DYNAMIC ANALYSIS OF FRAMED STRUCTURES

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

BACHELOR OF TECHNOLOGY
IN
CIVIL ENGINEERING

BY

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Ghamandi Lal Meena

Department of Civil Engineering
National Institute of Technology
Rourkela
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CERTIFICATE

This is to certify that the thesis entitled, "**Dynamic Analysis Of Framed Structures**" submitted by **Sri Kammula Viswanath** and **Sri Ghamandi Lal Meena** in partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Civil Engineering at the National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

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CONTENTS

ABSTRACT

All real physical structures, when subjected to loads or displacements, behave dynamically. The additional inertia forces, from Newton's second law, are equal to the mass times the acceleration. If the loads or displacements are applied very slowly then the inertia forces can be neglected and a static load analysis can be justified. Hence, dynamic analysis is a simple extension of static analysis.

In addition, all real structures potentially have an infinite number of displacements. Therefore, the most critical phase of a structural analysis is to create a computer model, with a finite number of mass less members and a finite number of node (joint) displacements, that will simulate the behavior of the real structure. The mass of a structural system, which can be accurately estimated, is lumped at the nodes. Also, for linear elastic structures the stiffness properties of the members, with the aid of experimental data, can be approximated with a high degree of confidence. However, the dynamic loading, energy dissipation properties and boundary (foundation) conditions for many structures are difficult to estimate. This is always true for the cases of seismic input or wind loads.

To reduce the errors that may be caused by the approximations summarized in the previous paragraph, it is necessary to conduct many different dynamic analyses using different computer models, loading and boundary conditions. Because of the large number of computer runs required for a typical dynamic analysis, it is very important that accurate and numerically efficient methods be used within computer programs

LIST OF FIGURES

LIST OF TABLES

TABLES **PAGE NOS.**

CHAPTER 1

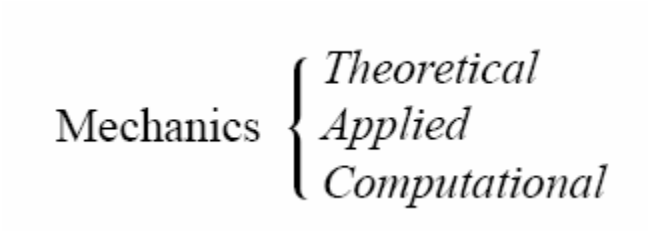
FINITE ELEMENT METHOD

FINITE ELEMENT METHOD

1.1 INTRODUCTION Theory

1.1.1Where this Material Fits

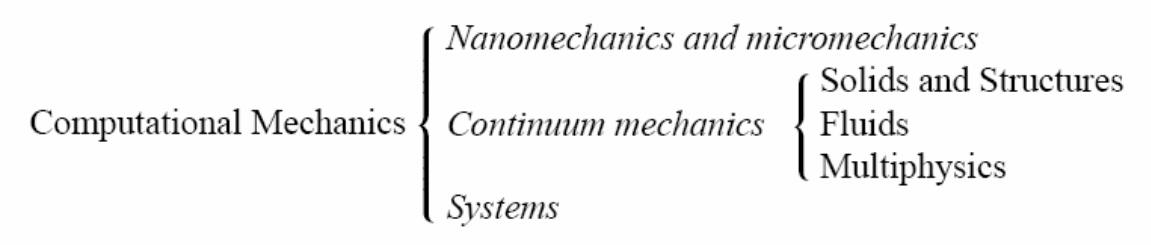
The field of Mechanics can be subdivided into three major areas:



Theoretical mechanics deals with fundamental laws and principles of mechanics studied for their intrinsic scientific value. *Applied mechanics* transfers this theoretical knowledge to scientific and engineering applications, especially as regards the construction of mathematical models of physical phenomena. *Computational mechanics* solves specific problems by simulation through numerical methods implemented on digital computers.

1.1.2Computational Mechanics

Several branches of computational mechanics can be distinguished according to the physical scale of the focus of attention:



Nano mechanics deals with phenomena at the molecular and atomic levels of matter. As such it is closely linked to particle physics and chemistry. Micro mechanics looks primarily at the crystallographic and granular levels of matter. Its main technological application is the design and fabrication of materials and micro devices.

Continuum mechanics studies bodies at the macroscopic level, using continuum models in which the microstructure is homogenized by phenomenological averages. The two traditional areas of application are solid and fluid mechanics. The former includes structures which, for obvious reasons, are fabricated with solids. Computational solid mechanics takes an applied sciences approach, whereas computational structural mechanics emphasizes technological applications to the analysis And design of structures.

Computational fluid mechanics deals with problems that involve the equilibrium and motion of liquid and gases. Well developed subsidiaries are hydrodynamics, aerodynamics, acoustics, atmospheric physics, shock, combustion and propulsion.

A system is studied by *decomposition*: its behavior is that of its components plus the interaction between components. Components are broken down into subcomponents and so on. As this hierarchical breakdown process continues, individual components become simple enough to be treated by individual disciplines, but component interactions get more complex.

1.1.3Statics vs. Dynamics

Continuum mechanics problems may be subdivided according to whether inertial effects are taken into account or not:

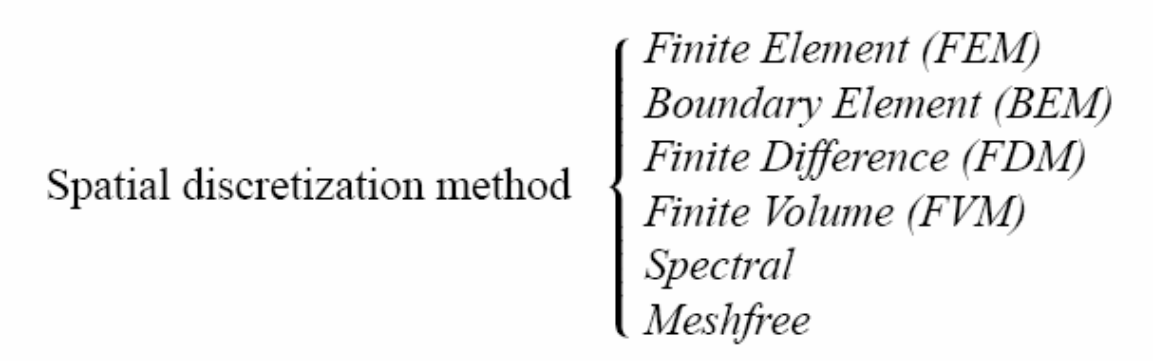
$$\text{Continuum mechanics} \begin{cases} \textit{Statics} \\ \textit{Dynamics} \end{cases}$$

In dynamics actual time dependence must be explicitly considered, because the calculation of inertial (and/or damping) forces requires derivatives respect to actual time to be taken.

Problems in statics may also be time dependent but with inertial forces ignored or neglected. Accordingly static problems may be classed into strictly static and quasi-static. For the former time need not be considered explicitly; any historical time-like response ordering parameter, if one is needed, will do. In quasi-static problems such as foundation settlement, metal creep, rate-dependent plasticity or fatigue cycling, a realistic measure of time is required but inertial forces are still neglected.

1.1.4 Discretization methods

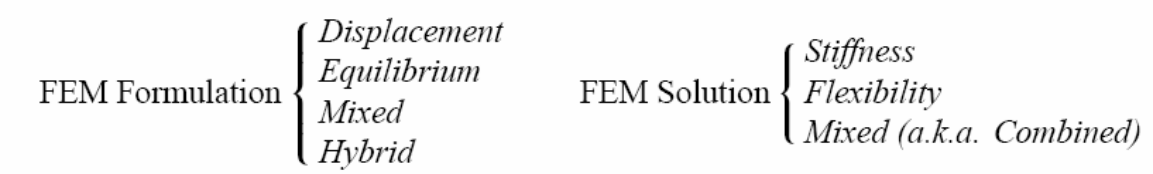
A final classification of CSM static analysis is based on the discretization method by which the continuum mathematical model is discretized in space, *i.e.*, converted to a discrete model with a finite number of degrees of freedom:



In CSM linear problems finite element methods currently dominate the scene as regards space discretization. Boundary element methods post a strong second choice in specific application areas. For *nonlinear* problems the dominance of finite element methods is overwhelming.

1.1.5 FEM Variants

The term Finite Element Method actually identifies a broad spectrum of techniques that share common features outlined in above sections. Two sub classifications that fit well applications to structural mechanics are



Of the variants listed above, emphasis is placed on the displacement formulation and stiffness solution. This combination is called the Direct Stiffness Method or DSM.

1.2 The Finite Element Method

The finite element method (FEM) is the dominant discretization technique in structural mechanics.

The basic concept in the physical FEM is the subdivision of the mathematical model into disjoint (non-overlapping) components of simple geometry called *finite elements* or *elements* for short. The response of each element is expressed in terms of a finite number of degrees of freedom characterized as the value of an unknown function, or functions, at a set of nodal points. The response of the mathematical model is then considered to be approximated by that of the discrete model obtained by connecting or assembling the collection of all elements.

1.2.1Element Attributes

The procedure involves the separation of elements from their neighbors by disconnecting the nodes, followed by referral of the element to a convenient local coordinate system.8 After that we can consider generic elements: a bar element, a beam element, and so on.

From the standpoint of the computer implementation, it means that you can write one subroutine or module that constructs, by suitable parametrization, all elements of one type, instead of writing a new one for each element instance.

1.2.2Dimensionality

Elements can have intrinsic dimensionality of one, two or three space dimensions. There are also special elements with zero dimensionality, such as lumped springs or point masses. The intrinsic dimensionality can be expanded as necessary by use of kinematic transformations. For example a 1D element such as a bar, spar or beam may be used to build a model in 2D or 3D space.

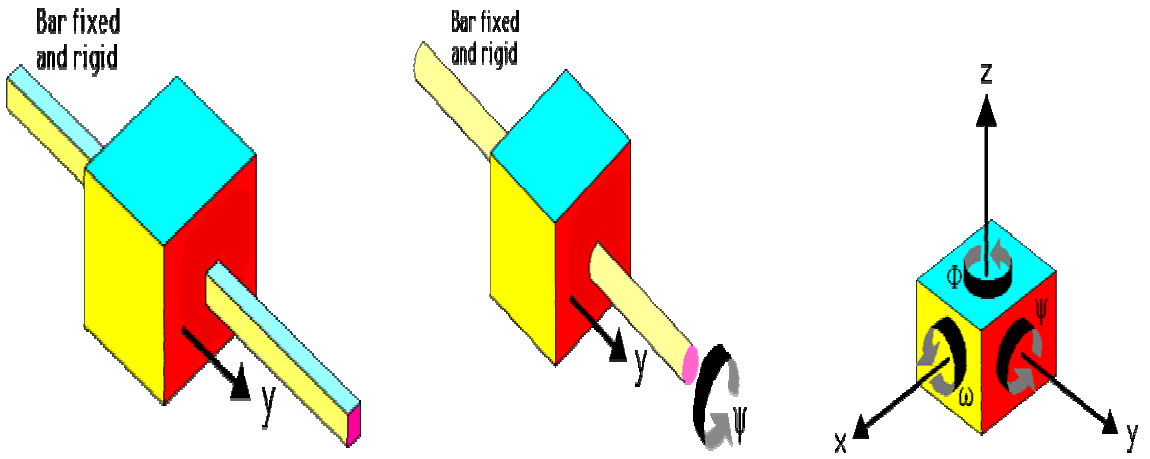
1.2.3Nodes

Each element possesses a set of distinguishing points called *nodal points* or *nodes* for short. Nodes serve a dual purpose: definition of element geometry, and home for degrees of freedom. When a distinction is necessary we call the former *geometric nodes* and the latter *connection nodes*. For most elements studied here, geometric and connector nodes coalesce.

1.2.4 Degrees of Freedom

The element degrees of freedom (DOF) specify the *state* of the element. They also function as “handles” through which adjacent elements are connected. DOFs are defined as the values (and possibly derivatives) of a primary field variable at connector node points.

A simple definition of "degrees of freedom" is - the number of coordinates that it takes to uniquely specify the position of a system.



