Dynamic Analysis of Aero-engine Rotors Supported on Ball bearing system

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***Abstract*— Present work deals with the numerical analysis of rotor carrying a central disk mounted over symmetrical ball-bearings, by considering the excitation forces using Muszynska's model along with linear unbalance force due to disk. Other nonlinearities considered in the model include: ball bearing contact forces and disk-stator rub-impact forces. Finite element model incorporating bending degrees of freedom is employed to mesh the system. Implicit Wilson-theta time-integration scheme is used to get the phase- plane, time-history plots and frequency spectra. Different periodic motions are studied by varying the different speeds.**

Keywords— Single disk rotor; Nonlinear forces; Time integration scheme; Effects of speed of rotation.

I. INTRODUCTION

Aero-engine rotors plays very important role and they are very expensive. There is a great need to understand the dynamics of the rotors in a better manner to design heavy machinery like aero engine rotors by considering the characteristics of the rotors dynamically. In general time or frequency domain techniques are used to analyze the nonlinear dynamics analysis of the rotor supported over ball bearings. The coupled nonlinear equations of motion are iterated until the steady-state was attained with a finite time steps in time domain techniques but in frequency domain techniques the equations are solved and periodic state is assumed in the given frequency values. In the analysis of engine rotors, very large number of modes needs to be considered in time and frequency domain techniques.

A typical engine model requires consideration of many hundreds of modes, posing demands on conventional time/frequency domain methods. Typical aero-engine rotors are often mounted on rolling element/SFD bearings. Hertzian contact forces, internal clearances and varying compliance of the ball bearings will affect the high speed rotor dynamic characteristics. Many researchers have derived ball bearing dynamic characteristics. The effects of bearing clearances were analyzed by Childs [1]. Saito [2] studied the response of unbalance nonlinearity of the Jeffcott rotor bearing system with bearing radial clearances. Akturk *et al.*[3] investigated rigid rotor shaft supported over ball bearings under radial and axial and radial vibrations. Bai and Xu [4] modeled rotor bearing system by considering the centrifugal force and gyroscopic moments of the ball bearing. Lee *et al.* [5] presented a design approach by considering the applied loads and spin speed of the ball bearing as the stiffness effecting parameters. Panda and Dutt [6] analyzed the rotor ball bearing system by varying the stability speed limit and the unbalance response. Tiwari *et al.* [7-9] simulated the bearing effects theoretically and studied the nonlinear dynamic response due to the radial internal clearance of the ball bearing.

Harsha *et al.* [10- 11] investigated the nonlinear dynamic behavior of ball bearing-rotor systems with different sources of nonlinearity. Bai et al. [12] studied the nonlinear dynamic characteristics of a rotor and bearing system under the application of axial preload. Yang *et al.* [13] presented the bifurcation analysis of a rotor supported over ball bearing system with different nonlinear excitation forces. Chen [14] modeled the unbalanced rotor dynamic system by considering the bearing clearance, Hertzian forces on the bearing. The bearing supported over ball bearings was studied by many works. But in very few works, there is a consideration of nonlinear external excitation forces. Jedrezjewski *et al.*[15] presented a spindle supported on angular contact ball bearing system for dynamic analysis with few external excitation forces.

Mario [16] analyzed the ball bearing system with an application of a known nonlinear force and presented the effect with different diagrams and introduced a new computational iterative procedure for the rotor system and compared the results of the system with the available literature. Patel *et al.* [17] formulated an analytical model by considering the ball and race contact forces to determine the localized defect effects on ball bearing vibration. Ghafari *et al.* [18] investigated the vibrations of balanced fault-free ball bearings.

Zhang *et al.* [19] presented a nonlinear model of rotor-seal system and analyzed nonlinear behavior of the system under the external excitation of the fluid force. Cheng *et al.*[20] investigated nonlinear phenomena of rotor supported on bearing with seal system using a lumped parameter model based on Jeffcott system. Here oil-film bearing were employed. Hua *et al.*[21] studied the dynamic behavior of rotor- seal system by considering the Muszynska's seal forces in the model. Here simply supported bearings are considered. Li *et al.*[22] illustrated the nonlinear dynamic behavior of rotor-labyrinth seal system using lumped parameter model. More recently, Li *et al*. [23] presented a nonlinear rotor bearing model subjected to steam forces and analyzed the system using finite element model. Here, the oil-film bearing nonlinearity was considered.

