OLYMPIAD PROBLEMS

2009-2010

DIVISION E

WITH
ANSWERS AND SOLUTIONS

MATH OLYMPIADS

Since 1979

Our Thirty-First Year

Mathematical Olympiads for Elementary and Middle Schools

A Nonprofit Public Foundation

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1A *Time: 3 minutes*

What is the value of the following?

$$(8 \times 4) + (8 \times 3) + (8 \times 2) + (8 \times 1)$$

1B *Time: 5 minutes*

A bag contains 18 jelly beans. 4 are red, 6 are white and 8 are blue. Amanda takes them out one at a time without looking. What is the fewest jelly beans she must take out to be certain that at least 2 of the jelly beans she takes out are blue?

1C *Time: 5 minutes*

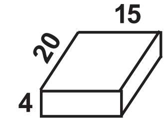
A prime number is a counting number with exactly two factors, the number itself and the number 1. In the sequence 2, 5, 11, 23, …, each number is obtained by doubling the previous number and adding 1. What is the first number in the sequence that is *not* a prime number?

1D *Time: 6 minutes*

A digital timer counts down from 5 minutes (5:00) to 0:00 one second at a time. For how many seconds does at least one of the three digits show a 2?

1E *Time: 6 minutes*

A rectangular box has a top that is 15 cm by 20 cm and a height of 4 cm. An ant begins at one corner of the box and walks along the edges. It touches all eight corners. What is the shortest distance, in cm, that the ant may travel?



Mathematical Olympiads

December 15, 2009

for Elementary and Middle Schools

2A ***Time: 3 minutes***

What is the three-digit number CAT?

$$\begin{array}{r} 3\ 4\ 5 \\ 6\ 7\ 8 \\ +\ \ C\ A\ T \\ \hline 1\ 2\ 0\ 5 \end{array}$$

2B ***Time: 5 minutes***

Suppose a *twinner* is a number that is both 1 more than a prime number and 1 less than another prime number. For example, 30 is a *twinner* because 29 and 31 are both prime numbers. What is the sum of the three least *twinners*?

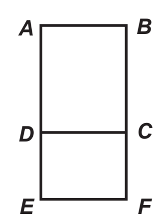
2C ***Time: 5 minutes***

Five standard dice are rolled on a flat surface and the numbers on the top faces are totaled. How many different totals are possible?

(Standard dice have 6 faces, each showing a different number from 1 through 6.)

2D ***Time: 5 minutes***

The area of rectangle *ABCD* is 63 square centimeters. The area of rectangle *DCFE* is 35 sq cm. In each rectangle, the length of each side is a counting number of cm. *AB* is longer than *DE*. How long is *AE*, in cm?



2E ***Time: 7 minutes***

Ashley, Brenda, and Cate play a game with marbles. The winner of each round of the game gets from *each* of the other players as many marbles as the winner had at the start of that round. After Round 2, Ashley has 5 marbles, Brenda has 6, and Cate has 7. How many marbles did Ashley have at the start of the game?

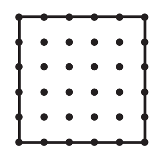
3A *Time: 4 minutes*
Joshua writes a four-digit number whose digits are 3, 5, 7, and 9, not necessarily in that order. The number is a multiple of 5. The first two digits and the last two digits have the same sum. The thousands digit is larger than the hundreds digit. What is Joshua's number?

3B *Time: 6 minutes*
One hat and two shirts cost $21. Two hats and one shirt cost $18. Megan has exactly enough money to buy one hat and one shirt. How much money does Megan have?

3C *Time: 6 minutes*
It takes 3 painters 4 hours to paint 1 classroom. How many hours does it take 1 painter to paint 2 classrooms of the same size as the first one?

Assume all painters work at the same rate for the full time.

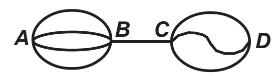
3D *Time: 5 minutes*
Mr. Wright wants to tile a 5 ft by 5 ft square floor. He has three kinds of square tiles: 1 ft by 1 ft, 2 ft by 2 ft, and 3 ft by 3 ft. Tiles may not overlap or be cut. What is the fewest tiles Mr. Wright may use to completely cover his floor?



