Distributed Coordination of Multiple Unknown Euler-Lagrange Systems

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***Abstract*—A robust consensus tracking problem is addressed for multiple unknown Euler-Lagrange systems where only a subset of the agents is informed of the desired time-varying trajectory. Challenging unstructured uncertainties, including unknown nonlinear dynamics and disturbances, are considered in the agent dynamics. A model-free, identifier-based, continuous, distributed robust control method is designed to solve this problem under both undirected and directed graphs. The control inputs and coupling gains depend only on local information and the consensus tracking errors are proven to converge to zero asymptotically. Under an undirected graph, a distributed nonlinear identifier is developed for each agent to compensate for the unknown nonlinear dynamics and disturbances. Based on this identifier, a continuous distributed control law is designed to enable asymptotic robust consensus tracking. By selecting the gains of the designed controller according to the derived conditions, closed-loop stability is proven using graph theory and Lyapunov analysis. Furthermore, the directed graph case is investigated via a distributed two-layer coordination scheme in which a model-free continuous distributed controller is designed by using information obtained from a distributed leader estimator. Numerical simulation results are given to illustrate the effectiveness of the proposed methods.**

***Index Terms*—Asymptotic convergence, distributed model-free identifier design, Lyapunov method, nonlinear multiagent systems, robust consensus tracking, undirected and directed graphs.**

I. Introduction

CONSENSUS of multiagent systems has attracted much attention due to its broad applications in control and robotics, such as transportation, multivehicle systems, and sensor networks. The objective is to design a distributed protocol using local information exchange to achieve an agreement. As an effective consensus seeking approach, the leader-following strategy has received significant interest (e.g., [1]–[5]). In [1], the authors studied a coordinated tracking problem by an observer design. The leader-following consensus of multiagent systems was studied in [3] by an internal model method. The authors in [5] used passivity for constructing a distributed stabilizing feedback law. The aforementioned results focus on single or double-integrator agent dynamics. However, almost all practical systems are inherently nonlinear, and distributed coordination of nonlinear multiagent systems is more challenging.

Distributed coordination of multiple Euler-Lagrange systems has received increasing attention (e.g., [2], [6]–[15]). Euler-Lagrange equations can be used to model a broad class of electromechanical systems, such as autonomous vehicles, robotic manipulators, mobile robots, and rigid bodies. Due to the inherent nonlinearity of Euler-Lagrange systems, the results for linear multiagent systems cannot be directly applied. [6] developed a model-based controller via nonlinear contraction analysis to analyze the coordinated tracking problem. Neural-network-based adaptive controllers were proposed in [8] and [9] to yield uniformly ultimately bounded (UUB) consensus tracking. In [10] and [11], the authors developed distributed algorithms for networked Euler-Lagrange systems in which a discontinuous sliding-mode design was used. Based on the "linearity-in-parameters" assumption, distributed adaptive controllers, combined with discontinuous sliding-mode estimators, were proposed to solve the problem. Note that few results design a continuous model-free distributed controller to deal with unstructured uncertainties of networked Euler-Lagrange systems, especially under a directed graph.

Distributed coordinated tracking under unknown dynamics and disturbances is a practical and challenging problem. The agents are not only affected by the interaction among neighboring agents, but also by its own dynamics. A variety of approaches, such as nonsmooth sliding-mode control [10]–[18], pinning control [19], and neural-network-based control [20]–[22] has been used to solve this problem. A robust finite-time coordinated tracking problem for multiple Euler-Lagrange systems with input disturbances was examined in [16] using a discontinuous sliding-mode controller. In practical implementations, discontinuous control methods may lead to chattering behavior and excite unmodelled high-frequency dynamics [17]. A pinning control scheme was introduced for synchronizing interconnected systems with identical dynamics in [19]. Practically, node dynamics are often nonidentical or even unknown. Based

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on neural-network methods, consensus problems for multiagent systems with uncertainties and external disturbances were proven to yield UUB tracking in [20]–[22]. In contrast to the aforementioned approaches, an identifier-dependent distributed design was proposed [24]. However, compared to the results in this paper, [24] considered only second-order multiagent systems. In previous work [12], consensus tracking of multiple Euler-Lagrange systems was studied under an undirected graph.

This paper addresses a robust consensus tracking problem for heterogeneous multiple Euler-Lagrange systems with unknown dynamics and disturbances. The objective is that a team of agents can achieve robust consensus tracking. A key challenge is to develop an identifier to compensate for the unknown dynamics and disturbances. We leverage a blend of Lyapunov methods and graph theory to resolve this challenge.

A contribution of this paper is that the agents in the network are modelled by nonlinear Euler-Lagrange dynamics. The uncertain nonlinear dynamics present a challenge that inhibits direct application of results developed for linear systems (cf. [1]–[5], [18]–[29]). Moreover, in comparison to results that have been developed for nonlinear dynamics or disturbed linear dynamics (cf., [6]–[29]), a contribution of this work is that the developed continuous distributed controller is model independent, and the model that is used for stability analysis does not include the typical linear in the uncertain parameters assumption. Compared with centralized approaches (cf., [1]–[29]), the developed controller does not require global information about the graph. The developed asymptotic consensus tracking result is achieved under the derived stability conditions. The novelty of this result is also that directed information topology is considered, compared with the results in [6]–[29] that only consider undirected graphs assuming that the upper bounds of the disturbances are exactly known constants, the developed distributed continuous scheme in this paper does not require this prior knowledge. This bidirectional case is nontrivial and brings significant theoretical challenges. Since the information-exchange matrix is asymmetric, it is unclear and challenging on how to develop a model-independent continuous distributed controller to achieve an asymptotic consensus tracking for multiple Euler-Lagrange systems with unknown nonlinear dynamics and disturbances. In this paper, by a distributed two-layer coordination design and by using information obtained from a distributed leader estimator, a novel model-free continuous robust distributed control law is developed to solve this problem.

This paper is organized to provide mathematical development and stability analysis along with numerical simulations. Specifically, in Section II, the relevant concepts on graph theory and nonsmooth analysis are given and a robust coordinated tracking problem is formulated. In Section III, the distributed controller design and stability analysis are both presented for asymptotic robust consensus tracking under an undirected graph. In Section IV, a novel control law via a distributed two-layer design is further presented for a directed graph case and the consensus tracking errors are proven to converge to zero asymptotically. Section V provides the numerical stimulations, followed by the conclusion in Section VI.

II. Problem Formulation

A. Graph Theory

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ represent a graph and $\mathcal{V} \in \{1, \ldots, N\}$ denote the set of vertices. The set of edges is denoted as $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. In this paper, we assume that there is no self loops in the graph, that is, $(i, i) \notin \mathcal{E}$. $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$ denotes the neighborhood set of vertex i. Graph $\mathcal{G}$ is said to be undirected if for any edge $(i, j) \in \mathcal{E}$, edge $(j, i) \in \mathcal{E}$. A matrix $A = [a_{ij}] \in \mathbb{R}^{N \times N}$ denotes the adjacency matrix of $\mathcal{G}$, where $a_{ij} > 0$ if and only if $(j, i) \in \mathcal{E}$, else $a_{ij} = 0$. A matrix $\mathcal{L} \triangleq D - A$ is called the Laplacian matrix of $\mathcal{G}$, where $D = [d_{ii}]$ is a diagonal matrix with $d_{ii} = \sum_{j=1}^{N} a_{ij} \in \mathbb{R}^{N \times N}$.

Consider a distributed coordinated tracking case where the access of agents to the desired trajectory is represented by a diagonal matrix $B = \text{diag}\{b_1, b_2, \ldots, b_N\}$. If $b_i > 0$, then the i-th agent has access to the desired trajectory; otherwise, $b_i = 0$. For further analysis, an information-exchange matrix $H = \mathcal{L} + B$ is defined. To facilitate the subsequent design, we define a virtual leader $\mathcal{V}_0$ whose states are equal to $q_d(t)$ and $\dot{q}_d(t)$.

B. Nonsmooth Analysis

A nonsmooth LaSalle-Yoshizawa Theorem in [33] is recalled to provide boundedness and convergence of solutions for the system $\dot{x} = f(x, t)$ with a discontinuous right-hand side, where $x \in \mathbb{R}^n$ is the state vector and $f(x, t)$ is Lebesgue measurable and essentially locally bounded, uniformly in t.

Lemma 1—[33]: For the above nonsmooth system, let $\mathcal{D} \subset \mathbb{R}^n$ be an open and connected set containing $x = 0$ and suppose that $f(x, t)$ is Lebesgue measurable and essentially locally bounded, uniformly in t. Let a function $V : \mathcal{D} \times [0, \infty) \to \mathbb{R}$ be locally Lipschitz and regular such that $\dot{V}(x, t) \overset{a.e.}{\in} \dot{\tilde{V}}(x, t)$[1] satisfies

$$W_1(x) \leq V(x, t) \leq W_2(x)$$
$$\dot{\tilde{V}}(x, t) \leq -W(x), \quad \forall\, t \geq 0, \quad \forall\, x \in \mathcal{D} \tag{1}$$

where $W_1(x)$ and $W_2(x)$ are continuous positive definite functions and $W(x)$ is a continuous positive semi-definite function on $\mathcal{D}$. Choose $r > 0$ and $c > 0$ such that $B_r \subset \mathcal{D}$ and let $c < \min_{\|x\|=r} W_1(x)$. Then, all Filippov solutions of $\dot{x} = f(x, t)$ with $x(t_0) \in \{x \in B_r | W_2(x) \leq c\}$ are bounded and satisfy $W(x) \to 0$ as $t \to \infty$.

C. Agent Model

Consider a team of N agents governed by the unknown Euler-Lagrange dynamics (e.g., [9])

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) + f_i(\dot{q}_i) + \delta_i = \tau_i \tag{2}$$

where $q_i, \dot{q}_i, \ddot{q}_i \in \mathbb{R}^l$ denote the link position, velocity, and acceleration vectors of generalized configuration coordinates,

[1] Based on the Chain Rule in [33], the function $V(x, t)$ is absolutely continuous and its time derivative exists almost everywhere (a.e), i.e., for almost all $t \in [0, \infty)$ and $\dot{V}(x, t) \overset{a.e.}{\in} \dot{\tilde{V}}(x, t)$ where $\dot{\tilde{V}}(x, t) \triangleq \cap_{\varsigma \in \partial V(x,t)} \varsigma^T \begin{pmatrix} \mathbb{k}[f](x, t) \\ 1 \end{pmatrix}$.

respectively, $M_i(q_i) \in \mathbb{R}^{l\times l}$ is the symmetric positive definite inertia matrix, $C_i(q_i,\dot{q}_i)\dot{q}_i \in \mathbb{R}^l$ is the vector of Coriolis and centrifugal torques, $G_i(q_i) \in \mathbb{R}^l$ is the vector of gravitational torque, $\tau_i \in \mathbb{R}^l$ is the vector of control inputs, $f_i(\dot{q}_i) \in \mathbb{R}^l$ represents the unknown unmodelled dynamics (or frictions), and $\delta_i \in \mathbb{R}^l$ is a time-varying input disturbance. The inertia, Coriolis, gravity and friction terms are all assumed unknown. To facilitate the subsequent analysis, the following results are based on the assumption that $q_i(t)$ and $\dot{q}_i(t)$ are available. Moreover, the following properties [37], [38] of system (2) will be exploited in the subsequent development.

