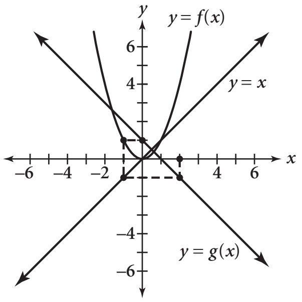
5. Sample answer: Using the graphs of $y = f(x) = x^2$ and $y = g(x) = -x + 1$, you can find $f(g(2)) = 1$. Start at 2 on the x-axis, move to the corresponding point on the graph of $y = g(x)$, then move to $y = x$, then to the graph of $y = f(x)$, and lastly to 1 on the y-axis.



Compositions are essentially a series of input-output functions. Drawing a vertical line up to the graph of $y = g(x)$ gives the value of $g(x)$. Drawing a horizontal line to the graph of $y = x$ makes that y-value into an x-value. Drawing a vertical line to the graph of $y = f(x)$ evaluates $f(x)$ for that output value, and the horizontal line to the y-axis reveals the answer.

CHAPTER 5

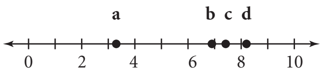
REFRESHING YOUR SKILLS • WORKING WITH POSITIVE SQUARE ROOTS

1. a. $\sqrt{9} < \sqrt{11} < \sqrt{16}$, $3 < \sqrt{11} < 4$, $\sqrt{11} \approx 3.3$

b. $\sqrt{36} < \sqrt{47} < \sqrt{49}$, $6 < \sqrt{47} < 7$, $\sqrt{47} \approx 6.9$

c. $\sqrt{49} < \sqrt{55} < \sqrt{64}$, $7 < \sqrt{55} < 8$, $\sqrt{55} \approx 7.4$

d. $\sqrt{64} < \sqrt{67} < \sqrt{81}$, $8 < \sqrt{67} < 9$, $\sqrt{67} \approx 8.2$



2. a. $\sqrt{24} = \sqrt{4} \cdot \sqrt{6} = 2\sqrt{6}$

b. $\sqrt{75} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$

c. $\sqrt{45} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

d. $\sqrt{40} = \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$

e. $\sqrt{300} = \sqrt{100} \cdot \sqrt{3} = 10\sqrt{3}$

3. a. iii. B.; **b.** i., C.; **c.** iv., D.; **d.** ii., A.

4. a. $\sqrt{17} + \sqrt{27} \approx 4.1 + 5.2 = 9.3$, $\sqrt{17 + 47} \approx \sqrt{44} \approx 6.6$, so $\sqrt{17} + \sqrt{27} > \sqrt{17 + 27}$

b. $8\sqrt{5} = \sqrt{64 \cdot 5} = \sqrt{320}$, $5\sqrt{8} = \sqrt{25 \cdot 8} = \sqrt{200}$, so $8\sqrt{5} > 5\sqrt{8}$

c. $\sqrt{30} - \sqrt{10} \approx 5.5 - 3.2 = 2.3$, $\sqrt{30 - 10} = \sqrt{20} \approx 4.5$, so $\sqrt{30} - \sqrt{10} < \sqrt{30 - 10}$

d. $\sqrt{6^2 + 11^2} = \sqrt{36 + 121} = \sqrt{157}$, or approximately 12.5, $\sqrt{6^2} + \sqrt{11^2} = 6 + 11 + 17$, so $\sqrt{6^2 + 11^2} < \sqrt{6^2} + \sqrt{11^2}$

LESSON 5.1

SUPPORT EXAMPLE

$y = -2^{x-2} - 5$

EXERCISES

1. a. $f(5) = 4.753(0.9421)^5 \approx 3.52738$

b. $g(14) = 238(1.37)^{14} \approx 19528.32$

c. $h(24) = 47.3(0.835)^{24} + 22.3 \approx 22.9242$

d. $j(37) = 225(1.0825)^{37-3} = 225(1.0825)^{34} \approx 3332.20$

2. a. 16, 12, 9; $y = 16(0.75)^x$. Starting with the initial value of 16, repeatedly multiply the previous term, u_{n-1}, by 0.75 to find the next term, u_n. Thus $u_0 = 16$, $u_1 = 16(0.75) = 12$, and $u_2 = u_1(0.75) = 12(0.75) = 9$. The explicit formula for the sequence is $u_n = 16(0.75)^n$. The exponent, n, represents the repeated multiplication of the common ratio, 0.75. The equation of the continuous function through the points of this sequence is $y = 16(0.75)^x$.

b. 24, 36, 54; $y = 24(1.5)^x$. Starting with the initial value of 24, repeatedly multiply the previous term, u_{n-1}, by 1.5 to find the next term, u_n. Thus $u_0 = 24$, $u_1 = u_0(1.5) = 36$, and $u_2 = u_1(1.5) = 54$. The explicit formula for the sequence is given by $u_n = 24(1.5)^n$. The equation of the continuous function through the points of this sequence is $y = 24(1.5)^x$.

3. a. 125, 75, 45; $u_0 = 125$ and $u_n = 0.6u_{n-1}$, where $n \geq 1$. Starting with the initial value of 125, $f(0)$, repeatedly multiply the previous term by 0.6 to find the next term. Thus $f(0) = 125(0.6)^0 = 125$, $f(1) = 125(0.6)^1 = 75$, and $f(2) = 125(0.6)^2 = 45$. The recursive formula for the pattern is $u_0 = 125$ and $u_n = 0.6u_{n-1}$, where $n \geq 1$.

b. 3, 6, 12; $u_0 = 3$ and $u_n = 2u_{n-1}$, where $n \geq 1$. Starting with the initial value of 3, $f(0)$, repeatedly multiply the previous term by 2 to find the next term. Thus $f(0) = 3(2)^0 = 3$, $f(1) = 3(2)^1 = 6$, and $f(2) = 3(2)^2 = 12$. The recursive formula for the pattern is given by $u_0 = 3$ and $u_n = 2u_{n-1}$, where $n \geq 1$.

4. a. The ratio of 36 to 48 is $\frac{36}{48} = 0.75$. $0.75 - 1 = -0.25$, so this is a 25% decrease.

b. The ratio of 72 to 54 is $\frac{72}{54} = 1.\overline{3}$. $1.\overline{3} - 1 = 0.\overline{3}$, so this is a 33.3% increase.

c. The ratio of 47 to 50 is $\frac{47}{50} = 0.94$. $0.94 - 1 = -0.06$, so this is a 6% decrease.

d. The ratio of 50 to 47 is $\frac{50}{47} \approx 1.0638$. $1.0638 - 1 = 0.0638$, so this is approximately a 6.38% increase.

5. a. $u_0 = 1.211$ and $u_n = (1 + 0.015)u_{n-1}$ where $n \geq 1$. The starting population in 1995, u_0, is 1.211 billion. Each year the population is 1.5% greater than the population of the previous year. Multiply the previous year's population, u_{n-1}, by $(1 + 0.015)$ to calculate the current year's population, u_n. Thus the recursive formula is $u_0 = 1.211$ and $u_n = (1 + 0.015)u_{n-1}$, where $n \geq 1$.

b.

Year	Population (in billions)
1995	1.211
1996	1.229
1997	1.248
1998	1.266
1999	1.285
2000	1.305
2001	1.324
2002	1.344

Using the recursive formula in 5a, the population in 1995 is $u_0 = 1.211$. The population in 1996 is $u_1 = (1 + 0.015)u_0 = 1.015(1.211) \approx 1.229$. Continue to use this formula to complete the table.

c. $y = 1.211(1 + 0.015)^x$. Let x represent the number of years since 1995, and let y represent the population in billions. The starting population in 1995 is 1.211, and each year the population is multiplied by $(1 + 0.015)$. Express the population growth with the explicit formula $y = 1.211(1 + 0.015)^x$. The exponent, x, represents the repeated multiplication by $(1 + 0.015)$. To see that this formula works for your table of values, choose two data points, such as 1998 and 2000. In 1998 it had been 3 years since 1995. Check that the population (in billions) in 1998 equals $1.211(1 + 0.015)^3$. Calculate this to get $1.211(1 + 0.015)^3 \approx 1.266$, which is the same population calculated using the recursive formula. In 2000 it had been 5 years since 1995. Check that the population (in billions) in 2000 equals $1.211(1 + 0.015)^5$. Calculate this to get $1.211(1 + 0.015)^5 \approx 1.305$, which also matches the table in 5b.

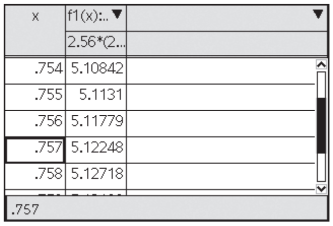
d. Using the recursive or exponential formula, the equations predict that the population of China in 2006 was 1.426 billion. This is larger than the actual value. This means that the population is growing at a slower rate that it was in 1995.

6. a. $y = 2.56(2.5)^x$; 250 cm; 625 cm. Calculate the ratio of the plant's height each day to its height the previous day: $\frac{6.4}{2.56} = 2.5$, $\frac{16}{6.4} = 2.5$, $\frac{40}{16} = 2.5$, and $\frac{100}{40} = 2.5$. Let x represent the number of the day, and let y represent the height in centimeters. The plant has height 2.56 cm on day 0 and its height increases by a factor of 2.5 every day, so the exponential equation is $y = 2.56(2.5)^x$. On the fifth day, $y = 2.56(2.5)^5 = 250$, so the plant's height is 250 cm. On the sixth day, $y = 2.56(2.5)^6 = 625$, so the plant's height is 625 cm.

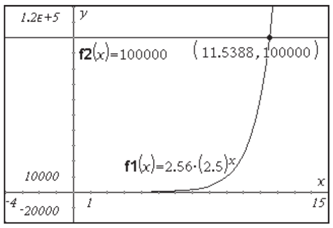
b. $y = 2.56(2.5)^{3.5} \approx 63.25$ cm. Jack measured the height of the plant at 8:00 A.M. every day, so when Jack's brother measured the plant at 8:00 P.M. on the third day, it had been 3 days 12 hours, or 3.5 days. Use the equation $y = 2.56(2.5)^x$ to find the height of the plant. $y = 2.56(2.5)^{3.5} \approx 63.25$, so its height is approximately 63.25 cm at 8:00 P.M. on the third day.

c. Approximately 728 cm. At noon on the sixth day, it has been 6 days 4 hours, or approximately $6.1\overline{6}$ days. The height is $y = 2.56(2.5)^{6.1\overline{6}} \approx 728.12$, or approximately 728 cm.

d. 0.76 day, or 18 hours. Using the equation $y = 2.56(2.5)^x$, find the value of x when $y = 2 \cdot 2.56 = 5.12$. Using a table on a graphing calculator, you can see that $y \approx 5.1225$ when $x = 0.757$, so the doubling time is approximately 0.76 day, or 18 hours.

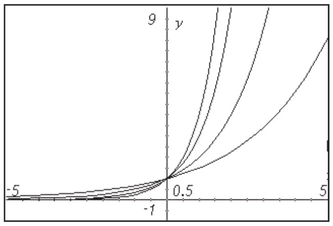


e. 11 days 13 hours, or 9 P.M. on day 11. To find the day and time when the stalk reached its final height of 1 km, or 100,000 cm, you can use guess-and-check, you can graph the equation and zoom in to find the time, or you can graph a second function, $y = 100{,}000$, and find the intersection point of the two graphs. Multiply the decimal part of your answer by 24 to get the number of hours past 8 A.M.



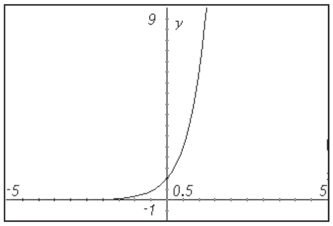
The plant will reach a height of 1 km at 11 days 13 hours, or at 9 P.M. on day 11.

7. a–d.



e. As the base increases, the graph becomes steeper. The curves all intersect the y-axis at (0, 1).

f. The graph of $y = 6^x$ should be the steepest of all of these. It will contain the points (0, 1) and (1, 6).



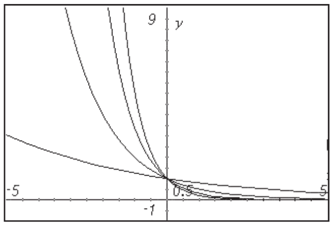
8. a. $y = 2^{x-3}$. This is the graph of $y = 2^x$ translated horizontally 3 units.

b. $y = 2^x + 2$, or $y - 2 = 2^x$. This is the graph of $y = 2^x$ translated vertically 2 units.

c. $y = 3 \cdot 2^x$, or $\frac{y}{3} = 2^x$. This is the graph of $y = 2^x$ dilated vertically by a factor of 3.

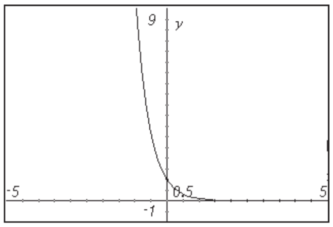
d. $y = 2^{x/3}$. This is the graph of $y = 2^x$ dilated horizontally by a factor of 3.

9. a–d.



e. As the base increases, the graph flattens out. The curves all intersect the y-axis at (0, 1).

f. The graph of $y = 0.1^x$ should be the steepest of all of these. It will contain the points (0, 1) and $(-1, 10)$.



10. a. $y = 5 + 0.5^x$, or $y - 5 = 0.5^x$. This is the graph of $y = 0.5^x$ translated vertically 5 units.

b. $y = -0.5^x + 5$, or $-(y - 5) = 0.5^x$. This is the graph of $y = 0.5^x$ reflected across the x-axis and then translated vertically 5 units.

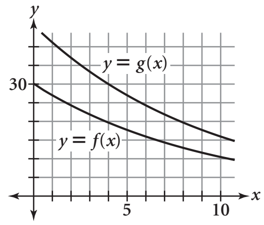
c. $y - 2 = 0.5^{(x-1)}$, or $y = 2 + 0.5^{(x-1)}$. This is the graph of $y = 0.5^x$ translated vertically –2 units and horizontally 1 unit.

d. $\frac{y}{3} = 0.5^{(x/2)}$ or $y = 3 \cdot 0.5^{(x/2)}$. This is the graph of $y = 0.5^x$ dilated vertically by a factor of 3 and dilated horizontally by a factor of 2.

11. a. The common ratio is $\frac{f(1)}{f(0)} = \frac{27}{30} = 0.9$.

b. $f(x) = 30(0.9)^x$. The general form of an exponential function is $f(x) = ab^x$, where a is $f(0)$ and b is the common ratio.

c.



d. $g(4) = f(4 - 4) = f(0) = 30$

e. Because $g(x) = f(x - 4)$, $g(x) = 30(0.9)^{x-4}$.

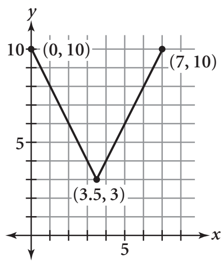
f. You can use the x- and y-values of any point on the curve, (x_1, y_1), and the common ratio to write the equation.

12. Answers will vary. All will have $b = 1.8$.

Use (0, 4) and (1, 7.2): $7.2 = 4 \cdot b^{1-0}$, $1.8 = b$, so $y = 4 \cdot 1.8^x$.

Use (1, 7.2) and (3, 23.328): $23.328 = 7.2 \cdot b^{3-1}$, $3.24 = b^2$, $1.8 = b$, so $y = 7.2 \cdot 1.8^{x-1}$.

13. a. Let x represent time in seconds, and let y represent distance in meters. Janell starts 10 m from the motion sensor at time 0 s, so the graph starts at (0, 10). Janell then moves at a constant rate of 2 m/s toward the motion sensor, so using the equation $d = rt$, or $t = \frac{d}{r}$, relating distance, rate, and time, it takes Janell $\frac{7}{2} = 3.5$ s to get 3 m from the sensor. Connect (0, 10) to the new point, (3.5, 3). When she turns around, it takes another 3.5 s for her to get back to where she started from, 10 m from the motion sensor. Connect (3.5, 3) to the new point, (7, 10).



b. The domain is $0 \le x \le 7$ because Janell starts at time 0 s and it takes 7 s for her round-trip walk. The range is $3 \le y \le 10$ because her distance from the motion sensor ranges between 3 m and 10 m.

c. $y = 2|x - 3.5| + 3$. This is the graph of $y = |x|$ stretched vertically by a factor of 2 and translated vertically 3 units and horizontally 3.5 units.

