DYNAMIC ANALYSIS And FIELD BALANCING Of 100 MW UTILITY GAS TURBINE - GENERATORS

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Abstract

This paper deals with some of the challenges in the dynamic analysis and field balancing of the W501D gas turbine-generator system currently used in a number of utilities. The current W501D model represents a power enhancement over the earlier Westinghouse 70 MW units which have been in service for many years. The utility gas turbines present a number of challenges as compared to conventional steam turbines. Because of the design of the gas turbine, the hot end of the turbine requires a very flexible support under the tilting pad bearing. This in turn causes a significant reduction of stiffness and particularly of damping. In addition, there is substantial rotor-casing interaction which is not present in conventional steam turbines. As a result the dynamic characteristics are totally unlike that encountered with conventional steam turbines. In 1985, papers were presented at Vibration Institute meetings on the dynamic analysis and field balancing of 70 MW gas turbine-generator systems. Extensive experimental data was collected on various units at that time. It was determined that computer assisted balancing was required to properly calculate the balance corrections necessary for field balancing. Unlike steam turbines, in which the casing is quite substantial, the W501 series of gas turbines have significant casing motion present in the operating speed range. The dynamics of the casing introduces rotor-casing modes which must be taken into consideration in order to perform proper field balancing. At the time of the earlier papers, it was not possible to compute the dynamics of the system including casing effects. Currently, with the modern finite element computer codes, the casing can be included in the dynamic analysis of the turbine-generator to produce a more realistic representation of the system. In the earlier analysis of the 70 MW gas turbine system, the transfer matrix method of rotor dynamics was employed. This has proven to be insufficient for the analysis of the gas turbine-generator systems including casing and foundation effects to compute stability and predicted unbalance response. It is only with the finite element approach that a complete analysis of the system can be accurately made. A significant amount of field data is presented and descriptions on recommended methods of balancing the W501D are presented.

Introduction and Statement of The Problem

Figure 1 Represents the Cross-Section of a modern W501D gas turbine which has been upgraded to 100 MW from the earlier Westinghouse 70 MW design. With the current emphasis on the reduction of CO_2 emissions from utilities, the W501D gas turbine generator systems have become very popular with various utilities. Although the W501 series of gas turbines have been very successful, and in use for many years, they are not without their problems.

One of the difficulties encountered with the gas turbines is the requirement for a very flexible support under the tilting pad bearing at the gas turbine hot end. This not only causes a reduction of the effective bearing stiffness, but can lead to over a 90% reduction in the tilting pad bearing effective damping. This can then cause the turbine to be very sensitive to unbalance conditions and thermal bowing in the turbine and distortion in the generator. In addition, there is considerable casing motion that interacts with the rotor that is not present in your typical steam turbine design. This then can lead to additional critical speeds in the operating speed range. This has led to difficulties in many instances of balancing.

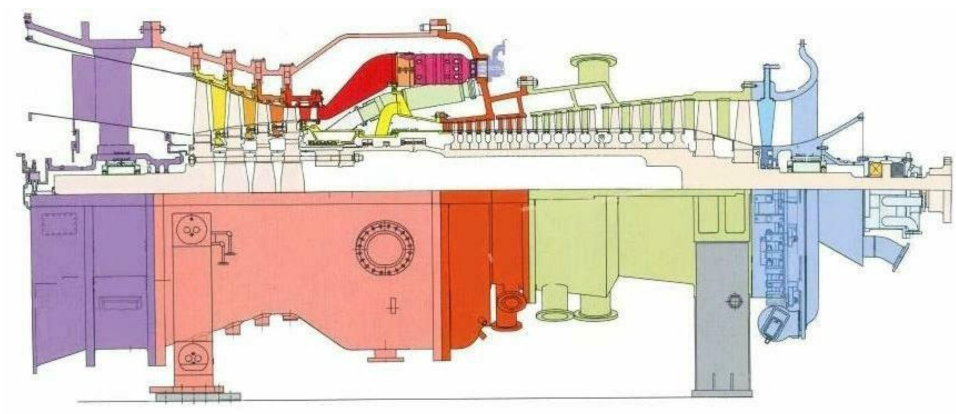


Fig. 1 Schematic Drawing of 100 MW Utility Gas Turbine

Figure 2 represents a W501D turbine which has been removed for maintenance. Visible in this picture are the turbine center plane balance holes. In the photograph are shown the four turbine stages and 17 smaller axial turbine blades. Additional planes available for balancing are located at the compressor inlet, the last large outboard turbine station, and also at the coupling.

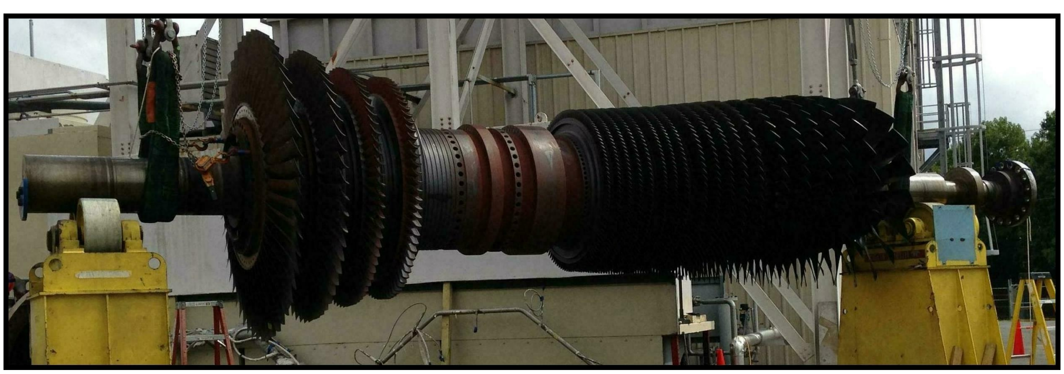


Fig. 2 100 MW Gas Turbine Being Removed For Servicing

Background on Analysis and Balancing Earlier 70 MW Gas Turbine-Generators

Figure 3 shows a cross-section of the 70 MW gas turbine showing the balance planes. The Rotor Dynamics Laboratory at the University of Virginia was contacted in 1983 by Carolina Power & Light to investigate the difficulties encountered with the field balancing of several of their W501 70 MW gas turbine-generators.

In particular one field engineer had spent three months in an attempt to balance one of the gas turbines. He was instructed by field service to perform a simultaneous two plane balance. However, in the field, he would place a balance weight on the turbine which in turn would cause high vibrations on the compressor side. A correction weight placed at the compressor balance plane would then in turn cause high turbine end vibrations.

Gas turbine balancing runs can be extremely time-consuming. For example, after a weight has been placed at the hot end of the gas turbine, it usually takes a day for the system to cool down before another balance weight may be applied at that location.

It is particularly embarrassing if a balance weight should be dropped into the gas turbine balancing location. This then requires a dismantling of the turbine section in order to remove the missing weight. Failure to do so can cause the missing balance weight to be ingested by the blades leading to extensive turbine damage.

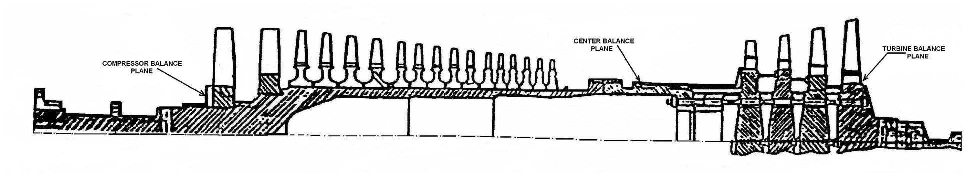


Fig. 3 Cross Section of 70 MW Gas Turbine Showing Balance Planes

The Bently Nevada Corporation, in support of the University of Virginia Rotor Dynamics Program, supplied instrumentation for the turbine generator System. Bill Pryor (currently PdM Solutions) was assigned to collect the data. Figure 4 shows the five locations along the turbine generator system where Bently 7000 series dual probes were installed on the system. The Bentley dual probes system uses a proximity probe for shaft relative motion and the velocity probe for the casing motion.

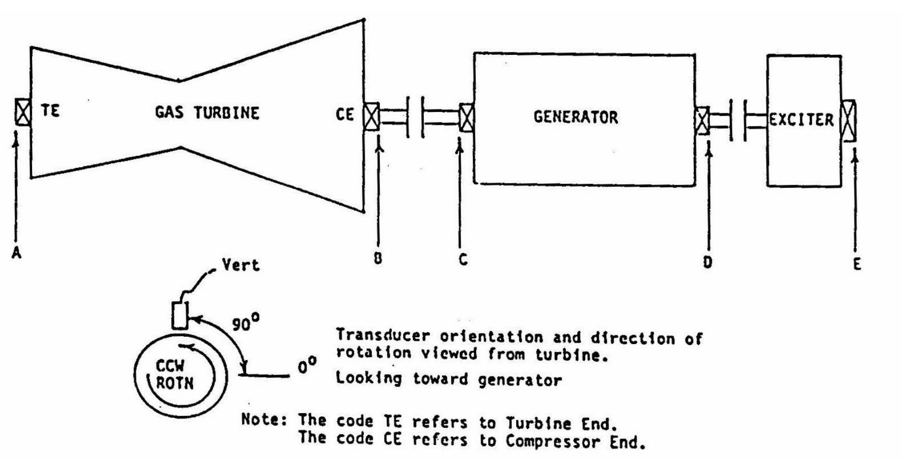
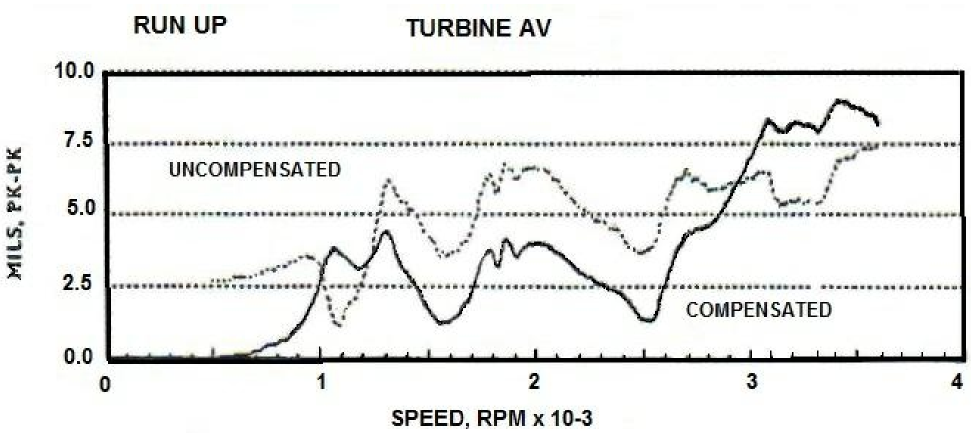


Fig. 4 Gas Turbine - Generator System Showing Measurement Planes

The casing velocity motion was integrated to determine displacement. The casing motion could then be added to the relative shaft motion to determine the absolute shaft motion in space. An additional probe referred to as a key phaser was provided to determine relative phase of motion. Accurate synchronous amplitude and phase are required for the balancing calculations.

70 MW Turbine-Generator Motion With Large Compressor Plane Unbalance:

Figure 5 represents the absolute vertical turbine motion at the hot end. Distinct turbine and generator modes are seen in the experimental data. The predicted turbine second critical speed is approximately 2700 RPM. It is noticed from the experimental data that the highest amplitudes are around 3100 RPM and 3448 RPM. These modes were not originally computed because of the inability of the transfer matrix computer program to include casing effects.



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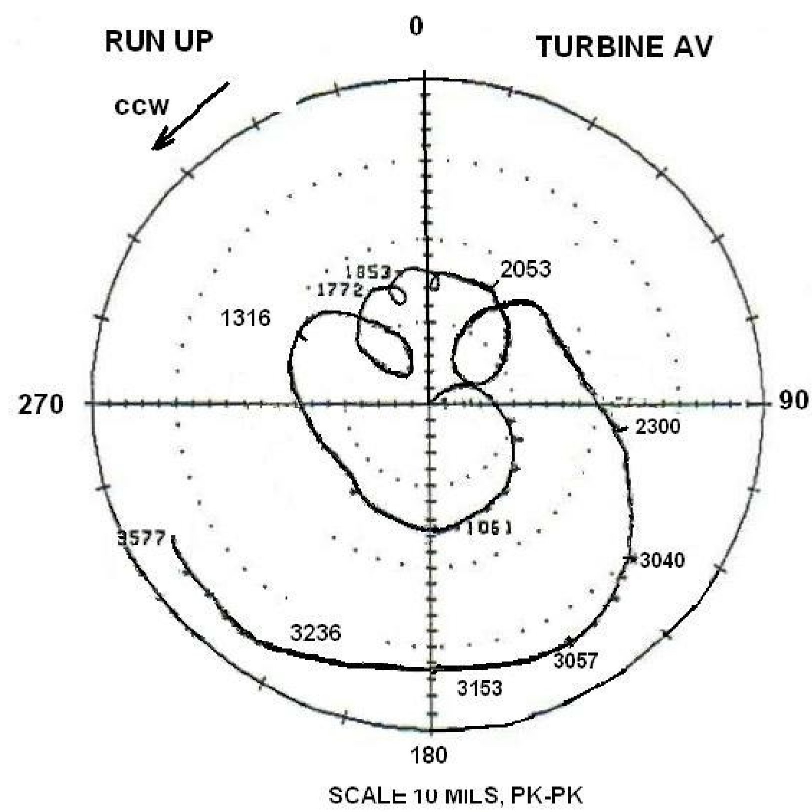


Fig. 6 Turbine Vertical Absolute Motion Polar Plot With Large Compressor Unbalance

Figure 6 represents the polar plot of the turbine vertical absolute motion with large compressor unbalance. From the examination of the polar plot, we can distinctly see various resonance frequencies in the system such as at 1051 RPM, 3016 RPM and 3057 RPM.
Of particular interest is the phase angle change that occurs from approximately 3000 RPM to 3600 RPM. This phase angle change is approximately 120°. This large phase angle change also helps to explain the difficulty in balancing the gas turbine at speed. If one considers balancing data only at running speed, then the mode at 3100 RPM will be excited.
Thus single plane balancing at the turbine location will cause high vibrations in the 3100 RPM speed range. This will result in excessive vibrations in the 3100 - 3200 RPM speed range such that it will not allow the system to come up to speed.
This mode is due to the interaction of the casing with the turbine. The mode generated is referred to as a bifurcated second critical speed. With bifurcated critical speeds, the casing can interact causing the second critical speed to split into higher and lower mode. Review of these modes will be shown in detail at a later section..

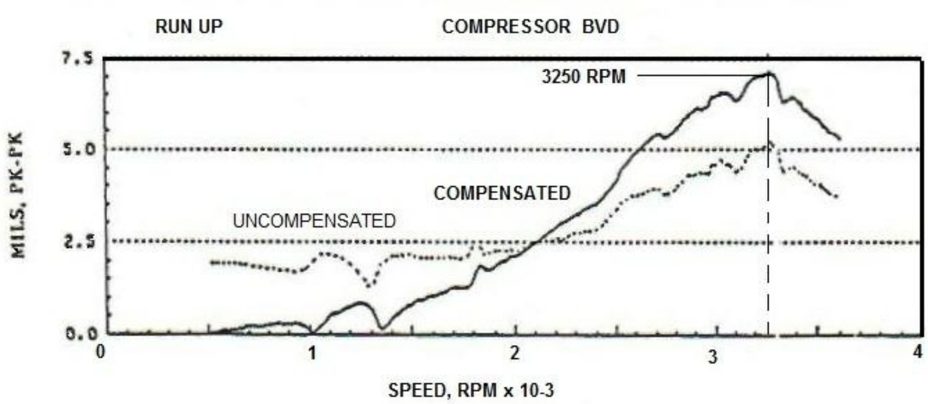


Fig. 7 Compressor Vertical Motion With Large Compressor Unbalance Showing Strong Resonance Frequency At 3,250 RPM

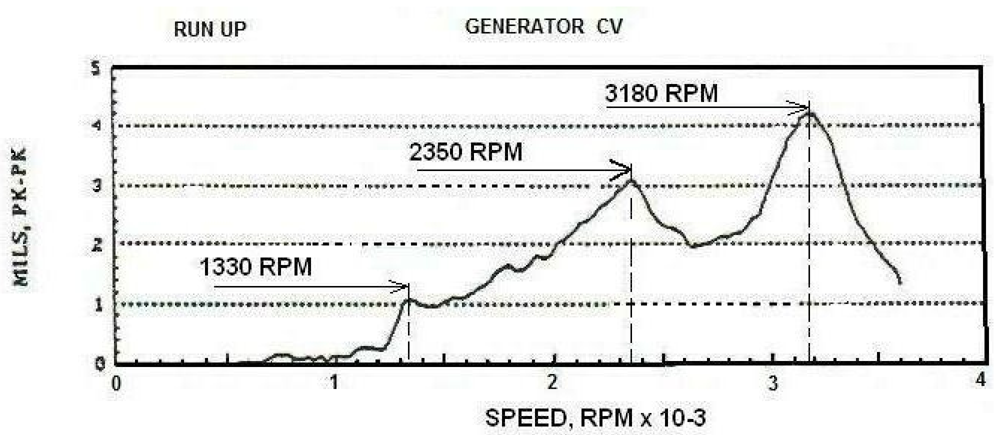


Fig. 8 Generator Inboard Vertical Motion With Large Compressor Plane Unbalance

Figure 8 represents the generator inboard vertical motion under the influence of a large unbalance weight at the gas turbine compressor inlet. The turbine rotor-casing mode is clearly seen in the vibration data at 3180 RPM. The response at 2350 RPM is the generator second mode. Also observed is the turbine mode at 1330 RPM. The computed critical speeds, including casing effects, and the corresponding mode shapes will be shown in The next section.

