Dynamic model of a heat pump based house heating system

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Abstract

Heat pumps have gained a huge popularity as an efficient indoor heating system including several other applications due to relatively low installation cost and energy consumption, and high potential mitigation of global warming than conventional heating systems. The present work aims to develop a modeling tool to investigate dynamic behavior of heat pumps for heating applications. A vapor-compression, air-source HP house heating system with a zeotropic refrigerant R-407c is developed using MATLAB[®] based Simscape[™] language. The system components such as the evaporator, compressor, condenser, expansion valve, house and the controller are modeled using a combination of SimscapeTM foundation libraries. Each component model has a set of tunable parameters that can be adjusted by user preferences in order to simulate the real-time operation. The dynamics of the system and its response to variable ambient temperature is investigated and discussed. The model depicts the expected behaviour under the simulated operating conditions and is capable of maintaining a comfortable user defined heating temperature inside the house. The model developed will be used to gain insight into performance characteristics of heat pump systems used for local production in the district heating. Keywords: Dynamic modeling, vapor-compression heat pump, Simscape, two-phase fluid, R-407c, coefficient of performance, house heating

1 Introduction

Heat pumps (HPs) are becoming one of the highly sustainable systems around the world as they can be operated in both, centralized and decentralized power generation. They are highly efficient and economical component of modern heating and cooling systems which not only use less energy but also mitigate greenhouse gas emissions. Due to the availability of a variety of external energy sources, HP based systems are widely used for both domestic and industrial applications. Air-source HPs are often used for air-conditioning in moderate climatic conditions. They absorb heat from the outside air at low temperatures into a fluid, which passes through a compressor, where its temperature is increased, and transfers heat at its highest temperature to the heating and hot water circuits.

account for approximately 79% of the total energy consumption (192.5 Mtoe¹) out of which exclusively 64.7% comes from the residential sector (Europa 2016). Reduction in the energy consumption, in buildings and industries, can be achieved by several means such as innovative construction and design, energy efficient heating systems, and smart control strategies. However, HP based heating can solely reduces the electricity than consumption by approximately 50% the conventional heating sources such as gas or oil boilers and direct electric heaters. Increasing popularity of HP systems for space heating applications has impelled the need to investigate HPs and their dynamic behavior in response to the varying load in order to make improvements in system design and control.

Mathematical modeling and simulation are one of the best tools for analyzing system's behavior and controlling it for the given operating conditions. Dynamic modeling of vapor-compression systems has been a topic of broad research interest since last three decades. Several modeling efforts, transient and steady-state, have been made to simulate HP and its components, and to approximate their behavior for heating applications. The transient response of evaporating and condensing flows was first investigated by Wedekind and Stoecker (Wedekind and Stoecker 1968). Later, more research focused mainly on two main modeling approaches, the moving boundary (MB) and finite volume (FV), was conducted to predict the accurate evolution of dynamics in the systems (Wedekind, Bhatt, and Beck 1978; Beck A detailed review of both and Wedekind 1981). approaches for dynamic modeling of vapor compression systems is provided by Bendapudi et al. (Bendapudi, Braun, and Groll 2008). It has also been reported that the FV formulation is more robust for start-up and load-change transients, however, computationally slower than MB method (Bendapudi, Braun, and Groll 2008). Rasmussen and Bhaskar (Rasmussen 2012: Rasmussen and Bhaskar 2012) reviewed the dynamic modeling of vapor-compression systems in detail and presented examples of physical models of the system components. (Koury, Machado, and Ismail 2001) Koury et al. presented two models to simulate the steady-state and transient behavior of water-water type refrigeration system where the condenser and evaporator were

In the European Union, space and water heating

¹million tonnes of oil equivalent

modeled as control volumes (CVs). Haberschill et al. (Haberschill, Guitari, and Lallemand 2007) developed a model to simulate the operation of a CO_2 air-to-water HP under transient conditions. Dynamic modeling of vapor-compression systems and maintaining the right balance between the complexity and accuracy is challenging. However, the evolution of advanced simulation softwares has significantly reduced the modeling effort.