SINGLE DOF TWO DOF SIX DOF

(Figure 1)

For mechanical elements, the primary variable is the displacement field and the DOF for many (but not all) elements are the displacement components at the nodes.

If the number of degrees of freedom is finite, the model is called *discrete*, and *continuous* otherwise. Because FEM is a discretization method, the number of DOF of a FEM model is necessarily finite. They are collected in a column vector called **u**. This vector is called the *DOF vector* or *state vector*. The term nodal displacement vector for **u** is reserved to mechanical applications.

1.2.5Nodal Forces

There is always a set of nodal forces in a one-to-one correspondence with degrees of freedom. In mechanical elements the correspondence is established through energy arguments.

1.2.6Assembly

The assembly procedure of the Direct Stiffness Method for a general finite element model follows rules identical in principle to those discussed for the truss example. As in that case the process involves two basic steps:

Globalization. The element equations are transformed to a common *global* coordinate system, if necessary.

Merge. The element stiffness equations are merged into the master stiffness equations by appropriate indexing and matrix-entry addition.

The master stiffness equations in practical applications may involve thousands or even millions of freedoms, and programming can become involved.

1.2.7 Essential and Natural B.C.

The key thing to remember is that boundary conditions (BCs) come in two basic flavors: essential and natural.

Essential BCs directly affect DOFs, and are imposed on the left-hand side vector **u**.

Natural BCs do not directly affect DOFs and are imposed on the right-hand side vector **f**.

The mathematical justification for this distinction requires use of concepts from variational calculus, and is consequently relegated to Part II. For the moment, the basic recipe is:

1. If a boundary condition involves one or more degrees of freedom in a *direct* way, it is essential. An example is a prescribed node displacement.

2. Otherwise it is natural.

The term "direct" is meant to exclude derivatives of the primary function, unless those derivatives also appear as degrees of freedom, such as rotations in beams and plates.

1.2.8Boundary Conditions in Structural Problems

Essential boundary conditions in mechanical problems involve *displacements* (but not strain-type displacement derivatives). Support conditions for a building or bridge problem furnish a particularly simple example. But there are more general boundary conditions that occur in practice.

The total potential energy in the body is

$$= 0.5 * Q^T K*Q - Q^T * F$$

Where K is the structural stiffness matrix is the global load vector, and Q is the global displacement vector. We now must arrive at the equations of equilibrium, from which we can determine nodal displacements, element stresses and support reactions. The minimum potential energy theorem is now invoked. This theorem is stated as follows: of all possible displacements that satisfy the boundary conditions of a structural system, those corresponding to equilibrium configurations make the total potential energy assume a minimum value.

It is noted that the treatment of boundary conditions in this sections is applicable to two and three dimensional problems as well.
It should be emphasized that improper specification of boundary conditions can lead to erroneous results. Boundary conditions eliminate the possibility of the structure moving as a rigid body.

There are two approaches to calculate displacements:

1. Elimination approach
2. Penalty approach

1.2.9 Elimination approach:

Considering the single boundary condition Q1=a1. The equilibrium equations are obtained by minimizing pi with respect to Q, subjected to the boundary condition

$Q_1=a_1$.for an N- dof structure, we have $Q=[Q_1,Q_2,\ldots\ldots Qn]$

$F= [F_1, F_2\ldots\ldots\ldots Fn]$

Steps involved :
Consider the boundary conditions $Q_1=a_1, Q_2=a_2\ldots\ldots\ldots Q_r=a_r$.

1. Store the p_1 th,p_2 th…and p_r th rows of the global stiffness matrix K and force vector F.these rows will be used subsequently.

2. Delete the p1th row and column, the p_rth row and column from the K matrix. The resulting stiffness matrix K is of dimension (N-r,N-r).similarly, the corresponding load vector F is of dimension(N-r,1).so

$$KQ=F$$

3. For each element, extract the element, displacement vector q from the q vector, element connectivity, and determine element stresses.
4. Using the information stored in step 1 ,evaluate the reaction forces at each support dof from

$Rp_1=Kp_1\ Q_1+Kp_2\ Q_2+\ldots\ldots\ldots Kp_N\ Q_N-Fp_1$

$Rp_2=Kp_1\ Q_1+Kp_2\ Q_2+\ldots\ldots\ldots Kp_N\ Q_N-Fp_2$

$Rp_r=Kp_{r1}\ Q_1+Kp_{r2}\ Q_2+\ldots\ldots\ldots Kp_{rN}\ Q_N-Fp_{r2}$

1.2.10 Penalty approach:

Consider the boundary condition $Q_1=a_1, Q_2=a_2\ldots\ldots\ldots Q_r=a_r$.

1. Modify the structural stiffness matrix K by adding a large number C to each of p_1 th,p_2 th…and p_r th diagonal elements of k . also modify the global load vector F by adding C_{a1} to F_{a1} , C_{a2} to F_{a2} …..and C_{ar} to F_{ar} . Solve KQ=F for the displacement Q, where K and F are the modified stiffness and load matrices.
2. For each element, extract the element displacement vector q from the Q vector, using element connectivity and determinate the element stresses.
3. Evaluate the reaction forces at each support from

$$R_{pi}=-C\ (Q_{pi}-a_i)\qquad i=1,2,3\ldots.r$$

It should be noted that the penalty approach presented here is an approximate approach. The accuracy of the solution particularly depends on choice of C.

1.3 STIFFNESS MATRIX

Properties:

1. The dimension of the global stiffness matrix K is (N*N), where N is the number of nodes .this follows from the fact that each node has only one DOF.

2. K is symmetric.

3. K is banded matrix. That is all elements outside of the band are zero. K(banded) is of dimension [N*NBW],where NBW is the half-bandwidth.in many one dimensional problems such as the example just considered ,the connectivity of element I,i+1.in such cases ,the banded matrix has only two columns (NBW=2).

NBW=max (difference between dof numbers connecting an element) +1

1.4 MASS MATRIX

1.4.1Introduction

To do dynamic and vibration finite element analysis, you need at least a mass matrix to pair with the stiffness matrix.

As a general rule, the construction of the master mass matrix **M** largely parallels that of the master stiffness matrix **K**. Mass matrices for individual elements are formed in local coordinates, transformed to global, and merged into the master mass matrix following exactly the same techniques used for **K**. In practical terms, the assemblers for **K** and **M** can be made identical. This procedural uniformity is one of the great assets of the Direct Stiffness Method

.

A notable difference with the stiffness matrix is the possibility of using a *diagonal* mass matrix based on direct lumping. A master diagonal mass matrix can be stored simply as a vector. If all entries are nonnegative, it is easily inverted, since the inverse of a diagonal matrix is also diagonal. Obviously a lumped mass matrix entails significant computational advantages for calculations that involve $\mathbf{M}^{-1}$.

1.4.2Mass Matrix Construction

The master mass matrix is built up from element contributions, and we start at that level. The construction of the mass matrix of individual elements can be carried out through several methods. These can be categorized into three groups: direct mass lumping, variational mass lumping, and template mass lumping. The last group is more general in that includes all others. Variants of the first two techniques are by now standard in the FEM literature, and implemented in all general purpose codes.

1.4.3Mass Matrix Properties

Mass matrices must satisfy certain conditions that can be used for verification and debugging. They are: (1) matrix symmetry, (2) physical symmetries, (3) conservation and (4) positivity.

1.4.4Globalization

Like their stiffness counterparts, mass matrices are often developed in a local or element frame. Should globalization be necessary before merge, a congruent transformation is applied:

$$\mathbf{Me} = (\mathbf{Te})^{\mathbf{T}} * \mathbf{M'}^{\mathbf{e}} * \mathbf{T}^{\mathbf{e}}$$

Here M'^{e} is the element mass referred to is the element mass referred to the local frame whereas T^{e} is the local-to-global displacement transformation matrix. Matrix $\mathbf{T}^{\mathbf{e}}$ is in principle that used for the stiffness globalization.

CHAPTER 2

Theory Of Vibration

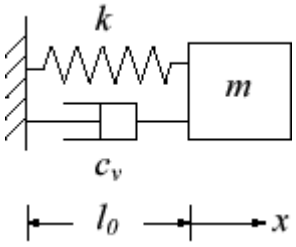
THEORY OF VIBRATION

2.1 SINGLE DEGREE OF FREEDOM SYSTEMS

2.1.1)Damped Vibration of Free SDOF Systems

Definition

Free vibration (no external force) of a single degree-of-freedom system with viscous damping can be illustrated as,



(Figure 2)

Damping that produces a damping force proportional to the mass's velocity is commonly referred to as "viscous damping", and is denoted graphically by a dashpot.

Time Solution for Damped SDOF Systems

For an unforced damped SDOF system, the general equation of motion becomes,

$$m\ddot{x} + c_v\dot{x} + kx = 0$$

with the initial conditions,

$$\begin{cases} x(t=0) = x_0 \\ \dot{x}(t=0) = v_0 \end{cases}$$

This equation of motion is a second order, homogeneous, ordinary differential equation (ODE). If all parameters (mass, spring stiffness and viscous damping) are constants, the ODE becomes a Linear ODE with constant coefficients and can be solved by the Characteristic Equation method. The characteristic equation for this problem is,

$$ms^2 + c_v s + k = 0$$

which determines the 2 independent roots for the damped vibration problem. The roots to the characteristic equation fall into one of the following 3 cases:

If $c_v^2 - 4mk < 0$, the system is termed under damped. The roots of the characteristic equation are complex conjugates, corresponding to *oscillatory motion* with an *exponential decay* in amplitude.

If $c_v^2 - 4mk = 0$, the system is termed critically damped. The roots of the characteristic equation are repeated, corresponding to *simple decaying motion* with at most *one overshoot* of the system's resting position.

If $c_v^2 - 4mk > 0$, the system is termed over damped. The roots of the characteristic equation are purely real and distinct, corresponding to simple *exponentially decaying* motion.

To simplify the solutions coming up, we define the critical damping c_c, the damping ratio ξ, and the damped vibration frequency ω_d as,

$$c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n$$

$$\zeta = \frac{c_v}{c_c}$$

$$\omega_d = \sqrt{1-\zeta^2}\,\omega_n$$

Where the natural frequency of the system ω_n is given by,

$$\omega_n = \sqrt{\frac{k}{m}}$$

Note that ω_d will equal ω_n when the damping of the system is zero (i.e. un damped). The time solution for the free SDOF system is presented below for each of the three case scenarios.