Problems with Generator Effecting Gas Turbine Balancing

In the course of attempting to balance the CP&L 70 MW gas turbine with a two plane balance correction, we were informed by Bill Pryor that an unusual occurrence had happened with the generator casing response. The vibration data indicated erratic behavior as well as a three times super synchronous vibration being generated. It was apparent at the time that an internal rub had occurred in the generator.

This was further verified later by examination of the amplitude and polar plots for the generator motion at station D. The turbine-casing mode at 3,200 RPM had sufficient response on the generator that it caused a horizontal generator rub. This caused a highly nonlinear jump as can be seen in Fig. 10 and Fig. 11. Balancing is not possible under these conditions and the generator must be realligned.

The problem is due in part to the poor 120° arc bearings used which provide little horizontal support.

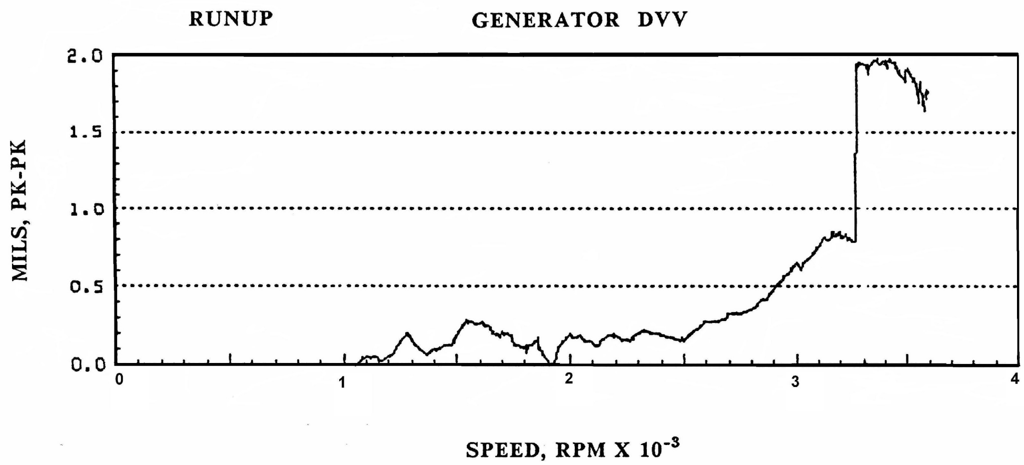


Fig. 9 Generator Casing Motion Showing Large Nonlinear Jump at 3,300 RPM

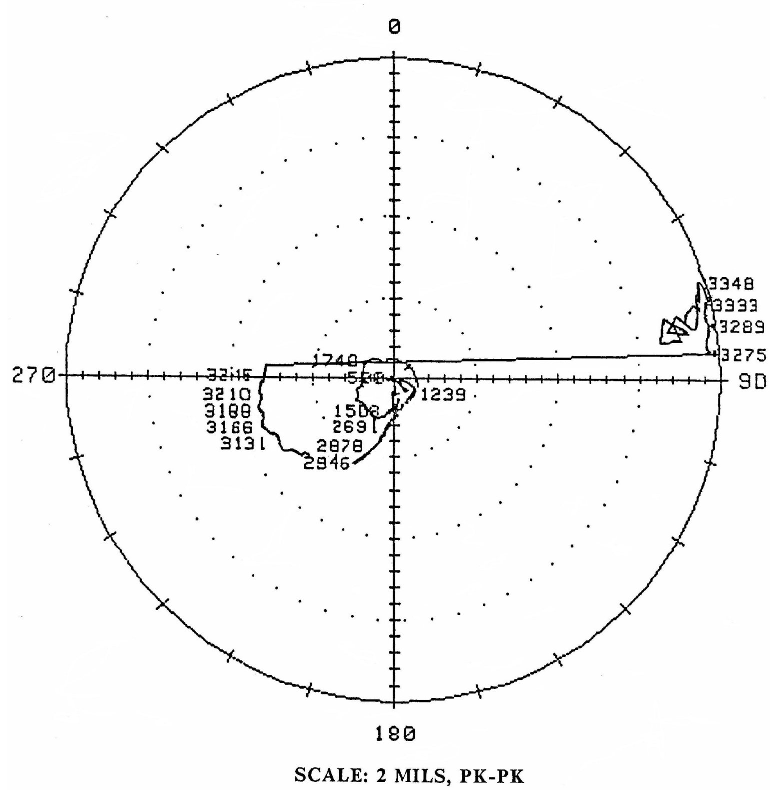


Fig. 10 Polar Plot of Generator Outboard Casing Motion Showing Large Nonlinear Jump at 3,300 RPM

Critical Speeds Of a W501D Gas-Turbine Generator

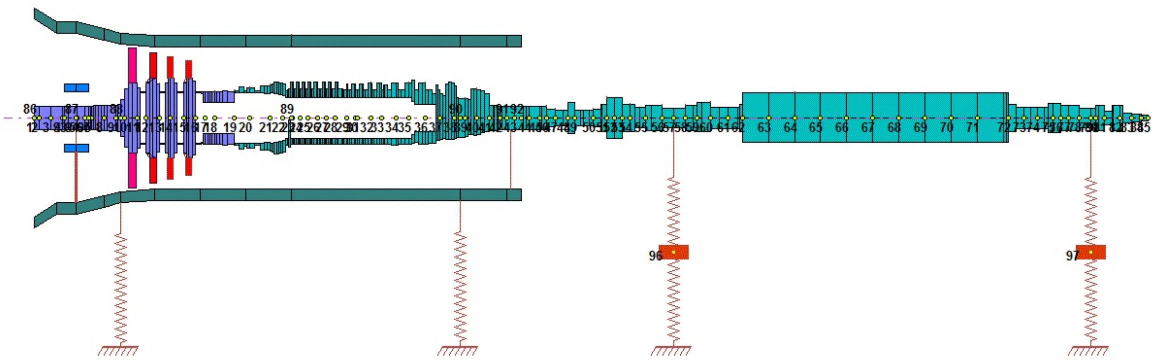


Fig. 11 Computer Model of the W501 Gas Turbine Generator

The model as shown in Figure 11 above has 92 elements and 378° of freedom. The solution of such a model would have been impossible with transfer matrix methods. In earlier reported papers on the W501, the critical speeds of the turbine-generator system were analyzed without casing effects. Other problems that are encountered with transfer matrix methods are the occurrence of missing and unconverged modes in the system. This is not the situation with the finite element method. There are no modes that are skipped in the system.

The gas turbine is supported by 2 4-pad tilting pad bearings. The generator is supported by 120° partial arc bearings. These generator partial arc bearings have been a continual source of problems as they provide low horizontal stiffness which can lead to seal rubs, provide low system damping, and are unstable with self excited whirling at running speed. Generator manufacturers are reluctant to change away from a bearing design that they have been using for over 100 years.

What is unusual about the gas turbine is the flexible webbing supporting the turbine end tilting pad bearing. In a previous paper presented on the W501 series gas turbines, it was shown that the flexible supporting web can reduce by as much as 95% the effective damping at the turbine end bearing. This combination of low turbine end damping with a flexible support and casing interaction leads to a number of unusual dynamical modes in the system.

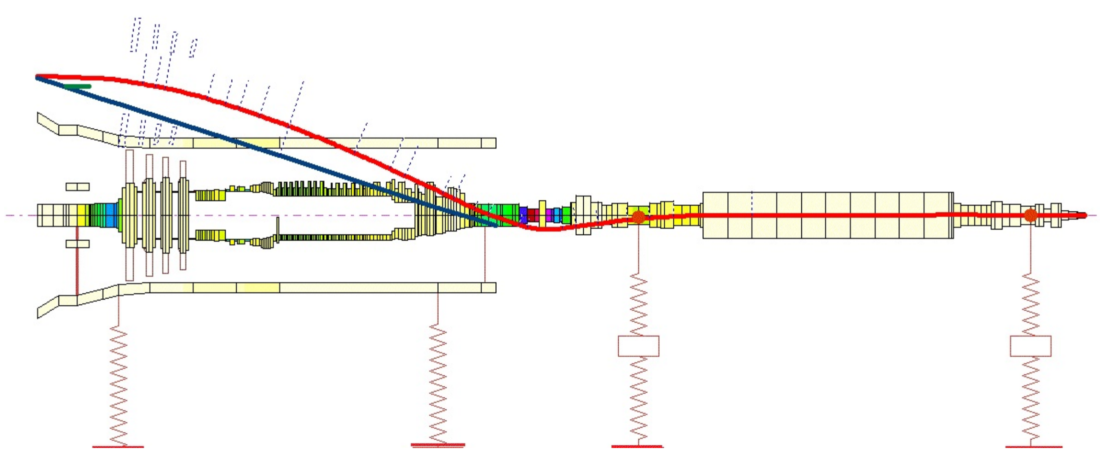


Fig. 12 Turbine-Generator 1st Undamped Mode, M1 = 688 RPM

Figure 12 represents the turbine-generator first mode at 688 RPM. These Critical Speeds are based on the average bearing horizontal and vertical stiffness values. For example, the horizontal resonance RPM could be as low as 430 RPM while the vertical could be 850 RPM.

The casing and rotor are moving in phase. This mode is caused by the low stiffness of the support at the hot end of the gas turbine.

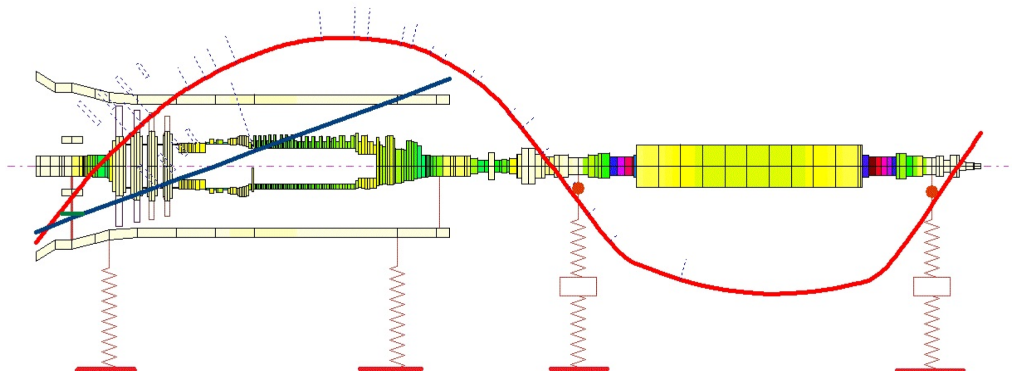


Fig. 13 Turbine-Generator 2nd Mode, M2 = 1131 RPM

Figure 13 represents a combined turbine-generator mode at 1131 RPM. In this case the casing is moving in phase with the turbine first mode. The effect of the casing has caused a bifurcated mode in which the casing at the lower speed is moving predominantly at the turbine end. In this mode the turbine end casing motion is reduced and the predominant motion as at the compressor location.

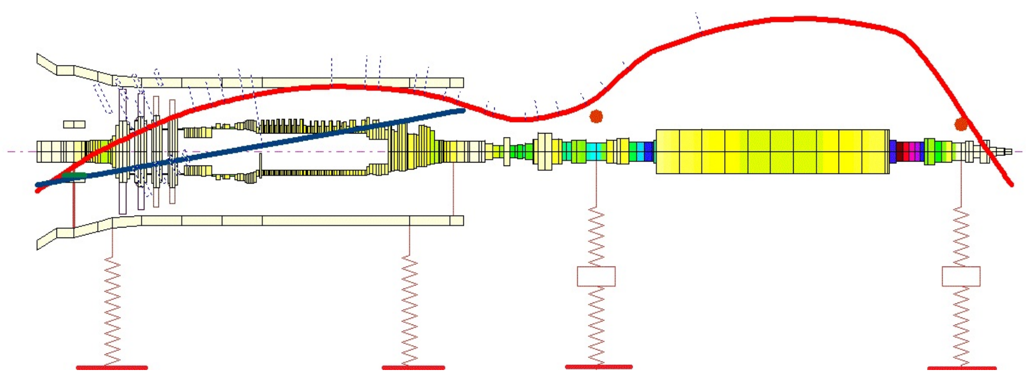


Fig. 14 Generator 1st Mode, M3 = 1171 RPM

Figure 14 for mode three is basically a generator mode at 1171 RPM. Note the close proximity of the frequencies between modes three and two. These two modes would be virtually impossible to compute by means of the transfer matrix method because of their close proximity. Closely spaced modes do not present a problem for the finite element method of computation.

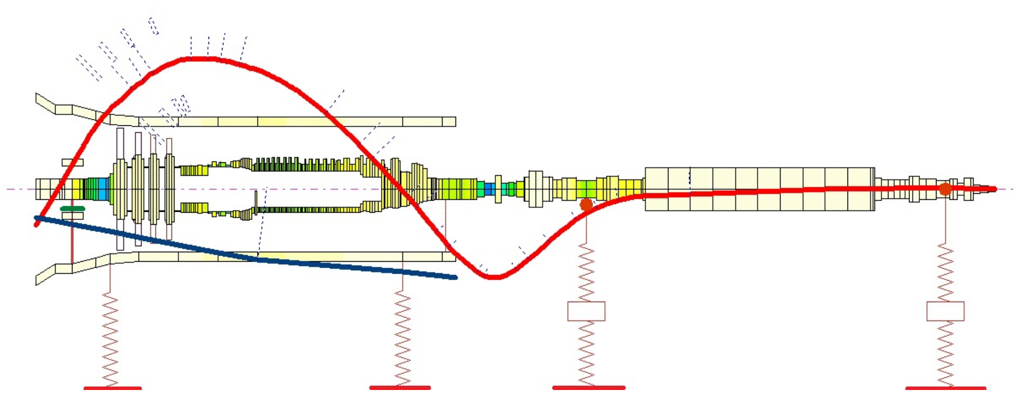


Fig. 15 Turbine-Casing Mode, M4 = 1618 RPM

Figure 15 for mode four at 1618 RPM is a bifurcated mode of the turbine mode shown as shown in Fig. 13, mode two at 1131 RPM. In the case of M4, the casing is moving out of phase with the first turbine mode. Without the influence of the casing, the first turbine mode would be somewhere between 1131 RPM and 1618 RPM.

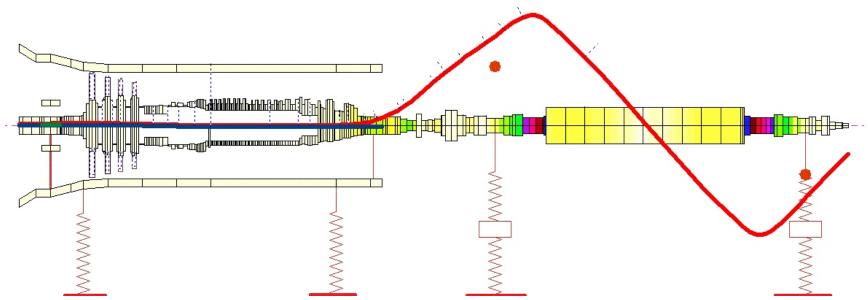


Fig. 16 Generator 2nd Mode, M5 = 2925 RPM

Fig. 16, system mode M5 at 2,925, represents the generator 2nd critical speed. It is little effected by either the turbine or the turbine casing.

