Dilemma not Trilemma? Capital Controls and Exchange Rates with Volatile Capital Flows

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We consider a standard New Keynesian model of a small open economy with nominal rigidities and study optimal capital controls. Consistent with the Mundellian view, we find that the exchange rate regime is key. However, in contrast with the Mundellian view, we find that capital controls are desirable even when the exchange rate is flexible. Optimal capital controls lean against the wind and help smooth out capital flows.

1 Introduction

Volatile capital flows have been extensively blamed for episodes of booms and busts in emerging markets (see e.g. Calvo, 1998). What sort of macroeconomic intervention, if any, is required to deal with these episodes?

Mundell's celebrated trilemma provides a powerful framework to analyze this question. It emphasizes the importance of the exchange rate regime. With fixed a exchange rate, there is a case for interfering with the free movement of international capital flows by imposing capital controls in order to regain monetary autonomy (see e.g. Farhi and Werning, 2012; Schmitt-Grohe and Uribe, 2012). By contrast, with a flexible exchange rate, monetary policy is already independent and there is no prima facie case for restricting international capital mobility.

This last conclusion has recently been challenged by policymakers and academics. According to this view, there is a *dilemma*, with independent monetary policies possible only when the capital account is managed (Rey, 2013). The goal of this paper is to investigate this argument using a model with nominal rigidities in the New Keynesian tradition, which is better suited for normative analysis than the traditional Mundell-Flemming models.[1,2] In our model, we capture capital inflow surges and sudden stops to a country by assuming a time-varying risk premium in the interest rates available this country.[3]

Our results share similarities and differences with the standard Mundellian conclusions. Consistent with the Mundellian view, we confirm that the exchange rate regime is crucial. The optimal management of capital flows depends importantly on the exchange rate regime. However, in contrast with the traditional Mundellian view, we find a case for capital controls even with flexible exchange rates. Optimal capital controls lean against the wind. Thus, dealing with a capital inflow requires a temporary tax on inflows and a subsidy on outflows; dealing with a sudden stop requires a temporary subsidy on inflows and a tax on outflows.

To understand the role for capital controls with flexible exchange rates, consider the case of a sudden stop. Without capital controls, optimal monetary policy responds by allowing a depreciation of the nominal exchange rate and an increase in the nominal interest rate. The rebalancing in the current account occurs by a drop in domestic spending. Optimal capital controls take the form of temporary subsidies on inflows and taxes on outflows to smooth out these responses. This mitigates the required depreciation of the exchange rate, the increase in nominal interest rate, the reversal in the current account, and of the drop in consumption. We trace back the rationale for these interventions to the desirability of smoothing the terms of trade and stabilizing the macroeconomy. This is best done using two imperfect instruments, monetary policy and capital controls, rather than a single instrument.

A large literature in international macroeconomics is motivated by the volatility of capital flows, especially "sudden stops", see Mendoza (2010) and the references therein. Models with financial frictions such as Caballero and Krishnamurthy (2004) emphasize domestic and international collateral constraints that create inefficiencies and a potential role for intervention in international borrowing, even without nominal rigidities. A related strand of work emphasizes pecuniary externalities that work through prices in borrowing constraints, for example Bianchi and Mendoza (2010), Bianchi (2011), Jeanne and Korinek (2010), Korinek (2011). All these papers provide a rationale for "pruden-

[1]For example, how should one evaluate two policies that lead to the same output, but different current accounts and inflation rates? The basic Mundell-Fleming model was not designed to answer these questions.

[2]It would be absurd to argue against the practical advantages of using the Mundell-Fleming model for a first pass positive analysis. We do not hold this view. However, it demands extreme Luddism to deny the benefits of going beyond the Mundell-Fleming model, especially for a normative analysis.

[3]An alternative is to model time-varying, country-specific borrowing constraints. Such constraint induce fluctuations in the *shadow* interest rates faced by a given country. Thus, we conjecture that the analysis and implications are similar in such a formulation.

tial" policies that attempt to prevent excessive borrowing. An important difference with our analysis of capital controls and more generally with the Mundellian logic, is that the models in these papers are real, and as a result, optimal capital controls are independent of the exchange rate regime.

2 A Small Open Economy

We build on Farhi and Werning (2012), which in turn builds on the framework by Gali and Monacelli (2005, 2008). The model is composed of a continuum of open economies. Our main focus is on policy in a single country, which we call Home, taking as given the rest of the world, which we call Foreign. However, we also explore the joint policy problem for the entire world when coordination is possible. In contrast to their simplifying assumption of complete markets, we prefer to assume international financial markets are incomplete. No risk sharing between countries is allowed, only risk free borrowing and lending. Given this assumption, to keep the analysis tractable, we limit our attention to one-time unanticipated shocks to the economy. Relative to the literature, this is not a limitation since most studies, including Gali-Monacelli, work with linearized equilibrium conditions, so that the response to shocks is unaffected by the presence of future shocks.

2.1 Households

There is a continuum measure one of countries $i \in [0,1]$. We focus attention on a single country, which we call Home, and can be thought of as a particular value $H \in [0,1]$. In every country, there is a representative household with preferences represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right], \tag{1}$$

where N_t is labor, and C_t is a consumption index defined by

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

where $C_{H,t}$ is an index of consumption of domestic goods given by

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $j \in [0,1]$ denotes an individual good variety. Similarly, $C_{F,t}$ is a consumption index of imported goods given by

$$C_{F,t} = \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}},$$

where $C_{i,t}$ is, in turn, an index of the consumption of varieties of goods imported from country i, given by

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Thus, ϵ is the elasticity between varieties produced within a given country, η the elasticity between domestic and foreign goods, and γ the elasticity between goods produced in different foreign countries. An important special case obtains when $\sigma = \eta = \gamma = 1$. We call this the Cole-Obstfeld case, in reference to Cole and Obstfeld (1991). This case is more tractable and has some special implications that are worth highlighting. Thus, we devote special attention to it, although we will also derive results away from it.

The parameter α indexes the degree of home bias, and can be interpreted as a measure of openness. Consider both extremes: as $\alpha \to 0$ the share of foreign goods vanishes; as $\alpha \to 1$ the share of home goods vanishes. Since the country is infinitesimal, the latter captures a very open economy without home bias; the former a closed economy barely trading with the outside world.

Households seek to maximize their utility subject to the sequence of budget constraints

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j)djdi + D_{t+1} + \int_0^1 E_{i,t}D_{t+1}^i di$$
$$\leq W_t N_t + \Pi_t + T_t + (1+i_{t-1})D_t + \int_0^1 \frac{1+\tau_{t-1}}{1+\tau_{t-1}^i} \frac{\Psi_{t-1}}{\Psi_{i,t-1}} E_{i,t}(1+i_{t-1}^i)D_t^i di$$

for $t = 0, 1, 2, \ldots$ In this inequality, $P_{H,t}(j)$ is the price of domestic variety j, $P_{i,t}$ is the price of variety j imported from country i, W_t is the nominal wage, Π_t represents nominal profits and T_t is a nominal lump sum transfer. All these variables are expressed in domestic currency. The portfolio of home agents is composed of home and foreign bond holding: D_t is home bond holdings of home agents, D_t^i is bond holdings of country i of home agents. The returns on these bonds are determined by the nominal interest rate in the home country i_t, the nominal interest rate i_t^i in country i, and the evolution of the nominal exchange rate $E_{i,t}$ between home and country i. Capital controls are modeled as follows: τ_t is a subsidy on capital outflows (tax on capital inflows) in the home country, and similarly τ_t^i is a subsidy on capital outflows (tax on capital inflows) in country i. The proceeds of these taxes are rebated lump sum to the households at Home and country i, respectively.

Importantly, we have introduced risk premium shocks Ψ_t and $\Psi_{i,t}$ as wedges between foreign investors and the home country, in addition to capital controls for all $i \in [0,1]$. We do not attempt to model these wedges endogenously. Although our model lacks uncertainty, it could stand in for the risks of investing in the home country, if these risks are not equally valued between borrowers and lenders. It may also represent investor's preferences for a particular country's bonds along the lines of portfolio-balance models a la Black (1973) and Kouri (1976). Risk premium shocks can also be thought of capturing time-varying and country-specific borrowing constraints—the risk premium shock is simply the multiplier on the borrowing constraint.

Capital controls and risk premium wedges enter the agent's budget constraint in a similar way. The key difference between the two is that the domestic subsidy on outflows is financed with a lump sum tax on domestic agents, while the risk premium wedge is financed with a lump sum tax at the world level: the lump sum rebate T_t is given by

$$T_t = -\int_0^1 \int_0^1 (\frac{\Psi_{i,t-1}}{\Psi_{j,t-1}} - 1)\frac{1}{1+\tau_t^j}(1+i_{t-1}^j)E_{j,t}D_t^{j,i}didj \\ - \int_0^1 \frac{\tau_{t-1}}{1+\tau_{t-1}^i}\frac{\Psi_{t-1}}{\Psi_{i,t-1}}E_{i,t}(1+i_{t-1}^i)D_t^i di + \tau_L W_t N_t,$$

where $D_t^{i,j}$ is bond holdings of country j of agents of country i, and τ_L is a constant labor tax. Actually, in most of our analysis, we consider a small open economy. We treat the rest of the world as symmetric countries which do not face risk premium shocks $\Psi_{i,t} = 0$ and do not impose capital controls $\tau_t^i = 0$. In that case, the lump sum rebate becomes

$$T_t = -\tau_{t-1}\Psi_{t-1}\int_0^1 E_{i,t}(1+i_{t-1}^i)D_t^i di + \tau_L W_t N_t,$$

and the difference between risk premium wedges and capital controls appears most clearly. Risk premium shocks affects equally the interest rate at which home agents perceive they can borrow and lend to the rest of the world, and the interest rate at which the home country as a whole can borrow and lend to the rest of the world.By contrast, capital controls only affect the interest rate at which home agents perceive they can borrow and lend to the rest of the world, but not the interest rate at which the home country as a whole can borrow and lend to the rest of the world.[4]

2.2 Firms

Technology. A typical firm in the home economy produces a differentiated good with a linear technology given by

$$Y_t(j) = AN_t(j). \tag{2}$$

Price-setting assumptions. We will consider a variety of price setting assumptions: flexible prices, one-period in advance sticky prices, and sticky prices a la Calvo.

As in Gali and Monacelli (2005), we maintain the assumption that the Law of One Price (LOP) holds so that at all times, the price of a given variety in different countries is identical once expressed in the same currency. This assumption is sometimes known as Producer Currency Pricing (PCP).[5]

First, consider the case of flexible prices. We allow for a constant employment tax $1+\tau^L$, so that real marginal cost deflated by Home PPI is given by $MC_t = \frac{1+\tau^L}{A}\frac{W_t}{P_{H,t}}$.We take this employment tax to be constant in our model. We explain below how it is determined. Firm j optimally sets its price $P_{H,t}(j)$ to maximize

$$\max_{P_{H,t}(j)} \left(P_{H,t}(j)Y_{t|t} - P_{H,t}MC_t Y_{t|t}\right)$$

where $Y_{t|t} = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} Y_t$, taking the sequences for MC_t, Y_t and $P_{H,t}$ as given. Second, we consider the case where prices are perfectly rigid. Third we consider the case where are set one period in advance as in Obstfeld and Rogoff (1995). Since we consider only one time-unanticipated shocks around the symmetric deterministic steady state, this simply means that prices are fixed at $t=0$ and flexible for $t \geq 1$. Fourth, we consider Calvo price setting, where in every period, a randomly selected fraction $1-\delta$ of firms can reset their prices. Those firms that get to reset their price choose a reset price P_t^r to solve

$$\max_{P_t^r} \sum_{k=0}^{\infty} \delta^k \left(\prod_{h=1}^{k} \frac{1}{1+i_{t+h}}\right) \left(P_t^r Y_{t+k|t} - P_{H,t} MC_t Y_{t+k|t}\right)$$

[4] This observation already already gives a sense of why the naive idea that capital controls should simply offset risk premium shocks $\tau_t = \Psi_t^{-1} - 1$ is not supported by our analysis.

[5] This is sometimes contrasted with the assumption of Local Currency Pricing (LCP), where each variety's price is set separately for each country and quoted (and potentially sticky) in that country's local currency. Thus, LOP does not necessarily hold. It has been shown by Devereux and Engel (2003) that LCP and PCP may have different implications for monetary policy.

where $Y_{t+k|t} = \left(\frac{P_t^r}{P_{H,t+k}}\right)^{-\epsilon} Y_{t+k}$.

2.3 Terms of Trade, Exchange Rates and UIP

It is useful to define the following price indices: home's Consumer Price Index (CPI) $P_t = [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$, home's Producer Price Index (PPI) $P_{H,t} = [\int_0^1 P_{H,t}(j)^{1-\epsilon} dj]^{\frac{1}{1-\epsilon}}$, and the index for imported goods $P_{F,t} = [\int_0^1 P_{i,t}^{1-\gamma} di]^{\frac{1}{1-\gamma}}$, where $P_{i,t} = [\int_0^1 P_{i,t}(j)^{1-\epsilon} dj]^{\frac{1}{1-\epsilon}}$ is country i's PPI.