Under damped Systems

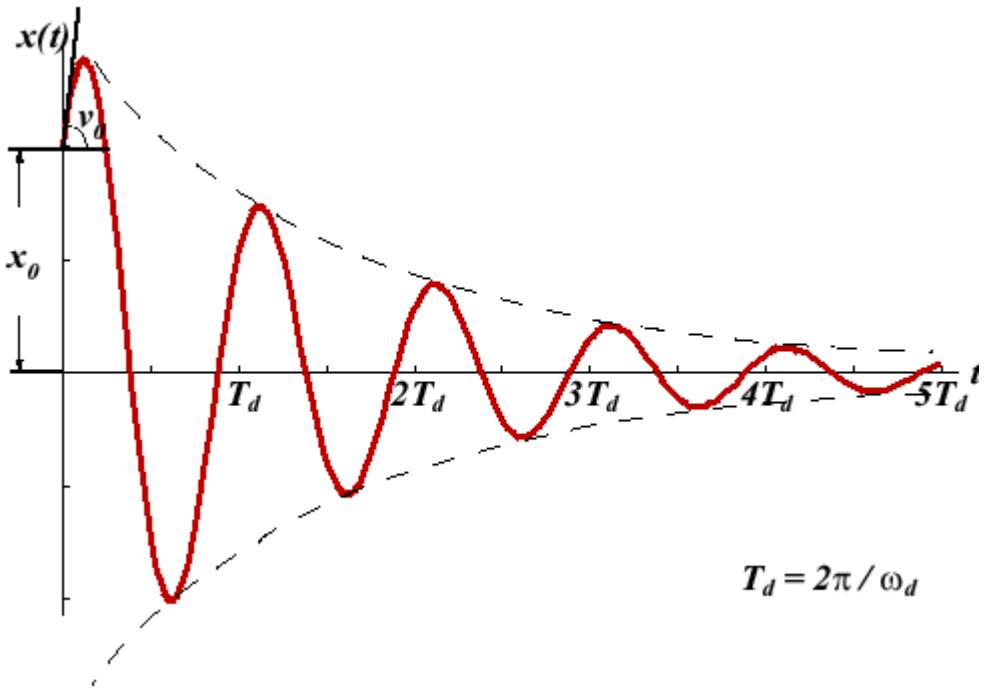
When $c_v^2 - 4mk < 0$ (equivalent to $\zeta < 1$ or $c_v < c_c$), the characteristic equation has a pair of complex conjugate roots. The displacement solution for this kind of system is,

$$\begin{aligned} x(t) &= c_1 e^{\left(-\zeta + i\sqrt{1-\zeta^2}\right)\omega_n t} + c_2 e^{\left(-\zeta - i\sqrt{1-\zeta^2}\right)\omega_n t} \\ &= e^{-\zeta\omega_n t}\left[d_1 \cos(\omega_d t) + d_2 \sin(\omega_d t)\right] \\ \Rightarrow x(t) &= \underbrace{e^{-\zeta\omega_n t}}_{\text{Exponentially decay}} \underbrace{\left[x_0 \cos(\omega_d t) + \frac{v_0 + \zeta\omega_n x_0}{\omega_d} \sin(\omega_d t)\right]}_{\text{Periodic motion}} \end{aligned}$$

An alternate but equivalent solution is given by,

$$x(t) = A_0 \underbrace{e^{-\zeta\omega_n t}}_{\text{Exponentially decay}} \underbrace{\cos(\omega_d t - \varphi_0)}_{\text{Periodic}}$$

The displacement plot of an under damped system would appear as,



(Figure 3)

Note that the displacement amplitude decays exponentially (i.e. the natural logarithm of the amplitude ratio for any two displacements separated in time by a constant ratio is a constant; long-winded!),

$$\frac{A_k}{A_{k+1}} = \frac{A_0 e^{-\zeta\omega_n(kT_d)}\cos(\varphi_0)}{A_0 e^{-\zeta\omega_n[(k+1)T_d]}\cos(\varphi_0)} = \frac{e^{-\zeta\omega_n(kT_d)}}{e^{-\zeta\omega_n[(k+1)T_d]}} = e^{\zeta\omega_n T_d}$$

$$\Rightarrow \ln\left(\frac{A_k}{A_{k+1}}\right) = \zeta\omega_n T_d = \zeta\omega_n \frac{2\pi}{\omega_d} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

where $T_d = \frac{1}{f_d} = \frac{2\pi}{\omega_d}$ is the period of the damped vibration.

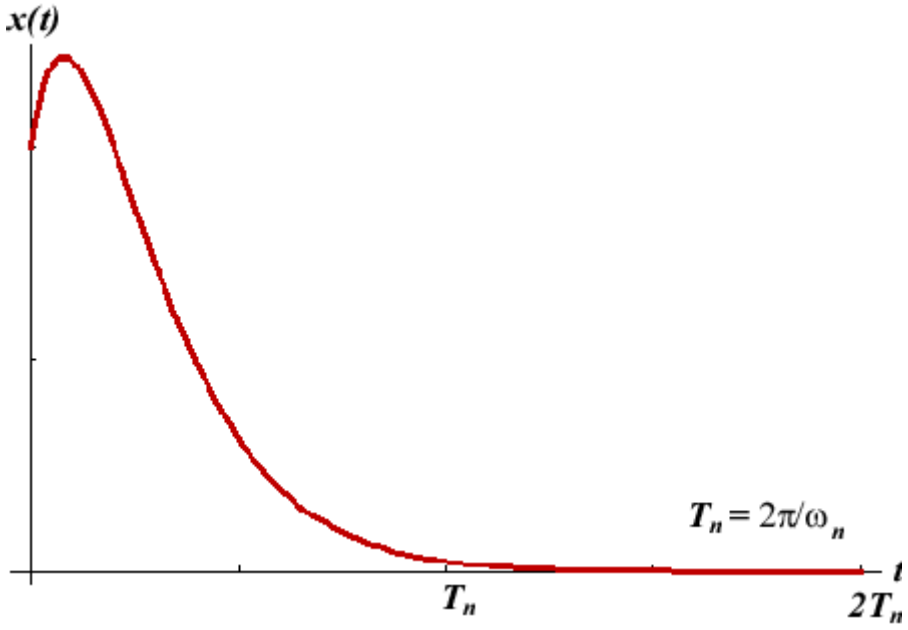
Critically-Damped Systems

When $c_v^2 - 4mk = 0$ (equivalent to $\zeta = 1$ or $c_v = c_c$), the characteristic equation has repeated real roots. The displacement solution for this kind of system is,

$$x(t) = (c_1 + c_2 t)e^{-\omega_n t}$$
$$\Rightarrow x(t) = e^{-\omega_n t}\left[x_0 + (v_0 + \omega_n x_0)t\right]$$

The critical damping factor c_c can be interpreted as the *minimum damping* that results in non-periodic motion (i.e. simple decay).

The displacement plot of a critically-damped system with positive initial displacement and velocity would appear as,



(Figure 4)

The displacement decays to a negligible level after one natural period, T_n. Note that if the initial velocity v_0 is negative while the initial displacement x_0 is positive, there will exist one overshoot of the resting position in the displacement plot.

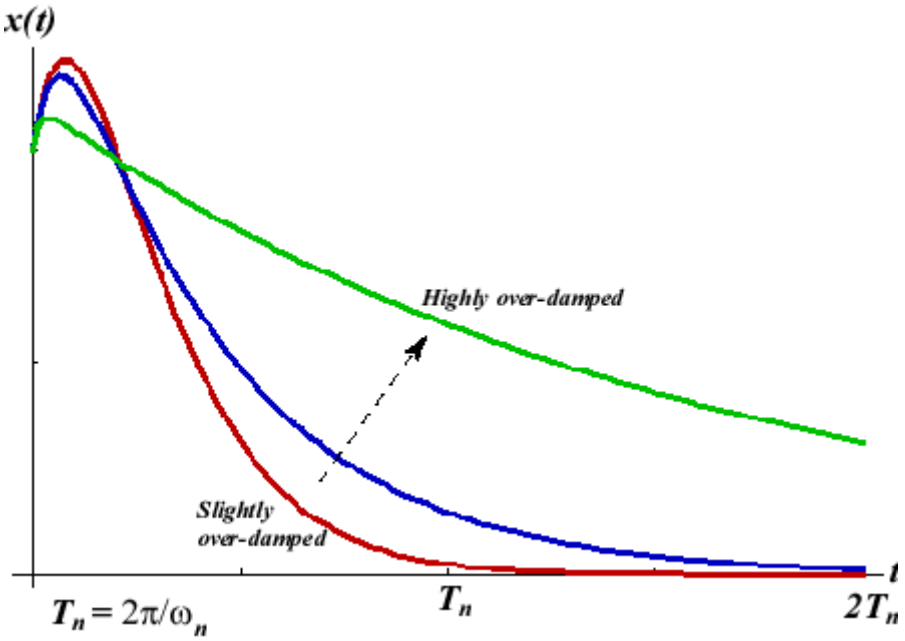
Over damped Systems

When $c_v^2 - 4mk > 0$ (equivalent to $\zeta > 1$ or $c_v > c_c$), the characteristic equation has two distinct real roots. The displacement solution for this kind of system is,

$$x(t) = c_1 e^{\left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n t} + c_2 e^{\left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n t}$$

$$\Rightarrow x(t) = \frac{x_0 \omega_n \left(\zeta + \sqrt{\zeta^2 - 1}\right) + v_0}{2\omega_n \sqrt{\zeta^2 - 1}} e^{\left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n t} + \frac{-x_0 \omega_n \left(\zeta - \sqrt{\zeta^2 - 1}\right) - v_0}{2\omega_n \sqrt{\zeta^2 - 1}} e^{\left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n t}$$

The displacement plot of an over damped system would appear as,



(Figure 5)

The motion of an over damped system is non-periodic, regardless of the initial conditions. The larger the damping, the longer the time to decay from an initial disturbance.

If the system is heavily damped, $\zeta \gg 1$, the displacement solution takes the approximate form,

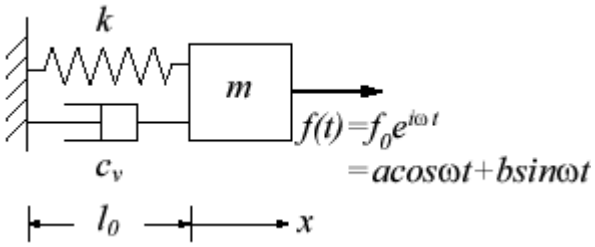
$$x(t) \approx x_0 + \frac{v_0}{2\zeta\omega_n}\left(1 - e^{-2\zeta\omega_n t}\right)$$

2.1.2) SDOF Systems under Harmonic Excitation

When a SDOF system is forced by f(t), the solution for the displacement x(t) consists of two parts: the complimentary solution, and the particular solution. The complimentary solution for the problem is given by the free vibration discussion. The particular solution depends on the nature of the forcing function.

When the forcing function is harmonic (i.e. it consists of at most a sine and cosine at the same frequency, a quantity that can be expressed by the complex exponential $e^{i\omega t}$), the method of undetermined coefficients can be used to find the particular solution. Non-harmonic forcing functions are handled by other techniques.

Consider the SDOF system forced by the harmonic function f(t),



(Figure 6)

The particular solution for this problem is found to be,

$$x_p(t) = \frac{f_0}{(k - m\omega^2) + ic\omega} e^{i\omega t}$$

The general solution is given by the sum of the complimentary and particular solutions multiplied by two weighting constants c_1 and c_2,

$$x(t) = c_1 \cdot x_c(t) + c_2 \cdot x_p(t)$$

The values of c_1 and c_2 are found by matching x(t = 0) to the initial conditions.

2.1.3) Undamped SDOF Systems under Harmonic Excitation

For an un damped system ($c_v = 0$) the total displacement solution is,

$$x(t) = d_1 \cos\omega_n t + d_2 \sin\omega_n t + \frac{f_0}{k - m\omega^2} e^{i\omega t}$$

$$\Rightarrow x(t) = \left(x_0 - \frac{f_0}{k - m\omega^2}\right)\cos\omega_n t + \left(\frac{v_0 - \frac{i\omega f_0}{k - m\omega^2}}{\omega_n}\right)\sin\omega_n t + \frac{f_0}{k - m\omega^2} e^{i\omega t}$$

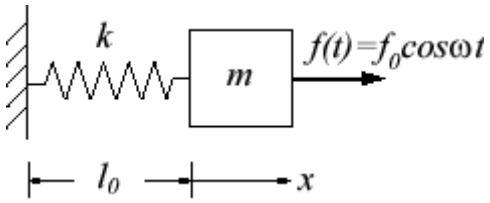
If the forcing frequency is close to the natural frequency, $\omega \cong \omega_n$, the system will exhibit resonance (very large displacements) due to the near-zeros in the denominators of x(t).

