DOCUMENT RESUME

ED 404 347 TM 026 100

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TITLE	The Importance of Structure Coefficients in Structural Equation Modeling Confirmatory Factor Analysis.
PUB DATE	8 Nov 96
NOTE	30p.; Paper presented at the Annual Meeting of the Mid-South Educational Research Association (Tuscaloosa, AL, November, 1996).
PUB TYPE	Reports - Evaluative/Feasibility (142) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	Ability; *Correlation; Heuristics; *Mathematical Models; Scores; Self Concept; *Structural Equation Models
IDENTIFIERS	*Confirmatory Factor Analysis; Linear Models; *Structure Coefficients

ABSTRACT

A general linear model (GLM) framework is used to suggest that structure coefficients ought to be interpreted in structural equation modeling confirmatory factor analysis (CFA) studies in which factors are correlated. The computation of structure coefficients in explanatory factor analysis and CFA is explained. Two heuristic data sets are used to make the discussion concrete, illustrating the calculation of pattern and structure coefficients in LISREL CFA studies investigating scores on ability batteries. The benefits from using CFA structure coefficients are illustrated using two additional studies. One involves nine ability variables from a previous CFA study, and the other involves a self-concept model tested in a study by B. M. Byrne (1989). (Contains 6 tables and 28 references.) (Author/SLD)

cfastruc.wp1 8/6/96

THE IMPORTANCE OF STRUCTURE COEFFICIENTS IN STRUCTURAL EQUATION MODELING CONFIRMATORY FACTOR ANALYSIS

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Σ sigma
Λ lambda
Φ phi
Θ theta
Ξ ksi

Paper presented at the annual meeting of the Mid-South Educational Research Association, Tuscaloosa, AL, November 8, 1997. The author appreciates the thoughtful comments of Barbara M. Byrne, University of Ottawa, and Richard L. Gorsuch, Fuller Theological Seminary, on a previous draft of this article.

Abstract

A general linear model (GLM) framework is employed to suggest that structure coefficients ought to be interpreted in structural equation modeling confirmatory factor analysis (CFA) studies in which factors are correlated. The computation of structure coefficients is explained. Two heuristic data sets are used to make the discussion concrete. The benefits from using CFA structure coefficients are illustrated using two additional studies.

The close association between factor analysis and measurement has been previously noted within Educational and Psychological Measurement (Thompson & Daniel, 1996). In this same journal 50 years earlier Guilford (1946) argued:

> The *factorial validity* of a test is given by its loadings in meaningful, common, reference factors. This is the kind of validity that is really meant when the question is asked "Does this test measure what it is supposed to measure?" (pp. 428, 437-438, emphasis added)

Similarly, Nunnally (1978) suggested that "factor analysis is intimately involved with questions of validity.... Factor analysis is at the heart of the measurement of psychological constructs" (pp. 112-113).

There are two basic factor analytic models, both of which are commonly used in measurement studies: exploratory factor analysis (EFA; cf. Gorsuch, 1983) and confirmatory factor analysis (CFA; cf. Byrne, 1989). CFA directly tests the fit of theoretically- or empirically-grounded models to data, and thus is especially useful, for at least three reasons.

First, CFA allows several rival models to be fit to data, and consequently better honors the role of falsification within scientific inquiry (Popper, 1962). This concept requires that a measure not be deemed credible until the underlying construct model has survived serious disconfirmation efforts. As Moss (1995) explained,

> Construct validation is most efficiently guided by the test of "plausible rival hypotheses" which suggests credible alternative explanations or meanings for the test score that are challenged and refuted by the evidence collected.... Essentially, test validation examines the fit between the meaning of the test score and the measurement intent, whereas construct validation entails the evaluation of an entire theoretical framework. (pp. 6-7)

Second, CFA forces us to be precise in defining our constructs. As Mulaik (1987, p. 301) emphasized, "It is we who create meanings for things in deciding how they are to be used. Thus we should see the folly of supposing that exploratory factor analysis will teach us what intelligence is, or what personality is." CFA requires us to accept the existential responsibility for defining our constructs. Of course, as Huberty (1994, p. 265) noted, our data can be used to guide our decisions as to what constructs are, that is, theory development and theory testing are "joint bootstrap operations" (Hendrick & Hendrick, 1986, p. 393).