The use of object-oriented modeling approach has increased during past few years due to the ease of implementation, computationally efficient and reasonable robustness (Rasmussen 2012). The most common simulation platforms available for dynamic modeling of complex physical systems are Dymola, Modelica, MATLAB-Simulink^{\mathbb{R}}, Simscape^{$^{TM}2$}. Torge and Gerhard (Torge and Gerhard 2004) developed a steady-state model of CO₂-refrigeration cycle with two different types of heat exchangers in Modelica and showed a fair agreement of the results with measured data. Gräber et al. (Graber et al. 2010) used an object-oriented thermodynamic library written in Modelica to model novel HP systems for domestic hot water supply. Bin and Alleyne (Bin and Alleyne 2010) developed a dynamic model in Simulink® to describe the transient behavior of heat exchangers (condenser/evaporator) for vapor compression cycle systems used in Air Conditioning and Refrigeration. They validated the model with the experimental system and concluded that the model well predicts the system dynamics in shut-down and start-up transients. Mortada et al. (Mortada et al. 2012) developed a dynamic model of a HP system in the Dymola environment and validated each component by comparison with test results of a heat pump prototype. In addition, Chamoun et al. (Chamoun et al. 2012) adopted both MB and FV approaches to develop a high temperature HP model in Modelica using water vapor as a refrigerant.

2 Problem definition and approach

A physical modeling approach provides several benefits among other approaches available for modeling multi-domain systems. It enables the design engineer to evaluate multiple system configurations and design options, predict system performance, and cost-effectively test control strategies. Models based on physical approach also provide a virtual prototype which can be scaled relatively faster than other conventional approaches. The sub-system or component models developed for one system can also be utilized in multi-domain system models of different application. The objective of the present work is to develop a platform to simulate the dynamic behavior of typical air-to-air vapor compression HP using a physical modeling

approach. Development of the system model in the chosen simulation environment for a house heating application is presented. Results from the model are discussed with the concluding remarks in the end. The main difference between the previous studies and the present work is mainly the choice of modeling platform and the controlling approach.

3 Dynamic modeling

The primary objective of developing a physical model is to capture the physical behavior of the HP system and its dynamic response to the heat consumption application. For the purpose here, air-to-air vapor-compression HP is simulated with R-407c as a refrigerant. The simulated application is useful as it captures the standard household utility of space heating.

3.1 Methodology

The modeling platform chosen in the present work is Simscape^{TM} (MATLAB 2018a) which is based on the physical modelling approach. Simscape offers a much stronger simulation environment in combination with MATLAB and Simulink functionalities. It strengthens the model capabilities and enables users to reuse the model in multiple physical domains and applications. For example, the physical system and controllers can be optimized simultaneously in one environment. Additionally, the physical system model can be converted into C-code which can be deployed for other tasks such as hardware-in-the-loop (HIL) testing. In the chosen approach, the whole system is simulated as a set of components (sub-systems) represented by physical blocks where the dynamic equations are expressed as partial differential equations. The model is developed by assembling the system components (sub-systems) of different domains using physical connections represented by lines and control signals into a single schematic. The output of each component is generated based on input parameters, states at a given time and the connection between them. The graphical scheme of the system and the interconnections between components can be seen in Figure 1.

The components of the HP system, i.e., evaporator, compressor, condenser expansion valve, along with the indoor space and the outdoor environment are created using the '*Two-phase fluid* (2P)' foundation library of Simscape. The model includes necessary sensors to measure the temperature, pressure, and mass flow rate at the desired locations. Simscape blocks that are used in developing the different components are also shown in Table 1. The description of each Simscape block with corresponding governing equations can be found in the reference (MATLAB 2018b). Table 2 illustrates the parameters used in developing the current model.