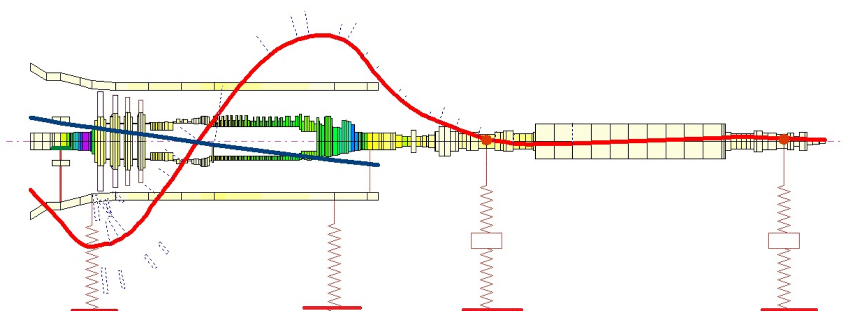


Fig. 17 Turbine-Casing Mode, M6 = 3,164 RPM

Fig. 17 represents the principal mode of concern with respect to the dynamics and the balancing of the gas turbine system. Due to the influence of the casing, the interaction of the turbine and casing causes a strong dynamical mode to fall in the operating speed range. Attempts to the balance the turbine by taking into consideration only the vibration at the operating speed of 3,600 RPM can result in an unbalance correction that will excite the system rotor dynamics response in the 3,200 RPM speed range. This may be to the extent that the turbine will trip off line before reaching operating speed.

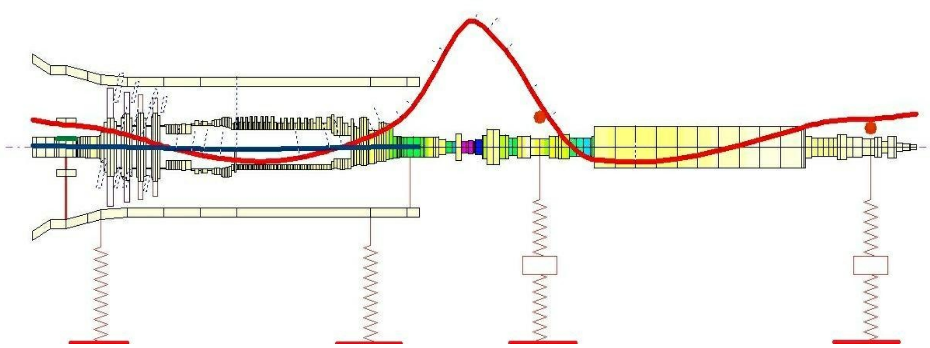


Fig. 18 Coupling Mode, M7 = 4651 RPM

Fig. 18 represents the response of the coupling supported between the turbine and generator bearings.

Gas Turbine - Generator Damped Eigenvalues

The analysis of the system undamped critical speeds is useful for the understanding of the mode shapes and the effective planes for balancing. In the following section, the system damped eigenvalues will be presented. In the evaluation of the damped system natural frequencies, all eight bearing stiffness and damping coefficients are taken into consideration. For example, for the case of the undamped critical speeds, the average of the horizontal and the vertical bearing stiffness coefficients was assumed for each bearing.

In this case, the bearing stiffness and damping characteristics at 3,600 RPM are used for the damped eigenvalue analysis.

The gas turbine is supported by 2-4 pad bearings with load between pads. The generator bearings are supported by 120° partial arc bearings. Fig. 19, as shown below represents the first system eigenvalue M1 at 485 CPM. This low frequency mode is a horizontal oscillation with the gas turbine and casing moving together. It is caused by the soft turbine horizontal casing support.

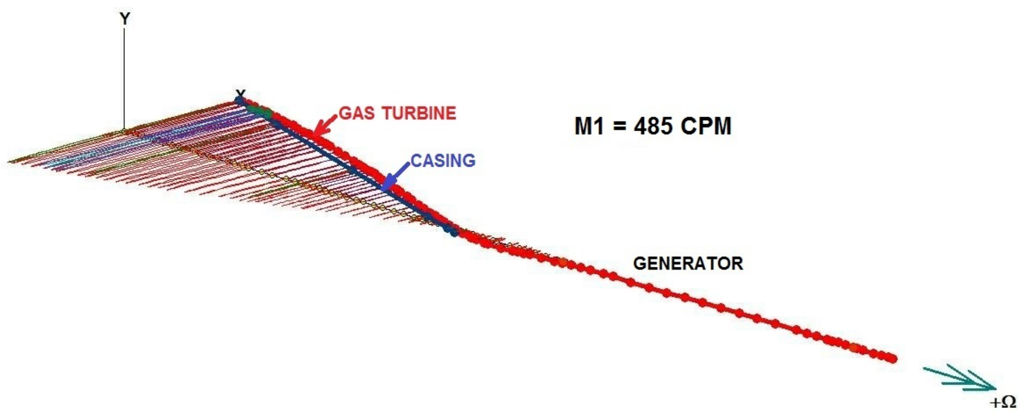


Fig. 19 Horizontal Gas Turbine-Casing Mode, M1= 485 CPM , Log Dec = 0.0038

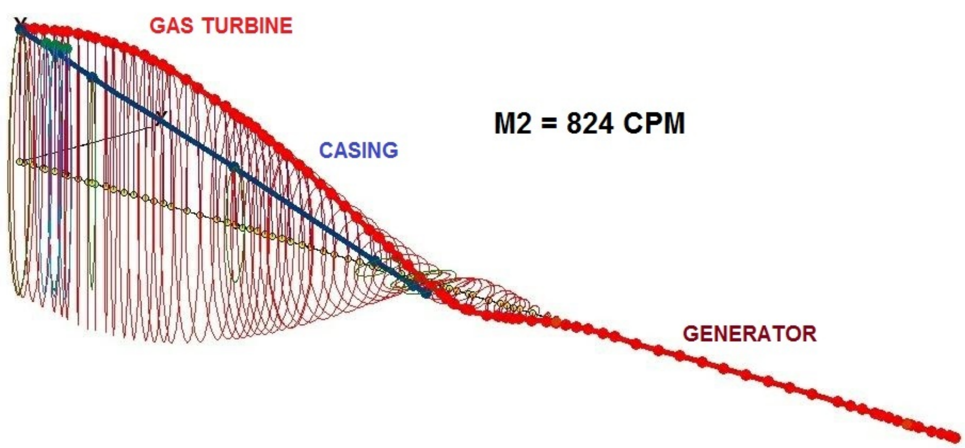


Fig. 20 Vertical Gas Turbine-Casing Mode, M2 = 824 CPM , Log Dec = 0.02

Figure 20 represents the second system mode M2 at 824 CPM. This mode is basically a vibration of the gas turbine and casing moving together in the vertical direction. The undamped critical speed for this mode was computed to be approximately 624 CPM since the stiffness in the vertical and horizontal directions were averaged.

Figure 21 represents the generator first critical speed. The damped eigenvalue analysis shows that the generator in 120° arc bearings is unstable. The system mode M4 at 3600 RPM is predicted to be at 1091 CPM. It would be highly desirable to have the generator bearings redesigned for tilting pad bearings. This would provide additional horizontal support and added system modal damping.

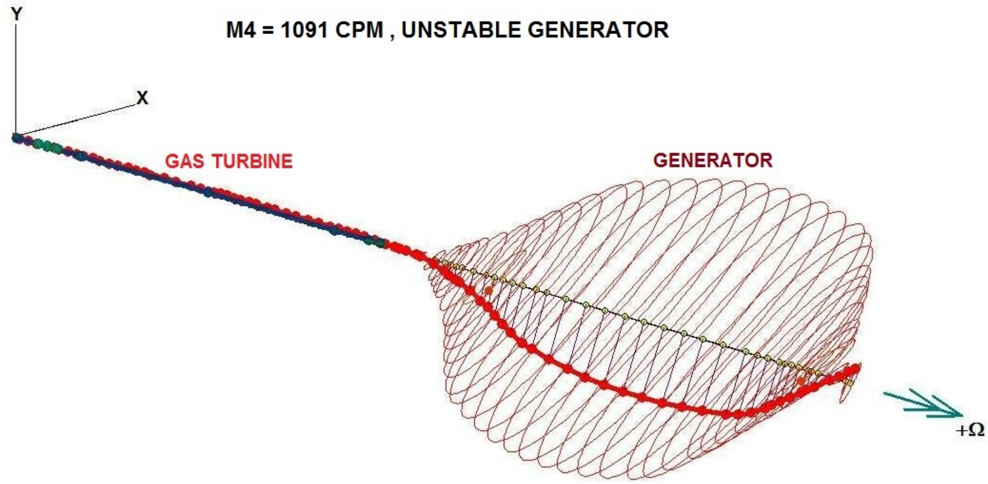


Fig. 21 Unstable Generator Mode M4 = 1091 CPM , Log Dec = -0.073

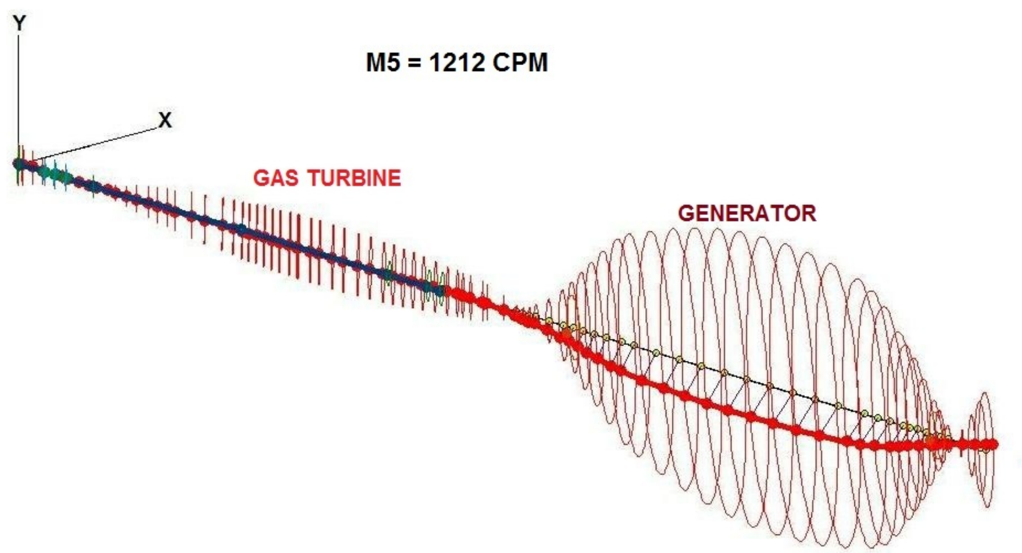


Fig. 22 Generator Vertical 1st Critical Speed M5 = 1212 CPM, Log Dec = 0.132

Figure 22 represents the generator vertical first critical speed. Because of the 120° partial arc bearings, the resonance frequency in the horizontal and vertical directions are different. The mode in the horizontal direction is unstable, however the resonance frequency in the vertical direction is stable because of the higher vertical bearing stiffness and damping values. Well-designed tilting pad bearings would be of benefit for the dynamical characteristics of the generator system. Generator manufacturers for years have resisted the call to employ superior bearings for their generators. This is in contrast to the well-designed tilting pad bearings employed on the gas turbine rotor.

Figure 23 represents the turbine vertical first critical speed mode M6 at 1274 CPM. Note that the casing motion is conical with the casing end displacements moving in phase with the turbine bearing motion. There is also some generator activity shown in this mode. The predicted amplification factor for this mode is over 40. Therefore this mode should be discernible in the vibration data.

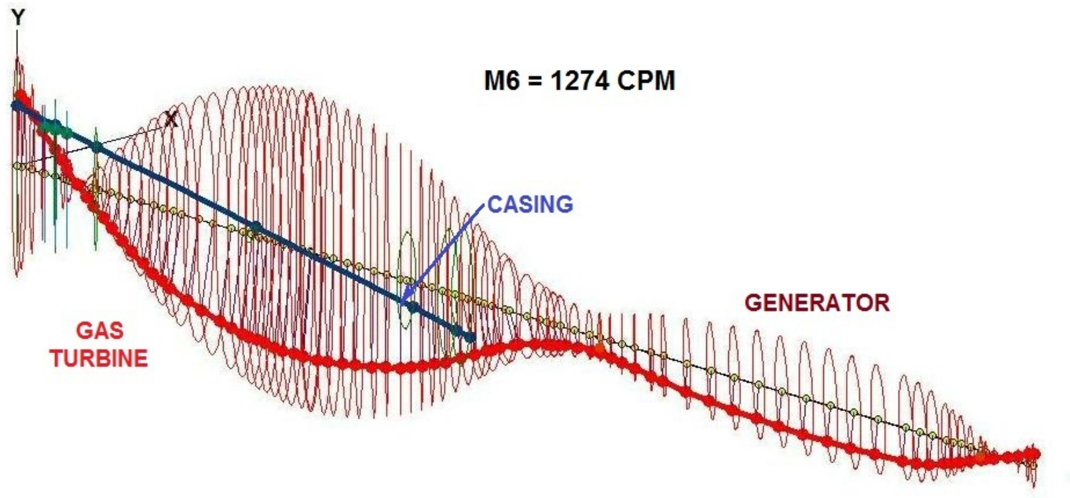


Fig. 23 Vertical Turbine Casing Mode M6 = 1274 CPM, Log Dec = 0.078

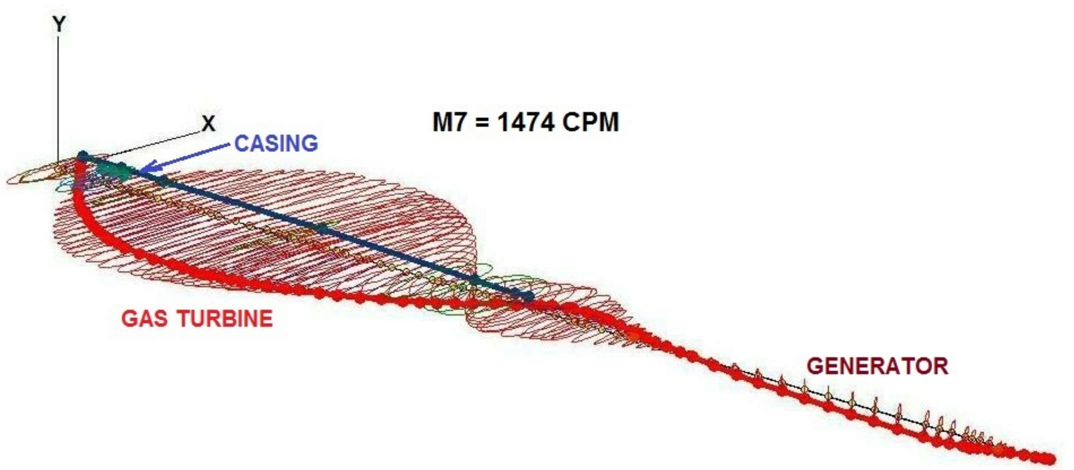


Fig. 24 Gas Turbine-Casing Horizontal Critical Speed, M7=1474 CPM, Log Dec = 0.145

Figure 24 represents the gas turbine horizontal critical speed mode M7 at 1474 CPM. In this mode the casing is moving out of phase to the turbine There is also some generator motion associated with this particular mode. The amplification factor for the horizontal mode is half the value as predicted for the vertical mode.

Figure 25 represents the turbine second critical speed mode M8 at 1798 CPM. In this mode, the casing displacement is following the gas turbine bearing motion. The log dec for this mode is 0.063 which is equivalent to an amplification factor of the almost 50.