Third, CFA models can be evaluated so as to reward parsimony (cf. Mulaik, James, van Alstine, Bennett, Lind, & Stilwell, 1989). It seems reasonable to emphasize parsimony in construct definition, since simpler models may be more likely to be true, and thus more parsimonious results are more likely to be replicable, the fit of rival models being roughly equal. In any case, simpler models--if useful--are easier to employ, and are therefore generally again preferable. In CFA when we "free" a parameter, we get an exact fit to the data for this estimate. Fit, then, is partially a function of how many parameters we free. The most rigorous tests of plausible alternative models arise when our models are most parsimonious.

Given these features, CFA has enjoyed considerable use especially within the Validity Studies section of Educational and Psychological Measurement. For example, the October and the December issues of the 1995 volume included four and three Validity Studies using CFA, respectively (e.g., Schau, Stevens, Dauphinee, & del Vecchio, 1995). However, in none of these articles did authors report and interpret factor structure coefficients for their analyses.

The purpose of the present paper is to argue that *both* factor pattern and factor structure coefficients should be interpreted in most CFA reports involving correlated factors. For the EFA case, (Gorsuch, 1983, p. 208, emphasis removed) argued that, "Indeed, proper interpretation of a set of factors can probably only occur if at least S [the factor structure coefficient matrix] and P [the factor pattern coefficient matrix] are both examined." It will be argued here that structure coefficients can also be useful in CFA research.

The argument is initially grounded in the framework of the general linear model (GLM). Next, the computation of structure coefficients in EFA and CFA is described. Finally, two illustrations of the calculation of pattern and structure coefficients in LISREL (Jöreskog & Sörbom, 1989) CFA studies are presented.

Structure Coefficients in the General Linear Model

It is well established that classical univariate and multivariate methods are all special cases (Fan, 1992; Knapp, 1978; Thompson, 1991) of canonical correlation analysis (cf. Thompson, 1984, but also see Bagozzi, 1981). It is also widely accepted that structure coefficients are important to interpret in classical parametric analyses.

Definitions

It has been previously noted that the language of statistics seems to have been chosen as if to maximally confuse everyone (Thompson & Daniel, 1996). As part of one general linear model, all these methods apply weights to (usually standardized) observed/measured variables to yield scores on the synthetic variables (e.g., factor scores, regression $\hat{Y}$ scores) actually of interest in the analysis.

Viewed differently, the system of weights (variously called "equations," "factors," or "functions") relate the unobserved synthetic variables (i.e., the constructs) back to the observed variables that the constructs presumably cause. This view is honored in structural equation graphic models by drawing arrows *from* the circles representing constructs *to* the rectangles representing observed variables (e.g., Byrne, 1989, p. 38). As Thompson (1995, p. 87) noted,

> These weights are all analogous, but are given

different names in different analyses (e.g., beta weights in regression, pattern coefficients in factor analysis, discriminant function coefficients in discriminant analysis, and canonical function coefficients in canonical correlation analysis), mainly to obfuscate the commonalities of parametric methods, and to confuse graduate students.

Factor *pattern coefficients* delineate "the weight matrix to calculate variable factor scores from factor standard scores" (Gorsuch, 1983, p. 207). Pattern coefficients are not necessarily correlation coefficients and need not range only between -1 and +1, just as regression beta weights when they are not correlation coefficients do not necessarily fall within that range.

The data from Holzinger and Swineford (1939, pp. 81-91) will be used here in one example to make this discussion concrete. These scores on ability batteries have classically been used in both EFA (Gorsuch, 1983, passim) and CFA examples (Jöreskog & Sörbom, 1989, pp. 97-104). Scores of 301 subjects on nine measured variables will be used to test a model positing three correlated factors; the model is diagrammed by Jöreskog and Sörbom (1989, p. 98).