²Simscape is a toolbox in Simulink which supports modeling and simulation of multi-domain physical systems.

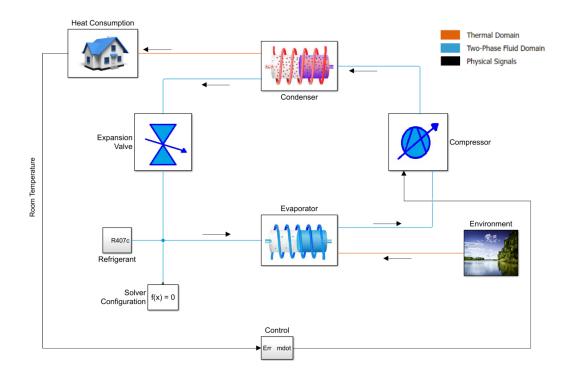


Figure 1. The graphical scheme of the system model in Simscape.

3.2 Physical component modeling

This section describes the modeling approach adopted to model different physical components of the heat pump system, the physical blocks adopted from the Simscape library, and the assumptions made to simulate system's dynamic behavior.

3.2.1 Evaporator

Evaporators are heat exchangers that are used in HPs to absorb heat from the low-temperature source and transfer it to the refrigerant. In the present study, outdoor ambient air is the low-temperature source and is simulated as the environment). For air-to-air systems, fin and tube type of heat exchangers are well suited evaporators as they have larger surface area and thus provide better heat transfer.

In the present work, a finned copper tube of the constant circular cross-sectional area is modeled using the '*Pipe* (2P)' block. The block models the flow dynamics of a two-phase fluid due to friction losses and the convective heat transfer within the rigid pipe wall. The evaporator

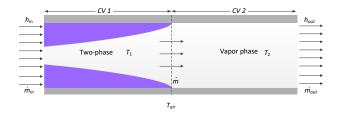


Figure 2. Illustration of the evaporator.

model is described by the geometrical data as shown in Table 2. The following assumptions are considered in the model:

- Two control volumes
- The liquid and vapor phase are in equilibrium
- Fully developed flow
- Negligible gravitational force
- Heat transfer at constant pressure

The refrigerant enters the evaporator in two-phase and leaves in the form of superheated vapor. To capture this dynamic change in the phase, the evaporator is modeled as two fluid CVs with a set of differential equations based on temperature and phase of the refrigerant in each segment (lumped model) which approximated a linear behavior. The illustration of the evaporator dynamics is shown in Figure 2. The pressure and temperature evolve according to the compressibility and thermal capacity of the fluid volumes. The model also accounts for heat convection from the ambient air i.e., the environment to the outer pipe wall, heat conduction through the pipe wall and heat convection from the evaporator pipe wall to the refrigerant. This approach satisfactorily describes the dynamics inside the heat exchanger.

3.3 Compressor

The compressor is a principal pressure source in HPs. When the superheated vapor leaves the evaporator, the

Physical	Simscape TM block	-
component		Table 2. Details of
Evaporator		Evaporator
	• H Pipe (2P)	Pipe length Pipe diameter Pipe thickness
Compressor	Controlled Mass Flow	Fin area Convective heat transfer co Phase-change time constar
	Rate Source (2P)	Condenser
Condenser	■ A ■ H Pipe (2P)	Pipe length Pipe diameter Pipe thickness Convective heat transfer co
		Phase-change time constan
Expansion valve	C AR B ■ Variable Local Restriction (2P)	Expansion valveMinimum restriction areaMaximum restriction area
	Thermal Mass	House
House	• A FB A B A Convective Heat Transfer from Condenser through Wall to Ambient	Height Width Length Wall thickness
Refrigerant	Two-Phase Fluid Properties (2P)	Set temperature Convective heat transfer from air to wall Convective heat transfer from wall to atmosphere
Environment		Environment
	Constant Sine Wave	Environment temperature

Table 1. Description of SimscapeTM blocks.