Property 1: The inertia matrix $M_i(q_i)$ is symmetric, positive definite and satisfies $\underline{m}_i\|y\|^2 \le y^T M_i(q_i) y \le \overline{m}_i(q_i)\|y\|^2$, $\forall\, y \in \mathbb{R}^l$, where $\underline{m}_i \in \mathbb{R}$ is a known positive constant, $\overline{m}_i(q_i) \in \mathbb{R}$ is a known positive bounded function.

Property 2: $M_i(q_i)$, $C_i(q_i,\dot{q}_i)$, $G_i(q_i)$ and $f_i(\dot{q}_i)$ are second-order differentiable such that their first and second order partial derivatives exist and are bounded, if q_i, $\dot{q}_i$, $\ddot{q}_i$, $\dddot{q}_i \in \mathcal{L}_\infty$.

The following assumptions are made on the disturbance term δ_i and the virtual leader $\mathcal{V}_0$, respectively.

Assumption 1: The nonlinear disturbance term $\delta_i(t)$ and its first two time derivatives are bounded by some unknown constants (i.e., $\delta_i(t), \dot{\delta}_i(t), \ddot{\delta}_i(t) \in \mathcal{L}_\infty$, $i \in \{1,\ldots,N\}$).

Assumption 2: The desired trajectory $q_d(t)$ of $\mathcal{V}_0$ and its m-order $(m = 1,2,3,4)$ time derivatives are assumed to be bounded by some unknown constants.

D. Control Objective

The control objective is to ensure that the states of all the agents modeled by the multiple unknown Euler-Lagrange systems (2) reach robust consensus asymptotically and track a desired time-varying trajectory which is only provided to a subgroup of agents. The model-free continuous distributed control law for each agent will be synthesized based on local information exchanged from neighboring agents despite the unknown nonlinear dynamics $f_i(\dot{q}_i)$ and disturbances δ_i.

The robust consensus tracking problem is defined as below.

Definition 1—(Robust Consensus Tracking): Design a distributed control law $\tau_i(t)$, $i \in \{1,\ldots,N\}$ such that the states $q_i(t)$ and $\dot{q}_i(t)$ of N Euler-Lagrange systems governed by (2) can reach asymptotic robust consensus, respectively, and track the corresponding desired time-varying signals $q_d(t)$ and $\dot{q}_d(t)$ in the sense that

$$q_i(t) - q_d(t) \to \mathbf{0}_l \quad \text{and} \quad \dot{q}_i(t) - \dot{q}_d(t) \to \mathbf{0}_l \text{ as } t \to \infty$$

for all the agents in the presence of the unknown nonlinear dynamics $f_i(\dot{q}_i)$ and disturbances δ_i.

III. Robust Consensus Tracking Under an Undirected Graph

Before presenting the main result in this section, an assumption on the undirected graph is given as follows.

Assumption 3: The graph $\mathcal{G}$ is undirected and $\mathcal{V}_0$ has directed paths to all the agents of $\mathcal{G}$.

Lemma 2—[2]: Suppose that Assumption 3 holds, then H is positive definite and symmetric.

Since not all the agents have access to the desired trajectory, the relative position and velocity consensus tracking errors $e_{\xi i}(t), e_{xi}(t) \in \mathbb{R}^l$, and $e_{fi}(t) \in \mathbb{R}^l$ are defined using only local information exchanged from neighboring agents as

$$e_{\xi i} = \sum_{j=1}^{N} a_{ij}(q_j - q_i) + b_i(q_d - q_i) \tag{3}$$

$$e_{xi} = \sum_{j=1}^{N} a_{ij}(\dot{q}_j - \dot{q}_i) + b_i(\dot{q}_d - \dot{q}_i) \tag{4}$$

$$e_{fi} = e_{xi} + \alpha_{1i} e_{\xi i},\ i = 1,2,\ldots,N \tag{5}$$

where $\alpha_{1i} \in \mathbb{R}$ is a positive constant gain to be designed.

A. Robust Consensus Tracking Control Law Design

The distributed control protocol is designed as

$$\tau_i(t) = b_i \dot{q}_d(t) - \alpha_{1i}\dot{q}_i(t) + \alpha_{2i} e_{fi}(t) + \hat{f}_i(t) \tag{6}$$

where α_{1i}, $\alpha_{2i} \in \mathbb{R}$ are two control gains and $\hat{f}_i(t) \in \mathbb{R}^l$ denotes a subsequently designed term to compensate for the unknown dynamics and disturbances.

The distributed estimation law for $\hat{f}_i(t)$ is given by

$$\begin{aligned} \dot{\hat{f}}_i(t) &= (k_{si}+1)\left(\dot{e}_{fi}(t)+\alpha_{2i}e_{fi}(t)\right)+b_i\beta_i(t)\odot\eta_i(t)+\theta_i(t)\\ \theta_i(t) &= \sum_{j=1}^{N} a_{ij}\left(\beta_i(t)\odot\eta_i(t) - \beta_j(t)\odot\eta_j(t)\right)\\ \eta_i(t) &= \operatorname{vsgn}\left(e_{fi}(t)\right),\ \hat{f}_i(0) = \hat{f}_{iO} \end{aligned} \tag{7}$$

where $\hat{f}_{iO}$ is an initial condition, k_{si} and α_{2i} are two positive constant control gains, $\beta_i(t) \in \mathbb{R}^l$ is a time-varying control term to be determined later, the symbol $\odot$ denotes the elementwise vector multiplication, and $\operatorname{vsgn}(e_{fi}) \triangleq [\operatorname{sgn}(e_{fi1}), \operatorname{sgn}(e_{fi2}),\ldots,\operatorname{sgn}(e_{\mathrm{fil}})]^T \in \mathbb{R}^l$.

Since $\dot{e}_{fi}(t)$ in (7) is not available, the following form is utilized to generate the estimation signal $\hat{f}_i(t)$[2]:

$$\begin{aligned} \hat{f}_i(t) &= (k_{si}+1)\left(e_{fi}(t) - e_{fi}(0)\right) + \upsilon_i(t),\\ \dot{\upsilon}_i(t) &= (k_{si}+1)\alpha_{2i}e_{fi}(t) + b_i\beta_i(t)\odot\eta_i(t) + \theta_i(t) \end{aligned} \tag{8}$$

where $\beta_i(t)$ introduced in (7) and (8) is defined as

$$\begin{aligned} \beta_i(t) &= \beta_{1i}(t) + \beta_{2i}\mathbf{1}_l,\ \beta_{1i}(0) = \mathbf{0}_l\\ \beta_{1i}(t) &= |e_{fi}(t)| - |e_{fi}(0)| + \alpha_{2i}\int_0^t |e_{fi}(\tau)|\,d\tau \end{aligned} \tag{9}$$

where $|e_{fi}(t)| = [|e_{fi1}(t)|, |e_{fi2}(t)|,\ldots,|e_{\mathrm{fil}}(t)|]^T \in \mathbb{R}^l$, $\beta_{1i}(t) \in \mathbb{R}^l$ and $\beta_{2i}\mathbf{1}_l \in \mathbb{R}^l$ is a positive constant part.

[2]Note that $\hat{f}_i(t)$ is the Filippov solution to the differential equation in (7) that is discontinuous due to the presence of the sign function. Using Filippov's theory of differential inclusions, the existence of solutions can be established for a system with a discontinuous right-hand side: $\dot{\varphi} \in \Bbbk[h](\sigma,t)$, where h is defined as the right-hand side of $\dot{\varphi}$ and $\Bbbk[h](\sigma,t) \triangleq \cap_{\delta>0}\cap_{\mu N=0}\overline{\mathrm{co}}h(B_\delta(\sigma)\setminus N, t)$ as shown in [33] for more details.

B. Filtered Error Dynamics

To facilitate the subsequent stability analysis, define an auxiliary error $r_i(t) \in \mathbb{R}^l$ as

$$r_i = \kappa_i^{-1}\left(\dot{e}_{fi} + \alpha_{2i} e_{fi} + \sum_{j=1,\, j\neq i}^{N} a_{ij} r_j\right) \tag{10}$$

where $\kappa_i = \sum_{j=1, j\neq i}^{N} a_{ij} + b_i$. Defining $r \triangleq \mathrm{col}(r_1, \ldots, r_N)$ and $E_f \triangleq \mathrm{col}(e_{f1}, \ldots, e_{fN}) \in \mathbb{R}^{Nl}$ yields $((D + B) \otimes I_l)r = \dot{E}_f + \Lambda_2 E_f + (A \otimes I_l)r$, where $\Lambda_2 \triangleq \mathrm{diag}(\alpha_{21}, \ldots, \alpha_{2N}) \otimes I_l \in \mathbb{R}^{Nl\times Nl}$ denotes a diagonal matrix, and I_l is a $l \times l$ identity matrix. Thus, $\tilde{H} r = \dot{E}_f + \Lambda_2 E_f$, $\tilde{H} = (\mathcal{L} + B) \otimes I_l$. According to Lemma 2, $\tilde{H}$ is positive definite and symmetric. Thus, it follows from (10) that:

$$r = \tilde{H}^{-1}(\dot{E}_f + \Lambda_2 E_f). \tag{11}$$

C. Closed-Loop System Development

The consensus tracking errors and the control law in (3)–(6), respectively, can be expressed as:

$$\begin{aligned} E_\xi &= -\tilde{H}(Q - Q_d), \quad \dot{E}_\xi = -\tilde{H}(\dot{Q} - \dot{Q}_d),\\ E_f &= \dot{E}_\xi + \Lambda_1 E_\xi, \qquad \Gamma = \tilde{B}\dot{Q}_d - \Lambda_1 \dot{Q} + \Lambda_2 E_f + \hat{F} \end{aligned} \tag{12}$$

which yields the time derivative of E_f

$$\begin{aligned} \dot{E}_f = \tilde{H} M^{-1}(F + C\dot{Q} + G + \delta - \tilde{B}\dot{Q}_d + \Lambda_1\dot{Q} - \hat{F} - \Lambda_2 E_f)\\ - \Lambda_1 \tilde{H}\dot{Q} + \Lambda_1 \tilde{B}\dot{Q}_d + \tilde{B}\ddot{Q}_d \end{aligned} \tag{13}$$

where $Q \triangleq \mathrm{col}(q_1, \ldots, q_N)$, $E_\xi \triangleq \mathrm{col}(e_{\xi 1}, \ldots, e_{\xi N})$, $F \triangleq \mathrm{col}(f_1, \ldots, f_N)$, $\hat{F} \triangleq \mathrm{col}(\hat{f}_1, \ldots, \hat{f}_N)$, $G \triangleq \mathrm{col}(G_1, \ldots, G_N)$, $\delta \triangleq \mathrm{col}(\delta_1, \ldots, \delta_N)$, $Q_d \triangleq (\mathbf{1}_N \otimes q_d)$; and $\tilde{B} \triangleq \mathrm{diag}(b_1, b_2, \ldots, b_N) \otimes I_l$, $\Lambda_1 \triangleq \mathrm{diag}(\alpha_{11}, \ldots, \alpha_{1N}) \otimes I_l$, $M \triangleq \mathrm{diag}(M_1, \ldots, M_N)$ and $C \triangleq \mathrm{diag}(C_1, \ldots, C_N)$.

