LESSON 7.3

Triangle Inequalities

Common Core Math Standards

The student is expected to:

COMMON CORE **G-SRT.B.5**

Use congruence … criteria for triangles to solve problems and to prove relationships in geometric figures. Also G-CO.C.10, G-CO.D.12

Mathematical Practices

COMMON CORE **MP.5 Using Tools**

Language Objective

Explain to a partner how to show the three inequalities generated for a triangle with side lengths a, b, and c.

ENGAGE

Essential Question: How can you use inequalities to describe the relationships among side lengths and angle measures in a triangle?

The sum of any two side lengths of a triangle will be greater than the length of the third side. If the sides of a triangle are not congruent, then the largest angle will be opposite the longest side and the smallest angle will be opposite the shortest side.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss the photo, making sure that students understand the objective of an orienteering competition and the tools used by competing teams. Then preview the Lesson Performance Task.

Name ____________ Class ____________ Date ________

7.3 Triangle Inequalities

Essential Question: How can you use inequalities to describe the relationships among side lengths and angle measures in a triangle?

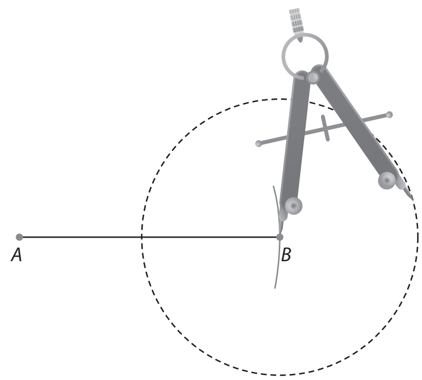
Resource Locker

Explore Exploring Triangle Inequalities

A triangle can have sides of different lengths, but are there limits to the lengths of any of the sides?

(A) Consider a $\triangle ABC$ where you know two side lengths, $AB = 4$ inches and $BC = 2$ inches. On a separate piece of paper, draw $\overline{AB}$ so that it is 4 inches long.

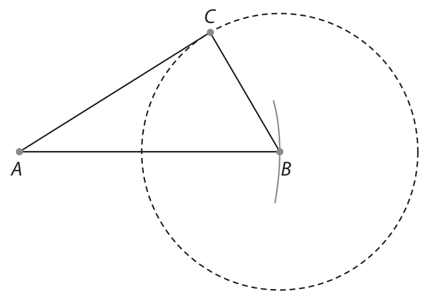
(B) To determine all possible locations for C with $\overline{BC} = 2$ inches, set your compass to 2 inches. Draw a circle with center at B.



(C) Choose and label a final vertex point C so it is located on the circle. Using a straightedge, draw the segments to form a triangle.

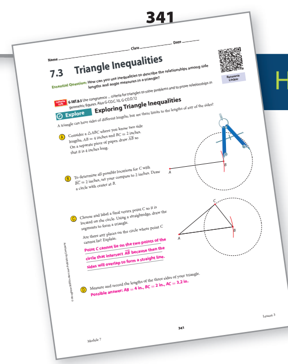
Are there any places on the circle where point C cannot lie? Explain.

Point C cannot lie on the two points of the circle that intersect $\overleftrightarrow{AB}$ because then the sides will overlap to form a straight line.



(D) Measure and record the lengths of the three sides of your triangle.

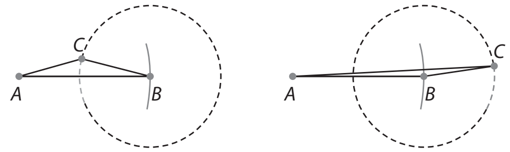
Possible answer: $AB = 4$ in., $BC = 2$ in., $AC = 3.2$ in.



HARDCOVER PAGES 293–302

Turn to these pages to find this lesson in the hardcover student edition.

Ⓔ The figures below show two other examples of $\triangle ABC$ that could have been formed. What are the values that $\overline{AC}$ approaches when point C approaches $\overline{AB}$?



AC approaches 2 inches or 6 inches.

Reflect

1. Use the side lengths from your table to make the following comparisons. What do you notice?

   $AB + BC \; ? \; AC$ $\quad BC + AC \; ? \; AB$ $\quad AC + AB \; ? \; BC$

   The sum of any of the two sides is greater than the third side.

2. Measure the angles of some triangles with a protractor. Where is the smallest angle in relation to the shortest side? Where is the largest angle in relation to the longest side?

   The smallest angle is opposite the shortest side; the largest angle is opposite the longest side.

3. **Discussion** How does your answer to the previous question relate to isosceles triangles or equilateral triangles?

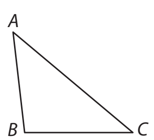
   When angles in a triangle have the same measure, the sides opposite those angles also have the same measure.

Explain 1 Using the Triangle Inequality Theorem

The Explore shows that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. This can be summarized in the following theorem.