compressor performs an isentropic work on it, which means that entropy at the evaporator outlet is equal to the entropy at the inlet of the condenser. It conveys superheated vapor of the refrigerant from a low pressure and low temperature state to a high pressure and high temperature state. In the present model, the compressor is modeled by using 'Controlled Mass Flow Rate Source (2P)' block, which is based on the following assumptions:

- Negligible friction losses
- Isentropic compression
- No heat exchange with the surroundings, i.e. adiabatic

The compressor is simulated as an ideal mechanical source that maintains the desired mass flow rate

Table 2. Details of n	nodel parameters.
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Evaporator	Value
Pipe length	30 m
Pipe diameter	0.01 <i>m</i>
Pipe thickness	0.005 m
Fin area	$1 m^2$
Convective heat transfer coefficient	$150 W/m^2 K$
Phase-change time constant	1 <i>s</i>
Condenser	
Pipe length	30 m
Pipe diameter	0.01 <i>m</i>
Pipe thickness	0.005 <i>m</i>
Convective heat transfer coefficient	$20W/m^2K$
Phase-change time constant	1 s
Expansion valve	
Minimum restriction area	$0.1mm^2$
Maximum restriction area	$1.5mm^2$
House	
Height	4 <i>m</i>
Width	5 m
Length	5 m
Wall thickness	0.15 <i>m</i>
Set temperature	294.15K
Convective heat transfer coefficient	$20 W/m^2 K$
from air to wall	
Convective heat transfer coefficient	$30 W/m^2 K$
from wall to atmosphere	
Environment	
Environment temperature	288.15 K
Atmospheric pressure	0.101325 MPa
Refrigerant	
Refrigerant type	R-407c
Initial pressure	0.6 <i>MPa</i>
Initial vapor quality	0.05
Mass flow rate	$0.005 \ kg/s$
Material properties	
Specific heat of air	1005.4 J/kgK
Density of air	$1.2250 \ kg/m^3$
Specific heat of Copper	390 J/kg.K
Thermal conductivity of copper	400 W/m.K
Density of copper	8940 kg/m^3
Thermal conductivity of brick wall	0.03 W/m.K

regardless of the pressure difference. The mass flow rate is provided as a physical input signal. It is worth to mention that the time-constant of the mass flow rate through the compressor is smaller than that for the heat exchangers. Since the start-up and shut-down period dynamics is not the focus of the present modeling, time-constant of 0.1 s is assumed sufficient to simulate the temperature variation through the compressor.

3.4 Condenser

The condenser is similarly modeled as the evaporator. However, necessary adjustments in parameters and construction modifications are made. The corresponding geometrical data is keyed into the '*Pipe (2P)*' block. The assumptions are similar to the evaporator but the condenser is simulated with one fluid CV (lumped). The dynamics of the condenser modeled is illustrated in Figure 3. The superheated high-pressure vapors enter the heat exchanger and condense at constant pressure leaving it in two-phase. The heat rejected by the condenser is transferred to house for heating consumption.

3.5 Expansion valve

The two-phase refrigerant from the outlet of condenser passes through the expansion-valve which reduces the refrigerant pressure to a level that maintains the superheating in the evaporator. The expansion is isenthalpic, which means the enthalpy at the condenser outlet is equal to the enthalpy at the evaporator input. The 'Variable Local Restriction (2P)' block is used to simulate the expansion valve in the present model. The block models the pressure loss across the restriction by controlling the restriction area as a physical signal. The model is based on the following assumptions:

- Negligible friction losses
- Isenthalpic expansion
- No heat exchange with the surroundings i.e., adiabatic

3.6 House

The house model is simplified in order to simulate a space heating consumption. The house is therefore simulated as a single, small-sized square room which needs to be heated up to a user desired temperature. The

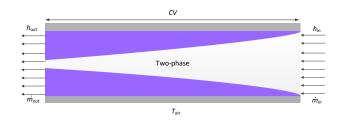


Figure 3. Illustration of the condenser.

model is characterized by the thermal mass of air and thermal properties of the materials. It is assumed that the house is empty and that no internal heat transfer takes place (i.e., no heat generation). However, losses to the ambient are considered. The indoor air exchanges heat with the environment (ambient air) which is simulated as a combination of thermal convection, and thermal conduction through exterior walls. The house model is developed using the '*Thermal* (*T*)' library of Simscape.

