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Double Tangent Sampling Method for Sinusoidal Pulse Width Modulation

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Abstract: The control method of inverter for low power single-phase motor mainly uses pulse width modulation technology. Based upon the traditional natural sampling method, the tangent sampling method and the secant sampling method, the switching time point calculation formulae are derived for the double tangent sampling method. Compared the switching time with above three methods and analyzing the error precision, the SPWM wave generated by the double tangent sampling method are closest to the wave generated by the natural sampling method. The double tangent sampling method can improve the accuracy of SPWM wave. The harmonic analysis indicates that the SPWM wave obtained by the double tangent sampling method has greater fundamental component, and its low order harmonic components are closer to the natural sampling method compared with the SPWM wave obtained by the secant sampling method. The double tangent method realizes easily in digital systems. It is feasible to obtain an ideal SPWM wave by the double tangent method.

Keywords: Sinusoidal Pulse Width Modulation (SPWM), Tangent Sampling Method, Secant Sampling Method, Double Tangent Sampling Method

1. Introduction

Sinusoidal pulse width modulation (SPWM) is widely used in power electronics technology recently [1]-[4]. SPWM waves are characterized by constant amplitude pulses with different duty cycle for each period. A sequence of voltage pulses can be generated by switching of the power electronic devices. The modulation of pulse width can control the output voltage of inverter, and reduce its harmonic component [5]-[7]. In variable frequency speed control system, SPWM is one of the most important modulation technique applied to motor control and inverter [8]-[10].

At present, the natural sampling method is an ideal SPWM technology since its waveform is closest to the sinusoidal wave [11]. The method can get good quality of waveform. The controlling precision of the fundamental wave is high. However, the natural sampling method needs to solve transcendental equations. This is difficult to realize by digital control devices in engineering application [12, 13]. In order to avoid handling the transcendental equations, some techniques such as symmetric regular sampling, asymmetric regular sampling and modified regular sampling have been presented. These methods can be designed flexible and easier to implement. However, their modulation waveforms contain higher harmonic contents to some extent. Therefore, researchers devote to develop the sampling methods which SPWM waves contain low harmonic content. The effect of modulation waves is more close to the natural sampling method of SPWM as possible. And these sampling methods are easy digital realization.

In this study, a double tangent sampling method to generate SPWM wave was explored on the basis of the natural sampling method, the tangent sampling method and the secant sampling method. At first, the principle of the double tangent sampling method was analyzed, Then the switching time calculation formula were derived. Finally, the precision of switching time and the characteristics of harmonics for the double tangent sampling method was analyzed, and compared them with those for the natural sampling method and the secant sampling method.

2. Several Sampling Methods

2.1. Natural Sampling Method

The natural sampling method is depicted in Figure 1. The pulse width of modulation wave varies with the frequency and amplitude of sinusoidal wave. Figure 1 indicates that the intersection of triangle waveform and sinusoidal waveform is asymmetrical to the median of triangular carrier waveform.

Suppose that the maximum amplitude of the sinusoidal voltage is U_{rm}. The sinusoidal voltage can be expressed as $u_{\mathrm{r}} = U_{\mathrm{rm}} \sin \omega t$. Given the maximum amplitude of bipolar triangular carrier wave $U_{\mathrm{tm}} = 1$, the modulation depth is defined $M = U_{\mathrm{rm}} / U_{\mathrm{tm}}$ $(0 < M < 1)$. Then the sinusoidal wave $u_{\mathrm{r}} = M \sin \omega t$. The turn-on time is denoted by t_{on}. The turn-off time is denoted by t_{off}. And the period of triangular carrier wave is denoted by T_{c}.

In a period of triangular carrier wave, the coordinates of point C is ($\frac{k}{N}\pi$, -1); the coordinates of point D is ($\frac{k-0.5}{N}\pi$, 1); and the coordinates of point E is ($\frac{k+0.5}{N}\pi$, 1). The linear equations of CD and CE can be obtained, respectively.

$$l_{CD} : y = -\frac{4N}{\pi}x + 4k - 1 \tag{1}$$

$$l_{CE} : y = \frac{4N}{\pi}x - 4k - 1 \tag{2}$$

In order to obtain the accurate SPWM waveform, the turn-on time and the turn-off time can be calculated by Equation (3) and (4).