When the forcing frequency is equal to the natural frequency, we cannot use the x(t) given above as it would give divide-by-zero. Instead, we must use L hospitals Rule to derive a solution free of zeros in the denominators,

$$x(t) = x_0 \cos\omega_n t + \frac{v_0}{\omega_n}\sin\omega_n t + \lim_{\omega \to \omega_n}\left\{\frac{f_0}{k - m\omega^2}\left(e^{i\omega t} - \cos\omega_n t - i\omega\sin\omega_n t\right)\right\}$$

$$= x_0 \cos\omega_n t + \frac{v_0}{\omega_n}\sin\omega_n t - \frac{f_0\omega_n}{2k}\left(ite^{i\omega_n t} - i\sin\omega_n t\right)$$

To simplify x(t), let's assume that the driving force consists only of the cosine function, $f(t) = f_0 \cos \omega t$,



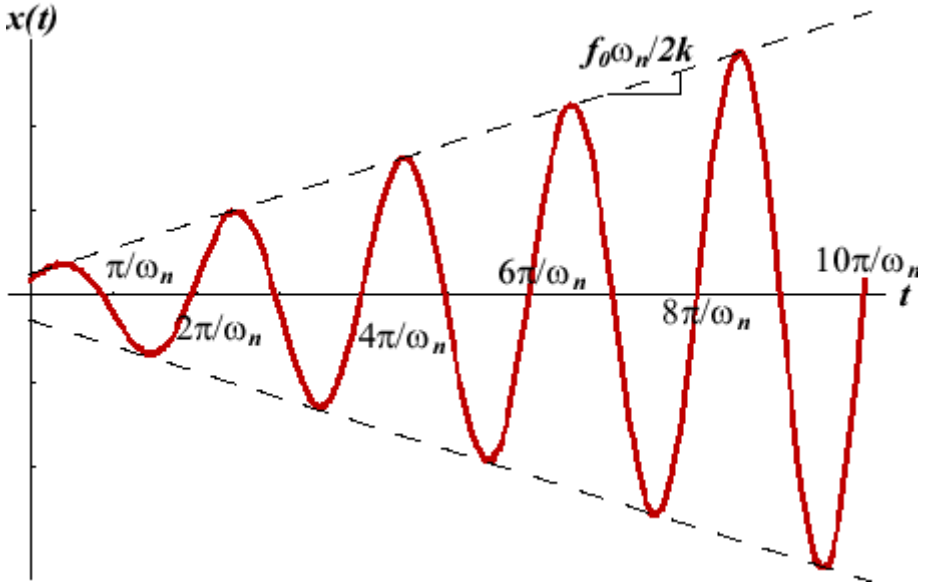
(Figure 7)

The displacement solution reduces to,

$$x(t) = x_0 \cos \omega_n t + \frac{v_0}{\omega_n} \sin \omega_n t + \lim_{\omega \to \omega_n} \left\{ \frac{f_0}{k - m\omega^2} \left(\cos \omega t - \cos \omega_n t \right) \right\}$$

$$= \underbrace{x_0 \cos \omega_n t + \frac{v_0}{\omega_n} \sin \omega_n t}_{\text{Free vibration (complementary) solution}} + \underbrace{\frac{f_0 \omega_n t}{2k}}_{\text{Amplitude linearly increased}} \sin \omega_n t$$

This solution contains one term multiplied by t. This term will cause the displacement amplitude to increase linearly with time as the forcing function pumps energy into the system, as shown in the following displacement plot,



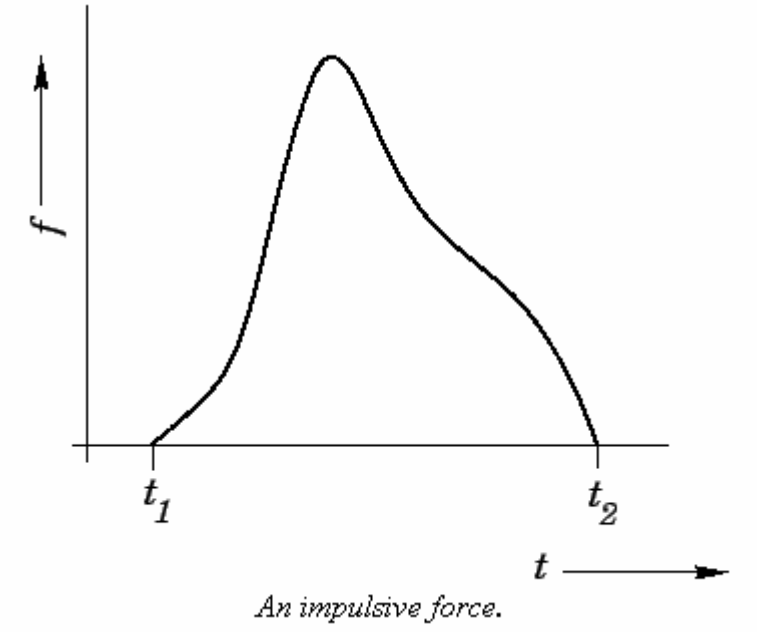
(Figure 8)

The maximum displacement of an un damped system forced at its resonant frequency will increase unbounded according to the solution for x(t) above. However, real systems will inject additional physics once displacements become large enough. These additional physics (nonlinear plastic deformation, heat transfer, buckling, etc.) will serve to limit the maximum displacement exhibited by the system, and allow one to escape the "sudden death" impression that such systems will immediately fail.

2.1.4) SDOF Systems under General Dynamic Loading

Impulsive Force

An Impulsive Loading is a Load which is applied during a short duration of time.



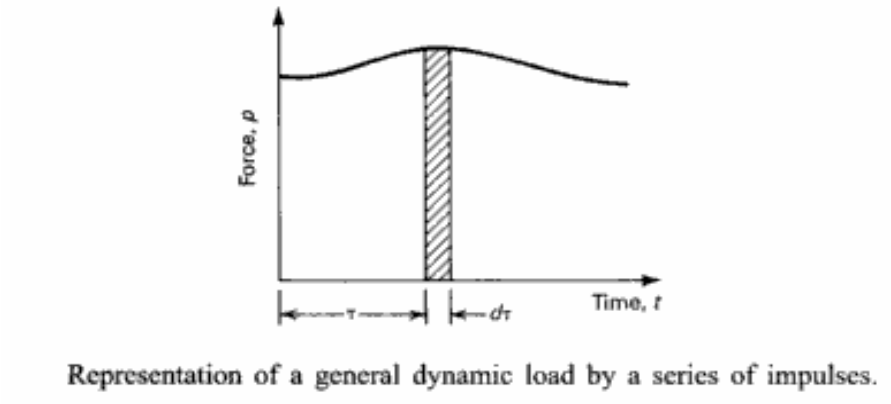
An impulsive force.

(Figure 9)

The above figure shows the typical time history of an impulsive force, f(t), It can be seen that the force is only non-zero in the short time interval t1 to t2. It is helpful to define a quantity known as the net *impulse*, I, associated with $f(t)$:

$$I = \int_{t_1}^{t_2} f(t)\, dt.$$

I is the total area under the $f(t)$ curve shown.



Representation of a general dynamic load by a series of impulses.

(Figure 10)

The Total Displacement of a UN damped Single degree of freedom system with any orbitary load is given by **CLASSIC SOLUTION** as

$$u(t) = e^{-\omega\xi t}\left(u_0 \cos\omega_d t + \frac{v_0 + \omega\xi u_0}{\omega_d}\sin\omega_d t\right) + \frac{1}{m\omega_d}\int_0^t p(\tau)\, e^{-\xi\omega(t-\tau)} \sin\omega_d(t-\tau)\, d\tau$$

2.2 Multiple Degree –of –Freedom SYSTEM

Multi degree-of-freedom system has the same basic form of the governing equation as a single degree-of-freedom system.

The difference is that it is a matrix equation:

$$\underset{\sim}{m}\ddot{q} + \underset{\sim}{k}q = \underset{\sim}{F}$$

~ = matrix

So apply the same solution technique as for a single degree-of-freedom system.

Free Vibration

$$\underset{\sim}{F} = \underset{\sim}{0}$$

$$\underset{\sim}{m}\ddot{\underset{\sim}{q}} + \underset{\sim}{k}\underset{\sim}{q} = \underset{\sim}{0}$$

Again assume a solution which has harmonic motion. It now has multiple components

$$\underset{\sim}{q}(t) = \underset{\sim}{A}e^{i\omega t}$$

where ω are the natural frequencies of the system

and

$\underset{\sim}{A}$ is a vector of constants $= \begin{Bmatrix} \vdots \\ A_i \\ \vdots \end{Bmatrix}$

Substituting the assumes solution in to the matrix set

of governing equations :

$$-\omega^2[m][A]\, e^{i\omega t} + [K]\,[A]\, e^{i\omega t} = [0]$$

CHAPTER 3

Dynamic Analysis By Numerical Integration

DYNAMIC ANALYSIS BY NUMERICAL INTEGRATION

3.1 INTRODUCTION

The most general approach for the solution of the dynamic response of structural Systems is the direct numerical integration of the dynamic equilibrium equations. This involves, after the solution is defined at time zero, the attempt to satisfy dynamic equilibrium at discrete points in time. Most methods use equal time intervals at D t, 2Dt, 3Dt.....NDt. Many different numerical techniques have previously been presented; however, all approaches can fundamentally be classified as either explicit or implicit integration methods.

Explicit methods do not involve the solution of a set of linear equations at each step. Basically, these methods use the differential equation at time " t " to predict a solution at time "t+Dt". For most real structures, which contain stiff elements, a very small time step is required in order to obtain a stable solution. Therefore, all explicit methods are conditionally stable with respect to the size of the time step.

Implicit methods attempt to satisfy the differential equation at time " t " after the solution at time "t-Dt" is found. These methods require the solution of a set of linear equations at each time step; however, larger time steps may be used. Implicit methods can be conditionally or unconditionally stable.

There exist a large number of accurate, higher-order, multi-step methods that have been developed for the numerical solution of differential equations. These multistep methods assume that the solution is a smooth function in which the higher derivatives are continuous. The exact solution of many nonlinear structures requires that the accelerations, the second derivative of the displacements, are not smooth functions. This discontinuity of the acceleration is caused by the nonlinear hysteresis of most structural materials, contact between parts of the structure, and buckling of elements.

3.2 The NUMERICAL SOLUTION can be calculated by various methods

- Duhamel Integral
- Newmark Integration method
- Central difference Method
- Houbolt Method
- Wilson θ Method

3.3The Newmark method

The Newmark method consists of the following equations [#! Newm1959!#]

$$\mathbf{M}\,{}^{t+\Delta t}\ddot{\mathbf{q}} + {}^{t+\Delta t}\mathbf{D}\,{}^{t+\Delta t}\dot{\mathbf{q}} + {}^{t+\Delta t}\mathbf{K}\,{}^{t+\Delta t}\mathbf{q} = {}^{t+\Delta t}\mathbf{F}^{\mathrm{ext}},$$

$$\begin{aligned} {}^{t+\Delta t}\mathbf{q} &= {}^{t}\mathbf{q} + \Delta t\dot{\mathbf{q}} + \frac{1}{2}\Delta t^2\left((1-2\beta)\,{}^{t}\ddot{\mathbf{q}} + 2\beta\,{}^{t+\Delta t}\ddot{\mathbf{q}}\right), \\ {}^{t+\Delta t}\dot{\mathbf{q}} &= {}^{t}\dot{\mathbf{q}} + \Delta t\left((1-\gamma)\,{}^{t}\ddot{\mathbf{q}} + \gamma\,{}^{t+\Delta t}\ddot{\mathbf{q}}\right), \end{aligned}$$

which are used for the determination of three unknowns ${}^{t+\Delta t}\ddot{\mathbf{q}}$, ${}^{t+\Delta t}\dot{\mathbf{q}}$ and ${}^{t+\Delta t}\mathbf{q}$.