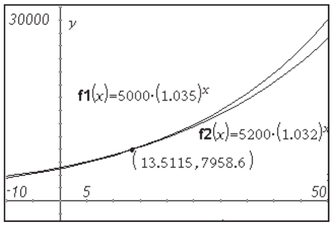
14. a. The point (3, 8.5) is on the graph of f, so $f(3) = 8.5$.

b. $y = 8.5 + 0.5(x - 3)$, $y = 10 + 0.5(x - 6)$, or $y = 7 + 0.5x$. The slope of the line is $\frac{10 - 8.5}{6 - 3}$, or 0.5. Using the point-slope form and the point (3, 8.5), the equation of the line is $y = 8.5 + 0.5(x - 3)$. Or using the point (6, 10), the equation is $y = 10 + 0.5(x - 6)$. Both equations are equivalent to $y = 7 + 0.5x$.

15. a. $A = 5000 \cdot (1 + 0.035)^5$

b. $S = 5200 \cdot (1 + 0.032)^5$

c. They will have the same amount at about 13.5 years, when $5000 \cdot (1 + 0.035)^x = 5200 \cdot (1 + 0.032)^x$. So, Austin will have more money after 14 years.



16. a. The rectangle diagram shows that the area of a rectangle with side lengths $(x + 6)$ and $(x - 4)$ is $x^2 + 6x - 4x - 24$, or $x^2 + 2x - 24$.

	x	6
x	x^2	$6x$
-4	$-4x$	-24

b. Distribute $(x - 4)$ across the sum $(x + 6)$.

$$\begin{aligned}(x - 4)(x + 6) &= x(x + 6) - 4(x + 6)\\ &= x^2 + 6x - 4x - 24\\ &= x^2 + 2x - 24\end{aligned}$$

c. Yes

d. A rectangle diagram also uses the distributive property. Each term in the first binomial is multiplied by each term in the second binomial.

EXTENSIONS

A. Here are some examples of growth and decay in the real world:

Decay: the intensity of a star's light as it travels to Earth; the applause of a crowd after a show has ended

Growth: the intensity of the sound of an airplane as it flies toward you; the interest on an unpaid credit card balance

B. Research results will vary.

LESSON 5.2

SUPPORT EXAMPLES

1. a. Power; $x^3 = \frac{512}{27}$

$$x^3 = \frac{8^3}{3^3}$$

$$x^3 = \left(\frac{8}{3}\right)^3$$

$$x = \frac{8}{3}$$

b. Exponential; $\left(\frac{1}{2}\right)^x = 32$

$$\left(\frac{1}{2}\right)^x = 2^5$$

$$\left(2^{-1}\right)^x = (2)^5$$

$$2^{-x} = 2^5$$

$$-x = 5$$

$$x = -5$$

2. $6x^{2/3} - (8x)^{2/3} = 32$

$$6x^{2/3} - 4x^{2/3} = 32$$

$$2x^{2/3} = 32$$

$$x^{2/3} = 16$$

$$x = 16^{3/2}$$

$$x = \pm 64$$

EXERCISES

1. a. $5^{-3} = \frac{1}{5^3} = \frac{1}{5 \cdot 5 \cdot 5} = \frac{1}{125}$

b. $-6^2 = -(6 \cdot 6) = -36$

c. $-3^{-4} = -\frac{1}{3^4} = -\frac{1}{3 \cdot 3 \cdot 3 \cdot 3} = -\frac{1}{81}$

d. $(-12)^{-2} = \frac{1}{(-12)^2} = \frac{1}{(-12)(-12)} = \frac{1}{144}$

e. $\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \left(\frac{4}{3}\right)\left(\frac{4}{3}\right) = \frac{16}{9}$

f. $\left(\frac{2}{7}\right)^{-1} = \left(\frac{7}{2}\right)^1 = \frac{7}{2}$

2. a. By the product property of exponents, $a^8 \cdot a^{-3} = a^{8+(-3)} = a^5$

b. By the quotient property of exponents, $\frac{b^6}{b^2} = b^{6-2} = b^4$

c. By the power of a power property, $\left(c^4\right)^5 = c^{4 \cdot 5} = c^{20}$.

d. By the definition of zero exponent and the definition of negative exponents, $\frac{d^0}{e^{-3}} = \frac{1}{e^{-3}} = e^3$.

3. a. False. Valid reasons: You must have the same base for the product property of exponents; $243 \cdot 16 \neq 35{,}831{,}808$.

b. False. You must raise to the power before multiplying.

c. False. Valid reasons: You must raise to the power before dividing; only one factor can be divided out; it should be $\frac{4^x}{4} = 4^{x-1}$.

d. True. $\frac{6.6 \cdot 10^{12}}{8.8 \cdot 10^{-4}} = 0.75 \cdot 10^{12-(-4)} = 0.75 \cdot 10^{16} = 0.75 \cdot 10 \cdot 10^{15} = 7.5 \cdot 10^{15}$

4. a. $3^x = \frac{1}{9} = \frac{1}{3^2} = 3^{-2}$, so $x = -2$

b. $\left(\frac{5}{3}\right)^x = \frac{27}{125} = \left(\frac{3}{5}\right)^3 = \left(\frac{5}{3}\right)^{-3}$, so $x = -3$

c. $x = -5$

$$\left(\frac{1}{3}\right)^x = 243 = 3^5$$
$$3^{-x} = 3^5$$
$$x = -5$$

d. $x = 0$

$$5\left(3^x\right) = 5$$
$$\frac{5\left(3^x\right)}{5} = \frac{5}{5}$$
$$3^x = 1$$
$$3^x = 3^0$$
$$x = 0$$

5. a. $x \approx 3.27$

$$x^7 = 4000$$
$$\left(x^7\right)^{1/7} = 4000^{1/7}$$
$$x = 4000^{1/7} \approx 3.27$$

b. $x = 784$

$$x^{0.5} = 28$$
$$\left(x^{0.5}\right)^2 = 28^2$$
$$x = 784$$

c. $x \approx 0.16$

$$x^{-3} = 247$$
$$\left(x^{-3}\right)^{-1/3} = 247^{-1/3}$$
$$x = 247^{-1/3} \approx 0.16$$

d. $x \approx 0.50$

$$5x^{1/4} + 6 = 10.2$$
$$5x^{1/4} = 4.2$$
$$x^{1/4} = 0.84$$
$$\left(x^{1/4}\right)^4 = (0.84)^4$$
$$x = 0.84^4 \approx 0.50$$

e. $x \approx 1.07$

$$3x^{-2} = 2x^4$$
$$3x^{-2} \cdot x^2 = 2x^4 \cdot x^2$$
$$3 = 2x^6$$
$$\frac{3}{2} = x^6$$
$$\left(\frac{3}{2}\right)^{1/6} = \left(x^6\right)^{1/6}$$
$$x = \left(\frac{3}{2}\right)^{1/6} \approx 1.07$$

f. $x = 1$

$$-3x^{1/2} + (4x)^{1/2} = -1$$
$$-3x^{1/2} + 4^{1/2} \cdot \left(x^{1/2}\right) = -1$$
$$-3x^{1/2} + 2x^{1/2} = -1$$
$$-x^{1/2} = -1$$
$$x^{1/2} = 1$$
$$\left(x^{1/2}\right)^2 = (1)^2$$
$$x = 1$$

6. a. $x^6 \cdot x^6 = x^{6+6} = x^{12}$

b. $4x^6 \cdot 2x^6 = (4 \cdot 2)\left(x^6 \cdot x^6\right) = 8x^{12}$

c. $\left(-5x^3\right) \cdot \left(-2x^4\right) = (-5)(-2)\left(x^3 \cdot x^4\right) = 10x^7$

d. $\frac{72x^7}{6x^2} = \frac{72}{6} \cdot \frac{x^7}{x^2} = 12x^5$

e. $\left(\frac{6x^5}{3x}\right)^3 = \left(2x^4\right)^3 = 2^3\left(x^4\right)^3 = 8x^{12}$

f. $\left(\frac{20x^7}{4x}\right)^{-2} = \left(5x^6\right)^{-2} = 5^{-2}\left(x^6\right)^{-2} = \frac{1}{5^2}x^{-12} = \frac{1}{25}x^{-12}$

7. Sample answer: $(a + b)^n$ is not necessarily equivalent to $a^n + b^n$. For example, $(2 + 3)^2 = 25$ but $2^2 + 3^2 = 13$. However, they are equivalent when $n = 1$, or when $a + b = 0$ and n is odd.

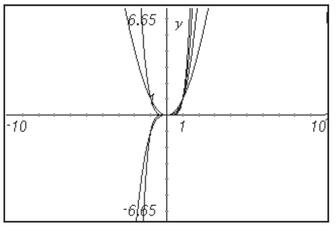
8. a. 49, 79.7023, 129.6418, 210.8723, 343

b. 30.7023; 49.9395; 81.2305; 132.1277. The sequence is not arithmetic because there is not a common difference.

c. 1.627; 1.627; 1.627; 1.627. The ratio of consecutive terms is always the same, so the difference is growing exponentially.

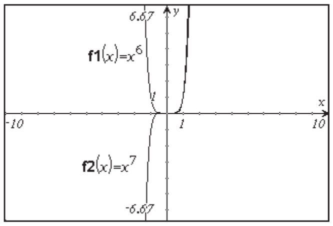
d. Possible answers: Non-integer powers may produce non-integer values. If the exponents form an arithmetic sequence, the decimal powers form a geometric sequence.

9. a–d.



e. Sample answer: As the exponents increase, the graphs get narrower horizontally. The even-power functions are U-shaped and always in the first and second quadrants, whereas the odd-power functions have only the right half of the U, with the left half pointed down in the third quadrant. They all pass through (0, 0) and (1, 1).

f. Sample answer: The graph of $y = x^6$ will be U-shaped, will be narrower than $y = x^4$, and will pass through (0, 0), (1, 1), (−1, 1), (2, 64), and (−2, 64).



Sample answer: The graph of $y = x^7$ will be in the first and third quadrants, will be narrower than $y = x^3$, $y = x^5$, and will pass through (0, 0), (1, 1), (−1, −1), (2, 128), and (−2, −128).

g. Power functions go through the origin and have long run values of ± infinity. Exponential functions have y-intercepts at 1 (or a) and go to zero either as x increases or decreases.

10. a. $y = x^3 + 4$, or $y - 4 = x^3$. This is the graph of $y = x^3$ translated up 4 units.

b. $y = (x + 2)^3$. This is the graph of $y = x^3$ translated left 2 units.

c. $y = \frac{1}{4}x^3$, or $4y = x^3$. This is the graph of $y = x^3$ dilated vertically by a factor of 4.

d. $y = \frac{1}{8}x^3 - 2$, $8(y + 2) = x^3$, or $y + 2 = \left(\frac{1}{2}x\right)^3$. This is the graph of $y = x^3$ dilated vertically by a factor of 8 and then translated down 2 units.

11. a. $47(0.9)^x = 47(0.9)^1(0.9)^{x-1} = 47(0.9)(0.9)^{x-1}$ by the product property of exponents; $47(0.9)(0.9)^{x-1} = [47(0.9)](0.9)^{x-1} = 42.3(0.9)^{x-1}$.

b. $47(0.9)(0.9)(0.9)^{x-2} = [47(0.9)(0.9)](0.9)^{x-2} = 38.07(0.9)^{x-2}$

c. The coefficients are equal to the values of $f_1(x)$ corresponding to the number subtracted from x in the exponent. If (x_1, y_1) is on the curve, then any equation $y = y_1 \cdot b^{(1x-x_1)}$ is an exponential equation for the curve.

12. a. $y = 30.0r^{x-3}$; $y = 5.2r^{x-6}$. The ball rebounds to 30 cm on the third bounce, so $y_1 = 30$ and $x_1 = 3$. The ball rebounds to 5.2 cm on the sixth bounce, so $y_1 = 5.2$ and $x_1 = 6$.

b. $30.0r^{x-3} = 5.2r^{x-6}$; $r \approx 0.5576$

$30.0r^{x-3} = 5.2r^{x-6}$	Set the two equations as equal.
$\frac{30.0}{5.2} = \frac{r^{x-6}}{r^{x-3}}$	Divide both sides by 5.2 and r^{x-3}.
$\frac{30.0}{5.2} = r^{x-6-x+3} = r^{-3}$	Divide r^{x-6} by r^{x-3}.
$\left(\frac{30.0}{5.2}\right)^{-1/3} = \left(r^{-3}\right)^{-1/3}$	Raise both sides to the power of $-\frac{1}{3}$.
$r \approx 0.5576$	Evaluate.

c. The ball was dropped from approximately 173 cm. The equation for the rebound height is $y = 30.0(0.5576)^{x-3}$. Before the ball is dropped, it is at the zero bounce, so set x equal to 0.

$y = 30.0(0.5576)^{0-3} \approx 173.0$

13. a. $x = 7$

$$(x - 3)^3 = 64$$
$$x - 3 = 64^{1/3}$$
$$x - 3 = 4$$
$$x = 7$$

b. $x = -\frac{1}{2}$

$$256^x = \frac{1}{16}$$
$$\left(16^2\right)^x = 16^{-1}$$
$$16^{2x} = 16^{-1}$$
$$2x = -1$$
$$x = -\frac{1}{2}$$

c. $x = 0$

$$\frac{(x + 5)^3}{(x + 5)} = x^2 + 25$$
$$(x + 5)^2 = x^2 + 25$$
$$x^2 + 10x + 25 = x^2 + 25$$
$$10x = 0$$
$$x = 0$$

14. a. $\frac{39.8}{42} \approx 0.9476$

b. $y = 42(0.9476)^{x-2002}$

c. $y = 39.8(0.9476)^{x-2003}$

d. Using the first equation,

$y = 42(0.9476)^{1980-2002} \approx 137.2$

Using the second equation,

$y = 39.8(0.9476)^{1980-2003} \approx 137.2$

Both equations give approximately 137.2 rads.

e. Using the first equation,

$y = 42(0.9476)^{2010-2002} \approx 27.3$

Using the second equation,

$y = 39.8(0.9476)^{2010-2003} \approx 27.3$

Both equations give approximately 27.3 rads.

f. The first equation is $y = 42(0.9476)^{x-2002}$. By the product property, $y = 42(0.9476)(0.9476)^{x-2002-1}$. By multiplying 42(0.9476), $y = 39.8(0.9476)^{x-2003}$. This is the second equation.

15. a. $x = 7$. Move the decimal in 3.7 seven places to the right to get 37,000,000.

b. $x = -4$. Move the decimal in 8.01 four places to the left to get 0.000801.

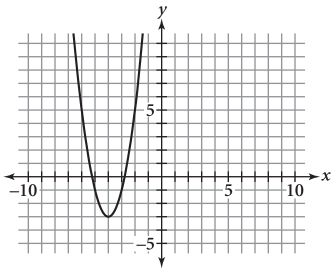
c. $x = 4$. Move the decimal in 4.75 four places to the right to get 47,500.

d. $x = 4.61$. Move the decimal in 4.61 two places to the left to get 0.0461.

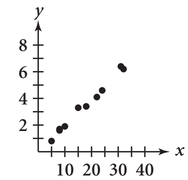
16. $\dfrac{y+3}{2} = (x+4)^2$

$y + 3 = 2(x+4)^2$

$y = 2(x+4)^2 - 3$



17. a. Let x represent time in seconds, and let y represent distance in meters.



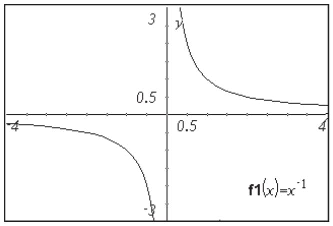
b. All you need is the slope of the median-median line, which is determined by $M_1(8, 1.6)$ and $M_3(31, 6.2)$. The slope is 0.2. The speed is approximately 0.2 m/s.

