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DISCOUNT RATES

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John H. Cochrane  
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### **ABSTRACT**

Discount rate variation is the central organizing question of current asset pricing research. I survey facts, theories and applications. We thought returns were uncorrelated over time, so variation in price-dividend ratios was due to variation in expected cashflows. Now it seems all price-dividend variation corresponds to discount-rate variation. We thought that the cross-section of expected returns came from the CAPM. Now we have a zoo of new factors. I categorize discount-rate theories based on central ingredients and data sources. Discount-rate variation continues to change finance applications, including portfolio theory, accounting, cost of capital, capital structure, compensation, and macroeconomics.

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# I. Introduction

Asset prices should equal expected discounted cashflows. 40 years ago, Gene Fama (1970) argued that the expected part, “testing market efficiency,” provided the framework for organizing asset-pricing research in that era. I argue that the “discounted” part better organizes our research today.

I start with facts: *How* discount rates vary over time and across assets. I turn to theory, *why* discount rates vary. I’ll attempt a categorization based on central assumptions and links to data, analogously to Fama’s “weak” “semi-strong” and “strong” forms of efficiency. Finally, I point to some applications, which I think will be strongly influenced by our new understanding of discount rates. In each case, I have more questions than answers. This paper is more of an agenda than a summary.

An apology: In the available space I cannot even cite let alone review all the deserving literature, or trace the development of all these ideas. Even my long reference list only gives examples of relevant work.

## II. Time-series facts

### A. Simple DP regression

Discount rates vary over *time*. (“Discount rate” “risk premium” and “expected return” are all the same thing here.) Start with a very simple regression of returns on dividend yields,<sup>1</sup> shown in Table I.

Horizon $k$	$b$	$t(b)$	$R^2$	$\sigma [E_t(R^e)]$	$\frac{\sigma[E_t(R^e)]}{E(R^e)}$
1 year	3.8	(2.6)	0.09	5.46	0.76
5 years	20.6	(3.4)	0.28	29.3	0.62

Table I. Return-forecasting regressions  $R_{t \rightarrow t+k}^e = a + b \times D_t/P_t + \varepsilon_{t+k}$ . Annual data, CRSP value-weighted return less three-month Treasury return 1947-2009. The five-year regression  $t$  statistic uses the Hansen-Hodrick (1983) correction.  $\sigma [E_t(R^e)]$  stands for  $\sigma(\hat{b} \times D_t/P_t)$ .

The one-year regression forecast doesn’t seem that important. Yes, the  $t$  statistic is “significant,” but there are lots of biases and fishing. The nine percent  $R^2$  isn’t impressive.

In fact, this regression has huge economic significance. First, the coefficient estimate is large. One percentage point more dividend yield forecasts nearly *four* percentage points more return. Prices rise by an additional three percentage points.

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<sup>1</sup>Fama and French (1988).

Second, five and a half percentage point variation in expected returns is a lot. A six-percent equity premium was already a “puzzle.”<sup>2</sup> The regression implies that expected returns *vary* by at least as much as their puzzling *level*.

By contrast,  $R^2$  is a poor measure of economic significance in this context<sup>3</sup>. The economic question is “How much do expected returns vary over time?” There will always be lots of unforecastable return movement, so the variance of ex-post returns isn’t a very informative comparison for this question.

Third, the slope coefficients and  $R^2$  rise with horizon. Figure 1 plots each year’s dividend yield along with the subsequent seven years of returns. Read the dividend yield as prices upside down: prices were low in 1980 and high in 2000. The picture then captures the central fact: *High prices, relative to dividends, have reliably led to many years of poor returns. Low prices have led to high returns.*

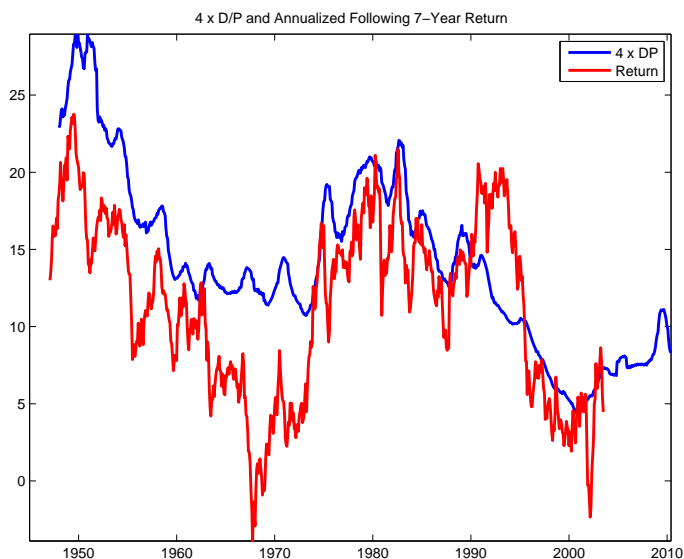


Figure 1: Dividend yield (multiplied by 4) and following annualized 7-year return. CRSP value-weighted market index.

## B. Present values, volatility, bubbles, and long-run returns

Long horizons are interesting, really, because they tie predictability to volatility, “bubbles,” and the nature of price movements. I make that connection via the Campbell-Shiller (1988) approximate present value identity,

$$dp_t \approx \sum_{j=1}^k \rho^{j-1} r_{t+j} - \sum_{j=1}^k \rho^{j-1} \Delta d_{t+j} + \rho^k dp_{t+k}, \quad (1)$$

<sup>2</sup>Mehra and Prescott (1985).

<sup>3</sup>Campbell (1991) makes this point, noting that a perpetuity would have very low short-run  $R^2$ .

where  $dp_t \equiv d_t - p_t = \log(D_t/P_t)$ ,  $r_{t+1} \equiv \log R$ , and  $\rho \approx 0.96$  is a constant of approximation. See the Appendix for details.

If we run regressions of weighted long-run returns and dividend growth on dividend yields, the present value identity (1) implies that the long-run regression coefficients must add up to one,

$$1 \approx b_r^{(k)} - b_{\Delta d}^{(k)} + \rho^k b_{dp}^{(k)}. \quad (2)$$

Just run both sides of the identity (1) on  $dp_t$ . Here,  $b_r^{(k)}$ ,  $b_{\Delta d}^{(k)}$  and  $b_{dp}^{(k)}$  denote long-run regression coefficients, i.e.

$$\sum_{j=1}^k \rho^{j-1} r_{t+j} = a_r + b_r^{(k)} dp_t + \varepsilon_{t+k}^r \quad (3)$$

$$\sum_{j=1}^k \rho^{j-1} \Delta d_{t+j} = a_d + b_d^{(k)} dp_t + \varepsilon_{t+k}^d$$

$$dp_{t+k} = a_{dp} + b_{dp}^{(k)} dp_t + \varepsilon_{t+k}^{dp}. \quad (4)$$

If we lived in an i.i.d. world, dividend yields would never vary in the first place. Expected future returns and dividend growth would never change. If dividend yields vary at all, they must forecast long-run returns, long-run dividend growth, or a “rational bubble” of ever higher prices.

The regression coefficients in (2) can be read as the fractions of dividend yield variation attributed to each source. To see this interpretation more clearly, multiply both sides of (2) by  $var(dp_t)$ , giving

$$var(dp_t) \approx cov \left[ dp_t, \sum_{j=1}^k \rho^{j-1} r_{t+j} \right] - cov \left[ dp_t, \sum_{j=1}^k \rho^{j-1} \Delta d_{t+j} \right] + \rho^k cov(dp_t, dp_{t+k}). \quad (5)$$

The empirical question is, which is it? Table II presents long-run regression coefficients.

	$b_r^{(k)}$	$b_{\Delta d}^{(k)}$	$\rho^k b_{dp}^{(k)}$
Direct regression , $k = 15$	1.01	-0.11	-0.11
Implied by VAR, $k = 15$	1.05	0.27	0.22
VAR, $k = \infty$	1.35	0.35	0.00

Table II. Long-run regression coefficients (3)-(4), for example  $\sum_{j=1}^k \rho^{j-1} r_{t+j} = a + b_r^{(k)} dp_t + \varepsilon_{t+k}^r$ . Annual data 1947-2009. “Direct” estimates are based on 15-year ex-post returns. The “VAR” estimates infer long-run coefficients from one-year coefficients, using estimates in the right-hand panel of Table III. (See the Appendix for details.)

The long-run return coefficients are all a bit larger than 1.0. The dividend-growth forecasts are small, insignificant, and positive point estimates go the “wrong” way – high prices relative to current dividends signal *low* future dividend growth. The 15-year dividend-yield forecast coefficient is also essentially zero, and has the “wrong” sign as well.

Thus the estimates say that *all price-dividend ratio volatility corresponds to variation in expected returns. None corresponds to variation in expected dividend growth, and none to “rational bubbles.”*

In the 1970s, we would have guessed exactly the opposite pattern. On the idea that returns are not predictable, we would have supposed that high prices relative to current dividends reflect expectations that dividends will rise in the future, and so forecast higher dividend growth. That pattern is completely absent. Instead, high prices relative to current dividends entirely forecast low returns.

*This* is the true meaning of return forecastability.<sup>4</sup> This is the real measure of “how big” the point estimates are – return forecastability is “just enough” to account for price volatility. This is the natural set of units with which to evaluate return forecastability. What we expected to be 0 is 1; what we expected to be 1 is 0.

Table II also reminds us that the point of the return-forecasting project is to understand *prices*, the *right hand* variable of the regression. We put return on the left because the forecast error is uncorrelated with the forecasting variable. That choice does not reflect “cause” and “effect,” nor does it imply that the point of the exercise is to understand ex-post return variation.

How you look at things matters. The long-run and short-run regressions are mathematically equivalent. Yet one transformation shows an unexpected economic significance. We will see this lesson repeated many times.

(Table II does not include standard errors, and sampling variation in long-run estimates is an important topic.<sup>5</sup> My point is the *economic* importance of estimates. One might still argue that we can’t reject the alternative views. But when point estimates are 1 and 0, arguing we should believe 0 and 1 because that view can’t be rejected is a tough sell.

The variance of dividend yields or price-dividend ratios corresponds entirely to discount-rate variation, but as much as half of the variance of price changes  $\Delta p_{t+1} = -dp_{t+1} + dp_t + \Delta d_{t+1}$  or returns  $r_{t+1} \approx -\rho dp_{t+1} + dp_t + \Delta d_{t+1}$  corresponds to current dividends  $\Delta d_{t+1}$ . This fact seems trivial but has caused a lot of confusion.

I divide by dividends for simplicity, to capture a huge literature in one example. Many other variables work about as well, including earnings and book values.)

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<sup>4</sup>Shiller (1981), Campbell and Shiller (1988), Campbell and Ammer (1993), Cochrane (1991a), (1992), (1994) and review in (2005c).

<sup>5</sup>See Cochrane (2006) and its references.

### C. A pervasive phenomenon

This pattern of predictability is pervasive across markets. For stocks, bonds, credit spreads, foreign exchange, sovereign debt and houses, a yield or valuation ratio translates one-for-one to expected excess returns, and does not forecast the cashflow or price change we may have expected. In each case our view of the facts have changed 100% since the 1970s.

- *Stocks.* Dividend yields forecast returns, not dividend growth.<sup>6</sup>
- *Treasuries.* A rising yield curve signals better one-year return for long-term bonds, not higher future interest rates. Fed fund futures signal returns, not changes in the funds rate.<sup>7</sup>
- *Bonds.* Much variation of credit spreads over time and across firms or categories signals returns not default probabilities.<sup>8</sup>
- *Foreign exchange.* International interest rate spreads signal returns, not exchange-rate depreciation.<sup>9</sup>
- *Sovereign debt.* High levels of sovereign or foreign debt signal low returns, not higher government or trade surpluses.<sup>10</sup>
- *Houses.* High price/rent ratios signal low returns, not rising rents or prices that rise forever.

Since houses are so much in the news, Figure 2 shows house prices and rents, and Table III presents a regression. High prices relative to rents mean low returns, not higher subsequent rents, or prices that rise forever. The housing regressions are almost the same as the stock market regressions. (Not everything about house and stock data is the same of course. Measured house price data are more serially correlated.)

	Houses			Stocks		
	$b$	$t$	$R^2$	$b$	$t$	$R^2$
$r_{t+1}$	0.12	(2.52)	0.15	0.13	(2.61)	0.10
$\Delta d_{t+1}$	0.03	(2.22)	0.07	0.04	(0.92)	0.02
$dp_{t+1}$	0.90	(16.2)	0.90	0.94	(23.8)	0.91

Table III. Left: Regressions of log annual housing returns  $r_{t+1}$ , log rent growth  $\Delta d_{t+1}$  and log rent/price ratio  $dp_{t+1}$  on the rent/price ratio  $dp_t$ ,  $x_{t+1} = a + b \times dp_t + \varepsilon_{t+1}$  1960-2010. Right: Regressions of log stock returns  $r_{t+1}$ , dividend growth  $\Delta d_{t+1}$  and dividend yields  $dp_{t+1}$  on dividend yields  $dp_t$ , annual CRSP value weighted return data 1947-2010.

<sup>6</sup>Fama and French (1988), (1989).

<sup>7</sup>Fama and Bliss (1987), Campbell and Shiller (1991), Piazzesi and Swanson (2008).

<sup>8</sup>Fama (1986), Duffie and Berndt (2011).

<sup>9</sup>Hansen and Hodrick (1980), Fama (1984).

<sup>10</sup>Gourinchas and Rey (2007).

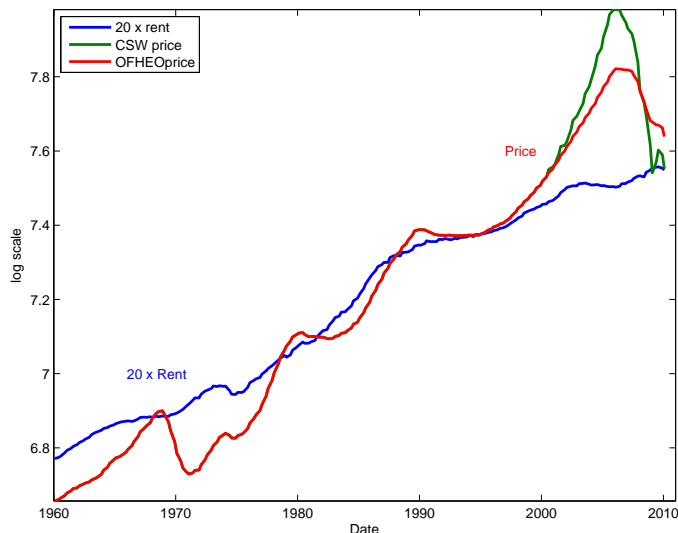


Figure 2: House prices and rents. OFHEO is the Office of Federal Housing Enterprise Oversight "purchase-only" price index. CSW are Case-Shiller-Weiss price data. Data from <http://www.lincolninst.edu/subcenters/land-values/rent-price-ratio.asp>

There is a strong common element and a strong business cycle association to all these forecasts.<sup>11</sup> Low prices and high expected returns hold in “bad times,” when consumption, output and investment are low, unemployment is high, businesses are failing, and vice versa.

These *facts* bring a good deal of structure to the debate over “bubbles” and “excess volatility.” High valuations correspond to low returns, and associated with good economic conditions. All a “price bubble” can possibly mean now is that the equivalent discount rate is “too low” relative to some theory. Regressions do not establish causality. But this equivalence channels us to a much more profitable discussion.

## D. The multivariate challenge

This empirical project has only begun. We see that one variable at a time forecasts one return at a time. We need to understand their *multivariate* counterparts, on both the left and right hand sides of the regressions.

For example the stock and bond regressions on dividend yield and yield spread ( $ys$ ) are

$$\begin{aligned} r_{t+1}^{\text{stock}} &= a_s + b_s \times dp_t + \varepsilon_{t+1}^s \\ r_{t+1}^{\text{bond}} &= a_b + c_b \times ys_t + \varepsilon_{t+1}^b. \end{aligned}$$

We have some additional predictor variables  $z_t$ , from similar univariate or at best bivariate

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<sup>11</sup>Fama and French (1989).



(hence  $[+b_s \times dp_t]$ ) explorations,

$$r_{t+1}^{\text{stock}} = a_s [+b_s \times dp_t] + d_s \times z_t + \varepsilon_{t+1}^s.$$

First, then, which of these variables are really important in a multiple regression sense? In particular, do the variables that forecast one return forecast another?

$$\begin{aligned} r_{t+1}^{\text{stock}} &= a_s + b_s \times dp_t + \boxed{c_s \times ys_t} + \boxed{d'_s z_t} + \varepsilon_{t+1}^s? \\ r_{t+1}^{\text{bond}} &= a_b + \boxed{b_b \times dp_t} + c_b \times ys_t + \boxed{d'_b z_t} + \varepsilon_{t+1}^b? \end{aligned} \quad (6)$$

(I put the variables we need to learn about in boxes.)

Second, how correlated are the right-hand terms of these regressions? What is the *factor structure of time-varying expected returns*? Expected returns  $E_t(r_{t+1}^i)$  vary across time  $t$ ; how correlated is such variation across assets and asset classes  $i$ , and how can we best express that correlation as factor structure? As an example to clarify the question, suppose we find the stock return coefficients are all double those of the bonds,

$$\begin{aligned} r_{t+1}^{\text{stock}} &= a_s + 2 \times dp_t + 4 \times ys_t + \varepsilon_{t+1}^s \\ r_{t+1}^{\text{bond}} &= a_b + 1 \times dp_t + 2 \times ys_t + \varepsilon_{t+1}^b \end{aligned}$$

We would see a one-factor model for expected returns, with stock expected returns always changing by twice bond expected returns,

$$\begin{aligned} E_t(r_{t+1}^{\text{stock}}) &= 2 \times \text{factor}_t \\ E_t(r_{t+1}^{\text{bond}}) &= 1 \times \text{factor}_t. \end{aligned} \quad (7)$$

Third, we need to relate time-varying *expected* returns to covariances with pricing factors or portfolio returns.

$$E_t(r_{t+1}^i) = \text{cov}_t(r_{t+1}^i \mathbf{f}'_{t+1}) \boldsymbol{\lambda}_t.$$

As a small step down this road, Cochrane and Piazzesi (2005) (2008) find that forward rates of *all* maturities help to forecast returns of *each* maturity – multiple regressions matter as in (6). We found that the right-hand sides are almost perfectly correlated across left-hand maturities.<sup>12</sup> A single common factor describes 99.9% of the variance of *expected* returns as in (7). Finally, we find that the spread in time-varying expected bond returns across maturities corresponds to a spread in covariances with a single “level” factor, and the market prices of risk of slope, curvature, and expected-return factors are zero.