Summary of the Newmark Method for Direct Integration

I. INITIAL CALCULATION

A. Form static stiffness matrix **K** , mass matrix **M** and damping matrix **C**

B. Specify integration parameters β and γ

C. Calculate integration constants

$$a_0 = 1/\left(\beta\Delta t^2\right), \quad a_1 = \gamma/\left(\beta\Delta t\right), \quad a_2 = 1/\left(\beta\Delta t\right),$$
$$a_3 = 1/\left(2\beta\right) - 1, \quad a_4 = \gamma/\beta - 1, \quad a_5 = \frac{1}{2}\Delta t\left(\gamma/\beta - 2\right),$$
$$a_6 = \Delta t\left(1-\gamma\right), \quad a_7 = \Delta t\gamma.$$

D. Form effective stiffness matrix

$$\mathbf{K}^{\text{eff}} = \mathbf{K} + a_0\mathbf{M} + a_1\mathbf{D};$$

E. Triangularize effective stiffness matrix

$$\mathbf{K}^{\text{eff}} = \mathbf{L}\mathbf{D}\mathbf{L}^{\mathrm{T}}$$

F. Specify initial conditions

II. FOR EACH TIME STEP t = Δt, 2Δt, 3Δt - - - - - -

A. Calculate effective load vector

$$\mathbf{F}^{\text{eff}} = {}^{t+\Delta t}\mathbf{F}^{\text{ext}} + \mathbf{M}\left(a_0\,{}^{t}\mathbf{q} + a_2\,{}^{t}\dot{\mathbf{q}} + a_3\,{}^{t}\ddot{\mathbf{q}}\right) + \mathbf{D}\left(a_1\,{}^{t}\mathbf{q} + a_4\,{}^{t}\dot{\mathbf{q}} + a_5\,{}^{t}\ddot{\mathbf{q}}\right)$$

B. Solve for node displacement vector at time **t**

C. Calculate node velocities and accelerations at time **tss**

$${}^{t+\Delta t}\mathbf{q} = {}^{t}\mathbf{q} + \Delta t\dot{\mathbf{q}} + \frac{1}{2}\Delta t^2\left(\left(1-2\beta\right){}^{t}\ddot{\mathbf{q}} + 2\beta\,{}^{t+\Delta t}\ddot{\mathbf{q}}\right),$$
$${}^{t+\Delta t}\dot{\mathbf{q}} = {}^{t}\dot{\mathbf{q}} + \Delta t\left(\left(1-\gamma\right){}^{t}\ddot{\mathbf{q}} + \gamma\,{}^{t+\Delta t}\ddot{\mathbf{q}}\right),$$

3.4Duhamel's integral

The response of a linear, viscously damped single degree of freedom (SDF) system to a time-varying mechanical excitation p(t) is given by the following second-orderordinary differential equation

$$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + kx(t) = p(t)$$

where m is the (equivalent) mass, x stands for the amplitude of vibration, t for time, c for the viscous damping coefficient, and k for the stiffness of the system or structure.

If a system is initially rest at its equilibrium position, from where it is acted upon by a unit-impulse at the instance t=0, i.e., p(t) in the equation above is a delta function δ(t), $x(0) = \left.\frac{dx}{dt}\right|_{t=0} = 0$, then by solving the differential equation one can get a fundamental solution (known as a unit-impulse response function)

$$h(t) = \begin{cases} \frac{1}{m\omega_d} e^{-\varsigma\omega_n t}\sin\omega_d t, & t > 0 \\ 0, & t < 0 \end{cases}$$

where $\varsigma = \frac{c}{2m\omega_n}$ is called the damping ratio of the system, ω_n is the natural angular frequency of the un damped system (when c=0) and $\omega_d = \omega_n\sqrt{1-\varsigma^2}$ is the angular frequency when damping effect is taken into account (when $c \neq 0$). If the impulse happens at t=τ instead of t=0, i.e. p(t) = δ(t − τ), the impulse response is

$$h(t-\tau) = \frac{1}{m\omega_d} e^{-\varsigma\omega_n(t-\tau)}\sin[\omega_d(t-\tau)] \quad t \geq \tau$$

CHAPTER 4

Plane frame Analysis

STEPS INVOLVED IN SOLVING 2 – D PLANE FRAME:

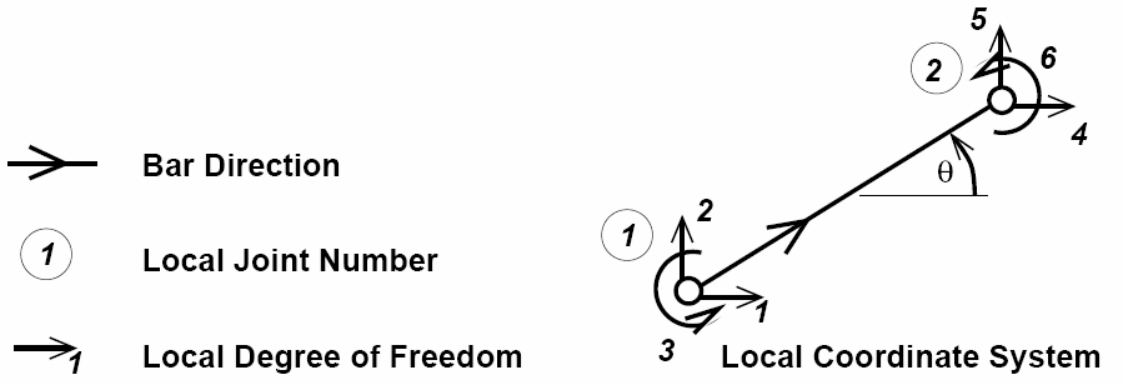
(Multi Degree Of Freedom System)

1. Identify Degrees of Freedom. Number all the global degrees of freedom in your frame. In a planar frame, every joint has three degrees of freedom: one in the global X-direction, one in the global Y-direction, and one rotation about the global Z-axis (counter-clockwise). Using the method in this handout, every joint gets three degrees of freedom

2. Number all of the elements.

3. Joint Coordinates: Write the coordinates of each joint using units consistent with E and I. In other words, if E and I are given in kN/cm2 and cm4, write the coordinates, (x; y), in terms of centimeters.

4. Define each element: Draw each element of your frame individually and draw the local coordinates in the global directions. For example if element number N is a diagonal beam element, and the global directions are X: horizontal and Y: vertical, draw element number N like this:



(Figure 11)

where 1,2,3,4,5,6 are the LOCAL coordinates of the beam element in the GLOBAL directions. The local coordinates are always numbered 1,2,3,4 with 1 and 4 pointing in

the global X direction (to the right), with 2 and 5 pointing in the global Y direction (up), and with 3 and 6 rotating about the global Z-axis (counter-clockwise). All of these six coordinates will line up with with the global degrees of freedom that you identified in step 1., above.

5. Element Stiffness Matrices in Global Coordinates, K.

For each element, find it's (6 x 6) element stiffness matrix, by evaluating the equations

$$\mathrm{K} = \begin{bmatrix} \frac{EA}{L}c^2 + \frac{12EI}{L^3}s^2 & \frac{EA}{L}cs - \frac{12EI}{L^3}cs & -\frac{6EI}{L^2}s & -\frac{EA}{L}c^2 - \frac{12EI}{L^3}s^2 & -\frac{EA}{L}cs + \frac{12EI}{L^3}cs & -\frac{6EI}{L^2}s \\ & \frac{EA}{L}s^2 + \frac{12EI}{L^3}c^2 & \frac{6EI}{L^2}c & -\frac{EA}{L}cs + \frac{12EI}{L^3}cs & -\frac{EA}{L}s^2 - \frac{12EI}{L^3}c^2 & \frac{6EI}{L^2}c \\ & & \frac{4EI}{L} & \frac{6EI}{L^2}s & -\frac{6EI}{L^2}c & \frac{2EI}{L} \\ & & & \frac{EA}{L}c^2 + \frac{12EI}{L^3}s^2 & \frac{EA}{L}cs - \frac{12EI}{L^3}cs & \frac{6EI}{L^2}s \\ & \text{SYM} & & & \frac{EA}{L}s^2 + \frac{12EI}{L^3}c^2 & -\frac{6EI}{L^2}c \\ & & & & & \frac{4EI}{L} \end{bmatrix}$$

where

$$c = \cos\theta = \frac{x_2 - x_1}{L}$$

$$s = \sin\theta = \frac{y_2 - y_1}{L}$$

6. Structural Stiffness Matrix, Ks.

The structural stiffness matrix is a square, symmetric, matrix with dimension equal to the number of degrees of freedom. In this step we will fill up the structural stiffness matrix using terms from the element stiffness matrices in global coordinates (from step 5.) This procedure is called matrix assembly.

Recall from step 4. how the LOCAL element degrees of freedom (1,2,3,4,5,6) line up with the GLOBAL degrees of freedom in your problem. For example, local coordinates (1,2,3,4,5,6) might line up with degrees of freedom (13,14,15,7,8,9) of the frame. In that case:
K(1,1) is added to Ks(13,13),
K(1,2) is added to Ks(13,14),
...
K(2,6) is added to Ks(14,9),
...
K(5,6) is added to Ks(8,9),
K(6,6) is added to Ks(9,9),
Add each element into the structural stiffness matrix in this way to build up Ks

7. Reactions: Ks must be modified to include the effects of the reactions, which have been ignored up until now. Set every element of each row and column corresponding to a restrained degree of freedom (reaction) equal to zero. Set every diagonal element that is zero equal to 1.

8. External Loads, p: Create the load vector p, by finding the fixed-end forces and moments of each member, and their components in the directions of the global degrees of freedom. Add the fixed end forces and moments to any point loads applied directly to the joints. Create the force vector by placing these force components into the force vector at the proper coordinates.

9. Deflections, d: Find the deflections by inverting the stiffness matrix and multiplying it by the load vector.

10. Internal beam forces, q: Again, recall how the global degrees of freedom line up with each element's coordinates (1,2 ,3,4, 5,6). For example, in element number "N" from step 6., the local element deflections v1,v2,v3,v4,v5,v6 line up with the global deflections d13,d14,d15,d7,d8,d9.

11. Formation of MASS Matrix: The mass matrix in local coordinate system for an element is given by $[M_C^{-e}]$, which is multiplied by the transformation matrix $[T^e]$ to get in global coordinate system

$$\bar{M}_C^e = \frac{\rho A \ell}{420}\begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22\ell & 0 & 54 & -13\ell \\ 0 & 22\ell & 4\ell^2 & 0 & 13\ell & -3\ell^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13\ell & 0 & 156 & -22\ell \\ 0 & -13\ell & -3\ell^2 & 0 & -22\ell & 4\ell^2 \end{bmatrix}, \quad T^e = \begin{bmatrix} c_\varphi & s_\varphi & 0 & 0 & 0 & 0 \\ -s_\varphi & c_\varphi & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_\varphi & s_\varphi & 0 \\ 0 & 0 & 0 & -s_\varphi & c_\varphi & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Similarly as described in step 6 the Global Mass Matrix is formed.

12. Dynamic Analysis: Now the dynamic analysis is carried out by substituting the Mass matrix [M] and stiffness matrix [K] in the Dynamic Equilibrium equation given by

$$\mathbf{M}\,\ddot{\mathbf{u}}(t)_a + \mathbf{C}\,\dot{\mathbf{u}}(t)_a + \mathbf{K}\,\mathbf{u}(t)_a = \mathbf{F}(t)$$

in which M is the mass matrix (lumped or consistent), C is a viscous damping matrix (which is normally selected to approximate energy dissipation in the real structure) and K is the static stiffness matrix for the system of structural elements.

The time-dependent vectors $u(t)_a$, $u'(t)a$,and $u''(t)a$ are the absolute node displacements, velocities and accelerations, respectively.

13. Free Vibration: In this case the Eigen values and Eigen vectors are calculated by using MATLAB program me and then the results are plotted and discussed.

14. Harmonic Vibration: In this case the displacement u(t), velocity $u'(t)$, acceleration $u''(t)$, vectors are calculated at different times by using the Newmark Integration Method (numerical method) for the given 2D plane frame system using MATLAB program me and the results are plotted and discussed.

