CHAPTER 1

Introduction

During the past several decades the field of digital signal processing (DSP) has grown to be important, both theoretically and technologically. A major reason for its success in industry is the development and use of low-cost software and hardware. New technologies and applications in various fields are now taking advantage of DSP algorithms. This will lead to a greater demand for electrical and computer engineers with background in DSP. Therefore, it is necessary to make DSP an integral part of any electrical engineering curriculum.

Two decades ago an introductory course on DSP was given mainly at the graduate level. It was supplemented by computer exercises on filter design, spectrum estimation, and related topics using mainframe (or mini) computers. However, considerable advances in personal computers and software during the past two decades have made it necessary to introduce a DSP course to undergraduates. Since DSP applications are primarily algorithms that are implemented either on a DSP processor [11] or in software, a fair amount of programming is required. Using interactive software, such as MATLAB, it is now possible to place more emphasis on learning new and difficult concepts than on programming algorithms. Interesting practical examples can be discussed, and useful problems can be explored.

With this philosophy in mind, we have developed this book as a *companion book* (to traditional textbooks like [18, 23]) in which MATLAB is an integral part in the discussion of topics and concepts. We have chosen MATLAB as the programming tool primarily because of its wide availability on computing platforms in many universities across the world. Furthermore, a low-cost student version of MATLAB has been available for several years, placing it among the least expensive software products for educational purposes. We have treated MATLAB as a computational and programming toolbox containing several tools (sort of a super calculator with several keys) that can be used to explore and solve problems and, thereby, enhance the learning process.

This book is written at an introductory level in order to introduce undergraduate students to an exciting and practical field of DSP. We emphasize that this is not a textbook in the traditional sense but a companion book in which more attention is given to problem solving and hands-on experience with MATLAB. Similarly, it is not a tutorial book in MATLAB. We assume that the student is familiar with MATLAB and is currently taking a course in DSP. The book provides basic analytical tools needed to process real-world signals (a.k.a. analog signals) using digital techniques. We deal mostly with discrete-time signals and systems, which are analyzed in both the time and the frequency domains. The analysis and design of processing structures called *filters* and *spectrum analyzers* are among of the most important aspects of DSP and are treated in great detail in this book. Two important topics on finite word-length effects and sampling-rate conversion are also discussed in this book. More advanced topics in modern signal processing like statistical and adaptive signal processing are generally covered in a graduate course. These are not treated in this book, but it is hoped that the experience gained in using this book will allow students to tackle advanced topics with greater ease and understanding. In this chapter we provide a brief overview of both DSP and MATLAB.

1.1 OVERVIEW OF DIGITAL SIGNAL PROCESSING

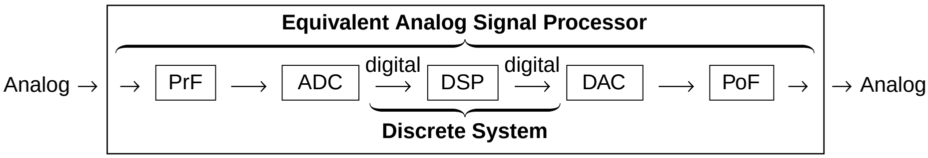
In this modern world we are surrounded by all kinds of signals in various forms. Some of the signals are natural, but most of the signals are manmade. Some signals are necessary (speech), some are pleasant (music), while many are unwanted or unnecessary in a given situation. In an engineering context, signals are carriers of information, both useful and unwanted. Therefore extracting or enhancing the useful information from a mix of conflicting information is the simplest form of signal processing. More generally, signal processing is an operation designed for extracting, enhancing, storing, and transmitting useful information. The distinction between useful and unwanted information is often subjective as well as objective. Hence signal processing tends to be application dependent.

1.1.1 HOW ARE SIGNALS PROCESSED?

The signals that we encounter in practice are mostly analog signals. These signals, which vary continuously in time and amplitude, are processed using electrical networks containing active and passive circuit elements. This approach is known as analog signal processing (ASP)—for example, radio and television receivers.

$$\text{Analog signal: } x_a(t) \longrightarrow \boxed{\text{Analog signal processor}} \longrightarrow y_a(t) \text{ :Analog signal}$$

They can also be processed using digital hardware containing adders, multipliers, and logic elements or using special-purpose microprocessors. However, one needs to convert analog signals into a form suitable for digital hardware. This form of the signal is called a digital signal. It takes one of the finite number of values at specific instances in time, and hence it can be represented by binary numbers, or bits. The processing of digital signals is called DSP; in block diagram form it is represented by



The various block elements are discussed as follows.

PrF: This is a prefilter or an antialiasing filter, which conditions the analog signal to prevent aliasing.

ADC: This is an analog-to-digital converter, which produces a stream of binary numbers from analog signals.

Digital Signal Processor: This is the heart of DSP and can represent a general-purpose computer or a special-purpose processor, or digital hardware, and so on.

DAC: This is the inverse operation to the ADC, called a digital-to-analog converter, which produces a staircase waveform from a sequence of binary numbers, a first step toward producing an analog signal.

PoF: This is a postfilter to smooth out staircase waveform into the desired analog signal.