Let $E_{i,t}$ be nominal exchange rate between home and i (an increase in $E_{i,t}$ is a depreciation of the home currency). Because the Law of One Price holds, we can write $P_{i,t}(j) = E_{i,t} P_{i,t}^i(j)$ where $P_{i,t}^i(j)$ is country i's price of variety j expressed in its own currency. Similarly, $P_{i,t} = E_{i,t} P_{i,t}^i$ where $P_{i,t}^i = [\int_0^1 P_{i,t}^i(j)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}$ is country i's domestic PPI in terms of country i's own currency. We therefore have

$$P_{F,t} = E_t P_t^*$$

where $P_t^* = [\int_0^1 P_{i,t}^{i 1-\gamma} di]^{\frac{1}{1-\gamma}}$ is the world price index and E_t is the effective nominal exchange rate.[6]

The terms of trade are defined by

$$S_t = \frac{P_{F,t}}{P_{H,t}} = \frac{E_t P_t^*}{P_{H,t}}.$$

Similarly let the real exchange rate be

$$\mathcal{Q}_t = \frac{E_t P_t^*}{P_t}.$$

The risk premium shocks Ψ_t and $\Psi_{i,t}$ introduce a wedge in the UIP condition

$$1 + i_t = \frac{\Psi_t}{\Psi_{i,t}} \frac{1+\tau_t}{1+\tau_t^i} (1 + i_t^i) \frac{E_{i,t+1}}{E_{i,t}}.$$

2.4 Equilibrium Conditions with Symmetric Rest of the World

We now summarize the equilibrium conditions. For simplicity of exposition, we focus on the case where all foreign countries are identical. We assume that there are no risk premium shocks in the foreign countries. Moreover, we assume that foreign countries

[6]The effective nominal exchange rate is defined as $E_t = [\int_0^1 E_{i,t}^{1-\gamma} P_{i,t}^{i 1-\gamma} di]^{\frac{1}{1-\gamma}} / [\int_0^1 P_{i,t}^{i 1-\gamma} di]^{\frac{1}{1-\gamma}}$.

do not impose capital controls. We denote foreign variables with a star. Taking foreign variables as given, equilibrium in the home country can be described by the following equations. We find it convenient to group these equations into two blocks, which we refer to as the demand block and the supply block.

The demand block is independent of the nature of price setting. It is composed of the Backus-Smith condition

$$C_t = \Theta_t C_t^* \mathcal{Q}_t^{\frac{1}{\sigma}}, \tag{3}$$

where Θ_t is a relative Pareto weight whose evolution is given by equation (7) below, by the equation relating the real exchange rate to the terms of trade

$$\mathcal{Q}_t = \left[(1-\alpha)\,(S_t)^{\eta-1} + \alpha\right]^{\frac{1}{\eta-1}}, \tag{4}$$

the goods market clearing condition

$$Y_t = (1-\alpha)\left(\frac{\mathcal{Q}_t}{S_t}\right)^{-\eta} C_t + \alpha S_t^{\gamma} C_t^*, \tag{5}$$

the labor market clearing condition

$$N_t = \frac{Y_t}{A_{H,t}}\Delta_t \tag{6}$$

where Δ_t is an index of price dispersion $\Delta_t = \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon}$, the Euler equation

$$1 + i_t = \beta^{-1}\frac{C_{t+1}^{\sigma}}{C_t^{\sigma}}\Pi_{t+1}$$

where $\Pi_t = \frac{P_{t+1}}{P_t} = \Pi_{H,t}\frac{S_t}{\mathcal{Q}_t}\frac{\mathcal{Q}_{t-1}}{S_{t-1}}$ is CPI inflation, the arbitrage condition between home and foreign bonds

$$\frac{\Theta_{t+1}^{\sigma}}{\Theta_t^{\sigma}} = \frac{1+i_t}{1+i^*}\frac{E_t}{E_{t+1}}, \tag{7}$$

and the country budget constraint

$$NFA_t = -C_t^{*-\sigma}\left(S_t^{-1}Y_t - \mathcal{Q}_t^{-1}C_t\right) + \beta\Psi_t^{-1}NFA_{t+1} \tag{8}$$

where NFA_t is the country's net foreign assets at t, which for convenience, we measure in the foreign price at home $P_{F,t}$ as the numeraire, and which we adjust by the foreign marginal utility of consumption $C_t^{*-\sigma}$. The country budget constraint is derived from the consumer's budget constraint after substituting out the lump-sum transfer. Under government budget balance the transfer equals the sum of the revenue from the labor tax and the tax on foreign investors, net of the revenue lost to subsidize domestic residents' investments abroad.[7] We also impose a No-Ponzi condition so that we can write the budget constraint in present-value form

$$0 = -\sum_{t=0}^{\infty} \beta^t \big(\prod_{s=0}^{t-1} \Psi_s^{-1} \big) C_t^{*-\sigma} \left(S_t^{-1} Y_t - \mathcal{Q}_t^{-1} C_t \right). \tag{9}$$

The supply block varies with the nature of price setting. With flexible prices, it boils down to the following condition, which combines the household and firm's first-order conditions,

$$C_t^{-\sigma} S_t^{-1} \mathcal{Q}_t = M \frac{1+\tau^L}{A} N_t^{\phi} \tag{10}$$

where $M = \frac{\epsilon}{\epsilon-1}$ is the desired markup of price over marginal cost, together with the no price dispersion assumption $\Delta_t = 1$. With one period in advance price stickiness, the only difference is that at $t = 0$, all prices are fixed. This means that $S_0 = E_0 \frac{P_0^*}{P_{H,0}}$ where P_0^* and $P_{H,0}$ are fixed. Finally with Calvo price setting, which will be our main focus, the supply block is more complex. It is composed of the equations summarizing the first-order condition for optimal price setting.

$$\frac{1-\delta \Pi_{H,t}^{\epsilon-1}}{1-\delta} = \left(\frac{F_t}{K_t} \right)^{\epsilon-1},$$

$$K_t = M \frac{1+\tau^L}{A} Y_t N_t^{\phi} \Pi_{H,t}^{\epsilon} + \delta \beta K_{t+1},$$
$$F_t = Y_t C_t^{-\sigma} S_t^{-1} \mathcal{Q}_t \Pi_{H,t}^{\epsilon-1} + \delta \beta F_{t+1},$$

together with an equation determining the evolution of price dispersion

$$\Delta_t = h(\Delta_{t-1}, \Pi_{H,t}),$$

where $h(\Delta, \Pi) = \delta \Delta \Pi^{\epsilon} + (1-\delta) \left(\frac{1-\delta \Pi^{\epsilon-1}}{1-\delta} \right)^{\frac{\epsilon}{\epsilon-1}}$. We will only analyze a log-linearized version of the model with Calvo price setting.

[7] We do not require budget balance but since Ricardian equivalence holds here, all other government financing schemes have the same implications.

2.5 Steady State Labor Tax

We allow for a constant tax on labor in each country. We pin this tax rate down by assuming that it is optimally set by each country and considering a symmetric steady state with flexible prices.[8] We refer the reader to Farhi and Werning (2012) for a derivation of the following result.

Proposition 1 (Steady State Tax)**.** *Suppose prices are flexible, that productivity is constant across time and countries and there are no export demand shocks. Then at a symmetric steady state,* $\tau^L = \frac{1}{M}\frac{(1-\alpha)(\eta-1)+\gamma}{(1-\alpha)(\eta-1)+\gamma-\alpha} - 1$ *and optimal capital controls are equal to zero.*

From each country's perspective, the labor tax is the result of a balancing act between offsetting the monopoly distortion of individual producers and exerting some monopoly power as a country. The two terms in the optimal tax formula reflect the two legs of this tradeoff.

2.6 Shocks

We assume that the economy is initially at the deterministic symmetric steady state and characterize the optimal use of capital controls for the home country in response to risk premium shocks. We choose to focus on these shocks because many discussions of capital controls, especially in developing countries, focus on capital inflow surges that are taken to be exogenous fluctuations in investor sentiments. It allows to flexibly capture episodes of capital flow surges (negative risk premium shocks) and sudden stops (positive risk premium shocks).

3 Flexible Prices and Rigid Prices in the Non-Linear Model

In this section, we treat the case of perfectly flexible and perfectly rigid prices in the non-linear model. In Section 4, we treat the intermediate case of sticky but not perfectly rigid prices using the Calvo model of price adjustment. There, we find it more convenient to work with a log-linearized version of the model.

We start the economy at a symmetric steady state. At $t = 0$, the economy is hit with an unanticipated risk premium shock. We focus on the Cole-Obstfeldt case $\sigma = \eta = \gamma = 1$. The reason is threefold. First, this parametrization is not unrealistic. Second, it is substantially more tractable and delivers clean results in the nonlinear model. Third, it

[8] The level of the tax is actually only relevant when we study the model under the Calvo pricing assumption. Our other results apply for any level of the tax rate.

is easier to derive a second order approximation of the loss function in the log linearized version of the model with Calvo price adjustment developed in Section 4.

3.1 Optimal Capital Controls with Perfectly Flexible Prices

The planning problem maximizes utility (1) subject to the equilibrium conditions (3), (4), (5), (6) with $\Delta_t = 1$, (9), and (10). The maximization takes place over $\{C_t, Y_t, N_t, \Theta_t, S_t, \mathcal{Q}_t\}$. Using the constraints to substitute out variables, the planning problem can be written in the following simple form

$$\max_{\{\Theta_t, S_t\}} \sum_{t=0}^{\infty} \beta^t \left[\log \Theta_t + (1-\alpha) \log S_t - \frac{1}{1+\phi} \frac{1}{A^{1+\phi}} S_t^{1+\phi} C^{*\phi+1} \left[(1-\alpha)\Theta_t + \alpha \right]^{1+\phi} \right] \tag{11}$$

subject to

$$0 = \alpha \sum_{t=0}^{\infty} \left(\Pi_{s=0}^{t-1} \Psi_s^{-1} \right) \beta^t \left(\Theta_t - 1 \right), \tag{12}$$

$$S_t = \left[\frac{1}{M(1+\tau^L)} \frac{A^{1+\phi}}{C^{*\phi+1}} \frac{1}{\Theta_t \left[(1-\alpha)\Theta_t + \alpha \right]^{\phi}} \right]^{\frac{1}{1+\phi}}. \tag{13}$$

Substituting S_t as a function of Θ_t from the second constraint, we can rewrite this problem as a function of the path for $\{\Theta_t\}$. The first order condition is then

$$\frac{\phi+\alpha}{1+\phi} - \frac{(1-\alpha)^2 \phi}{1+\phi} \frac{\Theta_t}{\alpha + (1-\alpha)\Theta_t} + \frac{(1-\alpha)\alpha}{1+\phi} \frac{1}{\Theta_t} = \Gamma \alpha \left(\Pi_{s=0}^{t-1} \Psi_s^{-1} \right) \Theta_t, \tag{14}$$

for some $\Gamma > 0$. The left-hand-side is decreasing in Θ_t and the right hand side is increasing in Θ_t. Hence $\frac{\Theta_{t+1}}{\Theta_t} \Psi_t^{-1} - 1$ has the opposite sign as $\Psi_t - 1$. Since $\frac{\Theta_{t+1}}{\Theta_t} \Psi_t^{-1} = 1 + \tau_t$, it follows that τ_t has the opposite sign as $\Psi_t - 1$ so that optimal capital controls with flexible prices rates lean against the wind. If $\Psi_t \geq 0$ for all t, then it is easy to see that the terms of trade S_t initially depreciate, but less than in the absence of capital controls, and eventually appreciate, but less than in the absence of capital of capital controls.

Proposition 2 (Perfectly Flexible Prices)**.** *If prices are completely flexible, the optimal capital controls τ_t have the opposite sign as $\Psi_t - 1$.*

One might naively have the intuition that one should just undo the risk premium shock by setting capital controls $\tau_t = \Psi_t^{-1} - 1$ so that Θ_t is constant over time. Indeed, the UIP condition then takes the same form as in the absence of risk premium shocks and capital controls $1 + i_t = (1 + i^*) \frac{E_{t+1}}{E_t}$. While this changes the terms at which private agents can borrow from abroad, this does not change the terms at which the country as a whole borrows from the abroad, because capital controls must be financed. Hence capital controls cannot completely undo risk premium shocks. Nor is it clear that they should and by implication that $\tau_t = \Psi_t^{-1} - 1$ should be a good benchmark.

In fact, the intuition that underlies their optimal determination is quite different, and has to do with terms of trade mnagement. This might seem surprising given the fact that we are considering a small open economy, with no ability to affect the world interest rate. The result can be understood by noting that capital controls allow a country to reallocate domestic consumption intertemporally. This in turn affects the terms of trade through two different channels. The first channel works through a wealth effect in labor supply: Home consumers demand higher wages to supply a given amount of labor when home consumption is high, which leads to higher home prices and appreciated terms of trade. The second channel works through a labor demand effect: Because of home bias in consumption, demand for home country's goods is high when home consumption is high, which pushes up labor demand, wages and home prices and leads to appreciated terms of trade. Through these two channels, capital controls allow a country to manage its terms of trade, raising them in some periods and lowering them in others.[9] In response to a positive risk premium shock for example, it is optimal to subsidize inflows and tax outflows in order to temporarily increase the demand for home goods and appreciate the terms of trade, while eventually decreasing the demand for home goods and depreciating the terms of trade.

In the limit where home bias in consumption disappears ($\alpha \to 1$), we get $\frac{\Theta_{t+1}}{\Theta_t}\Psi_t^{-1} = 1$ so that it is optimal to set capital controls to zero $\tau_t = 0$. In this limit, the steady state labor tax τ^L converges $+\infty$, and output and labor converge to zero. The reason can be understood as follows, focusing on the home country. Because the home country is a small open economy and there is no home bias, home consumers do not consume home products. Hence it is optimal for the home country to behave like a pure monopolist since increasing home's country's prices does not hurt home consumers. Because the country faces a unit-elastic demand for its products (an implication of the Cole-Obstfeldt parametrization), the country's export revenues are independent of scale and the pure monopoly problem is degenerate. The solution is to restrict home output to zero as possible by choosing $\tau^L = \infty$. Capital controls are then useless for terms of trade management purposes as they only distort consumption, and it is optimal to not use them. As this description makes clear, this result is special to the Cole-Obstfeld case.

