Does Classroom Instruction Predicts Students' Learning of Early Algebra: A cross-cultural opportunity-propensity analysis

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Abstract

This study examines whether instruction aligned with IES recommendations (i.e., use of worked examples, representations, deep questions) predicts student learning of early algebra in elementary classrooms. Instructional quality was determined in an opportunity-propensity analysis of cross-cultural data between United States and China, which show that teaching may play a stronger role in student learning (N = 589) than previously reported. After controlling for the covariates of antecedent (e.g., SES) and propensity factors (e.g., prior achievement) as well as the teacher characteristics (e.g., self-efficacy), teaching quality -- especially teachers' use of representations and deep questions – explains additional variance beyond highly predictive antecedent and propensity factors. The pattern held in both the US and China even though there were several interesting differences in responses.

Objective of Study

Algebra readiness is recognized as an important gatekeeper to future success in mathematics (National Mathematics Advisory Panel, 2008). Results from international studies indicate that a disproportionately large percentage of U.S. students are ill-prepared for the study of algebra, especially when compared with high-performing countries like China (e.g., Cai, 2004; PISA, 2006, 2009). Students' weak algebraic readiness mainly results from poor instruction in arithmetic where their teachers focus on surface features rather than underlying ideas that are essential for later learning of algebra (Carpenter, Franke, & Levi, 2003, Kaput, 1999). The Institute of Educational Sciences (IES) has recommended several instructional principles for improving students learning of fundamental concepts. Among these, teachers' use of worked examples, representations, and deep questions are particularly relevant to classroom instruction and thus we hypothesize that instruction that better addresses these aspects will provide better support for students' learning of early algebra. The purpose of this study is to examine this hypothesis based on cross-cultural data of teaching and learning of the early algebra topic of inverse relations. Specifically, we ask: does instruction that better aligns with the IES recommendations predict better learning of early algebra? Given that multiple factors affect student learning, our analysis follows an opportunity-propensity model which accounts for various factors beyond instruction.

Review of Literature

IES Recommendations of Quality Instruction

The IES recommendations are instructional principles gleaned from numerous high quality research studies (Pashler et al., 2007). A review of literature supporting the importance of the use of worked examples, representations, and deep questions follows below:

Worked Examples. Worked examples (problems with solutions given) help students acquire necessary schemas to solve new problems (Sweller & Cooper, 1985). Classroom experiments indicate that the use of worked examples is more effective than simply asking students to solve problems (Zhu & Simon, 1987). Fading examples into practices is also beneficial (Renkl, Atkinson, & Grobe, 2004). However, U.S. teachers often spend little time discussing one example before rushing to practice problems (Stigler & Hiebert, 1999).

Representations. Concrete representations, such as graphs or word problems, support initial learning because they provide familiar situations that facilitate students' sense-making (Resnick, Cauzinille-Marmeche, & Mathieu, 1987). However, overexposing students to concrete representations may hinder their transfer of learned knowledge because these representations contain irrelevant information (Kaminski, Sloutsky, & Heckler, 2008; Uttal, Liu, & Deloache, 1999). Thus, some researchers suggested fading the concreteness into abstract representations to promote transfer of learning (Goldstone & Son, 2005).

Deep Questions. Students can effectively learn new concepts through self-explanations (Chi, 2000; Chi et al., 1989). However, they themselves usually have little motivation to generate high-quality explanations. It is necessary for teachers to ask deep questions to elicit students' explanations of the underlying principles, causal relationships, and structural knowledge (Craig, Sullins, Witherspoon, & Gholson, 2006).

To explore the predictiveness of instructional quality along these three dimensions on algebraic learning, this study investigates the teaching and learning of *inverse relations*. Inverse relations are a ubiquitous mathematical concept emphasized by the Common Core standards across elementary grades (CCSSI, 2010). Elementary students can initially learn this relation through (a) fact families (e.g., 7+5=12, 5+7=12, 12-7=5, and 12-5=7), and (b) inverse word problems (the solutions form a fact family; Carpenter et al., 2003; Howe, 2009). An understanding of inverse relations contributes to a student's full comprehension of arithmetic (Wu, 2011a), algebraic thinking (Carpenter et al., 2003; Stern, 2005), and mathematical flexibility (Nunes, Bryant, & Watson, 2009). However, elementary students are often found to lack formal understanding of inverse relations, which may be associated with poor classroom instruction (Baroody, 1999; De Smedt et al., 2010). Accurately assessing the role of instruction in promoting achievement, however, requires contextualization, as instruction does not occur in a vacuum. This study uses the opportunity-propensity model, described below, for this purpose.

