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Dynamic Analysis of Composite Laminates Using Finite Element

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Abstract—In this paper a free vibration analysis of composite laminate is presented. Vibration is the most influencing parameter of life & performance of particular machine element or engineering structures, and invariably, damping is used to reduce that. Various types of damping mechanisms have been developed over time to control the undesired vibration of structures. Different composite laminates with symmetric and anti-symmetric laminates are solved in this paper. Finite element method is used by implementing in MATLAB using a four node quadrilateral element. Then, a set of results are presented to show the applicability of the present problem to various types of boundary conditions under free vibration conditions.

Index Terms—Composite Plate; Vibration Analysis; Finite Element Methods; Isotropic Plate; Boundary condition, shape function.

1 Introduction

Plates are of wide use in engineering industry like ships, containers, etc. In Aeronautics require complete enclosure of plates without use of additional covering, for which composite plates have been, used which consequently saves the material and labor [1]. Plates are of wide use in engineering industry. Many structures such as ships and containers require complete enclosure of plates without use of additional covering which consequently saves the material and labor. The analysis of plates first started in the 1800s. Euler [2] was responsible for solving free vibrations of a flat plate using a mathematical approach for the first time. Then it was the German physicist Chladni [3] who discovered the various modes of free vibrations. Then later on the theory of elasticity was formulated. Navier [4] can be considered as the originator of the modern theory of elasticity. Navier's numerous scientific activities included the solution of various plate problems. He was also responsible for deriving the exact differential equation for rectangular plates with flexural resistance. For the solution to certain boundary value problems Navier introduced exact methods which transformed differential equations to algebraic equations. Poisson in 1829 [5] extended the use of governing plate equation to lateral vibration of circular plates. Later, the theory of elasticity was extended as there were many researchers working on the plate and the extended plate theory was formulated. Kirchhoff [6] is considered as the one who formulated the extended plate theory. In the late 1900s, the theory of finite elements was evolved which is the basis for all the analysis on complex structures
Ungbhakorn and Singhatanadgid [7] investigated the buckling problem of rectangular laminated composite plates with various edge supports by using an extended Kantorovich method is employed. Setoodeh, Karami [8] investigated a three-dimensional elasticity approach to develop a general free vibration and buckling analysis of composite plates with elastic restrained edges. Luura and Gutierrez [9] studied the vibration of rectangular plates by a non-homogenous elastic foundation using the Rayleigh-Ritz method. Ashour [10] investigated the vibration analysis of variable thickness plates in one direction with edges elastically restrained against both rotation and translation using the finite strip transition matrix technique. Crisfield [11] derived a four-node quadrilateral element using discrete Kirchhoff constraints and a nine-node interpolator-y pattern for both transverse and rotational displacements. Kalita et al. [12] [13] [14] [15] [16] [17] has extensively studied the problem on vibration of plates. Patil et al. [18] has solved the problem for various boundary conditions. Grossi, Nallim [19] investigated the free vibration of anisotropic plates of different geometrical shapes and generally restrained boundaries. LU, et al [20] presented the exact analysis for free vibration of long-span continuous rectangular plates based on the classical Kirchhoff plate theory, using state space approach associated with joint coupling matrices. Chopra [21] studied the free vibration of stepped plates by analytical method.

2 Mathematical Formulation

2.1 Problem statement

Consider a three-dimensional body subjected to surface and body forces and temperature field. In addition, displacements are specified on some surface area. For given geometry of the body, applied loads, displacement boundary conditions, temperature field and material stress-strain law, it is necessary to determine the displacement field for the body. The corresponding strains and stresses are also of interest.
The displacements along coordinate axes *x*, *y* and *z* are defined by the displacement vector $\{u\}$

$\{u\} = \{u \ \nu \ w\}$

Six different strain components can be placed in the strain vector $\{\varepsilon\}$

$\{\varepsilon\} = \{\varepsilon_x \ \varepsilon_y \ \varepsilon_z \ \gamma_{xy} \ \gamma_{yz} \ \gamma_{zx}\}$

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For small strains the relationship between strains and displacements is

$\{\varepsilon\} = [D]\{u\}$

where

$$[D] = \begin{bmatrix} \partial/\partial x & 0 & 0 \\ 0 & \partial/\partial y & 0 \\ 0 & 0 & \partial/\partial z \\ \partial/\partial y & \partial/\partial x & 0 \\ 0 & \partial/\partial z & \partial/\partial y \\ \partial/\partial z & 0 & \partial/\partial x \end{bmatrix}$$

Six different stress components are formed the stress vector:

$\{\sigma\} = \{\sigma_x \ \sigma_y \ \sigma_z \ \tau_{xy} \ \tau_{yz} \ \tau_{zx}\}$

which are related to strains for elastic body by the Hook's law:

$\{\sigma\} = [E]\{\varepsilon^e\} = [E](\{\varepsilon\} - \{\varepsilon^t\})$

where

$$[E] = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}$$

Here $[E]$ is the elasticity matrix; $\{\varepsilon^e\}$ is the elastic part of strains; $\{\varepsilon^t\}$ is the thermal part of strains; λ and μ are elastic Lame constants which can be expressed through the elasticity modulus E and Poisson's ratio ν :

$\lambda = \nu E/((1 + \nu)(1 - 2\nu))$, $\mu = E/(2(1 + \nu))$

The purpose of finite element solution of elastic problem is to find such displacement field, which provides minimum to the functional of total potential energy:

$$\Pi = \int_V 1/2 \{\varepsilon^e\}^T\{\sigma\}dV - \int_V \{u\}^T\{p^V\}dV - \int_S \{u\}^T\{p^s\}dS,$$

where $\{p^V\} = \{p^V_x \ p^V_y \ p^V_z\}$ is the vector of body force and $\{p^S\} = \{p^S_x \ p^S_y \ p^S_z\}$ is the vector of surface force.

Prescribed displacements are specified on the part of body surface where surface forces are absent. Displacement boundary conditions are not present in the functional (3.7). Because of these, displacement boundary conditions should be implemented after assembly of finite element equations.

2.2 Finite element equations

Let us consider some abstract three-dimensional finite element having the vector of nodal displacements $\{q\}$:

$\{q\}= \{u_1 \ v_1 \ w_1 \ u_2 \ v_2 \ w_2 \ ...\}$

Displacements at some point inside a finite element $\{u\}$ can be determined with the use of nodal displacements $\{q\}$ and shape functions N_i:

$\{u\} = [N]\{q\}$

where

$$[N] = \begin{bmatrix} N_1 & 0 & 0 & N_2 & \dots \\ 0 & N_1 & 0 & 0 & \dots \\ 0 & 0 & N_1 & 0 & \dots \end{bmatrix}$$

Strains can also be determined through displacements at nodal points:

$\{\varepsilon\} = [B]\{q\}$

where

$[B] = [D][N] = [\, B_1 \ B_2 \ ... \,]$,

$$[B_i] = \begin{bmatrix} \partial Ni/\partial x & 0 & 0 \\ 0 & \partial Ni/\partial y & 0 \\ 0 & 0 & \partial Ni/\partial z \\ \partial Ni/\partial y & \partial Ni/\partial x & 0 \\ 0 & \partial Ni/\partial z & \partial Ni/\partial y \\ \partial Ni/\partial z & 0 & \partial Ni/\partial x \end{bmatrix}$$

Now using above eqns., we are able to express the total potential energy through nodal displacements:

$$\Pi = \int_V 1/2([B]\{q\}-\{\varepsilon^t\})^T[E]([B]\{q\}-\{\varepsilon^t\})dV - \int_V ([N]\{q\})^T\{p^V\}dV - \int_S ([N]\{q\})^T\{p^s\}dS$$

Nodal displacements $\{q\}$ which corresponds to the minimum of the functional Π are determined by the conditions:

$\{d\Pi/dq\} = 0$

Differentiation of above in respect to nodal displacements $\{q\}$ produces the following equilibrium equations for a finite element:

$$\int_V [B]^T[E][B]dV\{q\} - \int_V [B]^T[E]\{\varepsilon^t\}dV - \int_V [N]^T\{p^V\}dV - \int_S [N]^T\{p^s\}dS = 0$$

which is usually presented in the following form:

$[k]\{q\} = \{f\}$, $\{f\} = \{p\} + \{h\}$

where

$$[k] = \int_V [B]^T[E][B]dV$$

$$\{p\} = \int_V [N]^T\{p^V\}dV - \int_S [N]^T\{p^s\}dS$$

$$\{h\} = \int_V [B]^T[E]\{\varepsilon^t\}dV$$

Here $[k]$ is the element stiffness matrix; $\{f\}$ is the load vector; $\{p\}$

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is the vector of actual forces and {*h*} is the thermal vector which represents fictitious forces for modeling thermal expansion.