In aerospace applications, soft-mounted rolling element bearings using squirrel cage supports are commonly found. So, it needs to account nonlinear contact forces at the bearings along with the rotor-seal forces. Present work deals with the numerical analysis of rotor carrying a disk mounted on symmetrical ball-bearings by considering the excitation forces using Muszynska's model along with linear unbalance force at the disk. Finite element model incorporating Timoshenko beam elements is employed to mesh the system. Implicit Wilson-theta time- integration scheme is used to get the time response, phase plane diagrams and frequency spectra. Effects of different operating speeds are analyzed with periodic motion, quasi as well as chaotic motions. The organization of the paper is as follows: section 2 deals with the mathematical modeling employed for overall rotor dynamic system and expressions for the forces. Section 3 presents the results and discussions and section 4 gives the conclusions.

II. MATHEMATICAL MODELLING

In this section of mathematical modeling of rotor dynamic system using finite element analysis and descriptions of nonlinear excitation force and the ball bearing contact force are presented. Fig.1 is a geometric model representation of a rotor supported on ball bearings with a seal on the disk.

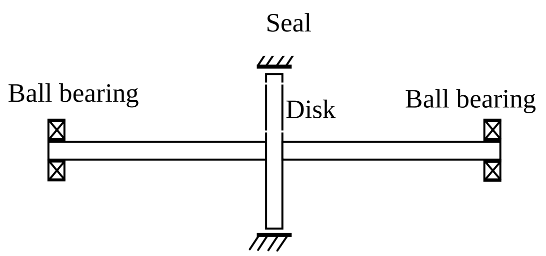

Fig.1.The structural diagram of a rotor supported on ball bearings with a seal at the disk position.

In this system the external excitation forces will act on the bearing and on the disk, the Jeffcott rotor is used to model the rotor and supported on two bearings on both the sides as shown in the above figure.

A. *Modelling of a rotor supported on ball bearings with a seal at the disk system*

The FE Model of the rotor supported on ball bearings with a seal at the disk system is constituted using two-noded Timoshenko beam elements as shown in Fig.2. Viscous damping and gyroscopic effects are considered. The system is modeled with four elements and five nodes and total of twenty Degrees of Freedom (DOF), in which half of them are translational and other half are rotational by considering two of rotational and two of translational DOF on each node. In our study, axial and torsional vibrations are ignored with only flexural behavior of rotor is taken into account.

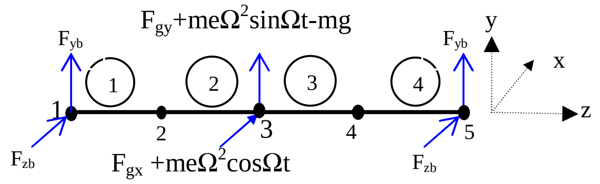

Fig.2. Finite Element Model of a rotor-ball bearing system

The disk and two ball bearing positions are at nodes 3,1 and 5 respectively. The nodes 2 and 4 are the mid nodes of 1, 3 and 1, 5 nodes. Kinetic and potential energies of the shaft unit in terms of displacements v and w along x and y directions and unit mass density ρA and diametral and polar moments of inertia of shaft I_s and I_p are [24]:

$$T_e=\int_0^{\ell}\frac{1}{2}\rho\left\{A\left(\dot{v}^2+\dot{w}^2\right)+I_s(\dot{\theta}^2+\dot{\phi}^2)+I_p\left[\Omega^2+\Omega(\dot{\theta}\phi-\dot{\phi}\theta)\right]\right\}ds \quad (1)$$

$$U_e=\int_0^{\ell}\frac{1}{2}\left\{EI\left((\theta')^2+(\phi')^2\right)+kGA\left[(v'-\phi)^2+(\theta+w')^2\right]\right\}ds \quad (2)$$

Here, Ω refers to the speed of operation and v and w are displacements in x and y directions respectively. In Finite element method, for the cross section of the shaft unit translational and rotational displacement can be determined and is given by [26]:

$$\begin{Bmatrix} v \\ w \end{Bmatrix}=[N]\{q_e\}=\begin{bmatrix} N_1 & 0 & 0 & N_2 & N_3 & 0 & 0 & N_4 \\ 0 & N_1 & -N_2 & 0 & 0 & N_3 & -N_4 & 0 \end{bmatrix}\{q_e\} \quad (3)$$