Triangle Inequality Theorem

The sum of any two side lengths of a triangle is greater than the third side length.



$AB + BC > AC$

$BC + AC > AB$

$AC + AB > BC$

To be able to form a triangle, each of the three inequalities must be true. So, given three side lengths, you can test to determine if they can be used as segments to form a triangle. To show that three lengths cannot be the side lengths of a triangle, you only need to show that one of the three triangle inequalities is false.

PROFESSIONAL DEVELOPMENT

Learning Progressions

In this lesson, students add to their knowledge of triangles by using a variety of tools to verify the Triangle Inequality Theorem. Students also explore how to order the side lengths given the angle measures of the triangle and how to predict the possible lengths of the third side of a triangle, given the lengths of two of the sides. Triangles have important uses in everyday life and in future mathematics study, including trigonometry. All students should develop fluency with the properties of triangles as they continue their study of geometry.

EXPLORE

Exploring Triangle Inequalities

INTEGRATE TECHNOLOGY

Students have the option of completing the triangle inequality activity either in the book or online.

QUESTIONING STRATEGIES

How do you decide if three lengths can be the side lengths of a triangle? **Check the sum of each pair of two sides. The sum must be greater than the third side.**

EXPLAIN 1

Using the Triangle Inequality Theorem

AVOID COMMON ERRORS

Some students may have difficulty understanding why all three inequalities must be checked for the Triangle Inequality Theorem. One example may be side lengths of 5 cm, 5 cm, and 10 cm. Straws with these lengths look like they will make a triangle, but they do not. Have them do several examples with different side lengths to test the theorem.

QUESTIONING STRATEGIES

For side lengths a, b, and c of a triangle, how many inequalities must be true? Write them. **3; $a < b + c$, $b < a + c$, $c < a + b$**

Example 1 **Use the Triangle Inequality Theorem to tell whether a triangle can have sides with the given lengths. Explain.**

Ⓐ 4, 8, 10

$4 + 8 \overset{?}{>} 10$ $\quad$ $4 + 10 \overset{?}{>} 8$ $\quad$ $8 + 10 \overset{?}{>} 4$

$12 > 10$ ✓ $\quad$ $14 > 8$ ✓ $\quad$ $18 > 4$ ✓

Conclusion: The sum of each pair of side lengths is greater than the third length. So, a triangle can have side lengths of 4, 8, and 10.

Ⓑ 7, 9, 18

$\boxed{7} + \boxed{9} \overset{?}{>} \boxed{18}$ $\quad$ $\boxed{7} + \boxed{18} \overset{?}{>} \boxed{9}$ $\quad$ $\boxed{9} + \boxed{18} \overset{?}{>} \boxed{7}$

$\boxed{16} > \boxed{18}$ $\boxed{x}$ $\quad$ $\boxed{25} > \boxed{9}$ $\boxed{✓}$ $\quad$ $\boxed{27} > \boxed{7}$ $\boxed{✓}$

Conclusion:

Not all three inequalities are true. So, a triangle cannot have these three side lengths.

Reflect

4. Can an isosceles triangle have these side lengths? Explain. 5, 5, 10

No; These numbers do not result in three true inequalities.

$5 + 5 \not> 10$, so no triangle can be drawn with these side lengths.

5. How do you know that the Triangle Inequality Theorem applies to all equilateral triangles?

Since all sides are congruent, the sum of any two side lengths will be greater than the third side.

Your Turn

Determine if a triangle can be formed with the given side lengths. Explain your reasoning.

6. 12 units, 4 units, 17 units

No; $12 + 4 \not> 17$

7. 24 cm, 8 cm, 30 cm

Yes; $24 + 8 > 30$, $8 + 30 > 24$, and $24 + 30 > 8$

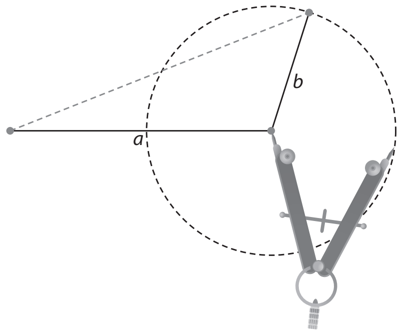
COLLABORATIVE LEARNING

Small Group Activity

Give all students in groups pieces of raw spaghetti. Have each student break three pieces of spaghetti in different lengths and measure the length of each piece. Then have a group member write the three inequalities for those lengths. Have another group member analyze the inequalities and conjecture if the three lengths will form a triangle. Ask the fourth student to position the pieces to show a triangle or to show no triangle. Switch roles and repeat the activity.