What similar patterns hold across broad asset classes? The challenge, of course, is that there are too many right hand variables, so we can’t just go run huge multiple regressions. But these are the vital *questions*.

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<sup>12</sup>Hansen and Hodrick (1983) and Stambaugh (1988) find similar structures.

## E. Multivariate prices

I advertised much of the point of running return regressions with prices on the right hand side was to understand those prices. How will a multivariate investigation change our picture of *prices and long-run* returns?

Again, the Campbell-Shiller present value identity

$$dp_t \approx \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} \quad (8)$$

provides a useful way to think about these questions. Since this identity holds ex-post, it holds for any information set. Dividend yields are a great forecasting variable because they reveal market expectations of dividend growth and returns. However, dividend yields combine the two sources of information. A variable can help the dividend yield to forecast long-run returns if it *also* forecasts long-run dividend growth. A variable can also help to predict *one* year returns  $r_{t+1}$  without much changing long-run expected returns, if it has an offsetting effect on longer-run returns  $\{r_{t+j}\}$ . Such a variable signals a change in the *term structure of risk premia*  $\{E_t r_{t+j}\}$ .

I examine Lettau and Ludvigson’s (2001a) (2005) consumption to wealth ratio *cay* as an example to explore these questions. Table IV presents forecasting regressions

	Coefficients		t-statistics		Other statistics		
	$dp_t$	$cay_t$	$dp_t$	$cay_t$	$R^2$	$\sigma [E_t(y_{t+1})] \%$	$\frac{\sigma[E_t(y_{t+1})]}{E(y_{t+1})}$
$r_{t+1}$	0.12	0.071	(2.14)	(3.19)	0.26	8.99	0.91
$\Delta d_{t+1}$	0.024	0.025	(0.46)	(1.69)	0.05	2.80	0.12
$dp_{t+1}$	0.94	-0.047	(20.4)	(-3.05)	0.91		
$cay_{t+1}$	0.15	0.65	(0.63)	(5.95)	0.43		
$r_t^{lr} = \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}$	1.29	0.033				0.51	
$\Delta d_t^{lr} = \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}$	0.29	0.033				0.12	

Table IV. Forecasting regressions using dividend yield and consumption-wealth ratio, 1952-2009, annual data. Long-run coefficients are computed using a first-order VAR with  $dp_t$  and  $cay_t$  as state variables. Each regression includes a constant. *Cay* is rescaled so  $\sigma(cay) = 1$ .  $\sigma(dp) = 0.42$ .

*Cay* helps to forecast one-period returns. The t statistic is large, and it raises the variation of expected returns substantially. *Cay* only marginally helps to forecast dividend growth. (Lettau and Ludvigson report that it works better in quarterly data.)

Figure 3 graphs the one-year return forecast using *dp* alone, the one-year return forecast using *dp* and *cay* together, and the actual ex-post return. Adding *cay* lets us forecast business-cycle frequency “wiggles” while not much changing the “trend.”

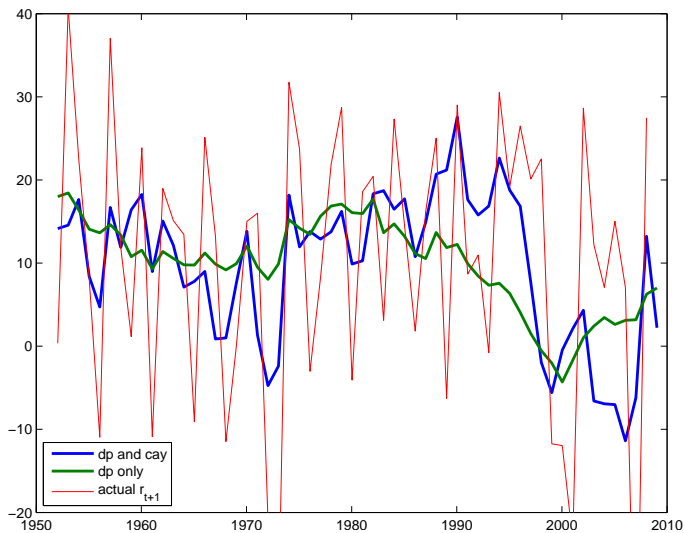


Figure 3: Forecast and actual 1 year returns. The forecasts are fitted values of regressions of returns on dividend yield and cay. Actual returns are plotted on the same date as their forecast, i.e.  $r_{t+1}$  is plotted at the same date as  $a + b \times dp_t$ .

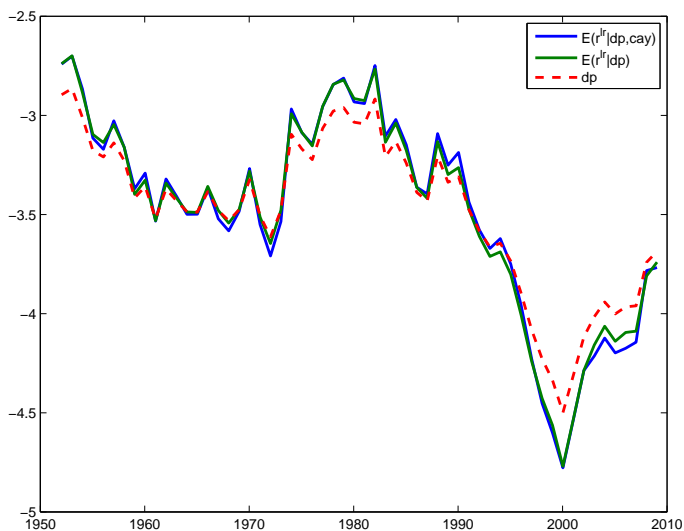


Figure 4: Dividend yield  $dp$  and forecasts of long-run returns  $\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}$ . Return forecasts are computed from a VAR including  $dp$ , and a VAR including  $dp$  and cay.

Long-run return forecasts are quite different. Figure 4 contrasts long-run return forecast with and without cay. Though cay has a dramatic effect on *one-period* return  $r_{t+1}$  forecasts in Figure 3, cay has almost no effect at all on *long-run* return  $\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}$  forecasts in Figure 4.

Figure 4 includes the actual dividend yield, to show (by (8)) how dividend yields break into long-run return vs. dividend growth forecasts. The last two rows of Table IV give the corresponding long-run regression coefficients. Essentially all price-dividend variation still

corresponds to expected-return forecasts.

How can cay forecast one-year returns so strongly, but have such a small effect on the terms of the dividend-yield present value identity? In the context of (8), cay alters the *term structure* of expected returns.

We can display this behavior with impulse-response functions. Figure 5 plots responses to a dividend growth shock, a dividend yield shock, and a cay shock. In each case, I include a contemporaneous return response to satisfy the return identity  $r_{t+1} = \Delta d_{t+1} - \rho dp_{t+1} + dp_t$ .

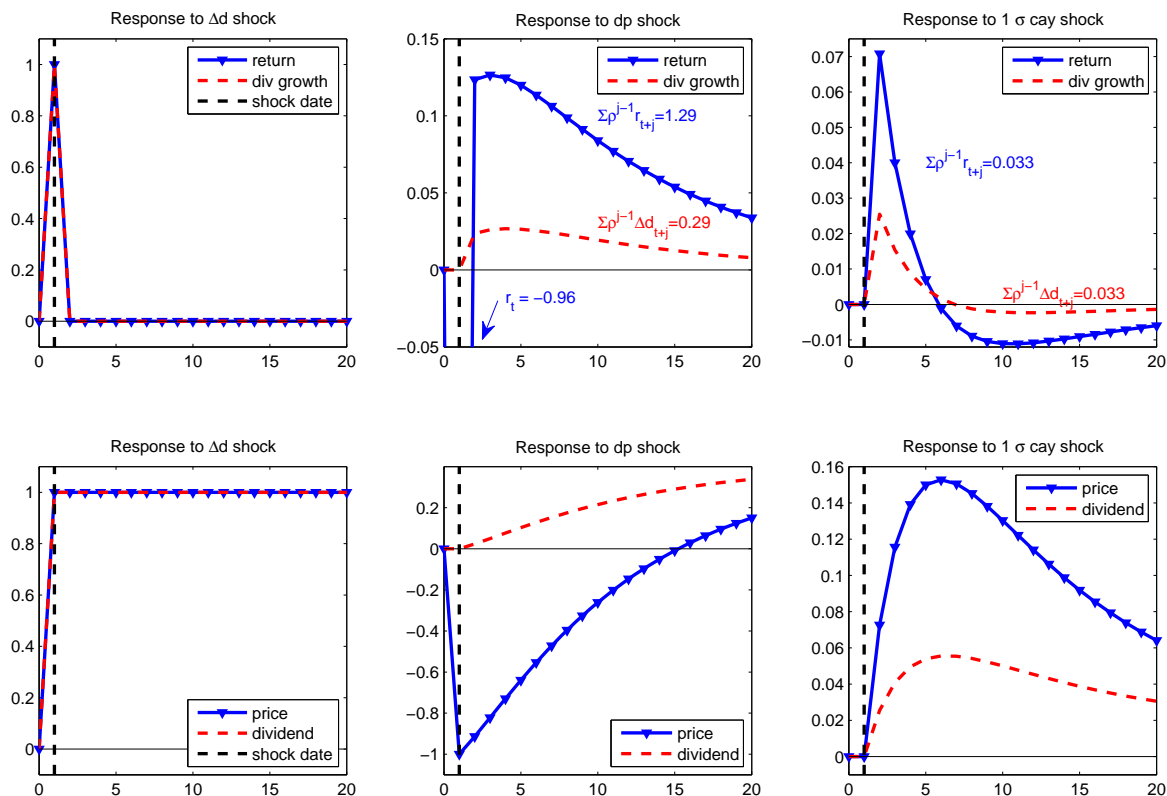


Figure 5: Response functions to dividend growth, dividend yield, and cay shocks. Calculations are based on the VAR of Table 4. Each shock changes the indicated variable without changing the others, and includes a contemporaneous return shock  $r_{t+1} = \Delta d_{t+1} - \rho dp_{t+1} + dp_t$ . The vertical dashed line indicates the period of the shock.

These plots answer the question, “what change in expectations corresponds to the given shock?” The dividend growth shock corresponds to permanently higher expected dividends with no change in expected returns. Prices jump to their new higher value and stay there. It is a pure “expected cashflow” shock. The dividend yield shock is essentially a pure discount rate shock. It shows a rise in expected returns with little change in expected dividend growth.

The cay shock in the rightmost panel of Figure 5 corresponds to a shift in expected returns from the distant future to the near future, with a small similar movement in the timing of a dividend growth forecast. It has almost no effect on long run returns or dividend

growth. We could label it *a shock to the term structure of risk premia*.<sup>13</sup>

So, cay strongly forecasts one-year returns, but has little effect on price-dividend ratio variance attribution. Does this pattern hold for other return forecasters? I don't know. In principle, consistently with the identity (8), other variables can help dividend yields to predict *both* long-run returns and long-run dividend growth. Consumption and dividends should be cointegrated, and since dividends are so much more volatile, the consumption-dividend ratio should forecast long-run dividend growth. Cyclical variables should work: at the bottom of a recession, both discount rates and expected growth rates are likely to be high, with offsetting effects on dividend yields. Reflecting both ideas, Lettau and Ludvigson (2005) report that “cdy” a cointegrating vector including dividends, does forecast long-run dividend growth in just this way. However, the lesser persistence of typical forecasters will work against much effect on price-dividend ratios. Cay's coefficient of only 0.65 on its own lag, and the fact that cay did not forecast dividend yields, are much of the story for cay's failure to affect long-run forecasts.

Even so, if additional variables help to forecast long-run dividend growth, they can only *raise* the contribution of long-run expected returns to price-dividend variation. It does not shift variance attribution from returns to dividends. A higher long-run dividend forecast must be matched by a higher long-run return forecast if it is not to affect the dividend yield.

This is a suggestive first step, not an answer. We have a smorgasbord of return forecasters to investigate, singly and jointly, including information in additional lags of returns and dividend yields (see the Appendix). The point is this: Multivariate long-run forecasts and consequent price implications can be quite different from one-period return forecasts. As we pursue the multivariate forecasting question using the large number of additional forecasting variables, we should look at pricing implications, not just focus on short-run  $R^2$  contests.

### III. The Cross Section

In the beginning, there was chaos; practitioners thought one only needed to be clever to earn high returns. Then came the CAPM. Every clever strategy to deliver high average returns ended up delivering high market betas as well. Then anomalies erupted, and there was chaos again. The “value effect” was the most prominent anomaly.

Figure 6 presents Fama-French 10 book/market-sorted portfolios. Average excess returns rise from growth (low book/market, “high price”) to value (high book/market, “low price”). This fact would not be a puzzle if the betas also rose. But the betas are about the same for all portfolios.

The absence of beta is really the heart of the value puzzle. It's perfectly natural that stocks which have fallen on hard times should have higher subsequent returns. If the market declines, these stocks should be particularly hard hit. They should have higher average returns – *and* higher betas. All puzzles are *joint* puzzles of expected returns and betas. Beta without expected return is just as much a puzzle – and profitable – as expected return

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<sup>13</sup>For impulse-responses, see Cochrane (1994). For the effect of cay, see Lettau and Ludvigson (2005).

without beta.<sup>14</sup>

Fama and French (1993), (1996) brought order once again with size and value factors. Figure 6 includes the results of multiple regressions on the market and Fama and French’s hml factor,

$$R_t^{ei} = \alpha_i + b_i \times r_{mrf}_t + h_i \times hml_t + \varepsilon_{it}.$$

The Figure shows the separate contributions of  $b_i \times E(r_{mrf})$  and  $h_i \times E(hml)$  in accounting for average returns  $E(R^{ei})$ . Higher average returns *do* line up well with larger values of the  $h_i$  regression coefficient

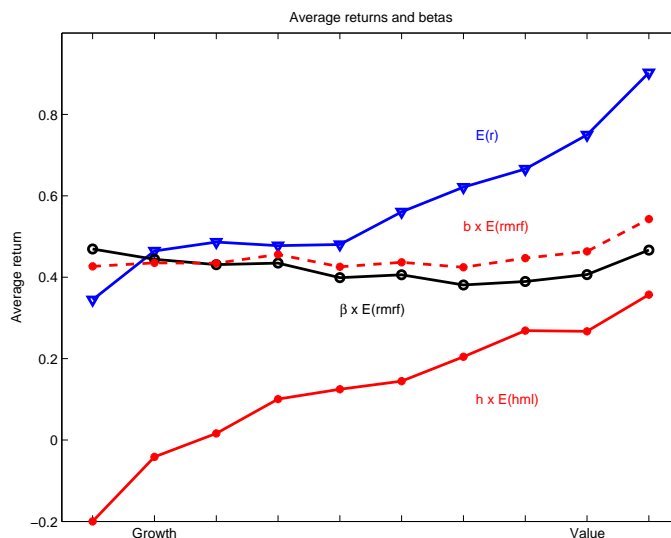


Figure 6: Average returns and betas for Fama - French 10 Book/Market sorted portfolios. Monthly data 1963-2010.

Fama and French’s factor model accomplishes a very useful data reduction. Theories now only have to explain the hml portfolio premium, not the expected returns of individual assets.<sup>15</sup> This lesson has yet to sink in to a lot of empirical work, which still uses the 25 Fama French portfolios to test deeper models.

Covariance is in a sense Fama and French’s central result: if the value firms decline, they all decline together. It’s a sensible result: Where there is mean, there must be comovement, so that Sharpe ratios do not rise without limit in well-diversified value portfolios.<sup>16</sup> But theories now must also explain this *common movement* among value stocks. It is not enough to simply generate temporary price movements, a “fad” that produces high or low prices, and then fades away rewarding contrarians. You need all the low-price securities to subsequently *rise and fall together* in the following month.

Finally, Fama and French found that *other* sorting variables, such as firm sales growth, did not each require a new factor. The three-factor model took the place of the CAPM for

<sup>14</sup>Frazzini and Pedersen (2010).

<sup>15</sup>Daniel and Titman (2006), Lewellen, Nagel, and Shanken (2010).

<sup>16</sup>Ross (1976), (1978).

routine risk-adjustment in empirical work.

Order to chaos, yes, but once again, the world changed 100%. None of the cross-section of average stock returns corresponds to market betas. 100% corresponds to *hml* (and size) betas.

Alas, the world is once again descending into chaos. Expected return strategies have emerged that *do not* correspond to market, value, and size betas. These include, among many others, momentum<sup>17</sup>, accruals, equity issues and other accounting-related sorts,<sup>18</sup> beta arbitrage, credit risk, bond and equity market-timing strategies, foreign exchange carry trade, put option writing, and various forms of “liquidity provision.”

## A. The multidimensional challenge

We’re going to have to repeat Fama and French’s anomaly digestion, but with many more dimensions. We have a lot of questions to answer:

First, which characteristics really provide *independent* information about average returns? Which are subsumed by others?

Second, does each new anomaly variable also correspond to a new factor formed on those same anomalies? Momentum returns correspond to regression coefficients on a winner-loser momentum “factor.” Carry-trade profits correspond to a carry-trade factor.<sup>19</sup> Do accruals return strategies correspond to an accruals factor? We should routinely look.

Third, how many of these new factors are really important? Can we again account for  $N$  independent dimensions of expected returns with  $K < N$  factor exposures? Can we account for accruals return strategies by betas on some other factor, as with sales growth?