The Opportunity-Propensity (O-P) Model

The basic idea of the O-P model is that achievement is a function of educational opportunities presented to students together with students' propensities to take advantage of these opportunities. This model has demonstrated a good fit to the data (accounting for 50-80% of the variance) in prior studies (Byrnes & Miller, 2007; Byrnes & Miller-Cotto, 2016; Byrnes & Wasik, 2009; Wang, Shen, & Byrnes, 2013). In this model, *opportunity* refers to high quality classroom instruction. For instance, differences in teachers' use of worked examples, representations, and deep questions may provide students with different learning opportunities. In contrast, *propensity* means students' willingness and ability to take advantage of these opportunities, such as their in-class attitudes, prior knowledge, self-concept, and innate mathematical talent. Student opportunity and propensity may interact. For example, higher instructional quality (opportunity) may promote better student attitudes towards math (propensity). In addition, other *antecedent* factors such as socioeconomic status (SES), parent aspirations, gender, and ethnicity are also predictors of students' learning. They operate earlier and may cause opportunity and propensity factors to emerge. It is expected that holding constant the covariates (e.g., student and teacher characteristics, antecedent factors), teachers' higher quality of teaching (opportunity) will lead to better student learning. Figure 1 illustrates a modified O-P model for this study. Note that in prior O-P studies (e.g., Byrnes & Miller, 2007; Byrnes & Miller-Cotto, 2016), the opportunity factor was largely based on teachers' self-report of data rather than actual classroom observations. Since self-report may yield biased estimates of instructional quality, we expect that our study using actual video data will contribute new insights. In contrast to prior studies that have explored student learning as a subject (e.g., math, science, literacy), our focus on one topic that aligns teaching with the corresponding learning may also provide more precise measures. Previously, the model has also only been tested on US samples.

Methods

Participants and Project

This study is part of a five-year NSF supported project identifying high quality instructional features in early algebra topics based on US and Chinese data. The current study explores the year 1 data focusing on inverse relations between addition and subtraction (grades 1-2 both US and China) and between multiplication and division (grades 3-4 US, grades 2-3 China). As such, a total of 8 US and 8 Chinese teachers and their students were involved in this study. All Chinese teachers have received teaching awards, three US teachers were national board certified teachers (NBCT), and the other five were recommended by their school district. A total of 589 students participated in this study ($N_{\text{US}} = 236$; $N_{\text{China}} = 353$). The average class size for US was smaller than China ($N_{\text{US}} = 30$; $N_{\text{China}} = 44$).

Data Sources and Coding

Each teacher in this study taught 4 videotaped lessons on inverse relations. Due to cross cultural differences in textbooks, teachers in both countries taught different lessons but with the same undergirding structures (e.g., fact family, inverse word problems). All 64 videotaped lessons were transcribed and coded for instructional quality based on a framework modified from a prior study (Ding & Carlson, 2013, see Appendix 1). This framework was further validated by independent coding of two US and China videos. Next, the first author coded all lessons focusing on teachers' use of worked examples , representations , and deep questions . Quality of these instructional aspects was coded at three levels (0=low, 1=medium, 2=high). The possible total score for each lesson is 12 points (4 point for each of the three aspects). An inter-rater reliability from a second coder exceeded 90%.

Covariate data was collected from student and teacher surveys modified from instruments validated by a prior NSF project (see Appendices 2 and 3). The student survey provided information about the antecedent factors (e.g., parent aspiration) and propensity factors (e.g., students' attitudes, self-efficacy, social adjustment). In addition, the teacher survey provided information about characteristics that are part of the opportunity factor (e.g., teacher preparedness perception, self-efficacy for teaching, belief in the impact of teaching on learning). In addition, student demographic information (e.g., ethnicity, IEP, disability) adds further data to the antecedent factor.

To measure student learning, we developed content-specific instruments based on inverse relations literature (e.g., Carpenter et al, 2003) and the common core state standards (CCSSI, 2010). The additive and multiplicative instruments contain parallel items (see Appendix 4). The structure of these items (e.g., fact family, inverse word problem) was consistent with the content covered by the videotaped lessons. Thus, our measures of teaching and learning were closely connected. The same instrument served as both the pretest (to index the propensity factor prior knowledge) and posttest. Students' responses were coded for correctness. Table 1 summarizes the variables tested and corresponding data sources.

Data Analysis

Hierarchical regression analyses were first conducted to analyze the overall data set. We entered the predictors (see Table 1) into four blocks in the following order: antecedent, propensity, opportunity-teacher characteristics, and opportunity-teaching quality. The rationale for this order was due to the main research question, that is, we are most-interested in exploring how much additional variation can be accounted for by student learning after the opportunity factor of classroom instruction is added. In addition, we employed the same data analysis procedures to analyze US and Chinese data sets, respectively, to examine whether there is a cross-cultural difference in terms of the predictability of instruction on student learning.

Result

Instruction Predicts Early Algebra Learning: An Overall Analysis

Table 2 summarizes the mean scores of non-categorical data for both US and China. Chinese students earned higher scores on inverse relations in both pre- and post-tests (Pre_{US}=3.64, Pre_{China}=6.67, $Post_{US}$=5.15, $Post_{China}$=7.47), which indicates their superior prior knowledge and learning outcomes. This findings is consistent with the existing literature on cross-cultural mathematics learning differences in mathematics (Cai, 2004; PISSA, 2006, 2009, 2012; TIMSS, 2003, 2007). Interesting differences in opportunity factors was also found. For instance, while US teachers demonstrated more positive teacher characteristics (e.g., attitude/beliefs toward teaching), Chinese teachers' instructional quality was rated higher. In addition, the variance of both the pre- and post-tests scores and teacher instructional quality were much greater in the US data than in the Chinese data.

