Disjunctive Logic Programming: Knowledge Representation Techniques, Systems, and Applications

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Topics

- Context and Motivation
- Datalog
- Theoretical Foundations of DLP
- Knowledge Representation and Applications
- Computational Issues
- DLP Systems
- ASP Development tools

MAIN FOCUS:

- Knowledge Representation and Applications

GOAL:

- Getting a Powerful Tool for Solving Problems in a Fast and Declarative way

Disjunctive Logic Programming (DLP)

Disjunctive Datalog

Disjunctive Databases

Answer Set Programming (ASP)

Roots – declarative programming

- Algorithm = Logic + Control (Kowalski, 1979)
- First-order logic as a programming language
- Expectations, hopes
  - easy programming, fast prototyping
  - handle on program verification
  - advancement of software engineering

Disjunctive Logic Programming (DLP)

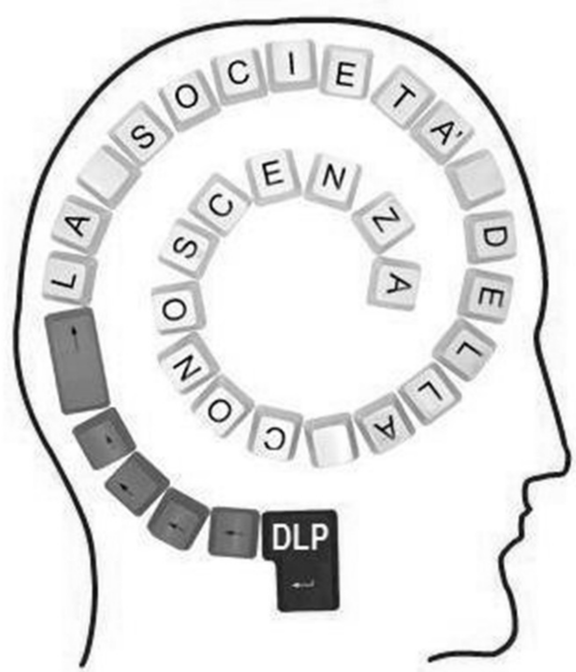
- Simple, yet powerful KR formalism
- Widely used in AI
  - Incomplete Knowledge
- Able to represent complex problems not (polynomially) translatable to SAT
- A declarative problem specification is executable

DLP Advantages

- Sound theoretical foundation (Model Theory)
- Nice formal properties (clear semantics)
- **Real** Declarativeness
  - Rules Ordering, and Goal Orderings is Immaterial!!!
  - Termination is always guaranteed
- High expressive power (Σ^P_2)

DLP Revolution

***INTELLIGENT* PROBLEM SOLVING**



COMPLEX DATA/*KNOWLEDGE* MANIPULATION

DLP Revolution

Why is DLP approach "revolutionary" ? :

DLP Declarative Programming

***vs* Traditional Procedural Programming**

- ***Traditional PROGRAMMING (OLD):***
  - ***Implement an Algorithm to solve the problem***
  - ***List commands or steps that need to be carried out In order to achieve the results***
  - ***Tell the computer "HOW TO" solve the problem***
- **DLP DECLARATIVE PROGRAMMING**
  - ***Specify the features of the desidered solution***
  - ***NO ALGORITHMS***
  - ***Simply Provide a "Problem Specification"***

Drawbacks

- Computing Answer Sets is rather hard (Σ^P_2)
- Very few solid and efficient implementations

  ...but this has started to change:
  - DLV, Clasp, …
  - Cmodels, IDP, …

What is DLP Good for? (Applications)

- Artificial Intelligence, Knowledge Representation & Reasoning
- Information Integration, Data cleaning, Bioinformatics, ...
- Employed for developing industrial applications

Applications

- Planning
- Theory update/revision
- Preferences
- Diagnosis
- Learning
- Description logics and semantic web
- Probabilistic reasoning
- Data integration and question answering
- Multi-agent systems
- Multi-context systems
- Natural language processing/understanding

Applications

- Argumentation
- Product configuration
- Linux package configuration
- Wire routing
- Combinatorial auctions
- Game theory
- Decision support systems
- Logic puzzles
- Bioinformatics
- Phylogenetics
- Haplotype inference

Applications

- System biology
- Automatic music composition
- Assisted living
- Robotics
- Software engineering
- Boundend model checking
- Verification of cryptographic protocols
- E-tourism
- Team building
- Data Cleaning
- Business Games

Datalog

Datalog Syntax: Terms

- Terms are either constants or variables
- Constants can be either symbolic constants (strings starting with some lowercase letter), string constants (quoted strings) or integers.
  - Ex.: pippo, “this is a string constant”, 123, …
- Variables are denoted by strings starting with some uppercase letter.
  - Ex.: X, Pippo, THIS_IS_A_VARIABLE, White, …

Datalog Syntax: Atoms and Literals

- A predicate atom has form $p(t_1,\ldots, t_n)$, where p is a <u>predicate name</u>, $t_1,\ldots, t_n$ are terms, and $n \geq 0$ is the <u>arity</u> of the predicate atom. A predicate atom p() of arity 0 is likewise represented by its predicate name p without parentheses.
  - Ex.: p(X,Y) - next(1,2) - q - i_am_an_atom(1,2,a,B,X)
- An atom can be negated by means of "not".
  - Ex: not a, not p(X), …
- A literal is an atom or a negated atom. In the first case it is said to be positive, while in the second it is said to be negative.

What is Datalog (I)

Datalog is the *non-disjunctive* fragment of DLP.

A (*general*) Datalog program is a set of rules of the form

$$\text{Rule:}\quad \underbrace{a}_{\text{head}} \text{ :- } \underbrace{\overbrace{b_1, \ldots, b_k}^{\text{positive body}},\ \overbrace{\text{not } b_{k+1}, \ldots, \text{not } b_m}^{\text{negative body}}}_{\text{body}} \qquad (1)$$

where "a" and each "b_i" are atoms.