Improving Your Reasoning Skills

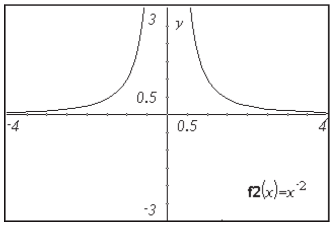
$\dfrac{(Eas)^{-1}(ter)^0 Egg}{y} = \dfrac{(ter)^0 Egg}{(Eas)y} = \dfrac{1 Egg}{Easy}$, or "one egg over easy."

Extensions

A. $y = x^{-1}$



$y = x^{-2}$



The even-power functions are in the first and second quadrants and go through $(-1, 1)$ and $(1, 1)$. The odd-power functions are in the first and third quadrants and go through $(-1, -1)$ and $(1, 1)$. The two parts of each graph get closer and closer to both axes. The smaller the absolute value of the exponent, the faster the graph approaches the axes.

B. See the solution to Take Another Look Activity 1 on page 312.

LESSON 5.3

Support Example

Possible answers: $x^{-2/3}$, $\left(\sqrt[3]{x}\right)^{-2}$, $\frac{1}{x^{2/3}}$, $\left(x^{-2}\right)^{1/3}$, $\left(x^{1/3}\right)^{-2}$

Exercises

1. a—e—j; $\sqrt[5]{x^2} = x^{2/5} = x^{0.4}$

b—d—g; $\left(\sqrt{x}\right)^5 = \sqrt[2]{x^5} = x^{5/2} = x^{2.5}$

c—i; $\sqrt[3]{x} = \sqrt[3]{x^1} = x^{1/3}$

f—h; $\left(\frac{1}{x}\right)^{-3} = \left(x^{-1}\right)^{-3} = x^3$

2. a. Power; the base is a variable.

b. Power; the base is a variable.

c. Exponential; the exponent is a variable.

d. Power; x is equivalent to x^1, so the base is a variable.

e. Power; a square root is equivalent to the power of $\frac{1}{2}$, so the base is a variable.

f. Power; $f(t) = t^2 + 4t + 3 = t^2 + 4t + 4 - 1 = (t+2)^2 - 1$, so it is a translation of $g(t) = t^2$.

g. Exponential; $\frac{12}{3^t}$ is equivalent to $12(3)^{-t}$.

h. Power; $\frac{28}{w-5}$ is equivalent to $28(w-5)^{-1}$.

i. Power; $\frac{8}{y^4}$ is equivalent to $8y^{-4}$.

j. Neither; the independent variable appears twice, raised to two different powers.

k. Power; $\sqrt[5]{4w^3}$ is equivalent to $4^{1/5}w^{3/5}$.

l. Exponential; the exponent contains a variable.

3. a. $a^{1/6}$

b. $b^{8/10}$, $b^{4/5}$, or $b^{0.8}$

c. $c^{-1/2}$, or $c^{-0.5}$

d. $d^{7/5}$, or $d^{1.4}$

4. a. $a = 5489.031744$

$a^{1/6} = 4.2$	$\sqrt[6]{a} = a^{1/6}$
$\left(a^{1/6}\right)^6 = 4.2^6$	Raise both sides to the power of 6.
$a = 5489.031744$	Evaluate.

b. $b \approx 27.808$

$b^{4/5} = 14.3$	$\sqrt[10]{b^8} = b^{4/5}$
$\left(b^{4/5}\right)^{5/4} = 14.3^{5/4}$	Raise both sides to the power of $\frac{5}{4}$.
$b \approx 27.808$	Evaluate.

c. $c \approx 3.306$

$c^{-1/2} = 0.55$	$\frac{1}{\sqrt{c}} = c^{-1/2}$
$\left(c^{-1/2}\right)^{-2} = 0.55^{-2}$	Raise both sides to the power of -2.
$c \approx 3.306$	Evaluate.

d. $d \approx 9.390$

$d^{7/5} = 23$	$\left(\sqrt[5]{d}\right)^7 = d^{7/5}$
$\left(d^{7/5}\right)^{5/7} = 23^{5/7}$	Raise both sides to the power of $\frac{5}{7}$.
$d \approx 9.390$	Evaluate.

5. 490 W/cm^2. Let x represent the number of gels, and let $f(x)$ represent the intensity in W/cm^2. Express f as an exponential function. Start by substituting the two points into the point-ratio form, $y = y_1 \cdot b^{x-x_1}$:

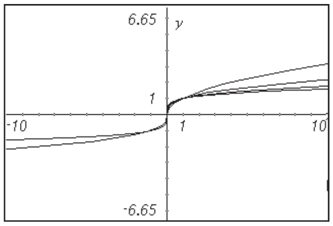
$y = 900b^{x-3}$ and $y = 600b^{x-5}$

Equating the two expressions and solving,

$$900b^{x-3} = 600b^{x-5}$$
$$\frac{b^{x-3}}{b^{x-5}} = \frac{600}{900}$$
$$b^{(x-3)-(x-5)} = \frac{2}{3}$$
$$b^2 = \frac{2}{3}$$
$$b = \sqrt{\frac{2}{3}}$$

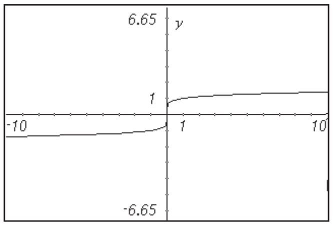
So $y = 900\left(\sqrt{\frac{2}{3}}\right)^{x-3}$. At $x = 6$, $y = 900\left(\sqrt{\frac{2}{3}}\right)^{6-3} = 900\left(\sqrt{\frac{2}{3}}\right)^{3} \approx 490$ W/cm^2.

6. a–d.



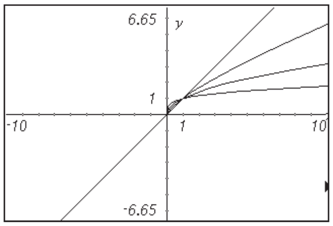
e. Each curve is less steep than the prior one. The graphs of $y = x^{1/2}$ and $y = x^{1/4}$ are only in the first quadrant, whereas the graphs of $y = x^{1/3}$ and $y = x^{1/5}$ are in the first and third quadrants. They all pass through the points (0, 0) and (1, 1), and the graphs of $y = x^{1/3}$ and $y = x^{1/5}$ both pass through the point $(-1, -1)$.

f. $y = x^{1/7}$ will be less steep than the others graphed and will be in the first and third quadrants. It will pass through the points (0, 0), (1, 1), and $(-1, -1)$.



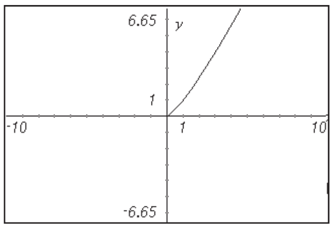
g. The domains of $y = x^{1/2}$ and $y = x^{1/4}$ are $x \geq 0$ because you can't take a square root or fourth root of a negative number. The domains of $y = x^{1/3}$ and $x^{1/5}$ are all real numbers.

7. a–d.



e. Each graph is steeper and less curved than the previous one. All of the functions go through (0, 0) and (1, 1).

f. $y = x^{5/4}$ should be steeper than the others and curve upward.



8. Sample answer: Power functions with rational exponents can have limited domain. When the exponent is between 0 and 1, the curve increases slowly with a shape similar to the graph of $y = \sqrt{x}$. Exponential curves always have a steadily increasing or decreasing slope, unlike power functions.

9. a. Exponential. The graph has a steadily increasing slope.

b. Neither. The graph looks like a parabola with the middle section reflected across a horizontal line. This is like neither an exponential function nor a power function.

c. Exponential. The graph has a steadily decreasing slope.

d. Power. The graph appears in the first and third quadrants. The shape in the first quadrant is similar to the graph of $y = \sqrt{x}$. The shape in the third quadrant is similar to the graph of $y = \sqrt{x}$ reflected across both axes.

10. a. $y = 3 + (x - 2)^{3/4}$. The black graph is translated horizontally 2 units and vertically 3 units to get the red graph.

b. $y = 1 + \left[-(x - 5)\right]^{3/4}$. The black graph is reflected across the y-axis, then translated horizontally 5 units and vertically 1 unit to get the red graph.

c. $y = 4 + \left(\frac{x}{4}\right)^{3/4}$. The black graph is dilated horizontally by a factor of 4, and then translated vertically 4 units to get the red graph.

d. $\frac{y}{4} = \left(\frac{x-3}{2}\right)^{3/4}$, or $y = 4\left(\frac{x-3}{2}\right)^{3/4}$. The black graph is dilated horizontally by a factor of 2 and vertically by a factor of 4. Then it is translated horizontally 3 units to get the red graph.

11. a. $x \approx 6.29$

$9\sqrt[5]{x} + 4 = 17$	Original equation.
$9\sqrt[5]{x} = 13$	Subtract 4 from both sides.
$\sqrt[5]{x} = \frac{13}{9}$	Divide both sides by 9.
$x = \left(\frac{13}{9}\right)^5 \approx 6.29$	Raise both sides to the power of 5.

b. $x \approx 3.66$

$\sqrt{5x^4} = 30$	Original equation.
$5x^4 = 30^2 = 900$	Square both sides.
$x^4 = \frac{900}{5} = 180$	Divide both sides by 5.
$x = 180^{1/4} \approx 3.66$	Raise both sides by the power of $\frac{1}{4}$.

c. $x \approx 180$

$4\sqrt[3]{x^2} = \sqrt{35}$	Original equation.
$\sqrt[3]{x^2} = \frac{\sqrt{35}}{4}$	Divide both sides by 4.
$x^{2/3} = \frac{\sqrt{35}}{4}$	$\sqrt[3]{x^2} = x^{2/3}$.
$x = \left(\frac{\sqrt{35}}{4}\right)^{3/2} \approx 1.80$	Raise both sides to the power of $\frac{3}{2}$.

12. a. Approximately 0.723 AU. Let t represent the orbital time in years, and let r represent the radius in AU. $r = t^{2/3}$. Substitute 0.615 for t and solve for r. $r = (0.615)^{2/3} \approx 0.723$

b. Approximately 29.475 yr. Substitute 9.542 for r and solve for t.

$9.542 = t^{2/3}$

$t = 9.542^{3/2} \approx 29.475$

c. See 12a and b for examples of how to fill in the blanks.

Planet	Mercury	Venus	Earth	Mars
Orbital radius (AU)	0.387	**0.7232**	**1.00**	1.523
Orbital time (yr)	**0.2408**	0.615	1.00	**1.8795**

Planet	Jupiter	Saturn	Uranus	Neptune
Orbital radius (AU)	**5.201**	9.542	**19.181**	30.086
Orbital time (yr)	11.861	**29.475**	84.008	**165.02**

13. a.

$P = kV^{-1}$	Original equation.
$P = \frac{k}{V}$	$V^{-1} = \frac{1}{V}$
$PV = k$	Multiply both sides by V.

b. $P = 40$ and $V = 12.3$, so using $PV = k$ from 13a, $k = (40)(12.3) = 492$.

c. 8.2 L. Substitute 60.0 for P and 492 for k in the equation $PV = K$.

$60.0V = 492$

$V = \frac{492}{60} = 8.2$

d. 32.8 mm Hg. Substitute 15 for V and 492 for k in the equation $P = kV^{-1}$.

$P = 492(15)^{-1} = 32.8$

14. a. $\left(3x^3\right)^3 = 3^3 \cdot \left(x^3\right)^3 = 27x^9$

b. $\left(2x^3\right)\left(2x^2\right)^3 = 2x^3\left[2y^3 \cdot \left(x^2\right)^3\right] = 2x^3 \cdot 8x^6 = 16x^9$

c. $\frac{6x^4}{30x^5} = \frac{6}{30}x^{4-5} = 0.2x^{-1}$

d. $\left(4x^2\right)\left(3x^2\right)^3 = 4x^2\left[3^3 \cdot \left(x^2\right)^3\right] = 4x^2 \cdot 27x^6 = 108x^8$

e. $\frac{-72x^5y^5}{-4x^3y} = \frac{-72}{-4}x^{5-3}y^{5-1} = 18x^2y^4$

15. a. $y = (x + 4)^2$. This is $y = x^2$ translated left 4 units.

b. $y = x^2 + 1$. This is $y = x^2$ translated up 1 unit.

c. $y = -(x + 5)^2 + 2$. This is $y = x^2$ reflected across the x-axis and translated left 5 units and up 2 units.

d. $y = (x - 3)^2 - 4$. This is $y = x^2$ translated right 3 units and down 4 units.

e. $y = \sqrt{x + 3}$. This is $y = \sqrt{x}$ translated left 3 units.

f. $y = \sqrt{x} - 1$. This is $y = \sqrt{x}$ translated down 1 unit.

g. $y = \sqrt{x + 2} + 1$. This is $y = \sqrt{x}$ translated left 2 units and up 1 unit.

h. $y = -\sqrt{x - 1} - 1$. This is $y = \sqrt{x}$ reflected across the x-axis and translated right 1 unit and down 1 unit.

16. About 840. The dart players in at least the 98th percentile are the best 2% of all registered dart players. 2% of 42,000 is $0.02 \cdot 42000 = 840$.

17. a. $u_1 = 20$ and $u_n = (1 + 0.2)u_{n-1} = 1.2u_{n-1}$, where $n \geq 2$

b. About 86 rats. You can use Home screen recursion or Sequence mode to find that $u_9 = 86$.

c. Follow the pattern in 17b to see that $u_n = 1.2^{n-1} \cdot 20$ where $n \geq 1$. Let x represent the year number, and let y represent the number of rats. $y = 20(1.2)^{x-1}$.

Extensions

A. See the Sketchpad demonstration Rational Exponents.

B. See the solution to Take Another Look Activity 2 on page 312.

LESSON 5.4

Support example

a. $x^3 = 12$, so $x = 12^{1/3} \approx 2.289$

b. $x^8 = 17$, so $x = 17^{1/8} \approx 1.425$

c. $x^{1/2} = 5$, so $x = 5^2 = 25$

d. $x^{3/4} = 12$, so $x = 12^{4/3} \approx 27.473$

Exercises

1. a. $x^5 = 50$, so $x = 50^{1/5} \approx 2.187$

b. $\sqrt[3]{x} = 3.1$, so $x = 3.1^3 = 29.791$

c. No real solution. $x^2 = -121$ and the square of a real number is never negative.

2. a. $x^{1/4} - 2 = 3$, so $x^{1/4} = 5$, and $x = 5^4 = 625$.

b. $x = 1$

$$4x^7 - 6 = -2$$
$$4x^7 = 4$$
$$x^7 = 1$$
$$x = 1^{1/7} = 1$$

c. $x = 512$

$$3\left(x^{2/3} + 5\right) = 207$$
$$x^{2/3} + 5 = 69$$
$$x^{2/3} = 64$$
$$x = 64^{3/2} = 512$$

d. $x \approx 0.951$

$$1450 = 800\left(1 + \frac{x}{12}\right)^{7.8}$$
$$1.8125 = \left(1 + \frac{x}{12}\right)^{7.8}$$
$$1.8125^{1/7.8} = 1 + \frac{x}{12}$$
$$x = 12\left(-1 + 1.8125^{1/7.8}\right) \approx 0.951$$

e. $x \approx 0.456$

$$14.2 = 222.1 \cdot x^{3.5}$$
$$\frac{14.2}{222.1} = x^{3.5}$$
$$x = \left(\frac{14.2}{222.1}\right)^{1/3.5} \approx 0.456$$

3. a. $\left(27x^6\right)^{2/3} = 27^{2/3} \cdot \left(x^6\right)^{2/3} = 9x^4$

b. $\left(16x^8\right)^{3/4} = 16^{3/4} \cdot \left(x^8\right)^{3/4} = 8x^6$

c. $\left(36x^{-12}\right)^{3/2} = 36^{3/2} \cdot \left(x^{-12}\right)^{3/2} = 216x^{-18}$

4. a. $100 \cdot r^6$

b. $100 \cdot r^6 = 50$; $r^6 = 0.5$, so $r = 0.5^{1/6} \approx 0.891$

c. The reduction rate is 100% minus the percent passed: $1 - 0.891 = 0.109$, or 10.9%.

