# Dynamics

J. W. Eischen

# Other Disciplines FE Specifications

| **Topic: Dynamics**<br>**7-11 FE exam problems** | **Exam Problem Numbers** |
|---|---|
| A. Kinematics | 53, 55 |
| B. Linear motion(e.g. force, mass, acceleration) | |
| C. Angular motion(e.g. torque, inertia, acceleration) | 54 |
| D. Mass moment of inertia | |
| E. Impulse and momentum(linear and angular) | 56 |
| F. Work, energy, and power | 58 |
| G. Dynamic friction | |
| H. Vibrations | |

We are grateful to NCEES for granting us permission to copy short sections from the FE Handbook to show students how to use Handbook information in solving problems. This information will normally appear in these videos as white boxes.

1. Which of the following statements best describes the area of study in Dynamics called Kinematics?

   A) Kinematics is the study of how forces and couples act on rigid bodies.

   B) Kinematics is the study of the interaction between multiple bodies to form more complex mechanisms.

   C) Kinematics is the study of the geometry of motion.

   D) Kinematics is the study of rigid bodies in 2D motion, whereas Kinetics is the study of bodies in 3D motion.

# Solution – Problem 1

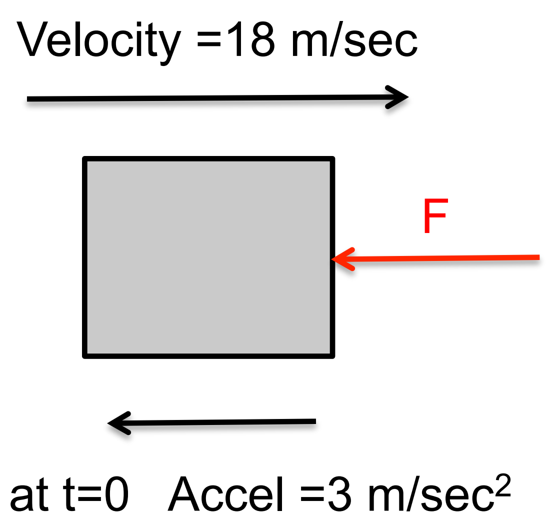
A) Kinematics is the study of how forces and couples act on rigid bodies. False

B) Kinematics is the study of the interaction between multiple bodies to form more complex mechanisms. False

C) Kinematics is the study of the geometry of motion. True

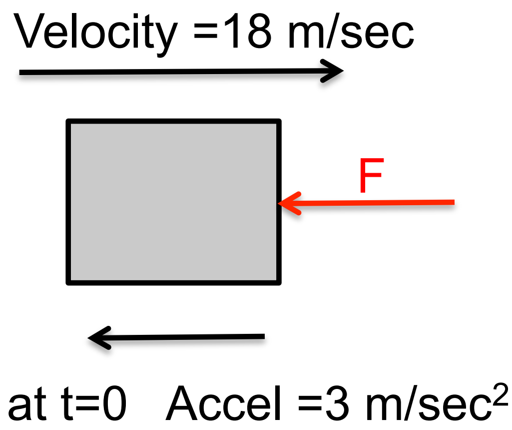
D) Kinematics is the study of rigid bodies in 2D motion, whereas Kinetics is the study of bodies in 3D motion. False

2. An object is moving to the right at 18 m/sec. Suddenly the object is subjected to a force that causes an acceleration of 3 m/sec$^2$ to the left. What is the position of the object 10sec after the force is applied? How far has the object traveled in 10 sec?




A) 24 m B) 30 m C) 54 m D) 78 m

# Solution – Problem 2 (Page 78 – Ref. Handbook)




Constant acceleration equations

$$a(t) = a_o$$

$$v(t) = a_o(t - t_o) + v_o$$

$$s(t) = a_o(t - t_o)^2 / 2 + v_o(t - t_o) + s_o$$

Problem data –

$$t_o = 0,\ s_o = o,\ v_o = +18\ m/sec,\ a_o = -3\ m/sec^2,\ t = 10\ sec$$

$$s(t) = a_o(t - t_o)^2 / 2 + v_o(t - t_o) + s_o$$

$$s = (-3)(10^2)/2 + (18)(10) = 30\ m$$

B)