After premultiplying (11) by M, the following expression is obtained by using (11) and (13) as

$$\begin{aligned} Mr &= F - \hat{F} + G + \delta - (I_{Nl} - M\tilde{H}^{-1}\Lambda_1)\tilde{B}\dot{Q}_d\\ &\quad + M\tilde{H}^{-1}\tilde{B}\ddot{Q}_d - (I_{Nl} - M\tilde{H}^{-1})\Lambda_2 E_f\\ &\quad + (C + \Lambda_1 - M\tilde{H}^{-1}\Lambda_1\tilde{H})\dot{Q}\\ &\triangleq \Omega - \hat{F} - (I_{Nl} - M\tilde{H}^{-1})\Lambda_2 E_f + \delta - \tilde{B}\dot{Q}_d \end{aligned} \tag{14}$$

where $\Omega \triangleq F + G + (C + \Lambda_1 - M\tilde{H}^{-1}\Lambda_1\tilde{H})\dot{Q} + M\tilde{H}^{-1}(\tilde{B}\ddot{Q}_d + \Lambda_1\tilde{B}\dot{Q}_d) \in \mathbb{R}^{Nl}$ and I_{Nl} is an identity block matrix.

According to (7)–(9), the time derivative of $\hat{F}$ is given by

$$\begin{aligned} \dot{\hat{F}} &= (K_s + I_{Nl})(\dot{E}_f + \Lambda_2 E_f) + \tilde{H}\mathrm{sgn}(E_f)\Pi_1\\ &\quad + \tilde{H}\mathrm{sgn}(E_f)\Pi_2 \mathbf{1}_{Nl} \end{aligned} \tag{15}$$

$$\dot{\Pi}_1 = \mathrm{sgn}(E_f)(\dot{E}_f + \Lambda_2 E_f) \tag{16}$$

where $\mathrm{sgn}(E_f) \triangleq \mathrm{diag}(\mathrm{dsgn}(e_{f1}), \ldots, \mathrm{dsgn}(e_{fN}))$, $\mathrm{dsgn}(e_{fi}) \triangleq \mathrm{diag}(\mathrm{sgn}(e_{fi1}), \ldots, \mathrm{sgn}(e_{\mathrm{fil}}))$, $K_s \triangleq \mathrm{diag}(k_{s1}, \ldots, k_{sN}) \otimes I_l$, $\Pi_2 \triangleq \mathrm{diag}(\beta_{21}, \ldots, \beta_{2N}) \otimes I_l \in \mathbb{R}^{Nl\times Nl}$ and $\Pi_1(t) \triangleq \mathrm{col}(\beta_{11}(t), \ldots, \beta_{1N}(t)) \in \mathbb{R}^{Nl}$.

To obtain the closed-loop error system, differentiating (15) and rewriting it utilizing (11) and (15) gives

$$\begin{aligned} M\dot{r} = -\frac{1}{2}\dot{M}r - (K_s + I_{Nl})\tilde{H}r - \tilde{H}E_f + \dot{\delta} - \tilde{B}\ddot{Q}_d\\ - \tilde{H}\mathrm{sgn}(E_f)(\Pi_1 + \Pi_2\mathbf{1}_{Nl}) + N(t) \end{aligned} \tag{17}$$

where $N(t) \in \mathbb{R}^{Nl}$ denotes an auxiliary term defined as

$$\begin{aligned} N(t) \triangleq \dot{\Omega} - \left[\frac{1}{2}\dot{M} + (I_{Nl} - M\tilde{H}^{-1})\Lambda_2\tilde{H}\right] r\\ + \left[\tilde{H} + \dot{M}\tilde{H}^{-1}\Lambda_2 + (I_{Nl} - M\tilde{H}^{-1})\Lambda_2^2\right] E_f. \end{aligned} \tag{18}$$

To facilitate the subsequent analysis, another unmeasurable auxiliary term $N_d(t) \in \mathbb{R}^{Nl}$ is defined as

$$\begin{aligned} N_d(t) &\triangleq N_d(Q_d, \dot{Q}_d, \ddot{Q}_d, \dddot{Q}_d, \mathbf{0}_{Nl}, \mathbf{0}_{Nl}, \mathbf{0}_{Nl})\\ &= \frac{\partial F(\dot{Q}_d)}{\partial \dot{Q}_d}\ddot{Q}_d + \frac{\partial G(Q_d)}{\partial Q_d}\dot{Q}_d + C(Q_d, \dot{Q}_d)\ddot{Q}_d\\ &\quad + \dot{C}(Q_d, \dot{Q}_d)\dot{Q}_d - \dot{M}(Q_d)\tilde{H}^{-1}\Lambda_1\tilde{H}\dot{Q}_d\\ &\quad + \left[\dot{M}(Q_d)\tilde{H}^{-1}\tilde{B} + (\Lambda_1 - M(Q_d)\tilde{H}^{-1}\Lambda_1\tilde{H})\right]\ddot{Q}_d\\ &\quad + M(Q_d)\tilde{H}^{-1}\tilde{B}\dddot{Q}_d + \dot{M}(Q_d)\tilde{H}^{-1}\Lambda_1\tilde{B}\dot{Q}_d\\ &\quad + M(Q_d)\tilde{H}^{-1}\Lambda_1\tilde{B}\ddot{Q}_d. \end{aligned} \tag{19}$$

After adding and subtracting $N_d(t)$ in (19), the closed-loop error system in (17) can be expressed as

$$\begin{aligned} M\dot{r} = -\frac{1}{2}\dot{M}r - (K_s + I_{Nl})\tilde{H}r + \tilde{\Delta}(t) + \tilde{H}\Delta_d(t)\\ - \tilde{H}E_f - \tilde{H}\mathrm{sgn}(E_f)(\Pi_1 + \Pi_2\mathbf{1}_{Nl}) \end{aligned} \tag{20}$$

where the unmeasurable auxiliary term $\tilde{\Delta}(t) \in \mathbb{R}^{Nl}$ is

$$\tilde{\Delta}(t) \triangleq N(t) - N_d(t),\ \Delta_d(t) \triangleq \tilde{H}^{-1}(N_d(t) + \dot{\delta} - \tilde{B}\ddot{Q}_d).$$

Based on Properties 1 and 2, Assumptions 1 and 2, the auxiliary term $\Delta_d(t)$ and its time derivative satisfy

$$\sup_{t\in[0,\infty)} |\Delta_{d_m}(t)| < c_{1_m}, \quad \sup_{t\in[0,\infty)} \left|\dot{\Delta}_{d_m}(t)\right| < c_{2_m} \tag{21}$$

where $m = 1, 2, \ldots, Nl$, $\Delta_{d_m}(t)$ and $\dot{\Delta}_{d_m}(t)$ denote the mth element of $\Delta_d(t)$ and $\dot{\Delta}_d(t)$, respectively. Let $c_1 = \mathrm{col}(c_{11}, c_{12}, \ldots, c_{1N})$, $c_2 = \mathrm{col}(c_{21}, c_{22}, \ldots, c_{2N}) \in \mathbb{R}^{Nl}$, where c_{1i}, $c_{2i} \in \mathbb{R}^l$, $i \in \mathcal{V}$. Then in (21), c_1 and c_2 can be used to denote some unknown upper bounds on the corresponding element of $\Delta_d(t)$ and $\dot{\Delta}_d(t)$, respectively. Furthermore, the Mean Value Theorem can be utilized to show that the auxiliary error $\tilde{\Delta}(t)$ is upper bounded as [25], [26], [36]–[39]

$$\left\|\tilde{\Delta}(t)\right\| \leq \rho\left(\|z(t)\|\right)\|z(t)\| \tag{22}$$

where $\rho(\cdot)$ is a positive, globally invertible, nondecreasing function and $z(t) \triangleq [E_\xi^T(t), E_f^T(t), r^T(t)]^T \in \mathbb{R}^{3Nl}$.

D. Stability Analysis and Sufficient Conditions

Theorem 1: Suppose that Assumptions 1–3 hold. Then, the proposed distributed control law in (6)–(9) ensures that the semi-global asymptotic robust consensus tracking objective for N agents with dynamics in (2) can be achieved as

$$\left[q_1^T,\ldots,q_N^T\right]^T - \mathbf{1}_N \otimes q_d(t) \to \mathbf{0}_{Nl}$$
$$\left[\dot{q}_1^T,\ldots,\dot{q}_N^T\right]^T - \mathbf{1}_N \otimes \dot{q}_d(t) \to \mathbf{0}_{Nl}, \quad \text{as } t\to\infty \tag{23}$$

provided that the gain k_{si} is selected sufficiently large[3] based on the stabilizing initial conditions (see the subsequent stability analysis) and α_{1i}, α_{2i} and β_{2i} in (6)–(9) are selected according to the following sufficient conditions:

$$\alpha_{1i} > 0.5,\ \alpha_{2i} > 1,\ \beta_{2i} > 0,\ \beta_{1i}(0) = \mathbf{0}_l. \tag{24}$$

Proof: Let $\mathcal{D} \subset \mathbb{R}^{4Nl+2}$ be a domain containing $y(t) \triangleq [z^T(t), \tilde{\Pi}_1^T(t), \sqrt{\mathcal{P}(t)}, \sqrt{\Phi(t)}]^T$, where $z(t)$ is given in (22) and $\mathcal{P}(t)$ is an auxiliary function, defined as

$$\mathcal{P}(t) = E_f^T(0)\hat{\Pi}_1 \mathrm{sgn}(E_f(0))\mathbf{1}_{Nl} - E_f^T(0)\Delta_d(0) - \mathcal{S}(t) \tag{25}$$

where $S(t)$ is the Filippov generalized solution to the following differential equations:

$$\dot{\mathcal{S}}(t) = r^T(t)\tilde{H}\left(\Delta_d(t) - \hat{\Pi}_1 \mathrm{sgn}\left(E_f(t)\right)\mathbf{1}_{Nl}\right),\quad \mathcal{S}(0)=0 \tag{26}$$

where $\hat{\Pi}_1 \triangleq \mathrm{diag}(\hat{\beta}_{11},\ldots,\hat{\beta}_{1N}) \otimes I_l \in \mathbb{R}^{Nl\times Nl}$ is a subsequently designed matrix introduced in (27) and $\hat{\beta}_{1i} > \|c_{1i}\|_\infty + (1/\alpha_{2i})\|c_{2i}\|_\infty$ such that $\mathcal{P}(t) \ge 0$.[4] Similarly, another auxiliary function $\Phi(t)$ is the Filippov generalized solution to the following defined differential equations:

$$\dot{\Phi} = \dot{E}_f^T \Pi_2 \mathrm{sgn}(E_f)\mathbf{1}_{Nl},\ \Phi(0) = E_f^T(0)\Pi_2 \mathrm{sgn}\left(E_f(0)\right)\mathbf{1}_{Nl}.$$