3E *Time: 7 minutes*
111,111 is the product of 5 different prime numbers. What is the sum of those 5 prime numbers?

4A *Time: 4 minutes*
Allie has half as much money as Ben. Ben has $3 more than Emma. Emma has 5 times as much money as Shauna. Shauna has $1. How much money do Allie and Ben have together?

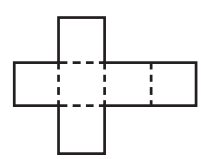
4B *Time: 5 minutes*
Following only the paths shown, what is the number of different paths that go from A to B to C to D and touch each of those points exactly once?



4C *Time: 5 minutes*
Sarah and Tyler ride their bikes. They start at the same time from the same point and ride in the same direction. Sarah travels 20 miles every hour, and Tyler travels 15 miles every hour. At the end of how many hours will Sarah be 30 miles ahead of Tyler?

4D *Time: 7 minutes*
Michael has some cards. If he puts them in 5 equal piles, there are 3 left over. If he puts them in 4 equal piles, there are 2 left over. If he puts them in 3 equal piles, there is 1 card left over. What is the fewest cards Michael may have?

4E *Time: 7 minutes*
The figure shown is made up of 6 congruent squares. The perimeter of the figure is 42 cm. It is folded along the dotted lines to form a box. How many 1-cm cubes can fit in the box?



5A ***Time: 4 minutes***

In the diagram below, what is the sum of the numbers in the shaded boxes?

1	**2**	**3**	4	5	**6**	**7**	8
9	10	11	**12**	**13**	14	15	**16**
17	**18**	**19**	20	21	**22**	**23**	24

5B ***Time: 5 minutes***

A toll bridge charges \$4 for a car and \$6 for a truck. One day 200 of these vehicles crossed the bridge and paid a total of \$860 in tolls. How many of these vehicles were trucks?

5C ***Time: 6 minutes***

Zach has 2 blue candies for every 1 red candy. After he eats 1 of the blues and 2 of the reds, Zach has 5 blue candies for every 2 red candies. How many candies does Zach start with?

5D ***Time: 7 minutes***

A cubical box without a top is 5 cm on each edge. The box is filled with 125 identical 1-cm cubes that exactly fill the box. For how many 1-cm cubes does exactly one face touch the box?

5E ***Time: 5 minutes***

The average of 6 consecutive odd numbers is 50. What is the least of these numbers?

ANSWERS AND SOLUTIONS

Note: Number in parentheses indicates percent of all competitors with a correct answer.

OLYMPIAD 1 NOVEMBER 17, 2010

Answers: [1A] **80** [1B] **12** [1C] **95** [1D] **120** [1E] **66**

87% correct

1A METHOD 1: *<u>Strategy</u>: Simplify using the Distributive Property.*
$(8 \times 4) + (8 \times 3) + (8 \times 2) + (8 \times 1) = 8 \times (4 + 3 + 2 + 1) = 8 \times 10$. **The value is 80.**

METHOD 2: *<u>Strategy</u>: Perform the operations as indicated.*
$(8 \times 4) + (8 \times 3) + (8 \times 2) + (8 \times 1) = 32 + 24 + 16 + 8 = 80$.

38%

1B *<u>Strategy</u>: Consider the worst case.*
We must avoid picking a second blue jelly bean as long as we can. Suppose Amanda picks all the red and white jelly beans first. She then has used 10 picks and her next two picks must be blue. **Without looking, she knows that among the 12 jelly beans she has picked, at least two must be blue.**

> **FOLLOW-UPS:** *(1) Suppose she wants two jelly beans of the same color, regardless of which color it is. What is the fewest jelly beans she must pick in this case?* [4] *(2) How many jelly beans would she need to take out to insure that she has at least two of each color?* [16]

40%

1C *<u>Strategy</u>: Examine each number in the sequence.*
Use divisibility tests to determine whether each number in the sequence, taken in ascending order, is prime or composite. The first five numbers (2, 5, 11, 23, and 47) are each prime. The next number, 95, ends in a 5 and is divisible by 5. **Thus, 95 is the first number in the sequence that is *not* a prime number.**

> **FOLLOW-UP:** *Consider the prime numbers less than 100. How many pairs of consecutive prime numbers have a difference that is odd?* [1]

11%

1D METHOD 1: *<u>Strategy</u>: Count in an organized way.*
The table shows the total count-down time separated into one-minute intervals. The second column specifies which times contain a “2” and the third column counts the total number of seconds “2” is displayed in each interval.
Thus, one of the digits shows a “2” for 120 seconds.