Total Distance Traveled Problem instead

Motion to the right (until 0 velocity)

$v(t) = -3t + 18$ note that when $t = 6$sec, $v = 0$

$s(t) = -\frac{3t^2}{2} + 18t$ so $s(6) = 54$m note that is position or distance traveled at $t = 6$sec

After 6 sec the object is moving to the left, v is negative for $t > 6$sec

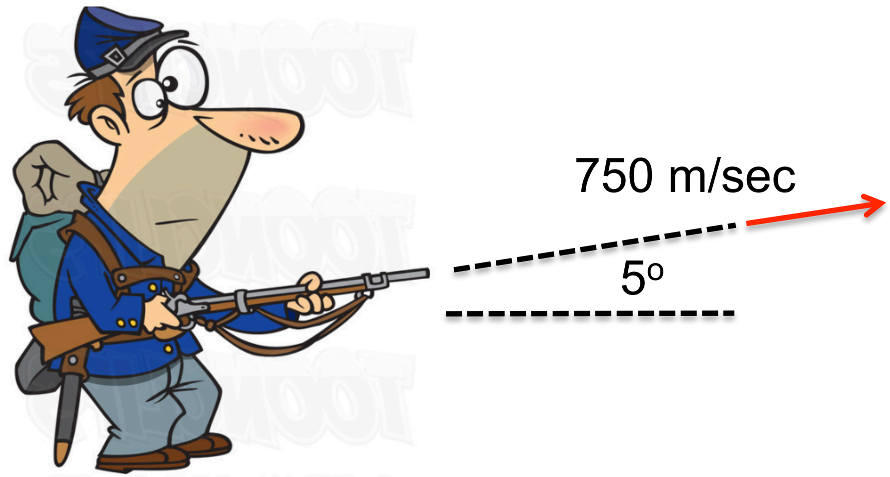
Motion to the left

$s(10) = 30$m we already know that

So between $t = 6$sec and 10sec the object has moved 24m to the left

Conclusion :Total distance traveled is $54 + 24 = 78$m D)

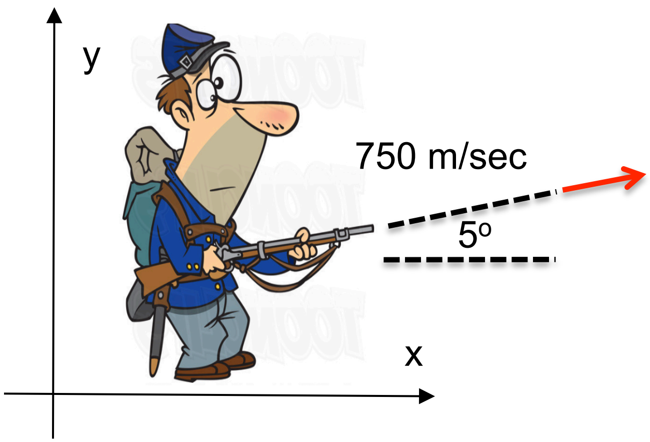
3. A rifle aimed $5^{o}$ above the horizon is fired. Neglecting air resistance and using 750 m/sec as the speed of the bullet as it leaves the rifle, to what altitude will the bullet reach during its flight?




