Discrete Maths: Exercises and Solutions

Basic Structures: Sets, Functions, Sequences, Sums and Matrices

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Much of discrete mathematics is devoted to the study of discrete structures, used to represent discrete objects. Many important discrete structures are built using sets, which are collections of objects.

2.1 Sets:

Introduction: In this section, we study the fundamental discrete structure on which all other discrete structures are built, namely, the set. Sets are used to group objects together. Often, but not always, the objects in a set have similar properties. For instance, all the students who are currently enrolled at any school/college, make up a set. Likewise, all the students currently taking a discrete mathematics course make up a set. In addition, those currently enrolled students, who are taking a course in discrete mathematics form a set that can be obtained by taking the elements common to the first two collections.

Definition: A ***set*** is an unordered collection of objects, called ***elements*** or ***members*** of the set. A set is said to ***contain*** its elements. We write $a \in A$ to denote that ***a* is an element** of the set *A*. The notation $a \notin A$ denotes that ***a* is not an element** of the set *A*.

It is common for sets to be denoted using uppercase letters. Lowercase letters are usually used to denote elements of sets.
There are several ways to describe a set. One way is to list all the members of a set, when this is possible. We use a notation where all members of the set are listed between braces. For example, the notation $\{a, b, c, d\}$ represents the set with the four elements *a, b, c*, and *d*. This way of describing a set is known as the **roster method**.

EXAMPLE 1 The set *V* of all vowels in the English alphabet can be written as $V = \{a, e, i, o, u\}$.

EXAMPLE 2 The set *O* of odd positive integers less than 10 can be expressed by

$$O = \{1, 3, 5, 7, 9\}.$$

Sometimes the roster method is used to describe a set without listing all its members. Some members of the set are listed, and then *ellipses* (. . .) are used when the general pattern of the elements is obvious.
EXAMPLE 3: The set of positive integers less than 100 can be denoted by $\{1, 2, 3, \ldots, 99\}$.

Another way to describe a set is to use **set builder** notation. We characterize all those elements in the set by stating the property or properties they must have to be members. For instance, the set O of all odd positive integers less than 10 can be written as

$O = \{x \mid x$ is an odd positive integer less than $10\}$, or, specifying the universe as the set of positive integers, as $O = \{x \in \mathbf{Z}+ \mid x$ is odd and $x < 10\}$.

We often use this type of notation to describe sets when it is impossible to list all the elements of the set. For instance, the set **Q+** of all positive rational numbers can be written as
$\mathbf{Q}+ = \{x \in \mathbf{R} \mid x = \frac{p}{q}$, for some positive integers p and $q\}$.

These sets (common Universal sets), each denoted using a boldface letter, play an important role in discrete mathematics:
$\mathbf{N} = \{0, 1, 2, 3, \ldots\}$, the set of **natural numbers**
$\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, the set of **integers**
$\mathbf{Z}+ = \{1, 2, 3, \ldots\}$, the set of **positive integers**
$\mathbf{Q} = \{p/q \mid p \in \mathbf{Z}, q \in \mathbf{Z}$, and q _= 0$\}$, the set of **rational numbers**
R, the set of **real numbers**
R+, the set of **positive real numbers**
C, the set of **complex numbers**.

Definition: Two sets are ***equal*** if and only if they have the same elements. Therefore, if A and B are sets, then ***A and B are equal if and only if*** **∀x(x ∈ A ↔ x ∈ B).** We write $A = B$ if A and B are equal sets.

EXAMPLE 4: The sets {1, 3, 5} and {3, 5, 1} are equal, because they have the same elements. Note that the order in which the elements of a set are listed does not matter. Note also that it does not matter if an element of a set is listed more than once, so {1, 3, 3, 3, 5, 5, 5, 5} is the same as the set {1, 3, 5} because they have the same elements.

The empty set: There is a special set that has no elements. This set is called the **empty set**, or **null set**, and is denoted by ∅. The empty set can also be denoted by { }

Definition: The ***set A is a subset of B*** if and only if every element of A is also an element of B. We use the notation $\boldsymbol{A \subseteq B}$ to indicate that A is a subset of the set B.

The Size of a Set

Sets are used extensively in counting problems, and for such applications we need to discuss the sizes of sets.