It appears from the above two approaches to signal processing, analog and digital, that the DSP approach is the more complicated, containing more components than the "simpler looking" ASP. Therefore one might ask, Why process signals digitally? The answer lies in the many advantages offered by DSP.

1.1.2 ADVANTAGES OF DSP OVER ASP

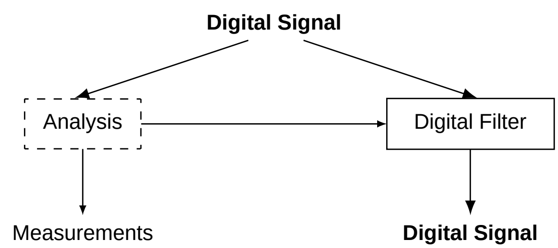
A major drawback of ASP is its limited scope for performing complicated signal-processing applications. This translates into nonflexibility in processing and complexity in system designs. All of these generally lead to expensive products. On the other hand, using a DSP approach, it is possible to convert an inexpensive personal computer into a powerful signal processor. Some important advantages of DSP are these:

1. Systems using the DSP approach can be developed using software running on a general-purpose computer. Therefore DSP is relatively convenient to develop and test, and the software is portable.
2. DSP operations are based solely on additions and multiplications, leading to extremely stable processing capability—for example, stability independent of temperature.
3. DSP operations can easily be modified in real time, often by simple programming changes, or by reloading of registers.
4. DSP has lower cost due to VLSI technology, which reduces costs of memories, gates, microprocessors, and so forth.

The principal disadvantage of DSP is the limited speed of operations limited by the DSP hardware, especially at very high frequencies. Primarily because of its advantages, DSP is now becoming a first choice in many technologies and applications, such as consumer electronics, communications, wireless telephones, and medical imaging.

1.1.3 TWO IMPORTANT CATEGORIES OF DSP

Most DSP operations can be categorized as being either signal *analysis* tasks or signal *filtering* tasks:



Signal analysis This task deals with the measurement of signal properties. It is generally a frequency-domain operation. Some of its applications are

- spectrum (frequency and/or phase) analysis
- speech recognition
- speaker verification
- target detection

Signal filtering This task is characterized by the signal-in signal-out situation. The systems that perform this task are generally called *filters*.

It is usually (but not always) a time-domain operation. Some of the applications are

- removal of unwanted background noise
- removal of interference
- separation of frequency bands
- shaping of the signal spectrum

In some applications, such as voice synthesis, a signal is first analyzed to study its characteristics, which are then used in digital filtering to generate a synthetic voice.

1.2 A BRIEF INTRODUCTION TO MATLAB

MATLAB is an interactive, matrix-based system for scientific and engineering numeric computation and visualization. Its strength lies in the fact that complex numerical problems can be solved easily and in a fraction of the time required by a programming language such as Fortran or C. It is also powerful in the sense that, with its relatively simple programming capability, MATLAB can be easily extended to create new commands and functions.

MATLAB is available in a number of computing environments: PCs running all flavors of Windows, Apple Macs running OS-X, UNIX/Linux workstations, and parallel computers. The basic MATLAB program is further enhanced by the availability of numerous toolboxes (a collection of specialized functions in a specific topic) over the years. The information in this book generally applies to all these environments. In addition to the basic MATLAB product, the Signal Processing toolbox (SP toolbox) is required for this book. The original development of the book was done using the professional version 3.5 running under DOS. The MATLAB scripts and functions described in the book were later extended and made compatible with the present version of MATLAB. Furthermore, through the services of www.cengagebrain.com every effort will be made to preserve this compatibility under future versions of MATLAB.

In this section, we will undertake a brief review of MATLAB. The scope and power of MATLAB go far beyond the few topics discussed in this section. For more detailed tutorial-based discussion, students and readers new to MATLAB should also consult several excellent reference books available in the literature, including [10], [7], and [21]. The information given in all these references, along with the online MATLAB's `help` facility, usually is sufficient to enable readers to use this book. The best approach to become familiar with MATLAB is to open a MATLAB session and experiment with various operators, functions, and commands until their use and capabilities are understood. Then one can progress to writing simple MATLAB scripts and functions to execute a sequence of instructions to accomplish an analytical goal.

1.2.1 GETTING STARTED

The interaction with MATLAB is through the command window of its graphical user interface (GUI). In the command window, the user types MATLAB instructions, which are executed instantaneously, and the results are displayed in the window. In the MATLAB command window the characters ">>" indicate the prompt which is waiting for the user to type a command to be executed. For example,

```
>> command;
```

means an instruction `command` has been issued at the MATLAB prompt. If a semicolon (;) is placed at the end of a command, then all output from that command is suppressed. Multiple commands can be placed on the same line, separated by semicolons ;. Comments are marked by the percent sign (%), in which case MATLAB ignores anything to the right of the sign. The comments allow the reader to follow code more easily. The integrated help manual provides help for every command through the fragment

```
>> help command;
```

which will provide information on the inputs, outputs, usage, and functionality of the command. A complete listing of commands sorted by functionality can be obtained by typing `help` at the prompt.