A) 220 m B) 440 m C) 880 m D) 1,760 m

# Solution – Problem 3

Page 79- Ref. Handbook




## Projectile Motion Equations

$$v_x = v_o \cos\theta$$

$$x = \left(v_o \cos\theta\right)t + x_o$$

$$v_y = -gt + v_o \sin\theta$$

$$y = -gt^2/2 + \left(v_o \sin\theta\right)t + y_o$$

## Problem Data

$$v_o = 750\ \text{m/sec},\ \theta = 5^o,\ \ y_o = 0,\ g = 9.8\ \text{m/sec}^2$$

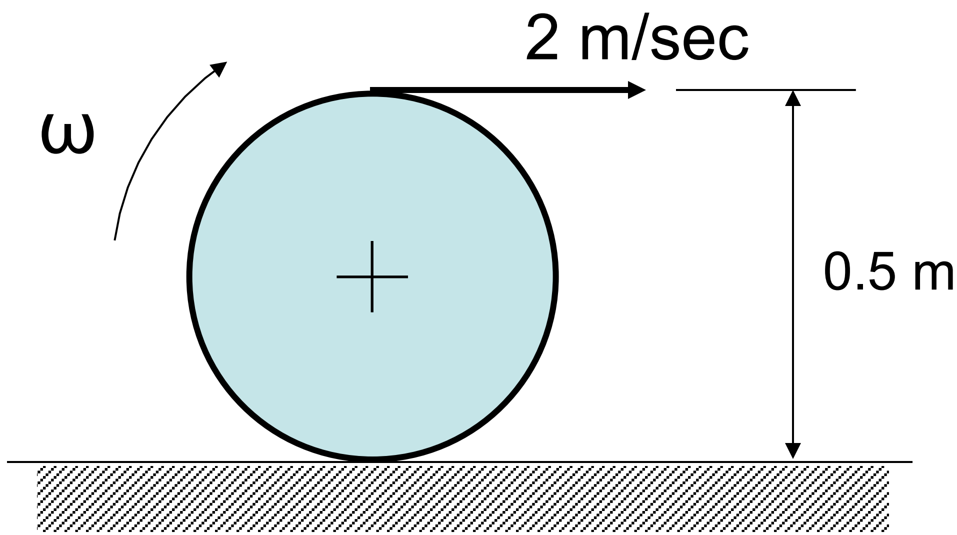
$$\text{at } y = y_{max}\quad v_y = 0\quad \text{then from } v_y - gt + v_o \sin\theta$$

$$t = v_o \sin\theta / g = 750(\sin 5^o)/9.8 = 6.67\,\text{sec}$$

$$\text{then}\quad y = -(9.8)(6.67^2)/2 + \left(750 \sin 5^o\right)(6.67) = 218\ \text{m}$$

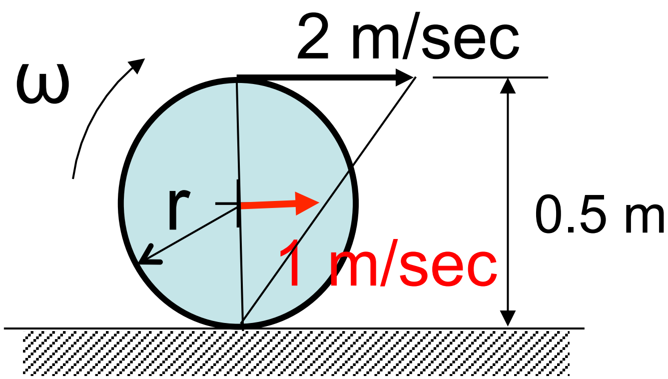
A)

4. For a wheel rolling without slipping on a horizontal surface, determine its angular speed (ω) if the velocity of the top of the wheel is a constant 2 m/sec.




A) 20 rpm B) 40 rpm C) 60 rpm D) 80 rpm

# Solution – Problem 4 (Page 78 – Ref. Handbook)




The kinematics of rolling contact that relates the linear velocity of the center of disk to its angular velocity is used to solve this problem

From kinematics of rolling contact

$$r\omega = v_{\text{center of disk}} = v_c$$

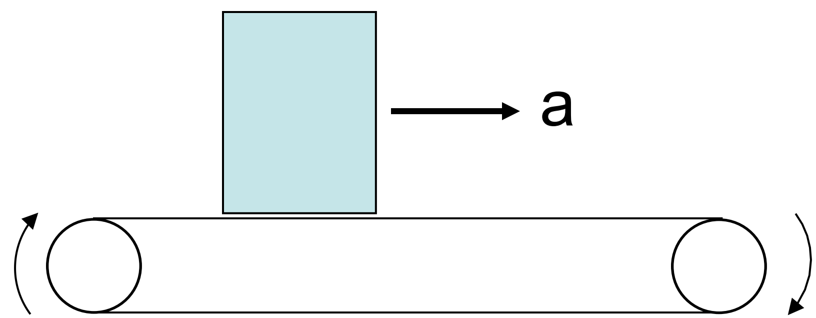
since horizontal velocity increases linearly from instantaneous center of rotation then

$$v_c = 1\,\text{m/sec} \qquad \text{and}$$

$$\omega = \frac{v_c}{r} = \frac{(1\,\text{m/sec})}{(0.25\,\text{m})}\frac{(60\,\text{sec})}{(1\,\text{min})}\frac{(1\,\text{rev})}{(2\pi\,\text{rad})} = 38.2\,\text{rpm}$$