Let V_L: $\mathcal{D}\times[0,\infty) \to \mathbb{R}$ be a continuously differentiable, non-negative function (i.e., a Lyapunov candidate), defined as

$$V_L(y,t) \triangleq E_\xi^T E_\xi + \frac{1}{2}E_f^T E_f + \frac{1}{2}r^T M r + \frac{1}{2}\tilde{\Pi}_1^T\tilde{\Pi}_1 + \mathcal{P}(t) + \Phi(t) \tag{27}$$

where $\tilde{\Pi}_1(t) = \Pi_1(t) - \hat{\Pi}_1\mathbf{1}_{Nl}$ and the positive constant time-varying term $\hat{\Pi}_1$ is an estimate of $\Pi_1(t)$. Under Property 1, the Lyapunov function in (27) is non-negative and satisfies: $\lambda_1\|y\|^2 \le V_L(y,t) \le \lambda_2\|y\|^2$, where $\lambda_1 \triangleq (1/2)\min\{1, \min_{j\in\mathcal{V}}(\underline{m}_j)\}$ and $\lambda_2 \triangleq \max\{(1/2)\sum_{j\in\mathcal{V}}\overline{m}_j, 1\}$.

Under Filippov's framework, the time derivative of $V(y,t)$ exists almost everywhere (a.e.), i.e., for almost all $t\in[0,\infty)$ and $\dot{V}_L \overset{a.e.}{\in} \dot{\tilde{V}}_L$ by Lemma 1, where

$$\dot{\tilde{V}}_L = \bigcap_{\varsigma\in\partial V_L} \varsigma^T \Bbbk\left[\dot{E}_\xi^T, \dot{E}_f^T, \dot{r}^T, \dot{\tilde{\Pi}}_1^T, \frac{1}{2}\mathcal{P}^{-\frac{1}{2}}\dot{\mathcal{P}}, \frac{1}{2}\Phi^{-\frac{1}{2}}\dot{\Phi}, 1\right]^T \tag{28}$$

where ∂V_L is the generalized gradient of V_L [33]. Since V_L is continuously differentiable in y, then it gives

$$\dot{\tilde{V}}_L \subset \nabla V_L^T \Bbbk\left[\dot{E}_\xi^T, \dot{E}_f^T, \dot{r}^T, \dot{\tilde{\Pi}}_1^T, \frac{1}{2}\mathcal{P}^{-\frac{1}{2}}\dot{\mathcal{P}}, \frac{1}{2}\Phi^{-\frac{1}{2}}\dot{\Phi}, 1\right]^T \tag{29}$$

where $\nabla V_L \triangleq [2E_\xi^T, E_f^T, r^T M, \tilde{\Pi}_1^T, 2\mathcal{P}^{1/2}, 2\Phi^{1/2}, (1/2)r^T\dot{M}r]^T$.

By using (11)–(15) and (20), the expression in (29) becomes

$$\begin{aligned}\dot{\tilde{V}}_L \subset & r^T\left\{-\frac{1}{2}\dot{M}r - (K_s + I_{Nl})\tilde{H}r - \tilde{H}E_f + \tilde{\Delta}(t)\right.\\ &\left. + \left(\tilde{H}\Delta_d(t) - \tilde{H}\Bbbk\left[\mathrm{sgn}(E_f)\right](\Pi_1 + \Pi_2\mathbf{1}_{Nl})\right)\right\}\\ & + 2E_\xi^T(E_f - \Lambda_1E_\xi) + E_f^T(\tilde{H}r - \Lambda_2E_f)\\ & + \frac{1}{2}r^T\dot{M}r + (\Pi_1 - \hat{\Pi}_1\mathbf{1}_{Nl})^T\Bbbk\left[\mathrm{sgn}(E_f)\right]\tilde{H}r\\ & + (\tilde{H}r - \Lambda_2E_f)^T\Pi_2\Bbbk\left[\mathrm{sgn}(E_f)\right]\mathbf{1}_{Nl}\\ & - r^T\left(\tilde{H}\Delta_d(t) - \tilde{H}\hat{\Pi}_1\Bbbk\left[\mathrm{sgn}(E_f)\right]\mathbf{1}_{Nl}\right)\end{aligned} \tag{30}$$

where $\Bbbk[\mathrm{sgn}(E_f)] = \mathrm{SGN}(E_f)$ such that $\mathrm{SGN}(E_{\mathrm{fik}}) = 1$ if $E_{\mathrm{fik}} > 0$, $[-1,1]$ if $E_{\mathrm{fik}} = 0$, and -1 if $E_{\mathrm{fik}} < 0$, $i = 1,2,\ldots,N$, $k = 1,2,\ldots,l$ [33]. After canceling the corresponding common terms and exploiting (22), the expression in (30) is rewritten as

$$\begin{aligned}\dot{V}_L \overset{\text{a.e.}}{\le} & \|r\|\rho\left(\|z(t)\|\right)\|z(t)\| + 2\|E_\xi\|\|E_f\|\\ & - \lambda_{\min}(\Theta)\|r\|^2 - \lambda_{\min}(\tilde{H})\|r\|^2\\ & - 2\lambda_{\min}(\Lambda_1)\|E_\xi\|^2 - \lambda_{\min}(\Lambda_2)\|E_f\|^2\end{aligned} \tag{31}$$

where $\Theta = (1/2)(K_s\tilde{H} + \tilde{H}K_s)$ and the set in (30) reduces to the scalar inequality in (31) since the RHS is continuous a.e., i.e., the RHS is continuous except for the Lebesgue negligible set of time when $r^T\tilde{H}\Bbbk[\mathrm{sgn}(E_f)](\Pi_1 + \Pi_2\mathbf{1}_{Nl}) - r^T\tilde{H}\Bbbk[\mathrm{sgn}(E_f)](\Pi_1 + \Pi_2\mathbf{1}_{Nl}) \neq \{0\}$ and $r^T\tilde{H}\hat{\Pi}_1\Bbbk[\mathrm{sgn}(E_f)]\mathbf{1}_{Nl} - r^T\tilde{H}\hat{\Pi}_1\Bbbk[\mathrm{sgn}(E_f)]\mathbf{1}_{Nl} \neq \{0\}$ [33].

Based on Young's Inequality and by completing the squares for r, the expression in (31) can be rewritten as

$$\begin{aligned}\dot{V}_L &\overset{\text{a.e.}}{\le} -\lambda_0\|z(t)\|^2 - \lambda_{\min}(\Theta)\|r\|^2 + \|r\|\rho\left(\|z(t)\|\right)\|z(t)\|\\ &\overset{\text{a.e.}}{\le} -\left(\lambda_0 - \frac{\rho^2\left(\|z(t)\|\right)}{4\lambda_{\min}(\Theta)}\right)\|z(t)\|^2\end{aligned} \tag{32}$$

where $\lambda_0 \triangleq \min\{2(\lambda_{\min}(\Lambda_1) - (1/2)), \lambda_{\min}(\Lambda_2) - 1, \lambda_{\min}(\tilde{H})\}$, and the bounding function $\rho(\|z(t)\|)$ is a positive globally invertible nondecreasing function. By selecting α_{1i} and α_{2i}, according to (24) and using the fact that H is of full rank, λ_0 is positive. Let $U_1(y) = -(\lambda_0 - (\rho^2(\|z(t)\|)/4\lambda_{\min}(\Theta)))\|z\|^2 = -u_1\|z\|^2$ be a continuous negative semi-definite function, which is defined on the following domain:

$$\mathcal{D} \triangleq \left\{y/\|y\| < \rho^{-1}\left[2\sqrt{\lambda_0\lambda_{\min}(\Theta)}\right]\right\}. \tag{33}$$

[3]A lower bound on the control gain k_{si} can be selected as $k_0 = \rho^2(c_0)/(4\lambda_0\lambda_{\min}(\tilde{H}))$ where $c_0 \ge \sqrt{\lambda_2/\lambda_1}\sum_{i\in\mathcal{V}}\|y_i(0)\|$ with $y_i(0)$ related to the initial values of states. Note that this lower bound requires global information. A large enough k_{si} is selected before the controller is implemented so that $k_{si} = k_s > k_0$ to yield a semi-global stability result [36]–[39].

[4]The proof is similar to Lemma 5 in [24], and is thus omitted here.

Let the set of stabilizing initial conditions be defined as

$$\Upsilon \triangleq \left\{ y \in \mathcal{D} \| \|y\| < \sqrt{\frac{\lambda_1}{\lambda_2}} \rho^{-1} \left[2\sqrt{\lambda_0 \lambda_{\min}(\Theta)} \right] \right\} \quad (34)$$

where λ_1, λ_2 and λ_0 are given in (27) and (31), respectively.

Under the condition (24) and the control gain k_{si} is selected sufficiently large such that $y(0) \in \Upsilon \subset \mathcal{D}$, it gives

$$\dot{V}_L \overset{\text{a.e.}}{\leq} -u_1 \|z\|^2 \quad (35)$$

for all $y \in \mathcal{D}$ and a positive constant $u_1 \in \mathbb{R}$. Thus, $\dot{V}_L$ is negative semi-definite and $V_L \leq V_L(0) \in \mathcal{L}_\infty$ holds in $\mathcal{D}$.

The details are given as below. Based on (3), (5) and (8), the initial errors $e_{\xi i}(0)$, $e_{fi}(0)$ and $r_i(0)$ are not related to k_{si}. Thus, k_{si} can be selected sufficiently large such that $y(0) \in \Upsilon$. That is, the ratio of the function $\rho^2(\|z(0)\|)$ to k_{si} in (32) can be made sufficiently small such that $\dot{V}_L(0) \leq 0$. For $t > 0$, a proof by contradiction is provided as follows to show that V_L is monotonically decreasing.

Proof by Contradiction: Let $4\lambda_{\min}(\Theta) = c$. Assume that there exists a time instant $t^* > 0$ such that $\lambda_0 < \rho^2(\|z(t^*)\|)/c$ and $\lambda_0 \geq \rho^2(\|z(t)\|)/c$ for $t \in [0, t^*)$. Thus, $\dot{V}_L(t) \leq 0$ for $t \in [0, t^*)$. That is, $\|z(t)\| \leq \|z(0)\|$ for $t \in [0, t^*)$.