There are three basic elements in MATLAB: numbers, variables, and operators. In addition, punctuation marks (`,`, `;`, `:`, etc.) have special meanings.

Numbers MATLAB is a high-precision numerical engine and can handle all types of numbers, that is, integers, real numbers, complex numbers, among others, with relative ease. For example, the real number 1.23 is represented as simply `1.23` while the real number 4.56×10^7 can be written as `4.56e7`. The imaginary number $\sqrt{-1}$ is denoted either by `1i` or `1j`, although in this book we will use the symbol `1j`. Hence the complex number whose real part is 5 and whose imaginary part is 3 will be written as 5+1j*3. Other constants preassigned by MATLAB are `pi` for π, `inf` for ∞, and `NaN` for not a number (for example, 0/0). These preassigned constants are very important and, to avoid confusion, should not be redefined by users.

Variables In MATLAB, which stands for MATrix LABoratory, the basic variable is a matrix, or an array. Hence, when MATLAB operates on this variable, it operates on all its elements. This is what makes it a powerful and an efficient engine. MATLAB now supports multidimensional arrays; we will discuss only up to two-dimensional arrays of numbers.

1. **Matrix:** A matrix is a two-dimensional set of numbers arranged in rows and columns. Numbers can be real- or complex-valued.
2. **Array:** This is another name for matrix. However, operations on arrays are treated differently from those on matrices. This difference is very important in implementation.

The following are four types of matrices (or arrays):

- **Scalar:** This is a 1×1 matrix or a single number that is denoted by the *variable* symbol, that is, lowercase italic typeface like

$$a = a_{11}$$

- **Column vector:** This is an $(N \times 1)$ matrix or a vertical arrangement of numbers. It is denoted by the *vector* symbol, that is, lowercase bold typeface like

$$\mathbf{x} = [x_{i1}]_{i:1,\ldots,N} = \begin{bmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{N1} \end{bmatrix}$$

  A typical vector in linear algebra is denoted by the column vector.
- **Row vector:** This is a $(1 \times M)$ matrix or a horizontal arrangement of numbers. It is also denoted by the vector symbol, that is,

$$\mathbf{y} = [y_{1j}]_{j=1,\ldots,M} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1M} \end{bmatrix}$$

  A one-dimensional discrete-time signal is typically represented by an array as a row vector.
- **General matrix:** This is the most general case of an $(N \times M)$ matrix and is denoted by the matrix symbol, that is, uppercase bold typeface like

$$\mathbf{A} = [a_{ij}]_{i=1,\ldots,N;j=1,\ldots,m} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1M} \\ a_{21} & a_{22} & \cdots & a_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NM} \end{bmatrix}$$

  This arrangement is typically used for two-dimensional discrete-time signals or images.

MATLAB does not distinguish between an array and a matrix except for operations. The following assignments denote indicated matrix types in MATLAB:

`a = [3]` is a scalar,
`x = [1,2,3]` is a row vector,
`y = [1;2;3]` is a column vector, and
`A = [1,2,3;4,5,6]` is a matrix.

MATLAB provides many useful functions to create special matrices. These include `zeros(M,N)` for creating a matrix of all zeros, `ones(M,N)` for creating matrix of all ones, `eye(N)` for creating an $N \times N$ identity matrix, etc. Consult MATLAB's help manual for a complete list.

Operators MATLAB provides several arithmetic and logical operators, some of which follow. For a complete list, MATLAB's help manual should be consulted.

`=`	assignment	`==`	equality
`+`	addition	`-`	subtraction or minus
`*`	multiplication	`.*`	array multiplication
`^`	power	`.^`	array power
`/`	division	`./`	array division
`<>`	relational operators	`&`	logical AND
`|`	logical OR	`~`	logical NOT
`'`	transpose	`.'`	array transpose

We now provide a more detailed explanation on some of these operators.

1.2.2 MATRIX OPERATIONS

Following are the most useful and important operations on matrices.

- **Matrix addition and subtraction:** These are straightforward operations that are also used for array addition and subtraction. Care must be taken that the two matrix operands be *exactly* the same size.
- **Matrix conjugation:** This operation is meaningful only for complex-valued matrices. It produces a matrix in which all imaginary parts are negated. It is denoted by $\mathbf{A}^*$ in analysis and by `conj(A)` in MATLAB.
- **Matrix transposition:** This is an operation in which every row (column) is turned into column (row). Let $\mathbf{X}$ be an $(N \times M)$ matrix. Then

$$\mathbf{X}^{'} = [x_{ji}]\,; \quad j = 1, \ldots, M,\ i = 1, \ldots, N$$

  is an $(M \times N)$ matrix. In MATLAB, this operation has one additional feature. If the matrix is real-valued, then the operation produces the usual transposition. However, if the matrix is complex-valued, then the operation produces a complex-conjugate transposition. To obtain just the transposition, we use the array operation of conjugation, that is, `A.'` will do just the transposition.