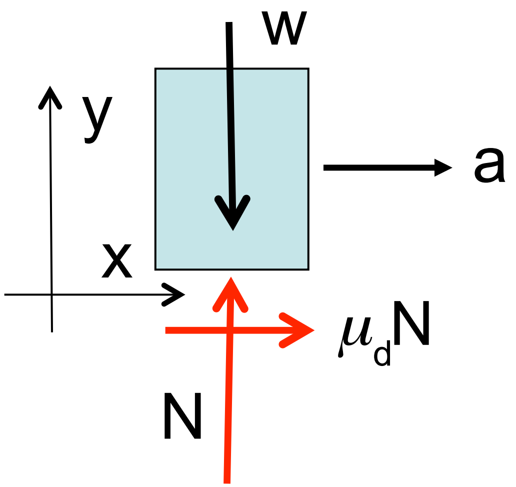
B)

5. What is the acceleration (a) of a box that is placed onto a moving conveyor until the box reaches the speed of the conveyor. The box weighs 200 N. The coefficient of static friction is 0.4 and the coefficient of dynamic friction is 0.2. (use $g = 9.8\ m/sec^2$)




A) $1\ m/sec^2$  B) $2\ m/sec^2$  C) $3\ m/sec^2$  D) $4\ m/sec^2$

# Solution – Problem 5 (Page 82 – Ref. Handbook)




The weight is accelerated to the right by a friction force due to the sliding of the weight relative to the conveyor until velocities are equal.

