Although speech and video signals have the privilege to be represented in both continuous and discrete time format; under certain circumstances, they are identical. Amplitudes also show discrete characteristics. Perfect example of this is a digital signal; whose amplitude and time both are discrete.



The figure above depicts a discrete signal's discrete amplitude characteristic over a period of time. Mathematically, these types of signals can be formularized as;

$$x = \{x[n]\}, \qquad -\infty < n < \infty$$

Where, *n* is an integer.

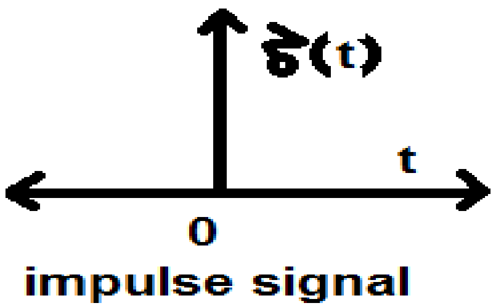
It is a sequence of numbers x, where nth number in the sequence is represented as x[n].

2. BASIC CT SIGNALS

To test a system, generally, standard or basic signals are used. These signals are the basic building blocks for many complex signals. Hence, they play a very important role in the study of signals and systems.

Unit Impulse or Delta Function

A signal, which satisfies the condition, $\delta(t) = \lim_{\epsilon \to 0} x(t)$ is known as unit impulse signal. This signal tends to infinity when t=0 and tends to zero when t $\neq$ 0 such that the area under its curve is always equals to one. The delta function has zero amplitude everywhere except at t=0.



Properties of Unit Impulse Signal

- δ (t) is an even signal.
- δ (t) is an example of neither energy nor power (NENP) signal.
- Area of unit impulse signal can be written as;

$$A = \int_{-\infty}^{\infty} \delta(t)dt = \int_{-\infty}^{\infty} \lim_{\epsilon \to 0} x(t)\, dt = \lim_{\epsilon \to 0} \int_{-\infty}^{\infty} [x(t)dt] = 1$$

- Weight or strength of the signal can be written as;

$$y(t) = A\delta(t)$$

- Area of the weighted impulse signal can be written as-

$$y(t) = \int_{-\infty}^{\infty} y(t)dt = \int_{-\infty}^{\infty} A\delta(t) = A[\int_{-\infty}^{\infty} \delta(t)dt\,] = A = 1 = wigthedimpulse$$

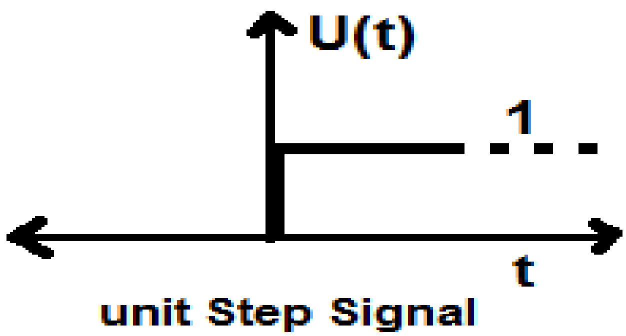
Unit Step Signal

A signal, which satisfies the following two conditions-

1. $U(t) = 1\ (when\ \ t \geq 0\ )and$
2. $U(t) = 0\ (when\ \ t < 0)$

is known as a unit step signal.

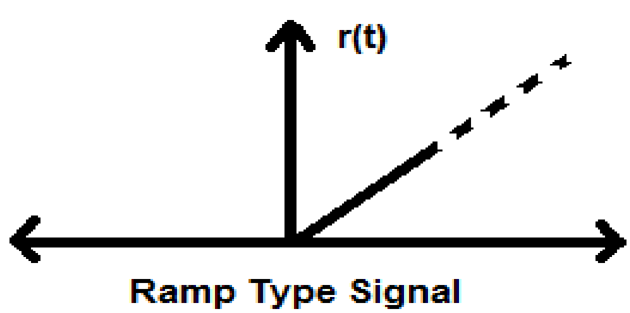
It has the property of showing discontinuity at t=0. At the point of discontinuity, the signal value is given by the average of signal value. This signal has been taken just before and after the point of discontinuity (according to Gibb's Phenomena).



If we add a step signal to another step signal that is time scaled, then the result will be unity. It is a power type signal and the value of power is 0.5. The RMS (Root mean square) value is 0.707 and its average value is also 0.5.

