

Problem-of-the-Week

The Problem: Miniature Amateurs

Mr. Ellis Hackworth has a miniature Pyramid of Egypt. It is 5 inches in height. Mr. Hackworth was invited to display it at an exhibition. Mr. Hackworth felt it was too small and decided to build a scaled-up model of the Pyramid out of material whose density is $(1/4)$ times the density of the material used for the miniature. He did a "back-of-the-envelope" calculation to check whether the model would be big enough.



If the mass (or weight) of the miniature and the scaled-up model are to be the same, how many inches in height will be the scaled-up Pyramid? Give your answer to two places of decimal.

It is important to note that

Mass = Density \times Volume; and
Volume of model / Volume of miniature = $(H \text{ of model} / H \text{ of miniature})^3$.

In the above equation, H is the characteristic dimension (say, height). If the mass is to be the same, then density is inversely proportional to volume. The volumes are directly proportional to the cubes of the heights for objects that are geometrically similar. Therefore, the heights are seen to be inversely proportional to the cube roots of the densities. Thus,

$$\text{Height of model} = \text{Height of miniature} \times (\text{Density of miniature} / \text{Density of model})^{1/3}$$

RAISING THE BAR

Guess the the problem of the week ACCURATELY by Friday, November 19th and your name will be included in the weekly raffle for a candy bar. Each Scholar only gets ONE submission.* GOOD LUCK!©

Study Guide

Student Edition
Pages 239–244**Direct and Inverse Variation**

If two variables x and y are related by the equation $y = kx$, where k is a nonzero constant, then the equation is called a **direct variation**, and k is called the **constant of variation**.

If two variables x and y are related by the equation $xy = k$, where $k \neq 0$, then the equation is called an **inverse variation**.

Example 1: If y varies directly as x , and $y = 12$ when $x = 4$, find y when $x = 7$.

$$\begin{array}{l|l} y = kx & y = kx \\ 12 = 4k & y = 3(7) \\ \frac{12}{4} = \frac{4k}{4} & y = 21 \\ 3 = k & \end{array}$$

Example 2: If y varies inversely as x , and $y = 3$ when $x = 12$, find x when $y = 4$.

$$\begin{array}{l|l} xy = k & xy = k \\ 12(3) = k & 4x = 36 \\ 36 = k & \frac{4x}{4} = \frac{36}{4} \\ & x = 9 \end{array}$$

Solve. Assume that y varies directly as x .

1. If $y = 4$ when $x = 2$, find y when $x = 16$.

2. If $y = 9$ when $x = -3$, find x when $y = 6$.

3. If $y = \frac{2}{5}$ when $x = \frac{1}{3}$, find y when $x = \frac{1}{4}$.

4. If $y = \frac{1}{4}$ when $x = \frac{1}{8}$, find x when $y = \frac{3}{16}$.

Solve. Assume that y varies inversely as x .

5. If $y = 9$ when $x = 7$, find y when $x = 2$.

6. If $y = 4.3$ when $x = 12.9$, find y when $x = 15.8$.

7. If $x = \frac{1}{2}$ when $y = \frac{1}{3}$, find y when $x = \frac{1}{4}$.

8. If $y = -6$ when $x = \frac{1}{4}$, find y when $x = \frac{3}{16}$.

PracticeStudent Edition
Pages 239–244***Direct and Inverse Variation******Solve. Assume that y varies directly as x .***

1. If $y = -4$ when $x = 2$, find y when $x = -6$.
2. If $y = 16$ when $x = 4$, find y when $x = 6$.
3. If $y = -5$ when $x = 12.5$, find x when $y = 15$.
4. If $y = 7$ when $x = 4$, find y when $x = 12$.
5. If $y = 80$ when $x = 32$, find x when $y = 100$.
6. If $y = 198$ when $x = 22$, find y when $x = 182$.
7. If $y = 28$ when $x = 168$, find y when $x = 108$.
8. If $y = 24$ when $x = 6$, find y when $x = -4$.

Solve. Assume that y varies inversely as x .