With extreme home bias in consumption ($\alpha = 0$), optimal capital controls are inde-

[9]The intuition is similar to that discussed in Costinot et al. (2011).

terminate. But the limit of optimal capital controls for $\alpha \to 0$ is determinate and non-zero, as can be confirmed by performing a first order Taylor expansion $\alpha(1+\frac{1}{\Theta_t}) = \alpha\Gamma\left(\Pi_{s=0}^{t-1}\Psi_s^{-1}\right)\Theta_t$, in α of condition (14). As a result, the limit of sequence $\{\Theta_t\}$ solves, together with Γ

$$(1+\frac{1}{\Theta_t}) = \Gamma\left(\Pi_{s=0}^{t-1}\Psi_s^{-1}\right)\Theta_t,$$

$$0 = \sum_{t=0}^{\infty}\beta^t\left(\Pi_{s=0}^{t-1}\Psi_s^{-1}\right)[\Theta_t - 1].$$

This implies that $\frac{\Theta_{t+1}}{\Theta_t}\Psi_t^{-1} - 1$ has the opposite sign as $\Psi_t - 1$, and hence that τ_t has the opposite sign as $\Psi_t - 1$ so that in the limit $\alpha \to 0$, optimal capital controls still lean against the wind.This might seem surprising since when home bias is extreme, the home country ends up exerting its monopoly power mostly on its own consumers (and consistent with this intuition, τ^L converges to $\frac{1}{M} - 1$). Indeed, the benefits of terms of trade management are of the order α. But so are the distortionary costs of capital controls. As a result, capital controls are still used in the limit $\alpha \to 0$. However, their impact on consumption C_t, output Y_t, labor N_t and hence welfare vanishes in the limit $\alpha \to 0$.

3.2 Optimal Capital Controls and Exchange Rates with Perfectly Rigid Prices

We now turn to the extreme opposite case, where prices are entirely rigid and fixed at their steady state values. In this case, the terms of trade can be perfectly managed with the exchange rate $S_t = E_t$. The planning problem is now

$$\max_{\{\Theta_t, E_t\}} \sum_{t=0}^{\infty}\beta^t\left[\log\Theta_t + (1-\alpha)\log E_t - \frac{1}{1+\phi}E_t^{1+\phi}\left(\alpha + \Theta_t(1-\alpha)\right)^{1+\phi}\left(\frac{C^*}{A}\right)^{1+\phi}\right] \quad (15)$$

subject to

$$0 = \alpha\sum_{t=0}^{\infty}\beta^t\left(\Pi_{s=0}^{t-1}\Psi_s^{-1}\right)[\Theta_t - 1].$$

The problem is convex: it features a concave objective and linear constraint in Θ_t. Putting a multiplier $\Gamma > 0$ on the left-hand side of the budget constraint, the necessary and sufficient first-order conditions are

$$(1-\alpha) = E_t^{1+\phi}\left(\alpha + \Theta_t(1-\alpha)\right)^{1+\phi}\left(\frac{C^*}{A}\right)^{1+\phi}, \quad (16)$$

$$1-(1-\alpha)^2\frac{\Theta_t}{\alpha+\Theta_t(1-\alpha)}=\Gamma\alpha\left(\Pi_{s=0}^{t-1}\Psi_s^{-1}\right)\Theta_t. \tag{17}$$

Once again, the left-hand-side is decreasing in Θ_t and the right hand side is increasing in Θ_t. It follows that $\frac{\Theta_{t+1}}{\Theta_t}\Psi_t^{-1}-1$ has the opposite sign as Ψ_t-1. Since $\frac{\Theta_{t+1}}{\Theta_t}\Psi_t^{-1}=1+\tau_t$, it follows that τ_t has the opposite sign as Ψ_t-1 so that optimal capital controls with rigid prices and flexible exchange rates lean against the wind. If $\Psi_t\geq 0$ for all t, then it is easy to see that the exchange rate E_t initially depreciates and eventually appreciates, but less so than under flexible prices and no capital controls.

Proposition 3 (Perfectly Rigid Prices)**.** *If prices are completely rigid, the optimal capital controls τ_t have the opposite sign as Ψ_t-1.*

The planning problem with rigid prices (15) is a relaxed version of the planning with flexible prices (11). The flexible price allocation with optimal capital controls can always be implemented with the appropriate exchange rate. But in general, this allocation does not make optimal use of capital controls and exchange rates—except, as we show below, in the limit $\alpha\to 0$. Rigid prices help exercise the country's monopoly power. Indeed, with rigid prices, the exchange rate E_t directly controls the terms of trade $S_t=E_t$. Capital controls allow additional control over the intertemporal path of domestic consumption and labor given this path for the exchange rate. This allows the planner to better optimize its joint objective of terms of trade management and macroeconomic stabilization. Both tools can be used in combination to achieve a better outcome.

To gain some intuition, we can also compare the allocation that makes optimal use of exchange rates and capital controls to the optimal allocation with flexible exchange rates but no capital controls, and to the allocation with fixed exchange rates and optimal capital controls. The optimal allocation with flexible exchange rates but no capital controls solves the same planning problem as (15) but with the extra constraint that $\Theta_t=\Theta_0\Pi_{s=0}^{t-1}\Psi_s^{-1}$ where $\Theta_0=(1-\beta)\sum_{t=0}^{\infty}\beta^t\left(\Pi_{s=0}^{t-1}\Psi_s^{-1}\right)$. It is easy to see that in the face of a positive risk premium shock where $\Psi_t\geq 0$ for all t, the exchange initially depreciates (more so than in the presence of capital controls) and then appreciates over time. The optimal allocation with fixed exchange rates and optimal capital controls solves the same planning problem as (15) but with the extra constraint that $E_t=1$. We have analyzed this problem in Farhi and Werning (2012). We refer the reader to this paper for a detailed analysis. There, we show that τ_t has the opposite sign as Ψ_t-1 so that capital controls also lean agains the wind. However the reason is very different than under flexible prices. With fixed exchange rates and rigid prices, the terms or trade are fixed. Capital controls are therefore not used for terms of trade management, but rather to regain monetary autonomy and perform macroeconomic stabilization. In other words, capital controls are used to alleviate Mundell's trilemma which famously states that it is impossible to have fixed exchange rates, an independent monetary policy, and free capital flows. Capital controls only achieve imperfect macroeconomic stabilization however, except in the limit $\alpha \to 0$ where the optimal allocation actually coincides with the flexible prices allocation.

It is easy to see that in the limit where home bias disappears ($\alpha \to 1$), and for the same reasons as under flexible prices, we get $\frac{\Theta_{t+1}}{\Theta_t}\Psi_t^{-1} = 1$ so that it is optimal not to use capital controls. In the limit of extreme home bias in consumption ($\alpha \to 0$), optimal capital controls have a well-defined non-zero limit. Interestingly, it turns out that in this limit, optimal capital controls are identical whether prices or flexible, or perfectly rigid but the exchange rate is flexible. Indeed, the first order approximations in α of the first order conditions (14) and (17) coincide. It is easy to see that the optimal exchange rate determined by condition (16) then coincides with the terms of trade of the flexible price allocation with optimal capital controls as determined by condition (13). This immediately implies the following proposition.

Proposition 4 (Dichotomy in the Limit $\alpha \to 0$)**.** *In the limit of extreme home bias ($\alpha \to 0$), the optimal allocation with rigid prices, flexible exchange rates and capital controls coincides with the optimal allocation with flexible prices and capital controls. In particular, capital controls are the same for both allocations.*

Hence in the limit of extreme home bias, the optimal allocation with rigid prices, flexible exchange rates and capital controls simply replicates the optimal allocation with flexible prices and capital controls. It features the same capital controls, and the exchange rate is chosen to reproduce the same terms of trade. In the limit of extreme home bias ($\alpha \to 0$), there is therefore a form of dichotomy between the real and nominal side. The real allocation (including capital controls) is determined as if prices were flexible. And exchange rate policy then ensures that the sticky price allocation replicates this allocation. Such a dichotomy breaks down away from extreme home bias (for $\alpha > 0$).

In Section 4, we analyze a log-linearized version of the model with sticky prices a la Calvo. We use this approximation to revisit these planning problems(15). Indeed, the log-linearization allows us to further characterize and compare the solutions of these planning problems. They also serve as useful comparison benchmarks for the general case of sticky but not perfectly rigid prices.

4 The Log-Linearized Economy

In this section, we study the standard New Keynesian version of the model, with staggered price setting a la Calvo. As is standard in the literature, we work with a log-linearized approximation of the model around a symmetric steady state. At $t = 0$, the economy is hit with an unanticipated risk premium shock. It is convenient to work with a continuous time version of the model. We denote the instantaneous discount rate by ρ, and the instantaneous arrival rate for price changes by ρ_δ.

From now on we focus on the Cole-Obstfeld case $\sigma = \eta = \gamma = 1$. This case is attractive because a tractable second order approximation of the welfare function around the symmetric deterministic steady state can be derived.

4.1 Summarizing the Economy

We first describe the *natural allocation* with no intervention, defined as the allocation that prevails if prices are flexible and capital controls are not used. We then summarize the behavior of the sticky price economy with capital controls in log-deviations (gaps) from the natural allocation. For both the natural and the sticky price allocation with capital controls, the behavior of the rest of the world is taken as given. We use lower cases variables to denote gaps from the symmetric deterministic steady state. We denote the natural allocation with bars, and the gaps from the natural allocation with hats. We refer the reader to (Gali and Monacelli, 2005, 2008) for details on the derivation.

The natural allocation. We confine ourselves to shocks to ψ_t, setting $c_t^* = a_t = \pi_t^* = 0$, $NFA_0 = 0$ and $i^* = \rho$. Without capital controls we have $\bar{\theta}_t = \bar{\theta}_0 + \int_0^t \psi_s ds$. The natural allocation is then

$$\bar{y}_t = -\frac{\alpha}{1+\phi}\bar{\theta}_t,$$
$$\bar{s}_t = -\frac{1+\phi(1-\alpha)}{1+\phi}\bar{\theta}_t,$$

where the budget constraint

$$\bar{\theta}_0 + \int_0^\infty \psi_t e^{-\rho t} dt = 0$$

pins down $\bar{\theta}_0$. We can also compute the natural levels of employment and consumption from the equations $\bar{y}_t = \bar{n}_t$ and $\bar{c}_t = \bar{\theta}_t + (1-\alpha)\bar{s}_t$.

We can compute the natural interest rate

$$\bar{r}_t - \rho = \frac{\alpha\phi}{1+\phi}\dot{\bar{\theta}}_t.$$

The natural allocation features trade imbalances. Indeed net exports are given by $\bar{nx}_t = -\alpha\bar{\theta}_t$, so that for positive risk premium shocks ($\psi_t \geq 0$ for all t), the home country initially runs a trade surplus, and eventually runs a trade deficit. It leads to a initial depreciation and an eventual appreciation of the terms of trade $\bar{s}_t = -\frac{1+\phi(1-\alpha)}{1+\phi}\bar{\theta}_t$. Hence, even with flexible prices, a positive risk premium shock captures some key aspects of a "sudden stop". The opposite patterns hold in response to negative risk premium shocks which capture some key aspects of a "capital inflow surge".

Summarizing the system in gaps. The equations summarizing an equilibrium are the log linearized analogues of the equilibrium conditions derived in Section 2.4. The demand block is summarized by three equations,

$$\dot{\hat{y}}_t = (1-\alpha)(i_t - i^* - \dot{e}_t - \psi_t) - \pi_{H,t} + i^* + \dot{e}_t + \psi_t - \bar{r}_t,$$

$$\dot{\hat{\theta}}_t = i_t - i^* - \dot{e}_t - \psi_t,$$

$$\int_0^\infty e^{-\rho t}\hat{\theta}_t dt = 0,$$

representing the Euler equation (after substituting out for consumption using the goods market clearing condition and the Backus-Smith condition), the UIP equation and the budget constraint, respectively.

The supply block consists of one equation, the New-Keynesian Philips Curve

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \hat{\kappa}\hat{y}_t - \lambda\alpha\hat{\theta}_t,$$

where $\lambda = \rho_\delta(\rho + \rho_\delta)$ and $\hat{\kappa} = \lambda(1+\phi)$.

Finally, we have the initial condition

$$\hat{y}_0 = (1-\alpha)\hat{\theta}_0 + e_0 - \bar{s}_0,$$

which formalizes the fact that prices are sticky so that the terms of trade at $t = 0$ are given by $\hat{s}_0 = e_0 - \bar{s}_0$. These equations are sufficient to pin down an equilibrium in the variables that are needed to evaluate welfare (see below).

Loss function. We can derive a simple second order approximation of the welfare function (see Appendix A.1 for the detailed derivation). The corresponding loss function (in consumption equivalent units) up to a constant independent of policy can be written as

$$(1-\alpha)(1+\phi)\int e^{-\rho t}\left[\frac{1}{2}\alpha_\pi \pi_{H,t}^2 + \frac{1}{2}(\hat{y}_t - \frac{\alpha}{1+\phi}\bar{\theta}_t)^2 + \frac{1}{2}\alpha_\theta(\hat{\theta}_t + \alpha_\psi\bar{\theta}_t + (1-\alpha_\psi)\bar{\theta}_0)^2\right]dt,$$

with $\alpha_\pi = \frac{\epsilon}{\lambda(1+\phi)}$, $\alpha_\theta = \frac{\alpha}{1+\phi}\left(\frac{2-\alpha}{1-\alpha}+1-\alpha\right)$, and $\alpha_\psi = \frac{1-\alpha}{\frac{2-\alpha}{1-\alpha}+1-\alpha}$.

Note that α_π is independent of α but that α_θ goes to zero when α goes to zero. Hence in the closed economy limit ($\alpha \to 0$), the cost of capital controls vanishes. The reason is that for a given path of $\hat{\theta}_t$, the associated trade balances $\hat{nx}_t = -\alpha\hat{\theta}_t$ and $\bar{nx}_t = -\alpha\bar{\theta}_t$ vanish as α goes to zero, and so do the distortions associated with the wedge between home and foreign intertemporal prices.