In the context of this example, the EFA analytic model is specified by equation 3.5.7 from Gorsuch (1983, p. 54, subscripts for the illustrative example added, original equation numbers are presented in brackets):

$$R_{(9 \times 9)} = P_{(9 \times 3)} R_{(3 \times 3)} P'_{(3 \times 9)} + U_{(9 \times 9)} \quad (1\ [3.5.7])$$

where $P_{(9 \times 3)}$ is the factor pattern coefficient matrix, $R_{(3 \times 3)}$ is the matrix of correlation coefficients among the three factors, $U_{(9 \times 9)}$ is the unique variance in the observed variables, and $R_{(9 \times 9)}$ is the matrix of correlation coefficients among the nine measured or observed variables. The related algorithm in CFA is specified by equation 3.10 from Jöreskog and Sörbom (1989, p. 97, illustrative subscripts added) in the context of population parameter matrices:

$$\Sigma_{(9 \times 9)} = \Lambda_{(9 \times 3)} \Phi_{(3 \times 3)} \Lambda'_{(3 \times 9)} + \Theta_{(9 \times 9)} \quad (2\ [3.10])$$

where $\Lambda_{(9 \times 3)}$ is the factor pattern coefficient matrix, $\Phi_{(3 \times 3)}$ is the matrix of correlation coefficients among the three factors, $\Theta_{(9 \times 9)}$ is the unique variance in the measured variables (usually associated with measurement error in a correctly specified model) and the covariances of these errors, and $\Sigma_{(9 \times 9)}$ is the matrix of covariance or correlation coefficients among the nine measured or observed variables.

Note that the term, "loading", was not used in labeling any coefficients (Thompson & Daniel, 1996). Too many people use this term to mean different and/or ambiguous things. As Gorsuch (1983, p. 25) noted, factor pattern and factor structure coefficients "are different even though terms such as 'factor coefficients' and 'factor loadings' may be used [by various people at various times] for both the pattern and the structure."

Factor scores on the latent constructs are usually estimated in EFA using the algorithm:

$$F_{(301 \times 3)} = Z_{(301 \times 9)} R_{(9 \times 9)}^{-1} P_{(9 \times 3)} \quad (3)$$

where $Z_{(301 \times 9)}$ is the matrix of z scores of the 301 people on the nine measured variables. Similarly, in CFA factor scores can be computed using:

$$\Xi_{(301 \times 3)} = Z_{(301 \times 9)} A_{(9 \times 3)} \qquad (4)$$

where $A_{(9 \times 3)}$ is a weight matrix computed using factor pattern coefficients. Jöreskog and Sörbom (1989, p. 93) employ a formula computing an arbitrarily transposed factor-score weight matrix (A'), but the identities between these EFA and CFA algorithms are again obvious:

$$A'_{(3 \times 9)} = \hat{\Phi}_{(3 \times 3)} \hat{\Lambda}'_{(3 \times 9)} \hat{\Sigma}_{(9 \times 9)}^{-1} \qquad (5)$$

where the estimated factor correlation, factor pattern coefficient, and covariance/correlation matrices are respectively designated as estimates using the caret ("^") symbol.

Unlike pattern coefficients, factor *structure coefficients* are always correlation coefficients; specifically, structure coefficients are "the correlations of the variables with the factors" (Gorsuch, 1983, p. 207). In the principal components EFA case, factor scores and factors have exactly the same correlations, and in this case the correlations of observed scores with the synthetic factor scores would equal the correlations of the observed variables with the factors (McMurray, 1987). Indeed, in most parametric methods the correlation of the n scores on an observed variable (e.g., a measured predictor variable in a regression analysis) with the n scores on estimated synthetic variables (e.g., the estimated scores on the $\hat{Y}$ variable in a regression analysis) are structure coefficients (Thompson, 1991; Thompson & Borrello, 1985).

However, in some EFA and most CFA applications the correlations of factors and the correlations of factor scores associated with the same data are not equal. Here the correlations of observed scores with factor scores cannot be used to compute structure coefficients.

In EFA the structure coefficients can be computed using formula 3.4.8 provided by Gorsuch (1983, p. 52, illustrative subscripts added):

$$S_{(9 \times 3)} = P_{(9 \times 3)} R_{(3 \times 3)} \qquad (6\ [3.4.8])$$

where $S_{(9 \times 3)}$ is the factor structure coefficient matrix, $P_{(9 \times 3)}$ is the factor pattern coefficient matrix, and $R_{(3 \times 3)}$ is the factor correlation matrix. Of course, when the factors are uncorrelated the factor correlation matrix is an Identity matrix [any matrix times I yields the original matrix, unchanged], and the pattern and structure coefficient matrices will then be equal.