9. If $y = -4$ when $x = 2$, find y when $x = -6$.
10. If $y = 16$ when $x = 4$, find y when $x = 6$.
11. If $y = -5$ when $x = 12.5$, find x when $y = 15$.
12. If $y = 7$ when $x = 4$, find y when $x = 12$.
13. If $y = 27$ when $x = 12$, find x when $y = -12$.
14. If $y = 6$ when $x = -4$, find x when $y = \frac{12}{5}$.
15. If $y = 60$ when $x = 80$, find x when $y = -20$.
16. If $y = 40$ when $x = 16$, find y when $x = 10$.

Study Guide

Student Edition
Pages 356–361**Graphing Linear Equations**

There are three methods you can use for graphing equations. You can find two ordered pairs that satisfy the equation, the x - and y -intercepts, or the slope and y -intercept.

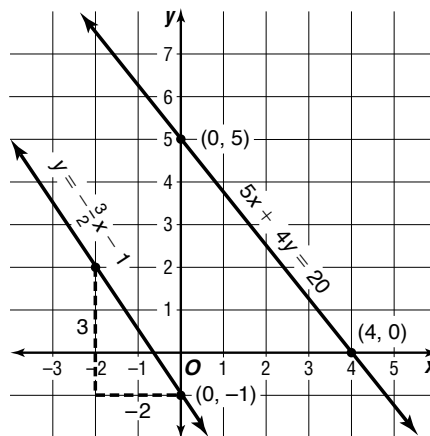
Example 1: Graph $5x + 4y = 20$ by using the x - and y -intercepts.

The equation is in standard form $Ax + By = C$.

The x -intercept is $\frac{C}{A}$, or 5.

The y -intercept is $\frac{C}{B}$, or 4.

Thus, the graph contains the points (4, 0) and (0, 5).

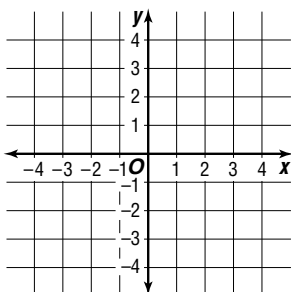


Example 2: Graph $y = -\frac{3}{2}x - 1$ by using the slope and y -intercept.

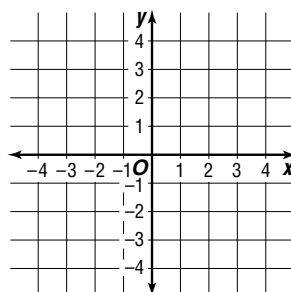
The y -intercept is -1 , the slope $-\frac{3}{2}$.

Graph each equation by using the x - and y -intercepts.

1. $-3x + 2y = 6$

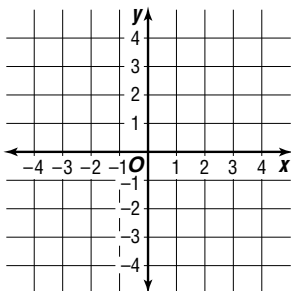


2. $3y + x = 3$

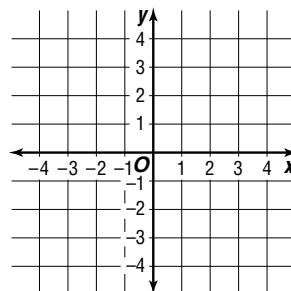


Graph each equation by using the slope and y -intercept.

3. $y = \frac{1}{3}x + 2$

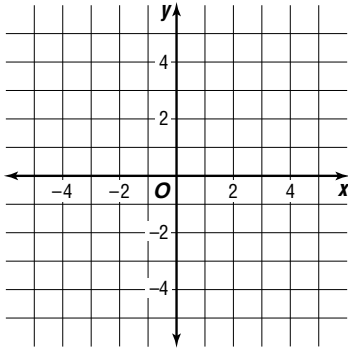


4. $y = \frac{1}{2}x + \frac{3}{4}$

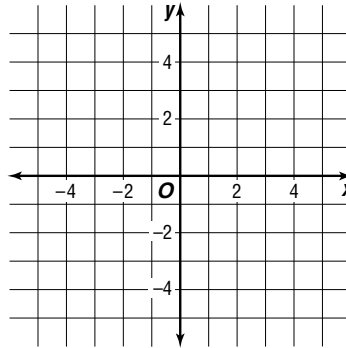


PracticeStudent Edition
Pages 356–361**Graphing Linear Equations***Graph each equation by using the x- and y-intercepts.*