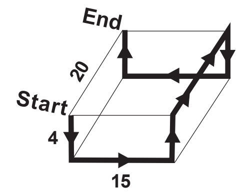
METHOD 2: *<u>Strategy</u>: Count the number of seconds that a 2 is <u>not</u> showing.*
Consider the times from 4:59 through 0:00, a total of 300 seconds.
The minutes digit is 4, 3, 2, 1, or 0. There are 4 values other than 2.
The 10-second digit is 5, 4, 3, 2, 1, or 0. There are 5 values other than 2.
The seconds digit is 9, 8, 7, 6, 5, 4, 3, 2, 1, or 0. There are 9 values other than 2.

We can form a reading that does not show 2 by choosing a non-2 for each of the 3 digits. This can be done in 4 × 5 × 9 = 180 different ways. There are then 180 seconds in which no 2 is showing and therefore 300 – 180 = 120 seconds in which a 2 is showing.

Follow-Ups: *(1) Which of the other digits will also be displayed for exactly 120 seconds?* [4,3,1] *(2) How many numbers between 200 and 600 are not divisible by 5?* [320]

9%

1E *Strategy: Minimize the use of the longest sides.*
By touching all 4 front corners first and then all 4 rear corners as shown, the ant can travel along a 20-cm side only once. If the ant starts along a 4-cm side when touching the 4 front corners, it travels only once along a 15 cm side. The same is true when the ant touches the 4 rear corners. **The shortest distance that the ant may travel is** (4 ×4) + (2 ×15) + (1 ×20) = **66 cm.** The diagram shows one of several possible paths.



Follow-Up: *How many different paths are 66 cm long?* [8, one starting at each corner]

OLYMPIAD 2 — December 15, 2010

Answers: [2A] **182** [2B] **22** [2C] **26** [2D] **14** [2E] **10**

78% correct

2A METHOD 1: *Strategy: Work from right to left.*
In the ones column, 5 + 8 + T ends in 5, so T = 2 (with a "carry" of 1). Then 1 + 4 + 7 + A ends in 0, so A = 8 (with a carry of 2). Finally, 2 + 3 + 6 + C is 12, and C = 1. **The three-digit number CAT is 182.**

$$\begin{array}{r} 3\ 4\ 5 \\ 6\ 7\ 8 \\ +\ \ C\ A\ T \\ \hline 1\ 2\ 0\ 5 \end{array}$$

METHOD 2: *Strategy: Add the first two numbers and subtract from the sum.*
1205 – (345 + 678) = 1205 – 1023 = 182.

Follow-Up: *Find digits A and B in the following multiplication: 12,345,679 × A = BBB,BBB,BBB.* [*A* = 9, *B* = 1; the digits of *BBB,BBB,BBB* add to 9 × *B*, a multiple of 9.]

31%

2B *Strategy: List the prime numbers.*
The first few primes are 2,3,5,7,11,13,17,19… . A "twinner" is surrounded by primes, so look for primes that differ by 2 (these are called *twin primes*). The first three pairs are 3 & 5, 5 & 7, and 11 & 13. The three least "twinners" are 4, 6 and 12, and **their sum is 22.**

Follow-Ups on next page.

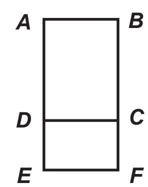
FOLLOW-UPS: *(1) Find three primes such that the sum of two of them equals the third.* [2 and any pair of twin primes] *(2) Can you find a solution without using 2 as one of the numbers? Explain.* [No. Primes other than 2 are odd, and the sum of two odd numbers is even.]