- **Multiplication by a scalar:** This is a simple straightforward operation in which each element of a matrix is scaled by a constant, that is,

$$ab \Rightarrow \texttt{a*b} \text{ (scalar)}$$

$$a\mathbf{x} \Rightarrow \texttt{a*x} \text{ (vector or array)}$$

$$a\mathbf{X} \Rightarrow \texttt{a*X} \text{ (matrix)}$$

This operation is also valid for an array scaling by a constant.

- **Vector-vector multiplication:** In this operation, one has to be careful about matrix dimensions to avoid invalid results. The operation produces either a scalar or a matrix. Let $\mathbf{x}$ be an $(N \times 1)$ and $\mathbf{y}$ be a $(1 \times M)$ vectors. Then

$$\texttt{x} * \texttt{y} \Rightarrow \mathbf{xy} = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \begin{bmatrix} y_1 & \cdots & y_M \end{bmatrix} = \begin{bmatrix} x_1y_1 & \cdots & x_1y_M \\ \vdots & \ddots & \vdots \\ x_Ny_1 & \cdots & x_Ny_M \end{bmatrix}$$

produces a matrix. If $M = N$, then

$$\texttt{y} * \texttt{x} \Rightarrow \mathbf{yx} = \begin{bmatrix} y_1 & \cdots & y_M \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix} = x_1y_1 + \cdots + x_My_M$$

- **Matrix-vector multiplication:** If the matrix and the vector are compatible (i.e., the number of matrix-columns is equal to the vector-rows), then this operation produces a column vector:

$$\texttt{y = A*x} \Rightarrow \mathbf{y} = \mathbf{Ax} = \begin{bmatrix} a_{11} & \cdots & a_{1M} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NM} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}$$

- **Matrix-matrix multiplication:** Finally, if two matrices are compatible, then their product is well-defined. The result is also a matrix with the number of rows equal to that of the first matrix and the number of columns equal to that of the second matrix. Note that the order in matrix multiplication is very important.

Array Operations These operations treat matrices as arrays. They are also known as *dot operations* because the arithmetic operators are prefixed by a dot (.), that is, `.*`, `./`, or `.^`.

- **Array multiplication:** This is an element by element multiplication operation. For it to be a valid operation, both arrays must be the same size. Thus we have

$$\texttt{x.*y} \rightarrow \text{1D array}$$

$$\texttt{X.*Y} \rightarrow \text{2D array}$$

- **Array exponentiation:** In this operation, a scalar (real- or complex-valued) is raised to the power equal to every element in an array, that is,

$$\texttt{a.\^{}x} \equiv \begin{bmatrix} a^{x_1} \\ a^{x_2} \\ \vdots \\ a^{x_N} \end{bmatrix}$$

is an $(N \times 1)$ array, whereas

$$\texttt{a.\^{}X} \equiv \begin{bmatrix} a^{x_{11}} & a^{x_{12}} & \cdots & a^{x_{1M}} \\ a^{x_{21}} & a^{x_{22}} & \cdots & a^{x_{2M}} \\ \vdots & \vdots & \ddots & \vdots \\ a^{x_{N1}} & a^{x_{N2}} & \cdots & a^{x_{NM}} \end{bmatrix}$$

is an $(N \times M)$ array.

- **Array transposition:** As explained, the operation `A.'` produces transposition of real- or complex-valued array `A`.

Indexing Operations MATLAB provides very useful and powerful array indexing operations using operator `:`. It can be used to generate sequences of numbers as well as to access certain row/column elements of a matrix. Using the fragment `x = [a:b:c]`, we can generate numbers from `a` to `c` in `b` increments. If `b` is positive (negative) then, we get increasing (decreasing) values in the sequence `x`.

The fragment `x(a:b:c)` accesses elements of `x` beginning with index `a` in steps of `b` and ending at `c`. Care must be taken to use integer values of indexing elements. Similarly, the `:` operator can be used to extract a submatrix from a matrix. For example, `B = A(2:4,3:6)` extracts a 3×4 submatrix starting at row 2 and column 3.

Another use of the `:` operator is in forming column vectors from row vectors or matrices. When used on the right-hand side of the equality (`=`) operator, the fragment `x=A(:)` forms a long column vector `x` of elements of `A` by concatenating its columns. Similarly, `x=A(:,3)` forms a vector `x` from the third column of `A`. However, when used on the right-hand side of the `=` operator, the fragment `A(:)=x` reformats elements in `x` into a predefined size of `A`.

Control-Flow MATLAB provides a variety of commands that allow us to control the flow of commands in a program. The most common construct is the `if-elseif-else` structure. With these commands, we can allow different blocks of code to be executed depending on some condition. The format of this construct is

```
 if condition1
   command1
elseif condition2
   command2
else
   command3
end
```

which executes statements in `command1` if `condition-1` is satisfied; otherwise statements in `command2` if `condition-2` is satisfied, or finally statements in `command3`.

Another common control flow construct is the `for..end` loop. It is simply an iteration loop that tells the computer to repeat some task a given number of times. The format of a `for..end` loop is

```
 for index = values
   program statements
          :
end
```

Although `for..end` loops are useful for processing data inside of arrays by using the iteration variable as an index into the array, whenever possible the user should try to use MATLAB's whole array mathematics. This will result in shorter programs and more efficient code. In some situations the use of the `for..end` loop is unavoidable. The following example illustrates these concepts.