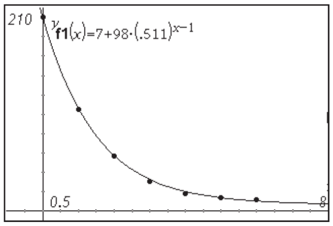
5. a. She must replace y with $(y - 7)$ and replace y_1 with $\left(y_1 - 7\right)$; $y - 7 = \left(y_1 - 7\right) \cdot b^{x - x_1}$.

b.

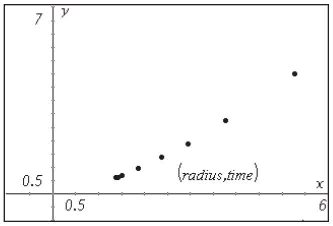
$$y - 7 = (105 - 7)b^{x-1}$$
$$\frac{y - 7}{98} = b^{x-1}$$
$$\left(\frac{y - 7}{98}\right)^{1/(x-1)} = b$$

c. Possible answers: $x = 0$, $y = 200$. $b = 0.508$; $x = 2$, $y = 57$, $b = 0.510$; $x = 3$, $y = 31$, $b = 0.495$

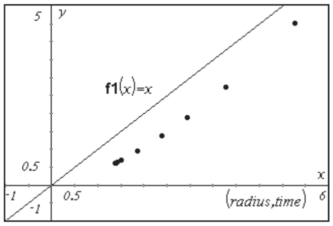
d. Possible answer: The mean of the b-values is 0.511. This gives the equation $\hat{y} = 7 + 98(0.511)^{x-1}$.



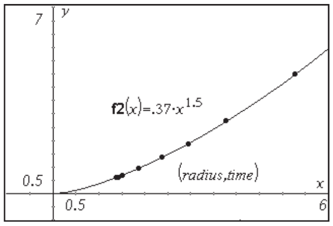
6. a.



b. Sample answer: $\hat{y} = 0.37x^{1.5}$. You might start by graphing the data with the line $y = x$.



The data curve upward slightly, so you know the exponent must be greater then 1, but the points lie below the line $y = x$, so you also need to apply a scale factor less than 1. The equation $\hat{y} = 0.37x^{1.5}$ appears to be a good fit.



c. Approximately 1,229,200 km. (Answers will vary depending on the equation found in 6b.) Substitute 15.945 for y in the equation found in 6b.

$$15.945 = 0.37x^{1.5}$$

$$43.0946 = x^{1.5}$$

$$x = 43.0946^{1/1.5} \approx 12.292$$

This is in units of 100,000 km, so the orbit radius is approximately 1,229,200 km.

d. Approximately 545.390 days. (Answers will vary depending on the equation found in 6b.) Substitute 129.52 for x in the equation found in 6b.

$$y = 0.37 \cdot (129.52)^{1.5} \approx 545.390$$

The orbital time is approximately 545.390 days.

7. a. Approximately 39.23 tons. Substitute 52 for L in the given equation.

$$W = 0.000137 \cdot 52^{3.18} \approx 39.23$$

b. Approximately 54.29 ft. Substitute 45 for W in the given equation.

$$45 = 0.000137L^{3.18}$$

$$328{,}467.153 = L^{3.18}$$

$$L = 328{,}467.153^{1/3.18} \approx 54.29$$

8. a. Approximately 19.58 cm. Substitute 18 for h in the given equation.

$$18 = \frac{5}{3}d^{0.8}$$

$$\frac{54}{5} = d^{0.8}$$

$$d = \left(\frac{54}{5}\right)^{1/0.8} \approx 19.58$$

b. Approximately 23.75 m. The circumference and diameter are related by $C = \pi d$, so $d = \frac{87}{\pi}$. Substitute $\frac{87}{\pi}$ for d in the given equation.

$$h = \frac{5}{3} \cdot \left(\frac{87}{\pi}\right)^{0.8} \approx 23.75$$

9. a. Approximately 1.9 g. Substitute 15 for M in the given equation.

$$F = 0.033 \cdot 15^{1.5} \approx 1.9$$

b. Approximately 12.8%. Divide $\left(0.033 \cdot 15^{1.5}\right)$ by 15. Use this unrounded number for better accuracy.

$$\frac{0.033 \cdot 15^{1.5}}{15} \approx 0.1278 \approx 12.8\%$$

10. Approximately 9% per year. Use the point-ratio equation, $y = 3.74(1 + r)^{x-2007}$, where r is the rate of inflation. A gallon of milk is predicted to be \$5.48 in 2017, so

$$5.48 = 3.74(1 + r)^{2017-2007}$$

$$1.4652 = (1 + r)^{10}$$

$$1 + r = 1.4652^{1/10} \approx 1.039$$

$$r \approx 0.039$$

To find the monthly inflation, change the exponent to reflect the number of months instead of the number of years and solve for r. This rate of inflation is approximately 0.319%.

11. a. 0.0466, or 4.66% per year. If r is the decay rate, then use the two data points to write the equation $5.2 = 6.0(1 - r)^3$. Solve for r.

$$(1 - r)^3 = \frac{5.2}{6}$$

$$r = 1 - \left(\frac{5.2}{6}\right)^{1/3} \approx 0.0466$$

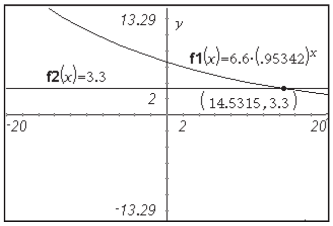
b. Approximately 6.6 g. If y_0 is the initial amount, then $6.0 = y_0 \cdot (1 - 0.0466)^2 = y_0 \cdot (0.9534)^2$. Solve for y.

$$y_0 = \frac{6.0}{0.9534^2} \approx 6.6$$

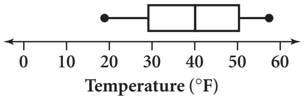
c. $y = 6.6(0.9534)^x$. The equation is in the form $y = y_0(1 - r)^x$.

d. Approximately 0.6 g. Substitute 50 for x in the equation from 11c. $y = 6.6(0.9534)^{50} \approx 0.607$.

e. About 14.5 yr. Look for an x-value for which the corresponding y-value is 3.3. Trace the graph on a graphing calculator to estimate the time.



12. a.



b. 18.9, 29.15, 40.1, 50.35, 57.4

c. Range $= 57.4 - 18.9 = 38.5$; $IQR = 50.35 - 29.15 = 21.2$

d. The data do not support Juan's conjecture. Ten cities fall below 32°F and only seven fall above 50°F.

13. $x = -4.5$, $y = 2$, $z = 2.75$. Eliminate one variable and then solve a system of equations in two variables.

$2x + y + 4z = 4$	First equation.
$x + y + z = \frac{1}{4}$	Second equation.
$x + 3z = 3.75$	Subtract.
$7x + 7y + 7z = 1.75$	Multiply the second equation by 7.
$-3x - 7y + 2z = 5$	Third equation.
$4x + 9z = 6.75$	Add.

Now you have two equations without y.

$4x + 12z = 15$	Multiply the first equation without y by 4.
$4x + 9z = 6.75$	Second equation without y.
$3z = 8.25$	Subtract.
$z = 2.75$	Divide by 3.

Substitute 2.75 for z in the first equation without y and solve for x.

$$x + 3(2.75) = 3.75$$
$$x = -4.5$$

Substitute -4.5 for x and 2.75 for z in the second original equation and solve for y.

$$-4.5 + y + 2.75 = \frac{1}{4}$$
$$y = 2$$

So $x = -4.5$, $y = 2$, and $z = 2.75$.

Improving Your Reasoning Skills

To take the half root is the same as squaring.

$\sqrt[\frac{1}{2}]{cin}\ nati = (cin)^2 nati = cin\ cin\ nati$

He should look in Cincinnati, Ohio.

Extensions

A. Answers will vary.

B. Answers will vary.

LESSON 5.5

Support Examples

1. $y = \pm\sqrt{x-1} + 3$

$$y = (x-3)^2 + 1$$
$$x = (y+3)^2 + 1$$
$$x - 1 = (y+3)^2$$
$$\pm\sqrt{x-1} = y - 3$$
$$\pm\sqrt{x-1} + 3 = y$$

2. 6

$$f(x) = -3x + 14$$
$$y = -3x + 14$$
$$x = -3y + 14$$
$$x - 14 = -3y$$
$$\frac{x-14}{-3} = y$$
$$\frac{x-14}{-3} = f^{-1}(x)$$

So, $f(6) = -3 \cdot 6 + 14 = -4$, and $f^{-1}(-4) = \frac{-4-14}{-3} = 6$.

Exercises

1. $(-3, -2)$, $(-1, 0)$, $(2, 2)$, $(6, 4)$. Switch the x- and y-coordinates of the known points.

2. a. $g(2) = 5 + 2(2) = 5 + 4 = 9$

b. $g^{-1}(9) = 2$. The inverse of $g(t) = 5 + 2t$ is $g^{-1}(t) = \frac{t-5}{2} \cdot g^{-1}(9) = \frac{9-5}{2} = 2$. Or, from 2a, $g(2) = 9$, so $g^{-1}(9)$ must equal 2.

c. $g^{-1}(20) = \frac{20-5}{2} = \frac{15}{2} = 7.5$

3. Graph c is the inverse because the x- and y-coordinates have been switched from the original graph so that the graphs are symmetric across line $y = x$.

4. a and e are inverses; b and d are inverses; c and g are inverses; f and h are inverses.

To find the inverse of a, switch x and y and then solve for y.

$$x = 6 - 2y$$
$$x - 6 = -2y$$
$$-\tfrac{1}{2}(x - 6) = y. \text{ This is e.}$$

To find the inverse of b, switch x and y and then solve for y.

$$x = 2 - \frac{6}{y}$$
$$x - 2 = -\frac{6}{y}$$
$$y(x - 2) = -6$$
$$y = \frac{-6}{x - 2}. \text{ This is d.}$$

To find the inverse of c, switch x and y and then solve for y.

$$x = -6(y - 2)$$
$$-\tfrac{1}{6}x = y - 2$$
$$2 - \tfrac{1}{6}x = y. \text{ This is g.}$$

To find the inverse of f, switch x and y and then solve for y.

$$x = \frac{2}{y - 6}$$
$$x(y - 6) = 2$$
$$xy - 6x = 2$$
$$xy = 6x + 2$$
$$y = \frac{6x + 2}{x} = 6 + \frac{2}{x}. \text{ This is h.}$$

5. a. $f(7) = -4 + 0.5(7 - 3)^2 = -4 + 0.5 \cdot 16 = -4 + 8 = 4$

$g(4) = 3 + \sqrt{2(4 + 4)} = 3 + \sqrt{16} = 3 + 4 = 7$

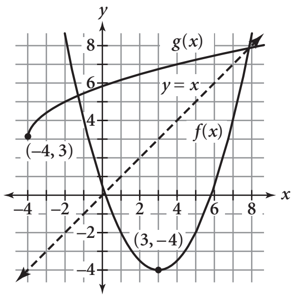
b. They might be inverse functions.

c. $f(1) = -4 + 0.5(1 - 3)^2 = -4 + 0.5 \cdot 4 = -4 + 2 = -2$

$g(-2) = 3 + \sqrt{2(-2 + 4)} = 3 + \sqrt{4} = 3 + 2 = 5$

d. They are not inverse functions, at least not over their entire domains and ranges.

e. $f(x)$ for $x \geq 3$ and $g(x)$ for $x \geq -4$ are inverse functions. Sketch a graph of the two functions and the line $y = x$.



$f(x)$ is a whole vertically oriented parabola, but $g(x)$ is only the top half of a horizontally oriented parabola. Only the right half of the graph of $f(x)$ is the inverse of $g(x)$, so the restricted domain of $f(x)$ is $x \geq 3$. The domain of $g(x)$ is $x \geq -4$.

6. a. $x = 34$

$4 + (x - 2)^{3/5} = 12$	Set $f(x)$ equal to 12.
$(x - 2)^{3/5} = 8$	Subtract 4 from both sides.
$x - 2 = 8^{5/3} = 32$	Raise both sides to the $\frac{5}{3}$ power and evaluate.
$x = 34$	Add 2 to both sides.

b. $f^{-1}(x) = 2 + (x - 4)^{5/3}$

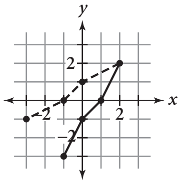
$4 + (y - 2)^{3/5} = x$	Switch x and y.
$(y - 2)^{3/5} = x - 4$	Subtract 4 from both sides.
$y - 2 = (x - 4)^{5/3}$	Raise both sides to the $\frac{5}{3}$ power.
$y = (x - 4)^{5/3} + 2$	Add 2 to both sides.

c. Sample answer: The steps are the same, but you don't have to do the numerical calculations when you find an inverse.

7. a. $f(-3) = -1, f(-1) = 0, f(0) = 1, f(2) = 2$

b. $(-1, 3), (0, -1), (1, 0), (2, 2)$

c. Yes, it is a function. It passes the vertical line test.



8. a. Original function: $f(x) = 2x - 3$. Inverse function: $f^{-1}(x) = \frac{x + 3}{2}$, or $f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$. To find the inverse, switch the x and y in the original function and solve for y.

$$x = 2y - 3$$
$$x + 3 = 2y$$
$$y = \frac{x + 3}{2}$$

The inverse is a function, so $f^{-1}(x) = \frac{x+3}{2}$, or $f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$.

b. Original function: $f(x) = \frac{-3x+4}{2}$. Inverse function: $f^{-1}(x) = \frac{-2x+4}{3}$, or $f^{-1}(x) = -\frac{2}{3}x + \frac{4}{3}$. To find the inverse, switch the x and y in the original function and solve for y.

$$3y + 2x = 4$$
$$3y = -2x + 4$$
$$y = \frac{-2x+4}{3}$$

The inverse is a function, so $f^{-1}(x) = \frac{-2x+4}{3}$, or $f^{-1}(x) = -\frac{2}{3}x + \frac{4}{3}$.

c. Original function: $f(x) = \frac{-x^2+3}{2}$. Inverse: $y = \pm\sqrt{-2x+3}$; not a function. To find the inverse, switch the x and y in the original function and solve for y.

$$y^2 + 2x = 3$$
$$y^2 = -2x + 3$$
$$y = \pm\sqrt{-2x+3}$$

The inverse isn't a function, so $f^{-1}(x)$ notation doesn't apply.