8%

2C *Strategy: Find the range of possible sums.*
If each die shows 1, the total is 5. If each die shows 6, the total is 30. All integral sums from 5 to 30 inclusive are possible. These are all the counting numbers up to 30, except for 1 through 4. Then **26 different sums are possible.**

34%

2D *Strategy: Determine the length of the common side.*
$\overline{DC}$ is a side of both rectangles *ABCD* and *DCFE* and its length is then a factor of both 63 and 35. The only common factors of 63 and 35 are 1 and 7. Suppose *DC* = 1. Then *AB* = 1 and *DE* = 35. But since *AB* is longer than *DE*, *DC* must be 7. Then *AD* = 9, *DE* = 5, and ***AE* is 14 cm long.**



8%

2E *Strategy: Working backwards, find the winner of each round.*
The winner of a round receives as many marbles as she already has from each of the others. **This triples what she has.** That is, after each round, the winner's total is a multiple of 3.

At the end of Round 2, the only multiple of 3 is Brenda's total, 6, so she won Round 2. Brenda started Round 2 with 2 marbles and received 2 more from each of the others. The table below shows how many marbles each had at the end of each round.

Similarly, at the end of Round 1, the only multiple of 3 is Cate's highlighted total, 9, so she won Round 1. Cate had started Round 1 with 3 marbles and received 3 more from each of the others. **At the start of the game, Ashley had** 7 + 3 = **10 marbles** as highlighted in the table.

Round	Ashley	Brenda	Cate
End of Round 2 — Brenda won 2 marbles from each.	5	**6**	7
End of Round 1 — Cate won 3 marbles from each.	7	2	**9**
Start.	**10**	5	3

OLYMPIAD 3 — JANUARY 12, 2010

Answers: [3A] **9375** [3B] **13** [3C] **24** [3D] **8** [3E] **71**

79% correct

3A *Strategy: Consider each condition in turn.*
The 4 digits form a sequence of consecutive odd numbers. To get the same sum, pair the greatest with the least, and the two middle ones with each other. Because Joshua's number is a multiple of 5, the last digit is a 5. The partner of 5 is 7, so the last two digits are 75. The thousands digit is greater than the hundreds, so the first two digits are 93.
Joshua's number is 9375.

62%

3B **METHOD 1:** *Strategy: Combine the given information.*
Suppose Megan has enough money to buy 1 hat and 2 shirts for $21 and then another 2 hats and 1 shirt for $18. In total, she has bought 3 hats and 3 shirts for $39. But she has enough for only 1 hat and 1 shirt and so **Megan has** 39 ÷ 3 = **$13.**

METHOD 2: *Strategy: Make a table.*
Try different values for the cost of a shirt. Use the first statement to find the cost of a hat. See which value also gives $18 for the second statement.

***Costs*:**

1 shirt:	$5	$6	$7	**$8**
2 shirts:	$10	$12	$14	$16
1 hat:	21–10 = $11	21–12 = $9	21–14 = $7	21–16 = **$5**
2 hats + 1 shirt:	2×11+5 = $27	2×9+6 = $24	2×7+7 = $21	**2×5+8 = $18**

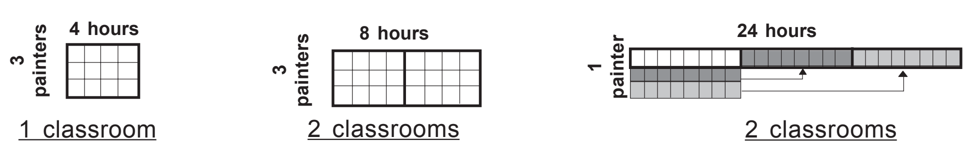
A shirt costs $8 and a hat costs $5. Megan has $13.

43%

3C **METHOD 1:** *Strategy: Find the time 1 painter needs to paint 1 room.*
Three painters each need 4 hours to paint one classroom, so one painter needs 3 × 4 = 12 hours to paint that classroom. **Then for one painter to paint two classrooms, it would take** twice as long, or **24 hours.**

METHOD 2: *Strategy: Find the part of a room done per hour by 1 painter.*
In 4 hours, 3 painters can paint 1 classroom, so in 1 hour the 3 painters can paint $\frac{1}{4}$ of a room. Then in 1 hour each painter paints $\frac{1}{12}$ of a room. So each painter working alone needs 12 hours to paint 1 classroom and therefore 24 hours to paint 2 classrooms.