□ **EXAMPLE 1.1** Consider the following sum of sinusoidal functions.

$$x(t) = \sin(2\pi t) + \tfrac{1}{3}\sin(6\pi t) + \tfrac{1}{5}\sin(10\pi t) = \sum_{k=1}^{3} \frac{1}{k}\sin(2\pi k t), \qquad 0 \le t \le 1$$

Using MATLAB, we want to generate samples of $x(t)$ at time instances `0:0.01:1`. We will discuss three approaches.

Approach 1 Here we will consider a typical C or Fortran approach, that is, we will use two `for..end` loops, one each on `t` and `k`. This is the most inefficient approach in MATLAB, but possible.

```
>> t = 0:0.01:1; N = length(t); xt = zeros(1,N);
>> for n = 1:N
>>     temp = 0;
>>     for k = 1:3
>>         temp = temp + (1/k)*sin(2*pi*k*t(n));
>>     end
>>     xt(n) = temp;
>> end
```

Approach 2 In this approach, we will compute each sinusoidal component in one step as a vector, using the time vector `t = 0:0.01:1` and then add all components using one `for..end` loop.

```
>> t = 0:0.01:1; xt = zeros(1,length(t));
>> for k = 1:3
>>    xt = xt + (1/k)*sin(2*pi*k*t);
>> end
```

Clearly, this is a better approach with fewer lines of code than the first one.

Approach 3 In this approach, we will use matrix-vector multiplication, in which MATLAB is very efficient. For the purpose of demonstration, consider only four values for $t = [t_1, t_2, t_3, t_4]$. Then

$$x(t_1) = \sin(2\pi t_1) + \tfrac{1}{3}\sin(2\pi 3t_1) + \tfrac{1}{5}\sin(2\pi 5t_1)$$

$$x(t_2) = \sin(2\pi t_2) + \tfrac{1}{3}\sin(2\pi 3t_2) + \tfrac{1}{5}\sin(2\pi 5t_2)$$

$$x(t_3) = \sin(2\pi t_3) + \tfrac{1}{3}\sin(2\pi 3t_3) + \tfrac{1}{5}\sin(2\pi 5t_3)$$

$$x(t_4) = \sin(2\pi t_4) + \tfrac{1}{3}\sin(2\pi 3t_4) + \tfrac{1}{5}\sin(2\pi 5t_4)$$

which can be written in matrix form as

$$\begin{bmatrix} x(t_1) \\ x(t_2) \\ x(t_3) \\ x(t_4) \end{bmatrix} = \begin{bmatrix} \sin(2\pi t_1) & \sin(2\pi 3t_1) & \sin(2\pi 5t_1) \\ \sin(2\pi t_2) & \sin(2\pi 3t_2) & \sin(2\pi 5t_2) \\ \sin(2\pi t_3) & \sin(2\pi 3t_3) & \sin(2\pi 5t_3) \\ \sin(2\pi t_4) & \sin(2\pi 3t_4) & \sin(2\pi 5t_4) \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$= \sin\left(2\pi \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{bmatrix} \begin{bmatrix} 1 & 3 & 5 \end{bmatrix} \right) \begin{bmatrix} 1 \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

or after taking transposition

$$\begin{bmatrix} x(t_1) & x(t_2) & x(t_3) & x(t_4) \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{5} \end{bmatrix} \sin\left(2\pi \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \begin{bmatrix} t_1 & t_2 & t_3 & t_4 \end{bmatrix} \right)$$

Thus the MATLAB code is

```
>> t = 0:0.01:1; k = 1:3;
>> xt = (1./k)*sin(2*pi*k'*t);
```

Note the use of the array division (`1./k`) to generate a row vector and matrix multiplications to implement all other operations. This is the most compact code and the most efficient execution in MATLAB, especially when the number of sinusoidal terms is very large.

1.2.3 SCRIPTS AND FUNCTIONS

MATLAB is convenient in the interactive command mode if we want to execute few lines of code. But it is not efficient if we want to write code of several lines that we want to run repeatedly or if we want to use the code in several programs with different variable values. MATLAB provides two constructs for this purpose.

Scripts The first construct can be accomplished by using the so-called block mode of operation. In MATLAB, this mode is implemented using a *script* file called an m-file (with an extension `.m`), which is only a text file that contains each line of the file as though you typed them at the command prompt. These scripts are created using MATLAB's built-in editor, which also provides for context-sensitive colors and indents for making fewer mistakes and for easy reading. The script is executed by typing the name of the script at the command prompt. The script file must be in the current directory on in the directory of the `path` environment. As an example, consider the sinusoidal function in Example 1.1. A general form of this function is

$$x(t) = \sum_{k=1}^{K} c_k \sin(2\pi k t) \tag{1.1}$$

If we want to experiment with different values of the coefficients c_k and/or the number of terms K, then we should create a script file. To implement the third approach in Example 1.1, we can write a script file

```
% Script file to implement (1.1)
t = 0:0.01:1; k = 1:2:5; ck = 1./k;
xt = ck * sin(2*pi*k'*t);
```

Now we can experiment with different values.