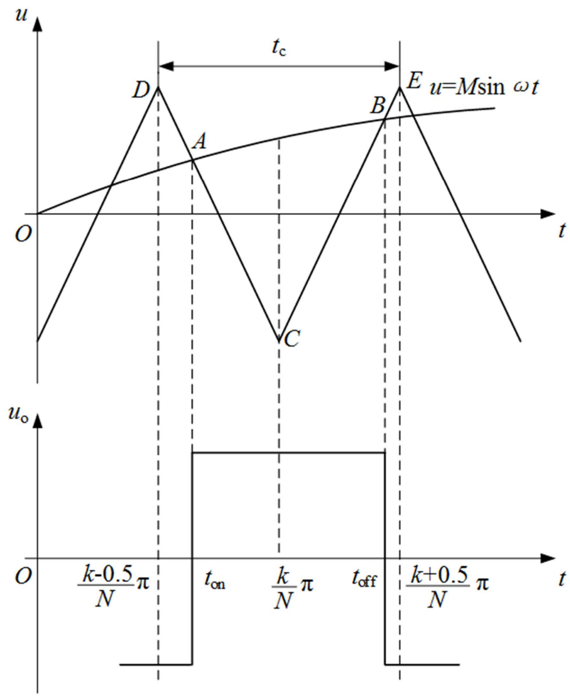


Figure 1. *Natural sampling method.*

$$M \sin \omega t + 4Nt / \pi - 4k + 1 = 0 \tag{3}$$

$$M \sin \omega t - 4Nt / \pi + 4k + 1 = 0 \tag{4}$$

Where is the sampling point number of sinusoidal wave in half period; k is the sequence number of sampling points.

The natural sampling method controls power electronic devices on-off at the natural intersection points of the sinusoidal modulation wave and high-frequency triangular carrier wave. The switching point time is solved by numerical iteration since Equation (1) and (2) are transcendental equation. Therefore, this method is not suitable for real-time digital control [10].

Among pulse width modulation methods, although the regular sampling method is simple to calculate, the harmonic contents are relatively greater. Tangent sampling method and secant sampling method can obtain higher accurate SPWM waveform, and effectively reduce harmonic contents [14].

2.2. Tangent Sampling Method

Figure 2 shows tangent sampling method. The method samples at the negative peak of triangle wave. A tangent of sinusoidal wave is drawn at the sampling point F. The switching time is determined by the intersection points of the tangent and triangular carrier wave.

The coordinates of point F are ($\frac{k}{N}$, $M \sin \frac{k}{N}\pi$). Let $\omega t = \theta(k)$, the tangent equation is

$$l_{AB} : u = \theta(k) M \cos(\frac{k}{N}\pi) - M\frac{k}{N}\pi \cos\left(\frac{k}{N}\pi\right) + M \sin\left(\frac{k}{N}\pi\right) \tag{5}$$

The formula of the switching time can be obtained by solving the tangent equation and two linear equations of triangular carrier wave simultaneously.

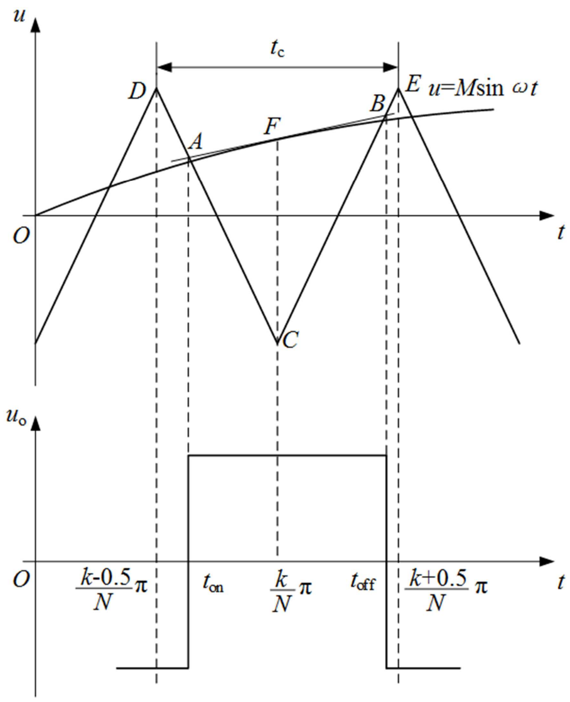


Figure 2. *Tangent sampling method.*

$$t_{on}=\pi\frac{4kN-N+\pi kM\cos(\frac{k\pi}{N})-M\sin\left(\frac{k\pi}{N}\right)N}{N\left[\pi M\cos\left(\frac{k\pi}{N}\right)+4N\right]} \tag{6}$$