9. a. i. $f^{-1}(x) = \frac{x+140}{6.34}$. To find $f^{-1}(x)$, switch the dependent and independent variables.

$$x = 6.34y - 140$$
$$x + 140 = 6.34y$$
$$y = \frac{x+140}{6.34}, \text{ or } f^{-1}(x) = \frac{x+140}{6.34}$$

ii. $f\left(f^{-1}(15.75)\right) = 15.75$

iii. $f^{-1}(f(15.75)) = 15.75$

iv. $f\left(f^{-1}(x)\right) = f^{-1}\left(f(x)\right) = x$

$$f\left(f^{-1}(x)\right) = f\left(\frac{x+140}{6.34}\right)$$
$$= 6.34\left(\frac{x+140}{6.34}\right) - 140$$
$$= (x+140) - 140 = x$$
$$f^{-1}(f(x)) = f^{-1}(6.34x - 140)$$
$$= \frac{(6.34x - 140) + 140}{6.34}$$
$$= \frac{6.34x}{6.34} = x$$

b. i. $f^{-1}(x) = \frac{x-32}{1.8}$. To find $f^{-1}(x)$, switch the dependent and independent variables.

$$x = 1.8y + 32$$
$$x - 32 = 1.8y$$
$$y = \frac{x-32}{1.8}, \text{ or } f^{-1}(x) = \frac{x-32}{1.8}$$

ii. $f\left(f^{-1}(15.75)\right) = 15.75$

iii. $f^{-1}(f(15.75)) = 15.75$

iv. $f\left(f^{-1}(x)\right) = f^{-1}(f(x)) = x$

$$f\left(f^{-1}(x)\right) = f\left(\frac{x-32}{1.8}\right)$$
$$= 1.8\left(\frac{x-32}{1.8}\right) + 32$$
$$= (x-32) + 32 = x$$
$$f^{-1}(f(x)) = f^{-1}(1.8x + 32)$$
$$= \frac{(1.8x+32) - 32}{1.8}$$
$$= \frac{1.8x}{1.8} = x$$

10. a. Enter the data for altitude in meters and temperature in °C into lists in your calculator, and calculate the equation of the median-median line. The equation is $f(x) = -0.006546x + 14.75$.

b. $f^{-1}(x) = \frac{x - 14.75}{-0.006546x}$, or $f^{-1}(x) = -152.76x + 2252.76$. Exchange x and y to find the inverse and solve for y.

$$x = -0.006546y + 14.75$$
$$x - 14.75 = -0.006546y$$
$$y = \frac{x - 14.75}{-0.006546}, \text{ or}$$
$$f^{-1}(x) = -152.77x + 2253.28$$

c. Enter the data for altitude in feet and temperature in °F into lists in your calculator, and calculate the equation of the median-median line. The equation is $g(x) = -0.003545x + 58.81$.

d. $g^{-1}(x) = \frac{x - 58.81}{-0.003545}$, or $g^{-1}(x) = -282.1x + 16{,}591$. Exchange x and y to find the inverse and solve for y.

$$x = -0.003545y + 58.81$$
$$x - 58.81 = -0.003545y$$
$$y = \frac{x - 58.81}{-0.003545}, \text{ or}$$
$$g^{-1}(x) = -282.1x + 16{,}591$$

e. $-14.44°F$. Because the altitude is given in meters, use the function in 10a to find the temperature in °C first. $f(x) = -0.006546(6194) + 14.75 = -25.80°C$. Then use the function from 9b to change °C to °F: $y = 1.8x + 32$, so $y = 1.8(-25.80) + 32 = -14.44°F$.

11. a. $y = 100 - C$

b. $y = 100 - \dfrac{F - 32}{1.8}$

$F = 1.8C + 32$, $F - 32 = 1.8C$, and $C = \frac{F-32}{1.8}$. So substitute for C in the equation for part a to get $y = 100 - \frac{F-32}{1.8}$.

12. Your friend's score is 1. Sample answers are given for explanations of incorrect answers.

Problem 1 is correct.

Problem 2 is incorrect. The notation $f^{-1}(x)$ indicates the inverse function related to $f(x)$, not the exponent -1.

Problem 3 is incorrect. The expression $9^{-1/5}$ can be rewritten as $\dfrac{1}{9^{1/5}}$.

Problem 4 is incorrect. The expression 0^0 is undefined.

13. a. i, ii, iii

b. ii, v

c. i, iv

d. i, ii, iii

14. a. $y = c(x) = 7.18 + 3.98x$, where $c(x)$ is the cost in dollars and x is the number of thousand gallons.

b. $c(8) = 7.18 + 3.98(8) = \$39.02$

c. $y = g(x) = \frac{x - 7.18}{3.98}$, where $g(x)$ is the number of thousands of gallons and x is the cost in dollars. This is the inverse function of c, so switch x and y in $y = 7.18 + 3.98x$ and solve for y.

$$x = 7.18 + 3.98y$$

$$x - 7.18 = 3.98y$$

$$\frac{x - 7.18}{3.98} = y\text{, so } c^{-1}(x) = \frac{x - 7.18}{3.98}$$

d. 12,000 gallons. $g(54.94) = \dfrac{54.94 - 7.18}{3.98} = 12$

e. $g(c(x)) = g(7.18 + 3.98x) = \dfrac{7.18 + 3.98x - 7.18}{3.98}$

$$= \frac{3.98x}{3.98} = x$$

$$c(g(x)) = c\left(\frac{x - 7.18}{3.98}\right) = 7.18 + 3.98\left(\frac{x - 7.18}{3.98}\right)$$

$$= 7.18 + x - 7.18 = x$$

f. Approximately \$6. (50 gallons/day)(30 days) = 1500 gallons; $3.98(1.5) \approx 6.00$.

g. Answers will vary, but volume should equal $231 \cdot 1500$, or 346,500 in^3 (approximately 200 ft^3). For example, the container could be 30 in. by 50 in. by 231 in.

15. Possible answers: $\sqrt[3]{125^2}$, $\left(\sqrt[3]{125}\right)^2$, 5^2, $\sqrt[3]{15625}$, 25

16. $f(x) = 12.6(1.5)^{x-2}$, or $f(x) = 42.525(1.5)^{x-5}$. Use the point-ratio equation with the point (2, 12.6): $y = 12.6r^{x-2}$. Now use the second point, (5, 42.525), to solve for r.

$$42.525 = 12.6r^{5-2}$$

$$3.375 = r^3$$

$$r = 3.375^{1/3} = 1.5$$

17. a. $x = \frac{9}{2} = 4.5$

$(2^2)^x = (2^3)^3$	Replace 4 with 2^2 and 8 with 2^3.
$2^{2x} = 2^9$	Power of a power property.
$2x = 9$	Common base property of equality.
$x = \frac{9}{2} = 4.5$	Divide both sides by 2.

b. $x = -\frac{1}{2} = -0.5$

$3^{4x+1} = (3^2)^x$	Rewrite 9 as 3^2.
$3^{4x+1} = 3^{2x}$	Power of a power property.
$4x + 1 = 2x$	Common base property of equality.
$2x = -1$	Subtract $2x$ and 1 from both sides.
$x = -\frac{1}{2} = -0.5$	Divide both sides by 2.

c. $x = 1$

$2^{x-3} = (2^{-2})^x$	Rewrite $\frac{1}{4}$ as 2^{-2}.
$2^{x-3} = 2^{-2x}$	Power of a power property.
$x - 3 = -2x$	Common base property of equality.
$3x = 3$	Add $2x$ and 3 to both sides.
$x = 1$	Divide both sides by 3.

18. $y = 3(x - 3)^2 + 2$ and $x = \frac{1}{9}(y - 2)^2 + 3$

The vertical parabola has equation $y = a(x - 3)^2 + 2$. Use the point (4, 5) to find a.

$$5 = a(4 - 3)^2 + 2$$

$$3 = a$$

The equation is $y = 3(x - 3)^2 + 2$.

The horizontal parabola has equation $x = b(y - 2)^2 + 3$. Use the point (4, 5) to find b.

$$4 = b(5 - 2)^2 + 3$$

$$b = \frac{1}{9}$$

The equation is $x = \frac{1}{9}(y - 2)^2 + 3$.

19. $x = -1$, $y = 1$, $z = 0$. Rewrite the second equation as $x + y - 2z = 0$, and then add it to the first equation to eliminate x. Also, divide the third equation by 2.2.

$2z = x + y$ — Second equation.

$$\begin{array}{rl} x + y - 2z = 0 & \text{Subtract } 2z \text{ from both sides.} \\ -x + 3y - z = 4 & \text{First equation.} \\ \hline 4y - 3z = 4 & \text{Add the two equations.} \end{array}$$

Now use the third equation to solve for y and z.

$2.2y + 2.2z = 2.2$ — Third equation.

$y + z = 1$ — Divide both sides by 2.2.

$y = 1 - z$ — Subtract z from both sides.

$4(1 - z) - 3z = 4$ — Substitute $(1 - z)$ for y in $4y - 3z = 4$.

$4 - 4z - 3z = 4$ — Distribute 4.

$-7z = 0$ — Subtract 4 from both sides.

$z = 0$ — Divide both sides by -7.

Substitute 0 for z in the third equation.

$2.2y + 2.2(0) = 2.2$

$y = 1$

Substitute 0 for z and 1 for y in the first or second equation.

$2(0) = x + 1$ — Second equation.

$x = -1$ — Subtract 1 from both sides.

The solution is $x = -1$, $y = 1$, $z = 0$.

EXTENSIONS

A. Answers will vary.

B. Answers will vary.

LESSON 5.6

SUPPORT EXAMPLES

1. $6^x = 14$

2. $3^x = 16$

$$\log 3^x = \log 16$$

$$x \log 3 = \log 16$$

$$x = \frac{\log 16}{\log 3} \approx 2.5237$$

EXERCISES

1. a. $10^x = 1000$. Recall that $\log 1000 = \log_{10} 1000$, so $b = 10$ and $a = 1000$.

b. $5^x = 625$. $b = 5$ and $a = 625$

c. $7^{1/2} = x$. $b = 7$ and $a = x$

d. $x^3 = 8$. $b = x$ and $a = 8$

e. $5^{-2} = x$. $b = 5$ and $a = x$

f. $6^x = 1$. $b = 6$ and $a = 1$

2. a. $10^x = 1000 = 10^3$, so $x = 3$

b. $5^x = 625 = 5^4$, so $x = 4$

c. $7^{1/2} = x$, so $x = \sqrt{7} \approx 2.65$

d. $x^3 = 8 = 2^3$, so $x = 2$

e. $5^{-2} = \frac{1}{5^2} = \frac{1}{25} = x$

f. $6^x = 1 = 6^0$, so $x = 0$

3. a. $x = \log_{10} 0.001$; $x = -3$

b. $x = \log_5 100$; $x = \log_5 100 = \dfrac{\log 100}{\log 5} \approx 2.8614$

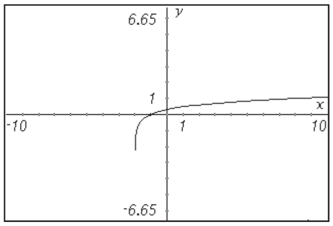
c. $x = \log_{35} 8$; $x = \log_{35} 8 = \dfrac{\log 8}{\log 35} \approx 0.5849$

d. $x = \log_{0.4} 5$; $x = \log_{0.4} 5 = \dfrac{\log 5}{\log 0.4} \approx -1.7565$

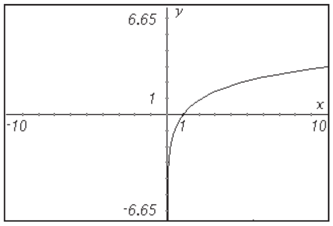
e. $x = \log_{0.8} 0.03$; $x = \log_{0.8} 0.03 = \dfrac{\log 0.03}{\log 0.8} \approx 15.7144$

f. $x = \log_{17} 0.5$; $x = \log_{17} 0.5 = \dfrac{\log 0.5}{\log 17} \approx -0.2447$

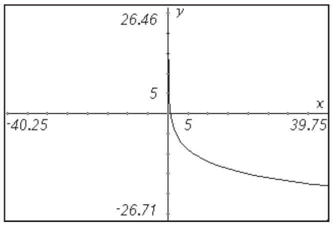
4. a. This is a translation of $y = \log x$ left 2 units.



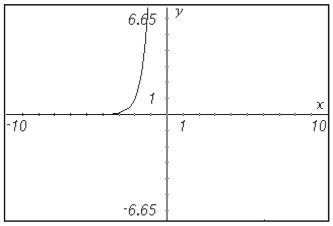
b. This is a vertical dilation of $y = \log x$ by a factor of 3.



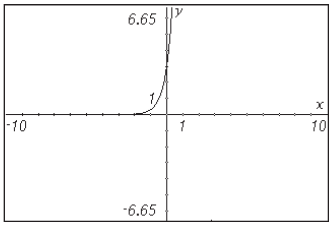
c. This is a reflection of $y = \log x$ across the x-axis, followed by a translation vertically -2 units.



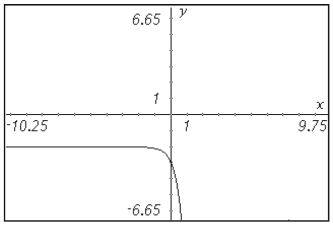
d. This is a translation of $y = 10^x$ left 2 units.



e. This is a vertical stretch of $y = 10^x$ by a factor of 3.



f. This is a reflection of $y = 10^x$ across the x-axis, followed by a translation down 2 units.



5. a. False; $x = \log_6 12$

b. False; $2^x = 5$

c. False; $x = \log_3 5.5 = \frac{\log 5.5}{\log 3}$ because $3^x = \frac{11}{2} = 5.5$.

d. False; $x = \log_3 7$

6. Approximately 25 min. Set $g(x)$ equal to 5 and solve for x.

$$5 = 23(0.94)^x$$

$$0.2174 \approx 0.94^x$$

$$x \approx \log_{0.94} 0.2174$$

$$x \approx \frac{\log 0.2174}{\log 0.94} \approx 24.66$$

7. a. 1977. Substitute 1000 for y in the given equation and solve for x.

$$1000 = 2.07596(1.083415)^x$$

$$481.705 \approx 1.083415^x$$

$$x \approx \log_{1.083415} 481.705$$

$$x \approx \frac{\log 481.705}{\log 1.083415}$$

$$x \approx 77.1$$

Because x represents years after 1900, according to the model the debt passed \$1 trillion in 1977.

b. Approximately 8.3%. $(1 + r) = 1.083415$, so $r \approx 0.083$

c. Approximately 5.6 yr. The doubling time depends only on the ratio, so you can ignore the 2.07596 and assume the function doubles from $y = 1$ to $y = 2$. Set y equal to 2, and solve for x.

$$2 = (1.083415)^x$$

$$x = \log_{1.083415} 2$$

$$x = \frac{\log 2}{\log 1.083415}$$

$$x \approx 8.562$$

The doubling time is about 8.7 yr.

8. a. $y \approx 100(0.999879)^x$. Start with the equation $y = ab^x$, and use the data points (0, 100) and (5730, 50) to find a and b. Using the first point, $100 = ab^0$, so $a = 100$. Now use the second point to find b.

$$50 = 100b^{5730}$$

$$0.5 = b^{5730}$$

$$b = 0.5^{1/5730} \approx 0.999879$$

The equation is $y \approx 100(0.999879)^x$.

b. About 6004 years ago. The technique is approximate and assumes that the carbon-14 concentration in the atmosphere has not changed over the last 6004 years.

$$48.37 \approx 100(0.999879)^x$$

$$0.4837 \approx 0.999879^x$$

$$x \approx \log_{0.999879} 0.4837$$

$$x \approx \frac{\log 0.4837}{\log 0.999879} \approx 6004$$

9. a. $y \approx 88.7(1.0077)^x$. Start with the equation $y = ab^x$, and use the data points (0, 88.7) and (6, 92.9) to find a and b. Using the first point, $88.7 = ab^0$, so $a = 88.7$. Now use the second point to find b.

$$92.9 = 88.7b^6$$

$$1.0474 = b^6$$

$$b = 1.0474^{1/6} \approx 1.0077$$

The equation is $y \approx 88.7(1.0077)^x$.

b. 23 or 24 clicks

$$106.3 \approx 88.7(1.0077)^x$$

$$1.1984 \approx 1.0077^x$$

$$x \approx \log_{1.0077} 1.1984$$

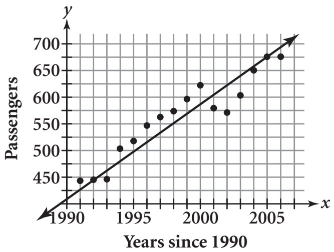
$$x \approx \frac{\log 1.1984}{\log 1.0077} \approx 23.475$$

10. a. $x - 2 = 49^{3/2} = 343$, so $x = 345$

b. $x = 7^{1/2.4} \approx 2.25$

$$3x^{2.4} - 5 = 16$$
$$3x^{2.4} = 21$$
$$x^{2.4} = 7$$
$$x = 7^{1/2.4} \approx 2.25$$

11. a.