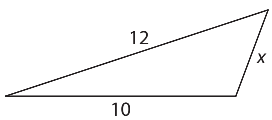
Explain 2 Finding Possible Side Lengths in a Triangle

From the Explore, you have seen that if given two side lengths for a triangle, there are an infinite number of side lengths available for the third side. But the third side is also restricted to values determined by the Triangle Inequality Theorem.



Example 2 **Find the range of values for x using the Triangle Inequality Theorem.**

(A) Find possible values for the length of the third side using the Triangle Inequality Theorem.

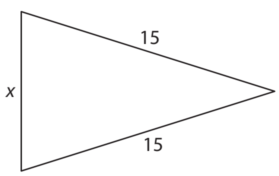


$x + 10 > 12$	$x + 12 > 10$	$10 + 12 > x$
$x > 2$	$x > -2$	$22 > x$

$2 < x < 22$

Ignore the inequality with a negative value, since a triangle cannot have a negative side length. Combine the other two inequalities to find the possible values for x.

(B)



$x + 15 > 15$	$x + 15 > 15$	$15 + 15 > x$
$x > 0$	$x > 0$	$30 > x$

$0 < x < 30$

EXPLAIN 2

Finding Possible Side Lengths in a Triangle

CONNECT VOCABULARY EL

Relate the *range of values* for a third side length of a triangle given two side lengths by stating that the third side length must be greater than the difference of the other two side lengths and also less than the sum of the other two side lengths.

DIFFERENTIATE INSTRUCTION

Manipulatives

Give students a number of straws and ask them to cut them into various lengths. Then have them measure each length. Ask them to make a table listing the measures of all possible combinations of the three lengths. Then have them manipulate the straws to see if they can form a triangle. Highlight sets of three measurements that do not form a triangle.

QUESTIONING STRATEGIES

How do find the range for the length of the third side of a triangle? **The range is $r < x < s$, where r is the difference of the two given side lengths and s is the sum of the two given side lengths.**

How do you interpret the compound inequality $a < x < b$ as individual inequalities? **$a < x$ and $x < b$**

EXPLAIN 3

Ordering a Triangle's Angle Measures Given Its Side Lengths

INTEGRATE MATHEMATICAL PRACTICES

Focus on Technology

MP.5 Have students use geometry software to create a scalene triangle. Ask them to use the measuring features to measure the side lengths. Then have them measure each angle and verify that the largest angle is opposite the longest side length and that the smallest angle is opposite the shortest side length. Ask them to drag the vertices to vary the side lengths and then observe that the angle measures are ordered in the same way as in the original triangle.

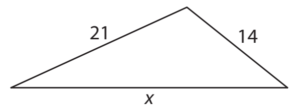
Reflect

8. **Discussion** Suppose you know that the length of the base of an isosceles triangle is 10, but you do not know the lengths of its legs. How could you use the Triangle Inequality Theorem to find the range of possible lengths for each leg? Explain.
Possible answer: If x represents the length of one leg, then by the Triangle Inequality Theorem, solve for $x + x > 10$ and $x + 10 > x$. The solution of the first inequality is $x > 5$. The solution of the second inequality is $10 > 0$, which is always true. So the range of possible lengths for each leg is $x > 5$.

Your Turn

Find the range of values for x using the Triangle Inequality Theorem.

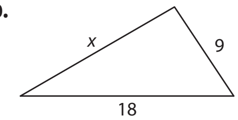
9.



$x + 14 > 21$
$x > 7$
$x + 21 > 14$
$x > -7$

$21 + 14 > x$
$35 > x$
$7 < x < 35$

10.

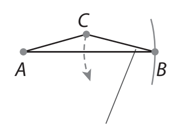


$x + 9 > 18$
$x > 9$
$x + 18 > 9$
$x > -9$

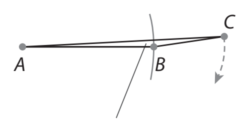
$18 + 9 > x$
$27 > x$
$9 < x < 27$

Explain 3 Ordering a Triangle's Angle Measures Given Its Side Lengths

From the Explore Step D, you can see that changing the length of $\overline{AC}$ also changes the measure of $\angle B$ in a predictable way.



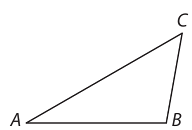
As side AC gets shorter, m$\angle B$ approaches 0°



As side AC gets longer, m$\angle B$ approaches 180°

Side-Angle Relationships in Triangles

If two sides of a triangle are not congruent, then the larger angle is opposite the longer side.



$AC > BC$
$m\angle B > m\angle A$

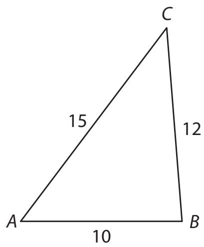
LANGUAGE SUPPORT EL

Visual Cues

Help students understand how to apply the inequality relationships in this lesson by suggesting they list all the angles and sides before doing an example or exercise. Then, they can list the angles in increasing order and write the side lengths opposite those angles in the same order.