Given a rule r of the form (1) above, we denote by:

- H(r): (head of r), the atom "a"
- B(r): (body of r), the set $b_1, \ldots, b_k$, not $b_{k+1}, \ldots,$ not b_m of all body literals
- $B^+(r)$: (positive body), the set $b_1, \ldots, b_k$ of positive body literals
- $B^-(r)$: (negative body), the set not $b_{k+1}, \ldots,$ not b_m of negative body literals

Positive Datalog

A *positive* (pure) Datalog rule has the following form:

head :- atom1, atom2, …., atom,…

where all the atoms are positive (non-negated).

Ex.: britishProduct(X) :- product(X,Y,P), company(P,“UK”,SP).

Facts

- A ground rule with an empty body is called a fact.
- A fact is therefore a rule with a True body (an empty conjunction is true by definition).
- The implication symbol is omitted for facts

```
parent(eugenio, peppe) :- true.
parent(mario, ciccio) :- true.
```

equivalently written by

```
parent(eugenio, peppe).
parent(mario, ciccio).
```

- Facts must always be true in the program answer!

What is Datalog (II)

We usually distinguish *EDB* predicates and *IDB* predicates

- EDB: predicates appearing only in bodies or in facts.

EDB's can be thought of as stored in a database.

- IDB: predicates defined (also) by rules. IDB's are intensionally defined, appear in both bodies and heads.

Intuitive meaning of a Datalog program:

- Start with the facts in the EDB and iteratively derive facts for IDBs.

Datalog as a Query Language

Datalog has been originally conceived as a query language, in order to overcome some expressive limits of SQL and other languages.

Exercise: write an SQL query retrieving all the cities reachable by flight from Lamezia Terme, through a direct or undirect connection.

Input: A set of direct connections between some cities represented by facts for connected(_,_).

Datalog as a Query Language

Exercise (2): write an SQL query retrieving all the cities indirectly reachable by flight from Lamezia Terme, with a stop/coincidence in a single city.

Exercise (3): write an SQL query retrieving all the cities indirectly reachable by flight from Lamezia Terme, with exactly 2 stops/coincidences in other cities.

Datalog and RECURSION

(original) Exercise: write a query retrieving all the cities reachable by flight from Lamezia Terme, through a direct or undirect connection.

A possible Datalog solution.

Input: A set of direct connections between some cities represented by facts for connected(_,_).

```
reaches(lamezia,B) :- connected(lamezia,B).
reaches(lamezia,C) :- reaches(lamezia,B), connected(B,C).
```

Transitive Closure

Suppose we are representing a graph by a relation *edge(X,Y).*

I want to express the query: *Find all nodes reachable from the others.*

path(X,Y) :- edge(X,Y).
path(X,Y) :- path(X,Z), path(Z,Y).

Recursion (ancestor)

If we want to define the relation of arbitrary ancestors rather than grandparents, we make use of recursion:

```
ancestor(A,B) :-  parent(A,B).
ancestor(A,C) :-  ancestor(A,B), ancestor(B,C).
```

An equivalent representation is

```
ancestor(A,B) :- parent(A,B).
ancestor(A,C) :- ancestor(A,B), parent(B,C).
```

Note the Full Declarativeness

The order of rules and of goals is immaterial:
ancestor(A,B) :- parent(A,B).
ancestor(A,C) :- ancestor(A,B), ancestor(B,C).

is fully equivalent to
ancestor(A,C) :- ancestor(A,B), ancestor(B,C).
ancestor(A,B) :- parent(A,B).

and also to
ancestor(A,C) :- ancestor(B,C), ancestor(A,B).
ancestor(A,B) :- parent(A,B).
NO LOOP!

Datalog Semantics

Later on, we will give the model-theoretic semantics for DLP, and obtain model-theoretic semantics of Datalog as a special case.

We next provide the operational semantics of Datalog, i.e., we specify the semantics by giving a procedural method for its computation.

Semantics: Interpretations and Models

Given a Datalog program P, an interpretation I for P is a set of ground atoms.

An atom “a” is true w.r.t. I if a $\in$ I; it is false otherwise.
A negative literal “not a” is true w.r.t. I if a $\notin$ I; it is false otherwise.

Thus, an interpretation I assigns a meaning to every atom: the atoms in I are true, while all the others are false.

An interpretation I is a MODEL for a ground program P if, for every rule r in P, the H(r) is True w.r.t. I, whenever B(r) is true w.r.t. I

Example: Interpretations

Given the program

```
a :- b, c.
c :- d.
d.
```

and the interpretation

$I = \{c,d\}$

the atoms c and d are true w.r.t. I, while the atoms a and b are false w.r.t. I.

Example: Models

Given the program

r_1: a :- b, c.
r_2: c :- d.
r_3: d.

and the interpretations

I_1 = {b,c,d} I_2={a,b,c,d} I_3={c,d}

we have that I_2 and I_3 are models, while I_1 is not, since the body of r_1 is true w.r.t. to I_1 and the head is false w.r.t. I_1.

Operational Semantics: ground programs

Given a <u>ground</u> <u>positive</u> Datalog program P and an interpretation I, the immediate consequences of I are the set of all atoms "a" such that there exists a rule "r" in P s.t. (1) "*a" is the head of "r",* and (2) *the body of "r" is true w.r.t. I.*

$$Tp(I) = \{ a \mid \exists\, r \in P \text{ s.t. } a = H(r) \text{ and } B(r) \subseteq I \}$$

where H(r) is the head atom, and B(r) is the set of body literals.

Example:
a :- b.   c :- d.   e :- a.   I = {b} → *T*p(I) = {a}.

THEOREM: On a positive Datalog program P, Tp always has a least fixpoint coinciding with the least model of P.

Thus: Start with I={facts in the EDB} and iteratively derive facts for IDBs, applying Tp operator.
Repeat until the least fixpoint is reached.

Operational Semantics: general case (non-ground)

What to do when dealing with a non-ground program?

Start with the EDB predicates, i.e.: “whatever the program dictates”, and with all IDB predicates empty.

Repeatedly examine the bodies of the rules, and see what new IDB facts can be discovered taking into account the EDB *plus* all IDB facts derived until the previous step.