2.3 Assembly of the global equation system

The aim of assembly is to form the global system of equations

$[K]\{Q\} = \{F\}$

using element equations

$[k_i]\{q_i\} = \{f_i\}$

Here $[k_i]$, $[q_i]$ and $[f_i]$ are the stiffness matrix, the displacement vector and the load vector of the *i*th finite element; $[K]$, $\{Q\}$ and $\{F\}$ are global stiffness matrix, displacement vector and load vector.

In order to derive an assembly algorithm, let us present the total potential energy for the body as a sum of element potential energies:

$\Pi = \sum\pi_i = \sum 1/2\{q_i\}^T[k_i]\{q_i\} - \sum 1/2\{q_i\}^T[\{f_i\}+ \sum E^0_i$

where E^0_i is the fraction of potential energy related to free thermal expansion:

$$E^0_i = \int_V 1/2\, \{\varepsilon^t\}^T[E]\, \{\varepsilon^t\}dV$$

Let us introduce the following vectors and a matrix where element vectors and matrices are simply placed:

$\{Q_d\} = \{\{q_1\}\{q_2\} \ldots \}$, $\{F_d\} = \{\{f_1\}\{f_2\} \ldots \}$

$$[K_d] = \begin{bmatrix} [k_1] & 0 & 0 \\ 0 & ..[k_2].. & 0 \\ 0 & 0 & \ldots \end{bmatrix}$$

It is evident that it is easy to find matrix [*A*] such that

$\{Q_d\} = [A]\{Q\}$, $\{F_d\} = [A]\{F\}$

The total potential energy for the body can be rewritten in the following form:

$\Pi = 1/2\{Q_d\}^T[K_d]\{Q_d\} - \{Q_d\}^T\{F_d\} + \sum E^0_i = 1/2\{Q\}^T[A]^T[K_d][A]\{Q\} - \{Q\}^T[A]^T\{F_d\} + \sum E^0_i$

Using the condition of minimum of the total potential energy

$\{d\Pi/dQ\} = 0$

we arrive at the following global equation system:

$[A]^T[K_d][A]\{Q\} - [A]^T\{F_d\} = 0$

The last equation shows that algorithms of assembly the global stiffness matrix and the global load vector are:

$[K] = [A]^T[K_d][A]$, $\{F\} = [A]^T\{F_d\}$

Here [*A*] is the matrix corresponding local and global enumeration. Fraction of nonzero (unit) entries in the matrix [*A*] is very small. Because of this the matrix [*A*] is never used explicitly in actual computer codes.

In present work, the numerical computations have been performed by using MATLAB 7.5.0 (R2007b) in DELL PRECISION T3500.

3 Results

3.1 Symmetric laminates

In Table 2 natural frequencies of four-layer symmetric laminate is shown. Different ratio of width (a) & thickness (h) of the plate is taken. The layup of the laminate considered in the study is shown in Fig. 1. The material properties in the study are detailed in Table 1.

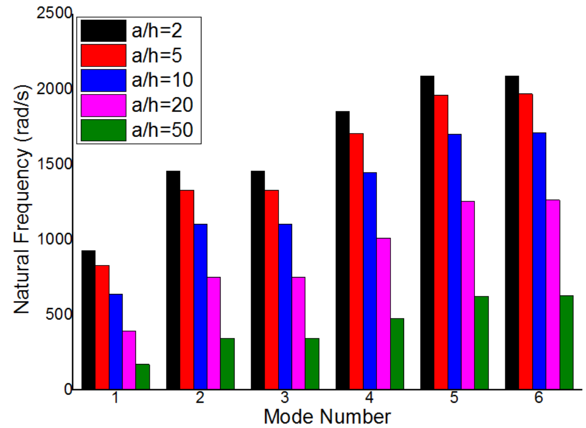

Fig. 5. Variation of natural frequency with different aspect ratios at a/h=50.

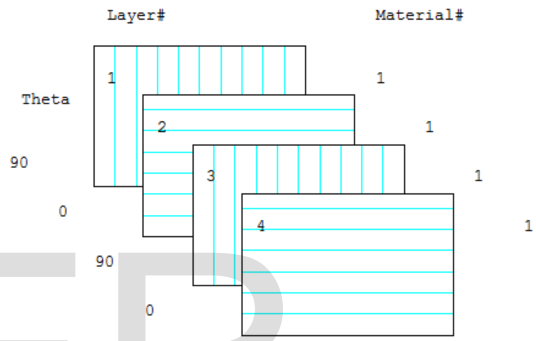

Fig. 2. Layup of the anti-symmetric laminate.

TABLE 1. Material properties of each laminate.