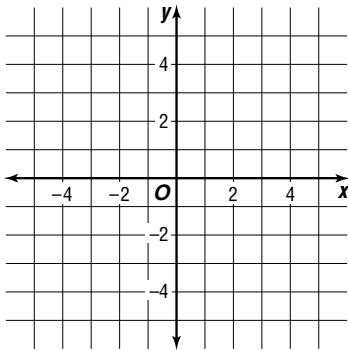
1. $3x - y = 3$



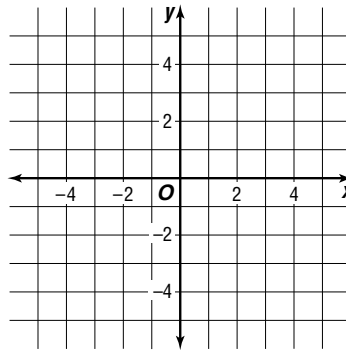
2. $3y - 2x = 6$



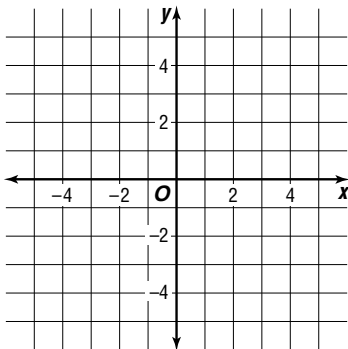
3. $4y + x = -2$



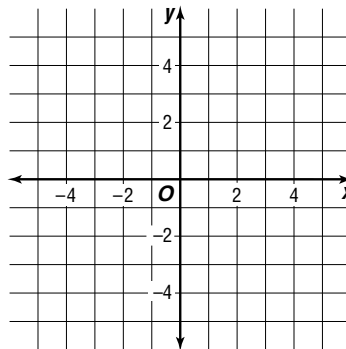
4. $x + y = -3$

*Graph each equation by using the slope and y-intercept.*

5. $y = -\frac{1}{2}x + 3$



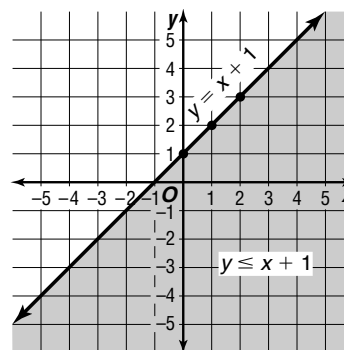
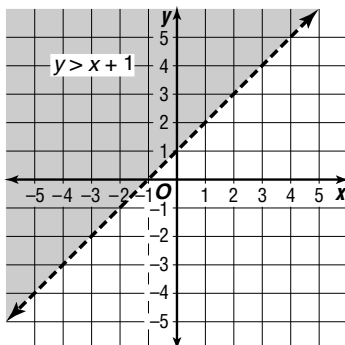
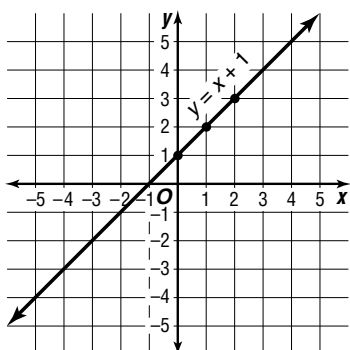
6. $3y + 6 = 2x$



Study Guide

Student Edition

Pages 436–441

Graphing Inequalities in Two Variables

The graph of the equation $y = x + 1$ is a line that separates the coordinate plane into two regions. Each region is called a **half-plane**. The line for $y = x + 1$ is called the **boundary** for each half-plane.

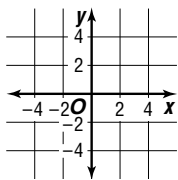
The boundary line in both regions is the line for $y = x + 1$.

In $y > x + 1$, the boundary is *not* part of the graph. The boundary is shown as a dashed line. All points above the line are part of the graph. This graph is called an **open half-plane**.