CHAPTER 5

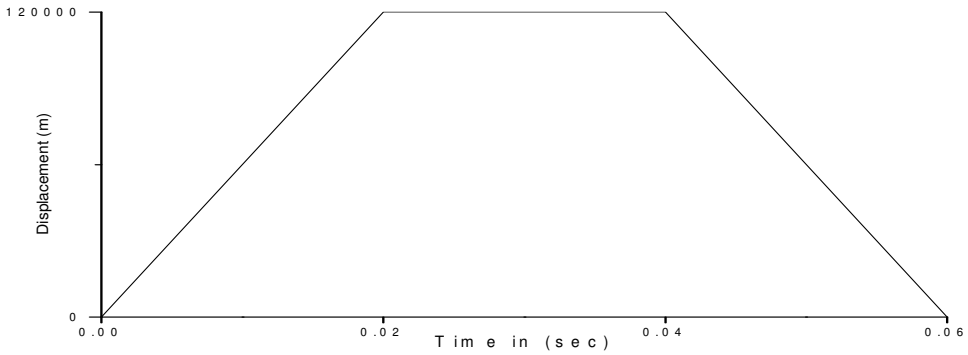
Problems Solved

PROBLEMS BEEN SOLVED DURING PROJECT:

COMPUTER CODE TO CALCULATE RESPONSE OF A STRUCTURE SUBJECTEDTOTHE FOLLOWING EXCITING FUNCTIONS USING Duhamel's INTEGRATION METHOD

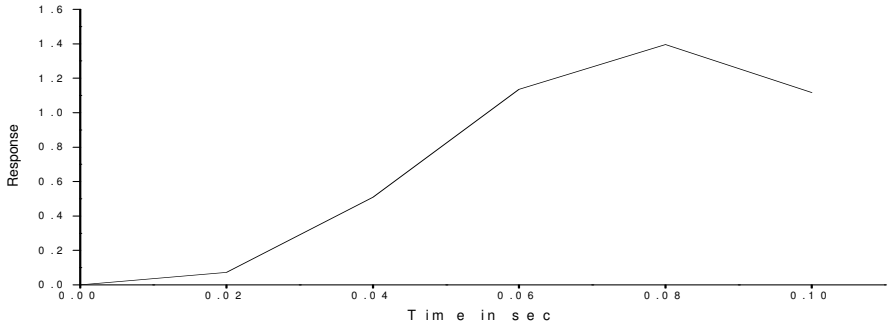
Data given K=100000 N/Sec-3 m=100kg

Problem 1) Forcing Function given by



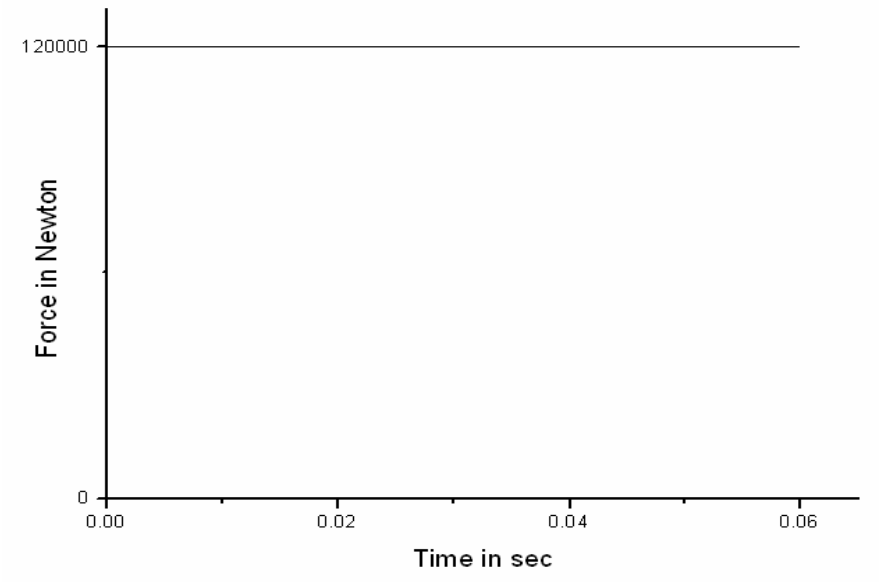
(Figure 12)

OUTPUT GRAPH CAN BE PLOTTED AS:



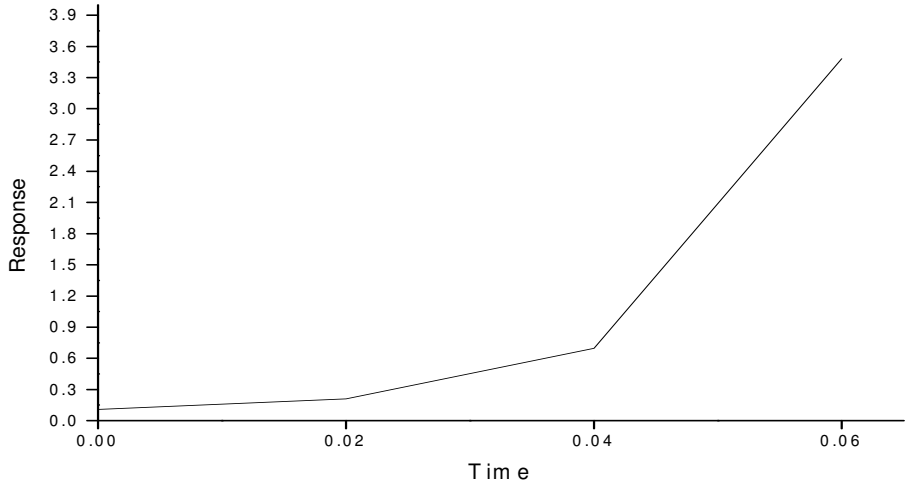
(Figure 13)

Problem 2) Forcing Function given by



(Figure 14)

OUTPUT GRAPH CAN BE PLOTTED AS:

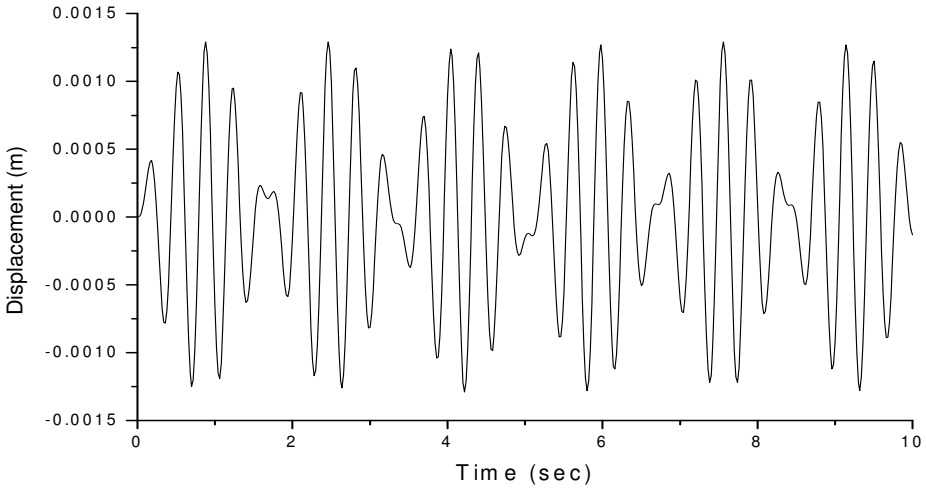


(Figure 15)

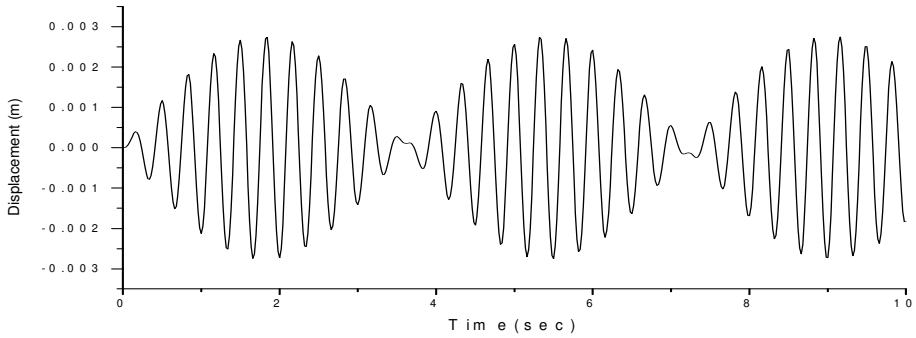
Problem 3) HARMONIC FORCING FUNCTION

F (t) = A Sin (ώ t)

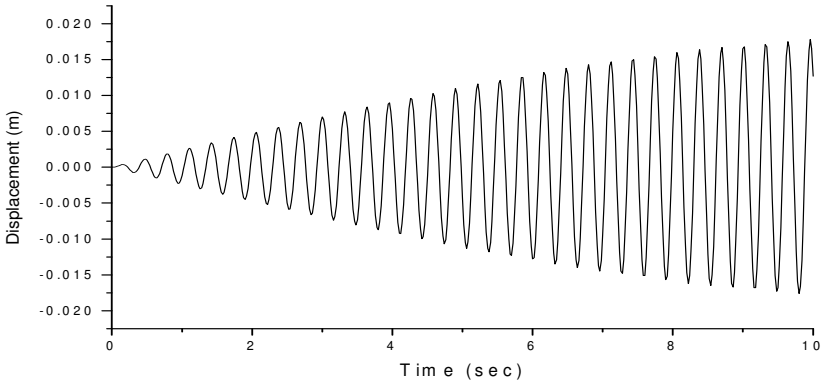
Where A= 1000N ; k=4000000N/m ; mass = 10000 kg ;natural frequency ω = 20 rad/s



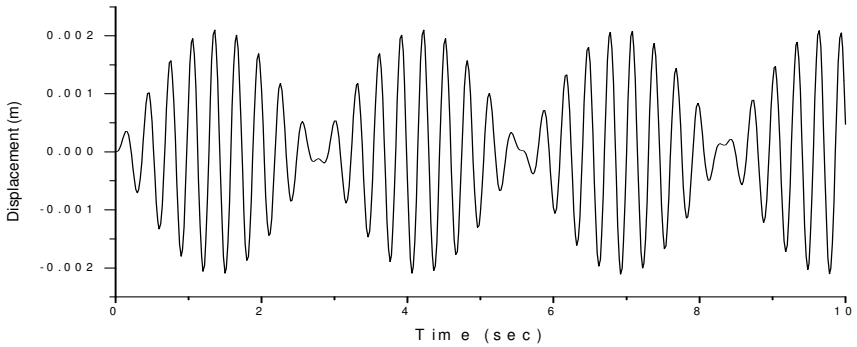
(Figure16) Forcing frequency ώ = 16 rad /sec



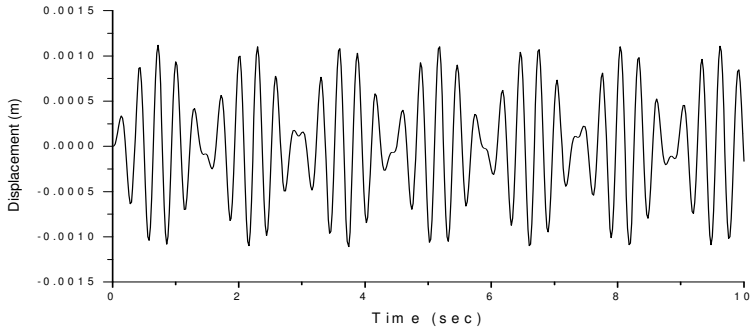
(Figure 17) Forcing frequency ώ = 18 rad /s



(Figure 18) Forcing frequency ώ = 20 rad /s

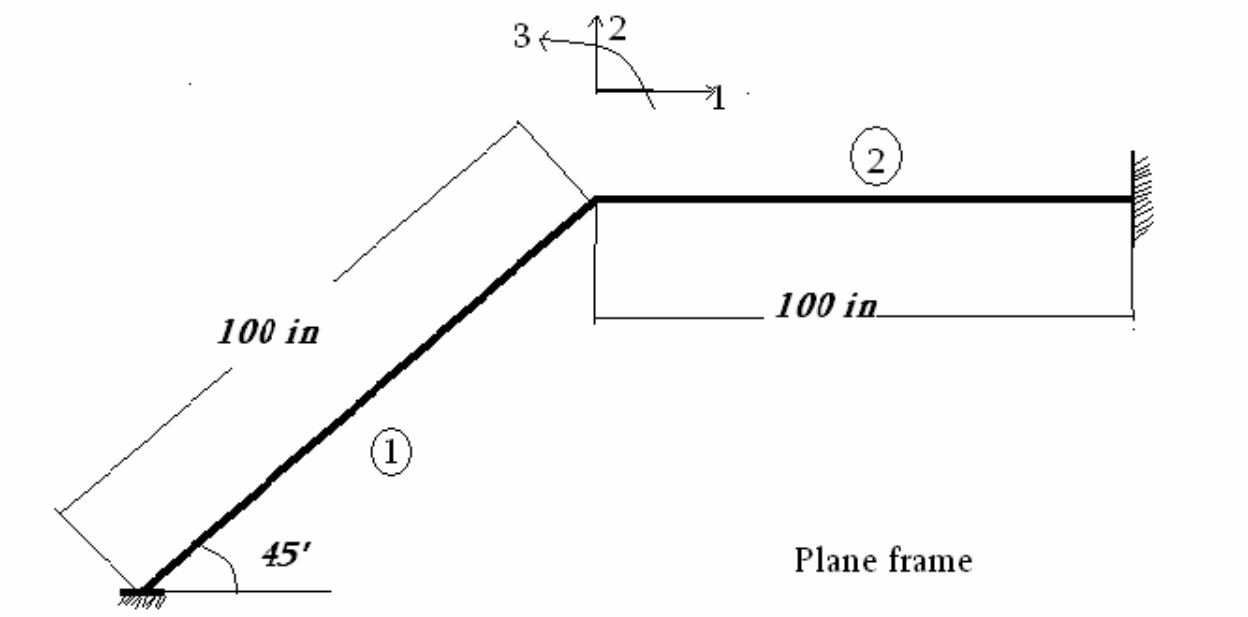


(Figure 19)Forcing frequency ώ = 22 rad/sec



(Figure 20)Forcing frequency ώ = 24 rad / sec

Problem 4) Free undamped vibration of 2 D plane Frame



(Figure 21)