The planning problem in gaps. We are led to the following planning problem

$$\min_{\{\pi_{H,t},\hat{y}_t,i_t,\hat{\theta}_t,e_t\}} \frac{1}{2}\int_0^\infty e^{-\rho t}\left[\alpha_\pi \pi_{H,t}^2 + (\hat{y}_t - \frac{\alpha}{1+\phi}\bar{\theta}_t)^2 + \alpha_\theta(\hat{\theta}_t + \alpha_\psi\bar{\theta}_t + (1-\alpha_\psi)\bar{\theta}_0)^2\right]dt$$

subject to

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \hat{\kappa}\hat{y}_t - \lambda\alpha\hat{\theta}_t,$$

$$\dot{\hat{y}}_t = (1-\alpha)(i_t - i^* - \dot{e}_t - \psi_t) - \pi_{H,t} + i^* + \dot{e}_t + \psi_t - \bar{r}_t,$$

$$\dot{\hat{\theta}}_t = i_t - i^* - \dot{e}_t - \psi_t,$$

$$\int_0^\infty e^{-\rho t}\hat{\theta}_t dt = 0,$$

$$\hat{y}_0 = (1-\alpha)\hat{\theta}_0 + e_0 - \bar{s}_0.$$

Note that this allows the planner to control $\hat{\theta}_t$ and $\hat{y}_t$ independently. We can drop the initial condition. We can therefore rewrite the problem as

$$\min_{\{\pi_{H,t},\hat{y}_t,\hat{\theta}_t\}} \frac{1}{2}\int_0^\infty e^{-\rho t}\left[\alpha_\pi \pi_{H,t}^2 + (\hat{y}_t - \frac{\alpha}{1+\phi}\bar{\theta}_t)^2 + \alpha_\theta(\hat{\theta}_t + \alpha_\psi\bar{\theta}_t + (1-\alpha_\psi)\bar{\theta}_0)^2\right]dt \tag{18}$$

subject to

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \hat{\kappa}\hat{y}_t - \lambda\alpha\hat{\theta}_t,$$

$$\int_0^\infty e^{-\rho t}\hat{\theta}_t dt = 0.$$

Capital controls are then given by $\tau_t = \dot{\hat{\theta}}_t$, and i_t and e_t can be found by solving the system of equations

$$\dot{\hat{y}}_t = (1-\alpha)(i_t - i^* - \dot{e}_t - \psi_t) - \pi_{H,t} + i^* + \dot{e}_t + \psi_t - \bar{r}_t,$$

$$\dot{\hat{\theta}}_t = i_t - i^* - \dot{e}_t - \psi_t,$$

$$\hat{y}_0 = (1-\alpha)\hat{\theta}_0 + e_0 - \bar{s}_0.$$

To build intuition for the full solution, we start with the case of perfectly flexible and perfectly rigid prices. We treated these cases in the non log-linearized version of the model in Section 3. The log-linearized model however makes it easier to characterize the solutions. These solutions also provide useful benchmarks for the intermediate case of sticky prices.

4.2 Optimal Capital Controls with Perfectly Flexible Prices

We start with the case where prices are flexible. With flexible prices, we have $\hat{y}_t = -\frac{\alpha}{1+\phi}\hat{\theta}_t$. Hence we are led to the planning problem

$$\min_{\{\hat{\theta}_t\}} \frac{1}{2}\int_0^\infty e^{-\rho t}\left[\left(\frac{\alpha}{1+\phi}\right)^2(-\hat{\theta}_t - \bar{\theta}_t)^2 + \alpha_\theta(\hat{\theta}_t + \alpha_\psi\bar{\theta}_t + (1-\alpha_\psi)\bar{\theta}_0)^2\right]dt$$

subject to

$$\int_0^\infty e^{-\rho t}\hat{\theta}_t dt = 0.$$

Proposition 5 (Perfectly Flexible Prices)**.** *Suppose prices are completely flexible and the economy is subject to risk-premium shocks ψ_t. Then optimal capital controls are given by*

$$\tau_t = -\frac{(\frac{\alpha}{1+\phi})^2 + \alpha_\theta\alpha_\psi}{(\frac{\alpha}{1+\phi})^2 + \alpha_\theta}\psi_t,$$

and the allocation can be expressed in closed form (see the appendix).[10]

Optimal capital controls are proportional to the current risk premium ψ_t shock. The tax τ_t has the opposite sign from ψ_t—policy leans against the wind. However, because $\alpha_\psi < 1$, capital controls react less than one for one to risk premium shocks.

[10]This result also applies in the limit to flexible prices, i.e. by taking $\lambda \to \infty$ while simultaneously varying $\alpha_\pi = \frac{\epsilon}{\lambda(1+\phi)}$ and $\hat{\kappa} = \lambda(1+\phi)$.

Optimal capital controls also have the property of stabilizing the trade balance. Since the trade balance with intervention equals $nx_t = \hat{nx}_t + \bar{nx}_t = -\alpha(\hat{\theta}_t + \bar{\theta}_t) = -\alpha \frac{\alpha_\theta(1-\alpha_\psi)}{(\frac{\alpha}{1+\phi})^2+\alpha_\theta}\bar{\theta}_t$ and without intervention equals $\bar{nx}_t = -\alpha\bar{\theta}_t$, the ratio $nx_t/\bar{nx}_t$ is constant and less than one. Optimal capital controls therefore mitigate the capital inflow surges associated with negative risk premium shocks, as well as the capital flight episodes associated with positive risk premium shocks.

In addition, optimal capital controls have the property of stabilizing the terms of trade. Indeed, the terms of trade with intervention equals $s_t = \hat{s}_t + \bar{s}_t = -\frac{\alpha_\theta(1-\alpha_\psi)}{(\frac{\alpha}{1+\phi})^2+\alpha_\theta}\frac{1+\phi(1-\alpha)}{1+\phi}\bar{\theta}_t$ and without intervention equals $\bar{s}_t = -\frac{1+\phi(1-\alpha)}{1+\phi}\bar{\theta}_t$. The ratio $s_t/\bar{s}_t$ is constant and less than one.

We introduce the following definition. When comparing a variables x_t across two allocations A and B, we will say that x_t is *constantly more stable* under allocation A than under allocation B if x_t^A and x_t^B have the same sign and the ratio x_t^A/x_t^B is constant and less than one. We will say that x_t is *more stable* under allocation A than under allocation B if x_t^A and x_t^B have the same sign and if x_t^A/x_t^B is less than one. Therefore, when prices are flexible, net exports nx_t and the terms of trade s_t are constantly more stable with optimal capital controls than without capital controls.

Capital controls allow our small open economy to reallocate domestic consumption demand intertemporally and therefore to manage its terms of trade, raising them in some periods and lowering them in others. In response to a positive risk premium shock for example, it is optimal to subsidize inflows and tax outflows in order to temporarily increase the demand for home goods and appreciate the terms of trade, while eventually decreasing the demand for home goods and depreciating the terms of trade.

4.3 Optimal Capital Controls and Exchange Rates with Perfectly Rigid Prices

We then deal with the case where prices are entirely rigid. Then the problem simplifies to

$$\min_{\{\hat{y}_t,\hat{\theta}_t\}} \frac{1}{2}\int_0^\infty e^{-\rho t}\left[(\hat{y}_t - \frac{\alpha}{1+\phi}\bar{\theta}_t)^2 + \alpha_\theta(\hat{\theta}_t + \alpha_\psi\bar{\theta}_t + (1-\alpha_\psi)\bar{\theta}_0)^2\right]dt \tag{19}$$

subject to

$$\int_0^\infty e^{-\rho t}\hat{\theta}_t dt = 0.$$

The solution is $\hat{y}_t = \frac{\alpha}{1+\phi}\bar{\theta}_t$ and $\hat{\theta}_t = -\alpha_\psi\bar{\theta}_t$. We can also compute the path for the exchange rate and the terms of trade $e_t = s_t = -(1-\alpha)(1-\alpha_\psi)\bar{\theta}_t$. Optimal capital controls can then easily be derived.

Proposition 6 (Perfectly Rigid Prices)**.** *Suppose prices are completely rigid and the economy is subject to risk-premium shocks ψ_t. Then optimal capital controls are given by*

$$\tau_t = -\alpha_\psi \psi_t,$$

and the allocation can be expressed in closed form.

Comparing with the flexible price allocation with no capital controls. The flexible price allocation with no capital controls (the natural allocation) can be implemented with rigid prices by setting capital controls to $\bar{\tau}_t = 0$ and the exchange rate to $\bar{e}_t = \bar{s}_t = -\frac{1+\phi(1-\alpha)}{1+\phi}\bar{\theta}_t$. With a positive risk premium shock, the exchange rate and the terms of trade initially depreciate and eventually appreciate, and the country initially runs a trade surplus and eventually a trade deficit. Comparing the optimal allocation with rigid prices to the natural allocation, the exchange rate e_t, the terms of trade s_t, and the trade balance nx_t are constantly more stable. The stabilization of the terms of trade and of the trade balance is also a feature of the optimal allocation with flexible prices (the allocation characterized in Proposition 5). But with rigid prices, optimal capital controls lean less against the wind than with flexible prices.

Comparing with the flexible price allocation with optimal capital controls. Note that the flexible price allocation with optimal capital controls can be implemented with rigid prices with the right capital controls and exchange rate. The required capital controls are $\tau_t = -\frac{(\frac{\alpha}{1+\phi})^2+\alpha_\theta\alpha_\psi}{(\frac{\alpha}{1+\phi})^2+\alpha_\theta}\psi_t$ and the required exchange rate is $e_t = s_t = -\frac{\alpha_\theta(1-\alpha_\psi)}{(\frac{\alpha}{1+\phi})^2+\alpha_\theta}\frac{1+\phi(1-\alpha)}{1+\phi}\bar{\theta}_t$. But this allocation does not make optimal use of the two instruments at the country's disposal, capital controls and exchange rates, except in the limit $\alpha \to 0$, as we have already established in Proposition 4. For $\alpha > 0$, capital controls lean less against the wind with rigid prices and flexible exchange rates than with flexible prices. Interestingly, the exchange rate is constantly more stable but the trade balance nx_t is constantly less stable (in the formal sense defined in Section 4.2).

Examining the roles of exchange rates and capital controls in isolation. To gain some intuition, it is useful to characterize how each of these tools is optimally used when the other is unavailable. To do so, we first look at optimal exchange rates when capital controls are constrained to be zero. We then look at optimal capital controls when exchange rates are constrained to be fixed.

The planning problem for optimal exchange rates with zero capital controls coincides with the planning problem (19) but with the extra constraint that $\hat{\theta}_t = 0$. It turns out that the optimal allocation is not the natural allocation. Instead the solution requires $e_t = -(1-\alpha)\bar{\theta}_t$ and hence $s_t = -(1-\alpha)\bar{\theta}_t$. This is because with rigid prices, the exchange rate allows to directly manage the terms of trade $s_t = e_t$. For example, in response to a positive risk premium shock, and compared to the natural allocation, it is optimal to use the exchange rate to initially appreciate and eventually depreciate the terms of trade, an outcome qualitatively similar to what is achieved with capital controls when the allocation in Proposition 5 is implemented. Note also that this allocation achieves $\hat{y}_t = \frac{\alpha}{1+\phi}\bar{\theta}_t$ just like the solution of the planning problem (19). Therefore it performs just as well on the first term in the loss function of (19). It is inferior only in as it performs worse on the second. Finally, it is possible to show (in the formal sense defined above) that the allocation with optimal capital controls and exchange rates has constantly more stable paths for the exchange rate e_t, the terms of trade s_t, and the trade balance nx_t, than the allocation with no capital controls but optimal exchange rates.

The planning problem for optimal capital controls with fixed exchange rates is more complex. We have analyzed this problem in Farhi and Werning (2012). We refer the reader to this paper for a detailed analysis. There, we show that the planning problem (19) but with three extra constraints

$$\dot{\hat{y}}_t = (1-\alpha)(i_t - i^* - \psi_t) + i^* + \psi_t - \bar{r}_t,$$

$$\dot{\hat{\theta}}_t = i_t - i^* - \psi_t,$$

$$\hat{y}_0 = (1-\alpha)\hat{\theta}_0 - \bar{s}_0.$$

The first extra constraint is simply the Euler equation. The second constraint is the UIP condition. With fixed exchange rates, the nominal interest rate i_t is pinned down unless capital controls are used. The third constraint simply expresses the fact that the terms of trade s_0 are predetermined at date 0 because the exchange rate is fixed. In Farhi and Werning (2012), we show that that optimal capital controls are then given by $\tau_t = -\frac{1}{1-\alpha+\frac{\alpha_\theta}{1-\alpha}}\frac{1+\phi(1-\alpha)}{1+\phi}\psi_t$. Optimal capital controls with fixed exchange rate also lean against the wind, more so than under flexible prices. But the role of capital controls is very different. Indeed, with rigid prices and a fixed exchange rate, terms of trade are fixed and cannot be managed. Instead, capital controls are used to regain monetary autonomy and perform macroeconomic stabilization. In other words, capital controls are used to alleviate Mundell's trilemma which famously states that it is impossible to have fixed exchange rates, an independent monetary policy, and free capital flows. Capital controls only achieve imperfect macroeconomic stabilization however, except in the limit $\alpha \to 0$ where the optimal allocation actually coincides with the natural allocation.

With rigid prices, the exchange rate e_t directly controls the terms of trade $s_t = e_t$. Capital controls allow additional control over the intertemporal path of domestic consumption demand (and hence also output because of home bias) given this path for the exchange rate. This allows the planner to better optimize its joint objective of terms of trade management and macroeconomic stabilization. Both tools can be used in combination to achieve a better outcome as illustrated by the solution of the planning problem (19).

4.4 Optimal Capital Controls and Exchange Rates with Sticky Prices

When prices are sticky but not perfectly rigid, we obtain the following characterization.

Proposition 7 (Sticky Prices)**.** *Suppose that the exchange rate is flexible. The optimal solution features nonzero inflation $\pi_{H,t} \neq 0$ and capital controls are given by*

$$\tau_t = -\alpha_\psi \psi_t - \frac{\lambda \alpha}{\alpha_\theta} \alpha_\pi \pi_{H,t},$$

and the solution can be expressed in closed form (see the appendix).