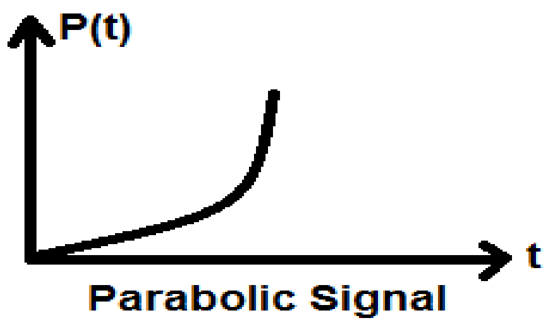
Ramp Signal

Integration of step signal results in a Ramp signal. It is represented by r(t). Ramp signal also satisfies the condition$r(t) = \int_{-\infty}^{t} U(t)dt = tU(t)$. It is neither energy nor power (NENP) type signal.



Parabolic Signal

Integration of Ramp signal leads to parabolic signal. It is represented by p(t). Parabolic signal also satisfies the condition$p(t) = \int_{-\infty}^{t} r(t)dt = (t^2/2)U(t)$. It is neither energy nor Power (NENP) type signal.

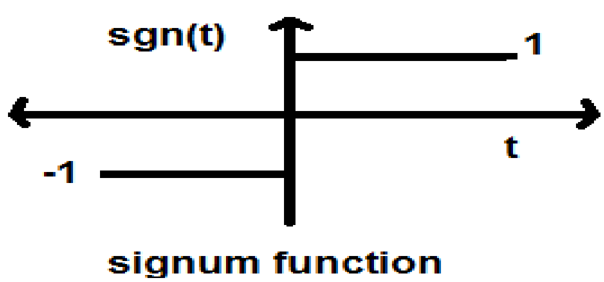


Signum Function

This function is represented as

$$sgn(t) = \begin{cases} 1 & for\ t > 0 \\ -1 & for\ t < 0 \end{cases}$$

It is a power type signal. Its power value and RMS (Root mean square) values, both are 1. Average value of signum function is zero.



Sinc Function

It is also a function of sine and is written as-

$$\mathrm{SinC(t)} = \frac{\mathrm{Sin}\Pi \mathrm{t}}{\Pi \mathrm{T}} = \mathrm{Sa}(\Pi \mathrm{t})$$

Properties of Sinc function

1. It is an energy type signal.

2. $\text{Sinc}(0) = \lim_{t \to 0} \frac{\text{Sin}\Pi t}{\Pi t} = 1$

3. $\text{Sinc}(\infty) = \lim_{t \to \infty} \frac{\text{Sin}\Pi\infty}{\Pi\infty} = 0$ (Range of $\sin\pi\infty$ varies between -1 to +1 but anything divided by infinity is equal to zero)

4. If $\text{Sinc}(t) = 0 \quad => \quad \text{Sin}\Pi t = 0$

   ⇨ $\Pi t = n\Pi$

   ⇨ $t = n \; (n \neq 0)$

Sinusoidal Signal

A signal, which is continuous in nature is known as continuous signal. General format of a sinusoidal signal is

$$x(t) = A\sin(\omega t + \Phi)$$

Here,

A = amplitude of the signal

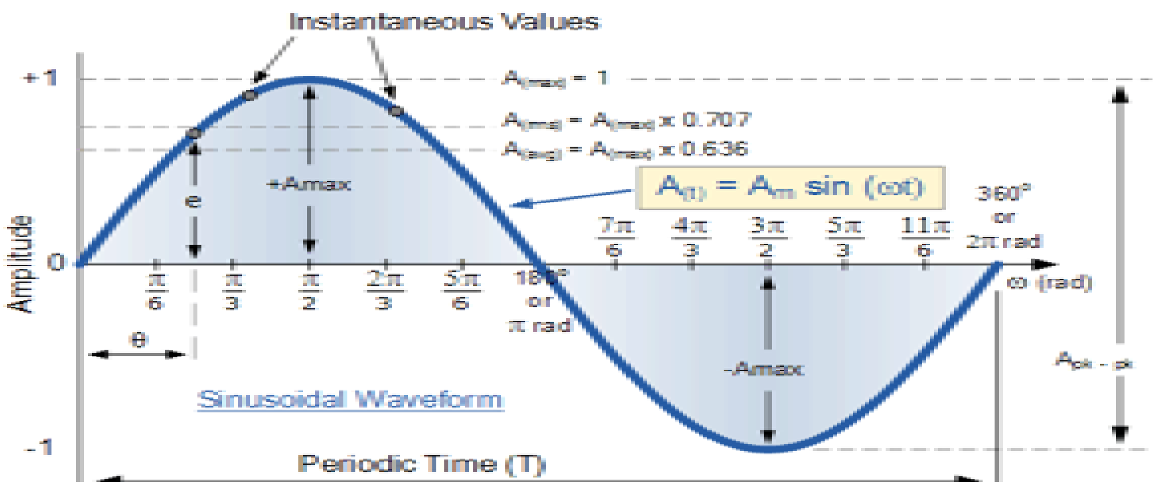
ω = Angular frequency of the signal (Measured in radians)