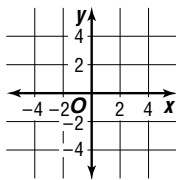
In $y \leq x + 1$, the boundary *is* part of the graph and is shown as a solid line. The graph also contains all points below the line. This graph is called a **closed half-plane**.

Graph each inequality.

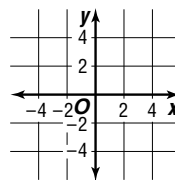
1. $y < 4$



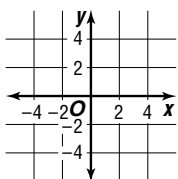
2. $3x < y$



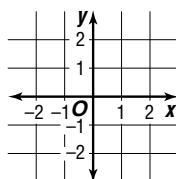
3. $2x - 3y \leq 6$



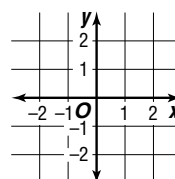
4. $-5x + 2 \geq y$



5. $x - y \geq 1$



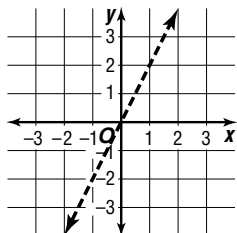
6. $-x > y$



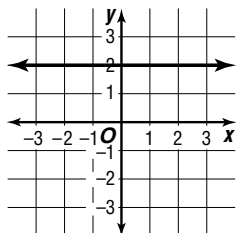
Practice

Student Edition
Pages 436–441**Graphing Inequalities in Two Variables****Shade the appropriate half-plane to complete the graph of each inequality.**