As with perfectly rigid prices, optimal capital controls lean against the wind. The fact that $\pi_{H,t} \neq 0$ indicates that the optimal allocation is not an allocation that is attainable under flexible prices and capital controls. Because all flexible price allocations are attainable can be replicated with no inflation when prices are sticky, flexible exchange rates and capital controls allow to achieve a strictly better outcome. Only in the limit $\alpha \to 0$ does this advantage disappear—an implication of Proposition 4 which extends to the sticky price case, since the planning problem with perfectly rigid prices is a relaxed version of the planning problem with sticky prices. However we show below that this advantage is quantitatively small.

Although we are able to solve for the the allocation in closed form, the expressions quickly become hard to handle. We therefore perform a numerical simulation. This simulation is meant to be an example and should not be thought of as a serious calibration exercise, for which our model is probably too stylized.

We follow Gali and Monacelli (2005) by setting $\phi = 3$, $\rho = 0.04$, $\delta = 1 - 0.75^4$, $\epsilon = 6$. We report results for an intermediate degree of openness $\alpha = 0.2$, somewhere between that of Brazil (where the ratio of imports to GDP is close to 15% and that of India where that ratio is close to 30%). We hit the economy with a 5% risk premium shock with a half-life of 2 years.

We can compare the allocation that makes optimal use of capital controls and exchange rates to a number of different allocations. First, we can compare the allocation that makes optimal use of capital controls and exchange rates to the optimal allocation without with the allocation with optimal exchange rates but no capital controls, which solves the planning problem (18) with the extra constraint that $\hat{\theta}_t = 0$. This comparison is performed in Figure 1. Under both allocations, there is a large depreciation of the exchange rate, a strong shift towards a trade surplus, little effect on output, a substantial drop in consumption, little inflation and a substantial increase in the nominal interest rate (of about 4% at impact). With optimal capital controls and exchange rates, capital controls lean against the wind, with an optimal tax on outflows of about 1%. The decrease in consumption is smaller (3.4% vs. 4.6% at impact), the shift towards a trade surplus is smaller (2% vs 2.6% at impact), the depreciation of the exchange rate is smaller (7.6% vs. 10.4% at impact).

Second, we can compare it to the optimal allocation with fixed exchange rates and optimal capital controls. Here again we refer the reader to Farhi and Werning (2012) for a detailed analysis. The planning problem coincides with the planning problem (18) but with three extra constraints

$$\dot{\hat{y}}_t = (1-\alpha)(i_t - i^* - \psi_t) - \pi_{H,t} + i^* + \psi_t - \bar{r}_t,$$

$$\dot{\hat{\theta}}_t = i_t - i^* - \psi_t,$$

$$\hat{y}_0 = (1-\alpha)\hat{\theta}_0 - \bar{s}_0.$$

Figure 2 depicts the optimal allocation with fixed exchange rates and capital controls, together with the allocation with fixed exchange rates and no capital controls. The allocation with fixed exchange rates and no capital controls features a pronounced recession (output drops by about 10% and consumption by about 13% at impact) and deflation, accompanied by a sharp rise in nominal interest rates. Capital controls allow a drastic improvement in this outcome. A large tax on outflows (of about 4%) is quickly phased in, which almost eliminates the impact of the risk premium shock on the nominal interest rate. Output, consumption, inflation, the terms of trade and the trade balance are almost perfectly stabilized. In this example, capital controls are very effective at regaining monetary autonomy and performing macroeconomic stabilization under a fixed exchange rate. However, this allocation essentially gives up on the other objective of terms of trade management.

Finally, we can compare it to the flexible price allocation, with optimal capital controls, which can be implemented with an appropriate exchange rate policy. This comparison is performed in Figure 3. We find that these two allocations are quite close, except in the very short run. In other words, the although the perfect dichotomy that we identified in Proposition 4 does not hold perfectly, it is not a bad approximation. Under both allocations, there is a large depreciation of the exchange rate (close to 8%), a strong shift towards a trade surplus (of the order of 2%), little effect on output, a substantial drop in consumption (approximately 3%), little inflation and a substantial rise in the nominal interest rate. Optimal capital controls lean against the wind, with an optimal tax on outflows of about 1%. Most of the difference between the two allocations occurs in the short run in the first 6 months of the shock. Under the optimal allocation, the exchange rate and the terms of trade depreciates a little less, consumption drops a little more, the trade balance shifts towards a slightly larger surplus. The biggest difference is that the optimal tax on outflows is initially substantially smaller and the increase in the domestic interest rate is initially substantially larger.[11]

5 Conclusion

We have found that both capital controls and flexible exchange rates are important tools to respond to sudden stops, modeled as risk premium shocks. Flexible exchange rates are perhaps the most important of the two. The ability to let the exchange rate depreciate drastically mitigates the consequences of the sudden stop and avoids a large recession. With fixed exchange rates, capital controls have an important macroeconomic stabilization role to play to regain some monetary autonomy and mitigate the impact of the recession. But capital controls also have an important yet different role to play when the exchange rate is flexible. They allow to better navigate the dual objective of macroeconomic stabilization and terms of trade management. They help mitigate the depreciation of the exchange rate and of the terms of trade, the drop in consumption, the outflow of capital and the associated trade surplus.

[11] The relative magnitudes of the increase in the nominal interest rate under both allocations can appear surprising to the naked eye. Both are however consistent with UIP (modified by the risk premium and the capital controls). Under the optimal allocation, the tax on outflows and the rate of appreciation of the exchange rate are smaller, enough to require this large difference in interest rates.

In future work, we intend to study capital controls in hybrid models that incorporate pecuniary externalities and nominal rigidities. Such models incorporate more details on the finance side of the model and offer a different rationale for terms of trade management, namely financial stability.

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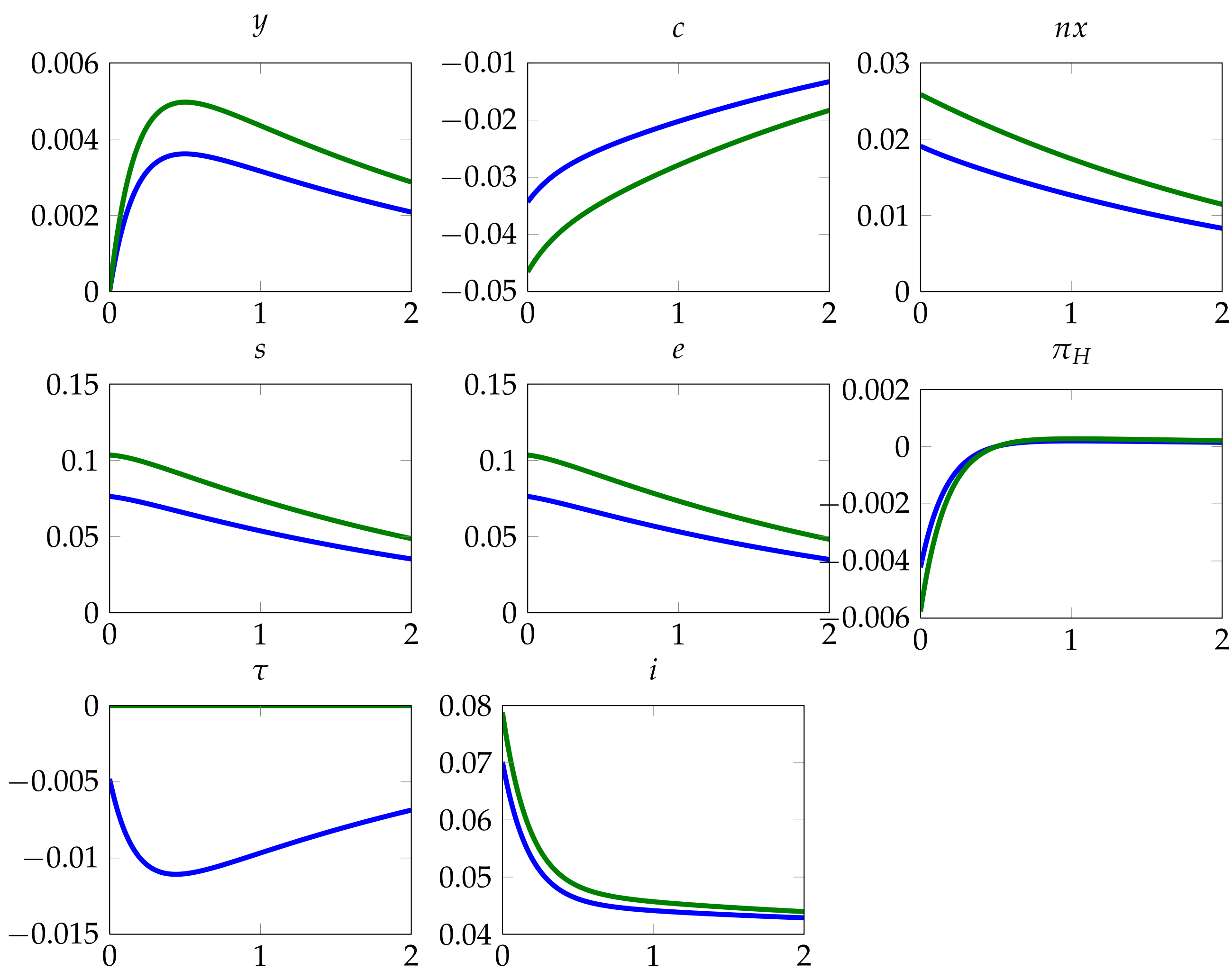

Figure 1: Capital controls (blue) and no capital controls (green) with flexible exchange rates.

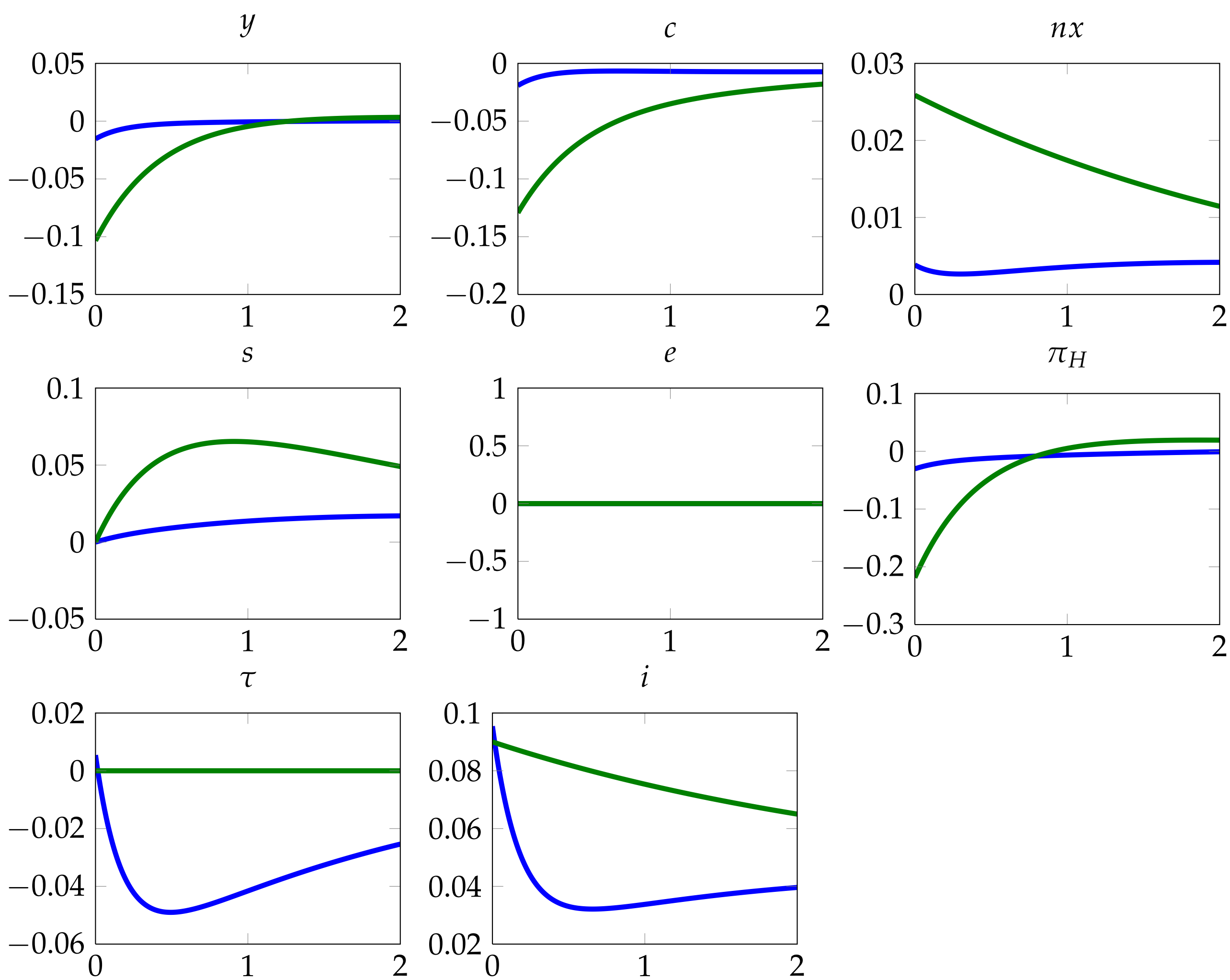

Figure 2: Capital controls (blue) and no capital controls (green) with fixed exchange rates.