And

$$\begin{Bmatrix} \theta \\ \phi \end{Bmatrix}=[D]\{q_e\}=\begin{bmatrix} 0 & -D_1 & -D_2 & 0 & 0 & -D_3 & -D_4 & 0 \\ D_1 & 0 & 0 & D_2 & D_3 & 0 & 0 & D_4 \end{bmatrix}\{q_e\} \quad (4)$$

where N_1, N_2,.. and D_1, D_2,.. are the shape functions and $\{q_e\}^T=\{v_1\ w_1\ \theta_1\ \phi_1\ v_2\ w_2\ \theta_2\ \phi_2\}$ is nodal displacement vector. The element mass, gyroscopic and stiffness matrices of shaft are obtained as follows [4]:

$$[M^e]=\int_0^{\ell}\rho A N^T N ds+\int_0^{\ell}\rho I_S D^T D ds \quad (5)$$

$$[G^e]=\int_0^{\ell}\rho I_P D^T\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} D ds \quad (6)$$

$$[K^e]=\int_0^{\ell}EI D'^T D' ds+\int_0^{\ell}kGA\left\{N'^T N'+D^T D+2N'\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} D\right\}ds \quad (7)$$

Likewise, The disk kinetic energy due mass of the disk m_d, mass and polar momts of inertia I_d and J_d written as follows.

$$T_d=\frac{1}{2}m_d(\dot{v}^2+\dot{w}^2)+\frac{1}{2}I_d(\dot{\theta}^2+\dot{\phi}^2)+\frac{1}{2}J_d\left(\Omega^2+\Omega(\dot{\theta}\phi-\theta\dot{\phi})\right) \quad (8)$$

And the work due to disk mass eccentricity in given by

$$W_d = m_d e\Omega^2 \, (v \cos \Omega t + w \sin \Omega t) \quad (9)$$

This gives mass and gyroscopic matrices of disk as follows:

$$[M_d] = \begin{bmatrix} m_d & 0 & 0 & 0 \\ 0 & m_d & 0 & 0 \\ 0 & 0 & I_d & 0 \\ 0 & 0 & 0 & I_d \end{bmatrix} \quad (10)$$

$$[G_d] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_d \\ 0 & 0 & -J_d & 0 \end{bmatrix} \quad (11)$$

Following Lagrangian approach, the consistent stiffness, mass and gyroscopic matrices can be derived. These matrices are assembled and using Guyan's static condensation scheme, the rotational degrees of freedom $\{\theta_1, \phi_1, \theta_1, \phi_2, \ldots\ldots.\}$ are eliminated and the following governing equations of motion for the system are obtained:

$$[M_r]\{\ddot{q}\} + [C_r]\{\dot{q}\} - \Omega[G_r]\{\dot{q}\} + [K_r]\{q\} = \{F\} \quad (12)$$

where $\{q_e\}^T = \{v_1 \;\; w_1 \; v_2 \;\; w_2 \;\; v_3 \;\; w_3 \;\; v_4 \;\; w_4 \; v_5 \;\; w_5\}$ are the overall translational degrees of freedom and [Mr], [Gr], [Cr],and [Kr] are reduced mass matrix, Gyroscopic matrix, viscous damping matrix and the stiffness matrix. The force equation can be written as in vector form

$$\{F\} = [F_{xb} \; F_{yb} \; 0 \; 0 \; F_{gx} + me\Omega^2 \cos\Omega t \;\; F_{gy} + me\Omega^2 \sin\Omega t - mg \; 0 \; 0 \; F_{xb} \; F_{yb}]^T \quad (13)$$

The numerical solution for these nonlinear equations is obtained from fourth order Runge-Kutta method. Effect of speed and seal clearance ratio on system stability is studied using bifurcation analysis. Nonlinear excitation force and the ball bearing contact forces are modeled as follows:

B. Nonlinear excitation force model

The clearance in the turbine and compressors is a non-axisymmetric, because of this there is a chance of having rotordynamic forces on the system and these are considered in very few authors. There are numerous models to explain the nature of nonlinear excitation forces, but Muszynska model is a well defined force model [25]. The expression for nonlinear character of seal- fluid force is given as:

$$\begin{Bmatrix} F_{gx} \\ F_{gy} \end{Bmatrix} = -\begin{bmatrix} K_g - m_g \gamma^2 \Omega^2 & \gamma\Omega D_g \\ -\gamma\Omega D_g & K_g - m_g \gamma^2 \Omega^2 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix}$$

$$-\begin{bmatrix} D_g & 2\gamma\Omega m_g \\ -2\gamma\Omega m_g & D_g \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} - \begin{bmatrix} m_g & 0 \\ 0 & m_g \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} \quad (14)$$