Operational Semantics: Seminaive Evaluation

Since the EDB never changes, on each round we get new IDB tuples only if we use at least one IDB tuple that was obtained on the previous round.

Saves work; lets us avoid rediscovering *most* known facts (a fact could still be derived in a second way…).

<u>Resuming:</u> a new fact can be inferred by a rule in a given round only if it uses in the body some fact discovered on the previous (last) round. But while evaluating a rule, *remember* to take into account also the rest (EDB + all derived IDB).

Operational Semantics: Derivation

Relation can be expressed intentionally through logical rules.

grandParent(X,Y) :- parent (X,Z), parent(Z,Y).
parent(a,b). parent(b,c).

Semantics: evaluate the rules until the *fixpoint* is reached:

<u>Iteration #0:</u> { parent(a,b), parent(b,c) }

<u>Iteration #1:</u> the body of the rule can be instantiated with "*parent(a,b)*", "*parent(b,c)*"
thus deriving { grandParent(a,c) }

<u>Iteration #2:</u> nothing new can be derived (it is easy to see that we derived only "grandParent(a,c)", and no rule having "grandParent" in the body is present). Nothing changes → we stop.

M= { grandParent(a,c), parent(a,b), parent(b,c) }

Operational Semantics: Ancestor

(i) ancestor(X,Y) :- parent (X,Z), parent(Z,Y).
(ii) ancestor(X,Y) :- parent (X,Z), ancestor(Z,Y).
parent(a,b). parent(b,c). parent(c,d).

<u>Iteration #0:</u> { parent(a,b), parent(b,c), parent(c,d) }

<u>Iteration #1:</u> { ancestor(a,c), ancestor(b,d) } (from rule (i))
- useless to evaluate rule (ii): no facts for "ancestor" are true.

<u>Iteration #2:</u>
- useless to evaluate rule (i): body contains only "parent" facts, and no new were derived at last stage;
- some "ancestor" facts were just derived, and "ancestor" appears in the body of rule (ii).

Thus we derive: { ancestor(a,d) } - *Note:* this is derived exploiting "*ancestor(b,d)*" but also "*parent(a,b)*", which was derived before last stage.

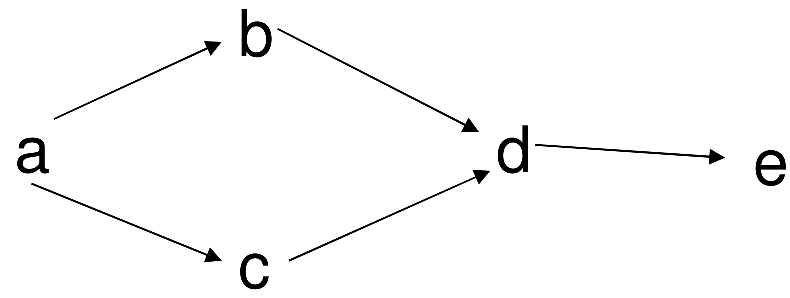
<u>Iteration #3:</u> nothing changes → we stop.

M= { parent(a,b), parent(b,c), parent(c,d), ancestor (a,c), ancestor(b,d), ancestor(a,d) }

Operational Semantics: Transitive Closure

(i) path(X,Y) :- edge(X,Y).
(ii) path(X,Y) :- path(X,Z), path(Z,Y).

edge(a,b). edge (a,c). edge(b,d).
edge(c,d). edge(d,e).



<u>Iteration #0:</u>	Edge: { (a,b), (a,c), (b,d), (c,d), (d,e) }
	Path: { }
<u>Iteration #1:</u>	Path: { (a,b), (a,c), (b,d), (c,d), (d,e) }
<u>Iteration #2:</u>	Path: { (a,d), (b,e), (c,e) }
<u>Iteration #3:</u>	Path: { (a,e) }
<u>Iteration #4:</u>	Nothing changes → We stop.

Note: number of iterations depends on the data. Cannot be anticipated by only looking at the rules!

Negated Atoms

We may put “*not*” in front of an atom, to negate its meaning.

Of course, programs having at least one rule in which negation appears aren’t said to be *positive* anymore.

Example: Think of arc(X,Y) as arcs in a graph.

s(X,Y) singles out the pairs of nodes <a,b> which are not symmetric, i.e., there is an arc from *a* to *b*, but no arc from *b* to *a*.

```
s(X,Y) :- arc(X,Y), not arc(Y,X).
```

Safety

A rule r is *safe* if

- each variable in the head, and
- each variable in a negative literal, and
- each variable in a comparison operator (<,<=, etc.)

also appears in a standard positive literal. In other words, all variables must appear at least once in the positive body.

Only safe rules are allowed.

Ex.: The following rules are unsafe:

- s(X) :- a.
- s(Y) :- b(Y), not r(X).
- s(X) :- not r(X).
- s(Y) :- b(Y), X<Y.

In each case, an infinity of *x*'s can satisfy the rule, even if "*r*" is a finite relation.

Problems with Negation and Recursion

Example:

IDB: p(X) :- q(X), not p(X).

EDB: q(1). q(2).

Iteration #0: q = {(1), (2)}, p = { }

Iteration #1: q = {(1), (2)}, p = {(1), (2)}

Iteration #2: q = {(1), (2)}, p = { }

Iteration #3: q = {(1), (2)}, p = {(1), (2)}

etc., etc. …

Recursion + Negation

"Naïve" evaluation doesn't work when there are negative literals.

In fact, negation wrapped in a recursion makes no sense in general.

Even when recursion and negation are separate, we can have ambiguity about the correct IDB relations.

Stratified Negation

Stratification is a constraint usually placed on Datalog with recursion and negation.

It rules out negation wrapped inside recursion.

Gives the sensible IDB relations when negation and recursion are separate.