Results from the hierarchical regression analysis indicate that instructional quality does add significant explanatory power for students' early algebraic learning. As indicated by Table 3, the full model explained a total of 58.4% of the variance. On the first step of the hierarchical regression, the antecedent factors (e.g., country, ethnicity, disability, parent expectation) were found to explain 42% of the variance. On the second step, indices of propensities (e.g., student characteristics, student prior knowledge) added an additional 9.3% of the variance. This change was significant. On the third step, indices of opportunity-teacher reported characteristics added only 0.6% of the variance, which was non-significant. On the final step, our primary predictor of interest, opportunity-teaching quality, added an additional 6.6% of the variance. This is also significant. As such, our finding suggests that after controlling all other predictors, teaching quality in terms of worked examples, representations, and deep questions does play a significant role in predicting student learning of early algebra.

A closer inspection of all predicators in the O-P model reveals interesting findings (see Table 4). First, with this overall data set, all predictors except for parent support (e.g., help with homework) and student attitude toward grades appeared to be significant. Second, several factors highly related to student outcomes – such as country (China), ethnicity (Asian), student prior knowledge, and teachers' questioning scores – are almost certainly correlated; for instance, Chinese students are more likely to be Asian and to have higher levels of prior knowledge, as well as teachers who tend to ask higher quality questions. Of course, such multicolinearity among predictor variables needs to be further diagnosed and taken into consideration when interpreting the results, and may make it difficult to determine the unique contribution of each predictor. Interestingly, even though all opportunity predictors were significant, teachers' self-reported characteristics (self-efficacy, beliefs) were negatively correlated with student learning while teaching quality (observed data) was positively correlated. A positive interpretation is that US teachers' self-report of teacher characteristics were more positive than Chinese teachers; yet the pattern of students learning in both countries was opposite (see Table 2). These patterns suggest the need for further exploration of the effect of teaching quality on students' early algebra learning for the US and Chinese cases, respectively.

Instruction Predicts Early Algebra Learning: Analysis Within Countries

Results from hierarchical regression analysis with US and Chinese data, respectively, indicate stronger accountability of the O-P model with the US data but not Chinese data (see Table 5).

(Insert Table 5 about here)

Overall, the O-P model explains 59.5% variance for US students' achievement but only 14.3% for Chinese students' achievement. This is reasonable due to the much smaller variance of students' learning and teaching quality in the Chinese data set (see Table 2). In both data sets, the antecedent factors did not provide significant explanations for variance (13% for US data and 0.4% for Chinese data). Given that the variation in Chinese student ethnicity, disability, and SES were small, this is unsurprising. However, the propensity factors, added on the second step, were significant: they additionally explained 31.6% of the variance for the US data ($p<0.00$) and 4.2% for the Chinese data ($p<0.05$). Interestingly, the "teacher characteristics" factor (1st opportunity predictor) adds significant explanations of variance for the Chinese data (additional 4.2%, $p<0.05$) but not the US data (additional 4.8%, $p>0.05$). In other words, it seems that Chinese teachers' self-reported teacher characteristics served as a significant predictor for student learning; yet, this did not apply for the US data, perhaps suggesting caution when interpreting the meaning of teachers' self-reported measures. As indicated by Table 2, the variation of US teachers' self-reported "teacher characteristics' were quite small for three of the four survey items (teachers being uniformly positive), which calls for consideration of other measures for the "opportunity" variable. Lastly, teaching quality, the opportunity variable most of-interest to the study, explains an additional 10.1% of the variance for the US data and 5.5% for the Chinese data. With both data sets, the changes of explanation for variance were significant ($p < 0.00$ for both data sets). Encouragingly, this indicates that despite cross-cultural differences in the predictability of O-P model, the factor of "teaching quality" in alignment with the IES recommendations (worked examples, representations, and deep questions) consistently plays a significant role in predicting students' early algebra learning across both countries in the sampled data sets.

Discussion

Does instruction that better aligns with the IES recommendations predict better learning of early algebra? Our findings from both the US and Chinese say "yes." That is, classroom instruction that better uses worked examples, representations and deep questions predicts better learning of inverse relations. This is an important finding because early algebra has long been recognized as a gatekeeper for students' mathematical learning (Carpenter et al., 2003). The mathematics education field also expects elementary teachers to develop students' algebraic thinking in classrooms (CCSSI, 2010), and has characterized classroom features that promote algebraic thinking (Blanton & Kaput, 2005). However, student learning is associated with many factors that go beyond classroom instruction. It is unclear, when other factors are controlled, to what extent teaching still plays a role in predicting students learning. More specifically, it is unclear what kinds of instructional features contribute to students' algebraic learning. Our cross-cultural findings indicates that instruction that aligns with the IES recommendation in quality use of worked examples, representations, and deep questions consistently contributes to students' early algebraic learning in both the US and China. This finding is particularly encouraging because, while the IES recommendations are instructional principles gleaned from various cognitive research and classroom experiments, these instructional principles are general guidance for the teaching of all subjects. Our study confirms that these instructional principles are robust in supporting students' learning of early algebra.