Example 3 **For each triangle, order its angle measures from least to greatest.**

Ⓐ



Longest side length: AC

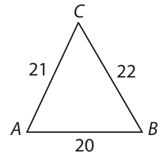
Greatest angle measure: $m\angle B$

Shortest side length: AB

Least angle measure: $m\angle C$

Order of angle measures from least to greatest: $m\angle C$, $m\angle A$, $m\angle B$

Ⓑ



Longest side length: **BC**

Greatest angle measure: **$m\angle A$**

Shortest side length: **AB**

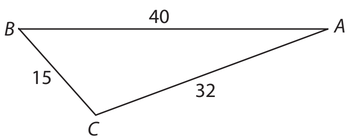
Least angle measure: **$m\angle C$**

Order of angle measures from least to greatest: **$m\angle C$, $m\angle B$, $m\angle A$**

Your Turn

For each triangle, order its angle measures from least to greatest.

11.

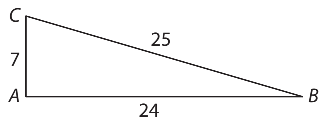


Longest side length: AB

Shortest side length: CB

$m\angle A$, $m\angle B$, $m\angle C$

12.



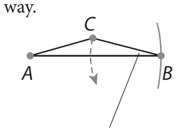
Longest side length: BC

Shortest side length: AC

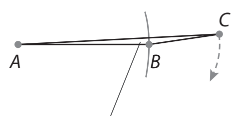
$m\angle B$, $m\angle C$, $m\angle A$

Explain 4 Ordering a Triangle's Side Lengths Given Its Angle Measures

From the Explore Step D, you can see that changing the the measure of $\angle B$ also changes length of $\overline{AC}$ in a predictable way.



As $m\angle B$ approaches 0°, side AC gets shorter



As $m\angle B$ approaches 180°, side AC gets longer

Angle-Side Relationships in Triangles
If two angles of a triangle are not congruent, then the longer side is opposite the larger angle.

QUESTIONING STRATEGIES

How do you know the greatest angle is opposite the longest side in a triangle? **If one side of a triangle is longer than another, then the angle opposite the longer side is larger than the angle opposite the shorter side.**

EXPLAIN 4

Ordering a Triangle's Side Lengths Given Its Angle Measures

AVOID COMMON ERRORS

Some students may think that they can use angle measures to compare the side lengths of different triangles. Explain that angle measures can be used to order the side lengths only within a single triangle. Give an example of why this must be the case, such as a very small triangle with an obtuse angle and a very large equilateral triangle. The obtuse angle is greater than the 60° angle of the equilateral triangle, but its opposite side may be shorter.

QUESTIONING STRATEGIES

How do you order the side lengths of a triangle given the angle measures? Explain. **The side lengths will be in the same order as the measure of the angles opposite the side lengths. For example, the greatest side length is opposite the greatest angle measure. Use the rule that if one angle of a triangle is larger than another, then the side opposite the larger angle is longer than the side opposite the smaller angle.**

ELABORATE

AVOID COMMON ERRORS

Some students may list angle measures in order and side lengths in order, but not make the connection that the largest side length must be opposite the largest angle measure. Suggest that they list the angles with the corresponding opposite side before they order their measures.

QUESTIONING STRATEGIES

? Can a triangle have side lengths of 7 cm, 12 cm, and 20 cm? Explain. **No; the Triangle Inequality Theorem states that the sum of each pair of lengths must be greater than the third length in order for 3 lengths to be side lengths of a triangle. Since $7 + 12 \not> 20$, these lengths cannot be lengths of sides of a triangle.**

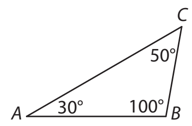
? A triangle has angle measures of 30°, 60°, and 90°. Which angle is opposite the longest side of the triangle? Explain. **The 90° angle, because the side lengths of a triangle are ordered in the same way as the angle measures.**

SUMMARIZE THE LESSON

? How do you know if three segment lengths can be the side lengths of a triangle? **Test the sum of each two pairs of segment lengths. By the Triangle Inequality Theorem, if the sum of each pair of segment lengths is greater than the third length, then the segments can be the side lengths of a triangle.**

Example 4 **For each triangle, order the side lengths from least to greatest.**

Ⓐ



Greatest angle measure: m∠*B*

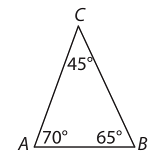
Longest side length: *AC*

Least angle measure: m∠*A*

Shortest side length: *BC*

Order of side lengths from least to greatest: *BC*, *AB*, *AC*

Ⓑ



Greatest angle measure: **m∠A**

Longest side length: **BC**

Least angle measure: **m∠C**

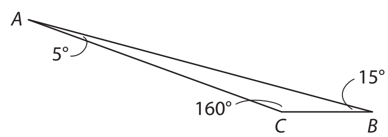
Shortest side length: **AB**

Order of side lengths from least to great: **AB, AC, BC**

Your Turn

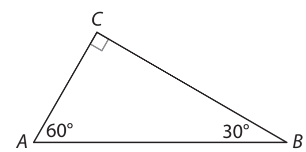
For each triangle, order the side lengths from least to greatest.