Stratified Negation: Definition

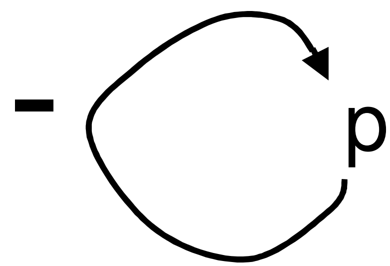
To formalize strata use the labeled *dependency graph*:

- Nodes = IDB predicates.
- Arc *b* -> *a* if predicate *a* depends on *b* (i.e., *b* appears in the body of a rule where *a* appears in the head), but label this arc "–" if the occurrence of *b* is negated.

A Datalog program is *stratified* if NO CYCLE of the labeled dependency graph contains an arc labeled "-".

Example: unstratified program

p(X) :- q(X), not p(X).



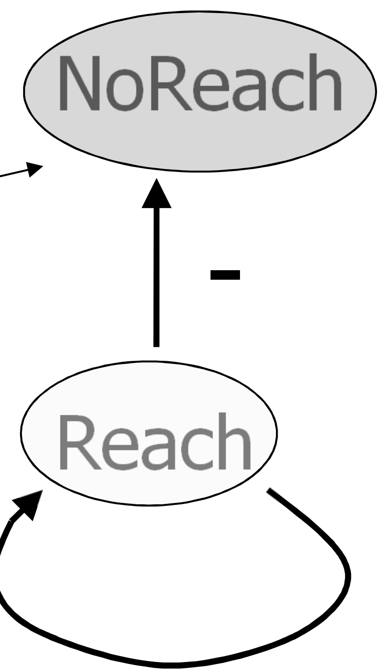
Unstratified: there is a cycle with a “-” arc.

Example: stratified program

EDB = source(X), target(X), arc(X,Y).

Define “targets not reached from any source”:

```
reach(X) :- source(X).
reach(X) :- reach(Y), arc(Y,X).
noReach(X) :- target(X), not reach(X).
```



Stratum 1:
some “-” arc
incoming from
Stratum 0

Stratum 0:
No “–” arcs on
any path in

Minimal Models

As already said, when there is no negation, a Datalog program has a unique minimal (thus minimum) model (one that does not contain any other model).

But with negation, there can be several minimal models.

Example: Multiple Models (1)

a :- not b.

Models: {a} {b}

Both are minimals. But stratification allows us to single out model {a}, which is indeed the (unique) answer set.

Subprograms

DEFINITION: Given a strongly-connected component *C* of the dependency graph of a given program *P*, the subprogram subP(C) is the set of rules with an head predicate belonging to *C*.

Evaluation of Stratified Programs 1

When the Datalog program is stratified, we can evaluate IDB predicates of the lowest-stratum-first.

Once evaluated, treat them as EDB for higher strata.

METHOD: Evaluate bottom-up the subprograms of the components of the dependency graph.

NOTE: The evaluation of a single subprogram is carried out by the (semi)NAÏVE method.

Evaluation of Stratified Programs 2

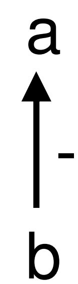
INPUT: EDB F, IDB P

- Compute the labeled dependency graph DG of P;
- Build a topological ordering C1,...,Cn of the components of DG;
- M= F;
- For i=1 To n Do
  - M = SemiNaive(*M* U *subP(Ci)*)
  - *% compute the least fixpoint of Tp on (M U subP(Ci))*
- OUTPUT M;

Stratified Model: example

a :- not b.

b :- d.



Two components: {a} and {b}.

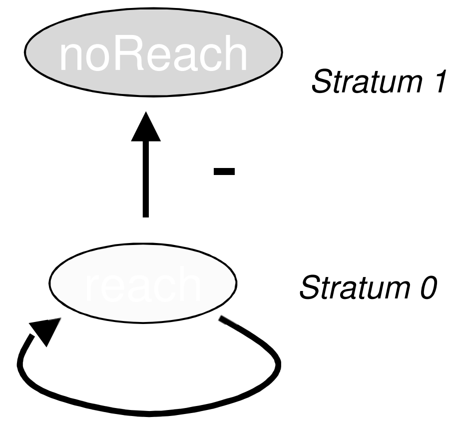
subP({b}) = {b :- d.}   subP({a}) = {a :- not b.}

- {b} is at the lowest stratum -> start evaluating subP({b}).
- The answer set of subP({b}) is AS(subP({b})) = {}.

  → “{}” is the input for subP({a}).
- The answer set of subP({a}) U {} is AS(subP({a})) = {a}, which is the (unique) answer set of the original program.

Example: Stratified Evaluation (2-1)

IDB: reach(X) :- source(X).
reach(X) :- reach(Y), arc(Y,X).
noReach(X) :- target(X), not reach(X).
EDB: node(1). node(2). node(3). node(4).
arc(1,2), arc(3,4). arc(4,3)
source(1), target(2), target(3).



We have two components:
C1 = {reach} C2 = {noReach}

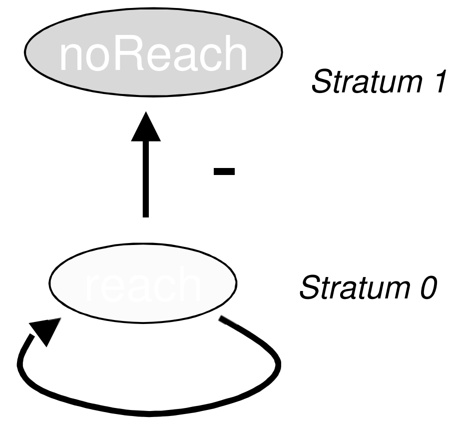
And the related subprograms are:
subP({reach}) = { reach(X) :- source(X).
reach(X) :- reach(Y), arc(Y,X). }
subP({noReach}) = { noReach(X) :- target(X), not reach(X). }

C1 is at a lower stratum w.r.t. C2, thus the subprogram of C1 has to be computed first.

Example: Stratified Evaluation (2-2)

IDB: reach(X) :- source(X).
reach(X) :- reach(Y), arc(Y,X).
noReach(X) :- target(X), not reach(X).

EDB: node(1). node(2). node(3). node(4).
arc(1,2), arc(3,4). arc(4,3)
source(1), target(2), target(3).