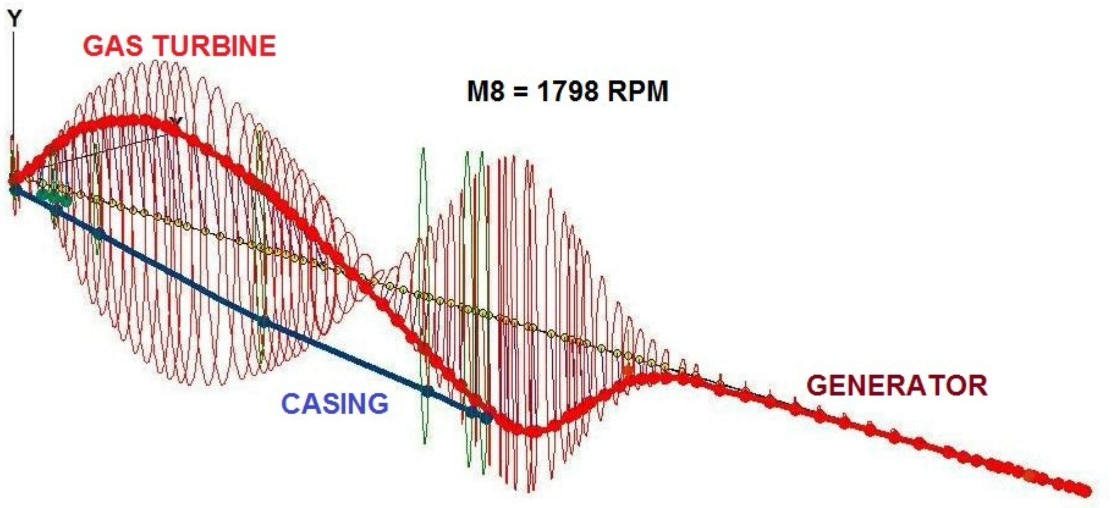


Fig. 25 Gas Turbine Vertical 2nd Critical Speed, M8 = 1798 CPM, Log Dec = 0.063

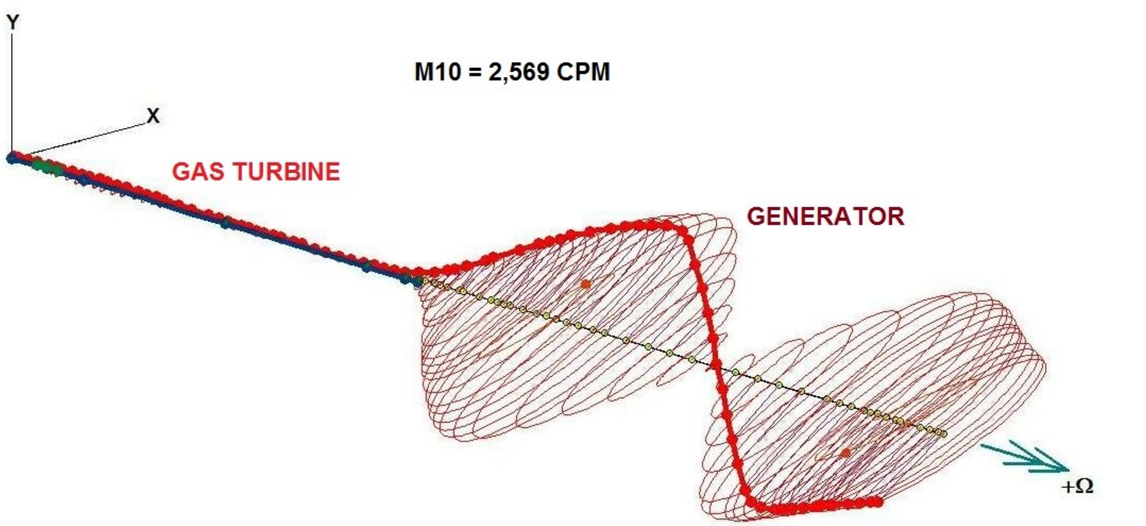


Fig. 26 Generator 2nd Critical Speed, M10 = 2569 CPM, Log Dec = 0.227

Figure 26 represents the generator second critical speed, M10 at 2569 CPM. The log dec of 0.227 is equivalent to a rotor amplification factor of approximately 14. In this mode, there is little gas turbine or casing participation.

Figure 27 represents the generator vertical second critical speed, M 12 equals 3163 CPM. Again this generator second critical speed has very little interaction with the gas turbine rotor and casing.

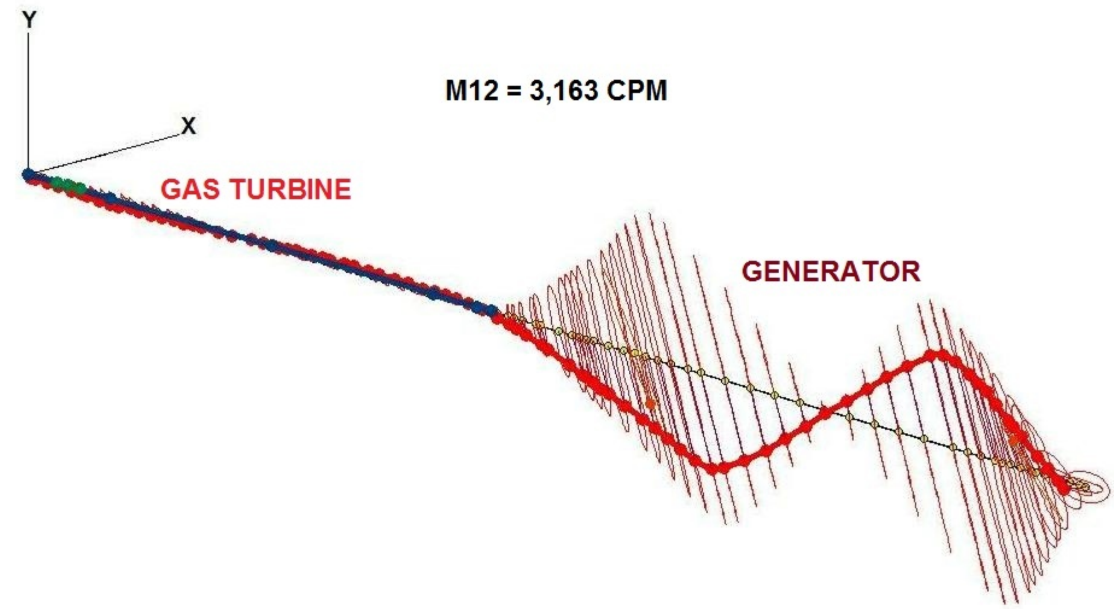


Fig. 27 Generator Vertical 2nd Critical Speed, M12 = 3163 CPM, Log Dec = 0.246

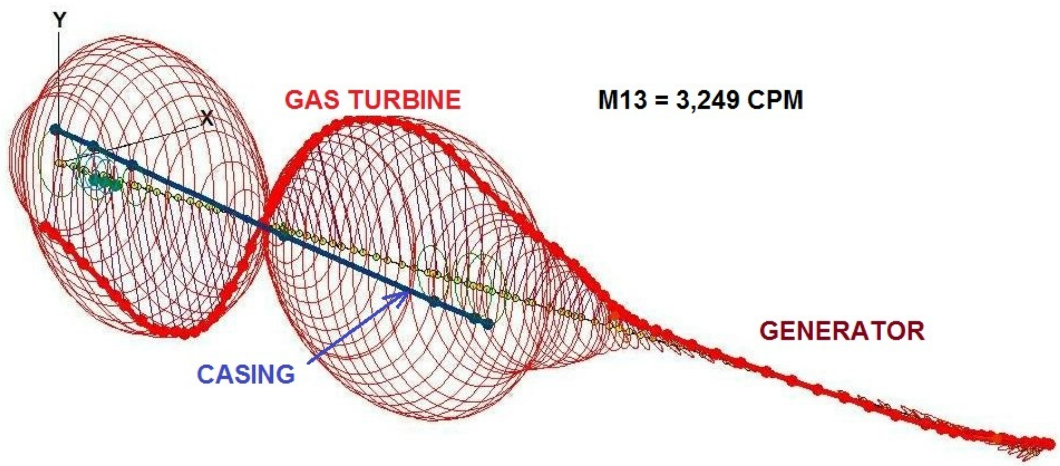


Fig. 28 Gas Turbine Casing Mode M13 = 3249 CPM, log Dec = 0.613

In Figure 28 represent one of the most significant modes of the system. This mode is actually a bifurcated turbine-casing mode and is related to the turbine second critical speed M8 at 1798 CPM. The action of the casing causes a pair of modes to occur due to the interaction of the casing acting in phase in mode M8 and out of phase to the turbine in mode M13. Without the influence of the casing, this mode M13 cannot be computed. Hence one would not know that this mode lies just below the operating speed range.

This mode plays a significant role in the difficulty of balancing the W501 series of gas turbines. For example, if balancing is applied only to vibration data at the 3600 RPM speed range, then this mode will be excited to the point that the turbine may not be able to reach operating speed due to the excitation of this mode at the lower speed.

Experimental W501D Vibration Data

Figure 30 as shown below represents the W501D with the turbine and compressor end probe locations identified. These are the locations at which the relative and absolute motion of the gas turbine may be identified at these two planes. Also identified in this figure are three planes that are available for field balancing. These are at the turbine exhaust, the center plane and the compressor balance planes. For the situation with the balancing of the earlier 70 MW gas turbines of CP&L, the center plane was not available due to the interference of the piping.

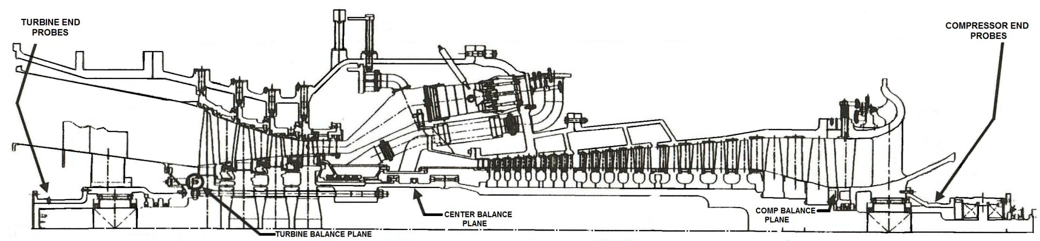


Fig. 29 **Gas Turbine Showing Field Balancing Planes and Probe Locations**

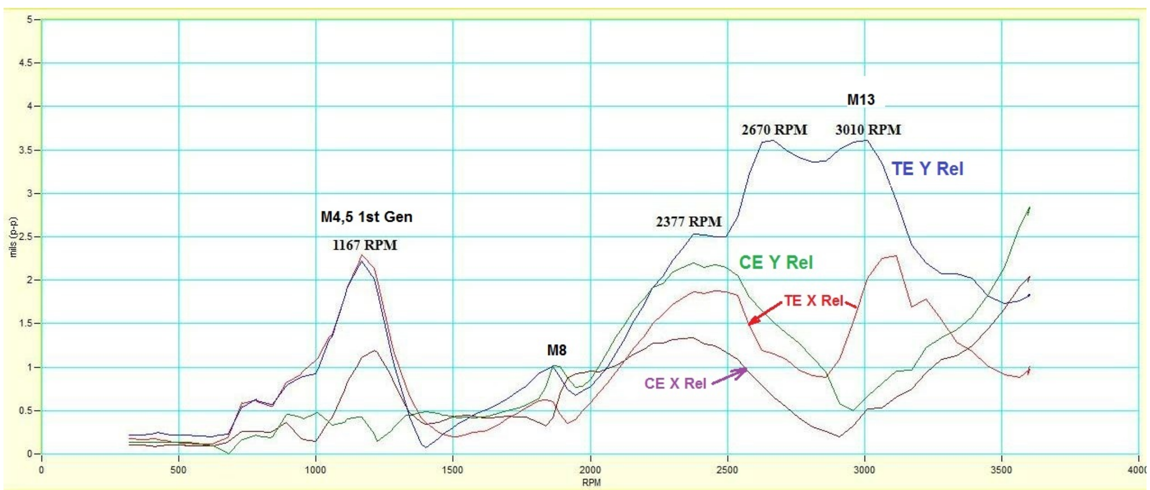


Fig. 30 Relative Probe Measurements at Compressor and Turbine Locations

Figure 30 above represents the relative vibrations measured at the compressor inlet location and at also at the turbine exhaust position. The compressor locations are referred to as CE-X and CE-Y. The two turbine locations are labeled TE-X and TE-Y. Figure 30, shows a substantial vibration at 1167 RPM. This vibration is labeled as M4 and M5 as it represents the generator first critical speed.

The most substantial relative vibrations are observed at TE Y at 2670 RPM and at 3010 RPM. The large response at 3010 RPM is believed to be the mode M13 which is the interaction of the gas turbine second mode with the rotor casing. Note that around 2800-2900 RPM, the compressor motion is at a minimum. As the speed increases above 3000 RPM, it is seen that the compressor relative motion increases while the gas turbine exhaust end reduces.

Thus in the balancing attempts of the 70 MW gas turbines, if single plane balancing was applied at the hot end of the turbine, then excessive vibrations at the compressor end would not allow the unit to come up to speed.

Attempts to apply a single plane balancing correction to the compressor end would then result in excessive vibrations at the turbine exhaust end.

Figure 31 represents the relative and absolute motion at the turbine probe location TE–X.
When the casing motion is included to determine the turbine end absolute motion at plane TE-X, it is seen that there is a very substantial vibration of six mils at 3272 CPM. This again is an example of mode M13 in which there is an interaction between the exhaust casing and the gas turbine second critical speed mode. As previously mentioned, this mode had not originally been predicted for the earlier 70 MW units or for the current improved 100 MW units.

There is a significant phase change between the vibrations around 3200 RPM and at the operating speed of 3600 RPM. It is therefore readily apparent that if a single plane balancing were attempted at speed on the turbine hot end, then mode M13 around 3200 could be significantly excited. The two plots as shown in Figures 30 and 31 then lead to some significant conclusions as to the procedure for field balancing the gas turbines.

It is apparent that single plane balancing at either the gas turbine exhaust or the compressor inlet end will be insufficient because of the occurrence of the strong rotor-casing mode that is encountered just below running speed. Therefore it is necessary to perform a two plane balancing in which vibration measurements are taken at running speed at both the compressor and turbine locations and also at the lower speed range around 3200 RPM. Balancing can then be accomplished using a lease squared error computer program.

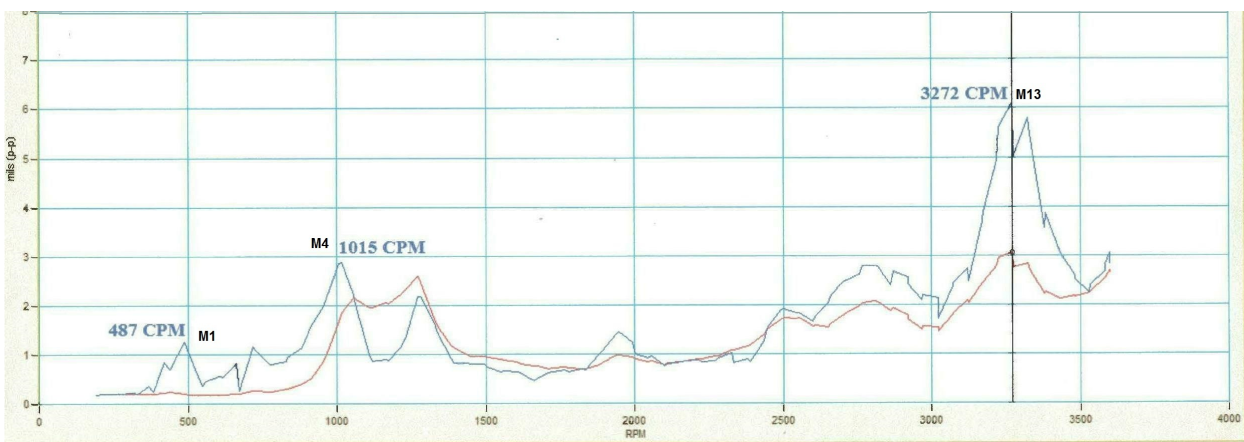
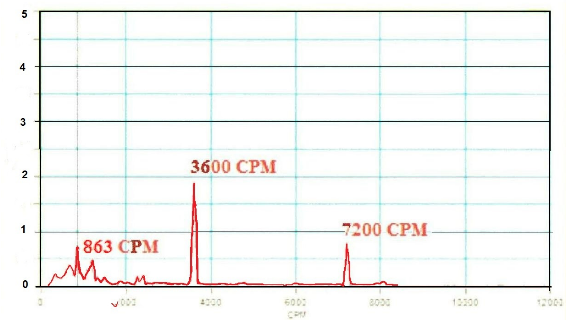


Fig. 31 TE-X Shaft Relative and Absolute Motion on Run Up

There is a small vibration observed around 487 CPM. It is shown that this is the rotor-casing mode M1. In this mode , the turbine and casing are moving in essentially a horizontal plane pivoting about the compressor bearing.

Figure 32 represents the frequency spectrum at the turbine exhaust location. The spectrum on the left represents the relative probe motion and the spectrum on the right is the absolute probe motion which includes the vibration of the casing.



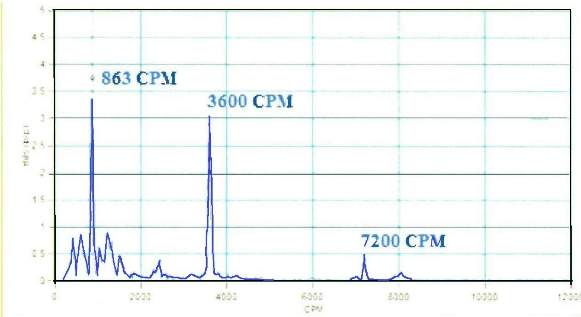


Fig. 32 TE-X Frequency Spectrum At 3600 RPM Showing Subsynchronous Whirling
A. Relative Motion Spectrum B. Absolute Motion Spectrum

Of particular significance in the vibration spectrum is the occurrence of the sub-synchronous component observed at 863 CPM. There is also a two times excitation as seen at 7200 CPM. This is quite typical for 3600 CPM turbine-generators. However, the occurrence of the 863 CPM resonance is quite unusual and was not observed earlier in the spectrum data of the 70 MW turbines.