13.



Greatest angle measure: m∠C
Least angle measure: m∠A
CB, AC, AB

14.



Greatest angle measure: m∠C
Least angle measure: m∠B
AC, BC, AB

Elaborate

15. When two sides of a triangle are congruent, what can you conclude about the angles opposite those sides?
They are also congruent.

16. What can you conclude about the side opposite the obtuse angle in an obtuse triangle?
It is the longest side of the triangle.

17. **Essential Question Check-In** Suppose you are given three values that could represent the side lengths of a triangle. How can you use one inequality to determine if the triangle exists?
If the sum of the two least values is greater than the remaining value, the triangle exists. Otherwise it does not exist.

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Use a compass and straightedge to decide whether each set of lengths can form a triangle.

1. 7 cm, 9 cm, 18 cm

No; if the base is 18 cm compass arcs of lengths 7 cm and 9 cm from each end of the base do not intersect.

2. 2 in., 4 in., 5 in.

Yes; if the base is 5 in. compass arcs of lengths 2 in. and 4 in. from each end of the base have two intersections, each forming a triangle.

3. 1 in., 2 in., 10 in.

No; if the base is 10 in. compass arcs of lengths 1 in. and 2 in. from each end of the base do not intersect.

4. 9 cm, 10 cm, 11 cm

Yes; if the base is 11 cm compass arcs of lengths 9 cm and 10 cm from each end of the base have two intersections, each forming a triangle.

Determine whether a triangle can be formed with the given side lengths.

5. 10 ft, 3 ft, 15 ft **No; $10 + 3 \not> 15$**

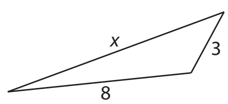
6. 12 in., 4 in., 15 in. **Yes; $12 + 4 > 15$, $4 + 15 > 12$, and $12 + 15 > 4$**

7. 9 in., 12 in., and 18 in. **Yes**

8. 29 m, 59 m, and 89 m **No; $29 + 59 \not> 89$**

Find the range of possible values for x using the Triangle Inequality Theorem.

9.

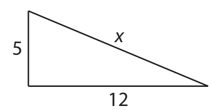


$5 < x < 11$

$3 + 8 > x$ $3 + x > 8$ $8 + x > 3$

$11 > x$ $x > 5$ $x > -5$

10.



$7 < x < 17$

$5 + 12 > x$ $5 + x > 12$ $12 + x > 5$

$17 > x$ $x > 7$ $x > -7$

11. A triangle with side lengths 22.3, 27.6, and x $5.3 < x < 49.9$

$22.3 + 27.6 > x$ $22.3 + x > 27.6$ $27.6 + x > 22.3$

$49.9 > x$ $x > 5.3$ $x > -5.3$

12. **Analyze Relationships** Suppose a triangle has side lengths AB, BC, and x, where $AB = 2 \cdot BC$. Find the possible range for x in terms of BC. $BC < x < 3 \cdot BC$

$AB + BC > x$ $BC + x > AB$ $x + AB > BC$

$2 \cdot BC + BC > x$ $BC + x > 2 \cdot BC$ $x + 2 \cdot BC > BC$

$3 \cdot BC > x$ $x > BC$ $x > -BC$

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
1–19	2 Skills/Concepts	MP.4 Modeling
20–27	2 Skills/Concepts	MP.4 Modeling
28	3 Strategic Thinking H.O.T.	MP.4 Modeling
29	3 Strategic Thinking H.O.T.	MP.3 Logic
30	3 Strategic Thinking H.O.T.	MP.3 Logic

EVALUATE

Personal Math Trainer

ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explore Exploring Triangle Inequalities	Exercises 1–4
Example 1 Using the Triangle Inequality Theorem	Exercises 5–8
Example 2 Finding Possible Side Lengths in a Triangle	Exercises 9–12
Example 3 Ordering a Triangle's Angle Measures Given Its Side Lengths	Exercises 13–15
Example 4 Ordering a Triangle's Side Lengths Given Its Angle Measures	Exercises 16–19

INTEGRATE MATHEMATICAL PRACTICES

Focus on Technology

MP.5 Students can check their solutions for correctness by using geometry software to create triangles with the same side lengths or angle measures. When checking solutions, remind students the order of the side lengths gives the order of the opposite angle measures.