Apply Newton's 2nd law

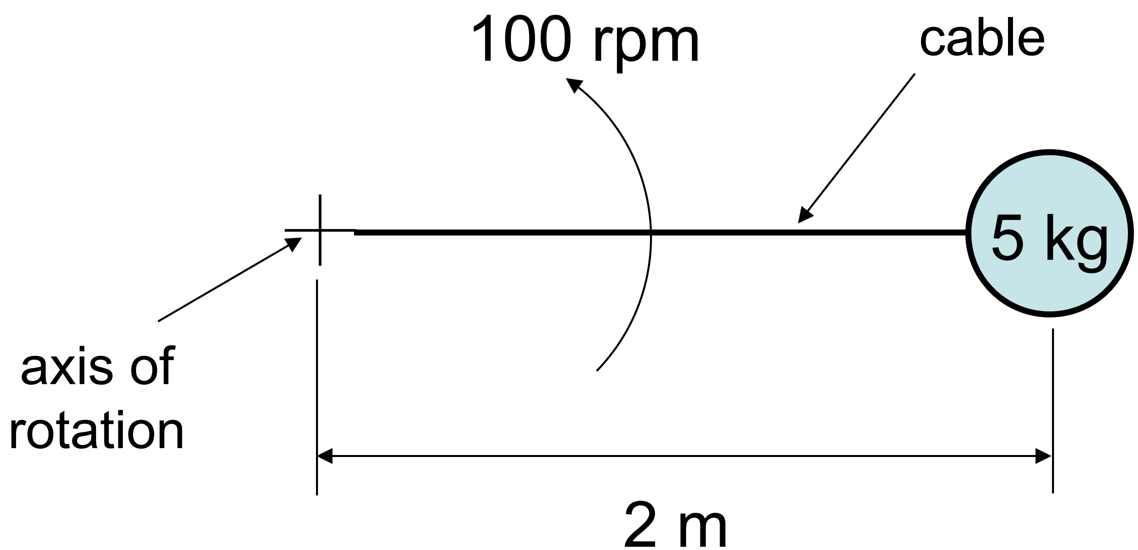
$$\sum F_y = 0 \quad N - W = 0$$

$$\sum F_x = \frac{W}{g} a \quad \mu_d N = \mu_d W = \frac{W}{g} a$$

$$\Rightarrow \quad a = \mu_d g = (0.2)(9.8) = 1.96 \text{ m/sec}^2$$

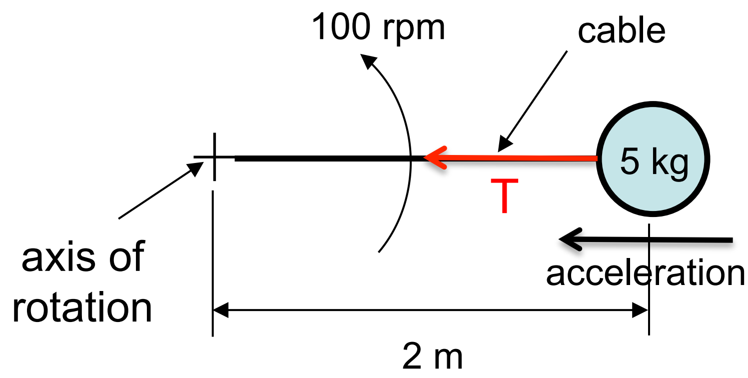
B)

6. If a mass of 5 kg is rotating in a horizontal plane at the end of a cable at a constant 100 rpm, then what is the tension in the cable?




A) 1,100 N B) 3,650 N C) 8,300 N D) 10,750 N

# Solution – Problem 6 (Page 79 – Ref. Handbook)




An object traveling in a circular path at a constant velocity is subject to an acceleration directed toward it center of rotation. This requires a radial force on the object.

Apply Newton's 2nd law in radial direction

$$\sum F_r = m\, a_r \quad \text{where } a_r = r\,\omega^2$$

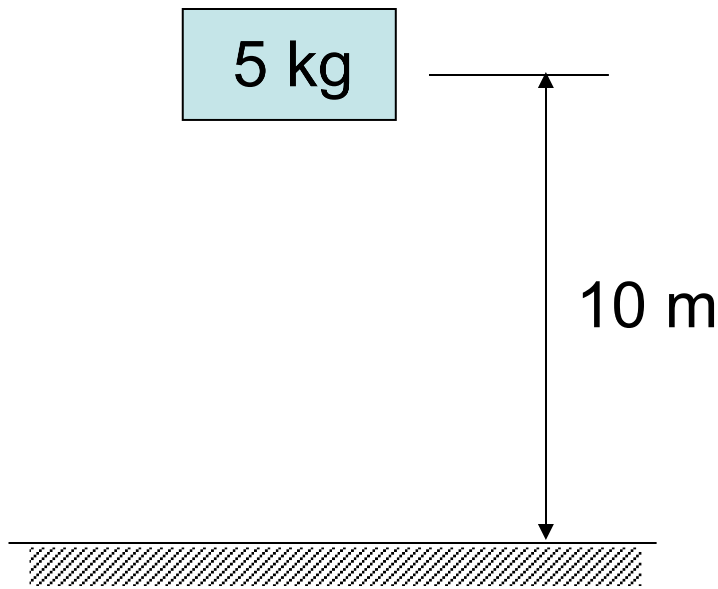
so that

$$T = m\,r\,\omega^2 = (5\text{ kg})(2\text{ m})\left[(100\,\frac{\text{rev}}{\text{min}})(\frac{1\text{ min}}{60\text{ sec}})(\frac{2\pi\text{ rad}}{\text{rev}})\right]^2$$

$$T = 1095\,\frac{\text{kg m}}{\text{sec}^2} = 1095\text{ N}$$

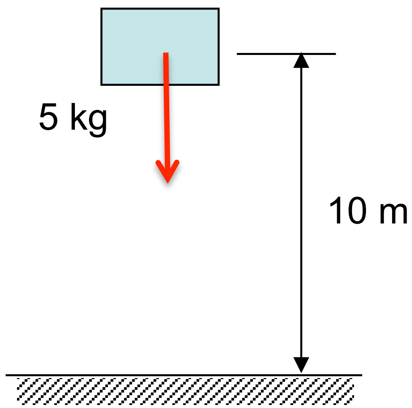
A)

7. If a 5 kg object is dropped from a height of 10 m, what is its velocity just before it hits the ground? (use $g = 9.8\ m/sec^2$)




A) 7 m/sec B) 14 m/sec C) 21 m/sec D) 28 m/sec

# Solution – Problem 7 (Page 78 – Ref. Handbook)




A falling body undergoes constant acceleration due to the force of gravity. Hence constant cceleration equations of motion apply.