A plane frame having two prismatic beam elements and three degrees of freedom as shown above

Using the consistent mass formulation determine the three natural frequencies and normal modes corresponding to discrete modal of frame

Given Area = 6 inch^2

Second moment of inertia = 100 inch^4

Young's modulus (E) = 10^7 lb/inch^2

After applying Boundary conditions the

Stiffness matrix is given by = [k] =

$$\begin{Bmatrix} 906000 & 294000 & 424000 \\ 294000 & 318000 & 176000 \\ 424000 & 176000 & 80000000 \end{Bmatrix}$$

Mass matrix is given by [M] =

$$\begin{Bmatrix} 288 & -8 & 1556 \\ -8 & 304 & 644 \\ 1556 & 644 & 80000 \end{Bmatrix}$$

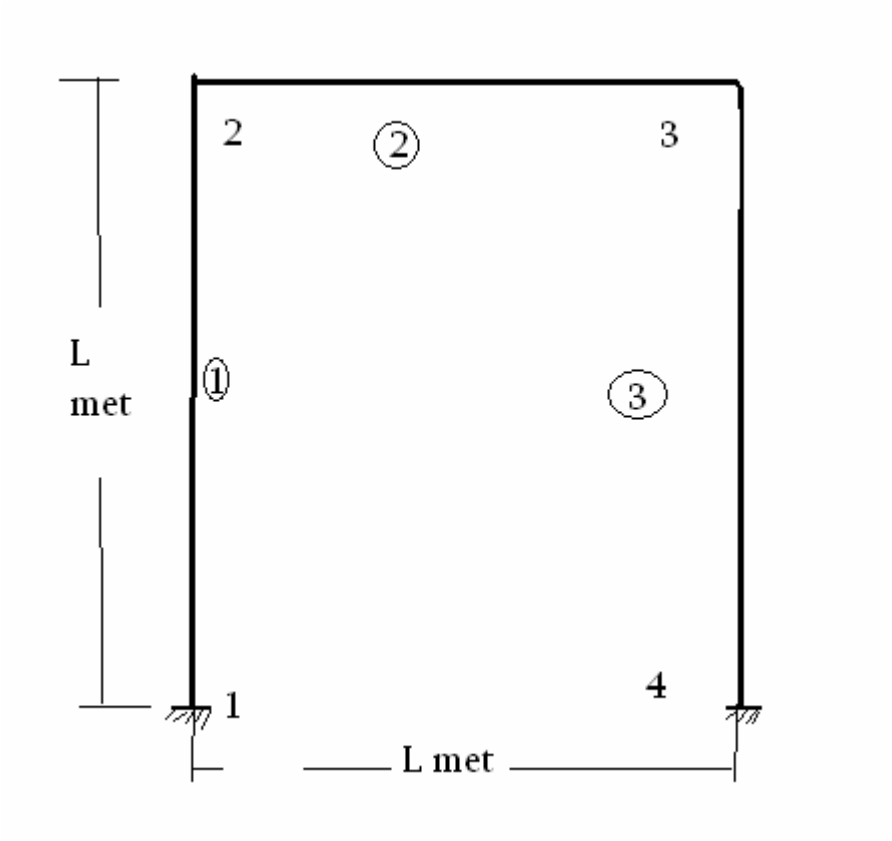
Eigen values as omega [ω] =

$$\begin{Bmatrix} 25.2686 & 0 & 0 \\ 0 & 31.2501 & 0 \\ 0 & 0 & 64.8984 \end{Bmatrix}$$

Eigen vectors as =

$$\begin{Bmatrix} 0.0218 & 0.0050 & -0.0583 \\ -0.0527 & 0.0021 & -0.0241 \\ 0.0000 & 0.0034 & 0.0016 \end{Bmatrix}$$

Problem 5)Free undamped vibration of 2 D plane Frame



(Figure 22)

A plane frame having three prismatic elements and three degrees of freedom as shown above

Using the consistent mass formulation determine the three natural frequencies and normal modes corresponding to discrete modal of frame

Given

Area = $1.85187*10^{-5}m^2$

E=$2.06829*10^{11}$ kg/cm^2

L= 0.2413 m

I=$2.85785*10^{-4}$

Density = 25613.5 kg/m^{-3}

After applying Boundary conditions the

$$K = 1.0e+012 * \begin{Bmatrix} 0.1578 & 0 & -0.0001 & -1.5870 & 0 & 0 \\ 0 & 1.5878 & 0.0001 & 0 & -0.0005 & 0.0001 \\ -0.0001 & 0.0001 & 0.0000 & 0 & -0.0001 & 0.0000 \\ -0.1570 & 0 & 0 & 1.5870 & 0 & -0.0001 \\ 0 & -0.0005 & -0.0001 & 0 & -1.5870 & -0.0001 \\ 0 & 0.0001 & 0.0000 & -0.0001 & -0.0001 & 0.0000 \end{Bmatrix}$$

$$M = 1.0e+003 * \begin{Bmatrix} 8.0663 & 0 & -0.1446 & 1.9070 & 0 & 0 \\ 0 & 8.0663 & 0.1446 & 0 & 1.4714 & -0.0855 \\ -0.1446 & 0.1446 & 0.0127 & 0 & 0.0855 & -0.0048 \\ 1.9070 & 0 & 0 & 8.0660 & 0 & -0.1446 \\ 0 & 1.4714 & 0.0855 & 0 & 8.0664 & -0.1446 \\ 0 & -0.0855 & -0.0048 & -0.1446 & -0.1446 & 0.0127 \end{Bmatrix}$$

Eigen values as omega [ω] =

1.0e+004 *

$$\begin{pmatrix} 2.0345 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 + 1.6044i & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.5504 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0252 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0918 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1983 \end{pmatrix}$$

Eigen vectors as =

$$\begin{pmatrix} -0.0024 & -0.3450 & -0.0000 & 0.0219 & -0.0117 & 0.0593 \\ 0.0012 & 0.0000 & 0.0001 & 0.0001 & -0.0997 & -0.0440 \\ 0.1464 & -1.0000 & 0.9991 & 1.0000 & 1.0000 & 1.0000 \\ -0.0100 & -0.0342 & 0.0000 & 0.0021 & 0.0272 & -0.0543 \\ -0.0837 & 0.0000 & -0.0001 & 0.0000 & 0.0048 & -0.0097 \\ -1.0000 & 0.1825 & -1.0000 & 0.8152 & 0.0677 & -0.650 \end{pmatrix}$$

Forced Vibration of 2D Plane Frame subjected to Harmonic Exciting Function

Problem 6) F(t)= 20000sin30(t) applied at node 2 in horizontal direction .for the frame shown in Problem 4 then calculate the displacement velocity and acceleration

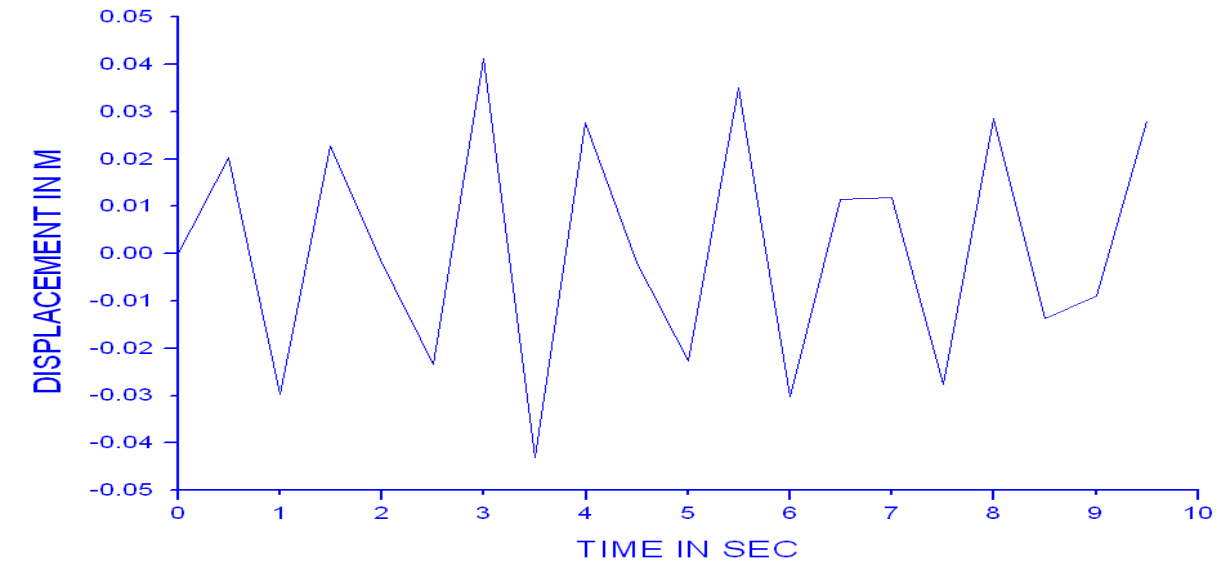
Output:

USING NEWMARK INTEGRATION METHOD AND MATLAB PROGRAM ME :