VISUAL CUES EL

Suggest that students label a side and its corresponding opposite angle in one color and then do the same with other side-angle combinations in different colors. This visual cue can help them to remember the order of the measures of the sides or angles.

AVOID COMMON ERRORS

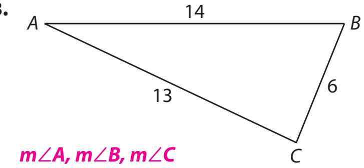
If students have trouble with compound inequalities, have them write the inequalities separately and then use a number line to help them combine the inequalities into one.

COLLABORATIVE LEARNING

Have students work in small groups to make a poster showing a triangle, the side-angle relationships, and the triangle inequality relationship they learned in this lesson. Give each group a different triangle to draw. Then have each group present its poster to the rest of the class, explaining each relationship they listed.

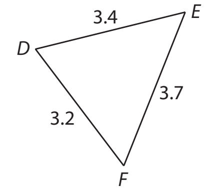
For each triangle, write the order of the angle measures from least to greatest.

13.



$m\angle A$, $m\angle B$, $m\angle C$

14.



$m\angle E$, $m\angle F$, $m\angle D$

15. **Analyze Relationships** Suppose a triangle has side lengths PQ, QR, and PR, where $PR = 2PQ = 3QR$. Write the angle measures in order from least to greatest.

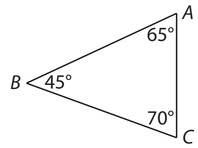
$PR = 2PQ$ $\quad$ $PR = 3QR$ $\quad$ $\frac{1}{3}PR < \frac{1}{2}PR < PR$

$\frac{1}{2}PR = PQ$ $\quad$ $\frac{1}{3}PR = QR$ $\quad$ $QR < PQ < PR$

So $m\angle P < m\angle R < m\angle Q$; Order from least to greatest: $m\angle P$, $m\angle R$, $m\angle Q$

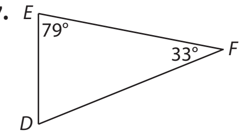
For each triangle, write the side lengths in order from least to greatest.

16.



AC, BC, AB

17.



$m\angle D = 68°$

DE, EF, DF

18. In $\triangle JKL$, $m\angle J = 53°$, $m\angle K = 68°$, and $m\angle L = 59°$. KL, JK, JL

19. In $\triangle PQR$, $m\angle P = 102°$ and $m\angle Q = 25°$.
$m\angle R = 53°$ PR, PQ, QR

20. **Represent Real-World Problems** Rhonda is traveling from New York City to Paris and is trying to decide whether to fly via Frankfurt or to get a more expensive direct flight. Given that it is 3,857 miles from New York City to Frankfurt and another 278 miles from Frankfurt to Paris, what is the range of possible values for the direct distance from New York City to Paris?

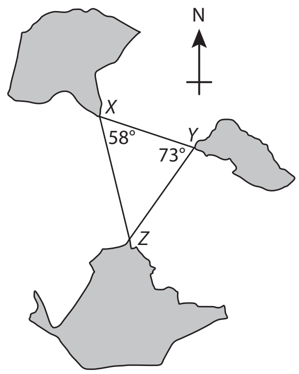
$3{,}857 + 278 > x$ $\quad$ $3{,}857 + x > 278$ $\quad$ $278 + x > 3{,}857$

$4{,}135 > x$ $\quad$ $x > -3{,}579$ $\quad$ $x > 3{,}579$

$3{,}579 < x < 4{,}135$

The direct distance is between 3,579 miles and 4,135 miles.

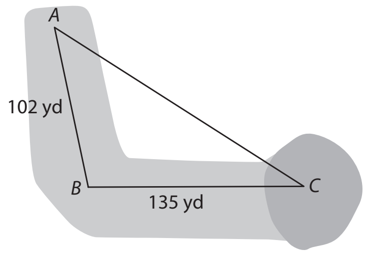
21. **Represent Real-World Problems** A large ship is sailing between three small islands. To do so, the ship must sail between two pairs of islands, avoiding sailing between a third pair. The safest route is to avoid the closest pair of islands. Which is the safest route for the ship?



$58° + 73° + m\angle Z = 180°$; $m\angle Z = 49°$

$m\angle Z < m\angle X < m\angle Y$, so $XY < YZ < XZ$. Therefore, the safest route is to avoid sailing between the islands at X and Y.

22. **Represent Real-World Problems** A hole on a golf course is a dogleg, meaning that it bends in the middle. A golfer will usually start by driving for the bend in the dogleg (from A to B), and then using a second shot to get the ball to the green (from B to C). Sandy believes she may be able to drive the ball far enough to reach the green in one shot, avoiding the bend (from A direct to C). Sandy knows she can accurately drive a distance of 250 yd. Should she attempt to drive for the green on her first shot? Explain. **Yes;**



$$102 + 135 > AC \qquad 102 + AC > 135 \qquad 135 + AC > 102$$
$$237 > AC \qquad AC > 33 \qquad AC > -33$$
$$33 < AC < 237$$

Since AC is less than 250 yd, Sandy has a good chance of reaching the green in one shot.