Equations of motion (at $t = 0, v = 0, s = 0$)

$$v = a_o t$$

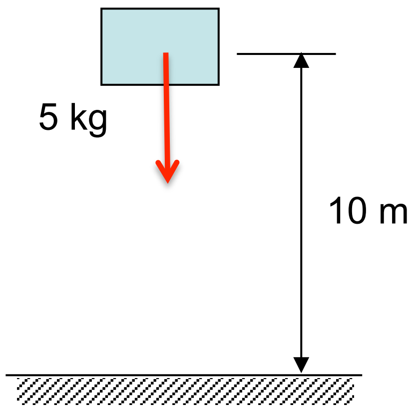
$$s = a_o t^2/2 \Rightarrow 10(m) = 9.8\,(m/sec^2)\,t^2/2$$

$$\Rightarrow t = \sqrt{20/9.8} = 1.43\,(sec) \text{ so that}$$

$$v = 9.8\,(m/sec^2)\,x\,1.43\,(sec) = 14\,(m/sec)$$

B)

# Solution – Problem 7 (alternate solution)




A falling body undergoes constant acceleration due to the force of gravity. Hence constant cceleration equations of motion apply.

Equations of motion (at $t = 0$, $v = 0$, $s = 0$)

$$v = a_o t = gt$$

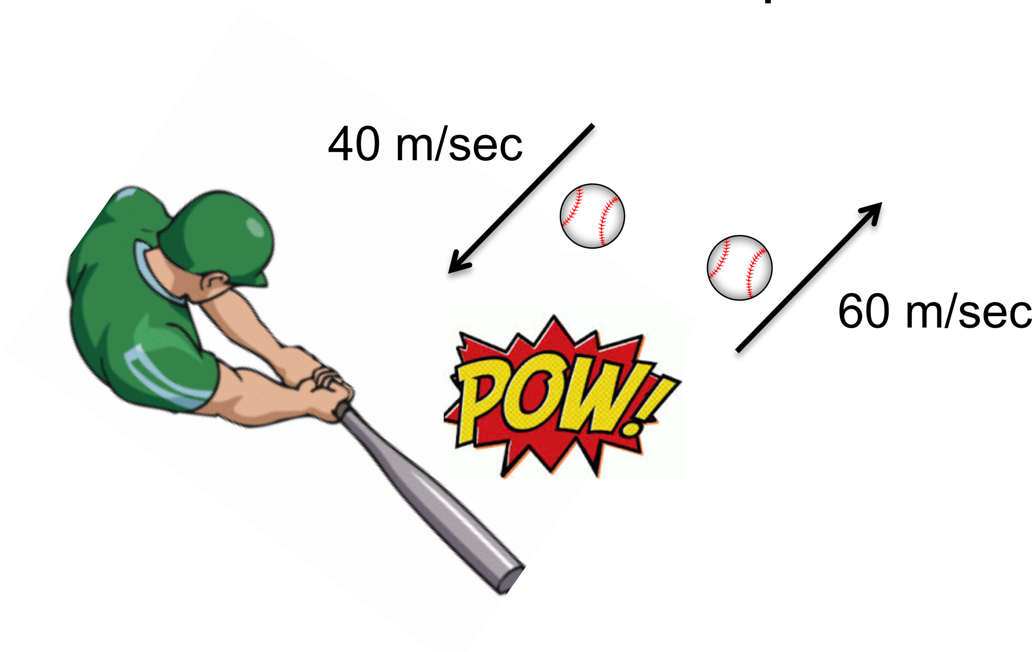
$$s = gt^2 / 2$$

$$\Rightarrow \; t = \sqrt{2s/g}$$

$$v = g\sqrt{2s/g} = \sqrt{2gs} = \sqrt{2(9.8)(m/sec^2)(10)(m)} = 14\,(m/sec)$$