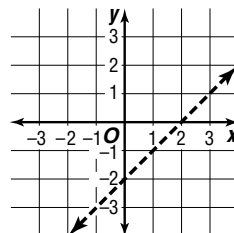
1. $y > 2x$



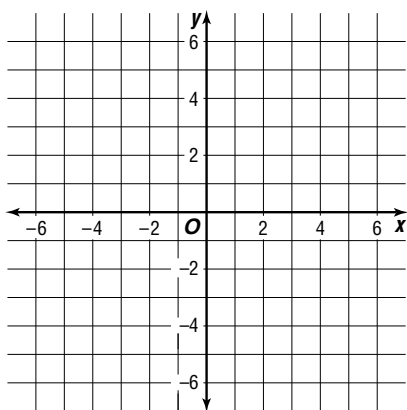
2. $y \leq 2$



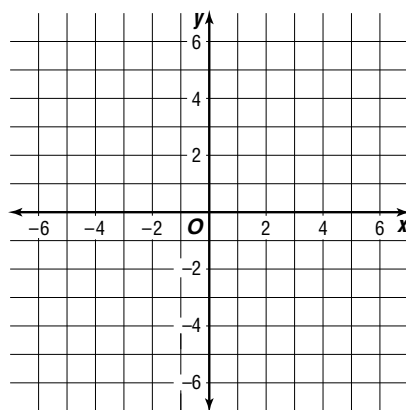
3. $y - x < -2$

**Graph each inequality.**

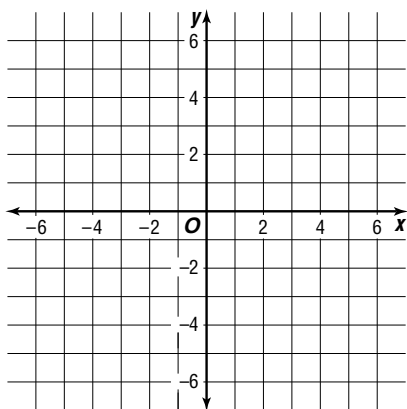
4. $y - x \leq 4$



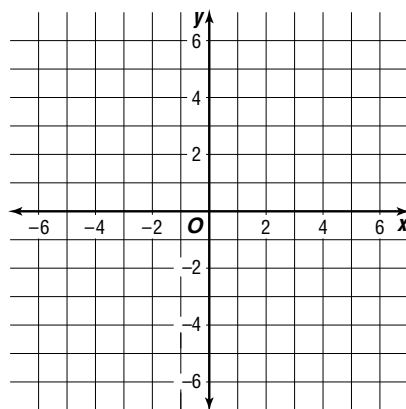
5. $2x + y > 5$



6. $x + 2y < -6$

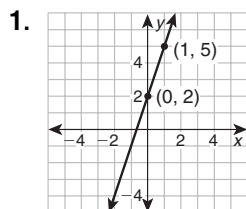


7. $2x - y \leq 4$

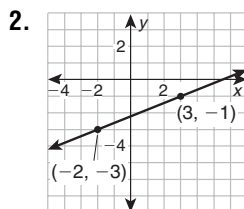


Topic Quiz: The Slope of a Line

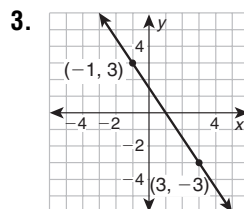
Find the slope of the line. Write your answer in simplest form.



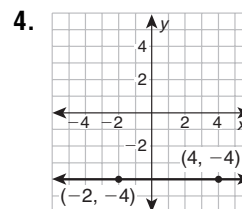
$$m = \underline{\hspace{2cm}}$$



$$m = \underline{\hspace{2cm}}$$



$$m = \underline{\hspace{2cm}}$$



$$m = \underline{\hspace{2cm}}$$

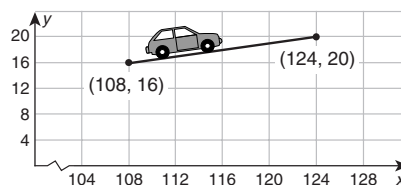
The slope of a driveway should be no more than $\frac{11}{50}$. The diagram shows a driveway.

5. Find the slope of the driveway. Write your answer in simplest form.

The slope of the driveway is _____.

6. Is the slope of the driveway acceptable? Circle yes or no.

yes no



Topic Quiz: Direct Variation

The variables x and y vary directly. Write an equation that relates x and y .

1. $x = 18$ and $y = 27$ $y = \underline{\hspace{2cm}}x$

2. $x = -4$ and $y = 48$ $y = \underline{\hspace{2cm}}x$

3. $x = -7$ and $y = -7$ $y = \underline{\hspace{2cm}}x$

4. $x = 36$ and $y = -8$ $y = \underline{\hspace{2cm}}x$

5. You pay \$20.52 for 12 gallons of gasoline. Use a direct variation model that relates the total price P to the number of gallons of gasoline g to find how much you would pay for 16.5 gallons of gasoline.

You would pay \$ _____ for 16.5 gallons of gasoline.

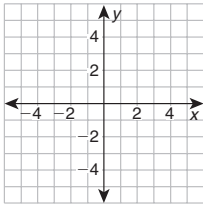
6. The length of your hair varies directly with the time. The longest hair on record belongs to Hoo Sateow of Thailand. His hair grew to 203 inches over a 68-year period. Use a direct variation model that relates the length L of his hair to the years y to find how long his hair would be after 75 years. Round your answer to the nearest inch.

His hair would be _____ inches long.

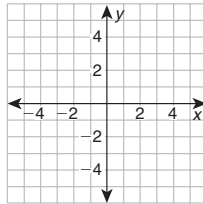
Topic Quiz: Graphing Horizontal and Vertical Lines

Graph the equation of the line.

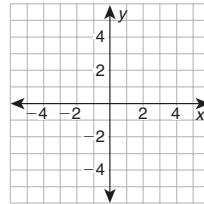
1. $y = -4$



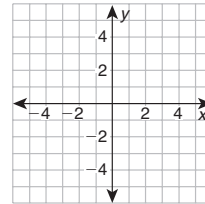
2. $x = 3$



3. $x = -5$



4. $y = 2$



The map shows the latitude and longitude lines near Santiago, Chile. Write equations for the latitude and longitude lines that describe the location of Santiago.

5. Latitude line equation: $y =$ _____

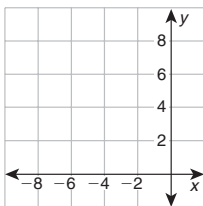
6. Longitude line equation: $x =$ _____



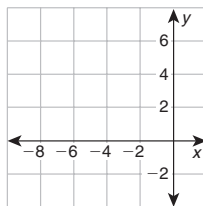
Topic Quiz: Graphing Lines Using Intercepts

Use the x -intercept and y -intercept to graph the equation of the line.