$$t_{off}=\pi\frac{-4kN-N+\pi Mk\cos\left(\frac{k\pi}{N}\right)-M\sin\left[\frac{k\pi}{N}\right]N}{N\left[\pi M\cos\left(\frac{k\pi}{N}\right)-4N\right]} \tag{7}$$

2.3. Secant Sampling Method

As shown in Figure 3, at the point C of the negative peak of the triangular carrier wave, a line perpendicular is drawn to abscissa axis. The line intersects the sinusoidal wave at point F. Through the point D of the positive peak of the triangular carrier wave, a line perpendicular is done to abscissa axis. The line intersects sinusoidal wave at point G. Similarly, the intersection point H is gotten. The point F is respectively connected with two intersection points G and H. There are two secants within a triangle carrier period. The switching time is determined by the intersection points of triangular carrier wave and the two secants.

The points G and F are connected. The secant line GF and the triangular carrier wave intersect at point A. The linear l_{GF} equation is

$$u=\frac{M(\sin\frac{2k-1}{2N}\pi-\sin\frac{k}{N}\pi)}{\frac{2k-1}{2N}\pi-\frac{k}{N}\pi}\left(\theta-\frac{k}{N}\pi\right)+M\sin\frac{k}{N}\pi \tag{8}$$

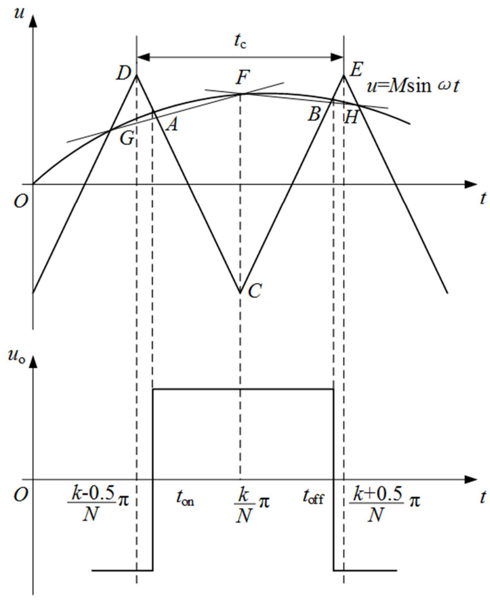


***Figure 3.** Secant sampling method.*

Solving Equation (1) and (8), the turn-on time can be obtained.

$$t_{on}=\frac{1}{2}\pi\frac{4k-1-M\sin(k\pi/N)+2Mk\sin(k\pi/N)-2Mk\sin\left[(2k-1)\pi/2N\right]}{N\left\{2+M\sin(k\pi/N)-M\sin\left[(2k-1)\pi/2N\right]\right\}} \tag{9}$$

$$t_{off}=\frac{1}{2}\pi\frac{4k+1+M\sin(k\pi/N)+2Mk\sin(k\pi/N)-2Mk\sin\left[(2k+1)\pi/2N\right]}{N\left\{2+M\sin(k\pi/N)-M\sin\left[(2k+1)\pi/2N\right]\right\}} \tag{10}$$

3. Double Tangent Sampling Method

Figure 4 shows double tangent sampling method. The method samples at the positive peak of the triangle wave. Two tangents for sinusoidal wave are done at two sampling point G and H, respectively. The switching time is determined by the intersection points of the tangents and triangular carrier wave. The formula of the switching time can be obtained by solving the two tangent equations and linear equations of triangular carrier wave.

There are two tangents within a period of the triangle carrier wave. Through the points D and E of the positive peak of the triangular carrier wave, two dotted lines are done perpendicular to abscissa axis. The lines intersect sinusoidal wave at point G and H, respectively. At the point G and H, tangents are done for the sinusoidal wave, respectively. The two tangents and the triangular carrier wave intersect at point A and B. The switching time is determined by the intersection A and B.