23. **Represent Real-World Problems** Three cell phone towers form a triangle, $\triangle PQR$. The measure of $\angle Q$ is 10° less than the measure of $\angle P$. The measure of $\angle R$ is 5° greater than the measure of $\angle Q$. Which two towers are closest together?

$m\angle Q = m\angle P - 10°$ and

$m\angle R = m\angle Q + 5° = (m\angle P - 10°) + 5° = m\angle P - 5°$

So, $m\angle Q < m\angle R < m\angle P$, and therefore $PR < PQ < QR$. The towers at Q and R are closest together.

24. **Algebra** In $\triangle PQR$, $PQ = 3x + 1$, $QR = 2x - 2$, and $PR = x + 7$. Determine the range of possible values of x. **First, each side length must be positive.**

$$3x + 1 > 0 \qquad 2x - 2 > 0 \qquad x + 7 > 0$$
$$x > -\frac{1}{3} \qquad x > 1 \qquad x > -7$$
$$(3x + 1) + (2x - 2) > x + 7 \qquad (3x + 1) + (x + 7) > 2x - 2 \qquad (2x - 2) + (x + 7) > (3x + 1)$$
$$x > 2 \qquad x > -5 \qquad 4 > 0$$

Since the last inequality is always true, $x > 2$.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 When writing side measures or side-angle measure relationships for a triangle, students should remember that there is a range of possible side lengths for making a triangle, and that the side lengths and angle measures must be in the same order.

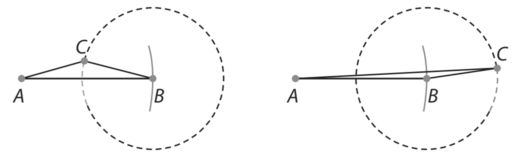
AVOID COMMON ERRORS

Some students may think that all three inequalities associated with the Triangle Inequality Theorem must be false. Point out that you need to show only that one of the three triangle inequalities is false to state that the three lengths are not side lengths of a triangle.

JOURNAL

Have students use diagrams in their journals to illustrate the angle-side relationships in triangles.

25. In any triangle ABC, suppose you know the lengths of $\overline{AB}$ and $\overline{BC}$, and suppose that $AB > BC$. If x is the length of the third side, $\overline{AC}$, use the Triangle Inequality Theorem to prove that $AB - BC < x < AB + BC$. That is, x must be between the difference and the sum of the other two side lengths. Explain why this result makes sense in terms of the constructions shown in the figure.



$AB + BC > x$

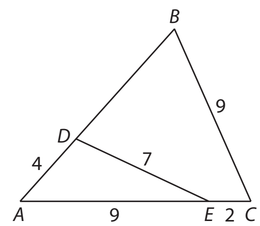
$AB + x > BC$
$x > BC - AB$

$BC + x > AB$
$x > AB - BC$

Since $AB > BC$, $BC - AB < 0$, so the second inequality is not relevant. Combining the first and last inequalities gives $AB - BC < x < AB + BC$.

The constructions show that AC approaches but is always greater than $AB - BC$, and that AC approaches but is always less than $AB + BC$.

26. Given the information in the diagram, prove that $m\angle DEA < m\angle ABC$.



In , $\triangle ADE$, $DA < DE$, so $m\angle DEA < m\angle DAE = m\angle BAC$. In $\triangle ABC$, $AC = 9 + 2 = 11$ (Segment Addition Postulate), so $BC < AC$, and therefore $m\angle BAC < m\angle ABC$. Therefore, $m\angle DEA < m\angle ABC$ (Transitive Property Of Inequality).

27. An isosceles triangle has legs with length 11 units. Which of the following could be the perimeter of the triangle? Choose all that apply. Explain your reasoning.

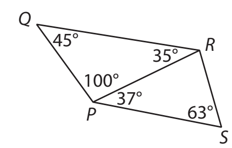
a. 22 units
b. 24 units
c. 34 units
d. 43 units
e. 44 units

B, C, D

If x represents the length of the third side of the triangle, then by the Triangle Inequality Theorem, solve for $11 + x > 11$ and $11 + 11 > x$. The solution of the first inequality is $x > 0$, which is always true. The solution of the second inequality is $22 > x$. So the range of possible lengths for the third side is $0 < x < 22$. Use both limits to solve for perimeter. $11 + 11 + 0 = 22$ and $11 + 11 + 22 = 44$. So, the perimeter for all possible triangles must be greater than 22 units and less than 44 units. So choices A and E are not possible.