Averaging of the circumferential flow which is rotating with the an angular velocity of $\gamma\omega$. This is an assumption made in Muszynska model. Here ω speed of the shaft and γ is the very important variable of Muszynska model, which will give the ratio of fluid average circumferential velocity. Here m_g is the inertia coefficients. In the Eq. (7), with the increase in the eccentricity of the rotor, there is increase in the parameters namely gas stiffness (K_g) and damping(D_g).

$$K_g = K_0 \left(1 - \varepsilon^2\right)^{-n_1},$$

$$D_g = D_0 \left(1 - \varepsilon^2\right)^{-n_1} \text{ where } n_1 = 0.5 - 3 \quad (15)$$

$$\gamma = \gamma_0 (1 - \varepsilon)^{n_2} \text{ where } 0 < n_2 < 1 \text{ and } \gamma_0 < 0.5$$

Here $\varepsilon = \dfrac{\sqrt{x^2 + y^2}}{c_s}$ is the relative eccentricity at the seal; c_s is seal clearance, n_1, n_2 and γ_0 vary for different types of seal materials; Characteristic factors K_0, D_0 and m_g can be obtained from Childs equation.

C. Ball Bearing contact forces

In the rotor, ball bearings have various components like inner race- which is fixed to the rotating journal rigidly. outer race- is fixed to the bearing housing and the angular contact rolling balls and cages. Due to the eccentricity of rotor the outer race and the inner race will not be at the middle position all the time and rolling balls will apply the force on inner race due to outer race displacements. This is a restoring force, which is generated by the contact deformation between balls and races. The bearing compliance and total stiffness will vary periodically with variation of the eccentricity of the rotor with the races and the balls and the bearing varying compliance is a parametric excitation of the rotor-balling bearing coupling.

In the ball bearing model, it is supposed that the balls are equispaced between the surfaces of the inner and the outer races and the contact angles are not considered. Fig. 3 shows the schematic of ball bearing model, where inner race center O_2 and outer race center O_1 are in same line joining the ball center.

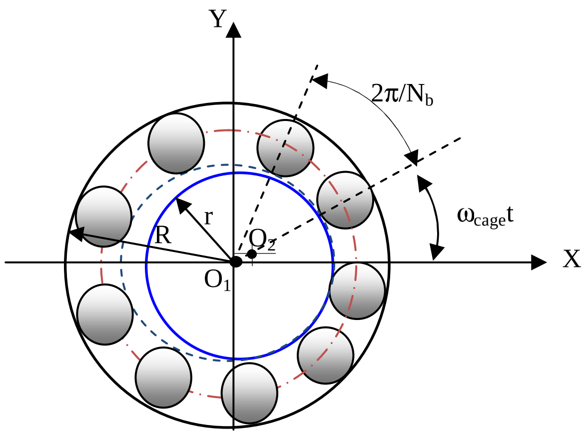

Fig.3 Ball bearing Schematic model

Let N_b, r and R are respectively the number of balls, radii of inner race and outer race respectively and r_0 is radial clearance. According to the nonlinear Hertzian contact theory, the contact forces between ball and race due to the rolling contact can be expressed in terms of Hertzian contact stiffness C_b as:

$$F_{xb}=\sum_{j=1}^{N_b}(f_{jx})=\sum_{j=1}^{N_b}\left(-C_b(x_t\cos\theta_j+y_t\sin\theta_j-r_0)^{3/2}H\right)\cos\theta_j \quad (16)$$

$$F_{yb}=\sum_{j=1}^{N_b}(f_{jy})=\sum_{j=1}^{N_b}\left(-C_b(x_t\cos\theta_j+y_t\sin\theta_j-r_0)^{3/2}H\right)\sin\theta_j \quad (17)$$

where heavy side function

$$H=\begin{cases}1, & \text{if } (x_t\cos\theta_j+y_t\sin\theta_j-r_0)>0\\ 0, & \text{if } x_t\cos\theta_j+y_t\sin\theta_j-r_0\le 0\end{cases} \quad (18)$$

and x_t=x+(R-r_0) cos θ_j and y_t= y+(R-r_0) sin θ_j are the coordinates of circumferential point on the ball.
Also, θ_j is jth ball angular position and is given by
$\theta_j=\omega_{cage}\times t+(2\pi j-2\pi))\ N_b$, here j is the number of ball in the bearing from 1 to N_b, and the cage angular velocity is $\omega_{cage}=\dfrac{\Omega\times r}{(R+r)}$

III. RESULTS AND DISCUSSIONS

Assembly and condensation procedure is implemented in MATLAB. The program could generate the nodal connectivity automatically based on the number of elements choosen. Table I shows the geometric and material properties of the rotor considered in the analysis.