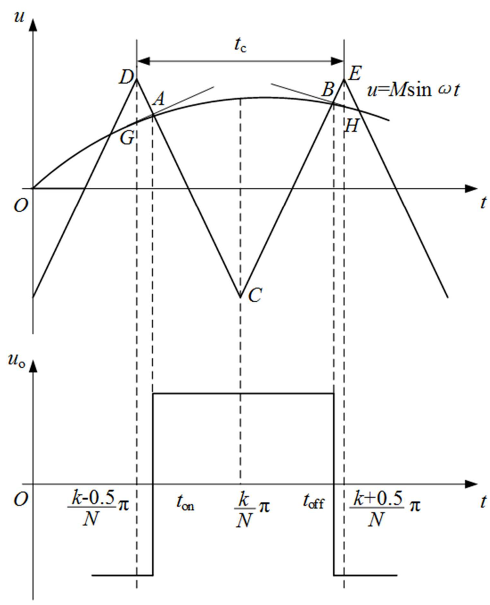


***Figure 4.** Double tangent sampling method.*

The coordinates of point G are ($\frac{k-0.5}{N}$, $M\sin\frac{k-0.5}{N}$). The coordinates of point H are ($\frac{k+0.5}{N}$, $M\sin\frac{k+0.5}{N}$). The two tangent equations are as following:

$$l_{\mathrm{GA}}:\ u=\theta M\cos(\frac{k-0.5}{N}\pi)-M\frac{k-0.5}{N}\pi\cos\left(\frac{k-0.5}{N}\pi\right)+M\sin\left(\frac{k-0.5}{N}\pi\right) \tag{11}$$

$$l_{\mathrm{HB}}:\ u=\theta M\cos(\frac{k+0.5}{N}\pi)-M\frac{k+0.5}{N}\pi\cos\left(\frac{k+0.5}{N}\pi\right)+M\sin\left(\frac{k+0.5}{N}\pi\right) \tag{12}$$

Solving equation (1) and (11), (2) and (12), the switching time can be obtained.

$$t_{on}=\pi\frac{4kN-N+M\left(k-0.5\right)\pi\cos\left(\frac{k-0.5}{N}\pi\right)-MN\sin\left(\frac{k-0.5}{N}\pi\right)}{N\left[\pi M\cos\left(\frac{k-0.5}{N}\pi\right)+4N\right]} \tag{13}$$

$$t_{off}=\pi\frac{-4kN-N+M\left(k+0.5\right)\pi\cos\left(\frac{k+0.5}{N}\pi\right)-MN\sin\left(\frac{k+0.5}{N}\pi\right)}{N\left[\pi M\cos\left(\frac{k+0.5}{N}\pi\right)-4N\right]} \tag{14}$$

The formula (13) and (14) are deduced by the double tangent sampling method. They are algebraic and easy to implement digitally. Setting the carrier ratio $2N$, $\sin\left(\frac{k-0.5}{N}\pi\right)$, $\cos\left(\frac{k-0.5}{N}\pi\right)$, $\sin\left(\frac{k+0.5}{N}\pi\right)$ and $\cos\left(\frac{k+0.5}{N}\pi\right)$ can be calculated by the microprocessor. While the modulation depth M changes, the switching time can be calculated to obtain SPWM wave.

4. Accuracy and Harmonic Analysis

4.1. Accuracy Analysis

Supposing, the carrier ratio is 18. The modulation depth is M=0.8. k is taken 1 to 9. Table 1 gives the switching time of tangent sampling method, secant sampling method and double tangent sampling method. The accuracy of these methods is compared with that of natural sampling method.

As stated in Table 1, the tangent sampling method and the secant sampling method can control the error in a small range. While k is taken 1 to 9, the tangent sampling method can control the error within -0.063~0.062%; the secant sampling method can make the error within -0.023%~0.025%. In comparison the tangent sampling method with the secant sampling method, it indicates that the accuracy of secant sampling method is higher than that of tangent sampling method. The error of double tangent sampling method can be controlled between 0.013%~0.012%. This verifies that the precision of the double tangent sampling method is higher than that of the secant sampling method and tangent sampling method. The double tangent sampling method is better than the tangent sampling and secant sampling method. The switching time of the double tangent sampling method is closest to that of the natural sampling method. The accuracy of the double tangent sampling method can meet the requirements of higher control.