METHOD 3: *Strategy: Draw a picture.*
In the pictures, each small square represents 1 painter's work for 1 hour. The first picture shows that 3 painters (rows) need 4 hours (columns) to paint 1 classroom. The next picture doubles the number of squares (by doubling the number of columns) to show the time the 3 painters need for 2 classrooms. The third picture rearranges the small squares into 1 column (1 painter) and shows that 1 painter needs 24 hours to paint the 2 classrooms.



42%

3D <u>Strategy</u>: *Place the largest tiles first.*
Start with a 3 by 3 tile. No matter where it is placed, the greatest number of squares remaining in a row or column is two. So only one 3 by 3 tile can be used. Put it in a corner position to allow maximum space for the 2 by 2 tiles.

Then 3 of the 2 by 2 tiles can be placed. One placement is shown. The remaining spaces must be filled by the 1 by 1 tiles. There are 4 of those spaces.

The fewest number of tiles is 1 + 3 + 4 = **8.**

> **FOLLOW-UP:** *What would be the fewest number of tiles Mr. Wright would need if his floor measured 6 ft by 6 ft? 7 ft by 7 ft? 8 ft by 8 ft?* [4; 12; 11]

5%

3E **METHOD 1:** <u>Strategy</u>: *Find a large factor first.*
Note that 111,111 consists of 2 blocks of the digits "111". Then 111 is a factor of 111,111, and upon division, 111,111 = 111 × 1001. To factor 111, note that the digit-sum is 3 and therefore 3 is a factor. Upon division, 111 = 3 × 37. These 2 factors are both prime.

The problem states that there are 3 more prime factors. To factor 1001, note that the divisibility test for 11 is satisfied (In 1001, 1 + 0 = 0 + 1). Upon division, 1001 = 11 × 91. 11 is prime, so 91 must be the product of the last 2 primes. To find them, it suffices to try primes that are less than 10. 2, 3, and 5 don't work, but 91 = 7 × 13, both of which are prime.
The sum of the 5 prime factors of 111,111 is 3 + 37 + 11 + 7 + 13 = **71.**

METHOD 2: <u>Strategy</u>: *Find a small factor first.*
The sum of the digits in 111,111 is 6, a multiple of 3, so 3 is a factor of 111,111, and 111,111 = 3 × 37,037. To factor 37,037, try 37 to get 37,037 = 37 × 1001. Proceed as in Method 1 to get the 5 prime factors 3, 37, 11, 7, and 13, whose sum is 71.

METHOD 3: <u>Strategy</u>: *Divide by each prime in order, starting with 2.*
111,111 ÷ **2** is not a whole number. 111,111 ÷ **3** = 37, 037. 37,037 ÷ **5** is not a whole number. 37,037 ÷ **7** = 5291. 5291 ÷ **11** = 481. 481 ÷ **13** = **37**. Then 3 + 7 + 11 + 13 + 37 = 71.

> **FOLLOW-UP:** *(1) In Method 1, we said that in order to factor 91, you only had to test primes less than 10. Why is this so?* [If both factors are greater than 10, the product is greater than 100.] *(2) To determine whether 421 is a prime number, you try to factor it. What is the greatest factor you have to try to show that it is prime?* [19]

OLYMPIAD 4 — FEBRUARY 9, 2010

Answers: [4A] **12** [4B] **12** [4C] **6** [4D] **58** [4E] **27**

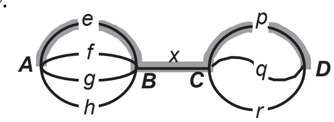
84% correct

4A METHOD 1: *Strategy: Work backwards.*
Shauna has \$1, so Emma has 1 × 5= \$5. Then Ben has 5 + 3= \$8.
Allie has ½ × 8 = \$4. **Allie and Ben have** 8 + 4= **\$12 together.**

> ***Follow-Up:*** *Lauren went to the mall with all of her birthday money. She spent half of it on a pair of designer jeans, a third of what was left on a T-shirt, and a sixth of what was left after that on a slice of pizza and a soda. She returned home with \$25. How much money did she get for her birthday?* [\$90]

54%

4B METHOD 1: *Strategy: Count paths to each letter separately.*
For each of the 4 paths from A to B, there is 1 path from B to C and 3 paths from C to D.
There are 4 × 1 × 3 = **12 different paths that go from A to B to C to D** and touch each point once.



exp exq exr
fxp fxq fxr
gxp gxq gxr
hxp hxq hxr

METHOD 2: *Strategy: Make an organized list.*
Label the individual paths by naming the three segments traveled. One such path, shown by the thick lines is *exp*. Paths from A to B to C to D can be represented by a tree diagram or by the list at the right:

There are 12 paths in all.