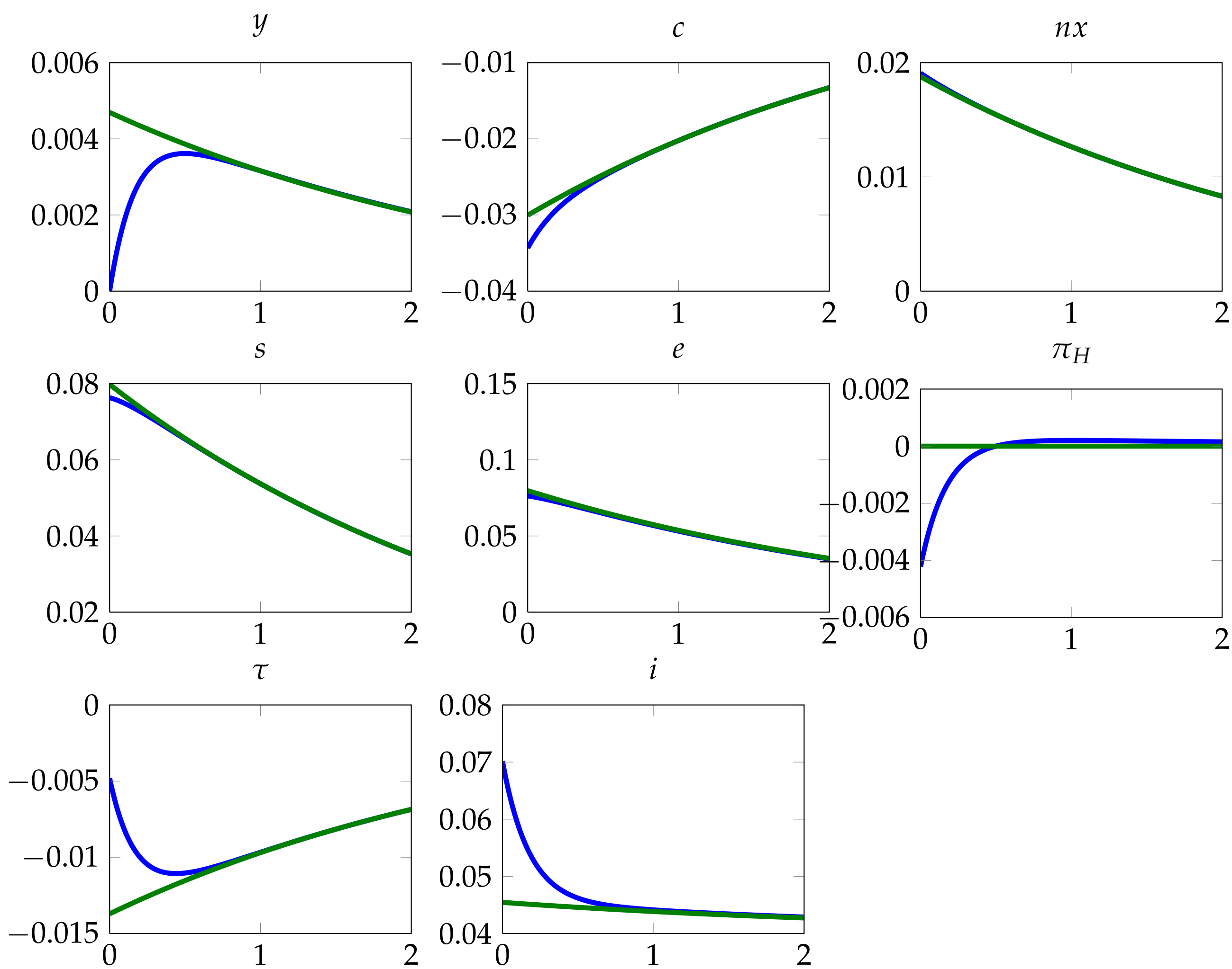

Figure 3: Optimal capital controls and exchange rates (blue) and capital controls and exchange rates that replicate the flexible price allocation with optimal capital controls (green).

A Appendix

A.1 Derivation of the Loss Function

We focus on Cole-Obstfeld case $\sigma = \gamma = \eta = 1$. We have the exact relationship

$$c_t = \theta_t + c_t^* + (1-\alpha)s_t$$

and the following second order approximation of the goods market clearing condition $Y_t = S_t C_t^* \left[(1-\alpha)\Theta_t + \alpha\right]$:

$$y_t = c_t^* + s_t + (1-\alpha)\theta_t + \frac{1}{2}\alpha(1-\alpha)\theta_t^2.$$

Using these equations, we can derive

$$c_t = \alpha c_t^* + \theta_t \alpha(2-\alpha) + (1-\alpha)y_t + \frac{1}{2} - \alpha(1-\alpha)^2\theta_t^2.$$

Hence in gaps,

$$\hat{c}_t = (1-\alpha)\hat{y}_t + \alpha(2-\alpha)\hat{\theta}_t + \frac{1}{2}(1-\alpha)^2\left[-\alpha\hat{\theta}_t(\hat{\theta}_t + 2\bar{\theta}_t)\right].$$

We can use this expression to derive

$$\begin{aligned}
\log C_t &= \bar{c}_t + \hat{c}_t \\
&= \bar{c}_t + (1-\alpha)\hat{y}_t + \alpha(2-\alpha)\hat{\theta}_t - \alpha\frac{1}{2}(1-\alpha)^2\hat{\theta}_t(\hat{\theta}_t + 2\bar{\theta}_t).
\end{aligned}$$

We have

$$\frac{N_t^{1+\phi}}{1+\phi} = \frac{\bar{N}_t^{1+\phi}}{1+\phi} + \bar{N}_t^{1+\phi}\left[\hat{y}_t + z_t + \frac{1}{2}(1+\phi)\hat{y}_t^2\right],$$

where

$$z_t = \log\int\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} \approx \frac{\epsilon}{2}\sigma^2_{p_{H,t}}.$$

Using the fact that $\bar{N}_t^{1+\phi} = (1-\alpha)(1-\alpha\bar{\theta}_t)$ for all t, we get the following expression for the objective function:

$$\int_0^\infty e^{-\rho t}\left(\frac{U_t-\bar{U}_t}{CU_c}\right)dt =$$

$$-\tfrac{(1-\alpha)(1+\phi)}{2}\int_0^\infty e^{-\rho t}\Big[\alpha_\pi \pi_{H,t}^2+\hat{y}_t^2-\frac{2\alpha}{1+\phi}\hat{y}_t\bar{\theta}_t - \tfrac{2\alpha(2-\alpha)}{(1-\alpha)(1+\phi)}\hat{\theta}_t+\alpha\tfrac{1-\alpha}{1+\phi}\alpha\hat{\theta}_t(\hat{\theta}_t+2\bar{\theta}_t)\Big]dt,$$

where $\alpha_\pi = \epsilon/[\lambda(1+\phi)]$.

We now use a second order approximation of the country budget constraint to replace the linear term in $\hat{\theta}_t$ in the expression above. We find that a second order approximation for nx_t:

$$nx_t = -\alpha(\theta_t + \tfrac{1}{2}\theta_t^2).$$

A first order approximation of the discount factor $e^{-\rho t}e^{-\int_0^t \psi_t dt}$ is $e^{-\rho t}\left[1+\bar{\theta}_0-\bar{\theta}_t\right]$. Combining the two, we get the following second order approximation for the budget constraint

$$\alpha\int_0^\infty e^{-\rho t}(\hat{\theta}_t+\frac{1}{2}\hat{\theta}_t(\hat{\theta}_t+2\bar{\theta}_t)+(\bar{\theta}_0-\bar{\theta}_t)\hat{\theta}_t)=0,$$

so that we can replace the linear term in $\hat{\theta}_t$ in the expression for welfare to get the following expression for the loss function:

$$(1-\alpha)(1+\phi)\int e^{-\rho t}\left[\frac{1}{2}\alpha_\pi\pi_{H,t}^2+\frac{1}{2}\hat{y}_t(\hat{y}_t-\frac{2\alpha}{1+\phi}\bar{\theta}_t)+\frac{1}{2}\alpha_\theta\hat{\theta}_t(\hat{\theta}_t+2(\alpha_\psi\bar{\theta}_t+(1-\alpha_\psi)\bar{\theta}_0))\right]dt,$$

or up to a constant

$$(1-\alpha)(1+\phi)\int e^{-\rho t}\left[\frac{1}{2}\alpha_\pi\pi_{H,t}^2+\frac{1}{2}(\hat{y}_t-\frac{\alpha}{1+\phi}\bar{\theta}_t)^2+\frac{1}{2}\alpha_\theta(\hat{\theta}_t+\alpha_\psi\bar{\theta}_t+(1-\alpha_\psi)\bar{\theta}_0)^2\right]dt,$$

where

$$\alpha_\psi = \frac{1-\alpha}{\frac{2-\alpha}{1-\alpha}+1-\alpha} \qquad \text{and} \qquad \alpha_\theta = \frac{\alpha}{1+\phi}\left(\frac{2-\alpha}{1-\alpha}+1-\alpha\right).$$

A.2 Proof of Proposition 5

With flexible prices, we have $\hat{y}_t = -\frac{\alpha}{1+\phi}\hat{\theta}_t$ and we can drop the initial condition since the price of home goods can jump. Hence we are led to the planning problem

$$\min_{\{\hat{\theta}_t\}}\frac{1}{2}\int_0^\infty e^{-\rho t}\left[\left(\frac{\alpha}{1+\phi}\right)^2(-\hat{\theta}_t-\bar{\theta}_t)^2+\alpha_\theta(\hat{\theta}_t+\alpha_\psi\bar{\theta}_t+(1-\alpha_\psi)\bar{\theta}_0)^2\right]dt$$

subject to

$$\int_0^\infty e^{-\rho t}\hat{\theta}_t dt = 0.$$

Let Γ be the multiplier on the budget constraint. The solution is given by

$$\left[\left(\frac{\alpha}{1+\phi}\right)^2 + \alpha_\theta\right]\hat{\theta}_t = -\left[\left(\frac{\alpha}{1+\phi}\right)^2 + \alpha_\theta\alpha_\psi\right]\bar{\theta}_t - \alpha_\theta(1-\alpha_\psi)\bar{\theta}_0 - \Gamma.$$

Since $\int_0^\infty e^{-\rho t}\bar{\theta}_t dt = 0$ we find $\Gamma = -\alpha_\theta(1-\alpha_\psi)\bar{\theta}_0$ so that the solution is

$$\hat{\theta}_t = -\frac{(\frac{\alpha}{1+\phi})^2 + \alpha_\theta\alpha_\psi}{(\frac{\alpha}{1+\phi})^2 + \alpha_\theta}\bar{\theta}_t,$$

$$\hat{y}_t = \frac{\alpha}{1+\phi}\frac{(\frac{\alpha}{1+\phi})^2 + \alpha_\theta\alpha_\psi}{(\frac{\alpha}{1+\phi})^2 + \alpha_\theta}\bar{\theta}_t.$$

A.3 Proof of Proposition 6

The problem simplifies to

$$\min_{\{\hat{y}_t,\hat{\theta}_t\}} \frac{1}{2}\int_0^\infty e^{-\rho t}\left[(\hat{y}_t - \frac{\alpha}{1+\phi}\bar{\theta}_t)^2 + \alpha_\theta(\hat{\theta}_t + \alpha_\psi\bar{\theta}_t + (1-\alpha_\psi)\bar{\theta}_0)^2\right]dt$$

subject to

$$\int_0^\infty e^{-\rho t}\hat{\theta}_t dt = 0.$$

The solution is $\hat{y}_t = \frac{\alpha}{1+\phi}\bar{\theta}_t$ and $\hat{\theta}_t = -\alpha_\psi\bar{\theta}_t$, which implies

$$\tau_t = -\alpha_\psi\psi_t.$$

We can also compute e_t. For that we use

$$\dot{\hat{y}}_t = (1-\alpha)(i_t - \rho) + \alpha(\dot{e}_t + \psi_t) - \frac{\alpha\phi}{1+\phi}\psi_t,$$

$$\dot{\hat{\theta}}_t = (i_t - \rho) - (\dot{e}_t + \psi_t),$$

$$\hat{y}_0 = (1-\alpha)\hat{\theta}_0 + e_0 - \bar{s}_0.$$

This yields

$$e_t = -(1-\alpha)(1-\alpha_\psi)\bar{\theta}_t,$$

i.e.

$$e_t = -(1-\alpha)(1-\alpha_\psi)\left[\int_0^t \psi_s ds - \int_0^\infty \psi_s e^{-\rho s} ds\right],$$

Hence in response to a negative risk premium shock that mean reverts to zero, the exchange rate initially appreciates and then depreciates over time.

We can compare the solution to the flexible price solution

$$\tau_t = -\frac{(\frac{\alpha}{1+\phi})^2 + \alpha_\theta \alpha_\psi}{(\frac{\alpha}{1+\phi})^2 + \alpha_\theta}\psi_t,$$

so that we see that capital controls are always used less with rigid prices and flexible exchange rate than with flexible prices. However, this difference disappears when $\alpha \to 0$.

A.4 Proof of Proposition 7

We have

$$\min_{\{\pi_{H,t}, \hat{y}_t, \hat{\theta}_t\}} \frac{1}{2}\int_0^\infty e^{-\rho t}\left[\alpha_\pi \pi_{H,t}^2 + (\hat{y}_t - \frac{\alpha}{1+\phi}\bar{\theta}_t)^2 + \alpha_\theta(\hat{\theta}_t + \alpha_\psi \bar{\theta}_t + (1-\alpha_\psi)\bar{\theta}_0)^2\right] dt$$

subject to

$$\dot{\pi}_{H,t} = \rho \pi_{H,t} - \hat{\kappa}\hat{y}_t - \lambda\alpha\hat{\theta}_t,$$

$$\int_0^\infty e^{-\rho t}\hat{\theta}_t dt = 0.$$

The FOCs are

$$-\dot{\mu}_{\pi,t} = \alpha_\pi \pi_{H,t},$$

$$\hat{y}_t - \frac{\alpha}{1+\phi}\bar{\theta}_t - \hat{\kappa}\mu_{\pi,t} = 0,$$

$$\alpha_\theta(\hat{\theta}_t + \alpha_\psi\bar{\theta}_t + (1-\alpha_\psi)\bar{\theta}_0) + \Gamma - \lambda\alpha\mu_{\pi,t} = 0.$$

Note that this implies the following formula for capital controls

$$\tau_t = -\alpha_\psi \psi_t + \frac{\lambda\alpha}{\alpha_\theta}\dot{\mu}_{\pi,t} = -\alpha_\psi\psi_t - \frac{\lambda\alpha}{\alpha_\theta}\alpha_\pi \pi_{H,t}.$$

This formula depends on the endogenous object $\pi_{H,t}$ which we determine in closed form below.