Functions The second construct of creating a block of code is through subroutines. These are called *functions*, which also allow us to extend the capabilities of MATLAB. In fact a major portion of MATLAB is assembled using function files in several categories and using special collections called *toolboxes*. Functions are also m-files (with extension `.m`). A major difference between script and function files is that the first executable line in a function file begins with the keyword `function` followed by an output-input variable declaration. As an example, consider the computation of the $x(t)$ function in Example 1.1 with an arbitrary number of sinusoidal terms, which we will implement as a function stored as m-file `sinsum.m`.

```
function xt = sinsum(t,ck)
% Computes sum of sinusoidal terms of the form in (1.1)
% x = sinsum(t,ck)
%
K = length(ck); k = 1:K;
ck = ck(:)'; t = t(:)';
xt = ck * sin(2*pi*k'*t);
```

The vectors `t` and `ck` should be assigned prior to using the `sinsum` function. Note that `ck(:)'` and `t(:)'` use indexing and transposition operations to force them to be row vectors. Also note the comments immediately following the `function` declaration, which are used by the `help sinsum` command. Sufficient information should be given there for the user to understand what the function is supposed to do.

1.2.4 PLOTTING

One of the most powerful features of MATLAB for signal and data analysis is its graphical data plotting. MATLAB provides several types of plots, starting with simple two-dimensional (2D) graphs to complex, higher-dimensional plots with full-color capability. We will examine only the 2D plotting, which is the plotting of one vector versus another in a 2D coordinate system. The basic plotting command is the `plot(t,x)` command, which generates a plot of `x` values versus `t` values in a separate figure window. The arrays `t` and `x` should be the same length and orientation. Optionally, some additional formatting keywords can also be provided in the `plot` function. The commands `xlabel` and `ylabel` are used to add text to the axis, and the command `title` is used to provide a title on the top of the graph. When plotting data, one should get into the habit of always labeling the axis and providing a title. Almost all aspects of a plot (style, size, color, etc.) can be changed by appropriate commands embedded in the program or directly through the GUI.

The following set of commands creates a list of sample points, evaluates the sine function at those points, and then generates a plot of a simple sinusoidal wave, putting axis labels and title on the plot.

```
>> t = 0:0.01:2; % sample points from 0 to 2 in steps of 0.01
>> x = sin(2*pi*t); % Evaluate sin(2 pi t)
>> plot(t,x,'b'); % Create plot with blue line
>> xlabel('t in sec'); ylabel('x(t)'); % Label axis
>> title('Plot of sin(2\pi t)'); % Title plot
```

The resulting plot is shown in Figure 1.1.

For plotting a set of discrete numbers (or discrete-time signals), we will use the `stem` command which displays data values as a stem, that is, a small circle at the end of a line connecting it to the horizontal axis. The circle can be open (default) or filled (using the option `'filled'`). Using Handle Graphics (MATLAB's extensive manipulation of graphics primitives), we can resize circle markers. The following set of commands displays a discrete-time sine function using these constructs.

```
>> n = 0:1:40; % sample index from 0 to 20
>> x = sin(0.1*pi*n); % Evaluate sin(0.2 pi n)
>> Hs = stem(n,x,'b','filled'); % Stem-plot with handle Hs
>> set(Hs,'markersize',4); % Change circle size
>> xlabel('n'); ylabel('x(n)'); % Label axis
>> title('Stem Plot of sin(0.2 pi n)'); % Title plot
```

The resulting plot is shown in Figure 1.2.

MATLAB provides an ability to display more than one graph in the same figure window. By means of the `hold on` command, several graphs can be plotted on the same set of axes. The `hold off` command stops the simultaneous plotting. The following MATLAB fragment (Figure 1.3)

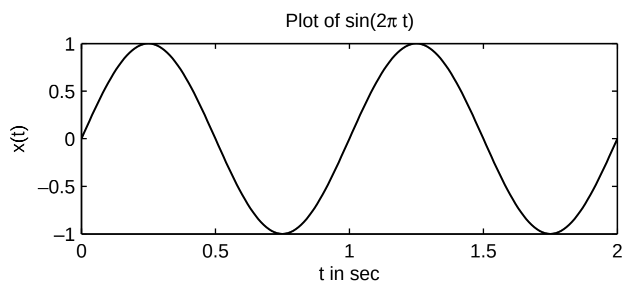


FIGURE 1.1 *Plot of the* $\sin(2\pi t)$ *function*

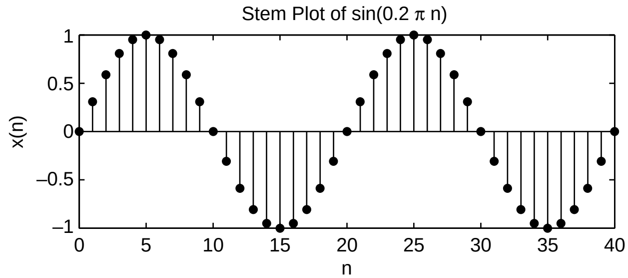


FIGURE 1.2 *Plot of the sin(0.2π n) sequence*

displays graphs in Figures 1.1 and 1.2 as one plot, depicting a "sampling" operation that we will study later.

```
>> plot(t,xt,'b'); hold on; % Create plot with blue line
>> Hs = stem(n*0.05,xn,'b','filled'); % Stem-plot with handle Hs
>> set(Hs,'markersize',4); hold off; % Change circle size
```