Gorsuch (1983, p. 23) also provides a formula (2.2.5) that does not require the use of matrix algebra. When a correlation matrix is analyzed, the measured variables are effectively standardized to have a variance of one; factors are typically standardized as well. Gorsuch's formula can then be expressed as:

$$r(X_j \times F_k) = P_k + [\sum_{m=1}^{nf} P_m * r(F_k \times F_m) \text{ where } k \neq m] \qquad (7)$$

where nf is the number of factors, X_j are the scores of the subjects on the j-th measured variable, F_k is the k-th factor, P_k is the j-th pattern coefficient on the k-th factor, P_m is the j-th pattern coefficient on the m-th factor, and $r(F_k \times F_m)$ is the correlation between the k-th factor and the m-th factor. The use of this algorithm will be illustrated shortly.

<u>The GLM Emphasis on Structure Coefficients</u>

Although there are a few researchers (Harris, 1989; Rencher, 1992) who do not recognize the usefulness of structure coefficients in multivariate parametric statistics, "The predominant method of identifying latent constructs in multivariate analyses... is to examine correlations between linear composite scores and scores on the individual variables in the composite" (Huberty, 1994, p. 209). Structure coefficients are also important in univariate analyses, as Thompson and Borrello (1985) argued in <u>Educational and Psychological Measurement</u> more than a decade ago. For example, Huberty (1994, p. 65, emphasis in original) noted, "Such *structure correlations* [structure coefficients] are also of interest in multiple correlation."

The emphasis on structure coefficients in the general linear model has historically been heavy. With respect to canonical correlation analysis, Meredith (1964, p. 55) suggested that, "If the variables within each set are moderately intercorrelated the possibility of interpreting the canonical variates by inspection of the appropriate regression weights [function coefficients] is practically nil." Similarly, Kerlinger and Pedhazur (1973, p. 344) argued that, "A canonical correlation analysis also yields weights, which, theoretically at least, are interpreted as regression [beta] weights. These weights [function coefficients] appear to be the weak link in the canonical correlation analysis chain." Levine (1977, p. 20, his emphasis) was even more emphatic:

> I specifically say that one has to do this [interpret structure coefficients] since I firmly believe as long as one wants information about the nature of the canonical correlation relationship, not merely the computation of the [synthetic function] scores, one must have the structure matrix.

Issues of "interpretation consistency" across the use of the related members of the GLM analytic family are important when determining which coefficients to interpret. As Huberty (1994, p. 263) noted,

> If a researcher is convinced that the use of structure *r*'s makes sense in, say, a canonical correlation context, he or she would also advocate the use of structure *r*'s in the contexts of multiple correlation, common factor analysis, and descriptive discriminant analysis.

This argument can be framed as a syllogism. The major premise is that structure coefficients are important interpretation aids throughout the general linear model. The minor premise is that the various parametric methods, including factor analysis, are part of the general linear model. The conclusion is that the one "basic matrix for interpreting the factors is the factor structure" (Gorsuch, 1983, p. 207).

Illustrative Applications

The utility of structure coefficients remains to be demonstrated in the context of CFA models positing correlated factors. The first example involves nine of the ability variables from the Holzinger and Swineford (1939, data on pp. 81-91), i.e., the same variables used in a CFA model tested by Jöreskog and Sörbom (1989, pp. 97-104). The second example involves a self-concept model tested by Byrne (1989, pp. 37-53, input correlation matrix on p. 45). The correlation matrix was analyzed in both illustrations, because no model modifications were undertaken, and the necessary conditions were met (Cudeck, 1989; Jöreskog & Sörbom, 1989, pp. 47-48).

An Ability CFA Model

Table 1 presents the correlation matrix associated with the ability battery scores of 301 subjects on nine measured variables. The results are reported to five decimal values to facilitate reanalysis of the data by interested readers. The model posited three factors, each associated with a different three of the nine measured variables, and the factors were presumed to be correlated.

INSERT TABLE 1 ABOUT HERE

Table 2 presents the maximum-likelihood estimates of the population factor pattern coefficients ($\Lambda_{(9 \times 3)}$) based on this model. Table 3 presents the estimated population factor-correlation matrix ($\Phi_{(3 \times 3)}$). The table also presents the correlations among the factor scores; these two correlation matrices are usually not exactly the same (principal components analysis provides an exception).