67%

4C METHOD 1: *Strategy: Compare the distances they ride each hour.*
Each hour, Sarah rides 5 miles more than Tyler. **Sarah will be 30 miles ahead of Tyler in** 30 ÷ 5 = **6 hours.**

METHOD 2: *Strategy: Use algebra.*
Let t = the number of hours that each rides. Sarah rides at 20 mph, so the distance she travels is $20t$. Likewise, Tyler rides a distance of $15t$ miles.
Then $20t = 15t + 30$. Solving, $t = 6$. Sarah will be 30 miles ahead of Tyler in 6 hours.

> ***Follow-Up:*** *Jake and Adam head off for the same ski lodge 270 miles away, but Adam starts one hour ahead of Jake. If Adam is traveling at 45 miles per hour, how fast must Jake travel to arrive at the ski lodge at the same time that Adam does?* [54 mph]

28%

4D METHOD 1: *Strategy: Use the pattern in the given information.*
Note that in each case the number of cards left over is 2 less than the number of piles. Suppose Michael gets 2 more cards. He can now put the cards into 3, 4, or 5 equal piles. Therefore the new number of cards is a multiple of 60, the Least Common Multiple (LCM) of 3, 4, and 5. Then before getting the extra 2 candies, **Michael has 58 cards.**

METHOD 2: *<u>Strategy</u>: Consider one condition at a time.*
The number of cards is 3 more than a multiple of 5, so it ends in 3 or 8.
This number is also 2 more than a multiple of 4, so it is even. The number ends in 8.
This number is 1 more than a multiple of 3; this multiple of 3 must end in 7.
Add 1 to the multiples of 3 that end in 7: 28, 58, 88, 118, … and test each.
28 satisfies two conditions, but is not 2 more than a multiple of 4. However, 58 satisfies all three conditions. Michael has 58 cards.

> **FOLLOW-UP:** *What is the least number that leaves a remainder of 3 when divided by 5, a remainder of 2 when divided by 6, a remainder of 1 when divided by 7, and is greater than 200? (Hint: Take some away.)* [218]

19%

4E *<u>Strategy</u>: Find the length of one side of the box.*
The perimeter of the figure is made up of 14 congruent segments. Each segment is 42 ÷ 14 = 3 cm. Folding the figure forms a box 3 cm high with a 3 cm by 3 cm base. 3 × 3 = 9 cubes can fit in one layer on the bottom and 3 such layers can fit in the box. In all, **27 one-cm cubes can fit in the box.**

OLYMPIAD 5 — MARCH 9, 2010

Answers: [5A] **150** [5B] **30** [5C] **24** [5D] **57** [5E] **45**

69% correct

5A METHOD 1: *<u>Strategy</u>: Look for a pattern.*
Notice that the numbers in the first shaded box and the last shaded box add to 25. Similarly, 3 + 22 = 25, 6 + 19 = 25, and so on. Six pairs of numbers each add to 25, so **the sum of the numbers in the shaded boxes is** 6 × 25 = **150.**

METHOD 2: *<u>Strategy</u>: Add in an organized way.*
Add by rows or columns. (See diagram.) 18 + 50 + 82 = 9 + 20 + … + 16 = 150.

1	**2**	**3**	4	5	**6**	**7**	8	18
9	10	11	**12**	**13**	14	15	**16**	50
17	**18**	**19**	20	21	**22**	**23**	24	82
9	20	22	12	13	28	30	16	**150**

> **FOLLOW-UP:** *What is the sum 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512? (Hint: Look for a pattern in the partial sums as terms are added left to right.)* [1023]

23%

5B **METHOD 1:** *<u>Strategy</u>: Start with a specific number of each vehicle.*
Suppose all 200 vehicles were cars. The toll total would be 4 ×200 = \$800, which is \$60 too low. Each car that is replaced by a truck increases the toll total by \$2. To increase the total by \$60, replace 60 ÷ 2 = 30 cars by trucks. Then **30 of the vehicles were trucks.** Checking, (30 × \$6) + (170 × \$4) = \$860.