$\hat{y} = -1121.25 + 17.1x$

b. $-1.85, 0.15, -6.75, 17.25, 14.45, 26.25, 25.25, 19.25, 24.75, 32.95, -26.45, -51.75, -36.65, -6.75, 1.75, -15.25$

	x	B y	C medy	D residuals	E
◆			=-1121.2!	=b[]-c[]	
1	91	433	434.85	-1.85	8952.9
2	92	452.1	451.95	.15	
3	93	462.3	469.05	-6.75	
4	94	503.4	486.15	17.25	
5	95	517.7	503.25	14.45	
E1	=sum(**residuals**^{2})				

c. $\sqrt{\dfrac{8952.9}{14}} \approx 25.288196$

d. 862.4 million riders; $-1121.25 + 17.1(116) = 82.35$

12. a. $C_1 = 32.7$, $C_2 = 65.4$, $C_3 = 130.8$, $C_6 = 1046.4$, $C_7 = 2092.8$, $C_8 = 4185.6$. To get the next frequency, multiply the previous term by 2, or to get a previous frequency divide by 2.

b. $y = 16.35(2)^x$, where x represents C-note number and y represents frequency in cycles per second. To find this equation, start with the point-ratio equation $y = 261.6(2)^{x-4}$. This is equivalent to $y = 261.6\left(2^{-4} \cdot 2^x\right) = 16.35(2)^x$.

13. a. $y + 1 = x - 3$, or $y = x - 4$. This is the graph of $y = x$ translated down 1 unit and right 3 units.

b. $y + 4 = (x + 5)^2$, or $y = (x + 5)^2 - 4$. This is the graph of $y = x^2$ translated down 4 units and left 5 units.

c. $y - 2 = |x + 6|$, or $y = |x + 6| + 2$. This is the graph of $y = |x|$ translated up 2 units and left 6 units.

d. $y - 7 = \sqrt{x - 2}$, or $y = \sqrt{x - 2} + 7$. This is the graph of $y = \sqrt{x}$ translated up 7 units and right 2 units.

14. a. $\begin{cases} 2l + 2w = 155 \\ l = 2w + 7 \end{cases}$

b. $l = 54$, $w = 23.5$; the length is 54 in. and the width is 23.5 in. Substitute the second equation into the first to get

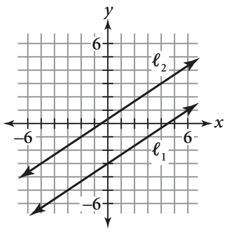
$$2(2w + 7) + 2w = 155$$
$$4w + 14 + 2w = 155$$
$$6w = 141$$
$$w = 23.5$$

Substitute 23.5 for w into the second equation to find l.

$l = 2(23.5) + 7$

$l = 54$

15. a. They are parallel.



b. Possible answer: $A(0, -3)$; $P(1, 1)$; $Q(4, 3)$

c. Possible answer: Translate right 1 unit and up 4 units; $2(x - 1) - 3(y - 4) = 9$.

d. Possible answer: Translate right 4 units and up 6 units; $2(x - 4) - 3(y - 6) = 9$.

e. Possible answer: $2(x - 1) - 3(y - 4) = 9 \rightarrow 2x - 2 - 3y + 12 = 9 \rightarrow 2x - 3y = -1$, which is the equation of ℓ_2. $2(x - 4) - 3(y - 6) = 9 \rightarrow 2x - 8 - 3y + 18 = 9 \rightarrow 2x - 3y = -1$, which is the equation of ℓ_2.

Extensions

A. Answers will vary.

B. Answers will vary.

LESSON 5.7

Support examples

1. $3\log 2 - \log 5 = \log 2^3 - \log 5 = \log 8 - \log 5 = \log \frac{8}{5}$

2. Possible answer: $\log 45 = \log (5 \cdot 9) = \log 5 + \log 9$

3. $\log 2^x = x \log 2$

Exercises

1. a. $\log 5 + \log 11 = \log (5 \cdot 11) = \log 55$

b. $3 \log 2 = \log 2^3 = \log 8$

c. $\log 28 - \log 7 = \log \frac{28}{7} = \log 4$

d. $-2 \log 6 = \log 6^{-2} = \log \frac{1}{6^2} = \log \frac{1}{36}$

e. $\log 7 + 2 \log 3 = \log 7 + \log 3^2 = \log(7 \cdot 9) = \log 63$

2. a. $\log 22 = \log (2 \cdot 11) = \log 2 + \log 11$

b. Answers will vary. Possible answer: $\log 13 = \log \frac{26}{2} = \log 26 - \log 2$

c. $\log 39 = \log (3 \cdot 13) = \log 3 + \log 13$

d. Answers will vary. Possible answer: $\log 7 = \log \frac{14}{2} = \log 14 - \log 2$

3. a. $\log 5^x = x \log 5$

b. $\log x^2 = 2 \log x$

c. $\log \sqrt{3} = \log 3^{1/2} = \frac{1}{2} \log 3$

d. $2 \log 7^x = x \cdot 2 \log 7 = 2x \log 7$

4. a. True; product property of logarithms

b. False; possible answer: $\log 5 + \log 3 = \log 15$

c. True; power property of logarithms

d. True; quotient property of logarithms

e. False; possible answer: $\log 9 - \log 3 = \log \frac{9}{3} = \log 3$

f. False; possible answer: $\log \sqrt{7} = \log 7^{1/2} = \frac{1}{2} \log 7$

g. False; possible answer: $\log 35 = \log 5 + \log 7$

h. True; $\log \frac{1}{4} = \log 1 - \log 4 = -\log 4$. $(\log 1 = 0)$

i. False; possible answer: $\log 3 - \log 4 = \log \frac{3}{4}$

j. True; $\log 64 = \log 2^6 = \log\left(2^4\right)^{6/4} = \log(16)^{3/2} = 1.5 \log 16$ by the power property of logarithms

5. a. $g^h \cdot g^k$; product property of exponents

b. $\log st$; product property of logarithms

c. f^{w-v}; quotient property of exponents

d. $\log h - \log k$; quotient property of logarithms

e. j^{st}; power property of exponents

f. $g \log b$; power property of logarithms

g. $k^{m/n}$; definition of rational exponents

h. $\log_u t$; change-of-base property

i. w^{t+s}; product property of exponents

j. $\frac{1}{p^h}$; definition of negative exponents

6. a. $y \approx 100(0.999879)^x$. Start with the equation $y = ab^x$, and use the data points (0, 100) and (5750, 50) to find a and b. Use the first point to find a. $100 = ab^0$, so $a = 100$. Now use the second point to find b.

$$50 = 100b^{5750}$$
$$0.5 = b^{5750}$$
$$b = 0.5^{1/5750} \approx 0.999879$$

The equation is $y \approx 100(0.999879)^x$.

b. Approximately 11,460 years old. Substitute 25 for y and solve for x.

$$25 \approx 100(0.999879)^x$$
$$0.25 \approx 0.999879^x$$
$$x \approx \frac{\log 0.25}{\log 0.999879} \approx 11460$$

c. 1911 B.C.E. Substitute 62.45 for y and solve for x.

$$62.45 \approx 100(0.999879)^x$$
$$0.6245 \approx 0.999879^x$$
$$x \approx \frac{\log 0.6245}{\log 0.999879} \approx 3891.968$$

Subtract this value from 1981 to get the year the ark was constructed: $1981 - 3891.968 \approx -1910.968$, or about 1911 B.C.E.

d. $y \approx 100(0.999879)^{100{,}000{,}000} \approx 0$. There is virtually nothing left to measure, so you could not use carbon-14 for dating coal unless you had very sensitive instruments to detect the radioactivity.

7. a. Let x represent the note's number of steps above middle C, and let y represent the note's frequency in hertz. $y = 261.6\left(2^{x/12}\right)$ because the starting value is 261.6 and there are 12 intermediate frequencies to get to the last C note, which has double that frequency.

b.

	Note	Frequency (Hz)
Do	C_4	261.6
	C#	**277.2**
Re	D	**293.6**
	D#	**311.1**
Mi	E	**329.6**
Fa	F	**349.2**
	F#	**370.0**
Sol	G	**392.0**
	G#	**415.3**
La	A	**440.0**
	A#	**466.1**
Ti	B	**493.8**
Do	C_5	523.2

8. a. $x \approx 3.3816$. Take the logarithm of both sides to get $\log 5.1^x = \log 247$. Use the power property of logarithms to get $x \log 5.1 = \log 247$. Then $x = \frac{\log 247}{\log 5.1} \approx 3.3816$.

b. $x \approx 11.495$. Subtract 17 from both sides to get $1.25^x = 13$. Take the logarithm of both sides to get $\log 1.25^x = \log 13$. Use the power property of logarithms to get $x \log 1.25 = \log 13$. Then $x = \frac{\log 13}{\log 1.25} \approx 11.495$.

c. $x \approx 11.174$. Divide both sides by 27 to get $0.93^x = \frac{12}{27} = \frac{4}{9}$. Take the logarithm of both sides to get $\log 0.93^x = \log \frac{4}{9}$. Use the power property of logarithms to get $x \log 0.93 = \log \frac{4}{9}$. Divide both sides by $\log 0.93$ to get $x \approx 11.174$.

d. $x \approx 42.739$. Subtract 23 from both sides and then divide both sides by 45 to get $1.024^x = \frac{124}{45}$. Take the logarithm of both sides to get $\log 1.024^x = \log \frac{124}{45}$. Use the power property of logarithms to get $x \log 1.024 = \log \frac{124}{45}$. Divide both sides by $\log 1.024$ to get $x \approx 42.739$.

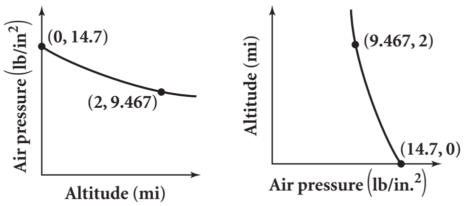
9. a. $y \approx 14.7(0.8022078)^x$. The point-ratio equation is $y = 14.7 \cdot r^x$. Use the data point (2, 9.46) to find r.

$$9.46 = 14.7 \cdot r^2$$

$$0.6435 \approx r^2$$

$$r \approx \sqrt{0.6435} \approx 0.8022078$$

b.



c. Approximately 8.91 lb/in^2. Convert the altitude to miles and substitute that value in the equation from 9a. $\frac{12{,}000 \text{ ft}}{5{,}280 \text{ ft/mi}} \approx 2.27$ mi; $y \approx 14.7(0.8022078)^{2.27} \approx 8.91$.

d. Approximately 6.32 mi. Substitute 3.65 for y and solve for x.

$$3.65 \approx 14.7\left(0.8022078^x\right)$$

$$0.248299 \approx 0.8022078^x$$

$$x \approx \frac{\log 0.248299}{\log 0.8022078} \approx 6.32$$

10. a. $100\% - 3.5\% = 96.5\%$ remains after 1 min.

b. $y = 100(0.965)^x$, where x is time in minutes and y is percentage of carbon-11 remaining. To find the equation, start with the point-ratio equation, $y = ab^x$, and use the data points (0, 100) and (1, 96.5) to find a and b. $100 = ab^0$, so $a = 100$, and $y = 100b^x$. $96.5 = 100b^1$, so $b = 0.965$, and $y = 100(0.965)^x$.

c. Approximately 19.456 min

$$50 = 100(0.965)^x$$

$$0.5 = 0.965^x$$

$$x = \frac{\log 0.5}{\log 0.965} \approx 19.456$$

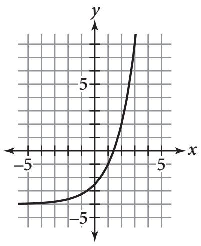
d. In one day the carbon-11 is virtually gone, so you could never date an archaeological find.

11. Graphs will vary. If a horizontal line intersects the graph in more than one point, its inverse is not a function.

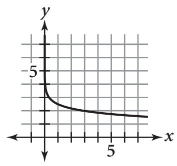
12. a. $y = 5 + 3x$. The common difference, or slope, is 3, so the y-intercept is $8 - 3 = 5$. Therefore the equation is $y = 5 + 3x$.

b. $y = 2 \cdot 3^x$. The y-intercept is 2 and the common ratio is 3, so the equation is $y = 2 \cdot 3^x$.

13. a. The graph has been vertically dilated by a factor of 3, then translated right 1 unit and down 4 units.



b. The graph has been horizontally dilated by a factor of 3, reflected across the x-axis, and translated up 2 units.



14. a. False. If everyone got a grade of 86% or better, one would have to have gotten a much higher grade to be in the 86th percentile.

b. False. Consider the data set {5, 6, 9, 10, 11}. The mean is 8.2; the median is 9.

c. False. Consider the data set {0, 2, 28}. The range is 28; the difference between the mean, 10, and the maximum, 28, is 18.

d. True

15. a. Let h represent the length of time in hours, and let c represent the driver's cost in dollars. $c = 14h + 20$. The domain is the set of possible values of the number of hours, $h > 0$. The range is the set of possible values of the cost paid to the driver, $c > 20$.

b. Let c represent the driver's cost in dollars, and let a represent the agency's charge in dollars. $a = 1.15c + 25$. The domain is the money paid to the driver if she had been booked directly, $c > 20$. The range is the amount charged by the agency, $a > 1.15(20) + 25 = 23 + 25$; $a > 48$.

c. $a = 1.15(14h + 20) + 25$, or $a = 16.1h + 48$

Extensions

A. Research results will vary.

B. Results will vary.

LESSON 5.8

SUPPORT EXAMPLE

a. $x \approx 7.2725$

$$3.15 \cdot (1.1)^x = 6.3$$
$$1.1^x = 2$$
$$x \log 1.1 = \log 2$$
$$x = \frac{\log 2}{\log 1.1}$$
$$x \approx 7.2725$$

b. $4 \cdot 5^x = 7$

$$5^x = 1.75$$
$$x \log 5 = \log 1.75$$
$$x = \frac{\log 1.75}{\log 5}$$
$$x \approx 0.3477$$

EXERCISES

1. a. $\log 800 = x \log 10$, so $x = \dfrac{\log 800}{\log 10} \approx 2.90309$

Check: $10^{2.90309} \approx 800$

b. $\log 2048 = x \log 2$, so $x = \dfrac{\log 2048}{\log 2} = 11$

Check: $2^{11} = 2048$

c. $\log 16 = x \log 0.5$, so $x = \dfrac{\log 16}{\log 0.5} = -4$

Check: $0.5^{-4} = 16$

d. $\frac{478}{18.5} = 10^x$, so $\log \frac{478}{18.5} = x \log 10$,

and $x = \dfrac{\log\left(\frac{478}{18.5}\right)}{\log 10} = \log\left(\frac{478}{18.5}\right) \approx 1.4123$

Check: $18.05(10)^{1.4123} \approx 478$

e. $\frac{155}{24.0} = 1.89^x$, so $\log\left(\frac{155}{24.0}\right) = x \log 1.89$, and

$$x = \frac{\log\left(\frac{155}{24.0}\right)}{\log 1.89} \approx 2.9303$$

Check: $24.0(1.89)^{2.9303} \approx 155$

f. $\frac{0.0047}{19.1} = 0.21^x$, so $\log\left(\frac{0.0047}{19.1}\right) = x \log 0.21$, and

$$x = \frac{\log\left(\frac{0.0047}{19.1}\right)}{\log 0.21} \approx 5.3246$$

Check: $19.1(0.21)^{5.3246} \approx 0.0047$

2. a.

$10^{n+p} = 10^n \cdot 10^p$	Original equation.
$\log\left(10^{n+p}\right) = \log\left(10^n \cdot 10^p\right)$	Take the logarithm of both sides.
$(n+p)\log 10 = \log 10^n + \log 10^p$	Power property and product property of logarithms.
$(n+p)\log 10 = n \log 10 + p \log 10$	Power property of logarithms.
$(n+p)\log 10 = (n+p)\log 10$	Combine terms.

b.

$\left(\frac{10^d}{10^e}\right) = 10^{d-e}$	Original equation.
$\log\left(\frac{10^d}{10^e}\right) = \log\left(10^{d-e}\right)$	Take the logarithm of both sides.
$\log 10^d - \log 10^e = \log\left(10^{d-e}\right)$	Quotient property of logarithms.
$d \log 10 - e \log 10 = (d-e)\log 10$	Power property of logarithms.
$(d-e)\log 10 = (d-e)\log 10$	Combine terms.