Now, factor structure is neither necessary nor sufficient for factor pricing. ICAPM and consumption-CAPM models do not predict or require that the multiple pricing factors will correspond to big common movements in asset returns. And big common movements, such as industry portfolios, need not correspond to any risk premium. There always is an equivalent single-factor pricing representation of any multifactor model, the mean-variance efficient portfolio. Still, the world would be much simpler if betas on only a few factors, important in the covariance matrix of returns, accounted for a larger number of mean characteristics.

Fourth, eventually, we have to connect all this back to the central question of finance, why do *prices* move?

## B. Asset pricing as a function of characteristics/unification

To address these questions in the zoo of new variables, I suspect we will have to use different *methods*. Following Fama and French, a standard methodology has developed: sort assets into portfolios based on a characteristic, look at the portfolio means, especially the 1-10 portfolio

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<sup>17</sup>Jegadeesh and Titman (1993).

<sup>18</sup>See Fama and French (2010).

<sup>19</sup>Lustig, Roussanov, and Verdelhan (2010a).

alpha, information ratio, and t-statistic, and then see if the spread in means corresponds to a spread of portfolio betas against some factor. But we can't do this with 27 variables.

Portfolio sorts are really the same thing as nonparametric cross sectional regressions, using rather inefficient non-overlapping histogram weights. Figure 7 illustrates the point. For one variable, portfolio sorts and regressions both work. But we can't chop portfolios 27

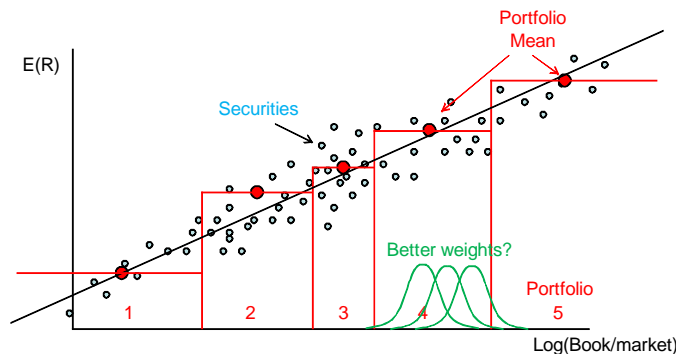


Figure 7: Portfolio means vs. cross-sectional regressions.

ways, so I think we will end up running multivariate regressions.<sup>20</sup> (The Appendix gives a simple cross-sectional regression to illustrate.)

Having said that, you see that “time series” forecasting regressions, “cross-sectional” regressions and portfolio mean returns are really the same thing. All we are ever really doing is understanding a big panel-data forecasting regression

$$R_{t+1}^{ei} = a + b' C_{it} + \varepsilon_{t+1}^i.$$

We end up describe expected returns as a function of characteristics,

$$E(R_{t+1}^e | C_t)$$

where  $C_t$  denotes some big vector of characteristics,

$$C_t = [\text{size, b/m, momentum, accruals, d/p, credit spread...}].$$

Is value a “time-series” strategy that moves in and out of a stock as that stock’s book/market changes, or a “cross-sectional” strategy that moves from one stock to another following the same signal? Well, both, obviously. They are the same thing. This is the managed-portfolio theorem:<sup>21</sup> an instrument  $z_t$  in a time series test  $0 = E[(m_{t+1} R_{t+1}^e) z_t]$  is the same as an unconditional test of a managed portfolio  $0 = E[m_{t+1} (R_{t+1}^e z_t)]$ .

Once we understand expected returns, we have to see if expected returns line up with covariances of returns with factors. Sorted-portfolio betas are a nonparametric estimate of

<sup>20</sup>Fama and French (2010) already run such regressions, despite evident reservations over functional forms.

<sup>21</sup>Cochrane (2005c).



this covariance function

$$\text{cov}_t(R_{t+1}^{ei}, f_{t+1}) = g(C_{it}).$$

Parametric approaches are natural here as well, to address a multidimensional world. For example, we can run regressions

$$[R_{t+1}^{ei} - E(R_{t+1}^{ei}|C_{it})] f_{t+1} = c + d' C_{it} + \varepsilon_{t+1}^i \Rightarrow g(C) = c + d' C.$$

(The errors may not be normal, but they are mean-zero and uncorrelated with the right hand variable.) We want to see if the mean return *function* lines up with the covariance *function*.

$$E(R^e|C) = g(C) \times \lambda?$$

Underlying everything we're doing is an assumption that expected returns, variances and covariances are stable functions of characteristics, not (say) security name. That's why we use portfolios in the first place. That is an incredibly useful assumption—or, fact about the world. Without it, it's hard to tell if there is any spread in average returns at all. It means however, that asset pricing really is about the equality of two *functions*; the function relating means to characteristics and the function relating covariance to characteristics.

Looking at portfolio average returns rather than forecasting regressions was really the key to understanding *economic* importance of many effects, as was looking at long-horizon returns. For example, serial correlation with an  $R^2$  of 0.01 doesn't seem that impressive. Yet is enough to account for momentum: The last year's winners went up 100%, so an annual autocorrelation of 0.1, meaning 0.01  $R^2$ , generates a 10% annual portfolio mean return. (An even smaller amount of time-series cross-correlation works as well.) As another classic example, Lustig, Roussanov, and Verdelhan (2010a) translated carry-trade return-forecasting regressions to means of portfolios formed on the basis of currency interest differentials. This step led them to look for and find a factor structure of country returns that depends on interest differentials, a “high minus low” factor. This step followed Fama and French (1996) exactly, but no one thought to look for it in 30 years of running country-by-country time-series forecasting regressions

But the equivalence of portfolio sorts and regressions goes both ways. We can still calculate these measures of *economic* significance if we *estimate* panel-data regressions for means and covariances. From the spread of lagged returns, we can calculate the momentum portfolio implications directly. The 1-10 portfolio information ratio is the same thing as the Sharpe ratio of the underlying factor, or t-statistic of the cross-sectional regression coefficient. (See the Appendix.) We could study the covariance structure of panel-data regression residuals as a function of the same characteristics (interest rate spread, for example) rather than actually form portfolios,

$$\text{cov}_t(R_{t+1}^i, R_{t+1}^j) = h(C_{it}, C_{jt}).$$

Running multiple panel-data forecasting regressions is full of pitfalls of course. One can end up focusing on tiny firms, or outliers. One can get the functional form wrong. Uniting time-series and cross section will yield new insights as well. For example, variation in book/market over time for a given portfolio has a larger effect on returns than variation

in book/market across the Fama-French portfolios, and a recent change in book/market also seems to forecast returns. (See the Appendix.) I didn't say it will be easy! But we must address the factor zoo, and I don't see how to do it by a high-dimensional portfolio sort.

## C. Prices

Then, we have to answer the central question, what is the source of *price* variation?

When did our field stop being “asset pricing” and become “asset expected returning?” Why are betas exogenous?<sup>22</sup> A lot of price variation comes from discount factor news. What sense does it make to “explain” expected returns by the covariation of expected return shocks with market expected return shocks? Market/book ratios should be our *left-hand* variable, the thing we're trying to *explain*, not a sorting characteristic for expected returns.

Focusing on expected returns and betas rather than prices and discounted cashflows makes sense in a two-period or i.i.d. world, since in that case betas are all cashflow betas. It makes much less sense in a time-varying discount rate world.

A long-run, price-and-payoff perspective may also end up being simpler. As a hint of the possibility, solve the Campbell-Shiller identity for long-run returns,

$$\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - dp_t.$$

So, *long-run* return uncertainty all comes from cashflow uncertainty. Long-run betas are all cashflow betas. The long run looks just like a simple one-period model with a liquidating dividend.

$$\begin{aligned} R_{t+1} &= \frac{D_{t+1}}{P_t} = \left( \frac{D_{t+1}}{D_t} \right) / \left( \frac{P_t}{D_t} \right) \\ r_{t+1} &= \Delta d_{t+1} - dp_t. \end{aligned}$$

A natural start is to forecast long-run returns and form price decompositions in the cross section, just as in the time-series; estimate forecasts such as

$$\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}^i = a + b' C_{it} + \varepsilon^i,$$

and then understand valuations with present value models as before.<sup>23</sup> (The Appendix includes two simple examples.)

In a formal sense, of course, it doesn't matter whether you look at returns or prices.  $1 = E_t(m_{t+1}R_{t+1})$  and  $P_t = E_t \sum_{j=1}^{\infty} m_{t,t+j} D_{t+j}$  each imply the other. But, as I found with return forecasts, our economic understanding may be a lot different in a price, long-run

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<sup>22</sup>Campbell and Mei (1993).

<sup>23</sup>Vuolteenaho (2002) and Cohen Polk and Vuolteenaho (2003) are a start, with too-few followers.

view than focusing on short-run returns. What constitutes a “big” or “small” error is also different if we look at prices vs. returns. At a 2% dividend yield,  $D/P = (r - g)$  implies that an “insignificant” 10bp/month expected return error is a “large” 12% price error, if it is permanent. For example, since momentum amounts to a very small time-series correlation and lasts less than a year, I suspect it has little effect on long-run expected returns and hence the level of stock prices. Long-lasting characteristics are likely to be more important. Conversely, small transient price errors can have a large impact on return measures. A tiny i.i.d. price error induces the appearance of mean reversion where there is none. Common procedures amount to taking many differences of prices, which amplify the error/signal ratio. For example, the forward spread  $f_t^{(n)} - y_t^{(1)} = p_t^{(n-1)} - p_t^{(n)} + p_t^{(1)}$  is already a triple-difference of price data.

## IV. Theories

Having viewed a bit of *how* discount rates vary, Let’s think now about *why* discount rates vary so much.

### A. A categorization, by ingredients and connection to data

It’s useful to classify theories by their main ingredient, and by which data they use to measure discount rates. My goal is to produce for discount rates something like Fama’s (1970) classification of informational possibilities.

1. Theories based on fundamental investors, with few frictions.
  - (a) Macroeconomics – tie to macro data.
    - i. Consumption, Aggregate risks.
    - ii. Risk sharing/background risks (Hedging outside income)
    - iii. Investment and production.
    - iv. General equilibrium, including macroeconomics
  - (b) Behavioral; Irrational expectations. Tie to price data. Other data?
  - (c) Finance. Expected return-beta, return-based factors, affine term structure models. Tie to price data, returns explained by covariances.
2. Theories based on frictions.
  - (a) Segmented markets – different investors in different markets; limited risk bearing of active traders.
  - (b) Intermediated markets. Prices set by leveraged intermediaries; funding difficulties.
  - (c) Liquidity.

- i. Idiosyncratic – easy to sell the asset.
- ii. Systemic – times of market illiquidity.
- iii. Information trading – value of securities in facilitating information trading.

“Macro” theories tie discount rates to macroeconomic data, such as consumption or investment, based on first-order conditions for the ultimate investors or producers.

For example, the canonical consumption-based model with power utility relates discount rates to consumption growth,

$$m_{t+1} = \beta \frac{u_c(t+1)}{u_c(t)} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma};$$

$$E_t(R_{t+1}^{ei}) = R^f \text{cov}(R_{t+1}^{ei}, m_{t+1}) \approx \gamma \text{cov}(R_{t+1}^{ei}, \Delta c_{t+1}).$$

High expected returns (low prices) correspond to securities that pay off poorly when consumption is low. This model combines frictionless markets, rational expectations and utility maximization, and risk sharing so that only aggregate risks matter for pricing. It evidently ties discount rate variation to macroeconomic data.

A vast literature has generalized this framework, including (among others)<sup>24</sup> 1) Nonseparability across goods – durable and nondurable<sup>25</sup>; traded and nontraded; 2) Nonseparability over time, such as habit persistence,<sup>26</sup> 3) Recursive utility and long-run risks<sup>27</sup> 4) Rare disasters, which alter measurements of means and covariances in “short” samples<sup>28</sup>.

A related category of theories adds incomplete markets or frictions preventing some consumers from participating. Though they include “frictions,” I categorize such models here because asset prices are still tied to some fundamental consumer or investor’s economic outcomes. For example, if non-stockholders do not participate, we still tie asset prices to the consumption decisions of stockholders who do participate.<sup>29</sup>

With incomplete markets, consumers still share risks as much as possible. The complete-market theorem that “all risks are shared,” marginal utility is equated across people  $i$  and  $j$ ,  $m_{t+1}^i = m_{t+1}^j$ , becomes “all risks are shared as much as possible.” The projection of marginal utility on asset payoffs  $X$  is the same across people  $\text{proj}(m_{t+1}^i | X) = \text{proj}(m_{t+1}^j | X) \equiv x^*$ . We can still aggregate *marginal utility* rather than aggregate *consumption* before constructing marginal utility. A discount factor  $m_{t+1} = E_{t+1}(m_{t+1}^i) = \int f(i) m_{t+1}^i di$  prices assets, where  $E_{t+1}$  takes averages across people conditional on aggregates. For example with power utility we have

$$m_{t+1} = \beta E_{t+1} \left[ \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\gamma} \right].$$

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<sup>24</sup>See Cochrane (2007a), Ludvigson (2011) for recent reviews.

<sup>25</sup>Recently, Yogo (2006),

<sup>26</sup>Campbell and Cochrane (1999) for example.

<sup>27</sup>Epstein and Zin (1989), Bansal and Yaron (2004), Hansen, Heaton and Li (2008).

<sup>28</sup>Rietz (1988), Barro (2006).

<sup>29</sup>For example, Mankiw and Zeldes (1991).

The fact that we aggregate nonlinearly across people means that variation in the *distribution* of consumption matter to asset prices. Times in which there is more *cross-sectional risk* will be high-discount factor events.<sup>30</sup>

Outside or nontradeable risks are a related idea. If a mass of investors has jobs or businesses that will be hurt especially hard by a recession, they avoid stocks that fall more than average in a recession.<sup>31</sup> Average stock returns then reflect the tendency to fall more in a recession, in addition to market risk exposure. Though in principle, given a utility function, one could see such risks in consumption data, individual consumption data will always be so poorly measured that tying asset prices to more fundamental sources of risk may be more productive.

If we ask the “representative investor” in December 2008 why he or she is ignoring the high premiums offered by stocks and especially fixed income, the answer might well be “that’s nice, but I’m about to lose my job, and my business might go under. I can’t take any more risks right now, especially in securities that will lose value or become hard to sell if the recession gets worse.” These extensions of the consumption-based model all formalize this sensible intuition – as opposed to the idea that these consumers have wrong expectations, or that they would have been happy to take risks but intermediaries were making all asset pricing decisions for them.

Investment-based models link asset prices to firms investment decisions, and general equilibrium models include production technologies and a specification of the source of shocks. This is clearly the ambitious goal towards which we are all aiming. The latter tries to answer the vexing questions, where do betas come from? What makes a company a “growth” or “value” company in the first place?<sup>32</sup>

I think “behavioral” asset pricing’s central idea is that people’s expectations are wrong.<sup>33</sup> It takes lessons from psychology to find systematic patterns to the “wrong” expectations. There are some frictions in many behavioral models, but these are largely secondary and defensive, to keep risk-neutral “rational arbitrageurs” from coming in and undoing the behavioral biases. Often, simple risk aversion by the rational arbitrageurs would serve as well. Behavioral models, like “rational” models, tie asset prices to the fundamental investor’s willingness, ability or (in this case) perception of risk.

Behavioral theories are also discount-rate theories. A distorted probability with riskfree discounting is mathematically equivalent to a different discount rate.

$$p = \sum_s \pi_s m_s x_s = \frac{1}{R^f} \sum_s \pi_s^* x_s$$

where

$$\pi_s^* \equiv \pi_s m_s R^f = \pi_s m_s / \sum_{s'} \pi_{s'} m_{s'},$$

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<sup>30</sup>Constantinides and Duffie (1996) .

<sup>31</sup>Fama and French (1996), Heaton and Lucas (2000).

<sup>32</sup>A few good examples: Gomes, Kogan and Zhang (2003), Gala (2010), Gourio (2007)..

<sup>33</sup>See Barberis and Thaler (2003) and Fama (1998) for reviews.

$s$  denote states of nature,  $\pi_s$  are true probabilities,  $m_s$  is a stochastic discount factor or marginal utility growth, and  $\pi_s^*$  are distorted probabilities.

It is pointless to argue “rational” vs. “behavioral” in the abstract. There is a discount rate and equivalent distorted probability that can rationalize any (arbitrage-free) data. “The market went up, risk aversion must have declined” is as vacuous as “the market went up, sentiment must have increased.” Any model only gets its bite by restricting discount rates or distorted expectations, ideally tying them to other data. The only thing worth arguing about is how persuasive those ties are in a given model and dataset, and whether it would have been easy for the theory to “predict” the opposite sign if the facts had come out that way.<sup>34</sup> And the line between recent “exotic preferences” and “behavioral finance” is so blurred<sup>35</sup> it describes academic politics better than anything substantive.

A good question for any theory is what *data* it uses to tie down discount factors. By and large, behavioral research so far largely ties prices to other prices; it looks for price patterns that are hard to understand with other models, such as “overreaction” or “underreaction” to news. Some behavioral research uses survey evidence, and survey reports of people’s expectations are certainly unsettling. However, surveys are sensitive to language and interpretation. It doesn’t take long in teaching MBAs to realize that the colloquial meanings of “expect” and “risk” are entirely different from conditional mean and variance. If people report the *risk-neutral* expectation, then expectations are in fact completely rational. An “optimistic” cash-flow growth forecast is the same as a “rational” forecast, already discounted at a low rate, and leads to the correct decision, invest more. And the risk-neutral expectation, i.e. the expectation weighted by marginal utility, is the right sufficient statistic for many decisions. Treat painful things as if they were more probable than they are in fact.

Of course, “rational” theories beyond the simple consumption-based model struggle as well. Changing expectations of consumption 10 years from now (long run risks) or changing probabilities of a big crash are hard to tell from changing “sentiment.” At least, one can aim for more predictions than assumptions, tying together several phenomena with a parsimonious specification.

“Finance” theories tie discount rates to broad return-based factors. That’s great for data reduction and practical applications. The more practical and “relative-pricing” the application the more “factors” we accept on the right hand side. For example, in evaluating a portfolio manager, hedging a portfolio, or finding the cost of capital for a given investment we routinely include momentum as a “factor” even though we don’t have a deep theory of why the momentum factor is priced.