Definition: Let S be a set. If there are exactly n distinct elements in S where n is a nonnegative integer, we say that S is a *finite set* and that n is the ***cardinality*** of S. The cardinality of S is **denoted by $|S|$.**

EXAMPLE 5 Let A be the set of odd positive integers less than 10. Then $|A| = 5$.

EXAMPLE 6 Let S be the set of letters in the English alphabet. Then $|S| = 26$.

EXAMPLE 7 Because the null set has no elements, it follows that $|\emptyset| = 0$.

Definition: Given a set S, the ***power set of S*** is the set of all subsets of the set S. The power set of S is **denoted by $P(S)$.**

EXAMPLE 8 What is the power set of the set $\{0, 1, 2\}$?
Solution: The power set $P(\{0, 1, 2\})$ is the set of all subsets of $\{0, 1, 2\}$. Hence,
$P(\{0, 1, 2\}) = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$.

Note: If a set has n elements, then its power set has 2^n elements.

Definition: Let A and B be sets. The *Cartesian product* of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. Hence, $A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$.

EXAMPLE 9 What is the Cartesian product of $A = \{1, 2\}$ and $B = \{a, b, c\}$?
Solution: The Cartesian product $A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$.

EXAMPLE 10 What is the Cartesian product $A \times B \times C$, where $A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{0, 1, 2\}$?
Solution: The Cartesian product $A \times B \times C$ consists of all ordered triples (a, b, c), where $a \in A$, $b \in B$, and $c \in C$.
Hence, $A \times B \times C = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2)\}$.

Note : When A, B, and C are sets, $(A \times B) \times C$ is not the same as $A \times B \times C$
We use the notation A^2 to denote $A \times A$, the Cartesian product of the set A with itself.
Similarly, $A^3 = A \times A \times A$, $A^4 = A \times A \times A \times A$, and so on. More generally,
$A^n = \{(a1, a2, \ldots, an) \mid ai \in A \text{ for } i = 1, 2, \ldots, n\}$.

EXAMPLE 11 Suppose that $A = \{1, 2\}$. It follows that $A^2 = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$ and
$A^3 = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)\}$.

Exercise 2.1

1. List the members of these sets.

a) $\{x \mid x$ is a real number such that $x^2 = 1\}$

b) $\{x \mid x$ is a positive integer less than $12\}$

c) $\{x \mid x$ is the square of an integer and $x < 100\}$

d) $\{x \mid x$ is an integer such that $x^2 = 2\}$

2. Use set builder notation to give a description of each of these sets.

a) $\{0, 3, 6, 9, 12\}$

b) $\{-3, -2, -1, 0, 1, 2, 3\}$

c) $\{m, n, o, p\}$

3. Determine whether each of these pairs of sets are equal.

a) $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}$, $\{5, 3, 1\}$

b) $\{\{1\}\}$, $\{1, \{1\}\}$

c) $\emptyset$, $\{\emptyset\}$

4. For each of the following sets, determine whether 2 is an element of that set.

a) $\{x \in \mathbf{R} \mid x$ is an integer greater than $1\}$

b) $\{x \in \mathbf{R} \mid x$ is the square of an integer$\}$

c) $\{2, \{2\}\}$

5. What is the cardinality of each of these sets?

a) $\{a\}$

b) $\{\{a\}\}$

c) $\{a, \{a\}\}$

d) $\{a, \{a\}, \{a, \{a\}\}\}$

6. Find the power set of each of these sets, where a and b are distinct elements.

a) $\{a\}$

b) $\{a, b\}$

c) $\{\emptyset, \{\emptyset\}\}$

7. Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$. Find

a) $A \times B$.

b) $B \times A$.

8. Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find

a) $A \times B \times C$.

b) $C \times B \times A$.

c) $C \times A \times B$.

d) $B \times B \times B$.

9. Find A^2 if

a) $A = \{0, 1, 3\}$.

b) $A = \{1, 2, a, b\}$.

10. Find A^3 if

a) $A = \{a\}$.

b) $A = \{0, a\}$.

2.2 Set Operations:

Introduction: Two, or more, sets can be combined in many different ways. For instance, starting with the set of Computer Science majors at your school and the set of Business majors at your school, we can form the set of students who are Computer Science majors or Business majors, the set of students who are joint majors in Business and Computer science, the set of all students not majoring in Computer Science, and so on.