We have the following system of differential equations

$$\dot{\pi}_{H,t} = \rho \pi_{H,t} - \left(\hat{\kappa}^2 + \frac{(\lambda\alpha)^2}{\alpha_\theta} \right) \mu_{\pi,t} + \lambda\alpha \left[\frac{\Gamma}{\alpha_\theta} - (1-\alpha_\psi)\bar{\theta}_t + (1-\alpha_\psi)\bar{\theta}_0 \right],$$

$$\dot{\mu}_{\pi,t} = -\alpha_\pi \pi_{H,t},$$

with $\mu_{\pi,0} = 0$. We can differentiate the first equation and use the second to substitute out $\dot{\mu}_{\pi,t}$. We find

$$\frac{d^2\pi_{H,t}}{dt} = \rho \frac{d\pi_{H,t}}{dt} + \left(\hat{\kappa}^2 + \frac{(\lambda\alpha)^2}{\alpha_\theta} \right) \alpha_\pi \pi_{H,t} - \lambda\alpha(1-\alpha_\psi)\psi_t.$$

The characteristic polynomial of this differential equation has exactly one negative eigenvalue ν^- and one positive eigenvalue ν^+ where

$$\nu^- = \frac{\rho - \sqrt{\rho^2 + 4\alpha_\pi \left(\hat{\kappa}^2 + \frac{(\lambda\alpha)^2}{\alpha_\theta} \right)}}{2} \quad \text{and} \quad \nu^+ = \frac{\rho + \sqrt{\rho^2 + 4\alpha_\pi \left(\hat{\kappa}^2 + \frac{(\lambda\alpha)^2}{\alpha_\theta} \right)}}{2}.$$

The solution is of the form

$$\pi_{H,t} = \lambda^- e^{\nu^- t} + x^+ \int_t^\infty e^{-\nu^+(s-t)} \psi_s ds + x^- \int_0^t e^{-\nu^-(s-t)} \psi_s ds.$$

We have

$$\frac{d\pi_{H,t}}{dt} = \nu^- \lambda^- e^{\nu^- t} - x^+ \psi_t + \nu^+ x^+ \int_t^\infty e^{-\nu^+(s-t)} \psi_s ds + x^- \psi_t + \nu^- x^- \int_0^t e^{-\nu^-(s-t)} \psi_s ds.$$

$$\begin{aligned}\frac{d^2\pi_{H,t}}{dt^2} = (\nu^-)^2 \lambda^- e^{\nu^- t} - x^+ \frac{d\psi_t}{dt} - \nu^+ x^+ \psi_t + (\nu^+)^2 x^+ \int_t^\infty e^{-\nu^+(s-t)} \psi_s ds \\ + x^- \frac{d\psi_t}{dt} + \nu^- x^- \psi_t + (\nu^-)^2 x^- \int_0^t e^{-\nu^-(s-t)} \psi_s ds.\end{aligned}$$

Hence

$$\frac{d^2\pi_{H,t}}{dt} - \rho \frac{d\pi_{H,t}}{dt} - \left(\hat{\kappa}^2 + \frac{(\lambda\alpha)^2}{\alpha_\theta} \right) \alpha_\pi \pi_{H,t} = (x^- - x^+) \frac{d\psi_t}{dt} + (\nu^- x^- - \nu^+ x^+) \psi_t - \rho(x^- - x^+) \psi_t.$$

We need

$$-\lambda\alpha(1-\alpha_\psi)\psi_t = (x^- - x^+) \frac{d\psi_t}{dt} + (\nu^- x^- - \nu^+ x^+) \psi_t - \rho(x^- - x^+) \psi_t,$$

Hence we must have

$$x^- = x^+,$$

$$(\nu^- - \rho)x^- - (\nu^+ - \rho)x^+ = -\lambda\alpha(1 - \alpha_\psi).$$

The solution is

$$x^- = x^+ = \frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+ - \nu^-}.$$

Hence the solution is

$$\pi_{H,t} = \lambda^- e^{\nu^- t} + \frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+ - \nu^-}\int_t^\infty e^{-\nu^+(s-t)}\psi_s ds + \frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+ - \nu^-}\int_0^t e^{-\nu^-(s-t)}\psi_s ds.$$

To determine λ^-, we use the requirement that $\mu_{\pi,0} = 0$. This implies that

$$\dot{\pi}_{H,0} = \rho\pi_{H,0} + \lambda\alpha\frac{\Gamma}{\alpha_\theta}.$$

Using

$$\dot{\pi}_{H,0} = \nu^-\lambda^- + \nu^+\frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+ - \nu^-}\int_0^\infty e^{-\nu^+ t}\psi_t dt,$$

$$\pi_{H,0} = \lambda^- + \frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+ - \nu^-}\int_0^\infty e^{-\nu^+ t}\psi_t dt,$$

this can be rewritten as

$$(\nu^- - \rho)\lambda^- + (\nu^+ - \rho)\frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+ - \nu^-}\int_0^\infty e^{-\nu^+ t}\psi_t ds = \lambda\alpha\frac{\Gamma}{\alpha_\theta}.$$

This determines Γ as a function of λ^-. We then use

$$\alpha_\theta(\hat{\theta}_t + \alpha_\psi\bar{\theta}_t + (1-\alpha_\psi)\bar{\theta}_0) + \Gamma - \lambda\alpha\mu_{\pi,t} = 0.$$

Integrating and using the country budget constraint, we find that

$$\frac{\alpha_\theta(1-\alpha_\psi)}{\rho}\bar{\theta}_0 + \frac{1}{\rho}\Gamma - \lambda\alpha\int_0^\infty e^{-\rho t}\mu_{\pi,t} dt = 0.$$

Integrating the last equation by parts using $\mu_{\pi,0} = 0$, we can rewrite this as

$$\frac{\alpha_\theta(1-\alpha_\psi)}{\rho}\bar{\theta}_0 + \frac{1}{\rho}\Gamma - \frac{\lambda\alpha}{\rho}\int_0^\infty e^{-\rho t}\dot{\mu}_{\pi,t} dt = 0.$$

Using $\dot{\mu}_{\pi,t} = -\alpha_\pi \pi_{H,t}$, we can rewrite this as

$$\frac{\alpha_\theta(1-\alpha_\psi)}{\rho}\bar{\theta}_0 + \frac{1}{\rho}\Gamma + \frac{\lambda\alpha\alpha_\pi}{\rho}\int_0^\infty e^{-\rho t}\pi_{H,t}dt = 0,$$

which holds if and only if

$$\frac{\alpha_\theta(1-\alpha_\psi)}{\rho}\bar{\theta}_0 + \frac{1}{\rho}\Gamma + \frac{\lambda\alpha\alpha_\pi}{\rho}\frac{\lambda^-}{\rho-\nu^-} + \frac{\lambda\alpha\alpha_\pi}{\rho}\frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+-\nu^-}\int_0^\infty e^{-\rho t}\int_t^\infty e^{-\nu^+(s-t)}\psi_s ds dt$$
$$+ \frac{\lambda\alpha\alpha_\pi}{\rho}\frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+-\nu^-}\int_0^\infty e^{-\rho t}\int_0^t e^{-\nu^-(s-t)}\psi_s ds dt = 0,$$

i.e.

$$\frac{\alpha_\theta(1-\alpha_\psi)}{\rho}\bar{\theta}_0 + \frac{1}{\rho}\Gamma + \frac{\lambda\alpha\alpha_\pi}{\rho}\frac{\lambda^-}{\rho-\nu^-} + \frac{1}{\rho}\frac{\lambda^2\alpha^2\alpha_\pi(1-\alpha_\psi)}{\nu^+-\nu^-}\int_0^\infty \left[\frac{e^{-\rho t}-e^{-\nu^+ t}}{\nu^+-\rho} + \frac{e^{-\rho t}}{\rho-\nu^-}\right]\psi_t dt = 0,$$

i.e.

$$\alpha_\theta(1-\alpha_\psi)\bar{\theta}_0 + \Gamma + \lambda\alpha\alpha_\pi\frac{\lambda^-}{\rho-\nu^-} + \frac{\lambda^2\alpha^2\alpha_\pi(1-\alpha_\psi)}{\nu^+-\nu^-}\int_0^\infty \left[\frac{e^{-\rho t}-e^{-\nu^+ t}}{\nu^+-\rho} + \frac{e^{-\rho t}}{\rho-\nu^-}\right]\psi_t dt = 0,$$

or

$$\Gamma = -\alpha_\theta(1-\alpha_\psi)\bar{\theta}_0 - \lambda\alpha\alpha_\pi\frac{\lambda^-}{\rho-\nu^-} - \frac{\lambda^2\alpha^2\alpha_\pi(1-\alpha_\psi)}{\nu^+-\nu^-}\int_0^\infty \left[\frac{e^{-\rho t}-e^{-\nu^+ t}}{\nu^+-\rho} + \frac{e^{-\rho t}}{\rho-\nu^-}\right]\psi_t dt,$$

or

$$\Gamma = -\alpha_\theta(1-\alpha_\psi)\bar{\theta}_0 - \lambda\alpha\alpha_\pi\frac{\lambda^-}{\rho-\nu^-} - \frac{\lambda^2\alpha^2\alpha_\pi(1-\alpha_\psi)}{\nu^+-\nu^-}\int_0^\infty \left[-\frac{e^{-\nu^+ t}}{-\nu^-} + \frac{\nu^+-\nu^-}{-\nu^-\nu^+}e^{-\rho t}\right]\psi_t dt.$$

Together with

$$(\nu^- - \rho)\lambda^- + (\nu^+ - \rho)\frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+-\nu^-}\int_0^\infty e^{-\nu^+ t}\psi_t dt = \lambda\alpha\frac{\Gamma}{\alpha_\theta},$$

this represents a linear system of two equations in two unknowns in Γ and λ^-. We find

$$(\nu^- - \rho + \frac{\lambda^2\alpha^2}{\alpha_\theta}\alpha_\pi \frac{1}{\rho-\nu^-})\lambda^- = \lambda\alpha(1-\alpha_\psi)\int_0^\infty \psi_t e^{-\rho t}dt -$$
$$\frac{\lambda^3\alpha^3\alpha_\pi(1-\alpha_\psi)}{\alpha_\theta(\nu^+-\nu^-)}\int_0^\infty \left[-\frac{e^{-\nu^+ t}}{-\nu^-} + \frac{\nu^+-\nu^-}{-\nu^-\nu^+}e^{-\rho t}\right]\psi_t dt - (\nu^+-\rho)\frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+-\nu^-}\int_0^\infty e^{-\nu^+ t}\psi_t dt.$$

We can rewrite this as

$$\frac{-\nu^+ + \frac{1}{\nu^+}\frac{\lambda^2\alpha^2\alpha_\pi}{\alpha_\theta}}{\lambda\alpha(1-\alpha_\psi)}\lambda^- = \int_0^\infty \psi_t e^{-\rho t}dt$$
$$-\frac{\lambda^2\alpha^2\alpha_\pi}{\alpha_\theta(\nu^+-\nu^-)}\int_0^\infty \left[-\frac{e^{-\nu^+ t}}{-\nu^-} + \frac{\nu^+-\nu^-}{-\nu^-\nu^+}e^{-\rho t}\right]\psi_t dt + \nu^- \frac{1}{\nu^+-\nu^-}\int_0^\infty e^{-\nu^+ t}\psi_t dt,$$

or

$$\frac{\nu^+ - \frac{1}{\nu^+}\frac{\lambda^2\alpha^2\alpha_\pi}{\alpha_\theta}}{\lambda\alpha(1-\alpha_\psi)}\lambda^- = \left[\frac{-\nu^- - \frac{\lambda^2\alpha^2\alpha_\pi}{-\nu^-\alpha_\theta}}{\nu^+-\nu^-}\right]\int_0^\infty e^{-\nu^+ t}\psi_t dt - \left[1 - \frac{\lambda^2\alpha^2\alpha_\pi}{-\nu^-\nu^+\alpha_\theta}\right]\int_0^\infty \psi_t e^{-\rho t}dt.$$

This gives us λ^-. To find the whole solution, we then solve the differential equation

$$\dot{\mu}_{\pi,t} = -\alpha_\pi \pi_{H,t},$$

with initial condition $\mu_{\pi,0} = 0$.From this we get $\hat{y}_t$ from

$$\hat{y}_t - \frac{\alpha}{1+\phi}\bar{\theta}_t - \hat{\kappa}\mu_{\pi,t} = 0,$$

and $\hat{\theta}_t$ from

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \hat{\kappa}\hat{y}_t - \lambda\alpha\hat{\theta}_t.$$