Findings in this study are based on the use of the Opportunity-propensity (O-P) model. As reviewed, this model contains three major categories: antecedent (e.g., SES, ethnicity), propensity (e.g., students' self-efficacy, prior knowledge), and opportunity (e.g., teacher characteristics, classroom instruction). Prior studies (e.g., Byrnes & Miller, 2007; Byrnes & Miller-Cotto, 2016; Byrnes & Wasik, 2009) consistently found opportunity factors to be of marginal predictive utility as compared with propensity factors (e.g., prior knowledge). While our study echoes the importance of prior knowledge, we find that opportunity factors remain significant even after accounting for antecedent and propensity factors. This may possibly be explained by the use of teacher self-reports for measurement of teacher-related opportunity factors in prior studies, which may not accurately reflect what actually goes on in the classroom. In this study, the "opportunity" category contains two predicators: self-reported teacher characteristics and the observed and coded lesson quality in terms of the use of worked examples, representations, and deep questions. When we separate the "self-reported" and "observed" opportunity data, our finding suggests the differences in predictability between these predictors. For instance, while the observed teaching quality plays a significant role in predicting students' learning in both countries, the self-reported teacher characteristics only achieves significance in the Chinese data. Our findings suggest the need for further research on the O-P model with external measures of instructional quality. Moreover, the difference in predictability of the O-P model with the US and Chinese data sets also provides an opportunity for further exploration and continuing development of the O-P model, a promising model of student learning.

Conclusion

This study has both theoretical and practical implications. First, our findings support cognitive research assertions on the importance of worked examples, representations and deep questions during instruction as recommended by IES. Second, our findings support the feasibility and predictability of the O-P model with a cross-cultural data. Future research may continuously integrate both lines of research to explore the relationship between teaching and learning of other mathematical topics. Whereas both the IES recommendations and O-P model are based on theory and prior evidence, they also provide insight into the causes of achievement and how to elevate performance. Our findings regarding cross-cultural differences in the teaching and learning of inverse relations call for increased effort to improve US classroom teaching so as to better support students' algebraic learning. Future study should explore how the IES recommendations as measured in this study are used differently in US and Chinese classrooms and how these components mattered in student learning. Future in-depth classroom video analyses focusing on these instructional dimensions are warranted. With continuing effort, improvements to students' early algebraic learning can be expected.

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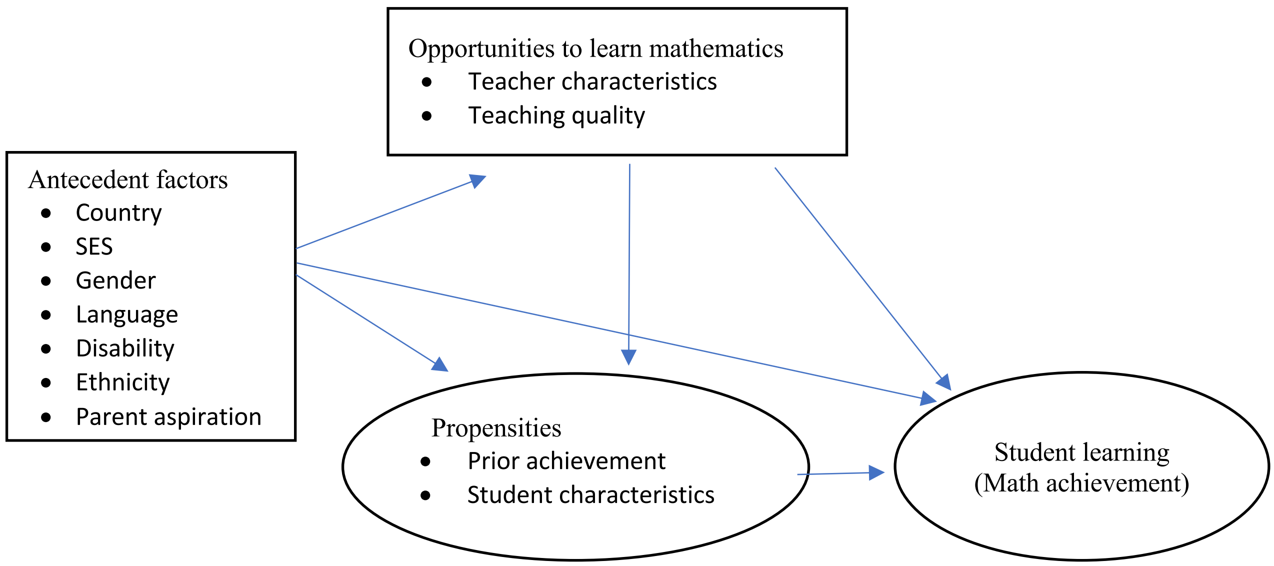


Figure 1. An Opportunity-propensity model used in this study

Table 1. *Predictors and Data Sources for the O-P Model*

Category	**Predictor**	**Data Source**
Antecedent	Country	Project information
	Family SES	Demography data
	Disability	Demography data
	Language proficiency	Demography data
	Gender	Demography data
	Ethnicity	Demography data
	Parent aspiration	Student survey #1 (sub 5,6)
Propensity	Attitude toward school	Student survey #3 (sub 4, 8, 9)
	Attitude toward grades	Student survey #4
	Student self-efficacy for math	Student survey #5 (all 7 sub)
	Student social adjustment	Student survey #6 (sub 1-2, 4-6)
	Prior knowledge	Student math pre-test
Opportunity	Teacher preparedness perception	Teacher survey #9 (all 7 sub)
	Teacher self-efficacy for teaching math	Teacher survey #14 (sub 1, 5, 6)
	Teacher self-efficacy for teaching math	Teacher survey #14 (sub 2, 4)
	Teacher belief in impact of teaching	Teacher survey #16 (sub 1-8, 11, 13, 18)
	Using worked examples	Video data
	Using representations	Video data
	Using deep questions	Video data
	Overall teaching quality	Video data
Outcome	Student learning	Student math post-test