METHOD 2: *<u>Strategy</u>: Use algebra.*
Let T = the number of trucks. Then 200 – T = the number of cars.
$4(200 - T) + 6T = 860$. Solving, $T = 30$. 30 of the vehicles were trucks.

16%

5C **METHOD 1:** *<u>Strategy</u>: Make two tables.*

Before eating some

Blue	2	4	6	…	14	16	18	20
Red	1	2	3	…	7	8	9	10

After eating some

Blue – 1	1	3	5	…	13	**15**	17	19
Red – 2		0	1	…	5	**6**	7	8

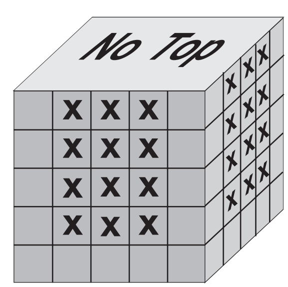
When Zach has 15 blue and 6 red candies, he has 5 blues for every 2 reds. Then **Zach starts with** 16 + 8 = **24 candies.**

METHOD 2: *<u>Strategy</u>: Group the candies two different ways.*
At first, Zach can form groups of 3 candies with 2 blue and 1 red in each group. The total number of candies is a multiple of 3. After he eats 3, the total is still a multiple of 3, but now the candies can also be grouped by 7s with 5 blues and 2 reds in each group. Thus the new total is now a multiple of both 3 and 7; that is, a multiple of 21. Test 21, 42, 63, … to see which multiple satisfies all conditions of the problem.

First test 21 candies. There are 3 groups of 7 and in each group 5 are blue and 2 are red. There are 3 × 5 = 15 blue and 3 × 2 = 6 red candies. Adding back the 1 blue and 2 red candies that were eaten, there were originally 16 blue and 8 red candies. This is 2 blues for every red, so the conditions of the problem are satisfied. Zach starts with 16 + 8 = 24 candies.

5%

5D *<u>Strategy</u>: Draw a picture. First consider the side faces of the box.*
The 12 cubes marked with an **X** in the picture each has exactly one face touching the front of the box. Likewise, there are another 12 cubes with exactly one face touching each of the other vertical sides. Only 9 cubes have exactly one face touching the bottom of the box, since each edge cube on the bottom also touches one or two vertical sides. (To visualize it, sketch of the bottom of the box.) In all there is **a total of** 4 × 12 + 9 = **57 one-cm cubes that have exactly one face touching the box.**



FOLLOW-UPS on next page.

FOLLOW-UP: (1) How many of the 125 cubes do not touch the box? [36] *(2) Suppose the box had a closed top. How many cubes would touch exactly one face?* [54] *(3) Now, suppose the box were 4 cm on an edge, with 64 cubes inside the box. How many cubes would touch exactly one face?* [24] *(4) Suppose the box were 6 cm on an edge, with 216 cubes inside the box. How many cubes would touch exactly one face?* [96] *Can we generalize these results?*

27%

5E METHOD 1: *Strategy: Work from the middle outward.*

Consider the numbers listed in order from smallest to largest:	__, __, __, __, __, __
Since 50 is the average, and all numbers are equally spaced, the two consecutive odd numbers in the middle are 49 and 51:	__, __, 49, 51, __, __
Once these are in place, write the odd numbers that precede 49 and that follow 51 to complete the list:	45, 47, 49, 51, 53, 55

The least of these numbers is 45.

METHOD 2: *Strategy: Group the candies two different ways.*

The sum of the six consecutive odd numbers is 6 × 50 = 300. Choose the simplest such set (1, 3, 5, 7, 9, and 11), and then add the same amount to each to reach a sum of 300. 36 is 264 short of 300, so we must increase each number in the set by 264 ÷ 6 = 44. The set then becomes 45, 47, 49, 51, 53, and 55. The least number in the set is 45.