3. About 195.9 mo, or about 16 yr 4 mo. The equation for money invested at an annual rate r, compounded n times per year, with initial amount P, is $A = P\left(1 + \frac{r}{n}\right)^t$, where t is the number of compoundings. In this case it is $A = 3000\left(1 + \frac{0.0675}{12}\right)^t$, where t is the number of months. Substitute 9000 for A and solve for t.

$$9000 = 3000\left(1 + \frac{0.0675}{12}\right)^t$$
$$3 = 1.005625^t$$
$$t = \frac{\log 3}{\log 1.005625} \approx 195.9$$

195.9 months is $\frac{195.9 \text{ mo}}{12 \text{ mo/yr}} = 16.321$ yr, and 0.321 yr is (0.325 yr)(12 mo/yr) = 3.9 mo. So, tripling your money will take about 195.9 months, or about 16 years 4 months.

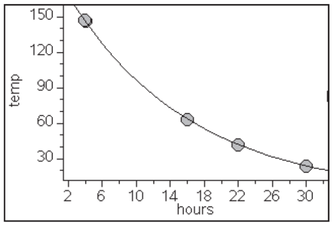
4. a. $h \approx 146(0.9331226)^{T-4}$. Two points on the curve are (4, 146) and (22, 42). Use the point-ratio equation $h = y_1 \cdot b^{T-x_1}$, where h represents the number of hours and T represents the temperature. Using the first point, $h = 146b^{T-4}$. Substitute (22, 42) into the equation to find b; $42 = 146b^{18}$, so $\frac{42}{146} = b^{18}$, and $b = \left(\frac{42}{146}\right)^{1/18} \approx 0.9331226$. The function is $h \approx 146(0.9331226)^{T-4}$.

b. $h \approx 146(0.9331226)^{30-4} \approx 24.1$ hr at 30°C;
$h \approx 146(0.9331226)^{16-4} \approx 63.6$ hr at 16°C.

c. Approximately 3.9°C

$$147 = 146(0.9331226)^{T-4}$$
$$1.00685 = 0.9331226^{T-4}$$
$$T - 4 = \frac{\log 1.00685}{\log 0.9331226}$$
$$T \approx -0.0986 + 4 \approx 3.9$$

d.



e. A realistic domain is 0° to 100°C; these are the freezing and boiling points of water.

5. a. $f(20) \approx 133.28$. After 20 days, 133 games have been sold.

b. $f(80) \approx 7969.17$. After 80 days, 7969 games have been sold.

c. $x \approx 72$. After 72 days, 6000 games have been sold. Make a table to approximate a value for x.

x	f1(x):...
	12000/(
70.	5460.96
71.	5718.24
72.	5976.56
73.	6234.98

$=f1(x):=\frac{12000}{1+499\cdot(1.09)^{-x}}$

$y = 6000$ when $x \approx 72$.

d.

$$6000 = \frac{12000}{1 + 499(1.09)^{-x}}$$

$$6000\left(1 + 499(1.09)^{-x}\right) = 12000$$

$$1 + 499(1.09)^{-x} = 2$$

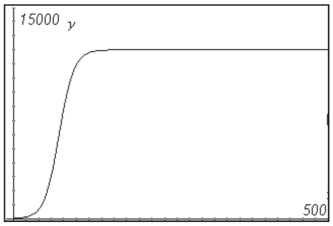
$$499(1.09)^{-x} = 1$$

$$1.09^{-x} = \frac{1}{499}$$

$$-x \log 1.09 = \log \frac{1}{499}$$

$$x = -\frac{\log \frac{1}{499}}{\log 1.09} \approx 72.09$$

e.



Sample answer: The number of games sold starts out increasing slowly, then speeds up, then slows down as everyone who wants the game has purchased one.

6. a. $D = 10\log\left(\frac{10^{-13}}{10^{-16}}\right) = 10\log 10^3 = 10 \cdot 3 = 30$ dB

b. $D = 10\log\left(\frac{3.16 \cdot 10^{-10}}{10^{-16}}\right) = 10\log 3{,}160{,}000 \approx$ 65 dB

c. Approximately 5.01×10^{-6} W/cm^2

$$107 = 10\log\left(\frac{I}{10^{-16}}\right)$$

$$10.7 = \log\left(\frac{I}{10^{-16}}\right)$$

$$10^{10.7} = \left(\frac{I}{10^{-16}}\right)$$

$$I = 10^{10.7-16} \approx 5.01 \times 10^{-6}$$

d. About 3.16 times as loud. Let I_1 be the power of the sound at 47 dB, and let I_2 be the power of the sound at 42 dB.

$$47 = 10\log\left(\frac{I_1}{10^{-16}}\right) \qquad 42 = 10\log\left(\frac{I_2}{10^{-16}}\right)$$

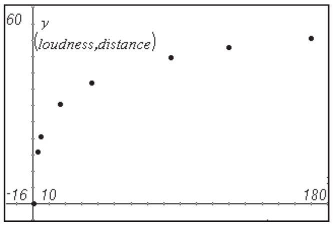
$$4.7 = \log\left(\frac{I_1}{10^{-16}}\right) \qquad 4.2 = \log\left(\frac{I_2}{10^{-16}}\right)$$

$$10^{4.7} = \frac{I_1}{10^{-16}} \qquad 10^{4.2} = \frac{I_2}{10^{-16}}$$

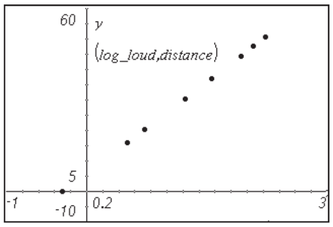
$$I_1 = 10^{4.7}\left(10^{-16}\right) = 10^{-11.3} \qquad I_2 = 10^{4.2}\left(10^{-16}\right) = 10^{-11.8}$$

$$\frac{I_1}{I_2} = \frac{10^{-11.3}}{10^{-11.8}} = 10^{0.5} \approx 3.162$$

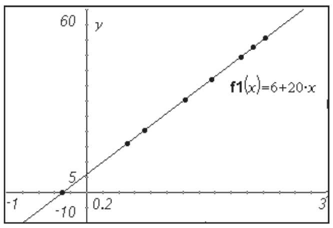
7. a.



b. Points in the form $(\log x, y)$ give a linear graph.

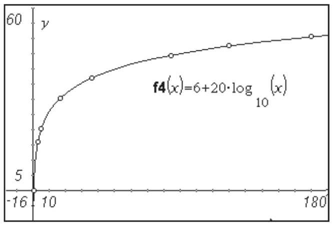


c. $\hat{y} = 6 + 20\log x$. Use your calculator to approximate a line of fit.



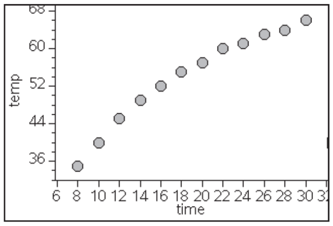
Two points on the line are (0.5, 16) and (2, 46), so the slope is 20. The y-intercept is 6, so the equation of the line is $y = 6 + 20x$. The x-values used to find this line were actually the logarithms of the x-values of the original data. Substitute $\log x$ for x. The final equation is $\hat{y} = 6 + 20 \log x$.

d.

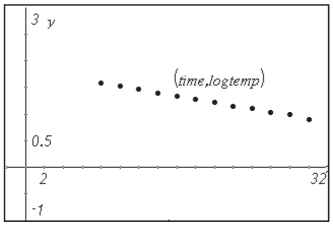


Sample answer: Yes; the graph shows that the equation is a good model for the data.

8. a. Let x represent time in minutes, and let y represent temperature in degrees Fahrenheit.

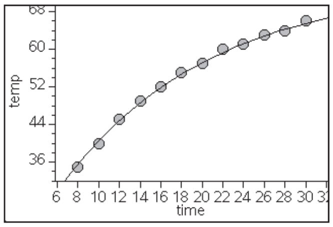


b. $\hat{y} \approx 74 - 68.55(0.9318)^x$. Because the curve is both reflected and translated, first graph points in the form $(x, -y)$ to account for the reflection. Then translate the points up so that the data approach a long-term value of zero. Then graph points in the form $(x, \log(-y + 74))$, which appear to be linear.



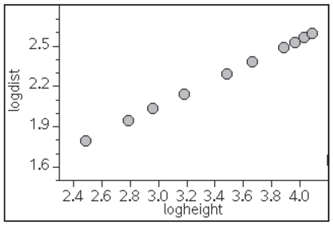
Use your calculator to find the median-median line through the points in the form $(x, \log(-y + 74))$, or approximately $\log(-y + 74) = 1.823 - 0.0298x$. Solving for y gives the exponential equation $\hat{y} \approx 74 - 10^{1.823 - 0.0298x}$, or $\hat{y} \approx 74 - 66.53(0.9337)^x$.

Check that this equation fits the original data.



9. Let x represent height in meters, and let y represent viewing distance in kilometers.

a. The data are the most linear when plotted in the form $(\log x, \log y)$.



b. $\hat{y} = 3.589x^{0.49909}$. Use your calculator to find that the median-median line through the points in the form $(\log x, \log y)$ is approximately $\log y = 0.555 + 0.49909 \log x$. Solving for y gives

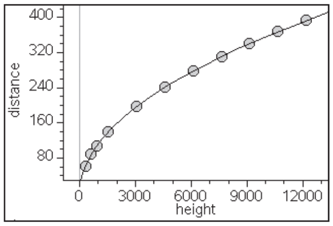
$$y = 10^{0.555 + 0.49909 \log x}$$

$$y = 10^{0.555} \cdot 10^{0.49909 \log x}$$

$$y = 3.589\left(10^{\log x}\right)^{0.49909}$$

$$\hat{y} = 3.589x^{0.49909}$$

Check that this equation fits the original data.

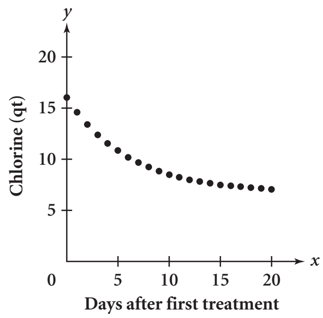


10. a. After 1 day: $16(1 - 0.15) + 1 \approx 14.6$ qt.
After 2 days: $14.6(1 - 0.15) + 1 \approx 13.41$ qt.
The recursive formula is $u_0 = 16$ and $u_n = u_{n-1}(1 - 0.15) + 1$, where $n \geq 1$.

b.

x	y
0	16
1	14.6
2	13.41
3	12.40
4	11.54
5	10.81
6	10.19
7	9.66
8	9.21
9	8.83
10	8.50

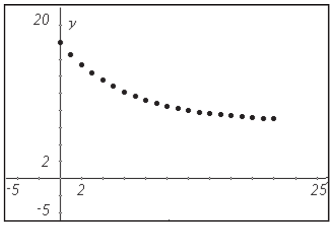
x	y
11	8.23
12	7.99
13	7.80
14	7.63
15	7.48
16	7.36
17	7.26
18	7.17
19	7.09
20	7.03



The sequence is a shifted geometric sequence, which corresponds to a translated exponential equation as an explicit model. To determine the amount of translation, find the long-run value of the sequence. Recall that you do this by assigning the same variable to u_n and u_{n-1}.

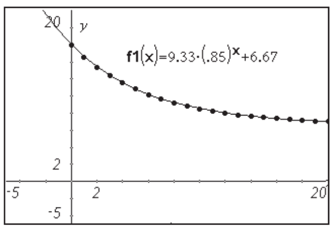
$c = 0.85c + 1$	Substitute c for both u_n and u_{n-1}, and rewrite $(1 - 0.15)$ as 0.85.
$c - 0.85c = 1$	Subtract $0.85c$ from both sides.
$0.15c = 1$	Subtract.
$c = 6.\overline{6}$	Divide both sides by 0.15.

On your calculator, plot points in the form $\left(x, y - 6.\overline{6}\right)$. These points form an exponential curve with long-run value 0.



The explicit exponential equation through these points will be in the form $y = ab^x$. The points still decay at the same rate, so b is still $(1 - 0.15)$, or 0.85. The starting value has been translated, so a is $\left(16 - 6.\overline{6}\right)$, or $9.\overline{3}$. The equation through the points in the form $\left(x, y - 6.\overline{6}\right)$ is $y - 6.\overline{6} = 9.\overline{3}(0.85)^x$. Solving for y gives $y = 9.\overline{3}(0.85)^x + 6.\overline{6}$.

Check that this equation fits the original data.



11. a. $y = 18\left(\sqrt{2}\right)^{x-4}$, $y = 144\left(\sqrt{2}\right)^{x-10}$, or $y = 4.5\left(\sqrt{2}\right)^{x}$. Start by using the point-ratio equation with the first point, $y = 18b^{x-4}$, and use the second point to find b.

$$144 = 18b^{10-4}$$
$$8 = b^6$$
$$b = 8^{1/6} = \left(2^3\right)^{1/6} = 2^{1/2} = \sqrt{2}$$

The equation is $y = 18\left(\sqrt{2}\right)^{x-4}$.

b. $y = \dfrac{\log x - \log 18}{\log\sqrt{2}} + 4$, $y = \dfrac{\log x - \log 144}{\log\sqrt{2}} + 10$, or $y = \dfrac{\log x - \log 4.5}{\log\sqrt{2}}$.

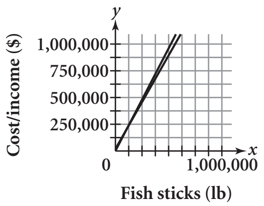
Notice that the points (18, 4) and (144, 10) are on the graph of the inverse of the function from 11a.

Start with $y = 18\left(\sqrt{2}\right)^{x-4}$, switch x and y, and solve for y.

$$x = 18\left(\sqrt{2}\right)^{y-4}$$
$$\frac{x}{18} = \left(\sqrt{2}\right)^{y-4}$$
$$\log\frac{x}{18} = (y-4)\log\sqrt{2}$$
$$\frac{\log\frac{x}{18}}{\log\sqrt{2}} = y - 4$$
$$y = \frac{\log\frac{x}{18}}{\log\sqrt{2}} + 4 = \frac{\log x - \log 18}{\log\sqrt{2}} + 4$$

12. a. Let x represent the amount of fish sticks in pounds, and let y represent cost or income in dollars. Cost: $y = 1.75x + 19{,}000$; income: $y = 1.92x$.

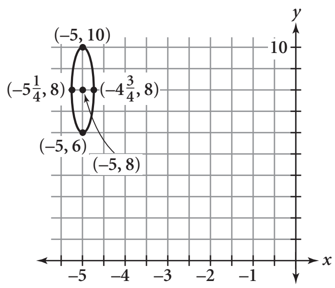
b.



c 111,765 pounds. Find the point where the two lines intersect. $y = 1.75x + 19{,}000$ and $y = 1.92x$, so $1.75x + 19{,}000 = 1.92x$. Therefore, $19{,}000 = 0.17x$, and $x \approx 111{,}765$.

d. \$66,000. The profit is the income minus the cost: *profit* $= 1.92x - (1.75x + 19{,}000) = 0.17x - 19{,}000$. At 500,000 fish, *profit* $= 0.17(500{,}000) - 19{,}000 = 66{,}000$.

13.



This is the ellipse $(4x)^2 + \left(\frac{y}{2}\right)^2 = 1$ translated left 5 units and up 8 units. A horizontal line segment from the center to either side of the ellipse is $\frac{1}{4}$ unit long, so two points are $\left(-5\frac{1}{4}, 8\right)$ and $\left(-4\frac{3}{4}, 8\right)$. A vertical line segment from the center to either the top or bottom of the ellipse is 2 units long, so the ellipse also contains the points $(-5, 10)$ and $(-5, 6)$.

14. a. $x \approx 5.09$

$x^5 = 3418$	Original equation.
$\left(x^5\right)^{1/5} = 3418^{1/5}$	Use the power property of equality and raise both sides to the power of $\frac{1}{5}$.
$x = 3418^{1/5} \approx 5.09$	Power of a power property.

b. $x \approx 9.1$

$(x - 5.1)^4 = 256$	Original equation.
$\left[(x - 5.1)^4\right]^{1/4} = 256^{1/4}$	Use the power property of equality and raise both sides to the power of $\frac{1}{4}$.
$x - 5.1 = 256^{1/4}$	Power of a power property.
$x = 256^{1/4} + 5.1$	Add 5.1 to both sides.
$x = 9.1$	Evaluate.