INSERT TABLES 2 AND 3 ABOUT HERE

Although they are currently rarely reported in published CFA research, the factor structure matrix can be requested in LISREL. The structure matrix is output with the label, "covariances X - KSI" (Ξ). When the factors and the variables are both standardized, these covariances are correlation coefficients. These structure coefficients are reported in Table 2 along with the pattern coefficients. Table 4 illustrates the calculation of selected structure coefficients using algorithm 7, described earlier.

INSERT TABLE 4 ABOUT HERE

A CFA Self-Concept Model

Table 5 presents the maximum-likelihood estimates of the population factor pattern coefficients ($\Lambda_{(11 \times 4)}$) for the self-concept model tested by Byrne (1989, pp. 37-53) when factor variances were each constrained to be one. [Byrne's application focused on testing whether self-concept is multidimensional, but the data can be employed to illustrate a hypothetical focus on the structure itself.] Table 6 presents the estimated factor correlation matrix ($\Phi_{(4 \times 4)}$).

INSERT TABLES 5 AND 6 ABOUT HERE

Discussion

When interpreting CFA results, it is important to remember that the variables with pattern coefficients fixed to zero nevertheless may have noteworthy correlations with these factors, when the factors are correlated. In the present illustrations, the two model tests resulted in some estimated factor correlations that were noteworthy. For example, the factor correlations presented in Table 3 for the ability model ranged from .283 to .470. The factor correlations presented in Table 6 for the self-concept model were more heterogenous, and ranged from .082 to .646.

The consequences of these factor correlations are honored in the structure coefficients presented in the factor parameter tables. For example, in the self-concept model presented in Table 5, on Factor I the structure coefficients for the eight variables with pattern coefficients fixed as zeroes were small, ranging from .223 to .319. Such values impact the interpretation of the factor.

Although the factor was labelled "General" in the original work, these coefficients suggest a particular meaning for the construct. Although the three observed variables measuring this construct may be general in their content, the construct here is <u>not</u> "general" in the particular sense of subsuming more specific types of self-concept as special cases. Rather, the "Academic" factor is most general in the sense of subsuming both the "English" and the "Math" factors, as indicated by the structure coefficients for the last six variables ranging from .492 to .595, even though these pattern coefficients were all constrained to be zeroes. As reported in Table 6, the Academic and English factors had an estimated correlation of .646, while the Academic and Mathematics factors had an estimated correlation of .634.

Conversely, the English and the Mathematics self-concept factors had an estimated correlation of .082, as reported in Table 6. The structure coefficients of the three English variables on the Mathematics factor were .064, .069, and .063, respectively, as reported in Table 5. Such dynamics ought to be honored when interpreting factors.

Consulting structure coefficients in conjunction with pattern coefficients, in presentations such as those reported in Tables 2 and 5, compels us to remember that measured variables are related to correlated factors even when these variables' pattern coefficients have been fixed to zero. Researchers reporting and interpreting only pattern and not structure coefficients may more readily neglect to honor this truism.

Examining CFA structure coefficients may also assist in identifying model misspecification. When structure coefficients for "fixed" variables are anomalously large and these variables' pattern modification coefficients are large, such results may suggest that parameters be freed, if a theoretical rationale for model modification can be identified. For example, in the ability model presented in Table 2 the structure coefficients for all six measured variables with fixed pattern coefficients on Factor I had structure coefficients (i.e., .390, .392, .384, .268, .340 and .313, respectively) approaching the value for one of the three freed values (.424) on this factor. The interpretation of this pattern/structure coefficient, in relation to both these other structure coefficients and the structure coefficients of the variable on Factors II and III (.194 and .199), might suggest that a fourth factor exists that could be delineated by using additional measured variables.

Of course, the procedures recommended here can also be applied to published research in which authors have not provided structure coefficients, provided that sufficient information is presented in such work. Examples from two studies recently reported in Educational and Psychological Measurement can be employed to illustrate this application and its value in interpretation.