At first, it was considered that this whirl frequency was the excitation of the generator first critical speed which was predicted to be unstable at a whirl frequency 1091 CPM. The observed frequency of 863 CPM is much lower than this and could not correspond to the generator first critical speed. What is different with the current W501D design as compared to the earlier 70 MW turbines is the increase in the horsepower.

In a paper written by Joel Alford of GE in 1965, all gas turbines can generate self excited rotor dynamic cross coupling forces leading to rotor instability. To simulate cross coupling, additional bearings were added in which Kxy equal to -Kxy of 50,000 Lb/In was applied to several turbine stations. The damped eigenvalue analysis now predicts the turbine to be unstable in the vertical mode M2 with a whirl frequency of 826 CPM.

In this mode, the turbine and casing are moving together in essentially a vertical plane as shown in Fig. 33. It would be of interest, therefore, to review the sub synchronous whirl component on the turbine at various power levels.
Figure 33 shows the damped eigenvalue of the system with aerodynamic cross coupling applied.
The stability analysis shows that the turbine is unstable with the applied level of aerodynamic cross coupling.

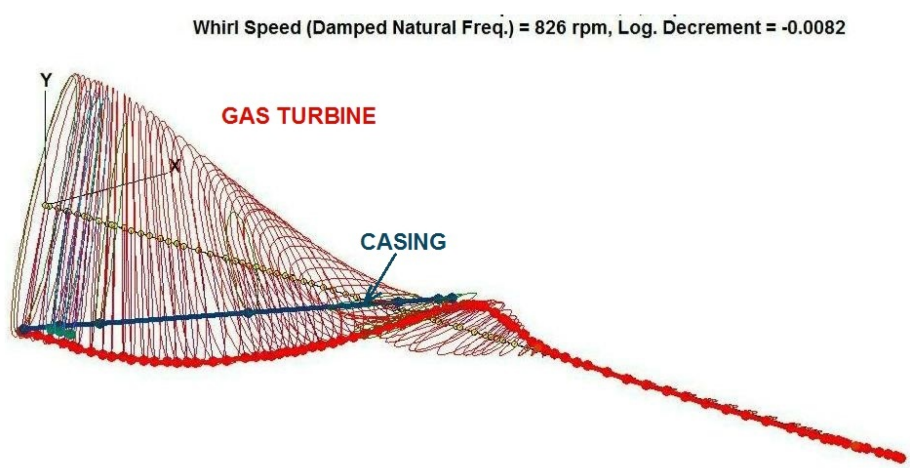


Fig. 33 Unstable Turbine Mode With Aerodynamics Cross Coupling Showing Whirling At 826 CPM, N = 3,600 RPM

Review of Field Balancing a CP&L 70 MW Gas Turbine-Generator

In 1982, the Rotor Dynamics Laboratory at the University Of Virginia was contacted by the research division of CP&L to investigate the balancing problems encountered with one of their W501 gas turbine-generator units. When reviewing the balancing data, we were informed that the young field engineer from Westinghouse had spent three months attempting to balance one of their problem gas turbines.

In his attempts to balance the CP&L unit, when high vibrations were encountered at the turbine exhaust, correction weights were applied at the turbine balance plane. This would result in inducing high vibrations at the compressor end. When balance correction weights were applied at the compressor location to correct excessive compressor vibrations, excessive vibrations would be induced at the turbine end often causing the turbine to trip off line when coming up to speed.

Before attempting to balance the W501 gas turbine, a critical speed analysis was performed using the current rotor dynamics codes based on the transfer matrix procedures of Lund. A preliminary critical speed analysis is extremely important if one is to successfully balance a large turbine-generator which operates through multiple critical speeds. It is important to have an understanding of the system mode shapes and also as to which planes will be the most effective for balancing. It is also important to understand how many balance planes will be required to properly balance a particular rotor-bearing system operating through multiple critical speeds.

An important paper was presented a number of years ago by Kellenberger in which he discussed the N or N +2 method of balancing. He stated that if a rotor operates through N critical speeds then a minimum of N balance planes must be supplied. If one also wishes to minimize the bearing forces transmitted, then an additional two planes is required. As can be seen from the experimental data and past experience, a complex system such as the W501 gas turbine can not be success successfully balanced at speed by the use of a single balance plane.

Attempts to balance the gas turbine-generator combination at speed with a single balance plane can lead to exciting various modes at lower speeds. This can lead to the unit tripping off line when coming up to speed due to excessive vibrations. There were some cases experienced in which the monitors were turned off in order to allow the unit to reach the operating speed. This, in general, is not a recommended procedure to use!

Detailed vibration measurements were recorded on the problem W501 unit. As a contribution to the Rotor Dynamics Research Program at the University of Virginia, the Bently Nevada Corporation supplied instrumentation and the assistance of Bill Pryor for data acquisition. The actual data used for the balancing calculations were obtained with the assistance of Ed Shepherd of Mechtel Analysis using their portable Turbo Balancer (www.Mechtell.com/turbobalancer). A thermal printout for each run would be obtained at 50 RPM increments recording the rotor speed, amplitude, and phase for the W501 unit at 4 vertical locations along the turbine and generator.

In 1984, a 46 page paper entitled '***Dynamic Analysis and Field Balancing of 70 MW Gas Turbine Generators'*** by E .J. Gunter and R. R. And Humphris was presented at the Vibration Institute. For more specifics of the dynamic analysis and the various balancing cases presented, the full paper may be obtained on download from the website www.RODYN.com/papers Several of the original balancing procedures for the 70 MW unit will be presented and reviewed.

The polar plot of Figure 6 shows the result of the large correction weight of 39.7 ounces placed at the compressor location. At that time, the rotor dynamics computer program based on the transfer matrix theory was not able to compute the apparent gas turbine rotor and casing mode observed around 3000 RPM. This mode appears to be closely related to the mode M13 as shown in Figure 28.

Table 5.1 Compressor End Balance Calculation of 70 MW Gas Turbine Using Turbine End T_e Data only at 3,600 RPM

RUN #7,-20 OZ @359 Deg LEAD AT COMPRESSOR END LOCATION ON GAS TURBINE
SINGLE PLANE BAL; 2 OF 4 BAL WEIGHTS(38.9 OZ) REMOVED AT Ce PLANE

NO. OF PROBES = 1 ; NO. OF SPEEDS = 1 ; NO. OF BALANCE PLANES = 1

THE SPEEDS FOR BALANCING DATA:

SPEED NO.	SPEED (RPM)
1	3600

INITIAL READINGS OF AMP & PHASE:

SPEED NO.	SPEED (RPM)	PROBE NO.	PROBE LOC	AMPLITUDE (MILS)	PHASE (DEG LEAD)
1	3600	1	Te	6.50	113.0

TRIAL UNBALANCES:

PLANE	MAGNITUDE	PHASE ANGLE (DEG LEAD)	BALANCE LOC	BALANCE (UNITS)	TRIAL WEIGHT AFTER RUN
1	20.00	359.0	COMP.	OZ	WEIGHT LEFT IN

READINGS AFTER A TRIAL WEIGHT HAS BEEN ADDED:

BALANCE PLANE	SPEED NO.	SPEED (RPM)	PROBE NO.	AMPLITUDE (MILS)	PHASE (DEG LEAD)
1	1	3600	1	3.10	99.0

THE BALANCE CORRECTION WEIGHTS:

PLANE	Ux (OZ)	Uy	MAGNITUDE ABSOLUTE VALUE	ANGLE (DEG) LEAD-CCW	(LAG-CW)	MAGNITUDE TRIM (+TRIAL)	ANGLE (DEG) LEAD	(LAG)
1	35.45	-8.26	36.40	346.9(	13.1)	17.36	332.9(	27.1)

SUMMARY TABLE OF THE SYSTEM INFLUENCE COEFFICIENTS

UNITS OF INFLUENCE COEF:MILS/OZ

BAL #	UBAL MAG	SPD #	SPEED (RPM)	PROBE #	AMP1 (MILS)	PHASE1 (LEAD)	AMP2 (MILS)	PHASE2 (LEAD)	INF. COEF (MAG)	(ANGLE) LEAD	(REL LAG)
1	20.00	1	3600	1	6.50	113	3.10	99	.1786	306	53

INITIAL READINGS AND PREDICTED COMP.RESPONSE

SPEED (RPM)	PROBE NO.	PROBE LOC	INITIAL AMP MILS	INITIAL PHASE LEAD	RESIDUAL AMP MILS	RESIDUAL PHASE LEAD	(LAG)
3600	1	Te	6.50	113	0.000	297	63

Table 5.1 was taken from the balancing paper presented in 1984. This table represents the compressor end balance calculations for the 70 MW gas turbine using only the turbine end data Te at 3600 RPM. In this balancing run, 20 ounce inches was removed from the compressor end balance location. After the weight was removed, the new amplitude at the gas turbine and has been reduced from 6.5 mils to 3.1 mils. Note that the influence coefficient for the response of the turbine at 3600 RPM for a balance weight at the compressor end is approximately 0.18 mils/oz.

The balancing calculation computes that the absolute magnitude of unbalance is 36.4 ounces @ 332°. Therefore, using only the turbine end data, it is seen that we would end up with approximately the same unbalance as was used in the original balancing case. From this calculation it becomes apparently clear that the young Westinghouse engineer was computing balancing based on turbine end vibration only at the operating speed of 3600 RPM.

Table 5.2 Compressor End Balance Calculation Using Compressor C_e and Turbine T_e Data at Various Speeds - U_c = 20 oz @ 359° - Initial Vibration Data

TRIAL WEIGHT LEFT IN PLACE AFTER RUN

NO. OF PROBES = 2 ; NO. OF SPEEDS = 6 ; NO. OF BALANCE PLANES = 1

THE SPEEDS FOR BALANCING DATA:

SPEED NO.	SPEED (RPM)
1	1066 TURBINE 1ST CRIT
2	1702 GENERATOR 1ST CRIT
3	2300 TURBINE 2ND CRIT
4	2756 TURBINE-GENERATOR CRIT
5	3600 RUNNING SPEED: COLD OMW
6	3600 RUNNING SPEED: HOT 70MW

Predicted Gas Turbine Response at Various Speeds
Correction Based on T_e Data Only at 3,600 RPM

ORIGINAL WEIGHT REMAIN IN PLANE(S),TRIM WEIGHT(S) ADDED

ORIGINAL TRIAL WEIGHT IN PLACE,TRIM WEIGHT ADDED

TRIAL UNBALANCES:

PLANE (OZ)	MAGNITUDE	PHASE ANGLE (DEG LEAD)	BALANCE LOC	BALANCE (UNITS)
1	17.36	333.0	COMP END	OZ

INITIAL READINGS AND PREDICTED COMP.RESPONSE

SPEED (RPM)	PROBE NO.	PROBE LOC	INITIAL AMP MILS	INITIAL PHASE LEAD	RESIDUAL AMP MILS	RESIDUAL PHASE LEAD	(LAG)
1066	1	Ce	2.60	313	3.304	296	64
1066	2	Te	5.90	7	7.708	339	21
1702	1	Ce	3.90	232	4.885	230	130
1702	2	Te	4.40	4	2.389	12	348
2300	1	Ce	4.20	160	4.961	167	193
2300	2	Te	2.80	340	2.391	357	3
2756	1	Ce	3.50	120	3.627	142	218
2756	2	Te	5.40	325	7.012	333	27
3600	1	Ce	4.10	73	3.079	81	279
3600	2	Te	3.70	97	1.116	74	286
3600	1	Ce	1.50	141	2.300	168	192
3600	2	Te	3.10	99	.702	79	281

Table 5.2 shows the five speed values that were recorded. The cases five and six represent the gas turbine at zero and at the full power level of 70 MW at 3600 RPM. Table 5.2 shows the initial and predicted response using balancing data based only on the turbine vibration at running speed. The turbine vibration is reduced from 3.12 mils to 0.702 mils at the full power level.

Of interest it is seen that using only turbine vibration data at full power for the balancing calculations results in a predicted turbine amplitude at 1066 RPM of 7.7 mils. This is a significant increase from the original value of 5.90 mils. This vibration level would be enough to trip the machine off line. The turbine data at 2756 RPM, which is a coupled turbine-casing 2nd critical speed, would increase from 5.40 to over 7 mils.

There were a number of cases in the balancing runs by the Westinghouse engineer in which the monitors were disconnected in order to allow the machine to come up to speed.

Table 5.3 represents the compressor end balance calculations and influence coefficients using all of the compressor data Ce and the turbine data Te at the 6 speed conditions. In this case the absolute magnitude of the unbalance is 22.6 ounces as shown in Fig. 35. If the original trial weight is left in place,then only a trim balance of 5 ounces in hole 34 is required as compared to the excessive predicted trim weight of 22 oz to be placed in balance hole 37 in the compressor balance plane.

Of particular interest is the examination of the influence coefficients for example the placement of the balance weight at the compressor causes a larger response at the turbine and at speeds of 106 six and also at running speed of 3600 RPM.

The more balancing data that is applied to the lease squared error balancing program, the greater is the reduction in the computed value of balance correction. The reason for this is that all of the additional readings act as constraints upon the amount of balance weight that should be placed at the balance location.

Thus we see at the initial and predicted response that the turbine response at zero power is 6.5 mils and is reduced to three mils at full power the initial response was 5.7 and with the reduced value of balancing this value is reduced to 2.5 mils is important to note that it is not always possible to reduce the vibration at all speeds going through more than two modes with only two planes of unbalance. For example we see that the turbine amplitude at 106 six RPM has increased from 4.62 5.5 mils at the second turbine casing mode of 2756 RPM the amplitude turbine amplitude has increased from the original value of 4.7 mils to 5.94 mil. However this value is not enough to trip the machine off line

Table 5.3 Compressor End Balance Calculations and Influence Coefficients Using Compressor C_e and Turbine T_e Data at Various Speeds

RUN #7 SHOT AT Ce: 20 OZ @ 359 DEG ON COMPRESSOR BALANCE PLANE
TRIAL WEIGHT LEFT IN PLACE AFTER RUN

THE BALANCE CORRECTION WEIGHTS:

PLANE	Ux (OZ)	Uy	MAGNITUDE ABSOLUTE VALUE	ANGLE (DEG) LEAD-CCW (LAG-CW)	MAGNITUDE TRIM (+TRIAL)	ANGLE (DEG) LEAD (LAG)
1	22.09	-4.97	22.64	347.3(12.7)	5.08	294.4(65.6)

SUMMARY TABLE OF THE SYSTEM INFLUENCE COEFFICIENTS

UNITS OF INFLUENCE COEF:MILS/OZ

BAL #	UBAL MAG	SPD #	SPEED (RPM)	PROBE #	AMP1 (MILS)	PHASE1 (LEAD)	AMP2 (MILS)	PHASE2 (LEAD)	INF. COEF (MAG)	(ANGLE) LEAD	(REL LAG)
1	20.00	1	1066	1	1.70	339	2.60	313	.0653	279	80
1	20.00	1	1066	2	4.60	54	5.90	7	.2177	317	42
1	20.00	2	1702	1	2.80	226	3.90	232	.0577	248	111
1	20.00	2	1702	2	6.70	10	4.40	4	.1185	202	157
1	20.00	3	2300	1	3.90	145	4.20	160	.0549	228	131
1	20.00	3	2300	2	3.70	333	2.80	340	.0491	134	225
1	20.00	4	2756	1	4.50	103	3.50	120	.0771	242	117
1	20.00	4	2756	2	4.70	302	5.40	325	.1064	26	333
1	20.00	5	3600	1	5.40	74	4.10	73	.0651	258	101
1	20.00	5	3600	2	6.50	113	3.70	97	.1558	313	46
1	20.00	6	3600	1	1.98	98	1.50	141	.0676	230	129
1	20.00	6	3600	2	5.70	114	3.10	99	.1411	312	47

INITIAL READINGS AND PREDICTED COMP.RESPONSE

SPEED (RPM)	PROBE NO.	PROBE LOC	INITIAL AMP MILS	INITIAL PHASE LEAD	RESIDUAL AMP MILS	RESIDUAL PHASE LEAD	(LAG)
1066	1	Ce	1.70	339	2.567	306	54
1066	2	Te	4.60	54	5.520	357	3
1702	1	Ce	2.80	226	4.095	229	131
1702	2	Te	6.70	10	4.018	10	350
2300	1	Ce	3.90	145	4.479	160	200
2300	2	Te	3.70	333	2.819	345	15
2756	1	Ce	4.50	103	3.729	125	235
2756	2	Te	4.70	302	5.938	325	35
3600	1	Ce	5.40	74	3.948	77	283
3600	2	Te	6.50	113	3.037	104	256
3600	1	Ce	1.98	98	1.820	145	215
3600	2	Te	5.70	114	2.530	108	252