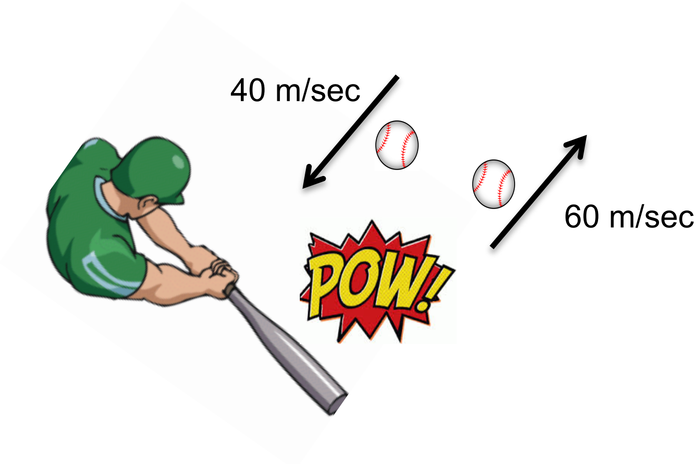
B)

8. What is the impulse imparted to a 200 gram baseball if the velocity the baseball arrives from the pitcher is 40 m/sec and leaves the bat towards the pitcher at 60 m/sec?




A) 4 N·sec  B) 10 N·sec  C) 16 N·sec  D) 20 N·sec

# Solution – Problem 8 (Page 80-81 – Ref. Handbook)




Assume inbound and outbound line of action is colinear and apply impulse momentum principle.

Linear impulse momentum principle

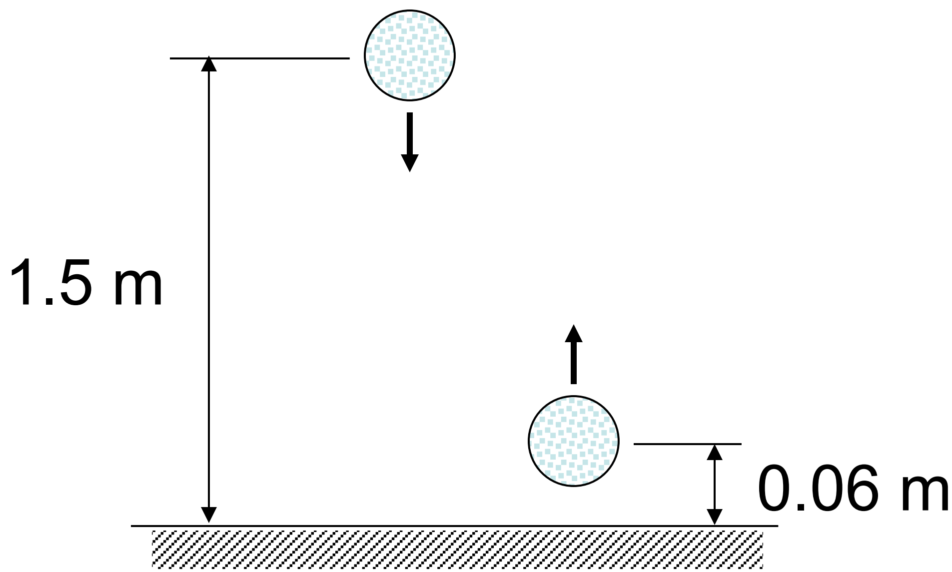
$$F\,dt = m\int_1^2 dv = m(v_2 - v_1)$$

$$F\,dt\,(N\,sec) = 200\,(gram)\left(\frac{kg}{1000\,gr}\right)\left[60(m/sec) - (-40)(m/sec)\right]$$

$$F\,dt\,(N\,sec) = 20\,(kg\,m/sec) = 20\,(N\,sec)$$

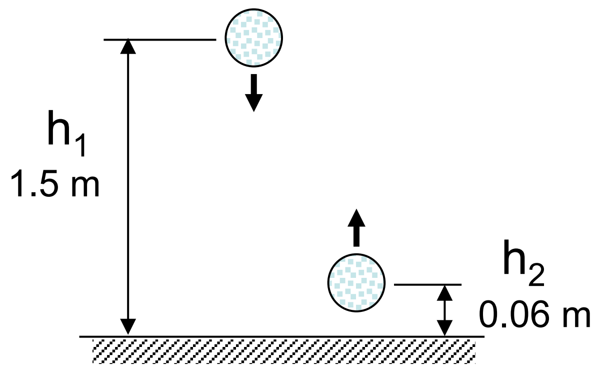
D)