Table 2. *Cross-cultural Difference in the Mean scores of Non-categorical Variables in the US and Chinese Data Sets*

Category	Predictor	US		China	
		Mean	SD	Mean	SD
Antecedent	Parent support – 3 points	2.41	0.44	2.39	0.41
	Parent disciplinary- 3 points	2.11	0.57	2.30	0.45
Propensity	Attitude toward school – 3 points	2.82	0.32	2.87	0.29
	Attitude toward grades – 3 points	2.84	0.40	2.79	0.52
	Student self-efficacy for math – 3 points	2.45	0.46	2.85	0.25
	Student social adjustment – 3 points	2.66	0.43	2.71	0.39
	Prior knowledge (pre-test) - 8 points	3.64	2.44	6.67	1.39
Opportunity	Teacher preparedness perception – 4 points	3.24	0.68	2.59	0.62
	Teacher self-efficacy for teaching (1) – 5 points	3.92	0.36	3.41	0.65
	Teacher self-efficacy for teaching (2) – 5 points	3.88	0.21	3.62	0.61
	Teacher belief in impact of teaching - 5points	3.81	0.44	3.04	0.57
	Using worked examples – 4 points	3.66	0.48	3.97	0.08
	Using representations – 4 points	3.24	0.64	3.84	0.21
	Using deep questions – 4 points	2.06	0.86	3.50	0.32
	Overall teaching quality – 12 points	8.96	1.54	11.31	0.35
Outcome	Student learning (posttest) – 8 points	5.15	2.08	7.47	0.66

Table 3. *Variation Explained by the O-P Model with the Overall Data in the Hierarchical Regression Analysis*

					Change Statistics				
Model	R	R Square	Adjusted R Square	Standard error of estimate	R Square Change	F Change	df1	df2	Sig. F Change
1	.648[a]	.420	.404	1.03157	.420	26.980	11	410	.000
2	.716[b]	.513	.493	.95136	.093	15.411	5	405	.000
3	.720[c]	.518	.494	.95050	.006	1.183	4	401	.318
4	.764	.584	.560	.88670	.066	20.926	3	398	.000

Table 4. *Correlation between Predictors and Student Learning Outcome in the Overall Data Set*

	Predictor	Pearson Correlation	Sig. (1-tailed)
Antecedent	Country+Family SES		
	US_low	-.422	.000
	US_NotLow	-.337	.000
	China_NotLow	.602	.000
	Ethnicity		
	White	-.332	.000
	Black	-.352	.000
	Asian	.589	.000
	Hispanic	-.104	.016
	Disability	-.262	.000
	Language Proficiency	-.120	.007
	Parent Aspiration		
	Parent support	-.057	.122
	Parent disciplinary	.128	.004
Propensity	Student characteristics		
	Attitude toward school	.159	.001
	Attitude toward grades	-.050	.152
	Self-efficacy for math	.377	.000
	Social adjustment	.136	.003
	Student prior knowledge	.569	.000
Opportunity	Teacher characteristics (self-report)		
	Preparedness perception	-.337	.000
	Self-efficacy for teaching1	-.281	.000
	Self-efficacy for teaching2	-.146	.001
	Belief in the impact of teaching	-.368	.000
	Teaching quality (observed)		
	Using worked examples	.241	.000
	Using representations	.435	.000
	Using deep questions	.573	.000
	Overall	.569	.000

Table 5. V*ariation Explained by the O-P Model with the US and Chinese Data in the Hierarchical Regression Analysis*

	Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
						R Square Change	F Change	df1	df2	Sig. F Change
US	1	.361	.130	.047	1.69388	.130	1.571	10	105	.125
	2	.668	.446	.363	1.38484	.316	11.419	5	100	.000
	3	.703	.494	.394	1.35101	.048	2.268	4	96	.067
	4	.772	.595	.505	1.22089	.101	11.777	2	94	.000
China	1	.064	.004	-.002	.66128	.004	.627	2	303	.535
	2	.215	.046	.024	.65250	.042	2.641	5	298	.024
	3	.298	.089	.055	.64219	.042	3.411	4	294	.010
	4	.379	.144	.103	.62562	.055	6.261	3	291	.000