However, we still need the deeper theories for deeper “explanation.” Even if the CAPM explained individual mean returns from their betas and the market premium, we would still have the equity premium puzzle – why is the market premium so large? (And why are betas what they are?) Conversely, even if we had the perfect utility function and a perfect consumption-based model, the fact that consumption data is poorly measured means we

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<sup>34</sup>Fama (1998).

<sup>35</sup>For example, which of Epstein and Zin (1989), Barberis, Santos, and Huang (2001), Hansen and Sargent (2005), Laibson (1997), Hansen, Heaton and Li (2008), and Campbell and Cochrane (1999) is really “rational” and which is really “behavioral?”

would still use finance models for most practical applications.<sup>36</sup>

A nice division of labor results. Empirical asset pricing in the Fama and French (1996) tradition boils down the alarming set of anomalies to a small set of large-scale systematic risks that generate rewards. “Macro” “behavioral” or other “deep” theories can then focus on why the factors are priced.

Models that emphasize frictions are becoming more and more popular, especially since the financial crisis. At heart, these models basically maintain the “rational” assumption. Admittedly, there are often “irrational” agents in such models. However, these agents are usually just convenient shortcuts rather than central to the vision. A model may want some large volume of trade,<sup>37</sup> or to include some “noise traders,” while focusing clearly on the delegated management problem, or the problem of leveraged intermediaries. For such a purpose, it’s easy to simply allude to a slightly irrational class of trader rather than spell out their motives from first principles. However those assumptions are not motivated by deep reading of psychology or lab experiments. The focus is on the frictions and behavior of intermediaries rather than the risk-bearing ability of ultimate investors, or their psychological misperceptions.

I think it’s useful to distinguish three categories of frictions: 1) Segmented markets and 2) Intermediated markets or “institutional finance”<sup>38</sup> and 3) Liquidity.

I distinguish “segmented markets” from “intermediated markets,” as shown in Figure 8. Segmentation is really about limited risk sharing among the pool of investors active in a particular market.<sup>39</sup> They can generate “downward sloping demands,” and average returns that depend on a “local” factor, little and poorly-linked CAPMs.<sup>40</sup> Given the factor zoo, that’s an attractive idea.

“Intermediated markets” or “institutional finance” refers to a different, vertical rather than horizontal, separation of investor from payoff. Investors use delegated managers. Then, agency problems in delegated management spill over into asset prices. For example, suppose investors split their investments to the managers into “equity” and “debt” claims. When losses appear, the managers stave off bankruptcy by trying to sell risky assets. But since all the managers are doing the same thing, prices fall and discount rates rise. Colorful terms like “fire sale,” “liquidity spirals” describe this process.<sup>41</sup>

Of course, we have to document and explain segmentation and intermediation. As suggested by the dashed arrows in Figure 8, there are strong incentives to undo any price anomaly induced by segmentation or intermediation. Models with these frictions often abstract from deep-pockets unintermediated investors – the sovereign wealth funds, pension funds, endowments, family offices, and Warren Buffets, or institutional innovation to bridge

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<sup>36</sup>Campbell and Cochrane (2000) give a quantitative example.

<sup>37</sup>Scheinkman and Xiong (2003).

<sup>38</sup>Markus Brunnermeier coined this useful term.

<sup>39</sup>Some important examples: Brunnermeier and Pedersen (2009), Brunnermeier (2009), Gabaix, Krishnamurthy and Vigneron (2007), Duffie and Strulovici (2011), Garleanu and Pedersen (2009), He and Krishnamurthy (2010), Krishnamurthy (2008), Froot and O’Connell (2008), Vayanos and Vila (2011).

<sup>40</sup>For example, Gabaix, Krishnamurthy and Vigneron (2007).

<sup>41</sup>Brunnermeier (2009), Brunnermeier and Pedersen (2009) for example.

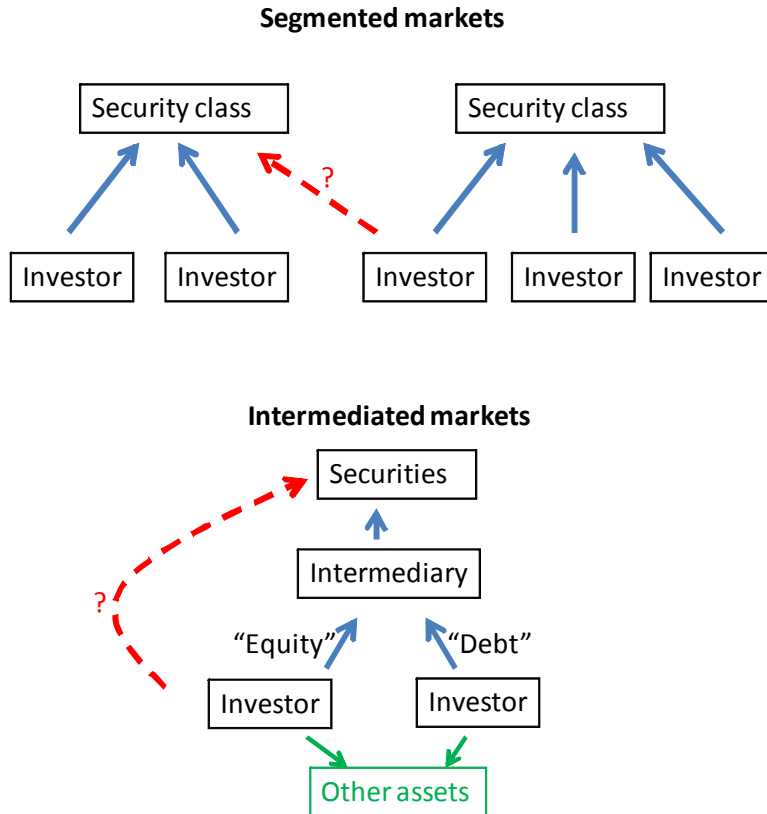


Figure 8: Segmented markets vs. intermediated markets.

the friction. Your “fire sale” is their “buying opportunity.” Thus, I think a little more attention to the reasons for segmentation and intermediation will help to define when and how long these models apply. For example, transactions costs, attention costs, or limited expertise suggest markets can be segmented until the “deep pockets” arrive, but that they do arrive eventually. So if this is why markets are segmented, that segmentation will be more important in the short run, after unusual events, or in more obscure markets. If I try to sell a truckload of tomatoes at 2 am in front of the Booth school, I am not likely to get full price. But if I do it every night, tomato buyers will start to show up. In the flash crash, it took about ten minutes for buyers to show up, which is either remarkably long or remarkably short, depending on your point of view.

A crucial question is, as always, what *data* will this class of theories use to measure discount rates? Arguing over puzzling patterns of prices is weak (the rational-behavioral debate has been doing that for 40 years). Ideally, one should tie price or discount-rate variation to central items in the models, such as the balance sheets of leveraged intermediaries, data on who is actually active in segmented markets, and so forth. Yet such data is hard to find.<sup>42</sup>

<sup>42</sup>Mitchell, Pedersen and Pulvino (2007) is a good example. They document who was active in convertible arbitrage markets through two episodes in which the specialized hedge funds left the market and it took months for the multi-strategy funds to move in.



We have long recognized that some assets have higher or lower discount rates in compensation for greater or lesser liquidity.<sup>43</sup> We have also long struggled to define and measure liquidity. There are (at least) three kinds of stories for liquidity that are worth distinguishing. Liquidity can refer to the ease of buying and selling an individual security. Illiquidity can also be systemic: assets will face a higher discount rate if their prices fall when the market as a whole is illiquid, whether or not the asset becomes more or less illiquid. Finally, assets can have lower discount rates if they facilitate information trading, as money facilitates physical trading.

I think of “liquidity” as different from “segmentation” in that segmentation is about limited risk-bearing ability, while liquidity is about trading. Liquidity is a feature of assets, not the risks to which they are claims. Many theories of liquidity emphasize asymmetric information, not limited risk-bearing ability – assets become illiquid when traders suspect that anyone buying or selling knows something, not because traders are holding too much of a well-understood risk. Understanding liquidity requires us to unravel the puzzle of why people and institutions trade so vastly more than they do in our models.

All of these facts and theories are really about discount rates, expected returns, risk bearing, risk sharing and risk premiums. None are fundamentally about slow or imperfect diffusion of cash-flow information, i.e. informational “inefficiency.” Informational efficiency isn’t wrong or disproved. Efficiency basically won, and we moved on. When we see information, it is quickly incorporated in asset prices. There is a lot of asset-price movement not related to visible information, but Hayek (1945) told us that would happen, and we learned that a lot of such price variation corresponds to expected returns. Little of the (large) gulf between the above models is really about information. Seeing the facts and the models as categories of discount-rate variation seems much more descriptive of most (not all) theory and empirical work.

Informational efficiency is much easier for markets and models to obtain than wide risk sharing or desegmentation, which is perhaps why it holds more broadly. A market can become efficient with only one informed trader, who doesn’t need to actually buy anything or take any risk. He should run in to a wall of indexers, and end up just bidding up the asset he knows is underpriced.<sup>44</sup> Though price discovery seems in reality to come with a lot of trading, it doesn’t have to do so. Risk sharing needs *everyone* to change their portfolios and bear a risk in order to eliminate segmentation. For example, if the small firm effect came from segmentation, the passively-managed small stock fund or total market fund should have ended it – but it also took the invention and marketing of such funds to end it. The actions of small numbers of arbitrageurs could not do so.

## B. Recent performance

This is not the place for a deep review of theory and empirical work supporting or confronting theories. Instead, I think it will be more productive to think informally about how these

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<sup>43</sup>Acharya and Pedersen (2005), Amihud, Mendelson and Pedersen (2005), Cochrane (2005b), Pastor and Stambaugh (2003), Vayanos and Wang (2011).

<sup>44</sup>Milgrom and Stokey (1982).

classes of models might be able to handle big recent events.

## C. Consumption

I still think the macro-finance approach is promising. Figure 9 presents the market price-dividend ratio, and aggregate consumption relative to a slow-moving “habit.” The habit is basically just a long moving average of lagged consumption, so the surplus consumption ratio line is basically detrended consumption.<sup>45</sup>

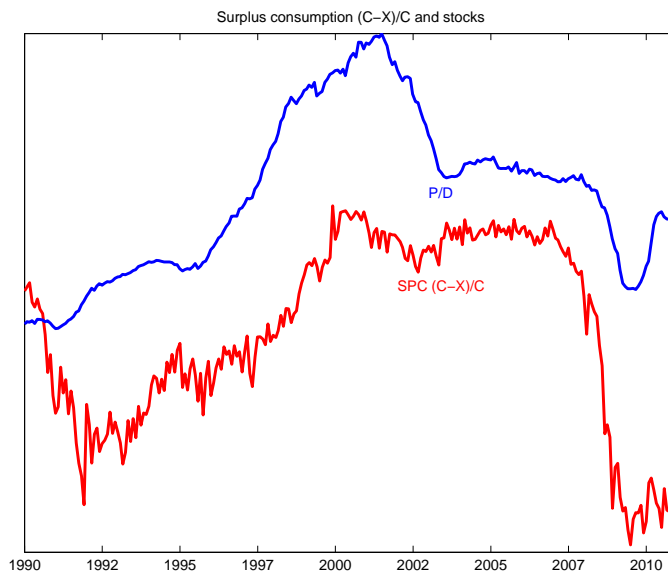


Figure 9: Surplus consumption ratio and price/dividend ratio. Surplus consumption is formed from real nondurable + services consumption using the Campbell and Cochrane (1999) specification and parameters. Price/dividend ratio is from the CRSP NYSE Value-Weighted portfolio.

As you can see, consumption and stock market prices *did* both collapse in 2008. Many high average-return securities and strategies (stocks, mortgage-backed securities, low-grade bonds, momentum, currency carry) collapsed more than low-average-return counterparts. The basic consumption-model logic – securities must pay higher returns, or fetch lower prices, if their values fall more when consumption falls – isn’t drastically wrong.

The habit model captures the idea that people become more risk averse as consumption falls in recessions. As consumption nears habit, people are less willing to take risks that involve the same proportionate risk to consumption. Discount rates rise, and prices fall. Lots of models have similar mechanisms, especially models with leverage.<sup>46</sup> In the habit model, the price-dividend ratio is a nearly log-linear function of the surplus consumption

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<sup>45</sup>Campbell and Cochrane (1999)

<sup>46</sup>For example, Longstaff (2008).

ratio. The fit isn't perfect, but the general pattern is remarkably good, given the hue and cry about how the crisis invalidates all traditional finance.

## D. Investment

The Q theory of investment is the off-the-shelf analogue to the simple power-utility model from the producer point of view. It predicts that investment should be low when valuations (market to book) are low, and vice versa,

$$1 + \alpha \frac{i_t}{k_t} = \frac{\text{market value}_t}{\text{book value}_t} = Q_t, \quad (9)$$

where  $i_t$  = investment and  $k_t$  = capital.

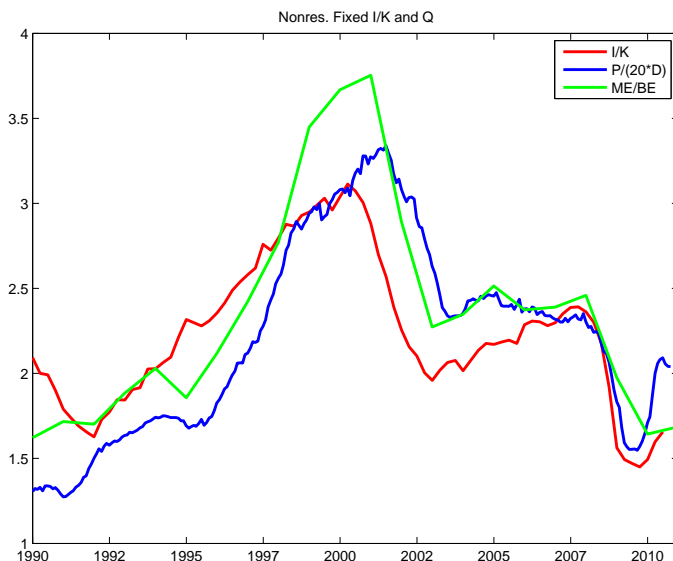


Figure 10: Investment/capital ratio, price/dividend ratio, and market/book ratio. Investment is real private nonresidential fixed investment. Capital is cumulated from investment with an assumed 10% annual depreciation rate. Price/dividend from CRSP, market/book from Ken French's website.

Figure 10 contrasts the investment/capital ratio, market/book ratio, and price/dividend ratio. The simple Q theory also links asset prices and investment better than you probably thought, both in the tech boom and the financial crisis.

Many finance puzzles are stated in terms of returns. To make that connection, one can transform (9) to a relation linking asset returns to investment growth. Many return puzzles are mirrored in investment growth as the q theory suggests.<sup>47</sup>

<sup>47</sup>Cochrane (1991b), (1996) (2007a), Lamont (2000) Li, Livdan and Zhang (2008), Liu, Whited and Zhang (2009), Belo (2010), Jermann (2010), Liu and Zhang (2011).

Q theory also reminds us that supply as well as demand matters in setting asset prices. If capital could adjust freely, stock values would never change, no matter how irrational investors are. Quantities would change instead.

I'm not arguing that consumption or investment *caused* the boom or the crash. Endowment-economy causal intuition does not hold in a production economy. These first-order conditions are happily consistent with a view, for example, that rather small losses on subprime mortgages were amplified by a run on the shadow banking system and flight to quality,<sup>48</sup> which certainly qualifies as a "friction". The first-order conditions are consistent with many other views of the fundamental determinants of both prices and quantities. But the graphs do argue that asset prices and discount rates are much better *linked* to big macroeconomic events than most people think (and vice versa). They suggest another important amplification mechanism: If people did not become more risk averse in recessions, and if firms could quickly transform empty houses into hamburgers, asset prices would not have declined as much.

I also don't pretend to have perfect versions of either of these first-order conditions, let alone a full macro model that captures value or the rest of the factor zoo. These are very simple and rejectable models. Each makes a 100%  $R^2$  prediction that is easy to formally reject. The point is only that research and further elaboration of these kinds of models, as well as using their basic intuition as an important guide to events, is not a hopeless endeavor.

## E. Comparisons

Conversely, I think the other kinds of models, though good for describing particular anomalies, will have greater difficulty accounting for recent big-picture asset pricing events.

We see a *pervasive, coordinated* rise in the premium for systematic risk, common across all asset classes, and present in completely unintermediated and unsegmented assets.<sup>49</sup> For example, Figure 11 plots government and corporate rates, and Figure 12 plots the baa-aaa spread with stock prices. You can see a huge credit spread open up and fade away along with the dip in stock prices.

Behavioral ideas – narrow framing, salience of recent experience, and so forth – are good at generating anomalous prices and mean returns in individual assets or small groups. They don't easily generate this kind of coordinated movement that looks just like a rise in risk premium. They don't naturally generate covariance either. For example, "extrapolation" generates the slight autocorrelation in returns that lies behind momentum. But why should all the momentum stocks then rise and fall together the next month, just as if they are exposed to a pervasive, systematic risk?

Finance models don't help, of course, because we're looking at variation of the factors which they take as given.

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<sup>48</sup>Cochrane (2011)..

<sup>49</sup>The "systematic" adjective is important. People don't seem to drive more carefully in recessions.

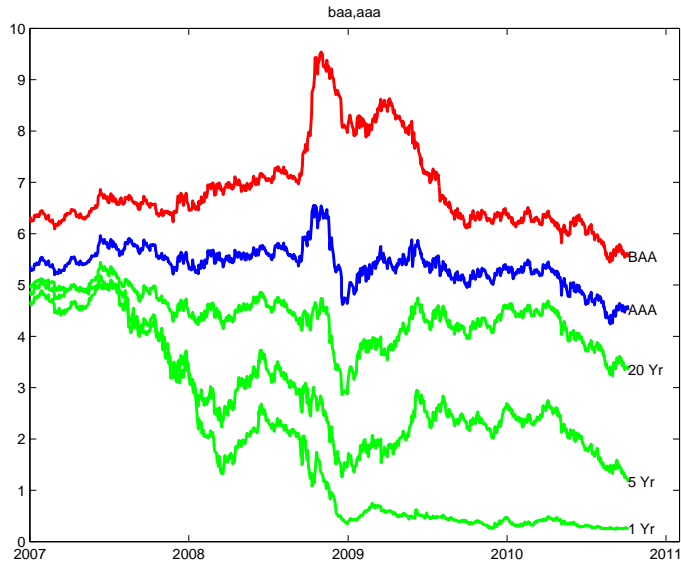


Figure 11: BAA, AAA, and Treasury yields.