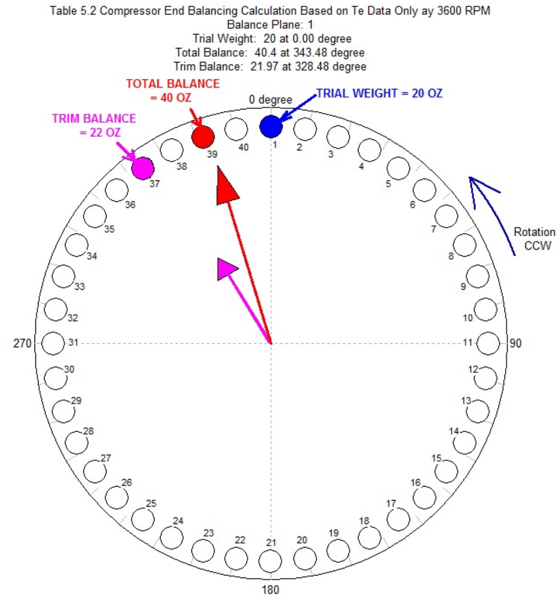


Fig. 34 Ce Balance Based on Turbine Data Only at 3600 RPM

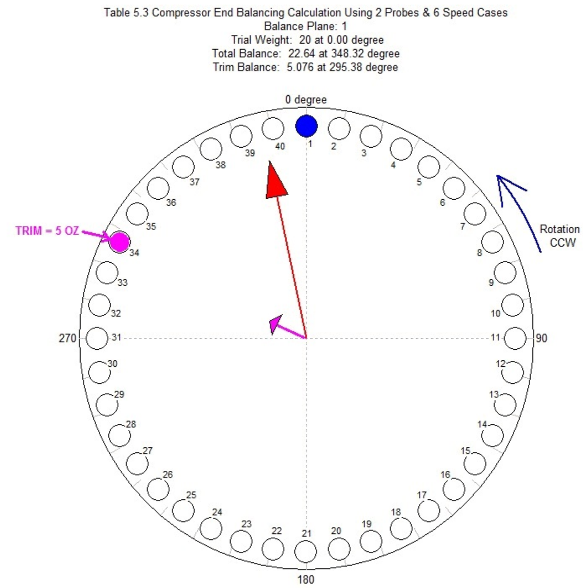


Fig. 35 Ce Balance Based on Compressor & Turbine Data at 6 Speeds

Figure 34 represents the balancing calculations based on only the Te data at 3600 RPM. In this case it is seen that a trim balance of 22 ounces would be applied at hole 37. In Table 5.3b below it is seen that at full power the amplitude of 5.7 mils would be reduced to zero and the amplitude at zero power of 6.5 mils would be reduced to 0.36 mils. However if this large correction weight were applied then it is seen that at the turbine first critical speed the amplitude would increase from 4.6 mils to 8.09 mils. This would trip the gas turbine off-line. With this large correction weight, the gas turbine could pass through the first critical speed.

Figure 35 shows the balance correction that would be added to the compressor balance plane if all of the data of the compressor and turbine were taken at the six speed cases. In this case the balance correction weight is reduced from 22 ounces to be placed at hole 37 to only 5 ounces placed at balance hole #34. As a result of this correction, the first critical speed amplitude increases only slightly to 5.5 mils. The amplitude at the second turbine critical speed is slightly increased to 5.9 mils which is acceptable for this initial single initial balance correction.

The important consideration is that at running speed at the zero power level, the amplitude is reduced to 3 mils and to 2.52 mils at full power. These values are acceptable compared to the original values of 6.5 and 5.7 mils observed at zero and full power.

Table 5.3 b

Speeds *(RPM)*	**Initial)** *(Mils)*	**Balance Te Only** *(Mils)*	**Balance With Ce & Te Data** *(MILS)*
1 1066	4.60	**8.09**	5.52
2 1702	**6.70**	1.99	4.02
3 2300	3.70	2.42	2.82
4 2756	4.70	**7.54**	5.94
5 3600 ***0 PW***	**6.50**	0.36	3.04
6 3600 ***Full PW***	5.70	0.0	2.52

Table 5.5 Turbine End Balance Calculation and Predicted Response Based on T_e and C_e Data at Various Speeds

BALANCE DATA WITH 4 SPEEDS WITH Ce & Te MEASUREMENTS,10 OZ @ 270 DEG
SINGLE PLANE CORRECTION,3600 ,2850 ,1054,1107 RPM- 4 SPEED SETS

THE BALANCE CORRECTION WEIGHTS:

PLANE	Ux (OZ)	Uy	MAGNITUDE ABSOLUTE VALUE	ANGLE (DEG) LEAD-CCW (LAG-CW)	MAGNITUDE TRIM (+TRIAL)	ANGLE (DEG) LEAD (LAG)
1	7.72	-23.37	24.61	288.3(71.7)	15.44	300.0(60.0)

SUMMARY TABLE OF THE SYSTEM INFLUENCE COEFFICIENTS

UNITS OF INFLUENCE COEF:MILS/OZ

BAL #	UBAL MAG	SPD #	SPEED (RPM)	PROBE #	AMP1 (MILS)	PHASE1 (LEAD)	AMP2 (MILS)	PHASE2 (LEAD)	INF. COEF (MAG)	(ANGLE) LEAD	(REL LAG)
1	10.00	1	3600	1	4.10	73	2.60	101	.2178	309	321
1	10.00	1	3600	2	3.70	97	3.40	95	.0325	28	242
1	10.00	2	2850	1	3.40	114	1.40	91	.2181	39	231
1	10.00	2	2850	2	5.50	320	4.30	327	.1339	207	63
1	10.00	3	1054	1	2.60	313	1.80	319	.0831	210	60
1	10.00	3	1054	2	5.90	7	4.30	15	.1748	257	13
1	10.00	4	1107	1	2.50	284	1.80	292	.0760	175	95
1	10.00	4	1107	2	4.50	351	3.40	4	.1412	228	42

INITIAL READINGS AND PREDICTED COMP.RESPONSE

SPEED (RPM)	PROBE NO.	PROBE LOC	INITIAL AMP MILS	INITIAL PHASE LEAD	RESIDUAL AMP MILS	RESIDUAL PHASE LEAD	(LAG)
3600	1	CE#4BRG	4.10	73	1.803	199	161
3600	2	Te#5BRG	3.70	97	3.128	88	272
2850	1	CE#4BRG	3.40	114	3.113	3	357
2850	2	Te#5BRG	5.50	320	2.233	327	33
1054	1	CE#4BRG	2.60	313	.592	295	65
1054	2	Te#5BRG	5.90	7	1.607	12	348
1107	1	CE#4BRG	2.50	284	.631	287	73
1107	2	Te#5BRG	4.50	351	1.431	28	332

Table 5.7 Compressor and Turbine Two Plane Balancing Calculation Without Removing Trial Weights - Balance Correction Weights and Predicted Turbine Response

RUN #7 SHOT AT Ce: 20 OZ @ 359 DEG; RUN #8 SHOT AT Te: 10 OZ @ 270 DEG
TRIAL WEIGHTS LEFT IN PLACE AFTER RUN

THE BALANCE CORRECTION WEIGHTS:

PLANE	Ux (OZ)	Uy	MAGNITUDE ABSOLUTE VALUE	ANGLE (DEG) LEAD-CCW	(LAG-CW)	MAGNITUDE TRIM (+TRIAL)	ANGLE (DEG) LEAD	(LAG)
1	26.24	-5.71	26.85	347.7(	12.3)	8.23	319.3(	40.7)
2	-1.00	-20.62	20.65	267.2(	92.8)	10.67	264.6(	95.4)

INITIAL READINGS AND PREDICTED COMP.RESPONSE

SPEED (RPM)	PROBE NO.	PROBE LOC	INITIAL AMP MILS	INITIAL PHASE LEAD	RESIDUAL AMP MILS	RESIDUAL PHASE LEAD	(LAG)
1066	1	Ce	1.70	339	1.093	310	50
1066	2	Te	4.60	54	2.594	358	2
1702	1	Ce	2.80	226	2.875	257	103
1702	2	Te	6.70	10	1.803	41	319
2300	1	Ce	3.90	145	4.850	66	294
2300	2	Te	3.70	333	.734	317	43
2756	1	Ce	4.50	103	1.610	154	206
2756	2	Te	4.70	302	4.331	341	19
3600	1	Ce O POWER	5.40	74	.542	343	17
3600	2	Te	6.50	113	2.112	114	246
3600	1	Ce FULL PW	1.98	98	1.637	207	153
3600	2	Te	5.70	114	2.086	118	242

Table 5.5 represents the turbine and balancing in which 10 oz was placed at 270°. The data at both the turbine and compressor locations with the six speed cases was used to compute a final 2 plane balance. Thus we have achieved an amplitude at the compressor location from 4.1 mils to 1.8 mils. The amplitude at the turbine is only reduced slightly from 3.7 to 3.1 mils. However there is a significant reduction in the amplitude at the first critical speed from 5.9 mils to 1.6 mils.

In table 5.7, the compressor and turbine data has been combined to perform a two plane balancing calculation without removing of either of the 2 trial weights. As can be seen from the above chart, this has proven to be highly successful with the peak turbine vibrations of 6.5 mils and 5.7 mils respectively been reduced to slightly over 2 mils. Of considerable importance is that the 6.7 mill turbine vibrations observed at 1702 RPM has now been reduced to 1.8 mils.
It is very apparent that by combining the influence coefficients from the two planes with six different speed cases, that a successful balance may be achieved on the gas turbine.

Review of a Balancing Case History For a 501D 100 MW Gas Turbine

In this section, the field balancing data generated for a 100 MW 501D gas turbine will reviewed. Balancing calculations based on this field data will be presented for various assumed probe vibration measurements and single and 2 plane balancing solutions will be presented. The balancing predictions will be generated using the Weighted Least Squared Error Balancing program of Dyrobes. The least squared error balancing procedure is well known and was first reported on by Goodman of GE in 1965. The first case to be considered is that of a single plane balancing shot applied to the gas turbine exhaust end.

The instrumentation for the current gas turbines consists eight probes placed along the gas turbine and generator at 45° from the vertical axis. There are 4 probes placed at the turbine inlet and exhaust sections and 4 more probes placed on the generator. The turbine probes measure the rotor relative shaft to casing motion. Casing pickups in series with the turbine probes may be used to determine the turbine absolute motion. Absolute motion is obtained by integrating the casing motion with the relative prox motion. For the earlier balancing cases with the 70 MW 501 gas turbines, superior balancing was achieved by using absolute probe measurements.

Review of Turbine Exhaust Balance Calculations

Table 1 represents the input table for a least squared error calculation for a trial weight placed at the turbine exhaust. In this example, a trial weight of 18.9 ounces was placed in hole number 13. This balance shot represents two normal balancing steel weights of approximately 9.45 ounces each. The balancing radius for the turbine end is 18 inches.

In Table 1 as shown below are the initial vibration readings before application of the balancing weight and the resulting vibrations after the balancing weight was applied. Note that the highest amplitudes before balancing occur at the turbine and compressor Ty and Cy probes of over four mils. It is seen that the trial weight has reduced the four mil responses to under three mils along the turbine. The table as shown below is for a least squared error balancing calculation. It is possible with the least squared error program to add weighting functions to the various probe readings. In the first case a weighting function of one is applied to only the relative turbine Ty vibration. This is equivalent to a single plane - single speed balancing calculation. By placing 0 for the weighting factors for the other probes, there amplitudes do not come into consideration for the balancing calculation.

Number of Balancing Planes: 1 Number of Speeds/Cases: 1
Number of Measured Probes: 8 Runout Compensation: No
Shaft Rotation: CCW (•) CW
Phase: (•) Lag Lead
0 degree at: (•) Y - Up X - Right
Balancing Holes: No. of Holes: 40 1st Hole Angle: 0
Numbering Direction: (•) CCW CW
Comment: Turbine Balance Using Turbine Ty Measurement Only
Comment: 18.9 oz @108.4 deg at Turbine Exhaust
Comment: 3600 RPM Full Load

	Condition	Speed	Description	Amplitude	Phase (deg)
1	Initial Readings	1	Ty	4.18	262
2	Initial Readings	1	Tx	3.9	15
3	Initial Readings	1	Cy	4.07	42
4	Initial Readings	1	Cx	3.15	146
5	Initial Readings	1	Gty	0.98	288
6	Initial Readings	1	Gtx	0.24	43
7	Initial Readings	1	Gey	1.84	128
8	Initial Readings	1	Gex	1.46	217
9	Trial Run < 1 >	---	Left-In Afterward	18.9	108.4
10	Response	1	Ty	2.46	262
11	Response	1	Tx	2.95	36
12	Response	1	Cy	2.7	78
13	Response	1	Cx	3.01	151
14	Response	1	Gty	0.63	183
15	Response	1	Gtx	0.11	275
16	Response	1	Gey	2.49	138
17	Response	1	Gex	1.32	132
18					
19					
20					

Weighting (Scale) Factors for probes and speeds

Probe	Factor
1	1
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

Speed	Factor
1	1
2	
3	
4	
5	
6	
7	
8	
9	
10	

New Open Save Save As Run Close

Table 1 Input table For Single Plane Turbine Exhaust Balancing With 8 Probe Inputs

Figure 36 shows the trial weight of 18.9 ounces placed at 108.4° which corresponds to balancing hole # 13. It is seen that a trim balance of 27 ounces is computed corresponding to the location of the placement of the trial weight at hole 13. It is therefore apparent that the original balancing calculation was based simply on the measurement of the vibration of 4.18 mils at 262°.

It will be shown later that if this computed trim balance weight of 27 ounces is applied, then the response at Ty of 4.18 mils will be reduced to zero amplitude. However it will be shown that this is an excessive amount of unbalance and would result in increasing the vibration amplitudes at the other points along the turbine and generator.

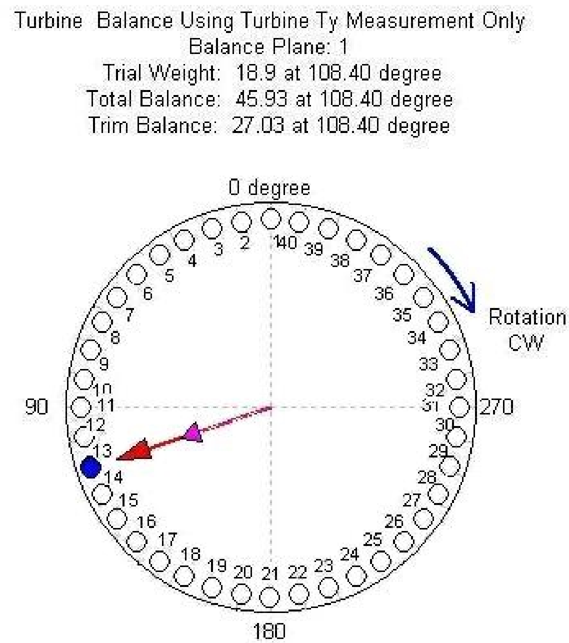


Fig. 36 Turbine Exhaust Balancing Based On Compressor Inlet Cy Measurement Only

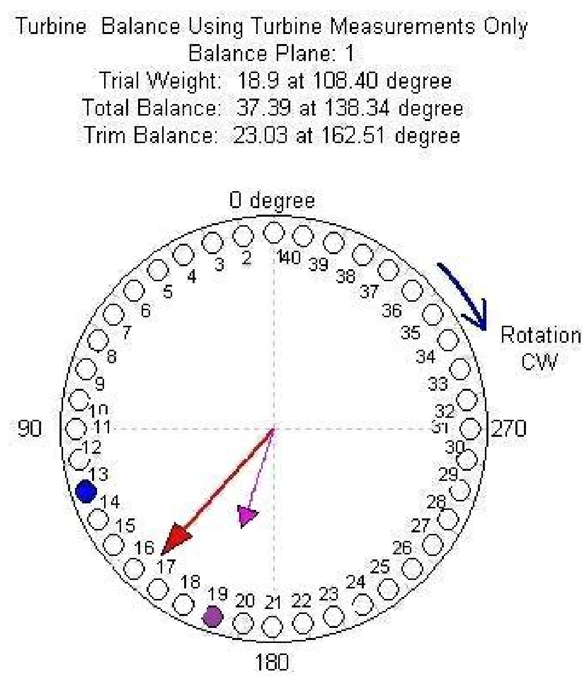


Fig. 37 Turbine Exhaust Balancing Based On Turbine and Compressor Inlet Measurements

Figure 37 represents the balancing calculation computed if the four turbine measurements are taken into consideration. This is done by assuming a weighting function of 1 for the 4 turbine probes which are labeled 1 to 4. By leaving the weighting factors for the generator probes at zero, they are not included in the calculations. By including the generator probes along for the ride, it will be seen later how much their amplitudes would change with the additional probe measurements taken into consideration.