Definition: Let A and B be sets. The ***union* of the sets *A* and *B***, denoted by $A \cup B$**,** is the set that contains those elements that are either in A or in B, or in both.

An element x belongs to the union of the sets A and B if and only if x belongs to A or x belongs to B. This tells us that $A \cup B = \{x \mid x \in A \vee x \in B\}$.

EXAMPLE 12 The union of the sets {1, 3, 5} and {1, 2, 3} is the set {1, 2, 3, 5};
that is, $\{1, 3, 5\} \cup \{1, 2, 3\} = \{1, 2, 3, 5\}$.

Definition: Let A and B be sets. The ***intersection* of the sets *A* and *B***, denoted by $A \cap B$, is the set containing those elements in both A and B.

An element x belongs to the intersection of the sets A and B if and only if x belongs to A and x belongs to B. This tells us that $A \cap B = \{x \mid x \in A \wedge x \in B\}$.

EXAMPLE 13 The intersection of the sets {1, 3, 5} and {1, 2, 3} is the set {1, 3};
that is, $\{1, 3, 5\} \cap \{1, 2, 3\} = \{1, 3\}$.

Definition: Let A and B be sets. The ***difference* of *A* and *B*,** denoted by $A - B$, is the set containing those elements that are in A but not in B. The difference of A and B is also called the ***complement of B with respect to A*.**

An element x belongs to the difference of A and B if and only if $x \in A$ and $x \notin B$. This tells us That $A - B = \{x \mid x \in A \wedge x \notin B\}$.

EXAMPLE 14 The difference of {1, 3, 5} and {1, 2, 3} is the set {5}; that is, $\{1, 3, 5\} - \{1, 2, 3\} = \{5\}$. This is different from the difference of {1, 2, 3} and {1, 3, 5}, which is the set {2}.

Definition: Let ***U* be the universal set**. The ***complement*** of the set *A,* denoted by $\bar{A}$, is the complement of *A* with respect to *U*. Therefore, the **complement of the set *A* is *U* – *A*.**

An element belongs to *A* if and only if $x \notin A$. This tells us that $A = \{x \in U \mid x \notin A\}$.

EXAMPLE 15 Let $A = \{a, e, i, o, u\}$ (where the universal set is the set of letters of the English alphabet). Then $A = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$.

Exercise 2.2

1. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find
   - **a)** $A \cup B$.
   - **b)** $A \cap B$.
   - **c)** $A - B$.
   - **d)** $B - A$.
2. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find
   - **a)** $A \cup B$.
   - **b)** $A - B$.
   - **c)** $B - A$.
   - **d)** $A \cap B$.
3. Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find
   - **a)** $A \cap B \cap C$.
   - **b)** $A \cup B \cup C$.
   - **c)** $(A \cup B) \cap C$.
   - **d)** $(A \cap B) \cup C$.
4. What can you say about the sets A and B if we know that
   - **a)** $A \cup B = A$?
   - **b)** $A \cap B = A$?
   - **c)** $A - B = A$?
   - **d)** $A \cap B = B \cap A$?

2.3 Functions

Introduction : In many instances we assign to each element of a set a particular element of a second set (which may be the same as the first). For example, suppose that each student in a discrete mathematics class is assigned a letter grade from the set {*A,B,C,D, F*}. And suppose that the grades are *A* for Adams, *C* for Chou, *B* for Goodfriend, *A* for Rodriguez, and *F* for Stevens. This assignment of grades is illustrated in Figure 1.

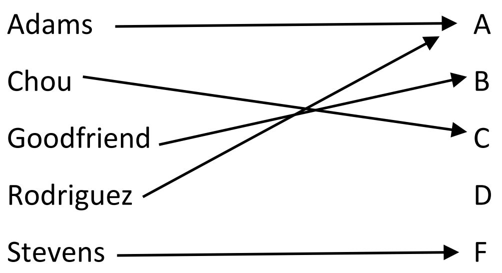

FIGURE 1: Assignment of Grades in a Discrete Mathematics Class.

This assignment is an example of a function. The concept of a function is extremely important in mathematics and computer science. For example, in discrete mathematics functions are used in the definition of such discrete structures as sequences and strings. Functions are also used to represent how long it takes a computer to solve problems of a given size.