Longitudinal Modulus E_1	Transverse Modulus E_2	Shear Modulus G_{12}	Shear Modulus G_{23}	Poisson's Ratio ν_{12}	Poisson's Ratio ν_{23}
(in GPa)					
108	10.3	7.13	4.03	0.28	0.28

TABLE 2. Natural frequencies (rad/s) obtained by theoretical prediction for different thickness.

Mode No.	a/h				
	2	5	10	20	50
1	940.2	836.92	649.29	410.72	181.02
2	1380.6	1233.2	976.77	626.78	278.32
3	1573.3	1452	1241.8	899.12	436.5
4	1884.1	1732.5	1463.2	991.74	456.24
5	1948.1	1776.1	1464.2	1041.7	502.2
6	2289.2	2163.4	1843.2	1319.9	639.11

TABLE 3. Natural frequencies (rad/s) obtained by theoretical

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prediction for different aspect ratio at a/h=5.

Mode No.	b/a					
	0.25	0.5	1	2	4	10
1	2522	1337.5	836.92	699.29	677.97	674.98
2	2796.4	1792.3	1233.2	792.13	691.47	676.17
3	3234.7	2383.9	1452	963.8	721.44	678.45
4	3775.7	2423.3	1732.5	1194.6	772.62	682.22
5	4381.1	2692.7	1776.1	1371.5	846.05	687.95
6	4850.8	3109.9	2163.4	1433.6	939.66	696.15

TABLE 4. Natural frequencies (rad/s) obtained by theoretical prediction for aspect ratio at a/h=50.

Mode No.	b/a					
	0.25	0.5	1	2	4	10
1	1131.4	351.99	181.02	159.3	156.72	156.32
2	1228.1	547	278.32	171.91	158.57	156.55
3	1440	833.62	436.5	200.48	162.3	156.94
4	1781.7	888.94	456.24	249.43	168.7	157.52
5	2237.8	964.02	502.2	319.39	178.67	158.31
6	2754.6	1226.2	639.11	408.96	193.01	159.37

TABLE 5. Natural frequencies (rad/s) obtained by theoretical

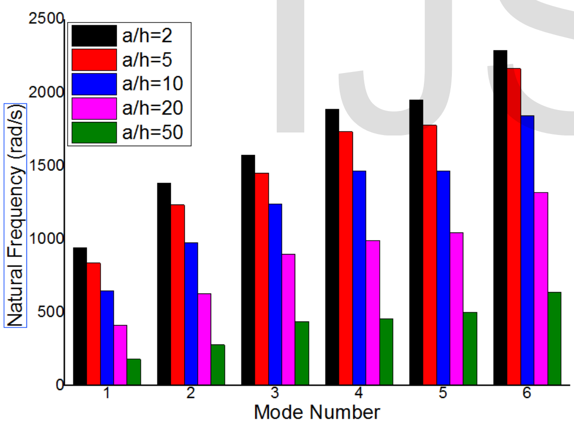

Fig. 3. Variation of frequency with different thickness.

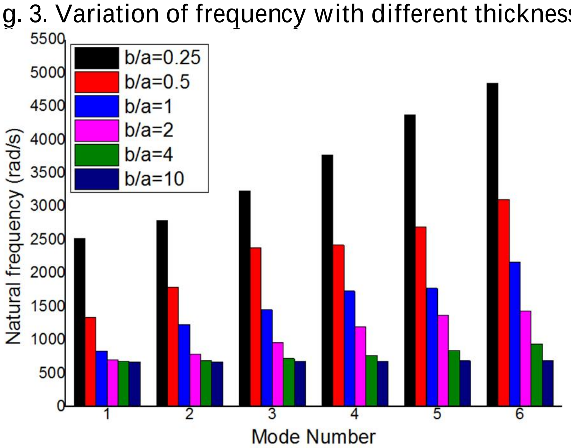

Fig. 4. Variation of natural frequency with different aspect ratios at a/h=5.

prediction for different thickness.

Mode No.	a/h				
	2	5	10	20	50
1	924.78	828.94	636.71	393.14	170.55
2	1455.3	1328.8	1102.1	750.51	346.07
3	1455.3	1328.8	1102.1	750.55	346.09
4	1850.1	1706.6	1443.2	1010.8	476.52
5	2087.4	1961.8	1701.8	1256.3	623.56
6	2088.5	1967	1710.5	1262.3	625.58

TABLE 6. Natural frequencies (rad/s) obtained by theoretical prediction for different aspect ratio at a/h=5.