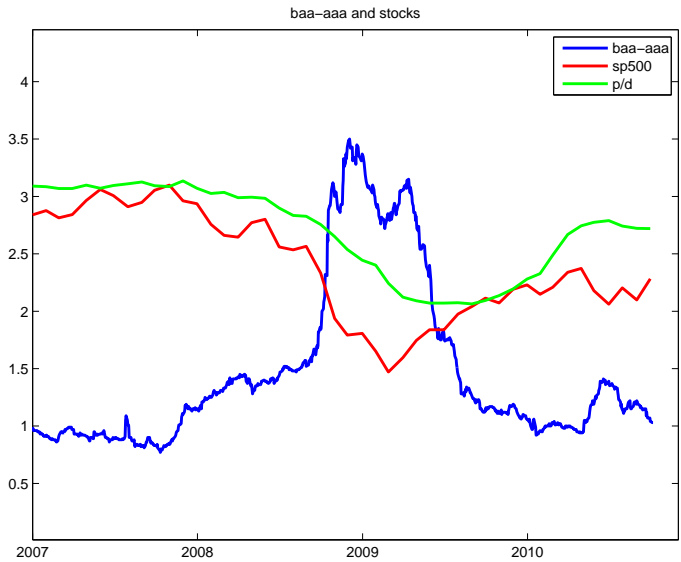


Figure 12: D/P, S&P500, BAA-AAA.

Segmented or institutional models aren't obvious candidates to understand broad market movements. Each of us can easily access stocks and bonds through low-cost indices.

And none of these models naturally describe the strong correlation of discount rates with macroeconomic events. Is it a coincidence that people become irrationally pessimistic when the economy is in a tailspin, and they could lose their jobs, houses, or businesses if systematic events get worse?

Again, macro isn't everything – understanding the smaller puzzles is important. The point is only that *looking* for macro underpinnings for discount rate variation, through fairly simple models, isn't as hopelessly anachronistic as many seem to think.

## F. Arbitrages?

One of the nicest pieces of evidence for segmented or institutional views is that arbitrage relationships were violated in the financial crisis.<sup>50</sup> Unwinding the arbitrage opportunities required one to borrow dollars, which intermediary arbitrageurs could not do.

Figure 13 gives one example. CDS plus Treasury should equal a corporate bond, and usually does. Not in the crisis.

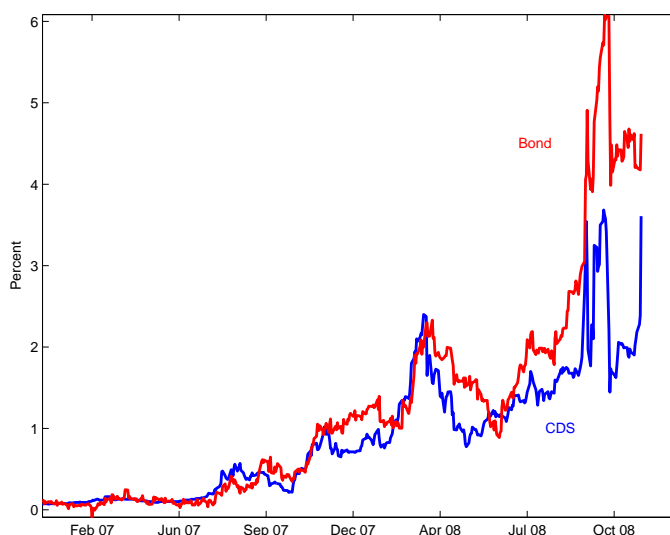


Figure 13: Citigroup CDS and Bond spreads. Source: Fontana (2010).

Figure 14 gives another example, covered interest parity. Investing in the US vs. investing in Europe and returning the money with forward rates should yield the same thing. Not in the crisis. In both cases, profiting from the arbitrage requires one to borrow dollars, which was difficult in the crisis.

Similar patterns happened in many other markets, including even US treasuries.<sup>51</sup> Now, any arbitrage opportunity is a dramatic event. But in each case here the *difference* between the two ways of getting the same cashflow is dwarfed by the *overall* change in prices. And, though an “arbitrage,” the price differences are not large enough to attract “long only” “deep pocket” money. If your precious cash is in a US money market fund, 20 basis points in the depth of a financial crisis is not enough to get you to listen to the salesman offering offshore investing with an exchange-rate hedging program.

<sup>50</sup>See also Fleckenstein, Longstaff, and Lustig (2010),

<sup>51</sup>Hu, Pan and Wang (2011).

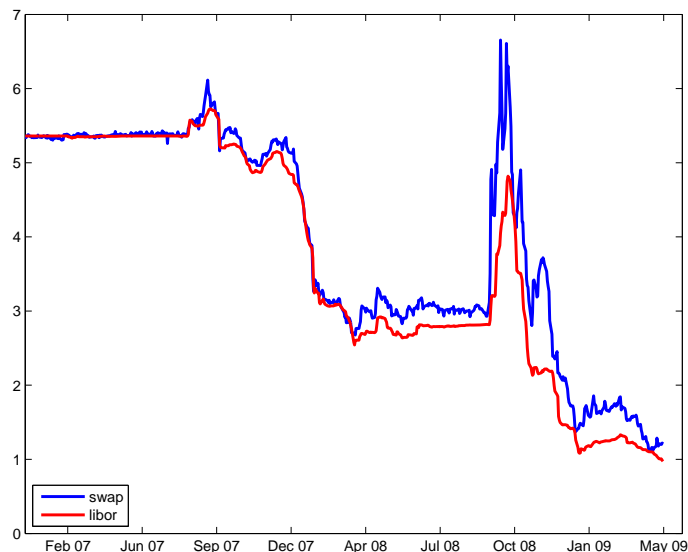


Figure 14: Three-month Libor and FX swap rate. Source: Baba and Packer (2009).

So maybe it’s possible that the “macro” view still builds the benchmark story of *overall* price change, with very interesting *spreads* opening up due to frictions. At least we have a theory for that. Constructing a theory in which the arbitrage spreads drive the coordinated rise in risk premium seems much harder.

The price of coffee displays arbitrage opportunities across locations at the ASSA meetings. (The AFA gave it away for free downstairs while Starbucks was selling it upstairs.) The arbitrage reflects an interesting combination of transactions costs, short-sale constraints, consumer biases, funding limits, and other frictions. Yet we don’t dream that this fact matters for big-picture variation in worldwide commodity prices.

## G. Liquidity premia; trading value

Trading-related liquidity does strike me as potentially important for the big picture, and a potentially important source of the low discount rates in “bubble” events.<sup>52</sup>

I’m inspired by one of the most obvious “liquidity” premiums: Money is overpriced – lower discount rate – relative to government debt, though they are claims to the same payoff in a frictionless market. And this liquidity spread can be huge – hundreds of percent in hyperinflations.

Now, money is “special” for its use in transactions. But many securities are “special” in trading. Trading needs a certain supply of their physical shares. We cannot support large trading volumes by recycling one outstanding share at arbitrarily high speed. Even short

<sup>52</sup>Cochrane (2001), (2003), (2005b), Garber (2000), Krishnamurthy (2002), O’Hara (2008), Scheinkman and Xiong (2003).

sellers must hold a share for some short period of time.

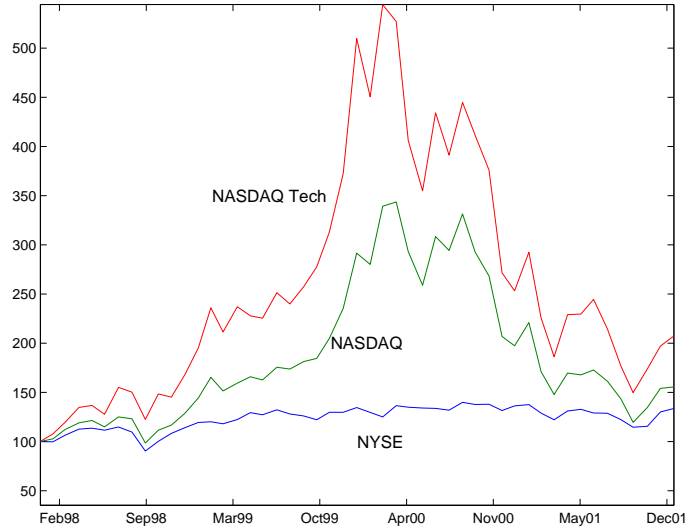


Figure 15: Nasdaq Tech, Nasdaq, and NYSE indices Source: Cochrane (2003).

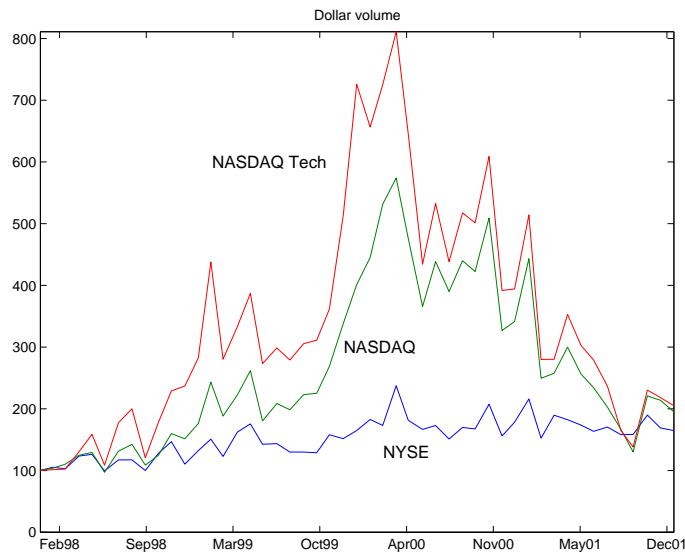


Figure 16: Dollar volume in Nasdaq tech, Nasdaq, and NYSE. Source: Cochrane (2003).

When share supply is small, and trading demand is large, markets can drive down the discount rate or drive up the price of highly-traded securities, as they do for money. These effects have long been seen in government bonds, for example in the Japanese “benchmark” effect, the spreads between on-the-run and off-the-run Treasuries, or the spreads between Treasury and agency bonds.<sup>53</sup> Could these effects extend to other assets?

<sup>53</sup>Boudoukh and Whitelaw (1991), Longstaff (2004), Krishnamurthy and Vissing-Jorgensen (2010).



Figures 15 and 16 are suggestive. The stock *price* rise and fall of the late 1990s was concentrated in Nasdaq and Nasdaq Tech. The stock *volume* rise and fall was concentrated in the same place. *Every* asset price “bubble” – defined here by people’s use of the label – has coincided with a similar trading frenzy, from Dutch tulips in 1620 to Miami condos in 2006.

Is this a coincidence? Do prices rise and fall for other reasons, and large trading volume follows, with no effect on price? Or is the high price – equivalently a low discount rate – explained at least in part by the huge volume; by the value of shares in facilitating a frenzy of information trading?

To make this a deep theory, we must answer why people trade so much. Verbally, we know the answer: The markets we study *exist* to support information-based trading. Yet, we really don’t have good models of information-based trading.<sup>54</sup> Perhaps the question how information is incorporated in asset markets will come back to the center of inquiry.

## V. Applications

Finance is about practical application, not just deep explanation. Discount rate variation will change applications a lot.

### A. Portfolio theory

A huge literature explores how investors should exploit the market-timing and intertemporal-hedging opportunities implicit in time-varying expected returns.<sup>55</sup>

But *the average investor must hold the market portfolio*. We can’t all market-time, we can’t all buy value, and we can’t all be smarter than average. We can’t even all rebalance. No portfolio advice other than “hold the market” can apply to everyone. A useful and durable portfolio theory must be consistent with this theorem. Our discount-rate facts and theories suggest one, built on *differences* between people.

Consider Fama and French’s (1996) story for value. The average investor is worried that value stocks will fall at the same time his or her human capital falls. But then some investors (“steelworkers”) will be more worried than average, and should *short* value despite the premium; some others (“tech nerds”) will have human capital correlated with growth stocks and buy lots of value, effectively selling insurance. A two-factor model implies a three-fund theorem, a three-dimensional multifactor efficient frontier as shown in Figure 17.<sup>56</sup> Investors have a difficult problem to figure out how much of three funds to hold.

And now we have *dozens* of such systematic risks for each investor to consider. Time-varying opportunities create more factors, as habits or leverage risk aversion shift some

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<sup>54</sup>Milgrom and Stokey (1982).

<sup>55</sup>Merton (1971), Barberis (2000), Brennan, Schwartz and Lagnado (1997), Campbell and Viceira (1999), (2002); Pastor (2000); see a review in Cochrane (2007b).

<sup>56</sup>See Fama (1996), Cochrane (2007b).

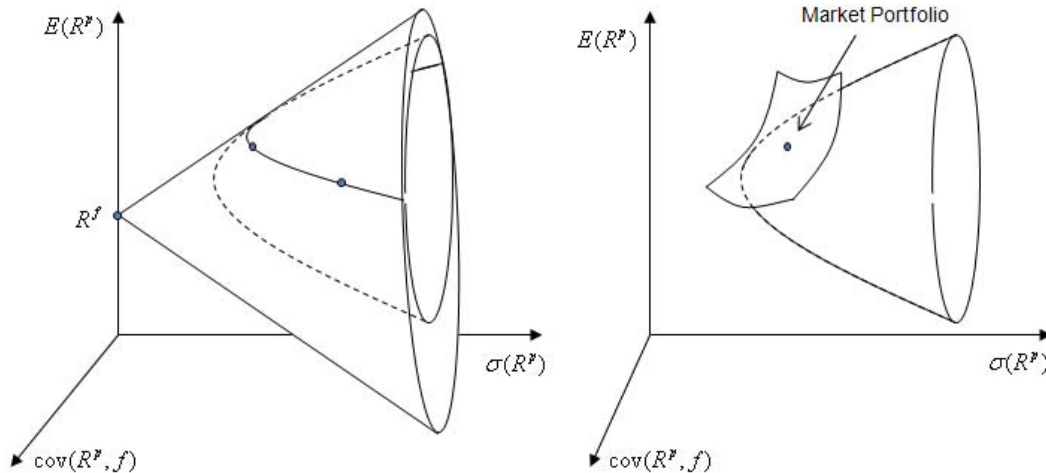


Figure 17: Multifactor efficient frontiers. Investors minimize variance given mean and covariance with the extra factor. A three-fund theorem emerges (left). The market portfolio is multifactor efficient, but not mean-variance efficient (right).

investor’s risk aversion through time more or less than others. *Unpriced* factors are even more important. Our steelworker should start by shorting a steel-industry portfolio, even if it has zero alpha. Such a zero-alpha factor is actuarially fair insurance. We academics should understand the variation across people in risks that are hedgeable by systematic factors, and find low-cost portfolios that span that variation.<sup>57</sup> Yet we’ve spent all our time looking for *priced* factors that are only interesting for the measure-zero mean-variance investor!

All of this sounds hard. That’s good! We finally have a reason for a fee-based “tailored portfolio” industry to exist, rather than just to deplore it as folly. We finally have a reason for us to charge fat tuitions to our MBA students. We finally have an interesting portfolio theory that is not based on chasing zero-sum alpha.

### A.1. State Variables

Discount-rate variation means that state-variable hedging should matter. It is almost completely ignored in practice. Almost all hedge funds, active managers, and institutions still use mean-variance optimizers. This is particularly striking given that they follow active strategies, predicated on the idea that expected returns and variances vary a lot over time!

Perhaps state variable hedging seems nebulous, and therefore maybe small and easy to ignore. Here’s a story to convince you otherwise. Suppose you are a highly risk averse investor, with a 10 year horizon. You are investing to cover a defined payment, say your 8 year old’s future tuition at the University of Chicago. The optimal investment is obviously

<sup>57</sup>Heaton and Lucas (2000).

a 10-year zero-coupon indexed Treasury (TIP).<sup>58</sup> Figure 18 tracks your investment through time.

Suppose now that bond prices plunge, and volatility surges, highlighted in the graph. Should you sell in a panic, to avoid the risk of further losses? No. You should tear up the statement. “Short term volatility” is irrelevant. Every decline in price comes with a corresponding rise in expected return. Evaluating bonds with a one-period mean-variance, alpha-beta framework is silly – though a surprising amount of the bond investing world does it!

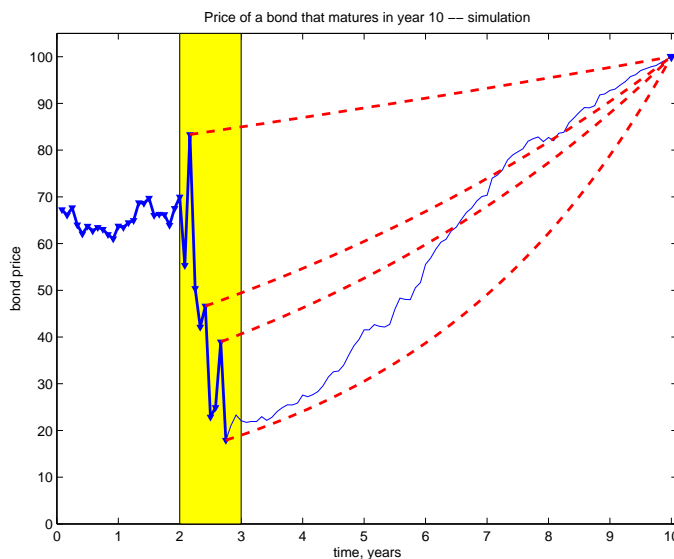


Figure 18: Bond price through time. A cautionary example.