TABLE I. GEOMETRIC AND MATERIAL PROPERTIES OF ROTOR

L_s mm	d_s(mm)	E(GPa)	G (GPa)	ρ(kg/m^3)
1000	50	197	80	7810
k	d_d(mm)	t_d(mm)	e(μm)	
0.65	500	80	60	

Without considering bearing dynamics, the rotor as a simply-supported beam first three bending modes as obtained from the present program are respectively: 98.311 Hz, 389.82 Hz and 865.7 Hz. These are close to the ANSYS solution (98.151 Hz, 387.46 Hz and 855.14 Hz) using 6-degree of freedom pipe elements. Further, the corresponding Campbell diagram showing the variation of natural frequency with shaft rpm is depicted in Fig.4.

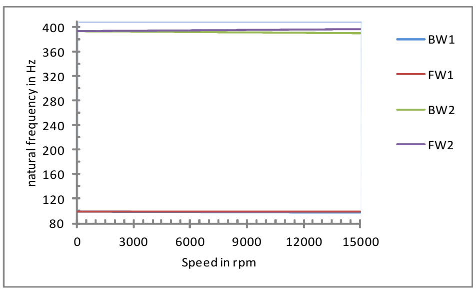

Fig.4. Campbell diagram variation in first two whirl modes

It is found that a four element model is sufficient to approximate the dynamics of rotor in lower frequency range. Table II gives the ball bearing and disc parameters employed in the analysis.

TABLE II. MAIN PARAMETERS OF THE BEARINGS AND DISK

N_b	C_b (N/m$^{3/2}$)	r(mm)	R mm)	r_o(μm)
8	13.34×10^9	31.0	49.52	20
Disk				
m_d (kg)		I_d(kgm^2)	J_d(kgm^2)	
15.95		0.64e-2	1.28e-2	

First, the ball bearing forces and unbalance at the disk are only considered. Figures 5 and 6 show the time history and phase-plane diagrams at the disk node for an operating speed equal to 100 rpm. It is observed that the motion in X and Y directions are not symmetrical at disk node, due to the gravity and the effect of speed on the Y bending mode. The motion is chaotic in Y direction.

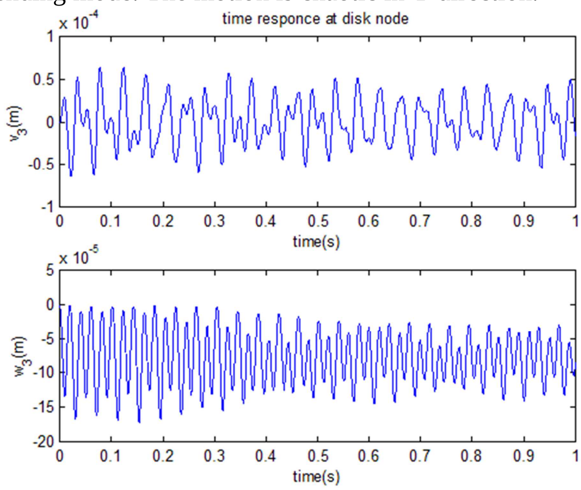

Fig.5. Time histories at disk in x and y directions

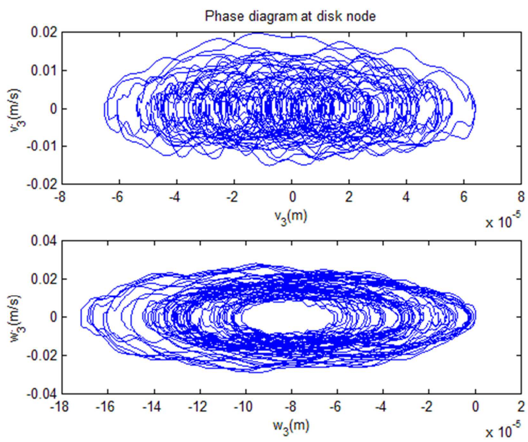

Fig.6. Phase diagrams in x and y directions at disk node

Figure 7 shows the frequency response as obtained from the FFT analysis at the bearing node 1. It is seen clearly the first few variable compliance (ball passing) frequencies clearly.