Another approach is to use the `subplot` command, which displays several graphs in each individual set of axes arranged in a grid, using the parameters in the `subplot` command. The following fragment (Figure 1.4) displays graphs in Figure 1.1 and 1.2 as two separate plots in two rows.

```
. . .
>> subplot(2,1,1); % Two rows, one column, first plot
>> plot(t,x,'b'); % Create plot with blue line
. . .
>> subplot(2,1,2); % Two rows, one column, second plot
>> Hs = stem(n,x,'b','filled'); % Stem-plot with handle Hs
. . .
```

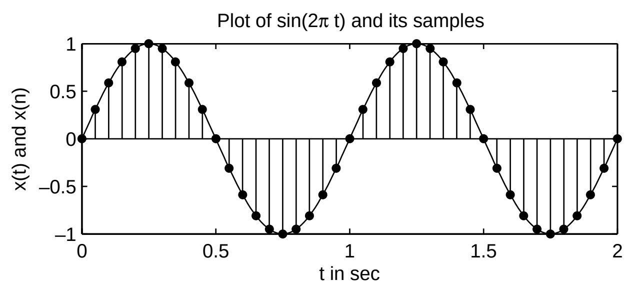


FIGURE 1.3 *Simultaneous plots of $x(t)$ and $x(n)$*

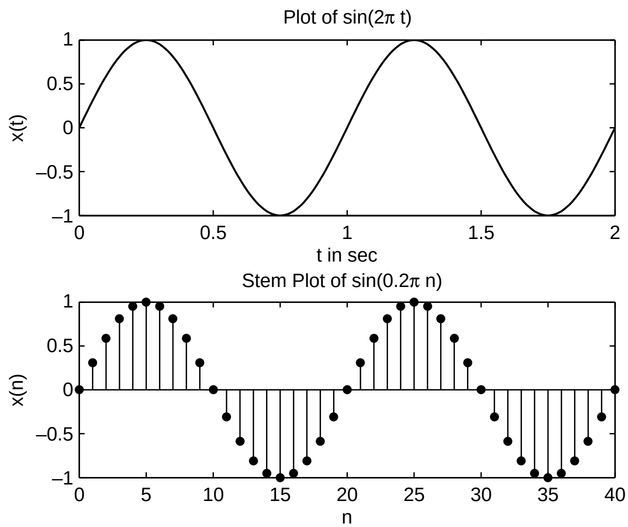


FIGURE 1.4 *Plots of $x(t)$ and $x(n)$ in two rows*

The plotting environment provided by MATLAB is very rich in its complexity and usefulness. It is made even richer using the handle-graphics constructs. Therefore, readers are strongly recommended to consult MATLAB's manuals on plotting. Many of these constructs will be used throughout this book.

In this brief review, we have barely made a dent in the enormous capabilities and functionalities in MATLAB. Using its basic integrated help system, detailed help browser, and tutorials, it is possible to acquire sufficient skills in MATLAB in a reasonable amount of time.

1.3 APPLICATIONS OF DIGITAL SIGNAL PROCESSING

The field of DSP has matured considerably over the last several decades and now is at the core of many diverse applications and products. These include

- speech/audio (speech recognition/synthesis, digital audio, equalization, etc.),
- image/video (enhancement, coding for storage and transmission, robotic vision, animation, etc.),
- military/space (radar processing, secure communication, missile guidance, sonar processing, etc.),
- biomedical/health care (scanners, ECG analysis, X-ray analysis, EEG brain mappers, etc.)

- consumer electronics (cellular/mobile phones, digital television, digital camera, Internet voice/music/video, interactive entertainment systems, etc) and many more.

These applications and products require many interconnected complex steps, such as collection, processing, transmission, analysis, audio/display of real-world information in near real time. DSP technology has made it possible to incorporate these steps into devices that are innovative, affordable, and of high quality (for example, iPhone from Apple, Inc.). A typical application to music is now considered as a motivation for the study of DSP.

Musical sound processing In the music industry, almost all musical products (songs, albums, etc.) are produced in basically two stages. First, the sound from an individual instrument or performer is recorded in an acoustically inert studio on a single track of a multitrack recording device. Then, stored signals from each track are digitally processed by the sound engineer by adding special effects and combined into a stereo recording, which is then made available either on a CD or as an audio file.

The audio effects are artificially generated using various signal-processing techniques. These effects include echo generation, reverberation (concert hall effect), flanging (in which audio playback is slowed down by placing DJ's thumb on the *flange* of the feed reel), chorus effect (when several musicians play the same instrument with small changes in amplitudes and delays), and phasing (aka phase shifting, in which an audio effect takes advantage of how sound waves interact with each other when they are out of phase). These effects are now generated using digital-signal-processing techniques. We now discuss a few of these sound effects in some detail.