That’s pretty obvious, but now imagine yourself a stock investor in December 2008 – say, your university’s endowment. Stocks plummeted, shown in Figure 19, and stock volatility in Figure 20 rose dramatically, from 16% to 70%.

Should you sell, to avoid the risks of further losses? The standard formula says so. Picking a mean return and risk aversion to justify 60% stocks in normal times, you should reduce the equity share to 4%:

$$\text{Equity Share} = \frac{1}{\gamma} \frac{E(R^e)}{\sigma^2(R^e)}. \quad 0.6 = \frac{1}{2} \frac{0.04}{0.18^2} \Rightarrow \frac{1}{2} \frac{0.04}{0.70^2} = \mathbf{0.04?}$$

(You might object that mean returns rose too. But they would have to have risen to  $4 \times 0.70^2 / 0.18^2 = 60\%$  for this formula to tell you not to change allocation. You also may

<sup>58</sup>Campbell and Viceira (2001), Wachter (2003).

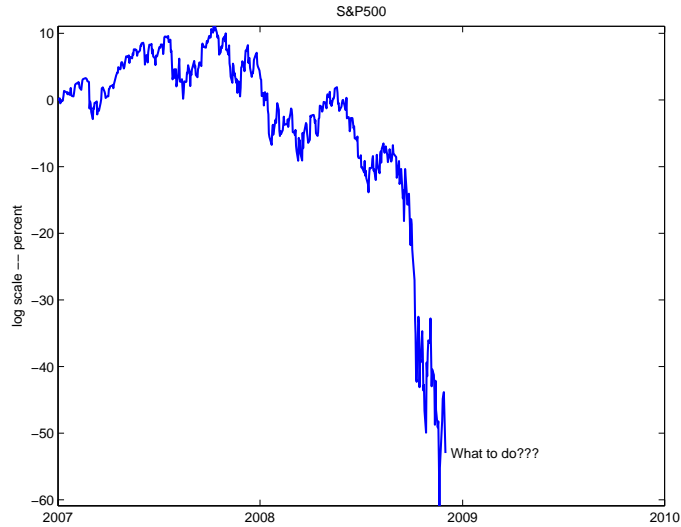


Figure 19: S&P500 price index in 2008

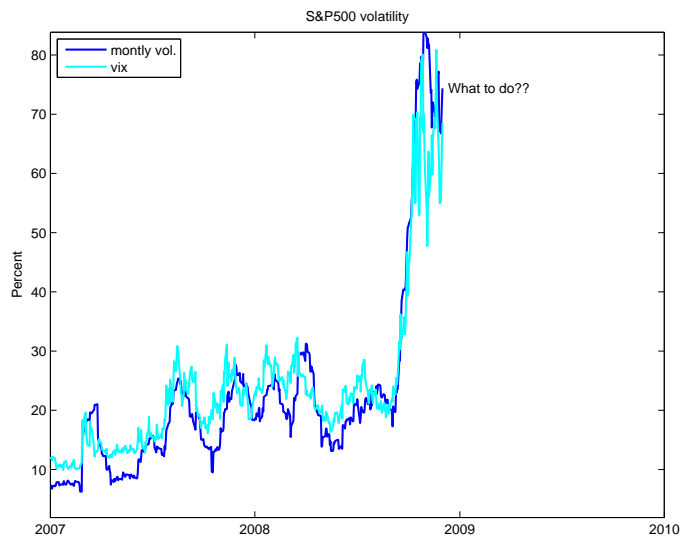


Figure 20: Volatility. VIX index, and 20 day realized daily volatility

object that many investors including endowments had leverage, tenured professor salaries to pay or other habit-like considerations for becoming more risk averse. Fair enough, but then one-period mean-variance theory is particularly inappropriate in the first place.)

But not everyone can do this – the market didn't fall 93%. If you're selling, who is buying? Is everyone else being stupid? Does it make sense to think that the market irrationally *overvalued* in the midst of the financial crisis?

The answer, of course, is that one-period mean-variance analysis is completely inappropriate. If the world were i.i.d., volatility couldn't change in the first place. Stocks are a

bit like bonds; price/dividend drops increase expected returns.<sup>59</sup> To some extent, “short run volatility” doesn’t matter to a long-run investor. *State-variable hedging matters a lot, even for simple real-world applications.* And, by ICAPM logic, we should therefore expect multiple priced factors. Time-series predictability should be a strong source of additional pricing factors in the “cross section,” and affect portfolios.

## A.2. Prices and payoffs

Or maybe not. Telling our bond investor to hold 10 year zeros because their price happens to covary properly with state variables for their investment opportunities just completely confuses the obvious. It’s much clearer to look at the final payoff and tell him to *ignore* price fluctuations. Maybe dynamic portfolio theory overall might get a lot simpler if we look at payoff streams rather than looking at dynamic trading strategies that achieve those streams.

If you look at payoff streams, it’s totally obvious that an indexed perpetuity (or annuity) is the risk-free asset for long-term investors, despite arbitrary time-varying return moments, just as the ten-year zero was obviously the riskfree asset for my bond investor. It’s interesting that coupon-only TIPS are perceived to be an exotic product, or a way to speculate on inflation, not the benchmark riskfree asset for every portfolio in place of a money-market investment.

How about risky investments? Here is a simple and suggestive step.<sup>60</sup> If utility is quadratic

$$\max_{\{c_t\}} E \sum_{t=0}^{\infty} \delta^t \left( -\frac{1}{2} \right) (c_t - c^*)^2$$

it turns out that we can still use two-period mean-variance theory to think about *streams of payoffs*, (loosely, streams of dividends) no matter how much expected returns vary over time.

*Every optimal payoff stream combines an indexed perpetuity and a claim to the aggregate dividend stream.* Less risk averse investors hold more of the claim to aggregate dividends, and vice versa.

*Optimal payoffs lie on a long-run mean / long-run variance frontier,* where I define “long run” means  $\tilde{E}(x)$  that sum over time as well as states,

$$\tilde{E}(x) = \frac{1}{1 - \beta} \sum_{j=0}^{\infty} \beta^j E(x_{t+j})$$

*State variables disappear from portfolio theory,* just as they did for our 10 year TIP investor, once he looked at the 10 year problem.

If our stock market investor thought this way, he would answer “I bought the aggregate dividend stream. Why should I buy or sell? I don’t look at the statements.” This is a lot

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<sup>59</sup>Campbell and Vuolteenaho (2004).

<sup>60</sup>Results from Cochrane (2008).

simpler to explain and implement than deep time series modeling, value function calculation, and optimal hedge portfolios!

If investors have outside income, they first short a payoff stream most correlated with their outside income stream, and then hold the mean-variance efficient payoffs. Calculating correlations of income streams this way may be easier than trying to impute discount-rate induced changes in the present value of outside income streams, in order to calculate return-based hedge portfolios.

If investors have no outside income, long-run expected returns (payoffs divided by initial prices) line up with long-run market betas. A CAPM emerges, despite arbitrary time-variation in expected returns and variances. ICAPM pricing factors fade away as we look at longer horizons. If investors do have outside income, an average-outside-income payoff emerges as a second priced factor, in the style of Fama and French's (1996) human capital story for the value effect.

Of course, quadratic utility is a troublesome approximation, especially for long-term problems. Still, this simple example captures the possibility that a price and payoff approach can give a much simpler view of pricing and portfolio theory than we get by focusing on the high-frequency dynamic trading strategy that achieves those payoffs in a given market structure.

## B. Alphas, betas, performance evaluation

In the 1970 view, there is one source of systematic risk, the market index. Active management chases “alpha,” which means uncovering inefficiently-priced assets.

Now we have dozens of dimensions of systematic risks. Many hedge fund strategies include an element of option writing. For example, Figure 21 shows the annual returns of “equity-market-neutral” hedge funds together with the market return. “Providing liquidity” looks a lot like writing out-of-the-money puts<sup>61</sup>!

I tried telling a hedge fund manager, “You don’t have alpha. I can replicate your returns with a value-growth, momentum, currency and term carry, and short-vol strategy.” He said, “Exotic beta’ *is* my alpha. I understand those systematic factors and know how to trade them. You don’t.” He has a point. How many investors have even thought through their exposures to carry trade or short volatility “systematic risks,” let alone actually can program computers to execute such strategies as “passive,” mechanical investments? To an investor who hasn’t heard of it and holds the market index, a new factor is alpha. And has nothing to do with informational inefficiency.

Most active management and performance evaluation just isn’t well described by the alpha-beta, information-systematic, selection-style split anymore. There is no “alpha.” There is just beta you understand and beta you don’t understand, and beta you are positioned to buy vs. beta you are already exposed to and should sell.

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<sup>61</sup>Mitchell and Pulvino (2001), Asness, Krail and Liew (2001), Agarwal, and Naik (2004).

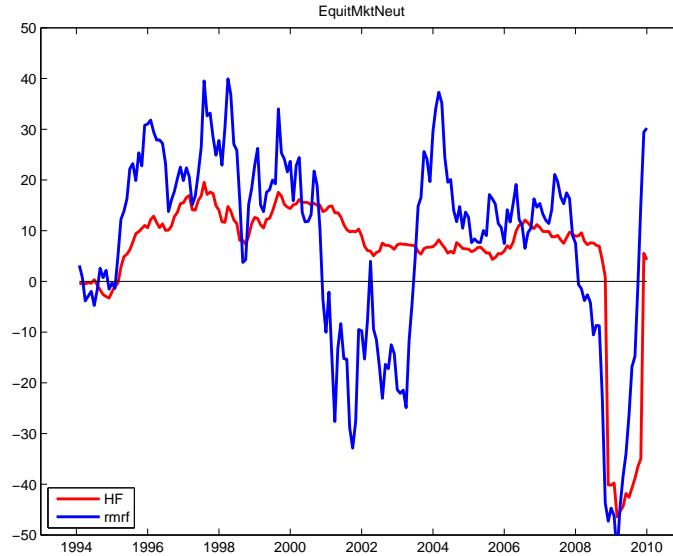


Figure 21: One year excess returns of the “equity market neutral” hedge fund index and the CRSP value-weighted portfolio. Data source: hedgeindex.com and CRSP.

### C. Procedures, corporate, accounting, regulation

Time-varying discount rates and multiple factors deeply change many applications.

The first slide in a capital budgeting lecture looks something like this

$$\text{Value of investment} = \frac{\text{Expected payout}}{R^f + \beta [E(R^m) - R^f]},$$

with a 6% market premium. All of which, we now know, is completely wrong. The market premium isn’t always 6%, but varies over time by as much as its mean. Expected returns don’t line up with CAPM betas, but rather with multifactor betas. And since expected returns change over time, the discount rate is different for cashflows at different horizons.

It’s interesting that investment decisions got so close to right anyway, with high investment following high stock prices. (Remember Figure 10.) Evidently, a generation of our MBAs figured out how to jiggle the numbers and get the right answer despite a dramatically wrong model. Perhaps what we often call “irrational” cash flow forecasts, optimistic in good times and pessimistic in bad times, are just a good way to offset artificially constant discount rates. Or perhaps they understood the Q theory lecture and just follow its advice.

I don’t think the answer lies in multifactor betas<sup>62</sup> or discounting with dynamic present value models and time-varying risk premia, at least not yet. Capital budgeting is a “relative pricing” exercise – we want to use available information in asset markets to help us decide what the discount rate for a given project should be. For this purpose, simply looking at average returns of “similar” securities is enough. Understanding discount rates as a function of characteristics – or, better, understanding valuations directly as a function of

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<sup>62</sup>Fama and French (1997) try.

characteristics (the use of “comparables”) – may end up being more fruitful. We don’t have to *explain* discount rates – relate expected returns to betas, and understand their deep economics – in order to *use* them. We don’t need an all-purpose model of everything to extend prices from known assets to a new one. Even when discount rates are explained, the characterization (characteristic models) may be a better measure for practical relative-pricing than the explanation (beta models). Conversely, capital budgeting gives the same answer if discount rates are “wrong.” When you shop for a salad, all you care about is the price of tomatoes. Whether tomatoes are expensive because the trucks got stuck in bad weather, or because of an irrational bubble in the tomato futures market makes no difference to your decision.

Many procedures in accounting, regulation and capital structure implicitly assume that returns are independent over time, and hence that prices only reflect cashflow information.

Suppose that a firm has a single cashflow in 10 years, and is funded by a zero-coupon bond and equity. In most accounting, capital structure, and regulation we would use the stock and bond prices to calculate the probability and distance to default. But if prices decline because discount rates rise, that fact has no implication for the probability or distance to default.

Perhaps banks’ complaint that low asset prices represent “illiquidity” or “temporarily depressed valuations” rather than insolvency – a lesser chance of making future interest and principal repayments – make some sense. Perhaps capital requirements do not have to respond immediately to such events. Perhaps “hold to maturity” accounting is not as silly as it sounds. Perhaps the fact that firms change capital structures very slowly in response to changes in equity valuations makes some sense.<sup>63</sup>

Of course, in such an event the *risk-neutral* probability of default has risen. Maybe regulators, bondholders, and capital structure *should* respond to a rise in the state-price of the default event exactly as they respond to a rise in the real probability of that event. Maybe, but at least it’s a very different issue and worth asking the question.

I am not arguing that mark-to-market accounting is bad, or that fudging the numbers is a good idea. The point is only that what you *do* with a mark-to-market number might be quite different in a world driven by discount-rate variation than one driven by cashflow variation.<sup>64</sup> The mark-to-market value is no longer a sufficient statistic. A loss of value with a rise in expected return has different implications than a loss of value with a decline in expected return. Decisions need to incorporate more information, not less

The view that the stock price is driven by expected earnings lies behind stock-based executive compensation as well. It’s already a bit of a puzzle that executives should be forced to hold the “systematic” risks due to recessions, market betas, or commodity-price exposures, about which they can do nothing. Understanding that a large fraction of stock returns reflect changes in discount rates or new-factor beta exposures makes the logic of such incentives even more curious. Perhaps stock-based compensation has less to do with effort and operating performance, but more with incentives for risk management or tax treatment.

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<sup>63</sup>Welch (2004).

<sup>64</sup>Heaton, Lucas and McDonald (2009).



## D. Macroeconomics

Large variation in risk premia implies exciting changes for macroeconomics.

Most of macroeconomics focuses on variation in a single intertemporal price, “the” interest rate, which intermediates saving and investment. Yet in the recent recession, as shown in Figure 11, interest rates paid by borrowers (and received by any investors willing to lend) spiked up, while short-term government rates went down. Recessions are all about changes in credit spreads, about the willingness to bear risk and the amount of risk to be borne, far more than they are about changes in the desire for current vs. future certain consumption. Most of the Federal Reserve’s response consisted of targeting risk premiums, not changing the level of interest rates or addressing a transactions demand for money.

Macroeconomics and finance have thought very differently about consumer (we call them investors) and firm behavior. For example, the consumers in the Campbell and Cochrane (1999) habit model balance very strong precautionary saving motives with very strong intertemporal substitution motives, and have large and time-varying risk aversion. Their behavior is very far from the permanent-income intuition (or constrained alternative) in macroeconomic thinking.

As one simple story, macroeconomists often think about how consumers will respond to a change in “wealth,” coming from a change in stock prices or house prices. Financial economists might suspect that consumers will respond quite differently to a decline in value coming from a discount rate rise – a temporary change in price with no change in capital stock or cashflow – than one that comes from a change in expected cashflows, or destruction of physical capital stock.

Financial models also emphasize adjustment costs or irreversibilities: If firms can freely transform consumption goods to capital, then stock prices ( $q$ ) are constant. Yet, most “real business cycle” literature following King, Plosser and Rebelo (1988) left out adjustment costs, because they didn’t need them to match basic quantity correlations. The first round of “new-Keynesian” literature abstracted from capital altogether, and much work in that tradition continues to do so. Figure 10 pretty strongly suggests  $q$  is not constant. Together with the regression evidence of Table I, and interest rates in Figure 11, it suggests that variations in the *risk premium* drive investment, not variation in the level of riskfree interest rates emphasized by macro policy-makers. Adjustment costs lead to basic differences in analysis. For example, without adjustment costs, the marginal product of capital  $\theta f'(k)$  can be negative, clashing with the zero bound on nominal rates. With adjustment costs, the price of capital can fall, giving a positive real rate of interest.

Formal macroeconomics has started to introduce some of the same ingredients that macro-finance researchers are using to understand discount rate variation, including “new” preferences, adjustment cost or other frictions in capital formation, and financial frictions in credit markets.<sup>65</sup> And macroeconomic models with financial frictions are all the rage since the

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<sup>65</sup>For example Christiano, Eichenbaum and Evans (2005). However, this is also a good example of remaining differences. They use a one-period habit, which does not generate slow-moving expected excess returns, and an adjustment cost tied to investment growth not the investment/capital ratio, which does not generate the  $q$  theory predictions of Figure 10 and related finance literature.

financial crisis. Still, we are a long way from a single general-equilibrium model that matches basic quantity and price facts.

Everyone is aware of the question. The job is just hard. Macroeconomic models are technically complicated. Macroeconomic models with time varying risk premia are much harder. Adding financial frictions while maintaining the models’ dynamic intertemporal character is harder still. At a deeper level, successful “grand synthesis” models do not consist of just mixing all the popular ingredients together and stirring the pot; they must maintain the clear quantitative-parable feature of good economic analysis.

An asset-pricing perspective also informs the interaction between monetary and fiscal policy. From a finance perspective, nominal government debt is “equity” in the government: it is the residual claim to primary fiscal surpluses. Hence, the price level must satisfy the standard asset pricing equation,

$$\frac{\text{Debt}_t}{\text{Price level}_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} (\text{real primary surplus}_{t+j}). \quad (10)$$

Inflation can absorb shocks to surpluses, just as equity absorbs shocks to profit streams. This fact is at least an important constraint on monetary policy, especially in a time of looming deficits.<sup>66</sup> Fiscal events can lead to inflation that the Fed is powerless to avoid. The analogy to stocks also suggests that variation in the *discount rate*  $m_{t,t+j}$  for government debt is important: A “flight to quality” lowers the discount rate for government debt, raising the right hand side of (10). People want to hold more government debt, which means getting rid of goods and services, i.e. lower aggregate demand. This story links the “rising risk premium” which finance people see as the core of a recession with the “decline in aggregate demand” which macroeconomists see. The standard corporate finance perspective also illuminates government debt maturity structure and denomination: Foreign currency debt is debt, that must be repaid or defaulted. Domestic-currency debt is equity, which can be inflated. Long-term debt allows bond prices to temporarily absorb shocks to future surpluses, postponing the immediate inflation when short-term debt cannot be refinanced.