Definition: Let *A* and *B* be nonempty sets. A *function f* from *A* to *B* is an assignment of exactly one element of *B* to each element of *A*. We write $f(a) = b$ if *b* is the unique element of *B* assigned by the function *f* to the element *a* of *A*. If *f* is a function from *A* to *B*, we write $f : A \to B$.

Note: Functions are sometimes also called **mappings** or **transformations**.

Definition: If *f* is a function from *A* to *B*, we say that *A* is the ***domain* of *f*** and *B* is the ***codomain* of *f***. If ***f (a)* = *b*,** we say that ***b* is the *image* of *a*** and ***a* is a *preimage* of *b***. The ***range***, or ***image***, of *f* is the set of all images of elements of *A*. Also, if *f* is a function from *A* to *B*, we say that ***f maps A* to *B*.**

EXAMPLE 16 What are the domain, codomain, and range of the function that assigns grades to students described in the first paragraph of the introduction of this section?

Solution: Let *G* be the function that assigns a grade to a student in our discrete mathematics class. Note that *G(*Adams*)* = *A*, for instance.
The domain of *G* is the set {Adams, Chou, Goodfriend, Rodriguez, Stevens}, and
the codomain is the set {*A,B,C,D, F*}.
The range of *G* is the set {*A,B,C, F*}, because each grade except *D* is assigned to some student.

Definition: Let ***f* be a function from *A* to *B*** and let ***S* be a subset of *A***. The ***image* of *S*** under the function f is the subset of B that consists of the images of the elements of S. We denote the image of S by $f(S)$, so $f(S) = \{t \mid \exists s \in S\ (t = f(s))\}$.
We also use the shorthand $\{f(s) \mid s \in S\}$ to denote this set.

EXAMPLE 17 : Let $A = \{a, b, c, d, e\}$ and $B = \{1, 2, 3, 4\}$ with $f(a) = 2$, $f(b) = 1$, $f(c) = 4$, $f(d) = 1$, and $f(e) = 1$. The image of the subset $S = \{b, c, d\}$ is the set $f(S) = \{1, 4\}$.

One-to-One and Onto Functions

Some functions never assign the same value to two different domain elements. These functions are said to be **one-to-one**.

Definition: A function f is said to be ***one-to-one,*** or an ***injunction,*** if and only if $f(a) = f(b)$ implies that $a = b$ for all ***a* and *b*** in the domain of f. A function is said to be ***injective*** if it is one-to-one.

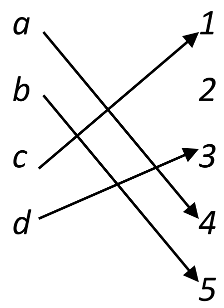

FIGURE 2: A One-to-One Function.

EXAMPLE 18 : Determine whether the function f from $\{a, b, c, d\}$ to $\{1, 2, 3, 4, 5\}$ with $f(a) = 4$, $f(b) = 5$, $f(c) = 1$, and $f(d) = 3$ is one-to-one.

Solution: The function f is one-to-one because f takes on different values at the four elements of its domain.

Definition: A function *f* from ***A* to *B* is called *onto***, or a ***surjection***, if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$. A function *f* is called ***surjective*** if it is onto.

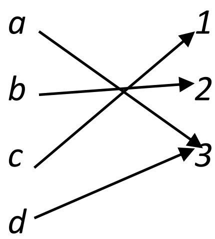

FIGURE 3: An Onto Function.

EXAMPLE 19 : Let *f* be the function from {*a, b, c, d*} to {1, 2, 3} defined by $f(a) = 3$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$. Is *f* an onto function?

Solution: Because all three elements of the codomain are images of elements in the domain, we see that *f* is onto. Note that if the codomain were {1, 2, 3, 4}, then *f* would not be onto.