Mode No.	b/a					
	0.25	0.5	1	2	4	10
1	2754.6	1483.6	924.78	722.49	666.84	654.58
2	2981.3	1864.8	1455.3	900.41	711.07	659.46
3	3332.7	2389.2	1455.3	1161.2	789.19	668.98
4	3772.4	2734.6	1850.1	1334.3	894.96	684.12
5	4273.4	2966.4	2087.4	1447.2	1020.1	705.33
6	4815.1	2970.8	2088.5	1454.8	1157.7	732.55

TABLE 7. Natural frequencies (rad/s) obtained by theoretical prediction for different aspect ratio at a/h=50.

Mode No.	b/a					
	0.25	0.5	1	2	4	10
1	1549.8	466.73	170.55	121.66	116.33	115.61
2	1596.8	570	346.07	148.55	119.66	115.94
3	1704	786.59	346.09	206.41	127.09	116.54
4	1894.1	1108.8	476.52	295.37	140.55	117.48
5	2176.6	1164.4	623.56	317.48	161.51	118.87
6	2546.5	1231	625.58	335.37	190.56	120.81

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3.2 Anti-Symmetric laminates

In Table 5 natural frequencies of four-layer anti-symmetric laminate is shown. Different ratio of width (a) & thickness (h) of the plate is taken. The layup in Fig. 2. The material properties in the study are detailed in Table 1.

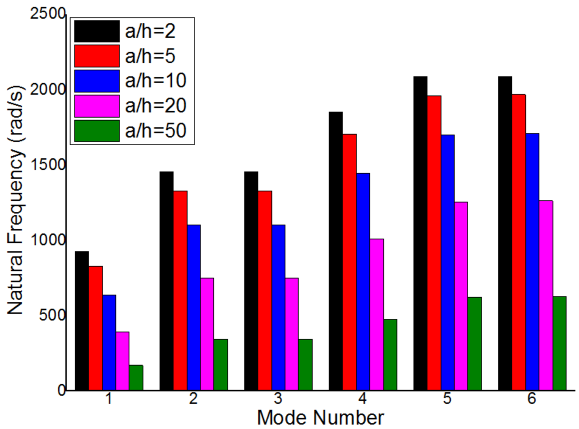

Fig. 6. Variation of frequency with different thickness.

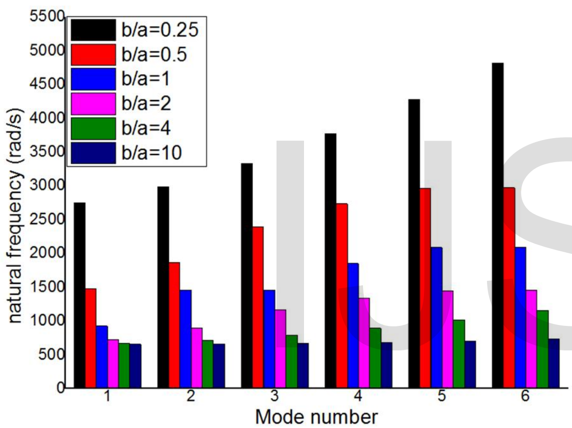

Fig. 7. Variation of natural frequency with different aspect ratios at a/h=5.

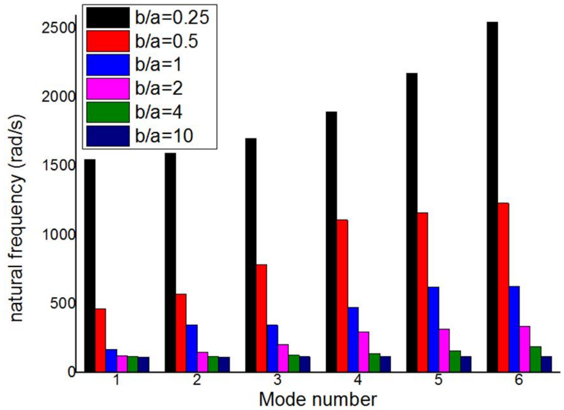

Fig. 8. Variation of natural frequency with different aspect ratios at a/h=50.

4 CONCLUSION

The variations of the first six natural frequencies with respect to thickness-length ratio and aspect ratio are presented under clamped condition. The present analysis is useful for the design of composites plates for dynamic response. FSDT gives very accurate results for thin plates as well as thick plates. CPT is known to produce accurate results only for thin plates. Hence the developed program is cable of taking into account any thickness. Based on the current study the following conclusions can be drawn. The natural frequency decreases with increase in thickness ratio. Frequency is found to be increasing with increase aspect ratio. The developed mathematical formulation is excellent in handling the both symmetric and anti-symmetric composite laminate problems. The wide variety of problems tackled further highlights the versatility of the developed formulation.

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