Note: $x = 1.1$ is also a valid solution. You would find this solution by recognizing that the 4th root of 256 could be 4 or -4. However, because the properties of fractional exponents are defined only for positive bases, this solution may be missed.

c. $x \approx 1.40$

$7.3x^6 + 14.4 = 69.4$	Original equation.
$7.3x^6 = 55$	Subtract 14.4 from both sides.
$x^6 = \frac{55}{7.3}$	Divide both sides by 7.3.
$\left(x^6\right)^{1/6} = \left(\frac{55}{7.3}\right)^{1/6}$	Use the power property of equality and raise both sides to the power of $\frac{1}{6}$.
$x = \left(\frac{55}{7.3}\right)^{1/6} \approx 1.40$	Power of a power property.

Note: $x = -\left(\frac{55}{7.3}\right)^{1/6} \approx -1.40$ is also a valid solution. You would find this solution by recognizing that a 6th root can be positive or negative. However, because the properties of exponents are defined for positive bases, this solution may be missed.

EXPLORATION • THE NUMBER *e*

QUESTIONS

1. Using the new equation, answers will probably be slightly more accurate than before.

2. a. $k \approx 0.0045$

$$0.80 = 1e^{-k(50)}$$

$$\log 0.80 = -50k \log e$$

$$k = \frac{\log 0.80}{-50 \log e} \approx 0.0045$$

b. Approximately 10.23 m

$$0.01 \approx 1e^{-0.0045t}$$

$$\log 0.01 \approx -0.0045t \log e$$

$$t \approx \frac{\log 0.01}{-0.0045 \log e} \approx 1023.37$$

The depth was approximately 1023 cm, or 10.23 m. If you use the unrounded value of k, you get 1031.89 cm $\approx$ 10.32 m.

CHAPTER 5 REVIEW

EXERCISES

1. a. $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

b. $(-3)^{-1} = \frac{1}{-3} = -\frac{1}{3}$

c. $\left(\frac{1}{5}\right)^{-3} = \left(\frac{5}{1}\right)^3 = 125$

d. $49^{1/2} = \sqrt{49} = 7$

e. $64^{-1/3} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}$

f. $\left(\frac{9}{16}\right)^{3/2} = \left[\left(\frac{9}{16}\right)^{1/2}\right]^3 = \left(\sqrt{\frac{9}{16}}\right)^3 = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$

g. $-7^0 = -1$. Calculate the exponent before multiplying.

h. $(3)(2)^2 = (3)(4) = 12$

i. $(0.6^{-2})^{-1/2} = 0.6^{(-2)(-1/2)} = 0.6^1 = 0.6$

2. a. $\log_3 7 = x$, or $x \log 3 = \log 7$

b. $\log_4 5 = x$, or $x \log 4 = \log 5$

c. $\log 7^5 = x$, or $x = 5 \log 7$

3. a. $10^{1.72} = x$

b. $10^{4.8} = x^2$, $10^{2.4} = x$

c. $5^{-1.47} = x$

d. $2^{15} = x^3$, $2^5 = x$

4. a. $\log xy$ by the product property of logarithms

b. $\log z - \log v$ by the quotient property of logarithms

c. $(7x^{2.1})(0.3x^{4.7}) = 2.1x^{2.1+4.7} = 2.1x^{6.8}$ by the associative property and the product property of exponents

d. $k\log w$ by the power property of logarithms

e. $x^{1/5}$ by definition of rational exponents

f. $\frac{\log t}{\log 5}$ by the change-of-base property

5. a. $x = \dfrac{\log 28}{\log 4.7} \approx 2.153$

$$4.7^x = 28$$
$$\log 4.7^x = \log 28$$
$$x\log 4.7 = \log 28$$
$$x = \frac{\log 28}{\log 4.7} \approx 2.153$$

Check: $4.7^{2.153} \approx 28$

b. $x = \pm\sqrt{\dfrac{\log 2209}{\log 4.7}} \approx \pm 2.231$

$$4.7^{x^2} = 2209$$
$$x^2\log 4.7 = \log 2209$$
$$x^2 = \frac{\log 2209}{\log 4.7}$$
$$x = \pm\sqrt{\frac{\log 2209}{\log 4.7}} \approx \pm 2.231$$

Check: $4.7^{\pm 2.231^2} \approx 2209$

c. $x = 2.9^{1/1.25} = 2.9^{0.8} \approx 2.344$. Start by writing the logarithmic equation in exponential form.

Check: $\log_{2.344} 2.9 = \dfrac{\log 2.9}{\log 2.344} \approx 1.25$

d. $x = 3.1^{47} \approx 1.242 \times 10^{23}$. Start by writing the logarithmic equation $\log_{3.1} x = 47$ in exponential form.

Check:

$$\log_{3.1}(1.242 \times 10^{23}) = \frac{\log(1.242 \times 10^{23})}{\log 3.1} \approx 47$$

e. $x = \left(\dfrac{101}{7}\right)^{1/2.4} \approx 3.041$

$$7x^{2.4} = 101$$
$$x^{2.4} = \frac{101}{7}$$
$$x = \left(\frac{101}{7}\right)^{1/2.4} \approx 3.041$$

Check: $7(3.041)^{2.4} \approx 101$

f. $18 = 1.065^x$, so $x = \log_{1.065} 18 = \dfrac{\log 18}{\log 1.065} \approx 45.897$

Check: $500(1.065)^{45.897} \approx 9000$

g. $x = 10^{3.771} \approx 5902$. Start by writing the logarithmic equation in exponential form.

Check: $\log 5902 \approx 3.771$

h. $x = 47^{5/3} \approx 612$

$$\sqrt[5]{x^3} = 47$$
$$x^{3/5} = 47$$
$$x = 47^{5/3} \approx 612$$

Check: $\sqrt[5]{612^3} \approx 47$

6. a. $x \approx 0.825$

$$\sqrt[8]{2432} = 2x + 1$$
$$\sqrt[8]{2432} - 1 = 2x$$
$$x = \frac{\sqrt[8]{2432} - 1}{2} \approx 0.825$$

b. $x \approx 5.779$

$$4x^{2.7} = 456$$
$$x^{2.7} = 114$$
$$x = 114^{1/2.7} \approx 5.779$$

c. $x \approx 9.406$

$$734 = 11.2(1.56)^x$$
$$\frac{734}{11.2} = 1.56^x$$
$$\log\left(\frac{734}{11.2}\right) = x\log 1.56$$
$$x = \frac{\log\left(\frac{734}{11.2}\right)}{\log 1.56} \approx 9.406$$

d. $x = 20.2$ because $f(f^{-1}(x)) = x$.

e. $x \approx 1.962$

$$147 = 12.1(1 + x)^{2.3}$$
$$\frac{147}{12.1} = (1 + x)^{2.3}$$
$$\left(\frac{147}{12.1}\right)^{1/2.3} = 1 + x$$
$$x = \left(\frac{147}{12.1}\right)^{1/2.3} - 1 \approx 1.962$$

f. $x \approx 36.063$

$$2\sqrt{x - 3} + 4.5 = 16$$
$$2\sqrt{x - 3} = 11.5$$
$$\sqrt{x - 3} = 5.75$$
$$x - 3 = 5.75^2$$
$$x = 5.75^2 + 3 \approx 36.063$$

7. About 39.9 h. Start with the equation $y = 45b^x$ by using the point-ratio equation with the initial point (0, 45). The half-life, 16 h, gives $22.5 = 45b^{16}$. Solve this equation for b. $0.5 = b^{16}$, so $b = 0.5^{1/16}$. Thus the equation is $y = 45(0.5)^{1/16^x} = 45(0.5)^{x/16}$. Now solve $8 = 45(0.5)^{x/16}$ for x.

$$\frac{8}{45} = 0.5^{x/16}$$

$$\log \frac{8}{45} = \frac{x}{16} \log 0.5$$

$$x = \frac{16 \log\left(\frac{8}{45}\right)}{\log 0.5} \approx 39.9$$

8. a. $f(2.5) = (4 \cdot 2.5 - 2)^{1/3} - 1 = 8^{1/3} - 1 = 2 - 1 = 1$

b. $f^{-1}(x) = \frac{(x+1)^3 + 2}{4}$. Switch x and y, and then solve for y.

$$x = (4y - 2)^{1/3} - 1$$

$$x + 1 = (4y - 2)^{1/3}$$

$$(x + 1)^3 = 4y - 2$$

$$(x + 1)^3 + 2 = 4y$$

$$y = \frac{(x + 1)^3 + 2}{4}$$

c. $f^{-1}(-1) = \dfrac{(-1 + 1)^3 + 2}{4} = \dfrac{2}{4} = \dfrac{1}{2}$

d. $f\left(f^{-1}(12)\right) = 12$ because $f\left(f^{-1}(x)\right)$ is always x.

9. $y = 5\left(\frac{32}{5}\right)^{(x-1)/6}$. Start with the point-ratio equation using the first point, $y = 5b^{x-1}$. Use the second point to find b.

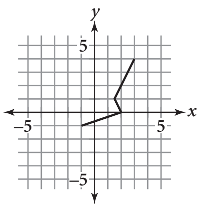
$$32 = 5b^{7-1}$$

$$\frac{32}{5} = b^6$$

$$b = \left(\frac{32}{5}\right)^{1/6}$$

The equation is $y = 5\left[\left(\frac{32}{5}\right)^{1/6}\right]^{x-1}$, or $y = 5\left(\frac{32}{5}\right)^{(x-1)/6}$.

10. Identify the coordinates of several poins on the graph of $y = f(x)$. Switch the x- and y-coordinates to find several points on the graph of $y = f^{-1}(x)$, and then connect them. Alternatively, imagine reflecting the graph across the line $y = x$.



11. a. $a = 0.50$

$$cost = a + b \log t$$

$$0.50 = a + b \log 1$$

$$0.50 = a + b(0)$$

$$a = 0.50$$

b. $b = \dfrac{2.94}{\log 15} \approx 2.4998$

$$cost = 0.50 + b \log t$$

$$3.44 = 0.50 + b \log 15$$

$$2.94 = b \log 15$$

$$b = \frac{2.94}{\log 15} \approx 2.4998$$

c. The t-intercept is about 0.63. The real-world meaning of the t-intercept is that the first 0.63 of a minute of calling is free.

From the work in 11a and b, the equation is $cost \approx 0.50 + 2.4998 \log t$. To find the t-intercept, set $cost = 0$ and solve for t.

$$0 \approx 0.50 + 2.4998 \log t$$

$$-0.50 \approx 2.4998 \log t$$

$$\log t \approx -0.2$$

$$t \approx 10^{-0.2} \approx 0.63$$

d. $cost \approx 0.50 + 2.4998 \log 30 \approx \$4.19$

e. About 4 min

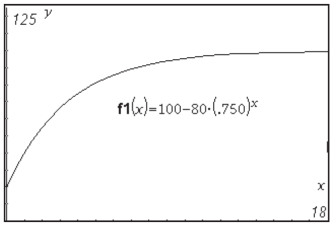
$$2.00 \approx 0.50 + 2.4998 \log t$$

$$1.5 \approx 2.4998 \log t$$

$$0.6 \approx \log t$$

$$t \approx 10^{0.6} \approx 3.98$$

12. a.



b Domain: $0 \leq x \leq 120$; range: $20 \leq y \leq 100$. The domain is limited to positive values because your age can only be a positive real value. A reasonable maximum for age is 120 yr. The range is limited between the size of your head as a newborn and the long-run size as an adult.

c. Vertically dilate by a factor of 80; reflect across the x-axis; translate up 100.

d. 55% of the average adult size. $y = 100 - 80(0.75)^2 = 55$.

e. About 4 years old

$$75 = 100 - 80(0.75)^x$$

$$-25 = -80(0.75)^x$$

$$\frac{5}{16} = 0.75^x$$

$$\log \frac{5}{16} = x \log 0.75$$

$$x = \frac{\log \frac{5}{16}}{\log 0.75} \approx 4.04$$

13. a. Approximately 37 sessions

$$t = -144 \log\left(1 - \frac{40}{90}\right) \approx 37$$

b. Approximately 47.52 wpm

$$47 = -144\log\left(1 - \frac{N}{90}\right)$$

$$-0.326 \approx \log\left(1 - \frac{N}{90}\right)$$

$$10^{-0.326} \approx 1 - \frac{N}{90}$$

$$0.472 \approx 1 - \frac{N}{90}$$

$$-0.528 \approx -\frac{N}{90}$$

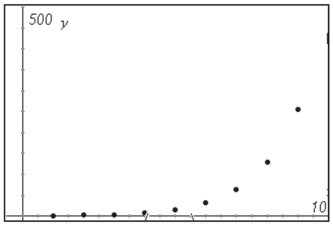
$$N \approx 47.55$$

c. Sample answer: It takes much longer to improve your typing speed as you reach higher levels. 60 wpm is a good typing speed, and very few people type more than 90 wpm, so $0 \le x \le 90$ is a reasonable domain.

14. a. $u_0 = 1$ and $u_n = (u_{n-1}) \cdot 2$, where $n \ge 1$. Humans start as one cell, so $u_0 = 1$. After each cell division, the number of cells doubles, so $u_n = (u_{n-1}) \cdot 2$, where $n \ge 1$.

b. $y = 2^x$. To get from any term in the sequence to the next, you multiply by a factor of 2. Thus the explicit equation is $y = 2^x$.

c.



d. Answers will vary but can include curving upward, increasing, increasing at an increasing rate, and discrete.

e. After 20 cell divisions

$$1{,}000{,}000 = 2^x$$

$$\log 1{,}000{,}000 = x\log 2$$

$$x = \frac{\log 1{,}000{,}000}{\log 2} \approx 19.93$$

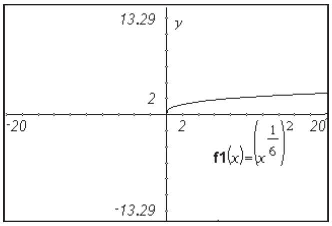
f. After 29 divisions. The number of cells doubles after each division, so one division before there were 1 billion cells, there were 500 million cells.

TAKE ANOTHER LOOK

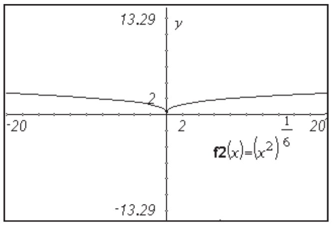
1. When m is even, you are taking an even root. You can take the even root only of a nonnegative number; therefore, the domain of $f(x) = \left(x^{1/m}\right)^n$ is $x \ge 0$ for all values of n. In the expression $\left(x^n\right)^{1/m}$, if n is even, then x^n is positive for all x; you can take an even or an odd root, so the domain is all values of x, no matter the value of m. When m is odd, or when m is even and n is odd, the domain of both equations will be all real numbers, and all y-values will be the same.

Here is an example of different graphs when both m and n are even:

$y = \left(x^{1/6}\right)^2$



$y = \left(x^2\right)^{1/6}$



2. $y = 695.04(0.4898)^x$. It is easier to solve for a in each equation. The first equation yields $a = \frac{40}{b^4}$, and the second equation yields $a = \frac{4.7}{b^7}$. Use substitution to set these two expressions equal to each other and solve for b.

$$\frac{40}{b^4} = \frac{4.7}{b^7}$$

$$\frac{b^7}{b^4} = \frac{4.7}{40}$$

$$b^3 = 0.1175$$

$$b = 0.1175^{1/3} \approx 0.4898$$

Now find a: $a = \frac{40}{b^4} = \frac{40}{0.1175^{4/3}} \approx 695.04$. The equation of the function is $y = ab^x = 695.04(0.4898)^x$.

CHAPTER 6

REFRESHING YOUR SKILLS • PROPERTIES OF REAL NUMBERS

1. A. d B. b C. a D. e E. c

2. a. Sometimes true; commutative property of subtraction; it is not a property for all real numbers.

b. Always true; distributive property of multiplication over addition (or subtraction); it is a property for all real numbers.

c. Never true; associative property of division; it is not a property for all real numbers.

d. Sometimes true; distributive property of addition over exponents; it is not a property for all real numbers.