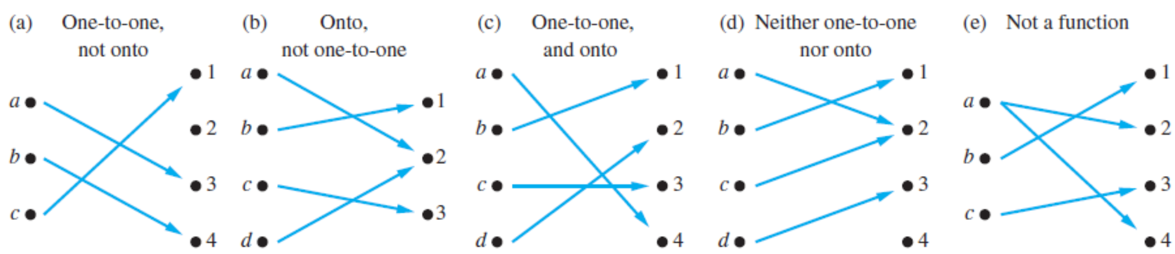

FIGURE 4: Examples of Different Types of Correspondences.

Definition: The function *f* is a ***one-to-one correspondence***, or a ***bijection***, if it is **both one-to-one** and **onto**. We also say that such a function is ***bijective***.

EXAMPLE 20: Let *f* be the function from {*a, b, c, d*} to {1, 2, 3, 4} with $f(a) = 4$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$. Is *f* a bijection?

Solution: The function *f* is one-to-one and onto. It is one-to-one because no two values in the domain are assigned the same function value. It is onto because all four elements of the codomain are images of elements in the domain. Hence, *f* is a bijection.

Definition: Let g **be a function** from the **set** A **to the set** B and let f **be a function** from the **set** B **to the set** C. The ***composition* of the functions** f **and** g, denoted for all $a \in A$ by $f \circ g$, is defined by $(f \circ g)(a) = f(g(a))$.

EXAMPLE 21: Let g be the function from the set $\{a, b, c\}$ to itself such that $g(a) = b$, $g(b) = c$, and $g(c) = a$.
Let f be the function from the set $\{a, b, c\}$ to the set $\{1, 2, 3\}$ such that $f(a) = 3$, $f(b) = 2$, and $f(c) = 1$. What is the composition of f and g, and what is the composition of g and f ?

Solution: The composition $f \circ g$ is defined by $(f \circ g)(a) = f(g(a)) = f(b) = 2$,
$(f \circ g)(b) = f(g(b)) = f(c) = 1$, and $(f \circ g)(c) = f(g(c)) = f(a) = 3$.
Note that $g \circ f$ **is not defined**, because the range of f is not a subset of the domain of g

EXAMPLE 22: Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of f and g? What is the composition of g and f ?

Solution: Both the compositions $f \circ g$ and $g \circ f$ are defined.
Moreover, $(f \circ g)(x) = f(g(x)) = f(3x + 2) = 2(3x + 2) + 3 = 6x + 7$

And $(g \circ f)(x) = g(f(x)) = g(2x + 3) = 3(2x + 3) + 2 = 6x + 11$.

Exercise 2.3

1. Determine whether each of these functions from $\{a, b, c, d\}$ to itself is
   a) one-to-one b) Onto

   i) $f(a) = b, f(b) = a, f(c) = c, f(d) = d$

   ii) $f(a) = b, f(b) = b, f(c) = d, f(d) = c$

   iii) $f(a) = d, f(b) = b, f(c) = c, f(d) = d$

2. Determine whether each of these functions from **Z** to **Z** is one-to-one.

a) $f(n) = n - 1$

b) $f(n) = n^2 + 1$

c) $f(n) = n^3$

3. Which functions in the above question are onto?

4. Determine whether each of these functions is a bijection from **R** to **R**.
   a) $f(x) = 2x + 1$

   b) $f(x) = x^2 + 1$

   c) $f(x) = x^3$

5. Find $\boldsymbol{f \circ g}$ and $\boldsymbol{g \circ f}$, where $\boldsymbol{f(x) = x^2 + 1}$ and $\boldsymbol{g(x) = x + 2}$, are functions from **R** to **R**.

2.4 Sequences and Summations

Introduction: Sequences are ordered lists of elements, used in discrete mathematics in many ways. They are also an important data structure in computer science. We will often need to work with sums of terms of sequences in our study of Discrete Mathematics.

The terms of a sequence can be specified by providing a formula for each term of the Sequence. Identifying a sequence when the first few terms are provided is a useful skill when solving problems in discrete mathematics.