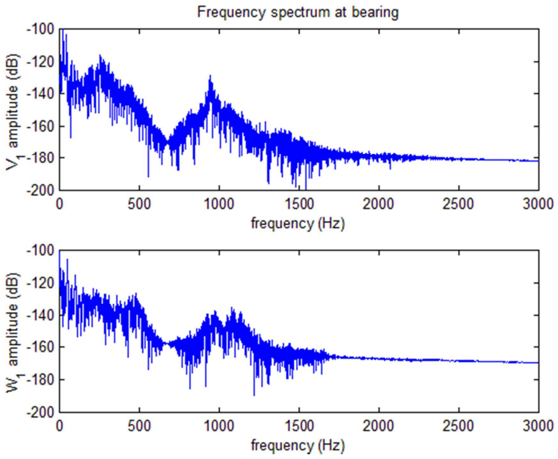

Fig.7. Frequency spectrum at the ball bearings

Beyond 1000 rpm, the motion is ergotic and the finally enters chaotic state. Fig.8 shows the whirl orbit of the shaft at bearing node.

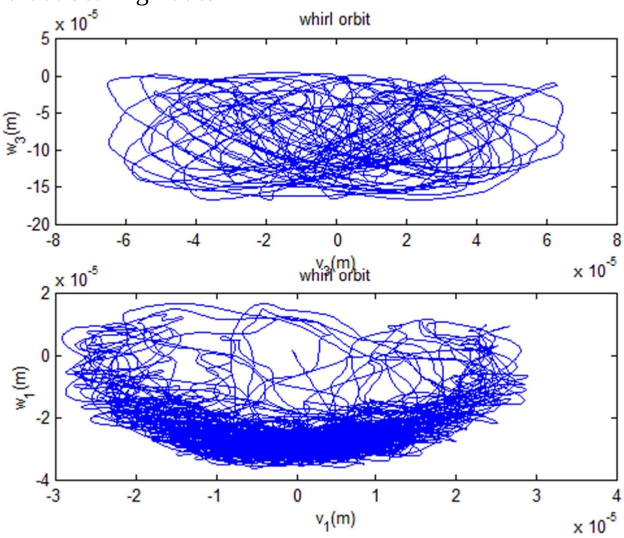

Fig.8. Whirl orbit of shaft at bearing node

It is seen that the intermittent parametric bearing forces change the dynamic response considerbly. As a next step, the following parameter values in Muszynska's force components are considered: n_1=2, n_2=1, γ_0=0.3, c_s=10 μm, K_o=1070.5 N/m, D_o=3.2849 Ns/m, m_f=0.0085 kg at Ω=100 rpm. Fig. 9 shows, the corresponding whirl orbits at the bearing node. It is clearly seen that these external seal forces are affecting the dynamic characteristics of rotor. The speed dependent seal forces are considerably higher comparative to the synchronous unbalance forces and the bearing reactions. It is obvious therefore, that the magnitudes of the responses are relatively high in this case. Further, the nonlinearity in the seal forces influences the dynamics of the system to a greater extent.

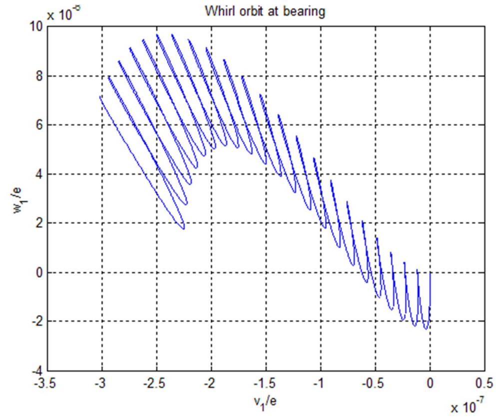

Fig.9. Orbits at bearing node

IV. Conclusions

This paper presented the dynamic modeling of a rotor supported on ball bearings with a seal at the disk system subjected to nonlinear parametric external excitations. A centrally supported symmetrical disk-shaft bearing system has been analyzed using Timoshenko beam elements. Intermittent ball bearing contact forces and Muszynska's force at seal-disk interface were considered in the model to simulate a real-time system. Results show that there was a marked effect of each type of nonlinear excitation on the overall system response. As a future scope of the work, the effects of variables such as speed and seal clearance on stability of system are to be studied.

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