Answer Set of subP(C1) U EDB
Iteration #0: facts = { source(1), target(2), target(3),... }
Iteration #1: { reach(1) }
Iteration #2: { reach(2) }
Iteration #3: { } → we stop.

→ M(subP(C1)) = { reach(1), reach(2) + facts }

Answer Set of subP(C2) U M(subP(C1))
Iteration #0: M(subP(C1) = { reach(1), reach(2) + facts }
Iteration #1: { noReach(3) }
Iteration #2: { } → we stop.

→ M(subP(C2)) = { noReach(3), reach(1), reach(2) + facts }

Evaluating through strata →
Answer Set: { reach(1), reach(2), noReach(3), + facts }.

Disjunctive logic programming

Disjunctive Datalog

Answer Set Programming

Foundations of DLP: Syntax and Semantics

a bit boring, but needed....

getFunTomorrow :- resistToday.

(Extended) Disjunctive Logic Programming

Datalog extended with

- full negation (even unstratified)
- disjunction
- integrity constraints
- weak constraints
- aggregate functions
- function symbols, sets, and lists

Disjunctive Logic Programming

SYNTAX

Rule: $a_1 \mid \ldots \mid a_n$:- $b_1, \ldots, b_k$, not b_{k+1} , …, not b_m

Constraints: :- $b_1 , \ldots, b_k$, not b_{k+1} , …, not b_m

Program: A finite Set ***P*** of rules and constraints.

- a_i b_i are atoms
- variables are allowed in atoms' arguments

mother(P,S) | father(P,S) :- parent(P,S).

Example Disjunction

In a blood group knowledge base one may express that the genotype of a parent ***P*** of a person ***C*** is either ***T1*** or ***T2***, if ***C*** is heterozygot with types ***T1*** and ***T2***:

```
genotype(P,T1) | genotype(P,T2) :-
                    parent(P,C), heterozygot(C,T1,T2).
```

In general, programs which contain disjunction can have more than one model.

Arithmetic Built-ins

Fibonacci

```
fib0(1,1).
fib0(2,1).
fib(N,X) :- fib0(N,X).
fib(N,X) :- fib(N1,Y1), fib(N2,Y2),
        +(N2,2,N), +(N1,1,N), +(Y1,Y2,X).
```

Unbound builtins

```
less(X,Y) :- #int(X), #int(Y), X < Y.
num(X) :- *(X,1,X), #int(X).
```

Note that an upper bound for integers has to be specified.

Default Negation

Often, it is desirable to express negation in the following sense: “ If we do not have evidence that X holds, conclude Y.” This is expressed by *default negation* (the operator not).

For example, an agent could act according to the following rule:
“At a railroad crossing, cross the rails if no train approaches”

```
cross_railroad(A) :- crossing(A), not train_approaches(A).
```

Strong Negation

However, in this example default negation is not really the right notion of negation.

It is possible that a train approaches, but that we don.t have any evidence for it (e.g. we do not hear the train). Rather, it would be desirable to definitely know that no train approaches.

This concept is called *strong negation*:

cross_railroad(A) :- crossing(A), -train_approaches(A).

The use of strong negation can lead to *inconsistencies:*

a. -a.

Informal Semantics

Rule: $a_1 \mid \ldots \mid a_n$:- $b_1, \ldots, b_k$, not $b_{k+1}, \ldots,$ not b_m

If all the $b_1 \ldots b_k$ are true and all the $b_{k+1} \ldots b_m$ are false, then at least one among $a_1 \ldots a_n$ is true.

isInterestedinDLP(john) | isCurious(john) :- attendsDLP(john).
attendsDLP(john).

Two (minimal) models, encoding two plausible scenarios:

M1: {attendsDLP(john), isInterestedinDLP(john) }

M2: {attendsDLP(john), isCurious(john) }

Disjunction

is *minimal*

a | b | c $\Rightarrow$ { a }, { b }, { c }

actually *subset minimal*

a | b.
a | c. $\Rightarrow$ {a}, {b,c}

but *not exclusive*

a | b.
a | c. $\Rightarrow$ {a,b}, {a,c}, {b,c}
b | c.

Informal Semantics

Constraints: $:- b_1 , \ldots, b_k , \text{not } b_{k+1} , \ldots, \text{not } b_m$

Discard interpretations which verify the condition

:- hatesDLP(john), isInterestedinDLP(john).

hatesDLP(john).

isInterestedinDLP(john) | isCurious(john) :- attendsDLP(john).

attendsDLP(john).

first scenario ({attendsDLP(john), isInterested(john) }) is discarded.

only one plausible scenario:

M: { attendsDLP(john), hatesDLP(john), isCurious(john) }

Integrity Constraints

When encoding a problem, its solutions are given by the models of the resulting program. Rules usually construct these models. *Integrity constraints* can be used to discard models.

:- L_1, … , L_n.

means: discard models in which L_1, … , L_n are simultaneously true.

a | b.
a | c. $\Rightarrow$ {a,b}, {a,c}, {b,c}
b | c.

:- a. $\Rightarrow$ {b, c}

(Formal) Semantics: Program Instantiation

Herbrand Universe, U_P= Set of constants occurring in program P
Herbrand Base, B_P= Set of ground atoms constructible from U_P and Pred.
Ground instance of a Rule R: Replace each variable in R by a constant in U_P
Instantiation ground(P) of a program P: Set of the ground instances of its rules.

Example: isInterestedinDLP(X) | isCurious(X) :- attendsDLP(X).
attendsDLP(john).
attendsDLP(mary).

U_P={ john, mary }

isInterestedinDLP(john) | isCurious(john) :- attendsDLP(john).
isInterestedinDLP(mary) | isCurious(mary) :- attendsDLP(mary).
attendsDLP(john).
attendsDLP(mary).

A program with variables is just a shorthand for its ground instantiation!

Interpretations and Models

Interpretation I of a program P:

set of ground atoms of P.

Atom q is true in I if q belongs to I; otherwise it is false.