Case i: $\|z(t)\| < \|z(0)\|$ for $t \in [0, t^*)$. From the assumption, $\rho^2(\|z(t^*)\|) > \lambda_0 c$. In addition, by $y(0) \in \Upsilon$, $\lambda_0 c > \rho^2(\|z(0)\|)$. Thus, $\rho^2(\|z(t^*)\|) > \rho^2(\|z(0)\|)$. Since $\rho(\|z(t)\|)$ is a positive nondecreasing function, $\|z(t^*)\| > \|z(0)\|$ holds, which contradicts with $\|z(t)\| < \|z(0)\|$ for $t \in [0, t^*)$ by the state continuity.

Case ii: $\|z(t)\| \equiv \|z(0)\|$ for $t \in [0, t^*)$. As $\lambda_0 c > \rho^2(\|z(0)\|)$ by $y(0) \in \Upsilon$, $\lambda_0 c > \rho^2(\|z(t)\|)$ for $t \in [0, t^*)$, which contradicts with $\rho^2(\|z(t^*)\|) > \lambda_0 c$ by the state continuity.

Hence, $V_L \leq V_L(0) \in \mathcal{L}_\infty$ holds in $\mathcal{D}$ for $\forall\, t \geq 0$.

Since $V_L \in \mathcal{L}_\infty$ in $\mathcal{D}$ and the fact that H is positive definite by Lemma 2, it follows from (12) and (27) that $E_\xi(t)$, $\dot{E}_\xi(t)$, $E_f(t)$, $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. From (11) and Properties 1 and 2, we have $\dot{F} \in \mathcal{L}_\infty$ in $\mathcal{D}$. From (17), $\dot{r}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. From (10) and (13), $\ddot{E}_\xi(t)$, $\dddot{E}_\xi(t)$, $\ddot{E}_f(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, then $\ddot{F} \in \mathcal{L}_\infty$ in $\mathcal{D}$. Lemma 1 can be used to show that $u_1\|z\|^2 \to 0$, as $t \to \infty$, $\forall\, y(0) \in \Upsilon$. That is $E_\xi(t) \to \mathbf{0}_{Nl}$, $E_f(t) \to \mathbf{0}_{Nl}$ and $r(t) \to \mathbf{0}_{Nl}$, as $t \to \infty$, $\forall\, y(0) \in \Upsilon$. Based on (12) and the fact that H is of full rank, it has $Q = Q_d$. Similarly, based on $E_\xi(t) = \mathbf{0}_{Nl}$ and $E_f(t) = \mathbf{0}_{Nl}$, it has $\dot{Q} = \dot{Q}_d$. Thus, $q_i(t) \to q_d(t)$ and $\dot{q}_i(t) \to \dot{q}_d(t)$ as $t \to \infty$, $\forall\, y(0) \in \Upsilon$. ■

Remark 1: According to Theorem 1, a large enough control parameter k_{si} in (8) can be selected to enlarge the domain of convergence to enclose any *a priori* given arbitrarily large bounded set as its subset.

Remark 2: Several consensus algorithms are developed for Euler-Lagrange systems (e.g., [2], [6]–[15]). However, most of them only deal with structured uncertainties via model-based parametric linearization. Challenging unstructured uncertainties, including unknown nonlinear dynamics and disturbances, is an important and challenging problem. The algorithms in [2], [6], and [7] are based on the exact knowledge of the dynamics and the leader's acceleration information was available to all the followers. Robust UUB coordinated tracking result was obtained in [9] by using the neural-network-based control. In [10] and [11], the proposed discontinuous distributed sliding-mode estimators required the parametric uncertainties to satisfy the "linearity-in-parameters" assumption. However, the proposed algorithm in this paper removes these constraints.

Remark 3: Several control gians α_{1i}, α_{2i}, k_{si} and a time-varying control term $\beta_i(t) = \beta_{1i}(t) + \beta_{2i}\mathbf{1}_l$ are used in (6)–(9). According to (24), sufficient conditions are derived to achieve robust consensus tracking. An agent based Lyapunov approach is used, where $\hat{\beta}_{1i}(t)$ is an estimate of the time-varying gain $\beta_{1i}(t)$ satisfying: $\hat{\beta}_{1i}(t) > \|c_{1i}\|_\infty + \alpha_{2i}^{-1}\|c_{2i}\|_\infty$. However, this gain $\hat{\beta}_{1i}(t)$ is only introduced to facilitate the Lyapunov analysis and not used in the controller. Thus, the designed distributed control law does not need the unknown upper bounds of c_1 and c_2. The proposed distributed controller shown in (6)–(9) is decentralized in the sense that only local feedback is required to compute the controller. However, by (33) the control gain k_{si} is related to $\tilde{H}$ containing global information. This k_{si} can thus be selected large enough before the control law is implemented to yield a semi-global tracking result as described in Theorem 1.

IV. Robust Consensus Tracking Under a Directed Graph

For a digraph, the information-exchange matrix H is asymmetric. In such a case, if we choose a Lyapunov function $r^T M r$, then the term $-r^T \tilde{H} r$ will appear in its derivative. Thus, it is unclear how (6) can be applied, especially when considering to compensate for $r^T \tilde{H}(\Delta_d - \hat{\Pi}_1 \text{sgn}(E_f)\mathbf{1}_{Nl})$ via the defined errors in (3)–(5) and the filter error: $\tilde{H} r = \dot{E}_f + \Lambda_2 E_f$. Hence, new distributed schemes are needed to achieve robust consensus tracking for a directed graph. In this section, a distributed two-layer coordination scheme will be proposed, where in the top layer, a distributed leader estimator is presented to estimate the leader's states. Under this distributed estimator, a novel distributed controller is developed.

Assumption 4: The graph $\mathcal{G}$ is directed and $\mathcal{V}_0$ has directed paths to all the agents of $\mathcal{G}$.

Lemma 3—[32]: Suppose that Assumption 4 holds, then H is nonsingular and all eigenvalues of H have positive real parts.

To facilitate the distributed leader estimator design, we assume that $\mathcal{V}_0$ is generated by the following leader system [42], [43]:

$$\ddot{q}_d(t) = S_1 q_d(t) + S_2 \dot{q}_d(t) \quad (36)$$

where S_1, $S_2 \in \mathbb{R}^{l \times l}$ are arbitrary constant matrices.

Define $S = \begin{bmatrix} 0_{l \times l} & I_l \\ S_1 & S_2 \end{bmatrix}$, (36) can be rewritten as

$$\begin{bmatrix} \dot{q}_d \\ \ddot{q}_d \end{bmatrix} = S \begin{bmatrix} q_d \\ \dot{q}_d \end{bmatrix}. \quad (37)$$

Assumption 5—[3]: The system in (37) is marginally stable.

Remark 4: Note that when $S_1 = S_2 = 0_{l \times l}$, (37) is a double integrator system. However, in this subsection, we do not require (37) to be a double integrator system.

A. Distributed Leader Estimator Design

For $i = 1, 2, \ldots, N$, the following leader estimation errors are defined by using only relative information as:

$$\varrho_{di1}(t) = \sum_{j=1}^{N} a_{ij}(\varrho_{j1} - \varrho_{i1}) + b_i(q_d - \varrho_{i1})$$
$$\varrho_{di2}(t) = \sum_{j=1}^{N} a_{ij}(\varrho_{j2} - \varrho_{i2}) + b_i(\dot{q}_d - \varrho_{i2}) \tag{38}$$

where $\varrho_{i1}(t)$, $\varrho_{i2}(t) \in \mathbb{R}^l$ are generated by

$$\dot{\varrho}_{i1}(t) = \varrho_{i2}(t) + \kappa_i \varrho_{di1}(t),$$
$$\dot{\varrho}_{i2}(t) = S_1 \varrho_{i1}(t) + S_2 \varrho_{i2}(t) + \kappa_i \varrho_{di2}(t) \tag{39}$$

where $\kappa_i > 0$ is an arbitrary constant.

Define new variables: $\varrho_i(t) \triangleq \text{col}(\varrho_{i1}, \varrho_{i2})$, $\varrho_{di}(t) \triangleq \text{col}(\varrho_{di1}, \varrho_{di2})$ and $\varrho_d(t) \triangleq \text{col}(q_d, \dot{q}_d)$. Then, the system composed of (38) and (39) can be further expressed as

$$\dot{\varrho}_i(t) = S\varrho_i(t) + \kappa_i \varrho_{di}(t)$$
$$\varrho_{di}(t) = \sum_{j=1}^{N} a_{ij}(\varrho_j - \varrho_i) + b_i(\varrho_d - \varrho_i) \tag{40}$$

where the matrix S is defined in (37).

Lemma 4: Suppose that Assumptions 4 and 5 hold. Then, under the proposed distributed leader estimator in (39), it follows that for all $\kappa_i > 0$, $i = 1, 2, \ldots, N$:

$$\varrho_{i1}(t) - q_d(t) \to \mathbf{0}_l \quad \text{and} \quad \varrho_{i2}(t) - \dot{q}_d(t) \to \mathbf{0}_l, \text{ as } t \to \infty. \tag{41}$$

Proof: Define a new variable $\tilde{\varrho}_i(t) = \varrho_i(t) - \varrho_d(t)$. Then it follows from (37) and (40) that the closed-loop estimate error system can be given by:

$$\dot{\tilde{\varrho}}_i(t) = \dot{\varrho}_i(t) - \dot{\varrho}_d(t) = S\varrho_i(t) - S\varrho_d(t) + \kappa_i \varrho_{di}(t)$$
$$= S\tilde{\varrho}_i(t) - \kappa_i \left(\sum_{j=1}^{N} a_{ij} \left(\tilde{\varrho}_i(t) - \tilde{\varrho}_j(t) \right) + b_i \tilde{\varrho}_i(t) \right)$$

which is rewritten in the following compact form for $\tilde{\varrho} \triangleq \text{col}(\tilde{\varrho}_1, \tilde{\varrho}_2, \ldots, \tilde{\varrho}_N)$ and $\kappa \triangleq \text{diag}(\kappa_1, \kappa_2, \ldots, \kappa_N) \otimes I_{2l}$:

$$\dot{\tilde{\varrho}}(t) = \left[(I_N \otimes S) - \kappa(H \otimes I_{2l}) \right] \tilde{\varrho}(t). \tag{42}$$

By Lemma 3, Lemma 1 in [41] and Assumption 5, if all the eigenvalues of S have nonpositive real parts, then for any $\kappa > 0$, the origin of the closed-loop estimate error system (42) is exponentially stable. That is, for all $i = 1, 2, \ldots, N$, $\lim_{t\to\infty} \tilde{\varrho}_i(t) = \mathbf{0}_l$. As a result, it can be seen that $\lim_{t\to\infty} \varrho_{i1}(t) - q_d(t) = \mathbf{0}_l$ and $\lim_{t\to\infty} \varrho_{i2}(t) - \dot{q}_d(t) = \mathbf{0}_l$ hold. ■