Consider the case where $\psi_t = \psi_0 e^{-\rho_\psi t}$. We then have

$$\Gamma + \lambda\alpha\alpha_\pi\frac{\lambda^-}{\rho-\nu^-} = \alpha_\theta(1-\alpha_\psi)\frac{\psi_0}{\rho+\rho_\psi} - \frac{\lambda^2\alpha^2\alpha_\pi(1-\alpha_\psi)}{\nu^+-\nu^-}\psi_0\left[\frac{1}{\nu^-}\frac{1}{\rho_\psi+\nu^+} + \frac{\nu^+-\nu^-}{-\nu^-\nu^+}\frac{1}{\rho+\rho_\psi}\right],$$

$$\lambda\alpha\frac{\Gamma}{\alpha_\theta} + \nu^+\lambda^- = -\nu^-\frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+-\nu^-}\frac{\psi_0}{\rho_\psi+\nu^+},$$

which gives us Γ and λ^{-1}. Then we have

$$-\frac{\mu_{\pi,t}}{\alpha_\pi} = \lambda^{-}\frac{e^{\nu^- t}-1}{\nu^-} + \frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+ - \nu^-}\left[\frac{\psi_0}{\rho_\psi+\nu^+}\frac{1-e^{-\rho_\psi t}}{\rho_\psi} + \frac{\psi_0}{\rho_\psi+\nu^-}\left(\frac{e^{\nu^- t}-1}{\nu^-} - \frac{1-e^{-\rho_\psi t}}{\rho_\psi}\right)\right],$$

$$\hat{y}_t = \frac{\alpha}{1+\phi}\left(-\frac{\psi_0}{\rho_\psi+\rho} + \frac{\psi_0(1-e^{-\rho_\psi t})}{\rho_\psi}\right) + \hat{\kappa}\mu_{\pi,t},$$

$$\hat{\theta}_t = -\alpha_\psi\left(-\frac{\psi_0}{\rho_\psi+\rho} + \frac{\psi_0(1-e^{-\rho_\psi t})}{\rho_\psi}\right) + (1-\alpha_\psi)\frac{\psi_0}{\rho_\psi+\rho} - \frac{\Gamma}{\alpha_\theta} + \frac{\lambda\alpha}{\alpha_\theta}\mu_{\pi,t}.$$

We can back out the terms of trade from

$$s_t = \hat{y}_t - (1-\alpha)\hat{\theta}_t - \frac{1+\phi(1-\alpha)}{1+\phi}\left(-\frac{\psi_0}{\rho_\psi+\rho} + \frac{\psi_0(1-e^{-\rho_\psi t})}{\rho_\psi}\right).$$

Finally inflation is

$$\pi_{H,t} = \lambda^{-}e^{\nu^- t} + \frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+-\nu^-}\frac{\psi_0 e^{-\rho_\psi t}}{\rho_\psi+\nu^+} + \frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+-\nu^-}\frac{\psi_0(e^{\nu^- t}-e^{-\rho_\psi t})}{\rho_\psi+\nu^-}.$$

and the exchange rate from

$$\begin{aligned} e_t &= s_t + \int_0^t \pi_{H,s}ds \\ &= s_t + \lambda^{-}\frac{e^{\nu^- t}-1}{\nu^-} + \frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+-\nu^-}\frac{\psi_0}{\rho_\psi+\nu^+}\frac{1-e^{-\rho_\psi t}}{\rho_\psi} + \frac{\lambda\alpha(1-\alpha_\psi)}{\nu^+-\nu^-}\frac{\psi_0}{\rho_\psi+\nu^-}\left(\frac{e^{\nu^- t}-1}{\nu^-} - \frac{1-e^{-\rho_\psi t}}{\rho_\psi}\right). \end{aligned}$$

We can also write

$$\dot{s}_t = \dot{\hat{y}}_t - (1-\alpha)\dot{\hat{\theta}}_t - \frac{1+\phi(1-\alpha)}{1+\phi}\psi_0 e^{-\rho_\psi t},$$

$$\dot{\hat{y}}_t = \frac{\alpha}{1+\phi}\psi_0 e^{-\rho_\psi t} - \hat{\kappa}\alpha_\pi\pi_{H,t},$$

so that

$$\dot{s}_t = -\hat{\kappa}\alpha_\pi\pi_{H,t} - (1-\alpha)\dot{\hat{\theta}}_t - (1-\alpha)\psi_0 e^{-\rho_\psi t},$$

$$\dot{e}_t = (1-\hat{\kappa}\alpha_\pi)\pi_{H,t} - (1-\alpha)\dot{\hat{\theta}}_t - (1-\alpha)\psi_0 e^{-\rho_\psi t},$$

$$i_t = \rho + \psi_t + \dot{\hat{\theta}}_t + \dot{e}_t = \rho + (1-\hat{\kappa}\alpha_\pi)\pi_{H,t} + \alpha\dot{\hat{\theta}}_t + \alpha\psi_0 e^{-\rho_\psi t}.$$

We now solve for the optimal allocation without capital controls. We have

$$\min_{\{\pi_{H,t},\hat{y}_t\}} \frac{1}{2}\int_0^\infty e^{-\rho t}\left[\alpha_\pi \pi_{H,t}^2 + (\hat{y}_t - \frac{\alpha}{1+\phi}\bar{\theta}_t)^2 + \alpha_\theta(\alpha_\psi\bar{\theta}_t + (1-\alpha_\psi)\bar{\theta}_0)^2\right] dt$$

subject to

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \hat{\kappa}\hat{y}_t.$$

The FOCs are

$$-\dot{\mu}_{\pi,t} = \alpha_\pi \pi_{H,t},$$

$$\hat{y}_t - \frac{\alpha}{1+\phi}\bar{\theta}_t - \hat{\kappa}\mu_{\pi,t} = 0.$$

We have the following system of differential equations (using $\hat{\kappa} = \lambda(1+\phi)$):

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \hat{\kappa}^2\mu_{\pi,t} - \lambda\alpha\bar{\theta}_t,$$

$$\dot{\mu}_{\pi,t} = -\alpha_\pi\pi_{H,t},$$

with $\mu_{\pi,0} = 0$. We can differentiate the first equation and use the second to substitute out $\dot{\mu}_{\pi,t}$. We find

$$\frac{d^2\pi_{H,t}}{dt} = \rho\frac{d\pi_{H,t}}{dt} + \hat{\kappa}^2\alpha_\pi\pi_{H,t} - \lambda\alpha\psi_t.$$

The characteristic polynomial of this differential equation has exactly one negative eigenvalue ν^- and one positive eigenvalue ν^+ where

$$\tilde{\nu}^- = \frac{\rho - \sqrt{\rho^2 + 4\alpha_\pi\hat{\kappa}^2}}{2} \quad \text{and} \quad \tilde{\nu}^+ = \frac{\rho + \sqrt{\rho^2 + 4\alpha_\pi\hat{\kappa}^2}}{2}.$$

The solution is of the form

$$\pi_{H,t} = \tilde{\lambda}^- e^{\tilde{\nu}^- t} + \frac{\lambda\alpha}{\tilde{\nu}^+ - \tilde{\nu}^-}\int_t^\infty e^{-\tilde{\nu}^+(s-t)}\psi_s ds + \frac{\lambda\alpha}{\tilde{\nu}^+ - \tilde{\nu}^-}\int_0^t e^{-\tilde{\nu}^-(s-t)}\psi_s ds.$$

To determine λ^-, we use the requirement that $\mu_{\pi,0} = 0$. This implies that

$$\dot{\pi}_{H,0} = \rho\pi_{H,0} - \lambda\alpha\bar{\theta}_0.$$

Using

$$\dot{\pi}_{H,0} = \tilde{\nu}^-\tilde{\lambda}^- + \tilde{\nu}^+\frac{\lambda\alpha}{\tilde{\nu}^+ - \tilde{\nu}^-}\int_0^\infty e^{-\tilde{\nu}^+ t}\psi_t dt,$$

$$\pi_{H,0} = \tilde{\lambda}^{-} + \frac{\lambda\alpha}{\tilde{\nu}^{+} - \tilde{\nu}^{-}} \int_0^\infty e^{-\tilde{\nu}^{+}t} \psi_t dt,$$

this can be rewritten as

$$(\tilde{\nu}^{-} - \rho)\tilde{\lambda}^{-} + (\tilde{\nu}^{+} - \rho)\frac{\lambda\alpha}{\tilde{\nu}^{+} - \tilde{\nu}^{-}} \int_0^\infty e^{-\tilde{\nu}^{+}t} \psi_t ds = -\lambda\alpha\bar{\theta}_0,$$

which determines $\tilde{\lambda}^{-}$. We can then determine $\hat{y}_t$ using

$$\hat{\kappa}\hat{y}_t = \rho\pi_{H,t} - \dot{\pi}_{H,t}.$$

We have

$$\pi_{H,t} = \tilde{\lambda}^{-} e^{\tilde{\nu}^{-}t} + \frac{\lambda\alpha}{\tilde{\nu}^{+} - \tilde{\nu}^{-}} \int_t^\infty e^{-\tilde{\nu}^{+}(s-t)} \psi_s ds + \frac{\lambda\alpha}{\tilde{\nu}^{+} - \tilde{\nu}^{-}} \int_0^t e^{-\tilde{\nu}^{-}(s-t)} \psi_s ds,$$

$$\frac{d\pi_{H,t}}{dt} = \tilde{\nu}^{-}\tilde{\lambda}^{-} e^{\tilde{\nu}^{-}t} + \tilde{\nu}^{+}\frac{\lambda\alpha}{\tilde{\nu}^{+} - \tilde{\nu}^{-}} \int_t^\infty e^{-\tilde{\nu}^{+}(s-t)} \psi_s ds + \tilde{\nu}^{-}\frac{\lambda\alpha}{\tilde{\nu}^{+} - \tilde{\nu}^{-}} \int_0^t e^{-\tilde{\nu}^{-}(s-t)} \psi_s ds.$$

We find

$$\hat{y}_t = (\rho - \tilde{\nu}^{-})\frac{\tilde{\lambda}^{-}}{\hat{\kappa}} e^{\tilde{\nu}^{-}t} + (\rho - \tilde{\nu}^{+})\frac{\frac{\lambda\alpha}{\hat{\kappa}}}{\tilde{\nu}^{+} - \tilde{\nu}^{-}} \int_t^\infty e^{-\tilde{\nu}^{+}(s-t)} \psi_s ds + (\rho - \tilde{\nu}^{-})\frac{\frac{\lambda\alpha}{\hat{\kappa}}}{\tilde{\nu}^{+} - \tilde{\nu}^{-}} \int_0^t e^{-\tilde{\nu}^{-}(s-t)} \psi_s ds.$$

We can back out the terms of trade using $\hat{y}_t = s_t - \bar{s}_t$, and the exchange rate from $e_t = s_t + \int_0^t \pi_{H,s} ds$.

Consider the case where $\psi_t = \psi_0 e^{-\rho_\psi t}$. We then have

$$(\tilde{\nu}^{-} - \rho)\tilde{\lambda}^{-} + (\tilde{\nu}^{+} - \rho)\frac{\lambda\alpha}{\tilde{\nu}^{+} - \tilde{\nu}^{-}}\frac{\psi_0}{\tilde{\nu}^{+} + \rho_\psi} = \lambda\alpha\frac{\psi_0}{\rho + \rho_\psi}.$$

$$\pi_{H,t} = \tilde{\lambda}^{-} e^{\tilde{\nu}^{-}t} + \frac{\lambda\alpha}{\tilde{\nu}^{+} - \tilde{\nu}^{-}}\frac{\psi_0 e^{-\rho_\psi t}}{\tilde{\nu}^{+} + \rho_\psi} + \frac{\lambda\alpha}{\tilde{\nu}^{+} - \tilde{\nu}^{-}}\frac{\psi_0(e^{\tilde{\nu}^{-}t} - e^{-\rho_\psi t})}{\tilde{\nu}^{-} + \rho_\psi},$$

$$\hat{y}_t = (\rho - \tilde{\nu}^{-})\frac{\tilde{\lambda}^{-}}{\hat{\kappa}} e^{\tilde{\nu}^{-}t} + (\rho - \tilde{\nu}^{+})\frac{\frac{\lambda\alpha}{\hat{\kappa}}}{\tilde{\nu}^{+} - \tilde{\nu}^{-}}\frac{\psi_0 e^{-\rho_\psi t}}{\tilde{\nu}^{+} + \rho_\psi} + (\rho - \tilde{\nu}^{-})\frac{\frac{\lambda\alpha}{\hat{\kappa}}}{\tilde{\nu}^{+} - \tilde{\nu}^{-}}\frac{\psi_0(e^{\tilde{\nu}^{-}t} - e^{-\rho_\psi t})}{\tilde{\nu}^{-} + \rho_\psi},$$

$$s_t = \hat{y}_t - \psi_0\frac{1 + \phi(1-\alpha)}{1+\phi}[\frac{1 - e^{-\rho_\psi t}}{\rho_\psi} - \frac{1}{\rho + \rho_\psi}],$$

$$e_t = s_t + \tilde{\lambda}^{-}\frac{e^{\tilde{\nu}^{-}t} - 1}{\tilde{\nu}^{-}} + \frac{\lambda\alpha}{\tilde{\nu}^{+} - \tilde{\nu}^{-}}\frac{\psi_0}{\tilde{\nu}^{+} + \rho_\psi}\frac{1 - e^{-\rho_\psi t}}{\rho_\psi} + \frac{\lambda\alpha}{\tilde{\nu}^{+} - \tilde{\nu}^{-}}\frac{\psi_0}{\tilde{\nu}^{-} + \rho_\psi}(\frac{e^{\tilde{\nu}^{-}t} - 1}{\tilde{\nu}^{-}} - \frac{1 - e^{-\rho_\psi t}}{\rho_\psi}).$$

We can also compute

$$\dot{s}_t = \dot{\hat{y}}_t - \psi_0 e^{-\rho_\psi t} \frac{1+\phi(1-\alpha)}{1+\phi},$$

$$\dot{\hat{y}}_t = \frac{\alpha}{1+\phi} \psi_0 e^{-\rho_\psi t} - \hat{\kappa} \alpha_\pi \pi_{H,t},$$

so that

$$\dot{s}_t = -\hat{\kappa} \alpha_\pi \pi_{H,t} - (1-\alpha) \psi_0 e^{-\rho_\psi t},$$

$$\dot{e}_t = (1-\hat{\kappa}\alpha_\pi) \pi_{H,t} - (1-\alpha) \psi_0 e^{-\rho_\psi t},$$

$$i_t = \rho + \psi_t + \dot{e}_t = \rho + (1-\hat{\kappa}\alpha_\pi) \pi_{H,t} + \alpha \psi_0 e^{-\rho_\psi t}.$$