Φ = Phase angle of the signal (Measured in radians)

The tendency of this signal is to repeat itself after certain period of time, thus is called periodic signal. The time period of signal is given as;

$$T = \frac{2\pi}{\omega}$$

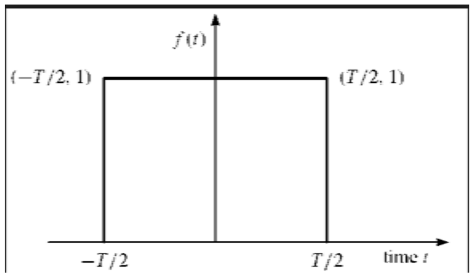
The diagrammatic view of sinusoidal signal is shown below.



Rectangular Function

A signal is said to be rectangular function type if it satisfies the following condition:

$$\pi\left(\frac{t}{\tau}\right) = \begin{cases} 1, & \text{for } t \leq \frac{\tau}{2} \\ 0, & \text{Otherwise} \end{cases}$$

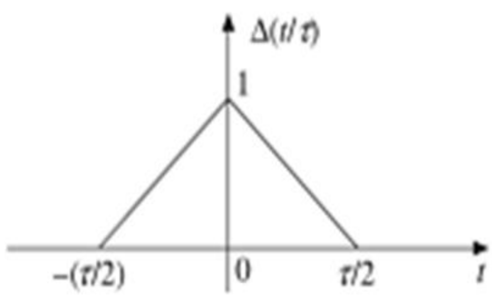


Being symmetrical about Y-axis, this signal is termed as even signal.

Triangular Pulse Signal

Any signal, which satisfies the following condition, is known as triangular signal.

$$\Delta\left(\frac{t}{\tau}\right) = \begin{cases} 1 - \left(\frac{2|t|}{\tau}\right) for\ |t| < \frac{\tau}{2} \\ 0\ for\ |t| > \frac{\tau}{2} \end{cases}$$



This signal is symmetrical about Y-axis. Hence, it is also termed as even signal.

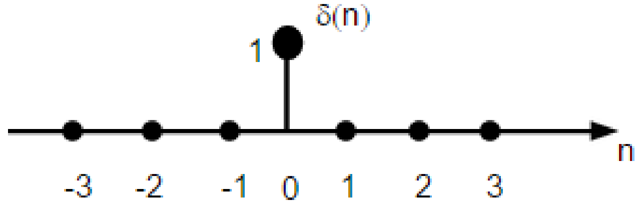
3. BASIC DT SIGNALS

We have seen that how the basic signals can be represented in Continuous time domain. Let us see how the basic signals can be represented in Discrete Time Domain.

Unit Impulse Sequence

It is denoted as $\delta(n)$ in discrete time domain and can be defined as;

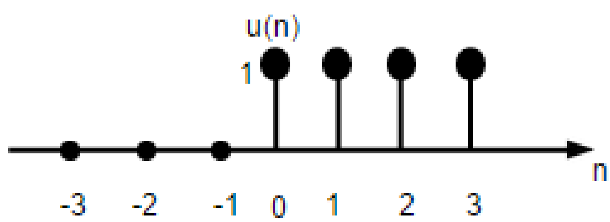
$$\delta(n) = \begin{cases} 1, for\ n = 0 \\ 0, otherwise \end{cases}$$



Unit Step Signal

Discrete time unit step signal is defined as;

$$\mathrm{U(n)} = \begin{cases} 1, for\ n \geq 0 \\ 0, for\ n < 0 \end{cases}$$

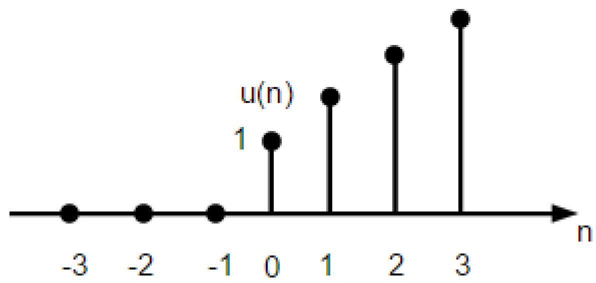


The figure above shows the graphical representation of a discrete step function.