B. Robust Consensus Tracking Control Law Design

To introduce the distributed controller, similar to (3)–(5) and (10), we define the following error signals based on information from the distributed leader estimator:

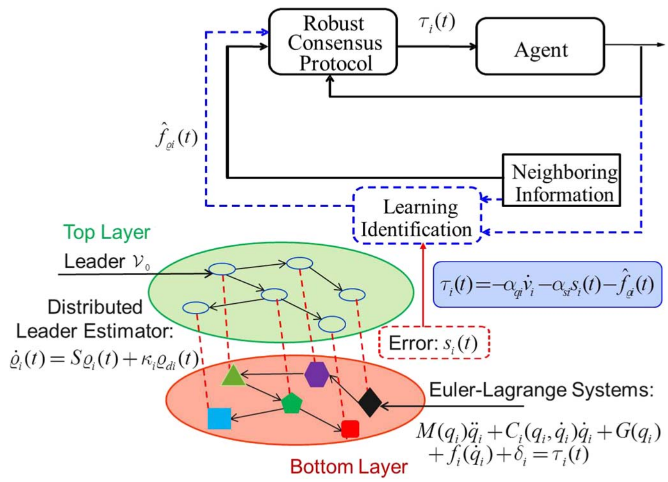

Fig. 1. Diagram of the two-layer based distributed control law design.

$$\dot{q}_{ri} = \varrho_{i2}(t) + \kappa_i \varrho_{di1}(t) - \alpha_{\varrho i}(q_i - \varrho_{i1})$$
$$s_i = \dot{q}_i - \dot{q}_{ri} = \dot{q}_i - \dot{\varrho}_{i1} + \alpha_{\varrho i}(q_i - \varrho_{i1}) \tag{43}$$

where s_i is the sliding surface, q_{ri} is the virtual reference, and similar to (10), a filter error signal $r_{si}(t) \in \mathbb{R}^l$ is defined as

$$r_{si} = \dot{s}_i + \alpha_{ri} s_i \tag{44}$$

where κ_i, $\alpha_{\varrho i}$, $\alpha_{ri} \in \mathbb{R}$ are some positive controller gains.

The two-layer based distributed control law shown in Fig. 1, for $i = 1, 2, \ldots, N$, is given by

$$\tau_i(t) = -\alpha_{qi}\dot{v}_i - \alpha_{si}s_i - \hat{f}_{\varrho i}(t) \tag{45}$$

where $v_i = q_i - \varrho_{i1}$, α_{qi}, α_{si} are two control gains, and similar to (8) and (9), the distributed estimation law $\hat{f}_{\varrho i}(t)$ is given by

$$\hat{f}_{\varrho i}(t) = (k_{\varrho si} + 1)\left(s_i(t) - s_i(0)\right) + v_{\varrho i}(t)$$
$$\dot{v}_{\varrho i}(t) = (k_{\varrho si} + 1)\alpha_{ri} s_i(t) + \beta_{\varrho i}(t) \odot \text{vsgn}\left(s_i(t)\right)$$
$$\beta_{\varrho i}(t) = \beta_{\varrho 1i}(t) + \beta_{\varrho 2i}\mathbf{1}_l, \ \beta_{\varrho 1i}(0) = \mathbf{0}_l$$
$$\beta_{\varrho 1i}(t) = |s_i(t)| - |s_i(0)| + \alpha_{ri} \int_0^t |s_i(\tau)|\, d\tau \tag{46}$$

where $|s_i(t)| \triangleq [|s_{i1}(t)|, |s_{i2}(t)|, \ldots, |s_{il}(t)|]^T \in \mathbb{R}^l$.

C. Closed-Loop System Development

The filter error signal (44) and the control law (43) together with (46) can be expressed in the following compact form:

$$r_s = \dot{s} + \Lambda_r s, \ s = \dot{v} + \Lambda_\varrho v$$
$$\Gamma = -\Lambda_q \dot{v} - \Lambda_s s - \hat{F}_\varrho \tag{47}$$

where given (2), the time derivative of s is written as

$$\dot{s} = M^{-1}\left[\Gamma - (F + C\dot{Q} + G + \delta)\right] - \ddot{\varrho}_1 + \Lambda_\varrho \dot{v} \tag{48}$$

where $s \triangleq \text{col}(s_1, \ldots, s_N)$, $r_s \triangleq \text{col}(r_{s1}, \ldots, r_{sN})$, $v \triangleq \text{col}(v_1, \ldots, v_N)$, $\hat{F}_\varrho \triangleq \text{col}(\hat{f}_{\varrho 1}, \ldots, \hat{f}_{\varrho N})$, $\varrho_1 \triangleq \text{col}(\varrho_{11}, \ldots, \varrho_{N1})$; and

$\Lambda_q \triangleq \text{diag}(\alpha_{q1}, \ldots, \alpha_{qN}) \otimes I_l$, $\Lambda_\varrho \triangleq \text{diag}(\alpha_{\varrho 1}, \ldots, \alpha_{\varrho N}) \otimes I_l$, $\Lambda_r \triangleq \text{diag}(\alpha_{r1}, \ldots, \alpha_{rN}) \otimes I_l$, $\Lambda_s \triangleq \text{diag}(\alpha_{s1}, \ldots, \alpha_{sN}) \otimes I_l$.

After premultiplying (48) by M, the following expression can be further obtained by using (47):

$$\begin{aligned} Mr_s &= -\Lambda_q \dot{v} - \Lambda_s s - \hat{F}_\varrho - (F + C\dot{Q} + G + \delta) \\ &\quad - M\ddot{\varrho}_1 + M\Lambda_\varrho \dot{v} + M\Lambda_r s \\ &\triangleq N_\varrho(t) + N_{d\varrho}(t) - \hat{F}_\varrho \end{aligned} \tag{49}$$

where $N_\varrho(t) \triangleq F_d - F + C_d\dot{Q}_d - C\dot{Q} + G_d - G + (M\Lambda_\varrho - \Lambda_q)\dot{v} + (M\Lambda_r - \Lambda_s)s + (M_d - M)\ddot{\varrho}_1$, $N_{d\varrho}(t) \triangleq -F_d - C_d\dot{Q}_d - G_d - M_d\ddot{\varrho}_1 - \delta$, and $F_d \triangleq F(\dot{Q}_d)$, $G_d \triangleq G(Q_d) \in \mathbb{R}^{Nl}$, and $C_d \triangleq C(Q_d, \dot{Q}_d)$, $M_d \triangleq M(Q_d) \in \mathbb{R}^{Nl \times Nl}$.

Similar to (15)–(17), differentiating (49) by using (46) yields

$$\begin{aligned} M\dot{r}_s = &-\frac{1}{2}\dot{M}r_s - (K_{\varrho s} + I_{Nl})r_s + \tilde{\Delta}_\varrho(t) + \Delta_{d\varrho}(t) \\ &- \text{sgn}(s)\left(\Pi_{\varrho 1}(t) + \Pi_{\varrho 2}\mathbf{1}_{Nl}\right) - s \end{aligned} \tag{50}$$

where the unmeasurable auxiliary terms $\tilde{\Delta}_\varrho(t)$ and $\Delta_{d\varrho}(t)$ are defined as: $\tilde{\Delta}_\varrho(t) \triangleq \dot{N}_\varrho(t) - (1/2)\dot{M}r_s + s$, $\Delta_{d\varrho}(t) \triangleq \dot{N}_{d\varrho}(t)$; and $K_{\varrho s} \triangleq \text{diag}(k_{\varrho s1}, \ldots, k_{\varrho sN}) \otimes I_l$, $\Pi_{\varrho 2} \triangleq \text{diag}(\beta_{\varrho 21}, \ldots, \beta_{\varrho 2N}) \otimes I_l$, $\Pi_{\varrho 1}(t) \triangleq \text{col}(\beta_{\varrho 11}(t), \ldots, \beta_{\varrho 1N}(t))$.

Under Assumption 5, q_d and $\dot{q}_d$ are bounded and so are ϱ_{i1} and ϱ_{i2} by (36), (39) and (41). In addition, based on Properties 1 and 2, Assumptions 4 and 5, similar to that in (21), the auxiliary term $\Delta_{d\varrho}(t)$ and its time derivative satisfy

$$\sup_{t\in[0,\infty)} |\Delta_{d\varrho_m}(t)| < c_{3_m}, \ \sup_{t\in[0,\infty)} \left|\dot{\Delta}_{d\varrho_m}(t)\right| < c_{4_m} \tag{51}$$

where $m = 1, 2, \ldots, Nl$, $c_3 \triangleq \text{col}(c_{31}, c_{32}, \ldots, c_{3N})$, $c_4 \triangleq \text{col}(c_{41}, c_{42}, \ldots, c_{4N}) \in \mathbb{R}^{Nl}$ with $c_{3i}, c_{4i} \in \mathbb{R}^l$, $i \in \mathcal{V}$.

Furthermore, similar to (22), the auxiliary error $\tilde{\Delta}_\varrho(t)$ can be upper bounded as [25], [26], [36]–[39]

$$\left\|\tilde{\Delta}_\varrho(t)\right\| \le \rho_\varrho\left(\|z_\varrho(t)\|\right)\|z_\varrho(t)\| \tag{52}$$

where $\rho_\varrho(\cdot)$ is a positive, globally invertible, nondecreasing function and $z_\varrho(t) \triangleq \left[v^T(t), s^T(t), r_s^T(t)\right]^T \in \mathbb{R}^{3Nl}$.