## VI. Conclusion

Discount rates vary a lot more than we thought. Most of the puzzles and anomalies that we face amount to discount rate variation we don’t understand. Our theoretical controversies are about how discount rates are formed. We need to recognize and incorporate discount rate variation in applied procedures.

We are really only beginning these tasks. The facts about discount rate variation need at least a dramatic consolidation. Theories are in their infancy. And most applications still implicitly assume i.i.d. returns and the CAPM, and therefore that price changes only reveal cashflow news. Throughout, I see hints that discount-rate variation may lead us to refocus analysis on prices and long-run payoff streams rather than one-period returns.

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<sup>66</sup>Sargent and Wallace (1981), Cochrane (2011).

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# Appendix

## I. Present values and identities

### A. Return identity

To keep the presentation self-contained, I start with a derivation of the Campbell-Shiller (1988) linearization. The return is by definition,

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\left(1 + \frac{P_{t+1}}{D_{t+1}}\right) D_{t+1}}{\frac{P_t}{D_t} D_t}$$

Therefore the log return is

$$r_{t+1} = \log\left(1 + e^{-dp_{t+1}}\right) + dp_t + \Delta d_{t+1}$$

where  $dp_t \equiv \log(D_t/P_t)$ ,  $r_t = \log(R_t)$ ,  $d_t = \log(D_t)$ . I Taylor expand the first term about a constant  $PD$  and  $dp = -\log(PD)$ . This constant need not be the mean. One can use the identity to examine cross-sectional variation in dividend yields without using a different constant of approximation for securities with different average dividend yield.

$$r_{t+1} = \log(1 + PD) - \frac{PD}{1 + PD} (dp_{t+1} - dp) + dp_t + \Delta d_{t+1}.$$

Denoting  $\rho \equiv \frac{PD}{1+PD}$ , we can write the approximation

$$r_{t+1} = \kappa - \rho dp_{t+1} + dp_t + \Delta d_{t+1} \tag{11}$$

where

$$\kappa = -(1 - \rho) \log(1 - \rho) - \rho \log(\rho).$$

In time-series applications where we will only consider second moments we interpret symbols as deviations from means and ignore  $\kappa$  leaving

$$r_{t+1} = -\rho dp_{t+1} + dp_t + \Delta d_{t+1}. \tag{12}$$

### B. Present value identity.

To derive the present value identity (1), rearrange the return identity as

$$dp_t = r_{t+1} - \Delta d_{t+1} + \rho dp_{t+1}.$$

Iterating forward,

$$dp_t = \sum_{j=1}^k \rho^{j-1} r_{t+j} - \sum_{j=1}^k \rho^{j-1} \Delta d_{t+j} + \rho^k dp_{t+k}.$$

Assuming the latter term goes to zero – the “transversality condition” which rules out “rational bubbles” – we have

$$dp_t = \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}. \quad (13)$$

This is an ex-post identity, so it also holds in conditional expectations using any information set.

### C. Dividend construction.

I create dividends from the CRSP annual return series with and without dividends,

$$R_t \equiv \frac{P_{t+1} + D_{t+1}}{P_t}; \quad RX_{t+1} \equiv \frac{P_{t+1}}{P_t}.$$

Then,

$$\frac{D_{t+1}}{P_{t+1}} = \frac{R_{t+1}}{RX_{t+1}} - 1.$$

By using an annual horizon, I avoid the strong seasonal in dividend payments.

Annual dividend growth also includes some return information, because this definition one reinvests dividends to the end of the year at the market return. Annual sums of dividends are a good deal less volatile. However, if one uses sums of dividends over the year, then the identity  $R_{t+1} = (P_{t+1} + D_{t+1})/P_t$  does not hold. It’s nice to use data definitions for which identities hold!

This definition of dividends has a second practical advantage. Consider the sharp fall in stocks in the Fall of 2008. Using a simple sum of past dividends, we would see a decline in price/dividend ratio as large as the decline in stock market values. But much of that decline surely reflected news that dividends in 2009 were going to fall dramatically, news reflected in the return. By reinvesting dividends to the end of the year, the “dividend” series is lower than the sum; this price-dividend ratio already includes the information that dividends will decline next year. The price-dividend ratio fell a good deal less than prices, and therefore better corresponds to expected returns.

I construct dividend growth by

$$\frac{D_{t+1}}{D_t} = \frac{(D_{t+1}/P_{t+1})}{(D_t/P_t)}.$$

For the VAR in Tables II-IV I use instead dividend growth implied by the identity (11),

$$\Delta d_{t+1} = \kappa + r_{t+1} + \rho dp_{t+1} - dp_t.$$

Actual dividend growth gives very similar results. However, this construction means that Campbell-Shiller approximate identities hold exactly, so it is easier to see the identities in the results. To make identities hold, it's better to use "pure" returns rather than infer returns from dividend growth, otherwise approximation errors can show up as magic investment opportunities.

## II. VARs

### A. Shock definition

I identified the shocks in Figure 5 by setting changes to the other variables in turn equal to zero. The return identity (12) means that therefore some of the shocks must come with contemporaneous shocks to returns.

The dividend growth shock is a shock to dividend growth with no change in dividend yield or cay. Hence, it must come with a contemporaneous return shock,

$$\varepsilon_{t+1}^d = 1, \varepsilon_{t+1}^{dp} = 0, \varepsilon_{t+1}^{cay} = 0, \varepsilon_{t+1}^r = 1.$$

If dividend growth rises and dividend yields do not change, you just made a good return. The dividend yield shock has no change in dividend growth or cay. If dividends don't change and  $dp$  rises, it means  $p$  and  $r$  fell a lot. Intuitively, a pure rise in discount rates lowers current return so it can raise subsequent returns.

$$\varepsilon_{t+1}^d = 0, \varepsilon_{t+1}^{dp} = 1, \varepsilon_{t+1}^{cay} = 0, \varepsilon_{t+1}^r = -\rho.$$

The cay shock is a change in  $cay_t$  with no change in dividend yield  $dp_t$  or dividend growth  $\Delta d_t$ , and hence no change in return  $r_t$ .

$$\varepsilon_{t+1}^d = 0, \varepsilon_{t+1}^{dp} = 0, \varepsilon_{t+1}^{cay} = 1, \varepsilon_{t+1}^r = 0.$$

I choose this definition of shocks because it leads to nicely interpretable responses, e.g. "cashflow" and "discount rate." Because dividends remain roughly unpredictable, this "short run" identification gives almost the same result as a "long run" identification. Precisely, if rather than define the first shock as  $\varepsilon_{t+1}^{dp} = 0, \varepsilon_{t+1}^{cay} = 0$ , we had identified it by

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = 0,$$

we would have gotten nearly the same result. The resulting shocks are nearly uncorrelated, which is also convenient.

This VAR is very simple, since I left dividend growth and returns out of the right hand side. My purpose is to distill the essential message of more complex VARs, not to deny that there may be some information in additional lags of these variables.

## B. Identities in the cay VAR.

The present-value identity (13), conditioned down and reproduced here,

$$dp_t = E \left[ \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} | I_t \right] - E \left[ \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} | I_t \right],$$

suggests that an extra variable can only help dp to forecast long horizon returns if it forecasts long horizon dividend growth; it can help to forecast one year returns by changing the term structure of return forecasts as well. Here I show how that intuition applies algebraically to multiple regression coefficients and the impulse response function.

Regressing both sides of (13) on  $dp_t$  and  $z_t$ , we obtain the generalized restriction on long-run multiple regression coefficients,

$$1 = b_r^{lr} - b_d^{lr} \tag{14}$$

$$0 = c_r^{lr} - c_d^{lr}, \tag{15}$$

where the notation refers to regressions

$$r_t^{lr} = \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = a_r + b_r^{lr} dp_t + c_r^{lr} z_t + \varepsilon^r$$

$$\Delta d_t^{lr} = \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} = a_d + b_d^{lr} dp_t + c_d^{lr} z_t + \varepsilon^d$$

Equation (14) is the same as before, now applied to the multiple regression coefficient. Equation (15) expresses the idea that a new variable can only help to forecast long-run returns if it also helps to forecast long-run dividend growth. The extra dividend growth and return forecasts will be perfectly negatively correlated. In this way, extra long-run dividend growth forecastability means *more* long-run return forecastability, not less.

In terms of individual long-horizon regressions

$$r_{t+j} = b_r^{(j)} dp_t + c_r^{(j)} z_t + \varepsilon_{t+j}^r$$

etc., (13) similarly implies

$$\begin{aligned}
1 &= \sum_{j=1}^{\infty} \rho^{j-1} b_r^{(j)} - \sum_{j=1}^{\infty} \rho^{j-1} b_d^{(j)} \\
0 &= \sum_{j=1}^{\infty} \rho^{j-1} c_r^{(j)} - \sum_{j=1}^{\infty} \rho^{j-1} c_d^{(j)}.
\end{aligned}$$

A variable can help to forecast *one-year* returns,  $c_r^{(1)} \neq 0$  only if it correspondingly changes the forecast of longer-horizon returns, or dividend growth.

Since impulse-response functions are the same as regression coefficients of future variables such as  $r_{t+j}$  on shocks at time  $t$ , the impulse-response functions must obey the same relation,

$$\begin{aligned}
1 &= \sum_{j=1}^{\infty} \rho^{j-1} e_{dp \rightarrow r}^{(j)} - \sum_{j=1}^{\infty} \rho^{j-1} e_{dp \rightarrow \Delta d}^{(j)} \\
0 &= \sum_{j=1}^{\infty} \rho^{j-1} e_{z \rightarrow r}^{(j)} - \sum_{j=1}^{\infty} \rho^{j-1} e_{z \rightarrow \Delta d}^{(j)},
\end{aligned}$$

where  $e_{dp \rightarrow r}^{(j)}$  denotes the response of  $r_{t+j}$  to a  $dp_t$  shock. This fact lets me easily interpret the change in forecastability by adding cay, in the context of the present value identity, by plotting the impulse responses. The numbers in Figure 5 are terms of this decomposition.

### C. Results using Welch-Goyal predictors

To see if the pattern of the cay VAR holds more generally, I tried a number of the forecasting variables in Welch and Goyal (2008). The results are in Table AI. Each of these variables helps substantially to forecast one-period returns. Yet they mean-revert quickly and don't forecast dividends much, so the contribution to the variance of dividend yields is still almost all from the variance of long-run expected returns.

	dp	cay	eqis	svar	ik	dfy
$c$		2.21	-0.71	1.48	-5.30	5.25
$t(c)$		(1.73)	(-2.53)	(3.40)	(-0.85)	(1.86)
$b$	0.13	0.10	0.19	0.15	0.11	0.13
$t(b)$	(2.61)	(1.82)	(3.75)	(3.05)	(2.16)	(2.53)
$R^2$	0.10	0.16	0.19	0.15	0.11	0.13
$\sigma(E_t \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j})$	0.52	0.46	0.49	0.42	0.53	0.49
$\sigma(E_t \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j})$	0.17	0.13	0.16	0.11	0.17	0.14

Table AI. Multiple return-forecasting regressions and implied variance of long-horizon returns.

$$r_{t+1} = a + b \times dp_t + c \times z_t + \varepsilon_{t+1}$$



Data are from Welch and Goyal, 1947-2009. I calculate the variance of long-horizon expected returns and dividend growth from a bivariate VAR, and using actual (not identity) dividend growth forecasts. Equis, Percentage Equity Issuance is the ratio of equity issuing activity as a fraction of total issuing activity. Svar is stock variance, computed as sum of squared daily returns on the S&P 500. Ik is the investment to capital ratio, the ratio of aggregate (private nonresidential fixed) investment to aggregate capital for the whole economy. Dfy, the default yield spread, is the difference between BAA and AAA-rated corporate bond yields.

## D. More lags of dividend growth and returns

An obvious first source of additional variables is less restrictive VARs than the simple first-order VAR that I presented in the text. Even in the information set of lagged  $\{dp_t, r_t, \Delta d_t\}$ , there may be more information. As usual in VARs individual coefficients largely enter insignificantly, so it takes some art or prior information to see robust patterns.

The second lag of dividend yields is at least economically important. Table AII presents the regressions.

	$dp_t$	$\Delta dp_t$	$t(dp_t)$	$t(\Delta dp_t)$	$R^2$	$\sigma(E_t(y))\%$	$\frac{\sigma(E_t(y))}{E(y)}$
$r_{t+1}$	0.13	0.26	(2.45)	(1.83)	0.15	6.76	0.65
$\Delta d_{t+1}$	0.03	0.35	(0.62)	(3.27)	0.14	4.98	0.90
$dp_{t+1}$	0.93	0.10	(24.7)	(0.85)	0.91		
$\Delta dp_{t+1}$	-0.07	0.10	(-1.85)	(0.85)	0.06		

Table AII. Forecasts using dividend yield and change in dividend yield. CRSP value weighted return 1947-2009.  $\Delta dp_t = dp_t - dp_{t-1}$

The change in dividend yield helps the return forecast, increasing  $R^2$  from 0.09 to 0.15, and correspondingly increasing the more interesting measures of expected return variation.

The change in dividend yield really helps to forecast dividend growth, with a 3.27 t statistic, 5% standard deviation of forecast and forecast that varies by 90% of the mean. The 0.10 autocorrelation in  $\Delta dp_t$  however suggests that this will be a very short-lived signal, one with little impact on forecasts of long-run dividend growth or returns, and thus to our view of the sources of price-dividend ratio volatility.

Similarly, while *individual*  $r_{t-j}$  and  $\Delta d_{t-j}$  coefficients don't look big and don't have much pattern, they can nonetheless help as a *group*, or by sensibly restricting the pattern of lagged coefficients. In this vein, Lacerda and Santa-Clara (2010) and Kojien and van Binsbergen (2009) find that moving averages of past dividend growth help to forecast both returns and dividend growth (as they must, given the present value identity), almost doubling the return-forecast  $R^2$ .

## E. VAR calculations

To find regression coefficients implied by a first-order VAR as in Table II, I run

$$r_{t+1} = b_r dp_t + \varepsilon_{t+1}^r \quad (16)$$

$$\Delta d_{t+1} = b_d dp_t + \varepsilon_{t+1}^d$$

$$dp_{t+1} = \phi dp_t + \varepsilon_{t+1}^{dp} \quad (17)$$

Then I report

$$b_r^{(k)} = b_r \frac{1 - (\rho\phi)^k}{1 - \rho\phi}.$$

To calculate long run regression coefficients as in Table IV, with  $z = cay$ , I write the VAR as

$$\begin{bmatrix} dp_{t+1} \\ z_{t+1} \end{bmatrix} = \Phi \begin{bmatrix} dp_t \\ z_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1}^{dp} \\ \varepsilon_{t+1}^z \end{bmatrix}$$

$$\begin{bmatrix} r_{t+1} \\ \Delta d_{t+1} \end{bmatrix} = B \begin{bmatrix} dp_t \\ z_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1}^r \\ \varepsilon_{t+1}^d \end{bmatrix}$$

then

$$E_t \begin{bmatrix} r_{t+1}^{lr} \\ \Delta d_{t+1}^{lr} \end{bmatrix} = B \sum_{j=1}^{\infty} \rho^{j-1} \Phi^{j-1} \begin{bmatrix} dp_t \\ z_t \end{bmatrix} = B(I - \rho\Phi)^{-1} \begin{bmatrix} dp_t \\ z_t \end{bmatrix}.$$

## F. Univariate implications

While extending the VAR to additional variables adds a lot, the univariate implications of the VARs are much less than one might think. The middle panel of Figure 5 implies a completely temporary component of prices; if prices fall and dividends do not, that price movement is expected completely to melt away. However, the implied univariate representation of returns is almost completely independent over time. Returns are predictable by dividend yields, but not by past returns. As a result, stock return volatility does not decline substantially with investment horizon and stocks are not safer in the long run. One has to see the state variable  $dp_t$  to have any effect on forecastability or classic portfolio theory.

The reason is easy to see in a simplified example. Write the VAR as

$$\begin{aligned} r_{t+1} &= b_r \times dp_t + \varepsilon_{t+1}^r \\ \Delta d_{t+1} &= 0 \times dp_t + \varepsilon_{t+1}^d \\ dp_{t+1} &= \phi \times dp_t + \varepsilon_{t+1}^{dp} \end{aligned}$$

and note that the return identity (12) means

$$\begin{aligned} \varepsilon_{t+1}^r &= \varepsilon_{t+1}^d - \rho \varepsilon_{t+1}^{dp} \\ b_r &= 1 - \rho\phi + 0. \end{aligned}$$

Examine  $(1 - \phi L)r_{t+1}$

$$\begin{aligned}(1 - \phi L)r_{t+1} &= b_r(1 - \phi L)dp_t + \varepsilon_{t+1}^r - \phi\varepsilon_t^r \\(1 - \phi L)r_{t+1} &= (1 - \rho\phi)\varepsilon_t^{dp} + \left(\varepsilon_{t+1}^d - \rho\varepsilon_{t+1}^{dp}\right) - \phi\left(\varepsilon_t^d - \rho\varepsilon_t^{dp}\right) \\(1 - \phi L)r_{t+1} &= -\rho\varepsilon_{t+1}^{dp} + \varepsilon_{t+1}^d + \varepsilon_t^{dp} - \phi\varepsilon_t^d\end{aligned}$$

Thus,  $r_{t+1}$  is an ARMA(1,1) in its univariate representation,

$$(1 - \phi L)r_{t+1} = (1 - \theta L)v_{t+1}.$$

Now,  $\rho \approx 0.96$  and  $\hat{\phi} \approx 0.94$ , and is biased down. Hence  $\rho = \phi$  is a good approximation. In that case, and with  $cov(\varepsilon^d, \varepsilon^{dp}) = 0$  which is also very close to the data,

$$\begin{aligned}var[(1 - \theta L)v_{t+1}] &= (1 + \theta^2)\sigma_v^2 = (1 + \rho^2)(\sigma_{dp}^2 + \sigma_d^2) \\cov[(1 - \theta L)v_{t+1}, (1 - \theta L)v_t] &= -\theta\sigma_v^2 = -\rho(\sigma_{dp}^2 + \sigma_d^2)\end{aligned}$$

Hence,  $\theta = \rho$  and returns are uncorrelated over time,

$$\begin{aligned}r_{t+1} &= v_{t+1}. \\ \sigma^2(r_{t+1} + r_{t+2} + \dots + r_{t+k}) &= k^2\sigma^2(r).\end{aligned}$$

In general, with  $\phi \neq \rho$ , we still have  $\theta$  very near  $\phi$  (between  $\phi$  and  $\rho$  in fact), so returns follow an ARMA with very slight mean-reversion and a large permanent component.