Table 1. *Switching time and error accuracy.*

k	t	natural sampling	tangent sampling	error precision	secant sampling	error precision	double tangent sampling	error precision
1	t_{on}	0.24487	0.24476	-0.0448	0.24494	0.0249	0.24484	-0.0129
	t_{off}	0.46781	0.46801	0.0428	0.46771	-0.0227	0.46787	0.0121
2	t_{on}	0.57302	0.57270	-0.0553	0.57313	0.0200	0.57297	-0.0073
	t_{off}	0.83726	0.83774	0.0577	0.83713	-0.0155	0.83729	0.0042
3	t_{on}	0.90503	0.90515	-0.0633	0.90515	0.0138	0.90500	-0.0030
	t_{off}	1.19952	1.20027	0.0623	1.19941	-0.0092	1.19954	0.0014
4	t_{on}	1.24290	1.24211	-0.0633	1.24301	0.0089	1.24289	-0.0009
	t_{off}	1.55333	1.55420	0.0557	1.55324	-0.0062	1.55334	0.0007
5	t_{on}	1.58826	1.58740	-0.0544	1.58836	0.0061	1.58825	-0.0007
	t_{off}	1.89869	1.89948	0.0416	1.89858	-0.0056	1.89870	0.0008

k	t	natural sampling	tangent sampling	error precision	secant sampling	error precision	double tangent sampling	error precision
6	t_{on}	1.94207	1.94133	-0.0384	1.94218	0.0057	1.94206	-0.0008
	t_{off}	2.23657	2.23714	0.0257	2.23644	-0.0055	2.23659	0.0012
7	t_{on}	2.30434	2.30385	-0.0210	2.30447	0.0056	2.30430	-0.0015
	t_{off}	2.56858	2.56889	0.0123	2.56846	-0.0045	2.56862	0.0016
8	t_{on}	2.67378	2.67358	-0.0075	2.67389	0.0040	2.67372	-0.0021
	t_{off}	2.89672	2.89683	0.0038	2.89666	-0.0021	2.89675	0.0011
9	t_{on}	3.04779	3.04778	-0.0003	3.04781	0.0008	3.04775	-0.0012
	t_{off}	3.22317	3.22319	-0.0002	3.22319	0.0006	3.22313	-0.0013

4.2. Harmonic Analysis

The bipolar SPWM wave is a sequence of periodic pulses. The function which the waveform expresses is periodic and odd. Its period is 2π. The positive and negative half period of sinusoidal wave is N equal divisions (N usually takes multiples of 3). Then the function can be defined on the interval [0, π].

$$f(\theta)=\begin{cases} U_{\mathrm{d}} & t_{on(k)} \le \theta < t_{of(k)f} \\ -U_{\mathrm{d}} & \dfrac{k-1}{N}\pi \le \theta < t_{on(k)} \text{ or } t_{off(k)} \le \theta < \dfrac{k}{N}\pi \end{cases} \tag{15}$$

where $k=1,2,3.....N$. Function (15) can be expanded in Fourier series.

$$f(\theta)=a_0+\sum_{n=1}^{\infty} a_n \cos(n\theta)+b_n \sin(n\theta) \tag{16}$$

$f(\theta)$ is due to an odd function on $[-\pi,\pi]$, SPWM wave does not contain even harmonics. Thus, $a_0=0$, $a_n=0$.

$$f(\theta)=\sum_{n=1,3,5...}^{\infty} b_n \sin(n\theta) \tag{17}$$

where

$$\begin{aligned} b_n &= \frac{2}{\pi}\int_0^{\pi} f(\theta)\sin(n\theta)d\theta \\ &= \frac{2U_{\mathrm{d}}}{\pi}\sum_{k=1}^{N}\left[\int_{\frac{k-1}{N}\pi}^{\theta_{on}(k)} -\sin(n\theta)d\theta+\int_{\theta_{on}(k)}^{\theta_{off}(k)}\sin(n\theta)d\theta-\int_{\theta_{off}(k)}^{\frac{k}{N}\pi}\sin(n\theta)d\theta\right] \\ &= \frac{2U_{\mathrm{d}}}{n\pi}\left\{2\sum_{k=1}^{N}\left[\cos(nt_{on}(k))-\cos(nt_{off}(k))\right]+\cos(n\pi)-1\right\} \end{aligned} \tag{18}$$