Echo Generation The most basic of all audio effects is that of *time delay*, or echoes. It is used as the building block of more complicated effects such as reverb or flanging. In a listening space such as a room, sound waves arriving at our ears consist of *direct* sound from the source as well as *reflected* off the walls, arriving with different amounts of attenuation and delays.

Echoes are delayed signals, and as such are generated using delay units. For example, the combination of the direct sound represented by discrete signal $y[n]$ and a single echo appearing D samples later (which is related to delay in seconds) can be generated by the equation of the form (called a difference equation)

$$x[n] = y[n] + \alpha y[n-D], \qquad |\alpha| < 1 \tag{1.2}$$

where $x[n]$ is the resulting signal and α models attenuation of the direct sound. Difference equations are implemented in MATLAB using the `filter` function. Available in MATLAB is a short snippet of Handel's hallelujah chorus, which is a digital sound about 9 seconds long, sampled at 8192 sam/sec. To experience the sound with echo in (1.2), execute the following fragment at the command window. The echo is delayed by $D = 4196$ samples, which amount to 0.5 sec of delay.

```
load handel; % the signal is in y and sampling freq in Fs
sound(y,Fs); pause(10); % Play the original sound
alpha = 0.9; D = 4196; % Echo parameters
b = [1,zeros(1,D),alpha]; % Filter parameters
x = filter(b,1,y); % Generate sound plus its echo
sound(x,Fs); % Play sound with echo
```

You should be able to hear the distinct echo of the chorus in about a half second.

Echo Removal After executing this simulation, you may experience that the echo is an objectionable interference while listening. Again DSP can be used effectively to reduce (almost eliminate) echoes. Such an echo-removal system is given by the difference equation

$$w[n] + \alpha w[n - D] = x[n] \tag{1.3}$$

where $x[n]$ is the echo-corrupted sound signal and $w[n]$ is the output sound signal, which has the echo (hopefully) removed. Note again that this system is very simple to implement in software or hardware. Now try the following MATLAB script on the signal $x[n]$.

```
w = filter(1,b,x);
sound(w,Fs)
```

The echo should no longer be audible.

Digital Reverberation Multiple close-spaced echoes eventually lead to reverberation, which can be created digitally using a somewhat more involved difference equation

$$x[n] = \sum_{k=0}^{N-1} \alpha^k y[n - kD] \tag{1.4}$$

which generates multiple echoes spaced D samples apart with exponentially decaying amplitudes. Another natural sounding reverberation is given by

$$x[n] = \alpha y[n] + y[n-D] + \alpha x[n-D], \quad |\alpha| < 1 \tag{1.5}$$

which simulates a higher echo density.

These simple applications are examples of DSP. Using techniques, concepts, and MATLAB functions learned in this book you should be able to simulate these and other interesting sound effects.

1.4 BRIEF OVERVIEW OF THE BOOK

The first part of this book, which comprises Chapters 2 through 5, deals with the signal-analysis aspect of DSP. Chapter 2 begins with basic descriptions of discrete-time signals and systems. These signals and systems are analyzed in the frequency domain in Chapter 3. A generalization of the frequency-domain description, called the z-transform, is introduced in Chapter 4. The practical algorithms for computing the Fourier transform are discussed in Chapter 5 in the form of the discrete Fourier transform and the fast Fourier transform.

Chapters 6 through 8 constitute the second part of this book, which is devoted to the signal-filtering aspect of DSP. Chapter 6 describes various implementations and structures of digital filters. It also introduces finite-precision number representation, filter coefficient quantization, and its effect on filter performance. Chapter 7 introduces design techniques and algorithms for designing one type of digital filter called *finite-duration impulse response (FIR) filters*, and Chapter 8 provides a similar treatment for another type of filter called *infinite-duration impulse response (IIR) filters*. In both chapters only the simpler but practically useful techniques of filter design are discussed. More advanced techniques are not covered.

Finally, the last part, which consists of the remaining four chapters, provides important topics and applications in DSP. Chapter 9 deals with the useful topic of the sampling-rate conversion and applies FIR filter designs from Chapter 7 to design practical sample-rate converters. Chapter 10 extends the treatment of finite-precision numerical representation to signal quantization and the effect of finite-precision arithmetic on filter performance. The last two chapters provide some practical applications in the form of projects that can be done using material presented in the first 10 chapters. In Chapter 11, concepts in adaptive filtering are introduced, and simple projects in system identification, interference suppression, adaptive line enhancement, and so forth are discussed. In Chapter 12 a brief introduction to digital communications is presented with projects involving such topics as PCM, DPCM, and LPC being outlined.

In all these chapters, the central theme is the generous use and adequate demonstration of MATLAB, which can be used as an effective teaching as well as learning tool. Most of the existing MATLAB functions for DSP are described in detail, and their correct use is demonstrated in many examples. Furthermore, many new MATLAB functions are developed to provide insights into the working of many algorithms. The authors believe that this hand-holding approach enables students to dispel fears about DSP and provides an enriching learning experience.