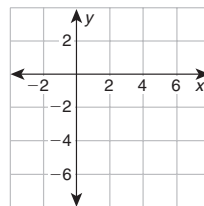
1. $6x - 8y = -48$



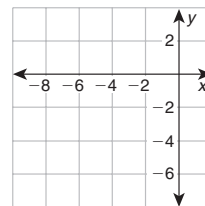
2. $-x + 2y = 6$



3. $5x - 4y = 20$



4. $3x + 6y = -24$



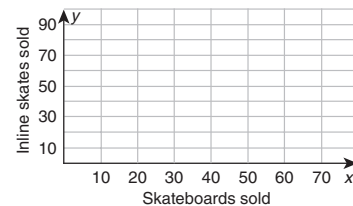
A company wants to make a profit of \$1600 selling skateboards and in-line skates. The company makes \$25 profit on each skateboard and \$20 profit on each pair of in-line skates. This is modeled by the equation $25x + 20y = 1600$, where x is the number of skateboards and y is the number of pairs of in-line skates.

5. Find the x -intercept and y -intercept of the line.

The x -intercept is _____.

The y -intercept is _____.

6. Graph the model.



Study Guide

Student Edition
Pages 346–353**Writing Linear Equations in Slope-Intercept Form**

The x -coordinate of the point where a line crosses the x -axis is called the **x -intercept**. Similarly, the y -coordinate of the point where the line crosses the y -axis is called the **y -intercept**.

Slope-Intercept Form of a Linear Equation

Given the slope m and the y -intercept b of a line, the slope-intercept form of an equation of the line is

$$y = mx + b.$$

If an equation is given in standard form $Ax + By = C$ and B is not zero, the slope of the line is $-\frac{A}{B}$ and the y -intercept is $\frac{C}{B}$. The x -intercept is $\frac{C}{A}$ where $A \neq 0$.

Example: Find the x - and y -intercepts of the graph of $5x - 2y = 10$. Then write the equation in slope-intercept form.

Since $A = 5$, $B = -2$, and $C = 10$,

$$\begin{array}{lll} \frac{C}{A} = \frac{10}{5} & \frac{C}{B} = \frac{10}{-2} & m = -\frac{A}{B} \\ = 2 & = -5 & = \frac{5}{2} \end{array}$$

Thus, the x -intercept is 2, and the y -intercept is -5 .

The equation of the line in slope-intercept form is

$$y = \frac{5}{2}x - 5.$$

Find the x - and y -intercepts of the graph of each equation.

1. $5x + 4y = 20$

2. $2x - 5y = -7$

3. $4x - 8y = 10$

4. $9x + y = -1$

Write an equation in slope-intercept form of a line with the given slope and y -intercept. Then write the equation in standard form.

5. $m = 6, b = 10$

6. $m = 4, b = 0$

7. $m = -1, b = 3$

8. $m = 2, b = -3$

Find the slope and y -intercept of the graph of each equation. Then write each equation in slope-intercept form.

9. $0.2x + 0.5y = 1.6$

10. $3x + 7y = 10$

11. $6x - y = 9$

12. $14x - 21y = 7$

Graphing 3 ways

Name _____
Pd _____

Graph each of the following equations on the graph below and label the equation. Check your answer with a graphing calculator.

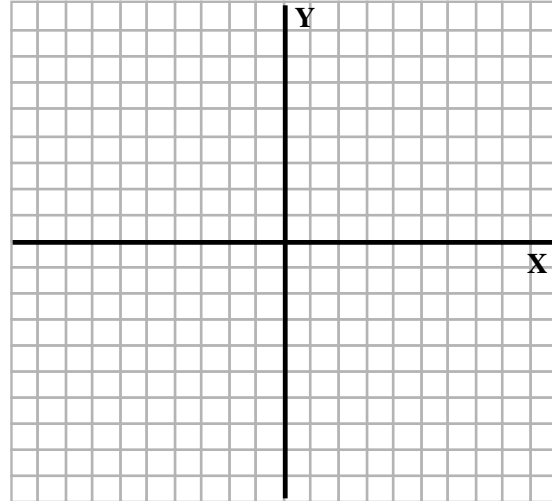
1. Graph using $y = mx + b$.