9. If a 45 gram golf ball is dropped from a height of 1.50 m onto a grassy surface and bounces back upward 0.06 m, then determine the coefficient of restitution between the golf ball and the surface. (use g = 9.8 m/sec$^2$)




A) 0.04 B) 0.08 C) 0.1 D) 0.2

# Solution – Problem 9 (Page 80-81 – Ref. Handbook)




The coefficient of restitution "e" is a measure of the energy dissapated when a body impacts a surface and then rebounds.

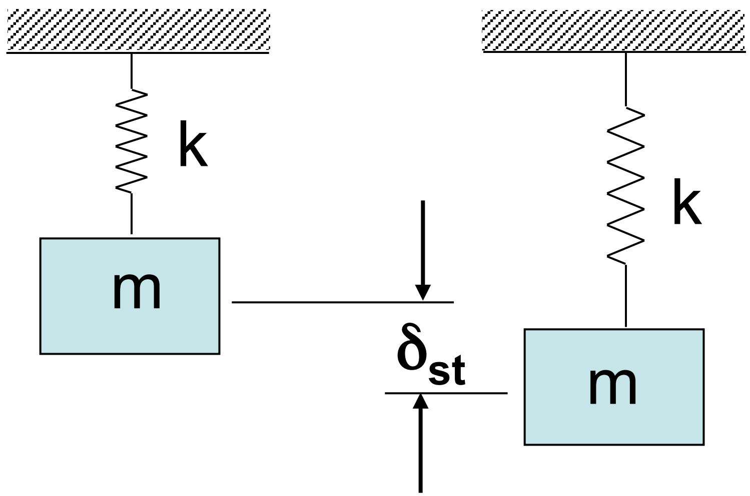
$$e = -\frac{v_{ball}^{just\ after}}{v_{ball}^{just\ before}} \quad \text{impacting stationary surface}$$

$$\text{where} \quad v_{ball}^{just\ after} = -\sqrt{2gh_2}, \quad v_{ball}^{just\ before} = \sqrt{2gh_1}$$

$$\Rightarrow \quad e = \frac{\sqrt{2gh_2}}{\sqrt{2gh_1}} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{0.06\,m}{1.5\,m}} = 0.2$$

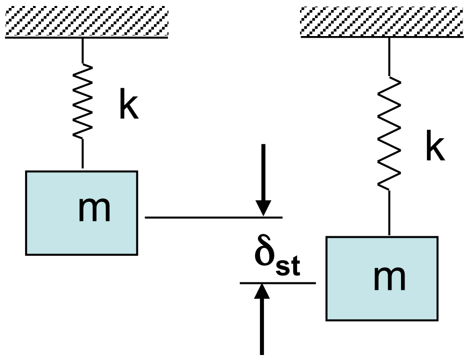
D)

10. What is the natural period of the spring-mass system shown? The mass (m) is 2 kg and the spring constant (k) is 10 N/m. The static deflection ($\delta_{st}$) is 2 m. (use g = 9.8 m/sec$^2$)




A) 2.8 sec B) 3.8 sec C) 4.8 sec D) 5.8 sec

# Solution – Problem 10 (Page 83 – Ref. Handbook)




Period is one over the frequency.
Calculate frequency for single degree mass on weightless spring.

Single degree of freedom frequency

$$\omega = \sqrt{\frac{k}{m}} \quad \text{or} \quad \omega = \sqrt{\frac{g}{\delta_{st}}}$$

$$\omega = \sqrt{\frac{10(N/m)}{2(kg)}} = \sqrt{5}\left(\frac{rad}{sec}\right), \ \omega = \sqrt{\frac{9.8(m/sec^2)}{2(m)}} \approx \sqrt{5}\left(\frac{rad}{sec}\right)$$

Period $$T = \frac{1}{\omega} = \frac{1}{\sqrt{5}}\left(\frac{sec}{rad}\right)\left(\frac{2\pi\ rad}{cycle}\right) = 2.8\ sec$$ A)