Appendix 1. *The Coding framework for Videotaped Lessons*

Grade: **Teacher Name:** **Lesson:** **Title:** **Score:**

Category	Subcategory	0	1	2
Worked Examples	Example	Examples and guided practice cannot be differentiated.	Worked examples are discussed in a brief manner	Worked example is sufficiently discussed
	Practice	Practice problems have no connection to the worked examples.	Practice problems have some connections to the worked example.	Practice problems have clear and explicit connection to the worked example.
Representations	Concrete	Discussions, especially of worked examples, are completely limited to the abstract. No manipulatives, pictures, or story situations are used.	- Concrete contexts (e.g., story problems) are involved but not utilized sufficiently for teaching the worked example; - Semi-abstract representations such as dots or cubes are used as a context for teaching the worked example	Discussions, especially of worked examples, are well situated in rich concrete contexts (e.g., pictures and story problems). Concrete materials are used to make sense of the target concepts.
	Abstract	Discussions are limited to the concrete and are not at all linked to the abstract representations of the target concept.	- Both concrete and abstract representations are involved but the link between both is lacked; - Since all discussions remain abstract, the link between the concrete and abstract is invisible; - Opposite: from abstract to concrete.	Concrete representations are used to purposefully link the abstract representations of the target concept.
Deep questions	Question	No deep questions are asked when discussing a worked example or guided practices.	Some deep questions are posed to elicit deep explanations/	Deep questions are sufficiently posed to elicit student explanation of the target concepts.
	Explanation	- No deep student explanations are elicited. - Teacher provides little or surface explanations.	- A few deep student responses are elicited. However, most of the student explanations still remain at a surface level. - Teacher rephrases students' explanations without promoting to a higher level. - Teacher directly provides deep explanations.	- Deep student explanations are elicited. In particular, these explanations are related to the target concepts. - Teacher rephrases student explanations to make them deep.

Note. The total score for each lesson has 12 points. Each category has 4 points and each subcategory has 2 points.

Appendix 2. *Student Survey Used in this Study (US version)*

Student Survey Instrument

Your Name: ______________________________Grade:______________ Age: _____________

Teacher's Name: ____

School: ____

Date: Month: ___________________ Day: ______________ Year: ______________________

1. How often do your parents do the following? *Check ONE box on each line.*

	Never	Sometimes	Often
Check on whether you have done your homework.	☐	☐	☐
Help you with your homework.	☐	☐	☐
Reward you for good grades.	☐	☐	☐
Limit your activities because of poor grades.	☐	☐	☐
Ask you to work or do chores.	☐	☐	☐
Limit your time watching TV/playing video games.	☐	☐	☐

2. How often does this happen in your mathematics lessons? *Check ONE box on each line.*

	Every day	Once a week	Once a month	Never
The teacher shows us how to do mathematics problems.	☐	☐	☐	☐
We copy notes from the board.	☐	☐	☐	☐
We have a quiz or test.	☐	☐	☐	☐
We work on mathematics projects.	☐	☐	☐	☐
We work from worksheets on our own.	☐	☐	☐	☐
We use calculators.	☐	☐	☐	☐
We use computers.	☐	☐	☐	☐
We work together in small groups.	☐	☐	☐	☐
The teacher gives us homework.	☐	☐	☐	☐
We can begin our homework in class.	☐	☐	☐	☐
The teacher checks homework.	☐	☐	☐	☐
We check each other's homework.	☐	☐	☐	☐
The teacher discusses homework from yesterday.	☐	☐	☐	☐
The teacher uses a computer.	☐	☐	☐	☐

3. Do you agree or disagree with the following statements about why you go to school?

	Agree	Disagree	Don't know
I think the subjects (e.g., math) are interesting	☐	☐	☐
I am satisfied with what I'm supposed to do in class.	☐	☐	☐
I have nothing better to do.	☐	☐	☐
It is important for getting a job later on.	☐	☐	☐
It's a place to meet my friends.	☐	☐	☐
I play on a team or belong to a club.	☐	☐	☐
I'm learning skills that I will need for a job.	☐	☐	☐
My teachers expect me to succeed.	☐	☐	☐
My parents expect me to succeed.	☐	☐	☐

4. How important are good grades to you? *Check One box.*

- ☐ Not important
- ☐ Somewhat important
- ☐ Very important

5. What do you think about the following? Check ONE box per line.

	Agree	Disagree	Don't know
I can do mathematical calculations.	☐	☐	☐
I would dislike doing mathematics after I leave school.	☐	☐	☐
It is hard for me to work on mathematics problems.	☐	☐	☐
I would dislike a job that uses mathematics.	☐	☐	☐
I know how to solve mathematics problems.	☐	☐	☐
A job that uses mathematics would be interesting	☐	☐	☐
A job as a mathematician would be boring.	☐	☐	☐

6. My school is a place where. *Check ONE box on each line.*

	Agree	Disagree	Don't know
I feel like an outsider (or left out of things).	☐	☐	☐
I make friends easily.	☐	☐	☐
I feel like I belong.	☐	☐	☐
I feel awkward and out of place.	☐	☐	☐
Other students seem to like me.	☐	☐	☐
I feel lonely.	☐	☐	☐

Appendix 3. *Teacher Survey Used in this Study (US version)*

Teacher Survey Instrument

This survey takes about 20 minutes. We ask for your name so that we can match your responses now with your responses at the end of the program. Your name will not be included with your responses when data is reviewed, analyzed and reported in aggregate form to understand the effects of the program.

1. **Your full name (last, first)** ________________________________

2. **Your school name** ________________________________

3. **Representative teaching honors or awards that you have received**

____

____

4. **How many years have you taught? Please check one box.**

☐ 6-10 ☐ 11-15 ☐ 16-20 ☐ 21-25 ☐ 26 and above

5. **What grade level are you teaching at your current school? Please check one box.**

☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

<u>Please tell us a bit about your own past experiences learning mathematics:</u>

6. Please indicate what kinds of mathematics you took during your post-secondary studies (e.g., college and your certification process). Also please indicate if it was required, if you liked it, and if you did well in it. (Circle one response in each applicable box.)