a) $y = 3x - 4$
m = b =

b) $y = x + 1$
m = b =

c) $y = -2x + 2$
m = b =

d) $y = -x - 3$
m = b =



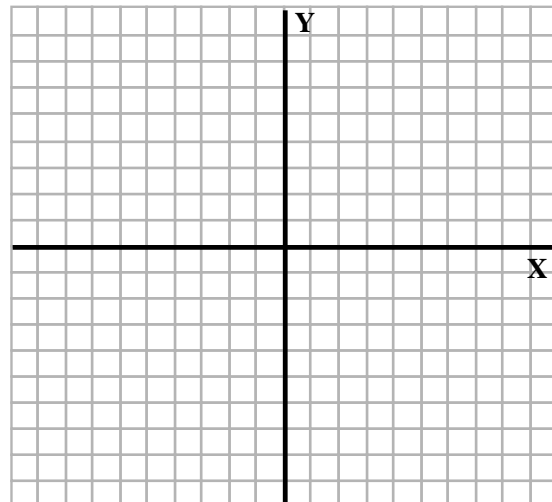
2. Graph using x- and y-intercepts.

a) $2x + 3y = 6$
x-int: y-int:

b) $x - 2y = -8$
x-int: y-int:

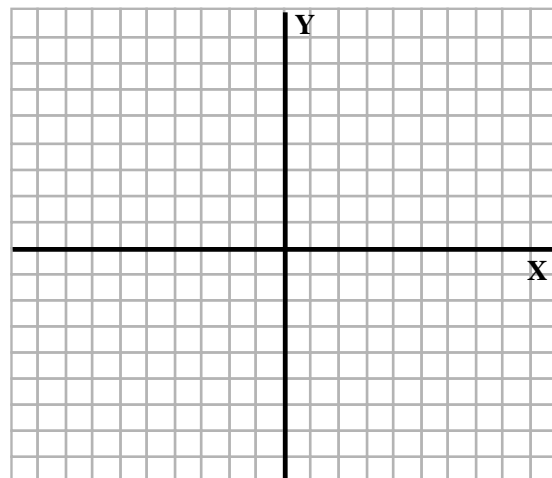
c) $-3x + y = -3$
x-int: y-int:

d) $-4x - 2y = 6$
x-int: y-int:



3. Graph using a table.

a) $y = 2x - 1$ b) $2x + y = 4$



Graphing Applications

Name _____

For each of the equations, answer the following questions.

1) The library charges 50¢ a week for a late book.

a) Write an equation for total fee.

b) Define the variables:

X = _____

Y = _____

c) Find the x- and y-intercept.

x-int:

y-int:

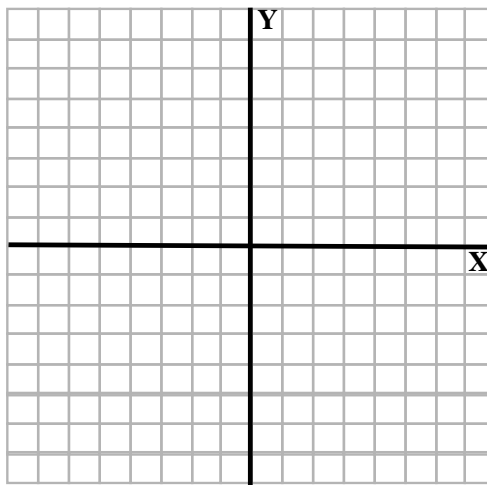
d) Complete the table and graph.

x	y

e) What is the slope of the line?

$$M = \text{SLOPE} = \frac{\text{RISE}}{\text{RUN}} =$$

f) Does the graph increase or decrease?



2) Flowers are \$1 each with a \$3 delivery charge.

a) Write an equation for total fee.

b) Define the variables:

X = _____

Y = _____

c) Find the x- and y-intercept.

x-int:

y-int:

d) Complete the table and graph.

x	y

e) What is the slope of the line?

$$M = \text{SLOPE} = \frac{\text{RISE}}{\text{RUN}} =$$

f) Does the graph increase or decrease?