### III. Asset Pricing as a function of characteristics

#### A. Portfolio spreads

In the text, I related 1-10 portfolio means to Sharpe ratios of underlying factors. Here is the result. Consider the ideal world for such an investigation: Expected returns rise with a characteristic  $C_i$  (for example b/m)

$$E(R^i) = a + b \times C_i,$$

and this variation corresponds exactly to a factor (for example, hml)

$$R_t^i = \beta^i \times f_t + \varepsilon_t,$$

with betas that also rise with the characteristic

$$\beta^i = \frac{a}{E(f)} + \frac{b}{E(f)} \times C_i,$$

and residuals are uncorrelated

$$\text{cov}(\varepsilon^i, \varepsilon^j) = 0.$$

Now, consider the usual 1-10 or 1-20 portfolio difference

$$\begin{aligned} E(R^i - R^j) &= b(C_i - C_j) \\ \sigma^2(R^i - R^j) &= (\beta^i - \beta^j)^2 \sigma_f^2 + 2 \frac{\sigma_\varepsilon^2}{N} = \frac{b^2}{E(f)^2} (C^i - C^j)^2 \sigma_f^2 + 2 \frac{\sigma_\varepsilon^2}{N} \end{aligned}$$

where  $N$  is the number of securities in each portfolio. Therefore, the Sharpe ratio, which is proportional to the t statistic  $\sqrt{T}E(f)/\sigma_f$  for the mean spread-portfolio return, is

$$\frac{E(R^i - R^j)}{\sigma(R^i - R^j)} = \frac{E(f)}{\sigma_f} \frac{b(C_i - C_j)}{\sqrt{b^2 (C^i - C^j)^2 + 2 \frac{\sigma_\varepsilon^2}{N} \frac{E(f)^2}{\sigma_f^2}}}$$

This Sharpe ratio rises as we look at further separated portfolios. As  $C_i - C_j$  increases, it approaches the pure Sharpe ratio of the factor  $E(f)/\sigma(f)$ . It does not increase forever. Splitting in to finer portfolios can get the magic 1% per month portfolio mean or alpha, but cannot arbitrarily raise Sharpe ratios or t statistics. Splitting the portfolio more finely reduces  $N$ , so splits that are too fine end up reducing the Sharpe ratio and t statistic by including too much idiosyncratic risk.

Having seen this analysis, of course, we see that it's more efficient simply to examine the statistical significance of the cross-sectional regression coefficient  $\hat{b}$ , which uses information in all the securities, not just the end portfolios. Since  $\hat{b} = \text{cov}(E(R^i), C^i) / \text{var}(C^i) = E(R^i \times [C^i - E(C^i)]) / \text{var}(C^i)$  this regression coefficient is the same thing as testing the mean  $E(f)$  of a factor which is also formed as a linear function of the characteristic  $f_t = R_t^i \times [C^i - E(C^i)]$ .

## B. Value, betas, and samples.

In the text, I emphasized that all puzzles are joint puzzles of expected returns and betas, and cautioned that the value puzzle does not hold in pre - 1963 US data. Figure 22 presents the CAPM in the Fama-French 10 book/market portfolios before and after 1963. In the left hand panel, you see the familiar failure of the CAPM – average returns are higher in the value portfolio, but there is no association between the wide spread in average returns and market betas. The right-and panel shows average returns and betas before 1963. Here the CAPM is working remarkably well. The big change is *not* in the pattern of average returns. Value still earns more than growth. The big change is betas – in the pre-63 period value firms have higher betas, just as they “ought” to do.<sup>67</sup>

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<sup>67</sup>Davis, Fama, and French (2000), Cambpell and Vuolteenaho (2004), Ang and Chen (2007), Fama and French (2006).

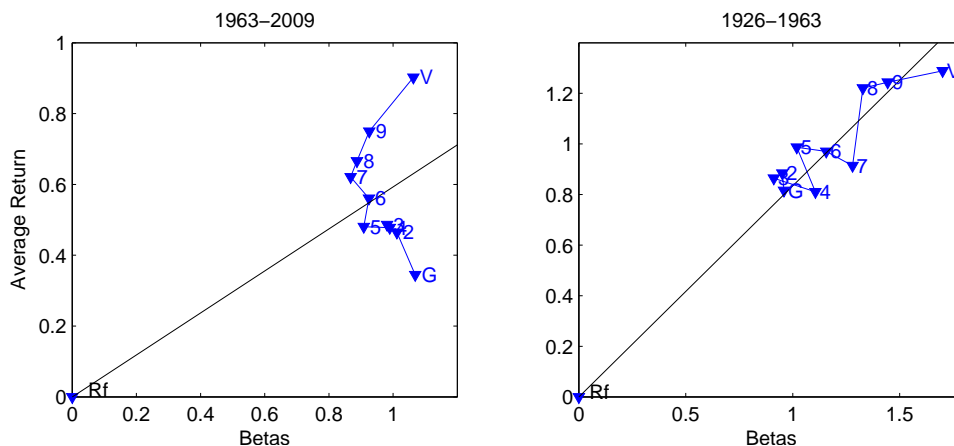


Figure 22: Value effect before and after 1963. Average returns on Fama - French 10 portfolios sorted by book/market equity vs. CAPM betas. Monthly data. Source: Ken French's website.

### C. Time series and cross section

As a first step towards understanding mean returns as a function of characteristics, and to help make the ideas concrete, Table AIII presents regressions using the Fama-French 25 size and book-market portfolios. I use log book/market and log size relative to the market portfolio.

The first row of Table AIII gives a pure cross-sectional regression. The fitted values of this regression fit the portfolio average returns quite well, with a 77%  $R^2$  (One does better still with a  $size \times bm$  cross-term, allowing the growth portfolios to have a different slope on size than the value portfolios.)

	$size_t$	$bm_t$	$\Delta size_t$	$\Delta bm_t$
1. Cross section	-0.030	0.27		
2. Pooled	-0.022	0.55		
3. Time dummies	-0.031	0.29		
4. Portfolio dummies	-0.087	1.48		
5. Pooled	-0.030	0.46	-0.38	1.11

Table AIII. Regressions of Fama-French 25 size and B/M portfolio returns on size and book/market characteristics. The regression specification is in the first row is

$$E(R_{t+1}^{ei}) = a + b \times E(size_{it}) + c \times E(bm_{it}) + \varepsilon_i; \quad i = 1, 2, \dots, 25$$

The remaining rows are

$$R_{t+1}^{ei} = a + (a_i) + (a_i) + b \times size_{i,t} + c \times bm_{i,t} + d \times (size_{i,t} - size_{i,t-12}) + e \times (bm_{i,t} - bm_{i,t-12}).$$

Terms in parentheses only appear in some regressions. *size* is  $\log(\text{market equity}/\text{total market equity})$ . *bm* is  $\log(\text{book}/\text{market})$ . Monthly data 1947-2009. Data from Ken French’s website.

The second row of Table AIII gives a pooled forecasting regression, which is the most natural way to integrate time series and cross section. The size coefficient is a little smaller, and the *bm* coefficient is much larger.

To diagnose the difference between the cross-section and pooled regressions, rows 3 and 4 present a regression with time dummies and a regression with portfolio dummies. Variation *over time* in a *given* portfolio’s *bm* is a much stronger signal of return variation than the same size variation *across portfolios* in average *bm*.

When we run such regressions for individual firms, we can’t use dummies, since the average return of a specific company over the whole sample is meaningless. The goal of this regression is to mirror portfolio formation and remove firm-name completely from the list of characteristics. The last line of Table AIII gives a way to capture the difference between time-series and cross section without dummies – it allows an independent effect of recent changes in the characteristics. This specification accounts quite well for the otherwise unpalatable time and portfolio dummies. The portfolio dummy regression coefficient that captures time-series variation is quite similar to the sum of the level and recent-change coefficients. It is also gratifyingly similar to the “recent-change” effect in aggregate dividend-yield regressions of Table AII. One could of course capture the same phenomenon with portfolios, by sorting based on level and recent change of characteristics. But my goal is to explore the other direction of this equivalence.

Next, we want to run regressions like this on individual data, and find similar characterization of the covariance matrix as a function of characteristics. Then, we can expand to multiple right-hand variables.

## D. Prices in the cross section

Section C suggested merging time-series and cross-sectional approaches, to understand the variation in prices (price-dividend ratios) across time and portfolios by exploring long-run return predictability in the cross-section. How much of the difference between one asset’s price-dividend, price-earnings, book-market, etc. ratio and another’s is due to variation in expected returns, and how much to expected dividend growth or other cashflow expectations?<sup>68</sup>

To explore this question and clarify the idea, I examine the 10 Fama-French book/market portfolios. Eventually, we’re looking for an estimates that are functions of size and other characteristics in individual data. Figure 23 presents the average return, dividend growth, and dividend yield of the portfolios.

Over long horizons, dividend yields are stationary so long-term average returns come from dividend yields and dividend growth. Taking unconditional means of the return identity (12),

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<sup>68</sup>Vuolteenaho (2002), Cohen, Polk and Vuolteenaho (2003).

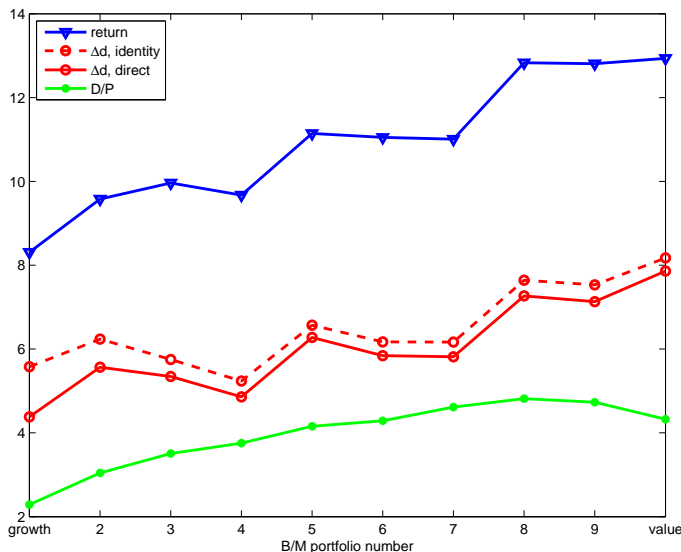


Figure 23: Average return  $r_{t+1}$ , dividend growth  $\Delta d_{t+1}$ , and dividend yield  $dp_t$  for the Fama - French 10 book/market portfolios, 1947-2008. The dashed  $\Delta d$  line gives mean dividend growth implied by the approximate identity  $\Delta d_{t+1} = r_{t+1} - \kappa + \rho dp_{t+1} - dp_t$ .

and imposing stationarity so  $E(dp_t) = E(dp_{t+1})$ ,

$$E(r^i) = (1 - \rho) E(dp^i) + E(\Delta d^i).$$

Figure 23 shows that value portfolio returns come roughly half from greater dividend growth and half from a larger average dividend yield.

Our objective is to produce variance decompositions across time and securities as with the market return. Flipping this around, we have

$$E(dp^i) = \frac{1}{1 - \rho} [E(r^i) - E(\Delta d^i)]. \quad (18)$$

The same observation corresponds to a fairly extreme version of the usual surprising result Low prices – high dividend yields – correspond to *high* dividend growth, and thus to *even higher* returns.<sup>69</sup>

The first column of Table AIV expresses the same idea in a purely cross-sectional regression. From (18), the coefficients in such a regression obey

$$1 = \frac{b_r^{cs}}{1 - \rho} - \frac{b_d^{cs}}{1 - \rho}, \quad (19)$$

where the  $b$  are the cross-sectional regression coefficients of the terms in (18). We can interpret these coefficients as the fraction of *cross-sectional* dividend yield variation driven

<sup>69</sup>Chen, Petkova and Zhang (2008) section 2.2.2 discuss this puzzle.

by discount rates and driven by dividend growth. (Vuolteenaho (2002) uses a different present value identity to understand variation in the book / market ratio directly, rather than use dividend yields as I have. This is a better procedure for individual stocks, which often do not pay dividends. I use dividend yields here for simplicity.) The results are quite similar to the time-series regressions for the market portfolio from Tables 2 - 4: More than all of the cross-sectional variation in average dividend yields of these portfolios comes from cross-sectional variation in expected returns (1.33). Expected dividend growth goes “the wrong way” – low prices correspond to high dividend growth, as seen in Figure 23. (Sample means obey the identity

$$E(dp_t^i) = \frac{1}{1-\rho} \left[ E(r_{t+1}^i) - E(\Delta d_{t+1}^i) + \rho \frac{1}{T} (dp_T^i - dp_1^i) \right].$$

The last term is not zero, which is why the coefficients in the  $b/(1-\rho)$  column do not add up following (19).)

	Cross section		Portfolio dummies		Time dummies		Pooled		Pooled	
	$b$	$\frac{b}{1-\rho}$	$b$	$\frac{b}{1-\rho\phi}$	$b$	$\frac{b}{1-\rho\phi}$	$b$	$\frac{b}{1-\rho\phi}$	$dp_{t-1}$	$\Delta dp_{t-1}$
$r$	0.053	1.33	0.107	0.90	0.044	0.33	0.095	0.97	0.090	0.074
$\Delta d$	0.026	0.64	-0.011	-0.10	-0.092	-0.68	-0.003	-0.03	-0.012	0.076
$dp$			0.92		0.90		0.94		0.94	0.002

Table AIV. Cross-sectional regression coefficients of average returns  $E(r_{t+1}^i)$ , average dividend growth  $E(\Delta d_{t+1})$ , and dividend yield change on dividend yields  $E(dp^i)$ , 10 Fama-French B/M portfolios. Implied dividend growth  $\Delta d$  is calculated from the approximate identity  $\Delta d_{t+1} = r_{t+1} - \kappa + \rho dp_{t+1} - dp_t$ . I use  $\rho = 0.96$

We can, of course, ask how much of the *time-variation* in these dividend yields around their *portfolio average* corresponds to return vs. dividend growth forecasts. A regression that includes portfolio dummies, shown next in Table AIV, addresses this question. The 0.11 return-forecasting coefficient for portfolios is almost the same as the return forecasting coefficient for the market as a whole seen in Tables II-IV. The dividend-growth forecast is also nearly zero. So *all variation in book/market sorted portfolio dividend yields over time, about portfolio means, corresponds to variation in expected returns*, much like that of market returns.

The regression with time dummies, next in Table AIV, paints a different picture. The return coefficient is smaller at 0.044, and  $\phi$  is smaller as well, so expected returns only account for 33% of the variation in dividend yields. Finally we see an important dividend growth forecast, with the right sign, -0.09, accounting for 68% of dividend yield volatility. The strong contrast of this result with the pure cross sectional regression means that *a time of unusually large cross-sectional dispersion in dividend yields corresponds to an unusually high dispersion in dividend growth forecasts*.



This is an important regression, in that it gives us a sense that there is a component of variation in valuations that does correspond to dividend growth forecasts. The unusual dispersion in dividend growth forecasts adds up to zero, so this kind of variation cannot be seen in the aggregate dividend yield and its forecasting relations. There is variation in forecastable dividend growth, which drives some individual variation in dividend/price ratios. But it averages out across all securities, so that the aggregate dividend yield is driven primarily by expected returns.

A pooled regression with no dummies looks much like the time-series regression with portfolio dummies. There is more time-variation in dividend yields than cross-sectional variation, so adding them up evenly the time-variation dominates the pooled regression.

The last column of Table AIV follows Table AIII, to try to unite time-series and cross-sectional variation without using dummies. It shows a very similar result, with the  $\Delta dp_{t-1}$  variable accounting for much of the dividend growth forecastability. The next step is to calculate the price implications of this multivariate regression, as I did with cay, but that takes us too far afield of this simple example.

The Fama-French size portfolios, shown in Table AV, paint a quite different picture. The pure cross-sectional regression (first column) shows cashflow effects: Higher pieces (low dividend yields) are associated with higher subsequent dividend growth, which by one measure fully accounts for the dividend yield variation! However, with portfolio dummies we again see that practically all dividend yield variation *over time for a given portfolio* comes from expected return variation, just as for the market as a whole. With time dummies, variation *across portfolios in a given time period* is split between return and dividend growth forecasts.

	Cross section		Portfolio dummies		Time dummies		Pooled	
	$b$	$\frac{b}{1-\rho}$	$b$	$\frac{b}{1-\rho\phi}$	$b$	$\frac{b}{1-\rho\phi}$	$b$	$\frac{b}{1-\rho\phi}$
$r$	-0.014	-0.36	0.077	1.02	0.023	0.27	0.067	0.95
$\Delta d$	-0.048	-1.20	0.002	0.02	-0.063	-0.73	-0.004	-0.05
$dp$			0.963		0.952		0.968	

Table AV. Cross-sectional regression coefficients of average returns  $E(r_{t+1}^i)$ , average dividend growth  $E(\Delta d_{t+1}^i)$ , and dividend yield change on dividend yields  $E(dp^i)$ , 10 Fama-French ME (size) portfolios. Implied dividend growth is calculated from the approximate identity  $\Delta d_{t+1} = r_{t+1} - \kappa + \rho dp_{t+1} - dp_t$ .