D. Stability Analysis and Sufficient Conditions

Theorem 2: Suppose that Assumptions 1, 4 and 5 hold. Then the proposed distributed control law in (45) together with the distributed leader estimator in (39) and the distributed nonlinear identifier in (46) ensure that the semi-global asymptotic robust consensus tracking objective for N agents with dynamics in (2) can be achieved in the sense: $\lim_{t\to\infty} q_i(t) - q_d(t) = \mathbf{0}_l$ and $\lim_{t\to\infty} \dot{q}_i(t) - \dot{q}_d(t) = \mathbf{0}_l$, provided that the control gain $k_{\varrho si}$ is selected sufficiently large, and κ_i, α_{qi}, α_{si}, $\alpha_{\varrho i}$, α_{ri}, $\beta_{\varrho 2i}$ in (39), (45) and (46) are selected as

$$\begin{aligned} \kappa_i &> 0, \ \alpha_{qi} > 0, \ \alpha_{si} > 0, \ \alpha_{\varrho i} > 0.5 \\ \alpha_{ri} &> 1, \ \beta_{\varrho 2i} > 0, \ \beta_{\varrho 1i}(0) = \mathbf{0}_l. \end{aligned} \tag{53}$$

Proof: Let $\mathcal{D}_\varrho \subset \mathbb{R}^{4Nl+2}$ be a domain containing $y_\varrho(t)$ defined as $y_\varrho(t) \triangleq [z_\varrho^T(t), \tilde{\Pi}_{\varrho 1}^T(t), \sqrt{\mathcal{P}_\varrho(t)}, \sqrt{\Phi_\varrho(t)}]^T$, where $z_\varrho(t)$ is given in (52) and $\mathcal{P}_\varrho(t)$ and $\Phi_\varrho(t)$ are two positive semi-definite auxiliary functions, defined as

$$\begin{aligned} \mathcal{P}_\varrho &= s^T(0)\hat{\Pi}_{\varrho 1}\text{sgn}\left(s(0)\right)\mathbf{1}_{Nl} - s^T(0)\Delta_{d\varrho}(0) - S_\varrho \\ \dot{S}_\varrho &= r_s^T(t)\left(\Delta_{d\varrho}(t) - \hat{\Pi}_{\varrho 1}\text{sgn}(s)\mathbf{1}_{Nl}\right), \quad S_\varrho(0) = 0 \\ \dot{\Phi}_\varrho &= \dot{s}\Pi_{\varrho 2}\text{sgn}(s)\mathbf{1}_{Nl}, \ \Phi_\varrho(0) = s^T(0)\Pi_{\varrho 2}\text{sgn}\left(s(0)\right)\mathbf{1}_{Nl} \end{aligned}$$

where $\hat{\Pi}_{\varrho 1} \triangleq \text{diag}(\hat{\beta}_{\varrho 11}, \ldots, \hat{\beta}_{\varrho 1N}) \otimes I_l \in \mathbb{R}^{Nl \times Nl}$ is a subsequently designed matrix introduced in (54) and $\hat{\beta}_{\varrho 1i} > \|c_{3i}\|_\infty + (1/\alpha_{ri})\|c_{4i}\|_\infty$ such that $\mathcal{P}_\varrho(t) \ge 0$.

Let $V_{\tilde{L}}$: $\mathcal{D}_\varrho \times [0, \infty) \to \mathbb{R}$ be a continuously differentiable, non-negative function (i.e., a Lyapunov candidate), defined as

$$V_{\tilde{L}}(y_\varrho, t) \triangleq v^T v + \frac{1}{2}s^T s + \frac{1}{2}r_s^T M r_s + \frac{1}{2}\tilde{\Pi}_{\varrho 1}^T\tilde{\Pi}_{\varrho 1} + \mathcal{P}_\varrho(t) + \Phi_\varrho(t) \tag{54}$$

where $\tilde{\Pi}_{\varrho 1}(t) = \Pi_{\varrho 1}(t) - \hat{\Pi}_{\varrho 1}\mathbf{1}_{Nl}$ and the positive constant time-varying term $\hat{\Pi}_{\varrho 1}(t)$ is an estimate of $\Pi_{\varrho 1}(t)$.

Next, the proof is similar to Theorem 1 to give $V_{\tilde{L}} \le V_{\tilde{L}}(0) \in \mathcal{L}_\infty$ in $\mathcal{D}_\varrho \triangleq \{y_\varrho/\|y_\varrho\| < \rho_\varrho^{-1}[2\sqrt{\lambda_{\varrho 0}\lambda_{\min}(K_s)}]\}$, $\lambda_{\varrho 0} \triangleq \min\{2(\lambda_{\min}(\Lambda_\varrho) - (1/2)), \lambda_{\min}(\Lambda_r) - 1, 1\}$. Based on (38), (39), (43) and (44), Lemma 1 can be used to show $v(t) \to \mathbf{0}_{Nl}$, $s(t) \to \mathbf{0}_{Nl}$ and $r_s(t) \to \mathbf{0}_{Nl}$, as $t \to \infty$, $\forall\, y_\varrho(0) \in \Upsilon_\varrho$.

To show that $\lim_{t\to\infty} q_i(t) - q_d(t) = \mathbf{0}_l$ and $\lim_{t\to\infty} \dot{q}_i(t) - \dot{q}_d(t) = \mathbf{0}_l$ hold, we need linear input-to-state stability, the conclusion (41) in Lemma 4 and the sliding-mode design in (43). Specifically, substituting (40) into (43) gives

$$s_i = \dot{q}_i - \dot{q}_{ri} = \dot{q}_i - (\varrho_{i2} + \kappa_i\varrho_{di1}) + \alpha_{\varrho i}(q_i - \varrho_{i1}) \tag{55}$$

i.e.,

$$\dot{q}_i - \dot{\varrho}_{i1} + \alpha_{\varrho i}(q_i - \varrho_{i1}) = s_i. \tag{56}$$

Since (56) can be viewed as a stable first-order differential equation with $s_i(t)$ as the input and this input is bounded for all $t \ge 0$ and tends to zero as $t \to \infty$, by exploiting the linear input-to-state stability, both $\dot{q}_i - \dot{\varrho}_{i1}$ and $q_i - \varrho_{i1}$ are bounded over $t \ge 0$ and will decay to zero. Thus, by the conclusion (41) and the sliding-mode design in (43), it follows from:

$$\begin{aligned} q_i(t) - q_d(t) &= q_i(t) - \varrho_{i1}(t) + \varrho_{i1}(t) - q_d(t) \\ \dot{q}_i(t) - \dot{q}_d(t) &= \dot{q}_i(t) - \dot{\varrho}_{i1}(t) + \dot{\varrho}_{i1}(t) - \dot{q}_d(t) \\ &= \dot{q}_i(t) - \varrho_{i2}(t) + \varrho_{i2}(t) - \dot{q}_d(t) \end{aligned} \tag{57}$$

that the robust consensus tracking holds, i.e., $\lim_{t\to\infty} q_i(t) - q_d(t) = \mathbf{0}_l$ and $\lim_{t\to\infty} \dot{q}_i(t) - \dot{q}_d(t) = \mathbf{0}_l$. ■

Remark 5: Based on the distributed two-layer design, all the gains are distributed. Moreover, $k_{\varrho si}$ is not related to $\tilde{H}$ containing global information, while k_{si} given in Theorem 1 does need this global information. Thus, this limitation is removed in Section IV for a directed graph case.

Remark 6: The continuous distributed controller in (6) for the undirected graph case is distributed and relies on neighboring error signals. The use of these signals in (8) provides asymptotic consensus tracking of a time-varying leader, although it requires two-hop communication. However, in Section IV, the two-hop communication requirement is reduced to single-hop communication based on a distributed two-layer coordination design

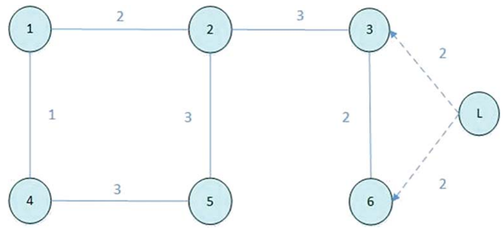

Fig. 2. An information-exchange undirected graph. Circles denote the labeled agents and the connecting lines denote weighted information edges among agents. The desired time-varying trajectory is denoted as a virtual leader.

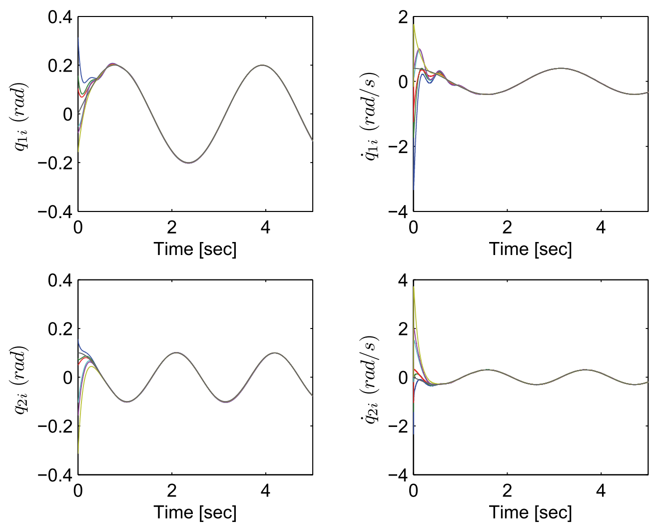

Fig. 3. Trajectories of the states of the followers and the leader under (6)–(9).

under a class of leader systems in (36) and (37). Sufficient conditions on the stability of closed-loop system are derived for both undirected and directed graph cases to achieve robust consensus tracking asymptotically.

V. Numerical Simulation

Two numerical simulations are presented to demonstrate the effectiveness of the proposed distributed control algorithms. As studied in [8]–[11], a group of robot manipulators are considered to model the networked Euler-Lagrange systems in (2). In this simulation, we investigate the distributed coordinated tracking problem for six networked two-link robotic manipulators to track a desired time-varying trajectory.

The classic manipulator's dynamics with unknown dynamics and disturbances are described as follows:

$$\begin{bmatrix} m_{1i}+2m_{3i}\cos(q_{2i}) & m_{2i}+m_{3i}\cos(q_{2i}) \\ m_{2i}+m_{3i}\cos(q_{2i}) & m_{2i} \end{bmatrix}\begin{bmatrix} \ddot{q}_{1i} \\ \ddot{q}_{2i} \end{bmatrix} + \begin{bmatrix} -m_{3i}\sin(q_{2i})\dot{q}_{2i} & -m_{3i}\sin(q_{2i})(\dot{q}_{1i}+\dot{q}_{2i}) \\ m_{3i}\sin(q_{2i})\dot{q}_{1i} & 0 \end{bmatrix} \times \begin{bmatrix} \dot{q}_{1i} \\ \dot{q}_{2i} \end{bmatrix} + \begin{bmatrix} g_{1i}(q_{1i},q_{2i}) \\ g_{2i}(q_{1i},q_{2i}) \end{bmatrix} + f_i(\dot{q}_i) + \delta_i = \tau_i \tag{58}$$

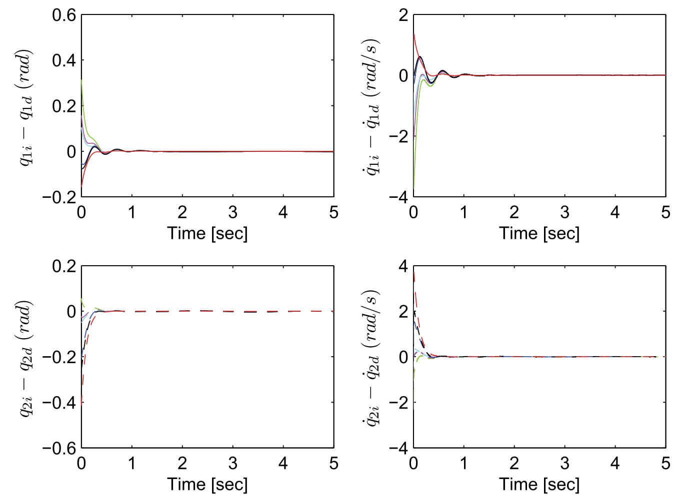

Fig. 4. Tracking errors between the followers and the leader under (6)–(9).

