



E-Fulfillment: Optimal Strategy and Operational Requirements for some Inventory Models

A Thesis

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By

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NOTATION TABLE

λ	Demand rate per unit time
c_0	Unit cost for producing or purchasing each unit
C_H	Holding cost per unit of time held in inventory
C_S	Shortage cost per unit of time short
K	The setup cost for producing or ordering one batch
S	Inventory level just after a batch of Q units is added to inventory
d	Total demand
i	$i=1$ refers to e-tail channel, and $i=2$ refers to retail channel
p	Market allocation for fulfilling Center and the supplier
p_i	Unit price for channel i , $i=1, 2$
Q_i	Inventory quantity for channel i , $i=1,2$

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SUMMARY

E-Fulfillment: Optimal Strategy and Operational Requirements for some Inventory Models

Submitted by: Sarah Ahmed Magdy Ali Mohamed Elsherif

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Inventory system concerns about the stock in hand and making sure that there is no sales loss due to shortage of stock. It also makes sure that relevant costs like: holding cost, capital cost are at the minimum. The goal of an inventory system is to determine the optimal stock levels and reorder point.

The thesis consists of five chapters, summary and list of publications.

Chapter I presents a brief review of the types of inventory systems and their properties for both retail and e-tail stores.

Chapter II presents inventory models like: Deterministic models where Replenishment lead time is known and rate of demand is known and constant too. It also presents continuous review uniform Demand – known also as EOQ, in which units are assumed to be withdrawn continuously at a known constant rate denoted by λ . The objective of this inventory problem is to determine how often to make a production run and what size it should be so that the cost per unit of time is minimized. It will first deals with the assumption that the shortages are not allowed and then this assumption will be relaxed.

Chapter III considers a single-product inventory system that serves both online and traditional customers where demands are independent Poisson processes. A (Q,r) replenishment policy is used and replenishment lead times are assumed to be constant.

The optimal parameters of the system are derived by minimizing the expected annual cost. The model has been illustrated with numerical examples and discuss the sensitivity of the results to variables such as demand lead time and the split between online and traditional orders.

Chapter IV presents a two-echelon dual channel supply chain where customers can purchase either from retail or e-tail channel. It also presents two different strategies: Centralized Inventory Strategy and Stackelberg Inventory Strategy. For each strategy we obtained the optimal inventory levels for both channels and the respective expected profits.

The material of this chapter has been accepted and published in the International Journal of Original Research, May 2016. India.

Chapter V deals with a two-echelon inventory model where manufacturer warehouse (upper echelon) which covers e-tail demand and retail store (lower echelon) which covers retail demand and the product is available in two supply channels: retail channel and e-tail direct channel. The system receives demand from two different segments: retail store loyal customers and internet based direct channel loyal customers. To develop operational measures of supply chain flexibility, we defined: inventory holding costs and sales lost costs. Computational outcomes indicate that the dual-channel strategy outperforms the other two channel strategies in most cases.

E-Fulfillment: Optimal Strategy and Operational Requirements for some Inventory Models

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Abstract

We look at some inventory models for dual channel retailers who fulfill demands from both channels from the same pool of stock. Since the arrival of the internet, companies, particularly retailers, have been trying to figure out how best to take advantage of this new channel. This became acute for retailers when Amazon.com became the pre-dominant bookseller overcoming rivals like Barnes and Noble and Borders who had been operating for several decades. Firms in other categories were worried that a web only retailer like Amazon.com could end up dominating their sectors as well. To improve efficiency and to take advantage of specific characteristics of this channel, firms had to modify the operations of their existing channels. In this thesis, we look at retailers who took advantage of the web channel to better their operations for the entire firm.

We introduce in Chapter I, a brief review of the types of inventory systems and their properties for both retail and e-tail stores.

In Chapter II, we provide a comprehensive literature survey of the topics that are covered in the thesis.

In Chapters III, IV, and V, we study the impact of product substitution differences exhibited by customers in the two channels on the inventory decisions of the dual channel retailer.

In Chapter III, we study a single-product inventory system that serves both online and traditional customers where demands

are independent Poisson processes. The optimal parameters of the system are derived by minimizing the expected annual cost.

Chapter IV presents a two-echelon dual channel supply chain where customers can purchase either from retail or e-tail channel. It also presents two different strategies: Centralized Inventory Strategy and Stackelberg Inventory Strategy. For each strategy we obtained the optimal inventory levels for both channels and the respective expected profits.

Chapter V deals with a two-echelon inventory model where manufacturer warehouse (upper echelon) which covers e-tail demand and retail store (lower echelon) which covers retail demand and the product is available in two supply channels. Computational outcomes indicate that the dual-channel strategy outperforms the other two channel strategies in most cases.

Key words

Inventory, inventory models, inventory strategies, centralized inventory strategy, stackelberg inventory strategy, stochastic demand, deterministic demand, Retail, e-tail, direct channel, traditional channel, upper echelon, lower echelon, dual channel, supply chain, shortage is permitted, shortage not permitted, quantity discount – shortage not permitted.

CHAPTER I

E-FULFILLMENT: A BRIEF REVIEW OF THE INVENTORY THEORY

1.1. INTRODUCTION

The control and maintenance of inventory is a problem common to all organizations in any sector of the economy. The problems of inventory do not confine themselves to profit-making institutions. Social and nonprofit institutions encounter the same types of problems. Inventories are common to agriculture, manufacturers, wholesalers, retailers, hospitals, prisons, zoos, universities and local governments. Indeed, inventories are also relevant to the family unit in relation to food, clothing, medicines and so forth. On an aggregate national basis, the total investment in inventory represents a sizable portion of the gross national product.

Inventory problems have been encountered by every society, but until the twentieth century the analytical techniques were not developed to study them. The initial impetus for analysis came from manufacturing industries. A concentrated effort on risk and uncertainty aspects of inventory was not made until after World War II. In theory, inventory is an area of organizational operation that is well developed. In practice, it is very backward. This gap will narrow as educational institutions integrate materials management into their course structures.

Historically, inventory management has often meant too much inventory and too little management or too little inventory and too much management. There can be severe penalties for excesses in either direction. Inventory problems have proliferated as technological progress has increased the organization's ability to produce goods in greater quantities, faster, and with multiple design variations. The public has compounded the problem by its receptiveness to variations and frequent design changes.

The relative significance of inventory management to an organization can be gaged by the overall investment in inventory and the magnitude of the material costs for all products. The overall investment in inventory can be ascertained by reviewing the balance sheet of an organization. If the investment in inventory is a large percentage of current assets or total assets, major emphasis should be placed on inventory management. Likewise, if material costs are a large percentage of total product costs, inventory management is critical.

1.2. E-COMMERCE

Electronic commerce (e-commerce) has risen considerably over the last years with the commoditization of internet. Although it started mostly associated with the financial and the airline industry, nowadays, e-commerce is present in virtually all industries, including the retail industry (e-tailing) (Gunasekaran et al., 2002).

E-commerce sales in Portugal in 2014 were approximately 626 million Euros, representing 3% of total retail sales. This figure is not transversal throughout all sectors. In office supplies,