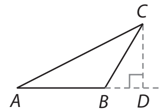
H.O.T. Focus on Higher Order Thinking

28. Communicate Mathematical Ideas Given the information in the diagram, prove that $PQ < PS$.



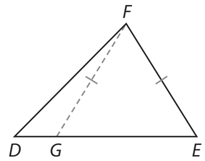
In, $\triangle PQR$, $m\angle PRQ < m\angle Q$, so $PQ < PR$. In $\triangle PRS$, $m\angle PRS = 180° - 37° - 63° = 80°$, so $m\angle S < m\angle PRS$, and therefore $PR < PS$. Therefore, $PQ < PS$ (Transitive Property of Inequality).

29. **Justify Reasoning** In obtuse $\triangle ABC$, $m\angle A < m\angle B$. The auxiliary line segment $\overline{CD}$ perpendicular to $\overleftrightarrow{AB}$ (extended beyond B) creates right triangles ADC and BDC. Describe how you could use the Pythagorean Theorem to prove that $BC < AC$.



Write two equations, $AD^2 + CD^2 = AC^2$ and $BD^2 + CD^2 = BC^2$. Equating expressions for CD^2, $AC^2 - AD^2 = BC^2 - BD^2$ and therefore $AC^2 - BC^2 = AD^2 - BD^2$. Since the right side is positive, so is the left side, which leads to $BC < AC$.

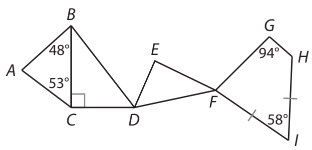
30. **Make a Conjecture** In acute $\triangle DEF$, $m\angle D < m\angle E$. The auxiliary line segment $\overline{FG}$ creates $\triangle EFG$, where $EF = FG$. What would you need to prove about the points D, G, and E to prove that $\angle DGF$ is obtuse, and therefore that $EF < DF$? Explain.



You would need to show that G lies on $\overline{DE}$, i.e. between D and E. In that case, since $\angle DGF$ and $\angle EGF$ are supplementary and $\angle EGF$ is acute, then $\angle DGF$ is obtuse. So $\angle DGF$ is the largest angle in $\triangle DGF$ and $FG < DF$. Since $EF = FG$, then by substitution, $EF < DF$.

Lesson Performance Task

As captain of your orienteering team, it's your job to map out the shortest distance from point A to point H on the map. Justify each of your decisions.



The shortest route is A-C-D-F-G-H.

In $\triangle ABC$ the smallest angle measures 48°, so the shortest side is $\overline{AC}$.

In $\triangle BCD$ the route from C to D is shorter than the route from C to B to D, because $\overline{BD}$ is the longest side of the triangle.

In $\triangle DEF$ the route from D to F is shorter than the route from D to E to F by the Triangle Inequality Theorem.

In quadrilateral $FGHI$, draw $\overline{FH}$. Since $FI = HI$, $\triangle FIH$ is isosceles, with base angles each measuring 61°. So, $\overline{FH}$ is the shortest side of $\triangle FIH$. $\overline{FH}$ is opposite the largest angle in $\triangle FGH$, so $\overline{FH}$ is the longest side in triangle in $\triangle FGH$ by the Triangle Inequality Theorem. So, $FI > FH > FG$ and $IH > FH > GH$. So, the path from F to G to H is shorter than the path from F to I to H.

EXTENSION ACTIVITY

Have students design orienteering courses like the one in the Lesson Performance Task. Courses should consist of at least four stages. At each stage of a course, angle measures or other information should be given that will allow an orienteer to apply the Triangle Inequality Theorem, angle-side relationships, and/or side-angle relationships, in order to gauge the shortest route to follow. Students may wish to work in teams of two or three and tackle routes other students have designed.

AVOID COMMON ERRORS

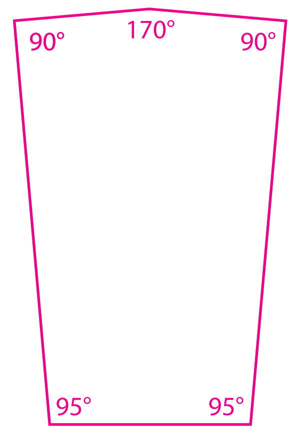
Students may attempt to apply the methods discussed in this lesson to figures other than triangles, an approach that is likely to lead to false conclusions. The solution, when confronted with a figure like quadrilateral *FGHI* in the Lesson Performance Task, is to draw one or more diagonals, dividing the figure into triangles whose inequalities can then be analyzed.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

MP.3 Define the side *opposite* an angle in a pentagon as the side that neither forms the angle nor is adjacent to a side that forms the angle.

Ask students to draw and label a pentagon in which, unlike a triangle, the side opposite the largest angle is the *shortest* side of the pentagon. **Sample figure:**



Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain.

0 points: Student does not demonstrate understanding of the problem.