Literal not q is true in I if q is false in I; otherwise it is false.

Interpretation I is a MODEL for a ground program P if, for every R in P, the head of R is True in I, whenever the body of R is true in I

Semantics for Positive Programs

We assume now that Programs are ground (just replace P by ground(P)) and Positive (not - free)

I is an answer set for a positive program P if it is a minimal model (w.r.t. set inclusion) for P

-> Bodies of constraint must be false.

Example (Answer set for a positive program)

isInterestedinDLP(john) | isCurious(john) :- attendsDLP(john).

isInterestedinDLP(mary) | isCurious(mary) :- attendsDLP(mary).

attendsDLP(john).

attendsDLP(mary).

I1 = { attendsDLP(john) } (not a model)

I2 = { isCurious(john), attendsDLP(john), isInterestedinDLP(mary), isCurious(mary), attendsDLP(mary) } (model, non minimal)

I3 = { isCurious(john), attendsDLP(john), isInterestedinDLP(mary), attendsDLP(mary) } (answer set)

I4={ isInterestedinDLP(john), attendsDLP(john), isInterestedinDLP(mary), attendsDLP(mary) } (answer set)

I5 = { isCurious(john), attendsDLP(john), isCurious(mary), attendsDLP(mary) } (answer set)

I6={ isInterestedinDLP(john), attendsDLP(john), isCurious(mary), attendsDLP(mary) } (answer set)

Example (Answer set for a positive program)

Let us ADD:

:- hatesDLP(john), isInterestedinDLP(john).

hatesDLP(john).

(same interpretations as before + hatesDLP(john))

I1 = { attendsDLP(john), hatesDLP(john) } (not a model)

I2 = { isCurious(john), attendsDLP(john), isInterestedinDLP(mary), isCurious(mary), attendsDLP(mary), hatesDLP(john) } (model, non minimal)

I3 = { isCurious(john), attendsDLP(john), isInterestedinDLP(mary), attendsDLP(mary) , hatesDLP(john) } (answer set)

I4={ isInterestedinDLP(john), attendsDLP(john), isInterestedinDLP(mary), attendsDLP(mary), hatesDLP(john) } (not a model)!!!

I5 = { isCurious(john), attendsDLP(john), isCurious(mary), attendsDLP(mary), hatesDLP(john) } (answer set)

I6={ isInterestedinDLP(john), attendsDLP(john), isCurious(mary), attendsDLP(mary), hatesDLP(john) } (not a model)!!!

Semantics for Programs with Negation

Consider general programs (with NOT)

The reduct or of a program P w.r.t. an interpretation I is the positive program P^I, obtained from P by

- deleting all rules with a negative literal false in I;
- deleting the negative literals from the bodies of the remaining rules.

An answer set of a program P is an interpretation I such that I is an answer set of P^I.

Answer Sets are also called Stable Models.

Example (Answer set for a general program)

P: a :- d, not b.
b :- not d.
d.

I = { a, d }

P^I : a :- d.
d.

I is an answer set of P^I and therefore it is an answer set of P.

Answer sets and minimality

An answer set is always a minimal model (also with negation).

In presence of negation minimal models are not necessarily answer sets

P: a :- not b.

Minimal Models: $I1 = \{ a \}$

$I2 = \{ b \}$

Reducts:

P^{I1} : a.

P^{I2} : {}

I1 is an answer set of P^{I1} while I2 is not an answer set of P^{I2} (it is not minimal, since empty set is a model of P^{I2}).

P^{I1} is the only answer set of P.

Datalog Semantics: a special case

The semantics of Datalog is the same as for DLP (Datalog programs are DLP programs).

Since Datalog programs have a simpler form, we can have for Datalog the following characterization:

- *the answer set of a positive datalog program is the least model of P*

  (i.e. the unique minimal model of P).

Why does this work?

THEOREM: A positive Datalog program has always a (unique) minimal model.

PROOF: The intersection of two models is guaranteed to be still a model; thus, only one minimal model exists.

Part II

A (Declarative) Methodology for Programming in DLP

DLP – How To Program?

Idea: encode a search problem ***P*** by a **DLP** program ***LP***.

The answer sets of ***LP*** correspond one-to-one to the solutions of ***P*** .

Rudiments of methodology

- Generate-and-test programming:
  - Generate (possible structures)
  - Weed out (unwanted ones) by adding constraints ("Killing" clauses)
- Separate data from program

“Guess and Check” Programming
Answer Set Programming (ASP)

- A disjunctive rule “guesses” a solution candidate.
- Integrity constraints check its admissibility.

From another perspective:

- The disjunctive rule defines the search space.
- Integrity constraints prune illegal branches.

3-colorability

Input: a Map represented by state(_) and border(_,_).

Problem: assign one color out of 3 colors to each state such that two neighbouring states have always different colors.

Solution:

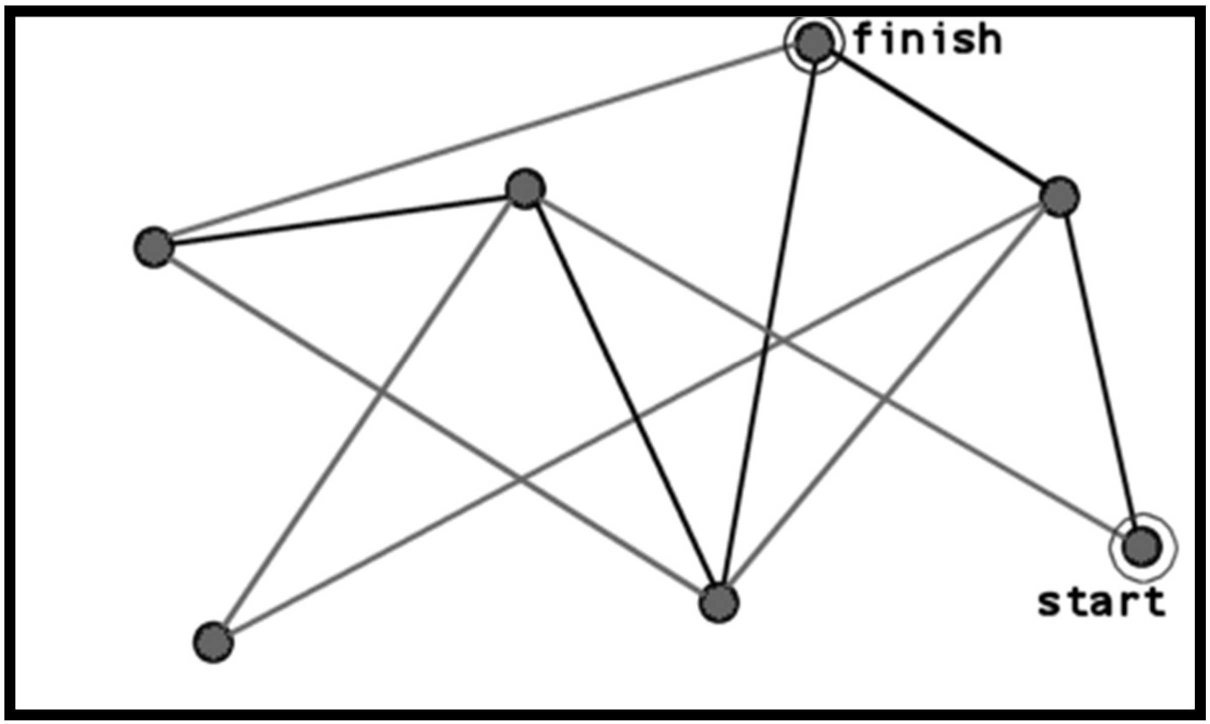
```
col(X,red) | col(X,green) | col(X,blue) :-state(X).   } Guess

:- border(X,Y), col(X,C), col(Y,C).                   } Check
```

Hamiltonian Path (HP) (1)

Input: A directed graph represented by node(_) and arc(_,_), and a starting node start(_).

Problem: Find a path beginning at the starting node which contains all nodes of the graph.



Hamiltonian Path (HP) (2)

Guess

```
inPath(X,Y) | outPath(X,Y) :- arc(X,Y).
```

Check

```
:- inPath(X,Y), inPath(X,Y1), Y <> Y1.
:- inPath(X,Y), inPath(X1,Y), X <> X1.
:- node(X), not reached(X).
:- inPath(X,Y), start(Y). % a path, not a cycle
```

Auxiliary Predicate

```
reached(X) :- start(X).
reached(X) :- reached(Y), inPath(Y,X).
```

Strategic Companies$_{(1)}$

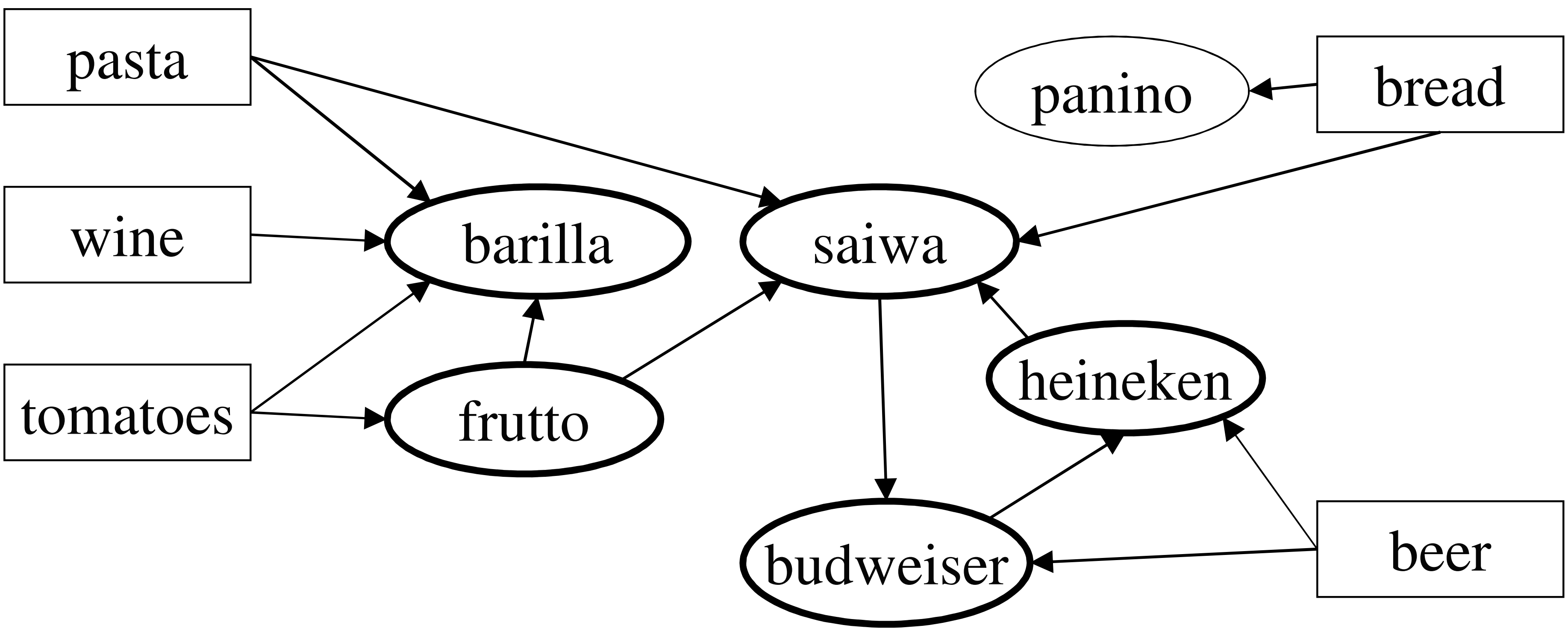
Input: There are various products, each one is produced by several companies.