Under the same conditions, the bipolar SPWM wave is generated by the secant sampling method and double tangent sampling method, respectively. The harmonic characteristics of the bipolar SPWM wave can be analyzed using Fourier series (18). Table 2 shows the fundamental and harmonic coefficients of SPWM waves achieved by the secant sampling method and the double tangent sampling method, respectively. The harmonics are within 40th. The fundamental coefficient of SPWM wave obtained by the double tangent sampling method is closest to that of SPWM wave obtained by the natural sampling method. The harmonic coefficient of SPWM wave obtained by the double tangent sampling method is lower than that of SPWM wave obtained by the secant sampling method. The voltage distortion of SPWM wave obtained by the double tangent sampling method is lighter than that obtained by the secant sampling method.

Table 2. *Harmonic coefficient.*

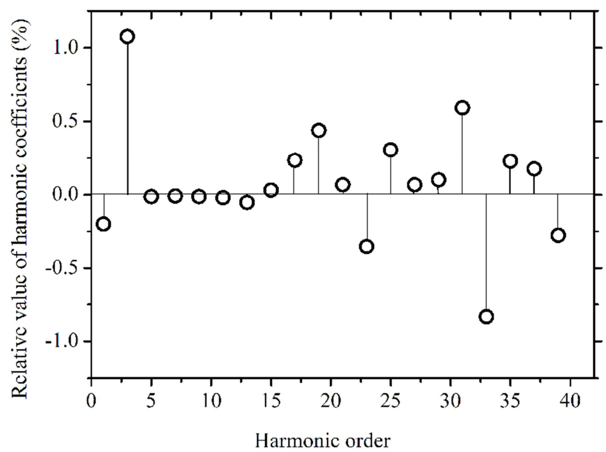
	Natural method	Secant method	Double tangent
1	0.78523	0.78365	0.78560
3	-0.04456	-0.04504	-0.04422
5	-0.07520	-0.07519	-0.07508
7	-0.10775	-0.10774	-0.10771

	Natural method	Secant method	Double tangent
9	-0.14435	-0.14433	-0.14433
11	-0.19015	-0.19011	-0.19015
13	-0.26210	-0.26196	-0.26207
15	-0.43723	-0.43736	-0.43701
17	-0.53851	-0.53977	-0.53817
19	0.30556	0.30690	0.30537
21	0.20875	0.20889	0.20865
23	0.04249	0.04234	0.04254
25	-0.01632	-0.01637	-0.01625
27	-0.04490	-0.04493	-0.04483
29	-0.05995	-0.06001	-0.05983
31	-0.05569	-0.05602	-0.05547
33	0.06834	0.06777	0.06852
35	0.24441	0.24497	0.24436
37	-0.37994	-0.38061	-0.3799
39	-0.19797	-0.19742	-0.19813

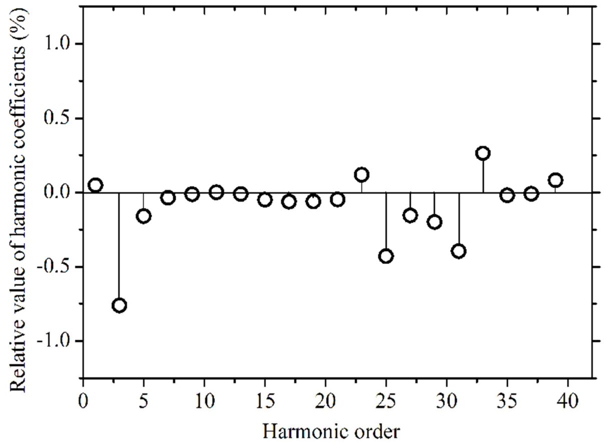
The harmonic coefficients generated by the natural sampling method, secant sampling method and double tangent sampling method is expressed as b_{Nn}, b_{Sn}, b_{Dn}, respectively. Taking $n = 1,3,5,7...39$, the relative value of harmonic coefficient σ is defined as

$$\sigma = \frac{b_{\mathrm{Sn}}(b_{\mathrm{Dn}}) - b_{\mathrm{Nn}}}{b_{\mathrm{Nn}}} \times 100\% \tag{19}$$