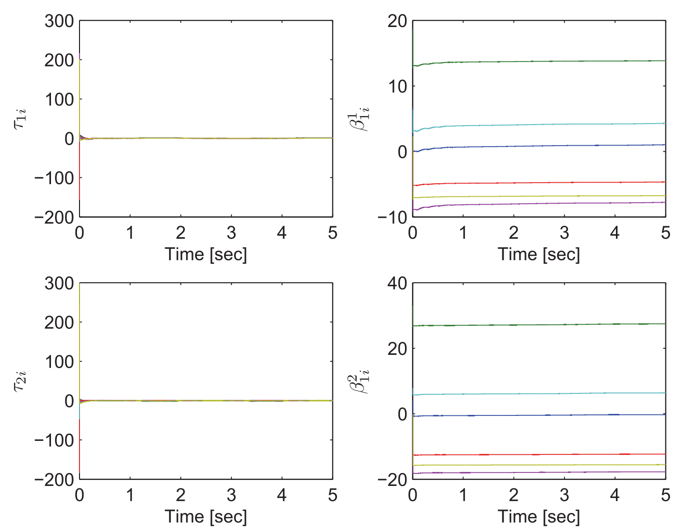

Fig. 5. The control input τ_i in (6) and the convergence of β_{1i} in (9).

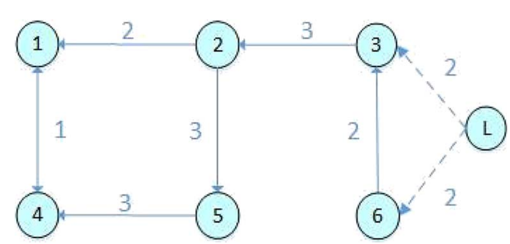

Fig. 6. A directed graph. Circles denote the labeled agents and the connecting lines denote weighted information edges among all agents and a virtual leader.

where $i=1,\ldots,6$, q_{1i}, q_{2i} denote joint angles, the uncertain dynamic parameters of each manipulator are set as $m_{1i}=i$, $m_{2i}=0.1i$, $m_{3i}=0.2i$, $g_{1i}(q_{1i},q_{2i}) = 0.2i*\cos(q_{1i})+0.1i*\cos(q_{1i}+q_{2i})$, $g_{2i}(q_{1i},q_{2i}) = 0.1i*\sin(q_{1i}+q_{2i})$, and the unknown dynamics and disturbances are given by $f_i(\dot{q}_i) = [0.1i*\dot{q}_{1i}, 0.2i*\dot{q}_{2i}]^T$ and $\delta_i = [0.1i*\sin(0.1i*t), 0.2i*\sin(0.2i*t)]^T$. The initial joint configuration and

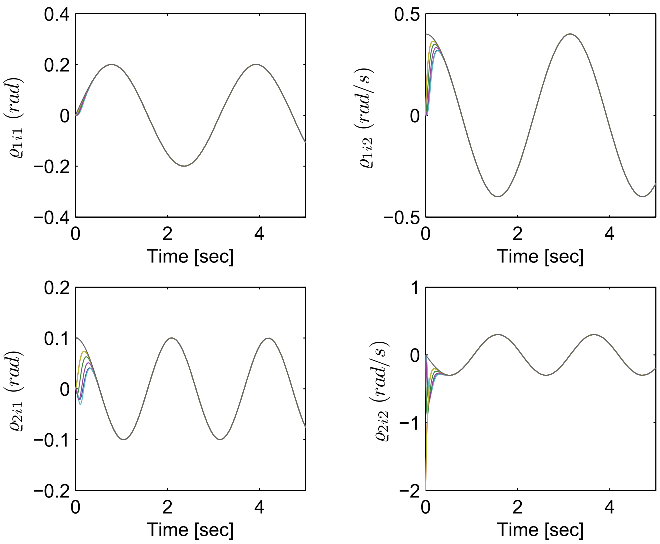

Fig. 7. Trajectories of the estimated states of the followers and the leader under the developed distributed leader estimator (39).

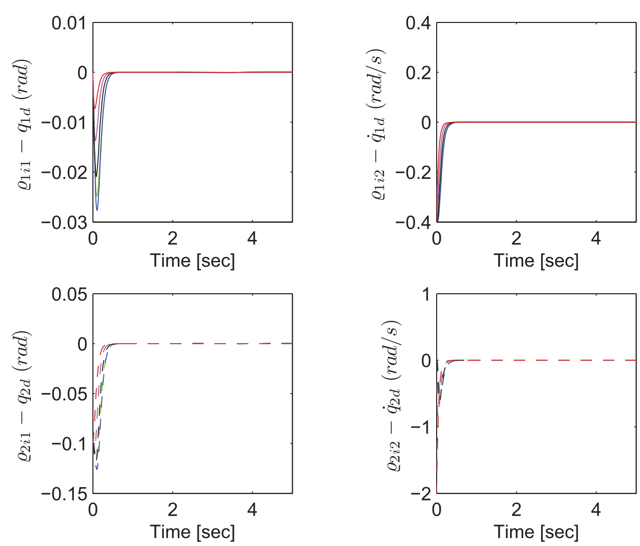

Fig. 8. Estimated state tracking errors between the followers and the leader under the developed distributed leader estimator (39).

velocity are selected as $q_i(0) = [\pi/10i, -\pi/20i]^T$ rad and $\dot{q}_i(0) = [0, 0]^T$ rad/s, respectively[5].

A. Robust Consensus Tracking Under an Undirected Graph

An undirected communication graph of a team of agents is shown in Fig. 2.

The controller gains in (6)–(9) are selected as $\alpha_{1i} = 10$, $\alpha_{2i} = 20$, $\beta_{2i} = 1$, $\beta_{1i}(0) = [0, 0]^T$, $k_{si} = 30$. Under the proposed distributed control algorithm, the state trajectories of the followers and the leader, and the tracking errors between the followers and the leader are shown in Figs. 3 and 4, respectively.

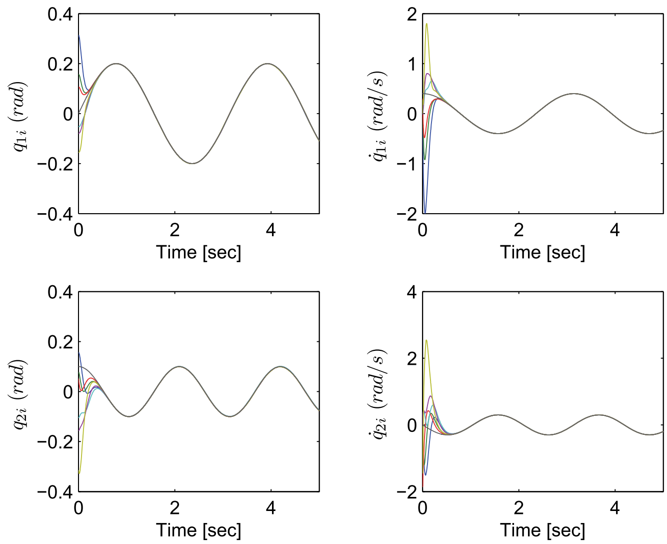

Fig. 9. Trajectories of the states of the followers and the leader under (45).

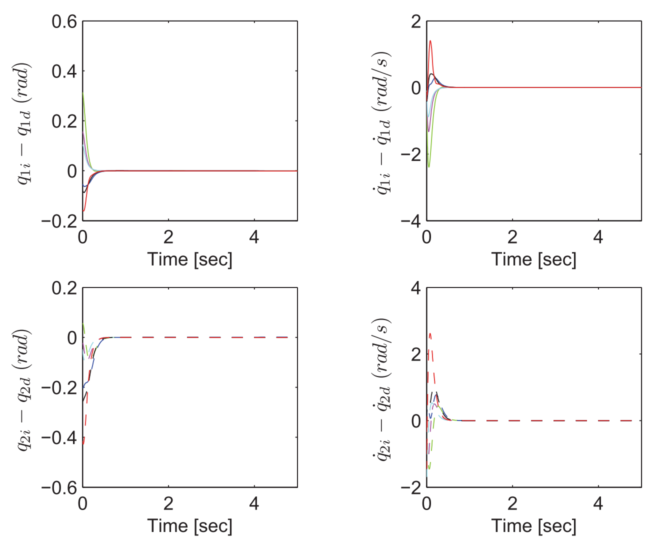

Fig. 10. Tracking errors between the followers and the leader under (45).

Fig. 5 shows the trajectories of τ_i in (6) and the convergence of a time-varying gain β_{1i} in (9), respectively. It can be seen that under the proposed distributed controller, robust consensus tracking is achieved for a group of unknown Euler-Lagrange systems under the undirected graph.

B. Robust Consensus Tracking Under a Directed Graph

The directed communication graph of the team of six agents and one leader is provided in Fig. 6. For the desired time-varying trajectory, the expression in (36) holds, provided that

$$S_1 = \begin{bmatrix} -4 & 0 \\ 0 & -9 \end{bmatrix} \quad \text{and} \quad S_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{59}$$

which implies that Assumption 5 is satisfied.

According to the derived sufficient conditions in Theorem 2, the distributed controller gains in (45) with (39) and (46) are selected as $\kappa_i = 10$, $\alpha_{qi} = 5$, $\alpha_{si} = 10$, $\alpha_{\varrho i} = 15$, $\alpha_{ri} = 20$,

[5]For each simulation, the desired time-varying trajectory $q_d(t)$ for consensus tracking is given by $q_d(t) = [0.2\sin(2t), 0.1\cos(3t)]^T$ rad, which is only provided to the agents labeled by 3,6 as indicated in the matrix $B = \text{diag}\{0, 0, 2, 0, 0, 2\}$.

$\beta_{\varrho 2i} = 1, k_{\varrho si} = 30$. Under the developed distributed leader estimator in (39), the estimated state trajectories of the followers and the leader and the estimated tracking errors between the followers and the leader are shown in Figs. 7 and 8, respectively. Moreover, under the developed distributed control algorithm in (45), the state trajectories of the followers and the leader and the tracking errors between the followers and the leader are shown in Figs. 9 and 10, respectively. As a result, it can be seen from Figs. 7–10 that robust distributed cooperative tracking can be achieved for a group of heterogeneous unknown Euler-Lagrange systems under the directed graph.

VI. Conclusion

A robust consensus tracking problem is considered for multiple unknown Euler-Lagrange systems under both undirected and directed graphs. The control objective is to enable all the agents to achieve robust consensus tracking asymptotically. A distributed nonlinear identifier is firstly developed to compensate for the unknown nonlinear dynamics and disturbances. Then, a robust distributed control law combined with this identifier is developed to enable all the agents to reach asymptotic robust consensus tracking. Furthermore, in the directed case a distributed two-layer coordination scheme is developed to solve the problem. For both undirected and directed graphs, the stability analysis of the closed-loop system is provided.

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Authors’ photographs and biographies not available at the time of publication.