Problem: We now have to sell some companies. What are the minimal sets of *strategic companies*, such that all products can still be produced? A company also belong to the set, if all its controlling companies belong to it.

strategic(Y) | strategic(Z) :- produced_by(X, Y, Z). **Guess**

strategic(W) :- controlled_by(W, X, Y, Z), strategic(X), strategic(Y), strategic(Z). **Constraints**

Strategic Companies - Example



Complexity Remark

The complexity is in **NP,** if the checking part does not “interfere” with the guess.

“Interference” is needed to represent Σ_2^P problems.

Testing and Debugging with GC

Develop DLP programs incrementally:

- Design the Data Model
  - The way the data are represented (i.e., design predicates and facts representing the input)
- Design the Guess module G first
  - test that the answer sets of G (+the input facts) correctly define the search space
- Then the Check module C
  - verify that the answer sets of G U C are the admissible problem solutions

Use small but meaningful problem test-instances!

Satisfiability

- Boolean, or propositional, satisfiability (abbreviated SAT) is the problem of determining if there exists an interpretation that satisfies a given Boolean formula.
- Conjunctive Normal form (CNF): a formula is a conjunction of clauses, where a clause is a disjunction of boolean variables.

$$\boxed{\Phi = \bigwedge_{i=1}^{n} (d_{i1} \vee \ldots \vee d_{ic_i})}$$

- 3-SAT: only 3-CNF formulas (i.e. exactly three variables for each clause)

$d_{i1} \vee \ldots \vee d_{ic_i}$

$$\boxed{\Phi = \bigwedge_{i=1}^{n} (d_{i1} \vee d_{i2} \vee d_{i3})}$$

- Problem: Find satisfying truth assignments of Φ (if any).

SAT: example

$$(d_1 \vee -d_2 \vee -d_3) \wedge (-d_1 \vee d_2 \vee d_3)$$

- Satisfying assignments:
  - $\{ d_1, \ d_2, \ d_3\}$
  - $\{ d_1, -d_2, \ d_3\}$
  - $\{ d_1, \ d_2, -d_3\}$
  - $\{-d_1, -d_2, \ d_3\}$
  - $\{-d_1, -d_2, -d_3\}$
  - $\{-d_1, \ d_2, -d_3\}$
- Non Satisfying assignments:
  - $\{ d_1, -d_2, -d_3\}$
  - $\{-d_1, \ d_2, \ d_3\}$

SAT: ASP encoding

Add a guessing rule for each propositional variable

$$\forall\ d_i \rightarrow d_i \mid nd_i.$$

Add a constraint for each clause, complementing the variables

$$\forall\ d_{i1} \vee d_{i2} \vee d_{i3} \rightarrow \ :\!-\ L_{i1}, L_{i2}, L_{i3}$$

where L_{ij} = a if d_{ij} = -a, and L_{ij} = not a if d_{ij} = a

Example: SAT → ASP

Formula

$$(d_1 \vee -d_2 \vee -d_3) \wedge (-d_1 \vee d_2 \vee d_3)$$

ASP encoding:

- d_1 | nd_1. :- not d_1, d_2, d_3.
- d_2 | nd_2. :- d_1, not d_2, not d_3.
- d_3 | nd_3.

Answer Sets

{ d1, d2, nd3}	{nd1, nd2, nd3}
{nd1, d2, nd3}	{nd1, nd2, d3}
{ d1, nd2, d3}	{ d1, d2, d3}

Part III

Computational Issues

Computational Issues

Problem: The complexity of DLP is very high (Σ^P_2 and even Δ^P_3), how to deal with that?

Tackle high complexity by isolating simpler sub-tasks

Tool: An in-depth Complexity Analysis

Main Decision Problems

[Brave Reasoning]

Given a DLP program P, and a ground atom A,
is A true in SOME answer sets of P?

[Cautious Reasoning]

Given a DLP program P, and a ground atom A,
is A true in ALL answer sets of P?

A relevant subproblem

[Answer Set Checking]

Given a DLP program P and an interpretation M, is M an answer set of Rules(P)?

Syntactic restrictions on DLP programs

- Head-Cycle Free Property

  [Ben-Eliyahu, Dechter]

- Stratification

  [Apt, Blair, Walker]

Level Mapping: a function || || from ground (classical) literals of the Herbrand Base B_P of P to positive integers.

Stratified Programs

Forbid recursion through negation.

P is (locally) stratified if there is a level mapping $\| \ \|_s$ of P such that for every rule r of P

- For any l in Body+(r), and for any l' in Head(r), $\| l \|_s <= \| l' \|_s$;
- For any not l in Body-(r), and for any l' in Head(r), $\| l \|_s < \| l' \|_s$

Example: A stratified program

P1: p(a) | p(c) :- not q(a).
p(b) :- not q(b).

P1 is stratified:

$||p(a)||_s = 2$, $||p(b)||_s = 2$, $||p(c)||_s = 2$

$||q(a)||_s = 1$, $||q(b)||_s = 1$

Example: An unstratified program

P2: p(a) | p(c) :- not q(b).
q(b) :- not p(a)

P2 is not stratified,
No stratified level mapping exists,
as there is recursion through negation!

Stratification Theorem

- If a program P is stratified and V-free, then P has at most one answer set.
- If, in addition, P does not contain strong negation and integrity constraint, then P has precisely one answer set.
- Under the above conditions, the answer set of P is polynomial-time computable.

Complexity of Answer-Set Checking

	{}	not_s	not
{}	P	P	P
V	coNP	coNP	coNP

Complexity of Brave Reasoning

	{}	not_s	not
{}	P	P	NP
V	Σ^P_2	Σ^P_2	Σ^P_2

Completeness under Logspace reductions

Intuitive Explanation

Three main sources of complexity:

1. the exponential number of answer set "candidates"
2. the difficulty of checking whether a candidate M is an answer set of Rules(P) (the minimality of M can be disproved by exponentially many subsets of M)
3. the difficulty of determining the optimality of the answer set w.r.t. the violation of the weak constraints

The absence of source 1 eliminates both source 2 and source 3

Complexity of Cautious Reasoning

	{}	not_s	not
{}	P	P	coNP
V	coNP	Π^P_2	Π^P_2

Note that < V, { } > is “only” coNP-complete!