3.7 Environment

The environment is modeled as a low-temperature heat source of infinite capacity to the heat pump with a defined ambient temperature. In the present work, blocks named '*Constant*' and '*Sine Wave*' are used to emulate ambient air temperature profiles.

3.8 Refrigerant properties

R-407c is preferred as a refrigerant to simulate the heating system as it is commonly used in both residential and commercial HPs. It is a non-flammable zeotropic blend of three refrigerants namely difluoromethane pentafluoroethane (R-32). (R-125). and 1,1,1,2-tetrafluoroethane (R-134a) with a mass ratio of 23%, 25% and 52% respectively (ASHRAE 2017). It is environmentally acceptable with zero Ozone Depletion Potential (ODP) however has a considerable Global Warming Potential (GWP) of 1530. The property tables of both phases, liquid and vapor, are generated using the REFPROP database developed by National Institute of Standard and Technology (NIST). The 'Two-Phase Fluid Properties (2P)' block is used to import the thermophysical properties of a two-phase refrigerant in the modeling platform and use during the simulation.

3.9 Operation mode and control

Since the present work focuses only on heat pump operation for space heating application, the critical feature of the model is heat flow control, which ensures the effective use of heat pump and its heating output. Two control strategies are utilized to simulate the desired behavior and are described in the following sections:

3.9.1 On/off control

In the model, the heating system is controlled by an on/off controller which keeps the indoor space temperature at a desired level by turning the system on and off to satisfy the load requirements. The on/off control is chosen because the heat pump system which will be later used to validate the model has the same controller. This is also the most common variation of thermostats for space heating. The temperature set-point is modeled using a '*Constant*' block which specifies the temperature that must be maintained inside the house. The set-point is 294.15 K with a differential of $\pm 1^{\circ}$ in the present simulation. When the temperature in the house falls below 293.15 K, the controller turns on and the

compressor runs with full power; when the temperature exceeds 295.15 K, the controller turns off the compressor.

3.9.2 Expansion valve control

The compressor in the heat pump is a core component, and also expensive which needs to be chosen carefully for the given application. Therefore, it must be controlled against any kind of failure under the operational limits. It is critical to control the temperature to ensure that only superheated vapors exit the evaporator to avoid malfunctioning and mechanical damage to the compressor. The control checks the evaporator temperature using a temperature sensor, and continuously feeds it back to control the restriction area of the valve desired to maintain the superheat.

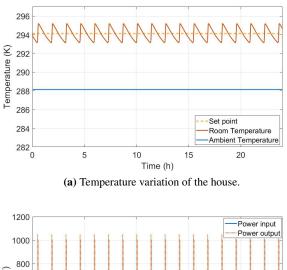
3.10 Solver

The complete model of the system is obtained by combining the sub-models of the components. Simulation time, step size, and solver type are defined as global model configuration parameters. The system is simulated for 24 hours ($\sim 86400s$) with a time step of 1 s. The implicit fixed-step solver ODE14X is selected as the solver for the simulation system. The present model simulates known physical system, such as heat exchanger and house designs. Whereas, unknown parameters like initial values, heat transfer coefficients are tuned to achieve the desired response and convergence.

4 Simulation results

In this section, the output results based on the simulated system design are presented and discussed. Since the ambient temperature has a significant influence on the thermal behavior of the house, results with two ambient temperature profile are analyzed.