By including the additional probe readings, each additional reading acts as a constraint on the total amount of balancing that will be computed. Thus the total unbalance is reduced from 45.9 ounces in the first case with only a single probe measurement to 37.4 ounces when the 4 turbine probes are included in the calculations. The trim balance, based on 4 turbine measurements at speed, has been now reduced to 23 ounces to be placed at balance location 19.

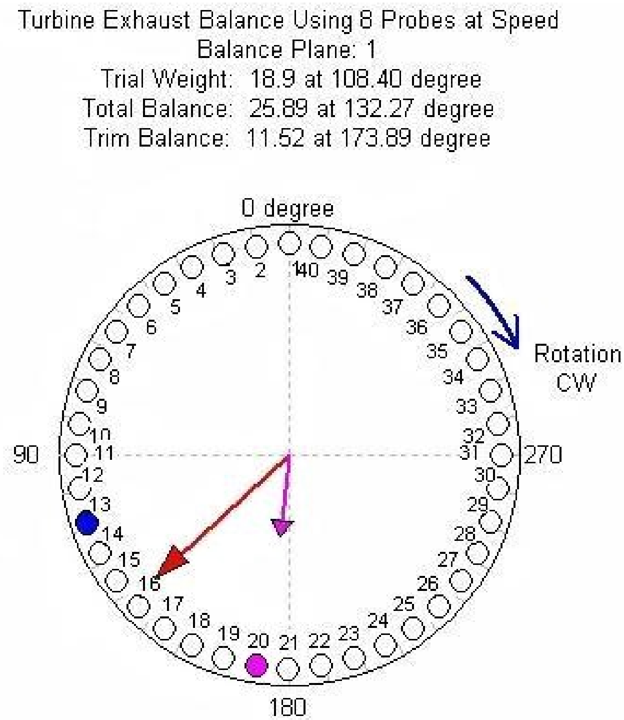


Fig. 38 Turbine Exhaust Balancing Based On 8 Turbine & Generator Measurements

Figure 38 represents the balancing calculation when all eight probes of the turbine and generator are taken into consideration. Again with the four additional generator measurements including in the balancing calculation, it is seen that the total balancing weight is reduced further to approximately 26 ounces and the correction trim weight is reduced to 11.5 ounces to be placed at balance hole number 20.

If we were to compare the difference in balancing based on a single measurement as compared to all eight probe readings, then it is seen that there is a substantial difference in the requested trim weight as shown between Figures 36 and 38. Using only a single plane measurement, a trim balance of 27 ounces is requested for hole 13 as compared to the greatly reduced value of 11.5 oz for hole 20.

Thus, when all eight turbine and generator measurements are taken into consideration, the trim weight is substantially reduced to less then one-half of the original value with the trim weight location shifted from balance hole 13 to balance hole 20. This represents a significant difference in the balancing strategy. This type of balancing calculation requires a computer simulation and can not be accomplished in the field by simple single plane vector plots.

Table 2 represents a summary table of the predicted final turbine and generator amplitudes of motion with the three various methods of balancing. In this table, the four turbine and four generator initial vibration values ,taken at speed under load, are shown along with the predicted results if the various computed correction weight were applied.

In the first balancing case in which only the Cy measurement was used, we see that if the full trim balance of 27 ounces is applied at hole 13, then the response at Cy would be reduced to zero.

The application of a single plane balancing to reduce a single vibration reading to zero is a highly undesirable procedure. Since the unit is operating above multiple critical speeds, the attempt to highly balance a single location will always result in higher amplitudes along the rest of the rotor. These increased amplitudes can be of such a significant magnitude that it may prevent the rotor from coming up to speed.

Of particular importance is that a high level of balancing applied at the turbine end will result in increased amplitudes of vibration at the other end of the machine on the generator. Note that the generator turbine end and exciter have increased to over 3.5 mils. In previous maintenance situations, the generator bearings have had to be re-Babbitted due to bearing over temperature. It is very easy to see that this damage may have been caused by excessive loadings placed on the generator bearings due to high exhaust turbine balance corrections.

In the second case, with four turbine measurements, we see that Tx is reduced to 2.65 mils and Cy is two mils, which is acceptable. The generator exciter motion Gex is unacceptable as its amplitude has increased to 3.6 mils.

When all eight turbine and generator measurements are taken into consideration, as explained before, this acts as a constraint and the maximum amount of requested trim unbalance is further reduced to 11 ounces. The turbine amplitudes are all acceptable as they are under three mils the generator amplitudes are now in the acceptable range even though the amplitudes have increased, they are still within 1.5 to 2.5 mils which is an acceptable range.

Table 2 Predicted Turbine-Generator Response Calculations

	(Trim Bal)	**27oz @ 108^0**	**23 oz @ 163^0**	**11 oz @ 174^0**
Probe	**Initial, Mils**	**Cy only**	**Turbine-4**	**Turbine & Gen**
Tx	3.15	2.89	2.65	2.80
Ty	4.01	3.89	1.13	1.20
Cx	3.90	2.80	1.12	2.00
Cy	4.18	**0.00**	2.10	2.34
Gtx	0.24	0.55	0.50	0.30
Gty	0.98	**2.32**	**2.80**	1.40
Gex	1.46	**3.67**	**3.60**	2.45
Gey	1.85	**3.49**	**2.8**	2.53

Review of Turbine Compressor Inlet Balancing

In this field balancing case, after a balancing correction weight had been applied to the turbine exhaust end, a trial weight was added to the compressor inlet section of the turbine. Two balancing weights of 9.5 ounces were added to balancing holes 21 and 22. This represents a total balancing trial weight of 19 ounces at 185° as shown in Figure 39.

Table 3 as shown below represents a section of the input table used to compute the balancing at compressor balance plane. This balancing plane has 40 holes at a radius of 15 inches. The balancing calculations were based on 12 vibration measurements taken at three speeds. These speeds corresponded to system resonances observed at 3070 RPM and 3490 RPM. A third speed case was taken at 3600 RPM at full power level.

The 12 measurements consisted of the four relative probe measurements taken on the turbine exhaust and inlet sections and the four absolute vibration measurements which incorporate casing response of the turbine. Four additional measurements from the generator were also included in this particular balancing calculation.

Number of Balancing Planes: 1 Number of Speeds/Cases: 3
Number of Measured Probes: 12 Runout Compensation: No
Shaft Rotation: CCW / CW (selected: CW)
Phase: Lag / Lead (selected: Lag)
0 degree at: Y - Up / X - Right (selected: Y - Up)
Balancing Holes — No. of Holes: 40 1st Hole Angle: 0 Numbering Direction: CCW / CW (selected: CCW)

Comment: Compressor Balance With 19 oz @ 185 Deg, 12 Measurements, 3 Speeds
Comment: Turbine relative & Abs Motion, generator motion
Comment: Speeds at 3070, 3490, & 3600 Full load

	Condition	Speed	Description	Amplitude	Phase (deg)
1	Initial Readings	3070	Trx	8.1	54
2	Initial Readings	3070	Try	2	276
3	Initial Readings	3070	Tx	1.6	265
4	Initial Readings	3070	Ty	2.1	159
5	Initial Readings	3070	Crx	1.1	143
6	Initial Readings	3070	Cry	3.7	268
7	Initial Readings	3070	Cx	1.7	287
8	Initial Readings	3070	Cy	0.8	156
9	Initial Readings	3070	Gtx	1.5	250
10	Initial Readings	3070	Gty	2.9	161
11	Initial Readings	3070	Gex	0.6	176
12	Initial Readings	3070	Gey	1.9	101
13	Initial Readings	3490	Probe: 1	5.7	326
14	Initial Readings	3490	Probe: 2	1.7	238
15	Initial Readings	3490	Probe: 3	2.9	23
16	Initial Readings	3490	Probe: 4	1.4	252
17	Initial Readings	3490	Probe: 5	2.5	70
18	Initial Readings	3490	Probe: 6	1.9	216
19	Initial Readings	3490	Probe: 7	2.9	23
20	Initial Readings	3490	Probe: 8	1.9	216

Weighting (Scale) Factors for probes and speeds

Probe	Factor
1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	1
11	1
12	1
13	
14	
15	
16	
17	
18	
19	
20	

Speed	Factor
1	1
2	1
3	1
4	
5	
6	
7	
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10	

New Open Save Save As Run Close

Table 3 Section of Input Table For Turbine Inlet Balancing Calculations
12 Turbine and Generator Vibration Measurements at 3 Speeds

In the least squared error calculation method of balancing, weighting functions may be added to any particular probe or speed data set. This allows one to examine the situation when vibration data is ignored in the balancing calculations. For example if one wishes to calculate a simple single plane balance based on only the value of the relative turbine exhaust x probe motion of Trx of 8.1 mils, then the weighting factor for probes 2 to 12 would be set to zero.

After a balancing calculation is performed, the results should be reviewed as there are cases in which one may wish to modify particular vibration values. These values may either be amplified or reduced by the use of the weighting factors. Later balancing studies showed that the generator vibration values had little influence on the compressor balance predictions. Hence the weighting factor F is set to zero for the 4 generator vibration readings.

Figure 39 represents the computed balancing to be applied to the compressor inlet plane based on 12 vibration measurements at three different speeds. The initial trial balancing weights of two 9.5 ounce weights were inserted in balancing holes 21 and 22. From the calculations generated from the balancing program, the total required unbalance is only 11 ounces at 132° which corresponds closely to hole 16. If the initial trial weight were left in place, then a large trim balance of 15 ounces would be required near hole 6. This trial weight was later removed.

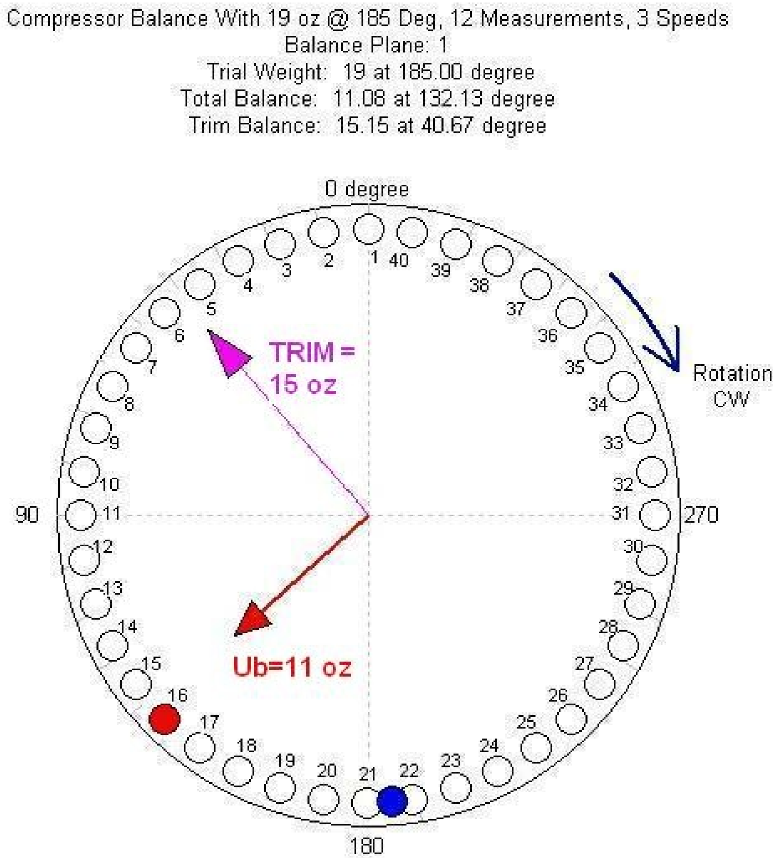
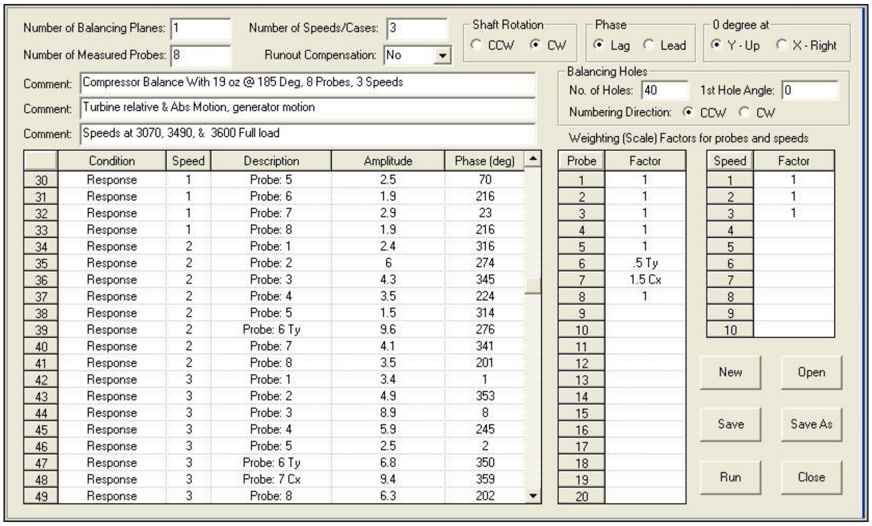


Fig 39 Initial Trial Weight, Trim and Total Computed Compressor End Balancing

It was seen that in reviewing the response of a trial weight placed at the compressor balance plane, that there was very little influence to the generator motion. Therefore when the weighting functions for the four generator probes was placed at zero, the balancing calculations were basically unchanged.

It was also determined that a fairly reasonable balance could be computed by using only the four absolute turbine exhaust and compressor inlet measurements. For example, using only the four turbine absolute measurements, the total balance was calculated to be 11.5 ounces at 124°. This compares favorably with the predicted value of 11 ounces at 132° as shown in Figure 39 using all 12 relative and absolute turbine and generator measurements.

As was reported earlier with the balancing of the 70 MW 501 gas turbines, the balancing was best accomplished by using the absolute turbine values in which the relative probe displacements and casing motions were combined.



Number of Balancing Planes: 1
Number of Speeds/Cases: 3
Shaft Rotation: CCW CW
Phase: Lag Lead
0 degree at: Y - Up X - Right
Number of Measured Probes: 8
Runout Compensation: No
Comment: Compressor Balance With 19 oz @ 185 Deg, 8 Probes, 3 Speeds
Comment: Turbine relative & Abs Motion, generator motion
Comment: Speeds at 3070, 3490, & 3600 Full load
Balancing Holes
No. of Holes: 40
1st Hole Angle: 0
Numbering Direction: CCW CW
Weighting (Scale) Factors for probes and speeds

	Condition	Speed	Description	Amplitude	Phase (deg)
30	Response	1	Probe: 5	2.5	70
31	Response	1	Probe: 6	1.9	216
32	Response	1	Probe: 7	2.9	23
33	Response	1	Probe: 8	1.9	216
34	Response	2	Probe: 1	2.4	316
35	Response	2	Probe: 2	6	274
36	Response	2	Probe: 3	4.3	345
37	Response	2	Probe: 4	3.5	224
38	Response	2	Probe: 5	1.5	314
39	Response	2	Probe: 6 Ty	9.6	276
40	Response	2	Probe: 7	4.1	341
41	Response	2	Probe: 8	3.5	201
42	Response	3	Probe: 1	3.4	1
43	Response	3	Probe: 2	4.9	353
44	Response	3	Probe: 3	8.9	8
45	Response	3	Probe: 4	5.9	245
46	Response	3	Probe: 5	2.5	2
47	Response	3	Probe: 6 Ty	6.8	350
48	Response	3	Probe: 7 Cx	9.4	359
49	Response	3	Probe: 8	6.3	202

Probe	Factor
1	1
2	1
3	1
4	1
5	1
6	.5 Ty
7	1.5 Cx
8	1
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

Speed	Factor
1	1
2	1
3	1
4	
5	
6	
7	
8	
9	
10	

Table 4 Compressor Balancing Input Table With Ty & Cx Weighting Functions Applied

Table 5 Vibration Measurements Before and After Compressor Plane Balancing Calculations With Weighting Functions Applied To Absolute Turbine Exhaust Measurement Tx and Absolute Compressor Measurement Cx

	3071 RPM			**3450 RPM**			**3600 RPM Full Power**		
(Wf)	***Initial***	***Trial***	**Final**	***Initial***	***Trial***	**Final**	***Initial***	***Trial***	**Final**
Txr	**8.1**	5.7	5.1	1.0	2.4	0.3	4.6	3.4	2.8
Tyr	2.0	1.7	1.6	5.4	6.0	3.2	4.9	4.9	3.9
Cxr	1.6	2.9	2.2	4.0	4.3	4.4	4.4	8.9	5.3
Cyr	2.1	1.4	2.3	2.9	3.5	3.3	4.8	5.9	5.1
Tx	1.1	2.5	0.5	1.5	1.5	0.9	3.7	2.5	2.3
Ty ***(.5)***	3.7	1.9	2.8	**9.1**	**9.6**	5.5	7.5	6.8	5.4
Cx***(1.5)***	1.7	2.9	2.4	3.5	4.1	3.9	3.9	**9.4**	5.1
Cy	0.8	1.9	1.3	2.5	3.5	3.0	4.2	6.3	4.