Did you take one course or more in the following subject matter? (Circle yes or no for each subject area.)			**If yes**, you did take at least one course…						
			Why did you take the course? Was it required, did it fulfill credit hours, or was it an elective? If you have taken more than one course in the subject, please circle ALL answers that apply.			Did you like the subject matter?		Did you consider that you did well in it?	
	Yes	No				Yes	No	Yes	No
Calculus	☐	☐	Required	Credit Hours	Elective	☐	☐	☐	☐
Linear Algebra	☐	☐	Required	Credit Hours	Elective	☐	☐	☐	☐
Modern Algebra	☐	☐	Required	Credit Hours	Elective	☐	☐	☐	☐
Probability and Statistics	☐	☐	Required	Credit Hours	Elective	☐	☐	☐	☐
Differential Equations	☐	☐	Required	Credit Hours	Elective	☐	☐	☐	☐
Numerical Analysis	☐	☐	Required	Credit Hours	Elective	☐	☐	☐	☐
Non-Euclidean geometry	☐	☐	Required	Credit Hours	Elective	☐	☐	☐	☐

7. **How many professional development sessions in mathematics have you attended during the past three years? Please check ONE box.**

☐ *None* ☐ *1-2* ☐ *More than three*

8. **Please indicate the areas where you would like to receive more professional development support in mathematics, ranking them (1 – 4) in order of importance to you, with 1 being the most important**

☐ Learn more content (subject-matter) knowledge.
☐ Learn more inquiry/investigation oriented strategies for the classroom.
☐ Learn more about understanding student thinking with regard to MATHEMATICS learning.
☐ Learn more about assessing student learning in mathematics.

9. **Please indicate how well prepared you feel to do each of the following. Please check ONE box per line.**

		Not Adequately Prepared	Somewhat Adequately Prepared	Fairly Well Prepared	Very Well Prepared
a.	Lead a class of students using investigative strategies.	☐	☐	☐	☐
b.	Manage a class of students engaged in hands-on/project-based work.	☐	☐	☐	☐
c.	Help students take responsibility for their own learning.	☐	☐	☐	☐
d.	Recognize and respond to student diversity.	☐	☐	☐	☐
e.	Encourage students' interest in mathematics.	☐	☐	☐	☐
f.	Use strategies that specifically encourage participation of females and minorities in mathematics.	☐	☐	☐	☐
g.	Involve parents in the mathematics education of their students.	☐	☐	☐	☐

10. **How many lessons per week do you typically teach mathematics in your class? Please check ONE box.**

☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 or more

11. **Approximately how many minutes is a typical mathematics lesson? Please check ONE box.**

☐ 20 or fewer ☐ 21-40 ☐ 41-60 ☐ 61-80 ☐ 81 or more

12. **How many mathematics units has your class (or a typical class if you have more than one) worked on so far this academic year? (We are defining a "unit" as a series of related activities, often on a single topic such as addition or subtraction) Please check ONE box.**

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

13. **How many weeks do your mathematics units typically last? (Circle one response.)**

☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10 or more weeks

We just have a few more questions about your view on mathematics teaching. Your responses are <u>very important</u> for our program evaluation, and we appreciate your time and thought.

14. **Please tell us how much you disagree or agree with the following statements about mathematics teaching and learning. Please check ONE box per line.**

		Strongly Disagree	Disagree	Not Sure	Agree	Strongly Agree
a.	When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort.	☐	☐	☐	☐	☐
b.	I am continually finding better ways to teach mathematics.	☐	☐	☐	☐	☐
c.	Even when I try very hard, I don't teach mathematics as well as I do most subjects.	☐	☐	☐	☐	☐
d.	When the mathematics grades of students improve, it is most often due to their teacher having found a more effective teaching approach.	☐	☐	☐	☐	☐
e.	I know the steps necessary to teach mathematics concepts effectively.	☐	☐	☐	☐	☐
f.	I am not very effective in monitoring mathematics experiments.	☐	☐	☐	☐	☐

15. **About how often do the students in your class (or typical class) take part in each of the following types of activities as part of their mathematics instruction? Please check ONE box per line.**

		Never	Rarely (e.g., a few times a year)	Sometimes (e.g., once or twice a month)	Often (e.g., once or twice a week)	All or almost all math lessons
a.	Work on solving a real-world problem.	☐	☐	☐	☐	☐
b.	Share ideas or solve problems with each other in small groups.	☐	☐	☐	☐	☐
c.	Engage in hands-on mathematics activities.	☐	☐	☐	☐	☐
d.	Interact with a professional scientist, engineer, or mathematician, either at school or on a field trip.	☐	☐	☐	☐	☐

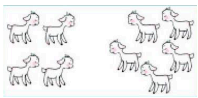
16. **Please tell us how much you disagree or agree with the following statements about mathematics teaching and learning. Please check ONE box per line.**