Figure 4(a-b) presents the performance results of the system at constant ambient temperature of 288.15 K. Figure 4(a) shows the house temperature (represented by solid line '-temperature (represented as solid line '-—') as predicted by the model. It can be seen that as the house temperature falls 1° below the user set temperature level, which is 294.15 K (represented by dashed line '----'), the HP starts and runs until the house temperature reaches 1° above the set-point, and then it stops until the temperature drops 1° below the set-point. The heating performance of the HP is assessed by the total heating effect produced and the amount of power consumed in producing that effect. Figure 4(b) shows the heat flow from the condenser to the house, i.e. power output (represented by dotted line'......') and the corresponding power input (represented as solid line '----'). The results show that approximately 1040 W heating effect is produced at a power consumption of 170 W by the system. The system behaves similarly over the simulated time and thus the heat consumption profile remains same for a period of 24 hours due to constant ambient



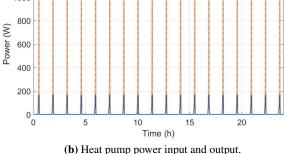
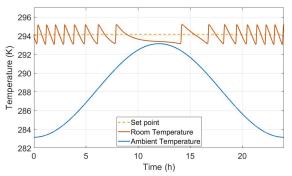


Figure 4. Performance characteristics at constant ambient temperature.

temperature.

The system's response is also analyzed with variable ambient temperature. To serve the purpose here, the sinusoidal temperature profile is chosen to emulate the variable ambient temperature. Figure 5(a-b) shows the corresponding results predicted by the model. The variation in the house temperature (represented by solid line'-—') with time is shown in Figure 5(a) where it is seen that when the ambient temperature (represented by solid line'——') is higher, the HP does not run or runs relatively less frequent than at lower ambient temperature. This effect is attributed to lower heat loss from the house due to higher ambient temperature which in turn reduces the house heating demand. As expected, the house temperature is maintained within 1° of the set temperature (represented by dotted line '- - - ') even when the outdoor temperature varies sinusoidally. The impact of varying ambient temperature on the power input (represented by solid line'----') and output (represented by dotted line'.....') is shown in Fig. 5(b). It is observed that the heating effect, i.e. power output is slightly lower, approximately 1000 W, at low ambient temperatures and increases gradually to approximately 1070 W as the ambient temperature increases leading to no flow of heat at the peak. The results confirm that the model developed satisfactorily predicts the dynamic behavior of the HP system, thus can be utilized at varying loads.



(a) Temperature variation of the house.

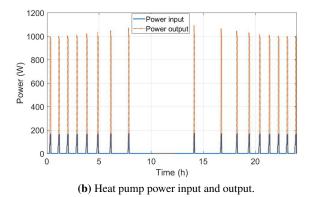


Figure 5. Performance characteristics at time varying ambient temperature.

5 Limitations and future work

The model developed demonstrates the dynamic behavior of a typical air-to-air vapor-compression HP for domestic space heating application. However, the results are yet to be verified by the laboratory experiments to ensure the reliability. The laboratory scale set-up of the HP system to test the accuracy of the model and various thermal effects based on the model are currently under construction. Once built, the model will be verified and used in developing and testing new control strategies, and the overall system design to achieve better performance and energy consumption results. It will further be extended using the same approach to simulate the dynamics at various load profiles. The model developed is considered useful in determining HPs feasibility to the heating demands in existing or new district heating system.

6 Conclusion

The present work focused on physical modeling of the HP based heating system dynamics. The model simulates the performance outputs such as evaporator, condenser and heating temperature, compressor, expansion valve operation as well as the thermal behavior of the heat consumption. The results obtained show that SimscapeTM is a robust tool for both evaluating and controlling the dynamic behavior of the system, however,

limited to the design simulated and the input parameters used. The developed model provides a framework to build multiple model strategies with different available sources and integrated renewable energy systems (buffer) that will deliver better heating performance and flexibility of a HP at a lower cost. Moreover, different scenarios of heating applications can be investigated by adopting different control designs in the model which can positively contribute to the district heating systems and power grid balancing.

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