Figure 5 depicts the relative value of harmonic coefficients of SPWM wave achieved by the secant sampling method and double tangent sampling method. To low order harmonics (order less than 20), the relative value of harmonic coefficients of the double tangent method is lower than those of the secant sampling method obviously. To high order harmonics, the relative value of harmonic coefficients of the double tangent method is also lower for most of harmonic orders. The harmonic characteristics of SPWM wave achieved by the double tangent sampling method is prior to those of SPWM wave achieved by secant sampling method within 40 orders. While the demand of output voltage is higher, it is a good choice that the SPWM wave is generated by the double tangent method.



(a) Secant sampling method



(b) Double tangent sampling method

Figure 5. *Relative value of harmonic coefficients.*

5. Conclusions

Compared with the tangent sampling method and the secant sampling method, the double tangent sampling method can generate the SPWM wave which error is smallest.

Harmonic analysis shows that the fundamental component of SPWM wave generated by double tangent sampling method is high. The harmonic components of SPWM wave obtained by double tangent sampling method are lower than those of the secant sampling method and the tangent sampling method.

The double tangent method can be implemented in digital control systems, and improve the accuracy of sinusoidal pulse width modulation. This method can be applied to industrial control and engineering application.

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References

[1] H. F. Xiao, K. Lan and L. Zhang, A quasi-unipolar SPWM full-bridge transformerless PV grid-connected inverter with constant common-mode voltage, IEEE Transactions on Power Electronics, 30 (6): 3122-3132, 2015.

[2] M. Lakka, E. Koutroulis and A. Dollas, Development of an FPGA-based SPWM generator for high switching frequency DC/AC inverters, IEEE Transactions on Power Electronics, 29 (1): 356-365, 2014.

[3] I. Colak and E. Kabalci, Developing a novel sinusoidal pulse width modulation (SPWM) technique to eliminate side band harmonics, International Journal of Electrical Power & Energy Systems, 44 (1): 861-871, 2013.

[4] M. M. Renge and H. M. Suryawanshi, Multilevel inverter to reduce common mode voltage in AC motor drives using SPWM technique, Journal of Power Electronics, 11 (1): 21-27, 2011.

[5] K. Jain and P. Chaturvedi, Matlab-based simulation & analysis of three-level SPWM inverter, International Journal of Soft Computing and Engineering, 2 (1): 56-59, 2012.

[6] T. H. Li, S. H. Chung and K. M. Chan, An active modulation technique for single-phase grid-connected CSI, IEEE Transactions on Power Electronics, 22 (4): 1373-1382, 2007.

[7] J. Huang and H. Shi, Reducing the common-mode voltage through carrier peak position modulation in an SPWM three-phase inverter, IEEE Transactions on Power Electronics, 29 (9): 4490-4495, 2014.

[8] I. Colak and E. Kabalci, Developing a novel sinusoidal pulse width modulation (SPWM) technique to eliminate side band harmonics, Electrical Power and Energy Systems, 44 (1): 861–871, 2013.

[9] Z. X. Zhang, H. Ma and X. N. He, Study on novel SPWM sampling methods, Journal of Zhejiang University: Engineering Science, 39 (4): 588-594, 2005.

[10] R. Nagarajan and M. Saravanan, Performance analysis of a novel reduced switch cascaded multilevel inverter, Journal of Power Electronics, 14 (1): 48-60, 2014.

[11] C. F. Zheng, B. Zhang and D. Y. Qiu, Digital natural sampling SPWM based on inverse operator method, in Proc. IEEE APESC, 792-797, 2007.

[12] A. R. Beig, G. Narayanan and V. T. Ranganathan, Modified SVPWM algorithm for three level VSI with synchronized and symmetrical waveforms, IEEE Transactions on Industrial Electronics, 54 (1): 486-494, 2007.

[13] B. Ismail, S. Taib, A. R. M. Saad, M. Isa and C. M. Hadzer, Development of a single phase SPWM microcontroller-based inverter, in Proc. IEEE PEC, 437-440, 2006.

[14] X. D. Liu and T. Xu, Precision analysis of SPWM switching time point, Electric Power Automation Equipment, 30 (12): 55-59, 2010.