		Strongly Disagree	Disagree	Not Sure	Agree	Strongly Agree
a.	If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.	☐	☐	☐	☐	☐
b.	I generally teach mathematics ineffectively.	☐	☐	☐	☐	☐
c.	The inadequacy of a student's mathematics background can be overcome by good teaching.	☐	☐	☐	☐	☐
d.	The low mathematics achievement of some students cannot generally be blamed on their teachers.	☐	☐	☐	☐	☐
e.	When a low achieving child progresses in mathematics, it is usually due to extra attention given by the teacher.	☐	☐	☐	☐	☐
f.	I understand mathematics concepts well enough to be effective in teaching elementary mathematics.	☐	☐	☐	☐	☐
g.	Increased effort in mathematics teaching produces little change in some students' mathematics achievement.	☐	☐	☐	☐	☐
h.	The teacher is generally responsible for the achievement of students in mathematics.	☐	☐	☐	☐	☐
i.	Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching.	☐	☐	☐	☐	☐
j.	If parents comment that their child is showing more interest in mathematics at school, it is probably due to the performance of the child's teacher.	☐	☐	☐	☐	☐
k.	I find it difficult to explain to students why mathematics procedures work.	☐	☐	☐	☐	☐
l.	I am typically able to answer students' mathematics questions.	☐	☐	☐	☐	☐
m.	I wonder if I have the necessary skills to teach mathematics.	☐	☐	☐	☐	☐
n.	Effectiveness in mathematics teaching has little influence on the achievement of students with low motivation.	☐	☐	☐	☐	☐
o.	Given a choice, I would not invite the principal to evaluate my mathematics teaching.	☐	☐	☐	☐	☐
p.	When a student has difficulty understanding a mathematics concept, I am usually at a loss as to how to help the student understand it better.	☐	☐	☐	☐	☐
q.	When teaching mathematics, I usually welcome student questions.	☐	☐	☐	☐	☐
r.	Even teachers with good mathematics teaching abilities cannot help some kids learn mathematics.	☐	☐	☐	☐	☐

Appendix 4. *Student Math Test of Inverse Relations (US version)*

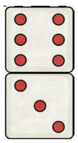
Name __________ Grade _____Age______ School __________Teacher__________

1. Write a group of related number facts suggested by the picture.



_____ + _____ = _____
_____ + _____ = _____
_____ – _____ = _____
_____ – _____ = _____

2. Write a group of related number facts suggested by the picture.



_____ + _____ = _____
_____ + _____ = _____
_____ – _____ = _____
_____ – _____ = _____

3. (a) Peggy had 7 balloons. Richard had 4 balloons. How many more balloons did Peggy have than Richard? Show how you found your answer.

   (b) Peggy had 3 more balloons than Richard. Richard had 4 balloons. How many balloons did Peggy have? Show how you found your answer.

   (c) Peggy had 7 balloons. She had 3 more balloons than Richard. How many balloons did Richard have? Show how you found your answer.

4. Please write a group of related number facts using 6, 7, and 13.

Fill in the blanks:

5. 9 + 3 = ( )
   12 – 3 = ( )

   How did you get the answer for 12 – 3 = ( )?

   ____

6. 81–79 = ( )

   How did you come up with your answer?

   ____

7. To solve 11–6=?, Mary's answer is 5, is this correct? ______________________

   How can you check if this is correct or not?

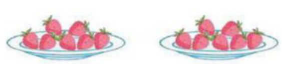
   ____

8. (a) Ali had some chocolate candies. He gave 2 of them to his sister, and then he had 6. How many candies did Ali have before giving his sister candies? Show how you found your answer.

   (b) Ali had some chocolate candies and his sister gave him 2, now he has 8. How many candies did he have before his sister gave him candies? Show how you found your answer.

Name ____________ Grade ______Age_____ School _________Teacher___________

1. Write a group of related number facts suggested by the picture.



____ × ____ = ____
____ ÷ ____ = ____
____ ÷ ____ = ____

2. Write a group of related number facts suggested by the picture.

____ × ____ = ____
____ × ____ = ____
____ ÷ ____ = ____
____ ÷ ____ = ____

3. (a) Hillary spent $9 on Christmas gifts for her family. Geoff spent 3 times as much money as Hillary. How much did Geoff spend? Show how you found your answer.

   (b) Hillary spent $9 on Christmas gifts for her family. Geoff spent $27. How many times as much did Geoff spend as Hillary? Show how you found your answer.

   (c) Hillary spent some money on Christmas gifts for her family. Geoff spent 3 times as much as Hillary. If Geoff spent $27, how much money did Hillary spend? Show how you found your answer.

4. Please write a group of related number facts using 63, 9, and 7.

Fill in the blanks.

5. Joe tried to solve 59÷8=?. His answer was 7 with a remainder of 2. Is this correct?

   ____

   How can you check if this is correct or not?

   ____

6. 3 × 7 = ( )
   21 ÷ 7 = ( )

   How did you get the answer for 21÷7 = ( )?

   ____

7. Use the equation 420 ÷ □ = 6 to answer the following question:

   What number should go in the □ to make this equation correct? ( )
   (A) 60 (B) 70 (C) 80 (D) 90

   How do you know if your answer is correct or not?

   ____

8. There are 3 tables. Each table has 2 plates. If 48 apples are split equally among the plates, how many apples does each plate have? Show how you found your answer.