Schau et al. (1995, p. 874) reported a study of students' attitudes toward statistics. They tested a model with four correlated factors. Scores on an affective item parcel ("A1") were constrained to have a zero pattern coefficient on Factor II, "Cognitive Competence." Using Gorsuch's (1983, p. 23) formula 2.2.5 (my equation 7), the structure coefficient of measured variable "A1" on the "Cognitive Competence" factor could be computed to be:

$$P_{(A1,II)} + P_{(A1,I)} \times r_{(I,II)} + P_{(A1,III)} \times r_{(III,II)} + P_{(A1,IV)} \times r_{(IV,II)}$$
$$0.000 + (0.818 \times 0.916) + (0.000 \times 0.374) + (0.000 \times 0.689)$$
$$0.000 + 0.749 + 0.000 + 0.000$$
$$= 0.749$$

Clearly, a structure coefficient of +.741 differs appreciably from the same variable's Factor II pattern coefficient of .000!

Marcoulides, Mayes, and Wiseman (1995, p. 808) reported a study of the computer anxieties of law enforcement officers and others. They tested a two-factor model in which two measured variables were freed on the two constructs being estimated. One such measured variable was "visiting computer store." This variable had pattern coefficients on Factors I and II of .69 and .20, respectively. The structure coefficient of the measured variable on Factor II could be computed to be:

$$P_{(V,II)} + P_{(V,I)} \times r_{(I,II)}$$
$$0.20 + (0.69 \times 0.60)$$
$$0.20 + 0.41$$
$$= 0.61$$

Again, a structure coefficient of +.61 differs appreciably from a pattern coefficient of +.20 for the same measured variable on the same factor.

Throughout parametric analyses within the general linear model, the weights used to estimate synthetic variable scores (or to predict observed variables using estimated synthetic/construct variable scores) partial out redundant variance measured by several observed variables. The decisions about which measured variables receive credit for redundant predictive ability can be conceptually arbitrary (Thompson & Borrello, 1985). Structure coefficients, on the other hand, do not correct for redundancy within the measured variables.

Neither perspective (i.e., pattern or structure) is intrinsically more valuable than the other. The joint use of both coefficients helps us to better understand our results. For example, in multiple regression measured variables with non-zero beta weights but structure coefficients of zero are examples of one kind of suppression effects. The form of such effects in GLM analyses may go unnoticed unless both types of coefficients are reported and interpreted.

In any case, it is vital to remember that non-zero structure coefficients are implicit within any factor analysis involving correlated factors, even for variables with pattern coefficients fixed to be zeroes. The estimation of these structure coefficients does not cost additional degrees of freedom, since the coefficients are fully determined by the pattern and the factor correlation coefficients already being estimated. And the failure to consult these structure coefficients can lead to interpretation errors.

In summary, a general linear model framework has been employed to suggest that structure coefficients ought to be reported in CFA studies in which factors are correlated. Although the estimation procedures of EFA and CFA differ, Gorsuch's (1983, p. 208, emphasis removed) argument--that "proper interpretation of a set of factors can probably only occur" if both pattern and structure coefficients are employed--may generalize to the CFA case.

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Table 1
Correlations of the Observed/Measured Ability Battery Variables

Var	Observed/Measured Variables								
	T1	T2	T4	T6	T7	T9	T10	T12	T13
T1	1.00000	0.29736	0.44068	0.37271	0.29345	0.35678	0.06687	0.22393	0.39035
T2	0.29736	1.00000	0.33986	0.15293	0.13939	0.19254	-0.07567	0.09228	0.20605
T4	0.44068	0.33986	1.00000	0.15864	0.07720	0.19766	0.07193	0.18602	0.32866
T6	0.37271	0.15293	0.15864	1.00000	0.73319	0.70449	0.17383	0.10690	0.20785
T7	0.29345	0.13939	0.07720	0.73319	1.00000	0.71998	0.10205	0.13868	0.22747
T9	0.35678	0.19254	0.19766	0.70449	0.71998	1.00000	0.12111	0.14962	0.21417
T10	0.06687	-0.07567	0.07193	0.17383	0.10205	0.12111	1.00000	0.48677	0.34065
T12	0.22393	0.09228	0.18602	0.10690	0.13868	0.14962	0.48677	1.00000	0.44903
T13	0.39035	0.20605	0.32866	0.20785	0.22747	0.21417	0.34065	0.44903	1.00000

Note. Values are reported to five decimal places to facilitate replication by readers.