Table 4 shows the application of weighting functions applied to the turbine absolute measurement Ty and the compressor measurement Cx. The above Table 5 indicates the importance of taking vibration measurements at speeds below the operating speed. For example it is seen that at 3071 RPM, the relative turbine exhaust motion Txr is 8.1 mils. At the higher speed of 3450 RPM, it is seen that the absolute turbine exhaust amplitude Ty is 9.1 mils. This is approaching shutdown values.

It is apparent that the choice of trial balancing weights of 19 oz was excessive as the trial amplitude on Ty has increased from 9.12 to 9.6 mils. In particular, it is also observed that the absolute compressor motion Cx at 3600 RPM has increased from 3.9 to 9.4 mils. When a balancing weight is applied, it is preferable that the chosen value will generally cause a reduction in overall vibration.

In the original calculations by the least squared error program, the large value of Ty encountered at 3450 RPM caused an excessive amount of unbalance to be applied to reduce this amplitude. The value of Cx at full power was also on the high side. To compensate for this, a weighting function of .5 was applied to the Ty, so as not to over balance, and a weighting factor of 1.5 was applied to Cx so that the absolute x amplitude at the compressor location at full power would be under control.

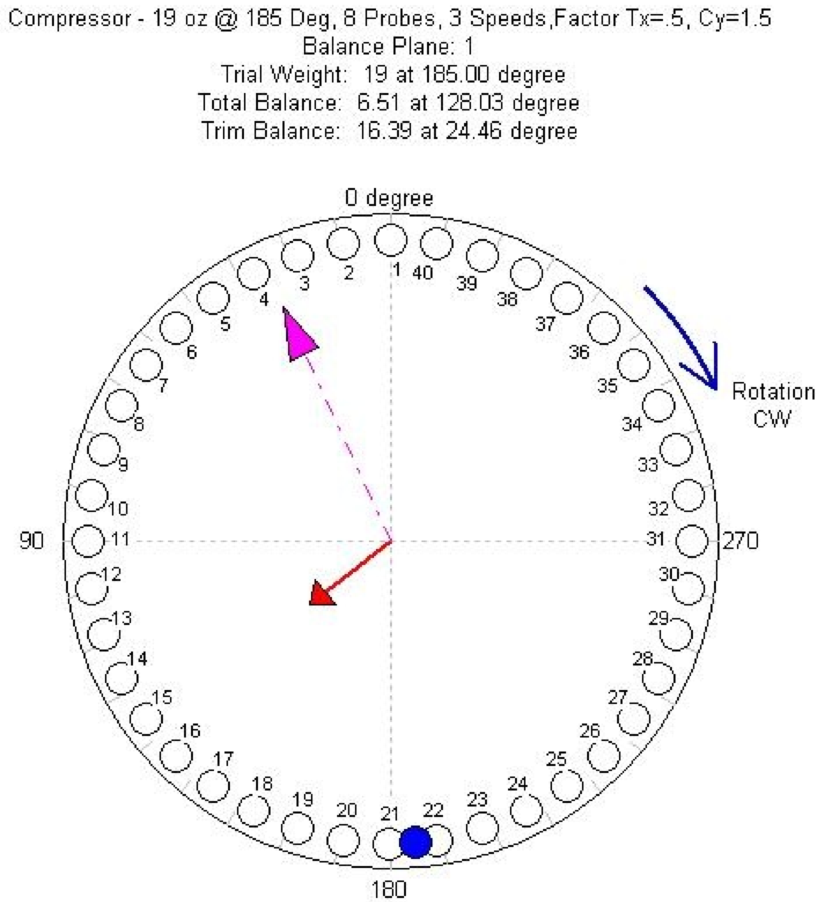


Fig. 40 Compressor Plane Balance Calculation With Tx and Cx Weighting Functions Applied

Figure 40 above shows the recomputed compressor end balancing calculations with the weighting functions applied to minimize over balancing of the turbine absolute exhaust motion Ty at 3450 RPM and to reduce the compressor absolute motion Cx at the full power level.

It is of interest to observe the final compressor balancing has been reduced to 6.5 ounces to be applied at hole 15 as compared to the earlier predicted value of 11 ounces to be applied at balance hole 16.

This represents a considerable shift from the initial trial balance of 19 ounces applied at holes 21 and 22. Such a balancing prediction can only be arrived at by the use of the computer to analyze the turbine vibrations at the lower system turbine-casing critical speed as well as the response at full power level at 3600 RPM. This predicted compressor end balancing value cannot be arrived at by simply examining the rotor polar plots.

W501D 2 Plane Balancing Calculations

Table 6 represents the input and results for a two plane balancing calculation using the initial trial weight of 19.27 oz applied at the compressor, balance plane 1 and also the results of the 7.4 oz trial weight applied at the turbine exhaust, balance plane 2.

By combining the results of these two initial unbalance trials, a combined final balance combination can be computed. These two balance corrections are then added simultaneously. For example, for the compressor inlet plane 1, the final balancing prediction is a correction weight of 16.45 oz at 70.4°. This balance would be achieved by removing the 19 oz weights applied at holes 21 and 23 and shifting the final reduced balancing weight over to hole 16 as shown in Figure 41.

The 2nd turbine exhaust balance plane correction is predicted to be 8.4 oz applied between holes 2 and 3. A trim weight of 7.6 oz could also be added to balance hole 36 as shown in Figure 42.

Table 6 2 Plane Balancing Prediction

Balance Plane 1 - Compressor Inlet , Ut = 19.27 oz@ 185^0 (Holes 21 & 22)
Ub1 = 16.45 oz@ 72^0 (Hole 16)
Balance Plane 2 - Turbine Exhaust , Ut = 7.4 oz @ 72^0 (Hole 9)
Ub2 = 8.4 oz @ 14^0 (Holes 2 & 3)

		3070 RPM			3600 Full Power		
Probe	**BAL Plane**	**Initial**	**Trial**	**Final**	**Initial**	**Trial**	***Final***
Tx	1	1.1	2.5	3.3	3.7	2.5	**1.6**
Ty	1	3.7	1.9	1.5	7.5	6.8	**2.2**
Cx	1	1.7	2.9	2.0	3.9	9.4	**3.7**
Cy	1	0.8	1.9	0.3	4.2	6.3	**2.3**
Tx	2	2.5	2.3	3.3	2.5	1.7	**1.6**
Ty	2	1.9	4.4	1.5	**6.8**	5.6	**2.2**
Cx	2	2.9	2.5	2.0	**9.4**	5.3	3.7
Cy	2	1.9	0.8	0.3	6.3	3.8	2.3

Figure 41 represents a vector plot of the initial trial, trim and final balance weight correction for the compressor plane 1. If the initial trial weight of 19 oz is left in the balance holes 21 and 22, then a fairly substantial trim balance of 16.5 oz would be required at balance hole 9.

It is of interest to note that the final balance correction is only 3 oz lower than the initial trial weight. The final balancing weight is to be applied to hole 16, which is only 60° away from the original guess. Although the original trial weight was high, it was of an acceptable amount and location.

The final balancing correction for the turbine balance plane 2 is shown to be around 8 oz as compared to the initial trial weight of 7.4 oz. Thus the initial turbine trial weight was of the proper magnitude and was off by approximately 60° from the final balance location. With regard to the balancing correction at the hot end of the gas turbine, it is sometimes desirable to leave the original trial weight in as it is often difficult to extract. Under these circumstances then a trim weight of 7.6 oz would be applied to balancing hole 36. This would seem to be a reasonable procedure.

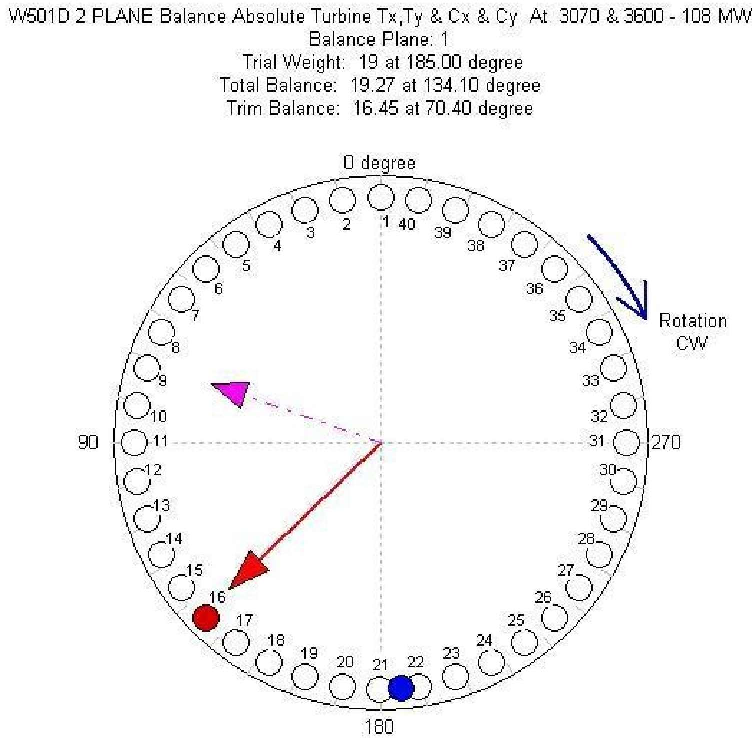


Fig. 41 Plane 1 - Compressor Initial Trial and Final Balance Correction

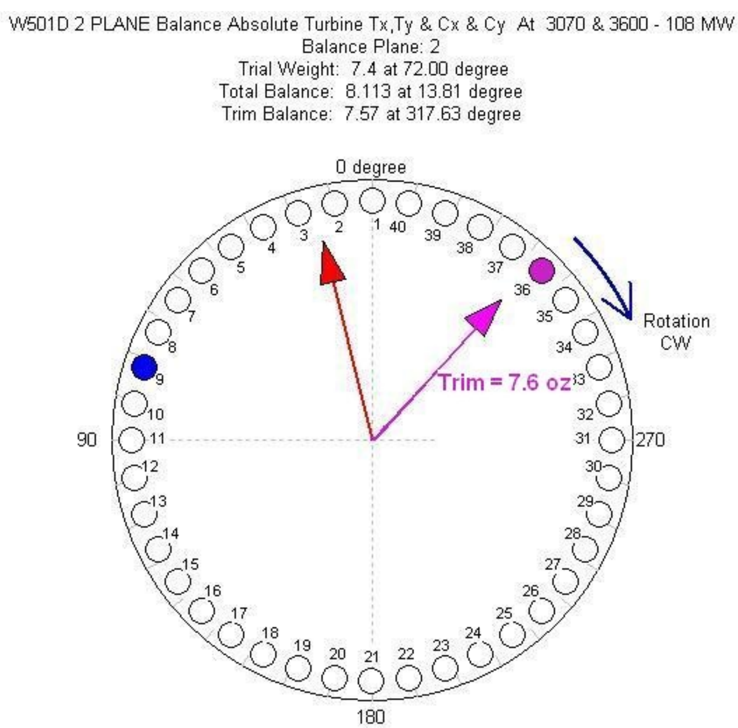


Fig. 42 Plane 2 - Turbine Initial Trial and Final Balance Correction

Discussion

The W501 series of gas turbine - generators represent a very complex system with challenging problems with respect to balancing and dynamic analysis. The situation is made more complicated by the thermal distortion of the gas turbine and generator under various load conditions. Of particular influence is the flexible support under the gas turbine exhaust tilting pad bearing. In a previous paper, it has been shown that the flexible turbine bearing support, not only reduces the effective bearing stiffness, but can also reduces significantly the turbine exhaust tilting pad bearing damping by over 90%.

In addition to the unusual behavior of the gas turbine due to the extremely flexible turbine bearing support, there is considerable turbine casing dynamics involved. The casing dynamics introduces additional modes into the operating speed range. This effect is not usually encountered so dramatically with steam turbines which have much more massive cases.

Other complications that can enter the picture is the fact that the design of the 120 deg partial arc generator bearings are over hundred years old! These partial arc bearings provide very little horizontal support and damping. In addition, the generator is predicted to be unstable at speed, exciting the generator first critical speed at 3600 RPM. This instability is also predicted by the OEM. Although not reported in practice, it is evidence of the poor damping provided by the generator bearings. One must also be careful that a generator rub does not occur as this will distort the influence coefficients and upset the balancing calculations. Hence, one must also monitor the full vibration spectrum as the rub can generate a 3x supersynchronous vibration.

Another problem that was not encountered with the 70 MW W501 gas turbines is the possibility of the occurrence of self excited subsynchronous turbine whirling. With the power level of the system increased from 70 MW to up to 208 MW, the Alford cross coupling forces are increased. There have been observed occurrences of self excited whirl instability in which the turbine 1st vertical critical speed around 860 RPM appears to be excited.

Conclusions

There have been situations with the W501 in which field balancing attempts have met with limited success. It appears that the selection of the initial compressor and turbine exhaust trial weights have been acceptable. These balancing trial weights are usually based on the polar plots of the vibration performance.

With respect to the number of planes required for balancing flexible rotors, Kellenberger had presented a paper on the N or N+2 method of balancing. The minimum requirement is N balance planes to balance N critical speeds. It was stated that N+2 balance planes are required if bearing forces are also to be minimized.

Therefore, it is impossible to have a single balance plane produce desirable results at both running speed and as well as at the additional critical speeds encountered through the entire speed range. If, for example, an aggressive single plane balancing correction is installed at full speed, full power, this balance correction runs the risk of causing excessive excitation of one or more of the lower system critical speeds. The high vibrations encountered may cause the unit to trip out and not allow the unit to reach full operating speed.

It is important to note that after two successful trial balancing weights have been applied to the turbine and compressor locations, then a computer program is required to crunch the numbers to further define a proper balance solution. It is not possible to simply use the polar plots to accurately compute the amount and particular the angular locations of the final balance corrections to be installed in the two balance locations. The system, due to the low turbine end damping and also operation near a rotor-casing mode, is very sensitive to the phase location of the balance weights.

When collecting balancing data, it is essential that the data before and after the placement of the trial weights, be taken under the same conditions of speed and power level. Due to the thermal effects in the gas turbine and also warpage of the generator at higher power levels, influence coefficients change with a change in the power level.

Recommendations

After the first two trial weights at the turbine and compressor location have been successfully installed, it is recommended that a least squared error computer program be used to compute the final balancing calculations. It should be noted that a trial balance at the coupling may also be incorporated with the other two planes to form a three plane balancing solution.

The W501 gas turbines have the unusual characteristics that the cross coupled influence coefficients are of greater magnitude then the direct coupled coefficients. This means that a balance weight applied at one end of the turbine causes a higher response at the other end. This again is due to the extreme flexibility under the turbine exhaust tilting pad bearing.

To further attempt to refine the balance by the use of polar plots only would not be productive.

One of the advantages of the least squared error method of balancing is that the influence coefficients collected from previous balancing cases may be used for future balancing situations. This approach has been reported to me to have been successfully used on several W501 units by Ed Shepard of Mechtell, the inventor of the Turbobalancer. The assistance of Mr. Shepard and the Turbobalancer was vital in the earlier successful balancing reported on the CP&L 70 MW 501 units.

There are several other recommendations concerning the modification of future W501 gas turbine-generator systems.

The generator 120° partial arc bearings should be replaced by 2 100° tilting pad or 2 fixed pad bearings with load between pads. This would add needed system damping and reduce the occurrence of horizontal generator rubs. The current generator bearing design provides very little horizontal support or damping. An increase in the generator horizontal stiffness and damping characteristics could aid in reducing some of the extreme rotor asymmetry characteristics observed.

The flexible support under the exhaust tilting pad bearing is required because of temperature considerations. However it would be of value to consider if this support could be stiffened. By stiffening up the turbine bearing support, the tilting pad effective damping will be greatly improved. This may also have the added effect of shifting the rotor casing mode to a more manageable speed. A detailed finite element analysis should be performed on the current tilting pad bearing supporting structure to review the feasibility of a design change.

With the improvement in electronic monitoring and miniature sensors, it is recommended that pressure sensors be embedded in future tilting pad and generator bearings. These bearings could monitor the rotor static and dynamic loads generated. The static and dynamic hydrodynamic bearing pressures generated may be used for both alignment and balancing. The use of static and dynamic bearing pressure measurements for alignment and balancing have been reported on earlier by both Lund and Gunter.

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