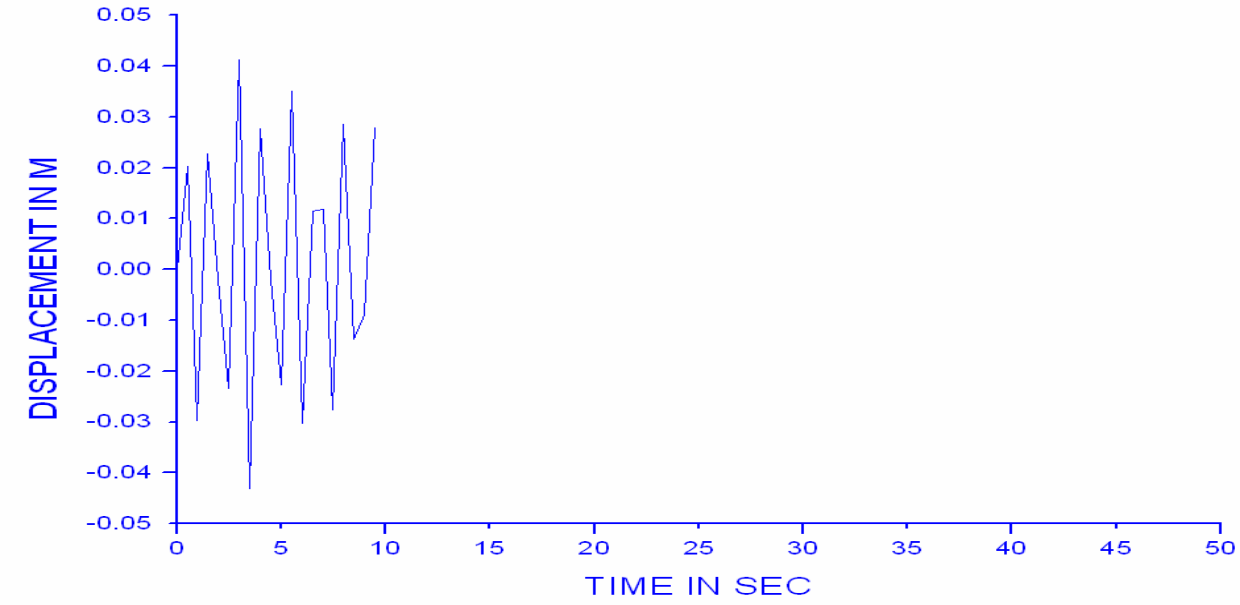
Table 1

Time(sec)	Displacement(m)	Velocity($ms^{-1)}$	Acceleration(ms-2)
0	0	0	0
0.5	0.0203	0.0810	0.3240
1.0	-0.0297	-0.2807	-1.7709
1.5	0.0227	0.4902	4.8544
2.0	-0.0019	-0.5885	-9.1692
2.5	-0.0234	0.5026	13.337
3.0	0.0412	-0.2442	-16.5209
3.5	-0.0431	-0.0929	17.1262
4.0	0.0277	0.3759	-15.2510
4.5	-0.0019	-0.4943	11.7701
5.0	-0.0226	0.4116	-8.1465
5.5	0.035	-0.1810	5.7761
6.0	-0.0302	-0.08	-5.3719
6.5	0.0114	0.2464	6.6773
7.0	0.0118	-0.2447	-8.6415
7.5	-0.0277	0.0867	9.9669
8.0	0.0285	0.1380	-9.7618
8.5	-0.0138	-0.3071	7.9815
9.0	-0.0089	0.3270	-5.4448

Graphs :



(Figure 23)



(Figure 24)

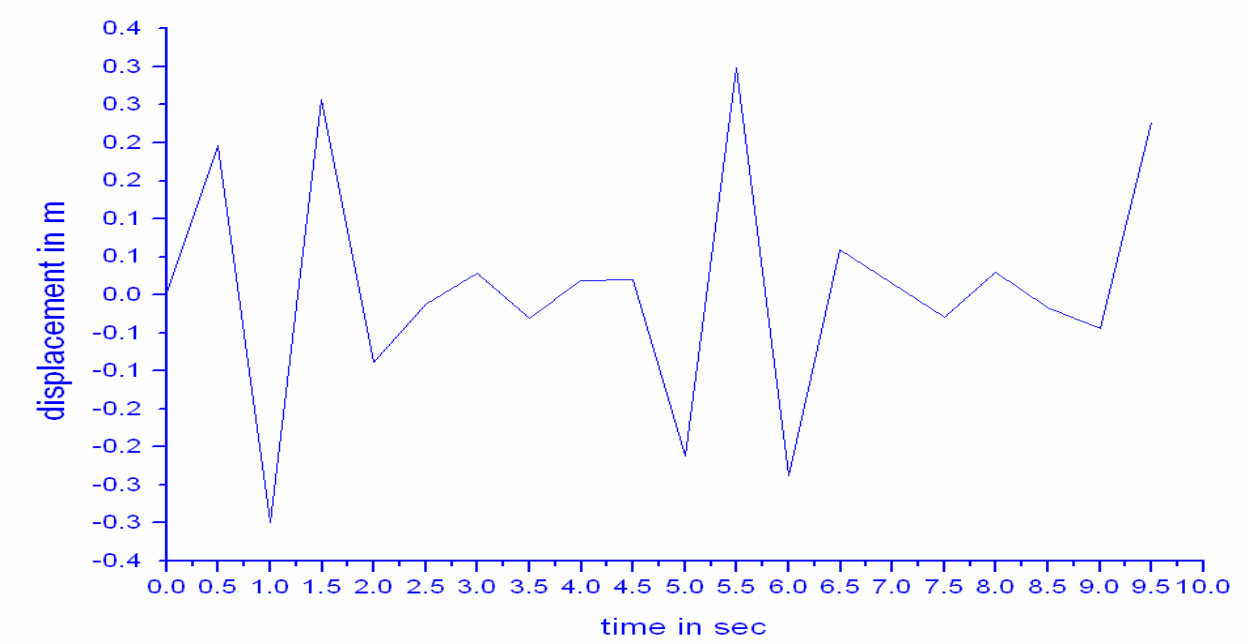
Problem 7) F(t)= 20000sin30(t) applied at node 2 in horizontal direction for the plane frame shown in Problem 5.then calculate the displacement velocity and acceleration ….

Output: USING NEWMARK INTEGRATION METHOD AND MATLAB PROGRAMME :

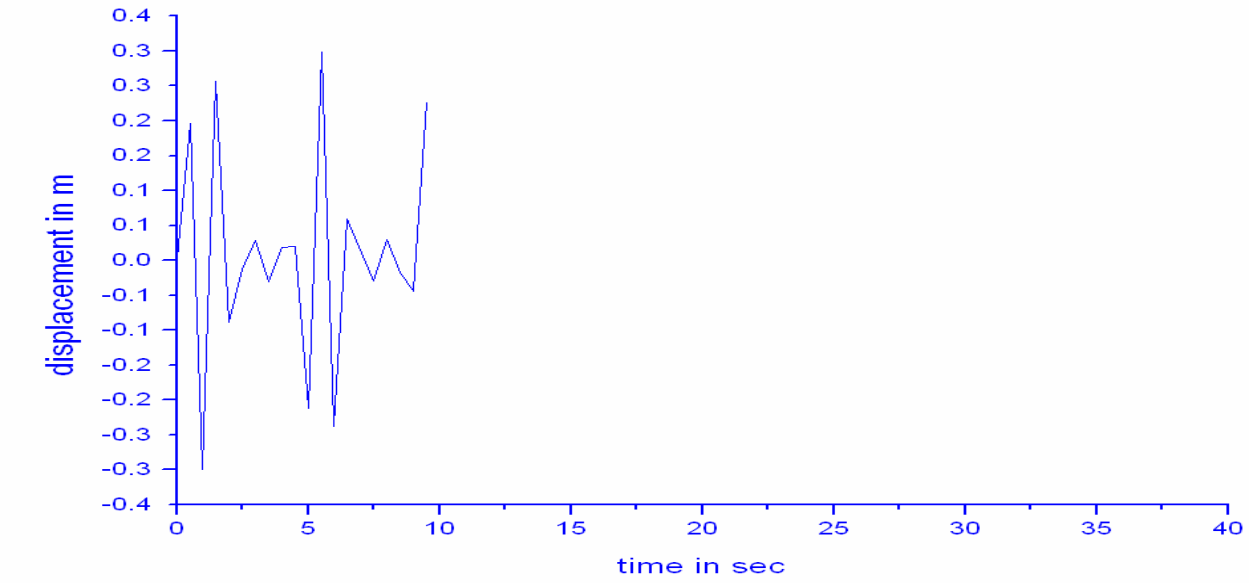
Table 2

Time	Displacement	Velocity	acceleration
0.0	$0.0031*10^{-4}$	$0.0012*10^{-3}$	0.00
0.5	$0.1959*10^{-4}$	$0.0759*10^{-3}$	0.0003
1.0	$-0.2990*10^{-4}$	-0.0003	-0.0017
1.5	$0.2565*10^{-4}$	0.0005	0.0048
2.0	$-0.0887*10^{-4}$	-0.0006	-0.0093
2.5	$-0.0124*10^{-3}$	0.0006	0.0143
3.0	$0.0280*10^{-3}$	-0.0005	-0.0186
3.5	$-0.0304*10^{-3}$	0.0002	0.0214
4.0	$0.0184*10^{-3}$	0.0	-0.0224
4.5	$0.0203*10^{-4}$	0.0	0.0224
5.0	$-0.2120*10^{-4}$	-0.0001	-0.0224
5.5	$0.2983*10^{-4}$	0.0003	0.0237
6.0	$-0.2376*10^{-4}$	-0.0005	-0.0267
6.5	$0.0589*10^{-4}$	0.0006	0.0309
7.0	$0.0152*10^{-3}$	-0.0006	-0.0355
7.5	$-0.0294*10^{-3}$	0.0004	0.0393
8.0	$0.0299*10^{-3}$	-0.0001	0.0414
8.5	$-0.0165*10^{-3}$	-0.00	0.0418
9.0	$-0.0438*10^{-4}$	0.0001	-0.0412
9.5	$0.2267*10^{-4}$	0.00	0.0409

Graphs :



(Figure 25)



(Figure 26)

CHAPTER 6

Computer Programmes

Computer Programmes

```
%Newmark's Direct Integration Method
%--------------------------------------------------------------------------
% Integrates  a N-DOF system with mass matrix "m", stiffness matrix "k" and damping
% matrix "c", when subjected to an external load harmonic in nature F(t).
% Returns the displacement, velocity and acceleration of the system with
% respect to an inertial frame of reference.
n=input('no. of time increments required n=');
dt=input('time interval dt =');
t(1)=0;
for(i=1:n)

      t(i+1)=t(i)+dt;
   end
end
disp('input the stiffness and mass matrices');
k=input('stiffness matrix=');
m=input('mass matrix=');
c=input('damping matrix c=');
disp('stiffness matrix k=');
disp(k);
disp('mass matrix m=');
disp(m);
disp('damping matrix c=');
disp(c);
disp('force matrix f=');
disp(f);
u=[0;0;0;0;0;0];
```

```
ud=[0;0;0;0;0;0];
udd=[0;10;10;0;0;0];
alpha=input('integration constantl alpha =');
gamma=input('integration constant gamma=');
a0=1/(alpha*dt^2);
a1=gamma/(alpha*dt);
a2=1/(alpha*dt);
a3=(1/(2*alpha))-1;
a4=(gamma/alpha)-1;
a5=((gamma/alpha)-2)*(dt/2);
a6=dt*(1-gamma);
a7=gamma*dt;
keff=k+a0*m+a1*c;

for(i=1:n)
   f=[0;2*sin(3*t(i));0;10;0;0];
s=m*(a0*u+a2*ud+a3*udd)
   +c*(a1*u+a4*ud+a5*udd);
   feff=f+s;
   disp('feffective ::')
   disp(feff)
   temp1=u;
   u=inv(keff)*feff;
   temp=udd;
      udd=a0*(u-temp1)-a2*(ud)-a3*udd;
   ud=ud+a6*temp+a7*udd;
   disp('upadated value of u')
   disp(u)
   disp('upadated value of u')
   disp(ud)
   disp('upadated value of udd:')
```

```
   disp(udd)

end
end
```

% Method to Display Eigen values and Eigen Vectors

```
K=10^4*[0.0052 0 -0.0026 0.0326;0 1.0880 -0.0326 0.2720;-0.0026 -0.0326 0.0026 -
0.0326;0.0326 0.2720 -0.0326 0.5440;]

M=[0.0929 0 0.0161 -0.0967;0 1.4881 0.0967 -0.5580;0.0161 0.0967 0.0464 -0.1637;-
0.0967 -0.5580 -0.1637 0.7440];

F=[0;0;0;0];

[V,D]=eig(K,M);

[lambda,K]=sort(diag(D));

V=V(:,K);

omega=sqrt(D);

pi=3.1414;

frequency=omega/(2*pi);

factor=diag(V'*M*V);

phi=V*inv(sqrt(diag(factor)));
```

CONCLUSION

We can observe the resonance phenomenon in the response plots when exciting frequency is equal to natural frequency of the applied harmonic exciting function.

Also it has been found that the type of Numerical Integration method used also affects the results. Duhamel's integration method can be applied single degree of freedom systems, while the Newmark integration method can be applied to multi degree of freedom systems. The Newmark integration method also very advantageous over other methods as its computer implementation is very easy and also the calculations involved are also very easy and can be computed very accurately.

In the Free Vibration analysis the lower modes are much more important than higher modes as it dominantly affects the structure than other modes,

The variation in Computer generated results and manually calculated results are in coincidence .The variation is very less because of the accurate approximations adopted in generating code.

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