Table 2
Factor Pattern and Structure Coefficients
for the Ability Model

	Factors					
	Visual		Verbal		Speed	
Observed/Measured Variables	Pattern	Structure	Pattern	Structure	Pattern	Structure
Visual perception test from Spearman VPT, Part III (T1)	0.77189	0.77189	0.00000	0.35392	0.00000	0.36319
Cubes, simplified Brigham's Spatial Relations Test (T2)	0.42361	0.42361	0.00000	0.19423	0.00000	0.19932
Lozenges from Thorndike--identify target shape (T4)	0.58114	0.58114	0.00000	0.26646	0.00000	0.27344
Paragraph comprehension test (T6)	0.00000	0.39046	0.85159	0.85159	0.00000	0.24098
Sentence completion test (T7)	0.00000	0.39206	0.85508	0.85508	0.00000	0.24197
Word meaning test (T9)	0.00000	0.38424	0.83802	0.83802	0.00000	0.23714
Speeded addition test (T10)	0.00000	0.26798	0.00000	0.16117	0.56953	0.56953
Speeded counting of dots in shape (T12)	0.00000	0.34022	0.00000	0.20467	0.72306	0.72306
Speeded discrimination of letters (T13)	0.00000	0.31290	0.00000	0.18818	0.66501	0.66501

Note. Fixed parameters are presented as zeroes. Factor variances were constrained to be unity, while factor correlations were freed to be estimated.

Table 3
Correlations of the Ability Factors and of the Factor Scores

Factor/ Scores	Factor/Factor Scores Visual	Verbal	Speed
Visual	1.00000	0.55160	0.59310
Verbal	0.45851	1.00000	0.34640
Speed	0.47052	0.28298	1.00000

Note. Correlations of the *factors* are presented *below* the diagonal. Correlations of the *factor scores* are presented *above* the diagonal.

Table 4
Illustrative Calculation of Structure Coefficients

j	k	P_k	+	P_m	x $r(F_k x F_m)$	+	P_m	x $r(F_k x F_m)$	= $S_{(j,k)}$
1	1	0.77189	+	(0.00000	x 0.45851)	+	(0.00000	x 0.47052)	
		0.77189	+	0.00000		+	0.00000		= 0.77189
2	1	0.42361	+	(0.00000	x 0.45851)	+	(0.00000	x 0.47052)	
		0.42361	+	0.00000		+	0.00000		= 0.42361
3	1	0.58114	+	(0.00000	x 0.45851)	+	(0.00000	x 0.47052)	
		0.58114	+	0.00000		+	0.00000		= 0.58114
4	1	0.00000	+	(0.85159	x 0.45851)	+	(0.00000	x 0.47052)	
		0.00000	+	0.39046		+	0.00000		= 0.39046
5	1	0.00000	+	(0.85508	x 0.45851)	+	(0.00000	x 0.47052)	
		0.00000	+	0.39206		+	0.00000		= 0.39206

Table 5
Factor Pattern and Structure Coefficients
for the Self-Concept Model

	Factors							
	General		Academic		English		Math	
Variable	Patt	Stru	Patt	Stru	Patt	Stru	Patt	Stru
SDQ-GSC	0.873	0.873	0.000	0.334	0.000	0.255	0.000	0.223
APIGSC	0.715	0.715	0.000	0.274	0.000	0.209	0.000	0.183
SES-GSC	0.899	0.899	0.000	0.344	0.000	0.263	0.000	0.229
SDQ-ASC	0.000	0.310	0.811	0.811	0.000	0.524	0.000	0.514
SCAASC	0.000	0.319	0.833	0.833	0.000	0.538	0.000	0.528
SDQ-ESC	0.000	0.229	0.000	0.506	0.783	0.783	0.000	0.064
APIESC	0.000	0.246	0.000	0.544	0.842	0.842	0.000	0.069
SCAESC	0.000	0.223	0.000	0.492	0.762	0.762	0.000	0.063
SDQ-MSC	0.000	0.240	0.000	0.595	0.000	0.077	0.938	0.938
API-MSC	0.000	0.234	0.000	0.581	0.000	0.075	0.916	0.916
SCAMSC	0.000	0.226	0.000	0.562	0.000	0.073	0.886	0.886

Note. Fixed parameters are presented as zeroes. Factor variances were constrained to be unity, while factor correlations were freed to be estimated.

Table 6
Correlations of the Self-Concept Factors

	Factors			
Factor	General	Academic	English	Math
General	1.000			
Academic	0.383	1.000		
English	0.293	0.646	1.000	
Math	0.255	0.634	0.082	1.000

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