Definition: A ***sequence*** is a function from a subset of the set of integers (usually either the set $\{0, 1, 2, . . .\}$ or the set $\{1, 2, 3, . . .\}$) to a set S. We use the notation a_n to denote the image of the integer n. We call $\boldsymbol{a_n}$ a ***term*** of the sequence

EXAMPLE 23: Consider the sequence $\{a_n\}$, where $a_n = \frac{1}{n}$

The list of the terms of this sequence, beginning with a_1, namely,
$a_1, a_2, a_3, a_4, . . .$, starts with $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, ……… *etc*

Definition: An *arithmetic progression* is a sequence of the form $a, a + d, a + 2d, . . . , a + nd, . . .$ where the *initial term a* and the *common difference d* are real numbers.

Definition: A *geometric progression* is a sequence of the form $a, ar, ar^2, . . . , ar^n, . . .$ where the *initial term a* and the *common ratio r* are real numbers.

EXAMPLE 24 Let $\{an\}$ be a sequence that satisfies the recurrence relation
$a_n = a_{n-1} + 3$ for $n = 1, 2, 3, . . .$, and suppose that $a_0 = 2$. What are a_1, a_2, and a_3?

Solution: We see from the recurrence relation that $a_1 = a_0 + 3 = 2 + 3 = 5$. It then follows that $a_2 = 5 + 3 = 8$ and $a_3 = 8 + 3 = 11$.

EXAMPLE 25 Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-1} - a_{n-2}$ for $n = 2, 3, 4, \ldots,$ and suppose that $a_0 = 3$ and $a_1 = 5$. What are a_2 and a_3?

Solution: We see from the recurrence relation that $a_2 = a_1 - a_0 = 5 - 3 = 2$ and $a_3 = a_2 - a_1 = 2 - 5 = -3$.We can find a_4, a_5, and each successive term in a similar way.

Summations :

Next, we consider the addition of the terms of a sequence. For this we introduce **summation notation**. We begin by describing the notation used to express the sum of the terms $a_m, a_{m+1}, \ldots, a_n$ from the sequence $\{a_n\}$.We use the notation

$$\sum_{j=m}^{n} a^j$$

(read as the sum from $j = m$ to $j = n$ of a_j) to represent $a_m + a_{m+1} + \cdot \quad \cdot \quad \cdot + a_n.$

Here, the variable j is called the **index of summation**, and the choice of the letter j as the variable is arbitrary; that is, we could have used any other letter, such as i or k.

EXAMPLE 26 Use summation notation to express the sum of the first 100 terms of the sequence $\{a_j\}$, where $a_j = \frac{1}{j}$ for j= $1, 2, 3, \ldots.$

Solution: The lower limit for the index of summation is 1, and the upper limit is 100.We write this sum as

$$\sum_{j=1}^{100} \frac{1}{j}$$

EXAMPLE 27: What is the value of $\sum_{j=1}^{5} j^2$

Solution: We have

$\sum_{j=1}^{5} j^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

$= 1 + 4 + 9 + 16 + 25$

$= 55.$

EXAMPLE 28: What is the value of $\sum_{k=4}^{8}(-1)^k$?

Solution: We have

$$\sum_{k=4}^{8}(-1)^k$$

$= (-1)^4 + (-1)^5 + (-1)^6 + (-1)^7 + (-1)^8$
$= 1 + (-1) + 1 + (-1) + 1$
$= 1.$

Exercise 2.4

1. Find these terms of the sequence $\{a_n\}$, where $a_n = 2 \cdot (-3)^n + 5^n$.

a) a_0 b) a_1 c) a_4 d) a_5

2. What is the term a^8 of the sequence $\{a^n\}$ if a^n equals

a) 2^{n-1}? b) 7?

3. What are the terms a_0, a_1, a_2, and a_3 of the sequence $\{a_n\}$, where a_n equals

a) $2^n + 1$?

b) $(-2)^n$?

c) $7 + 4^n$?

4. **4.** Find the first five terms of the sequence defined by each of these recurrence relations and initial conditions.

a) $a^n = 6a_{n-1}$, $a_0 = 2$

b) $a_n = a^2{}_{n-1}$, $a_1 = 2$

c) $a_n = a_{n-1} + 3a_{n-2}$, $a_0 = 1$, $a_1 = 2$

5. What are the values of these sums?

a) $\sum_{k=1}^{5}(k+1)$

b) $\sum_{j=0}^{4}(-2)^j$