Unit Ramp Function

A discrete unit ramp function can be defined as:

$$r(n) = \begin{cases} n, for\ n \geq 0 \\ 0, for\ n < 0 \end{cases}$$



The figure given above shows the graphical representation of a discrete ramp signal.

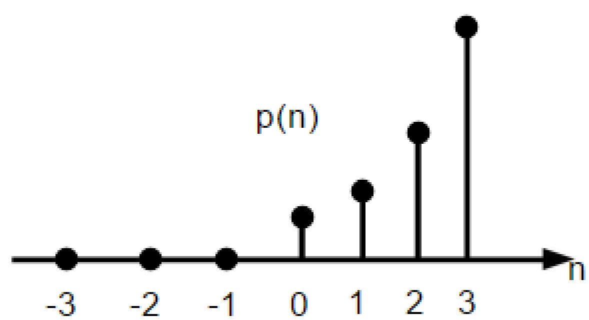
Parabolic Function

Discrete unit parabolic function is denoted as p(n) and can be defined as;

$$p(n) = \begin{cases} \frac{n^2}{2} & , for\ n \geq 0 \\ 0, for\ n < 0 \end{cases}$$

In terms of unit step function it can be written as;

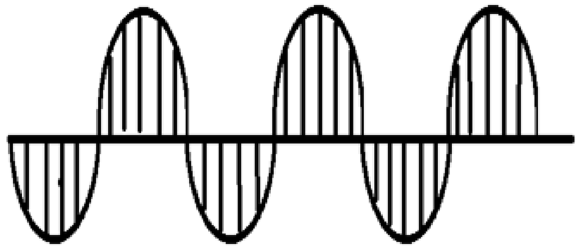
$$P(n) = \frac{n^2}{2} U(n)$$



The figure given above shows the graphical representation of a parabolic sequence.

Sinusoidal Signal

All continuous-time signals are periodic. The discrete-time sinusoidal sequences may or may not be periodic. They depend on the value of ω. For a discrete time signal to be periodic, the angular frequency ω must be a rational multiple of 2π.



Discrete sinusoidal signal

A discrete sinusoidal signal is shown in the figure above.

Discrete form of a sinusoidal signal can be represented in the format:

$$x(n) = A sin(\omega n + \Phi)$$

Here A,ω and Φ have their usual meaning and n is the integer. Time period of the discrete sinusoidal signal is given by:

$$N = \frac{2\pi m}{\omega}$$

Where, N and m are integers.

4. CLASSIFICATION OF CT SIGNALS

Continuous time signals can be classified according to different conditions or operations performed on the signals.

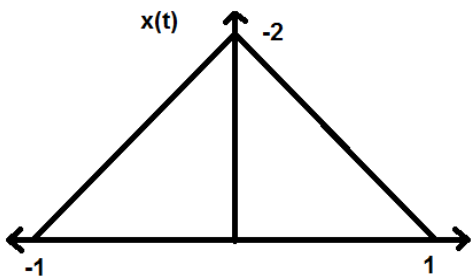
Even and Odd Signals

Even Signal

A signal is said to be even if it satisfies the following condition;

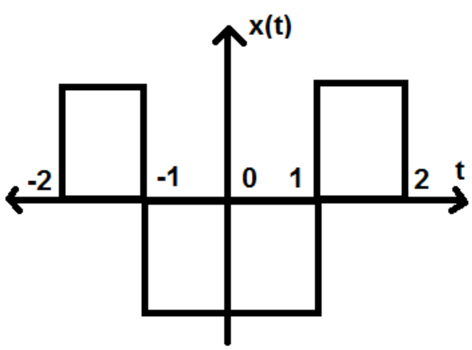
$$x(-t) = x(t)$$

Time reversal of the signal does not imply any change on amplitude here. For example, consider the triangular wave shown below.



The triangular signal is an even signal. Since, it is symmetrical about Y-axis. We can say it is mirror image about Y-axis.

Consider another signal as shown in the figure below.



We can see that the above signal is even as it is symmetrical about Y-axis.

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