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Modelling of multi-axial ultimate elastic wall stress (UEWS) test for glass fibre reinforced epoxy (GRE) composite pipes

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Abstract. This paper describes the modeling of multiaxial ultimate elastic wall stress (UEWS) at room temperature for glass fibre reinforced epoxy (GRE) composite pipes. The model developed, predicts the stress-strain response caused by the combined, static and cyclic of UEWS loading taking into effects of transverse matrix cracking within the laminates. The procedure, although not a standard method, seems to provide a good alternative to the current raw materials' re-qualification procedure delineated in ISO 14692 through ASTM D2992. The effective transverse and shear modulus of the lamina due to increasing presence of transverse matrix cracking were estimated. Classical laminate analysis was then applied to compute the corresponding ply properties as a function of increasing stress and strain. The model shows a good agreement with the experimental results of multiaxial UEWS tests on ±55° filament wound glass-reinforced epoxy pipes.

Introduction

The failure behaviour of filament wound GRE pipes subjected to biaxial load has been the subject of numerous experimental and modelling investigations spanning decades, as demonstrated in the literature [1-4]. The majority of such investigations have emphasized on the failure envelopes, fatigue strength, leakage and the associated deformation of angle ply laminates similar to those used in GRE pipes. However, whilst most of these studies concentrated on structural failure in composite pipes, the more significant issue of micro structural progressive damage, which leads to the final failure, is less clear.

Matrix cracking within composite laminates has been recognized as the major factor causing the reduction in stiffness of laminates. Various models have been presented to characterize such degradation in stiffness due to transverse matrix cracking under in-plane uniaxial and multiaxial loading. Among these models are the ply-discount approximation [5], the continuum damage model [6], shear lag model [7], self-consistent scheme [8], and the variational model proposed by Hashin [9]. Recently, Katerelos et al. [10] conducted an analysis of the effect of matrix cracking on the behaviour of angle ply laminates loaded statically using the equivalent constraint model (ECM). The approach showed a good agreement with the experimental results obtained by microscopic strain measurement using the laser Roman spectroscopy technique [11].

A finite element model was proposed by Tao and Sun [12] and Sun and Tao [13], who investigated the effects of matrix cracking on the stiffness degradation of laminates. The authors concluded that normalized crack density rather than crack density is a more appropriate parameter to be used in predicting cracking damage. This present investigation models the stress strain response of GRE pipes as results of transverse matrix cracking during multiaxial UEWS test. The results then were compared with the experimental UEWS data to achieve the closest fit.

Linear and non-linear stress strain modelling of UEWS test

The theoretical mechanical properties of the individual ply and the laminates or the pipe was calculated and later compared with the experimental results. First, for the calculation of reinforcement fibre, rule of mixtures was used to predict the E_1 and v_{12} to a good accuracy. However, the same treatment on predicting E_2 gives a large error due to the non-uniform distribution of stress and strain in transverse direction. Hence, Halpin-Tsai simplification was used instead to calculate the E_2 and G_{12} of the ply. Based on isotropic glass fibre reinforcement properties provided by FPI for the Wavistrong pipe product; $E_g = 73$ GPa and $v_g = 0.59$, epoxy matrix properties; $E_m = 3.6$ GPa and $v_m = 0.41$, the ply properties were calculated to be E_1=44.5 GPa, E_2=12.2 GPa, G_{12}= 4.33 and $v_{12} = 0.28$. The properties of the ±55° GRE pipe were then computed using laminate theory and given in the axial and hoop direction of the pipe. From the calculation,

$$E_{axial} = 11.52\ GPa \qquad E_{hoop} = 19.70\ GPa$$

$$\upsilon_{axial} = 0.40 \qquad \upsilon_{hoop} = 0.69$$

$$G_{12} = 11.76\ GPa$$

Also important to note that, radial component in this case is lower than that of axial and hoop components and therefore, ignored. For internal pressure loading only of filament wound GRE pipes, the stress is calculated from the following equation;

$$\sigma_H = \frac{Pd}{2t}, \sigma_A = \frac{Pd}{4t} \tag{1}$$

The corresponding strains produced by the these stresses generated in the tubes is then worked out from the following relations;

$$\varepsilon_H = \frac{\sigma_H}{E_H} - \upsilon_{AH}\frac{\sigma_A}{E_A}, \varepsilon_A = \frac{\sigma_A}{E_A} - \upsilon_{HA}\frac{\sigma_H}{E_H} \tag{2}$$

These strains were then transformed to the ply coordinate system by multiplying with the transformation matrix. Hence,

$$\begin{aligned}
\varepsilon_1 &= \varepsilon_A \cos^2\theta + \varepsilon_H \sin^2\theta \\
\varepsilon_2 &= \varepsilon_A \sin^2\theta + \varepsilon_H \cos^2\theta \\
\gamma_{12} &= 2\sin\theta\cos\theta(\varepsilon_H - \varepsilon_A)
\end{aligned} \tag{3}$$

Since the pipe wall is an angle ply laminates, the lamina can be considered of having orthotropic elastic properties, which are highly dependent on the winding angle θ. Thus, the stress-strain response at a low stress level of which the stress strain behaviour can be considered to be linear, the stresses in the unidirectional ply can be written as follows;

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{pmatrix} = \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_1 \\ \gamma_{12} \end{pmatrix} \tag{4}$$

Where *Q_{11}, Q_{12} and etc.* are the stiffness matrixes, which can be expressed in engineering terms as,

$$\begin{aligned}
Q_{11} &= \frac{E_1}{1-v_{12}v_{21}}; \qquad Q_{12} = \frac{v_{12}E_1}{1-v_{12}v_{21}} = \frac{v_{21}E_2}{1-v_{12}v_{21}} \\
Q_{22} &= \frac{E_2}{1-v_{12}v_{21}}; \qquad Q_{66} = G_{12}
\end{aligned} \tag{5}$$

Where, E_1 and E_2 are the modulus of elasticity in the lamina's principal axes. From finite element model developed by Sun and Tao, the deterioration in the transverse and shear modulus of composite laminates due to the increasing presence of matrix cracks can be estimated in the form of;

$$\begin{aligned}
\frac{E_2}{E_2^o} &= \exp(-\alpha_{E2}\rho^*) \\
\frac{G_2}{G_2^o} &= \exp(-\alpha_G\rho^*)
\end{aligned} \tag{6}$$

Where;
E_2 and E_2^o are effective and initial transverse modulus of ply respectively.
G_2 and G_2^o are effective and initial shear modulus of ply respectively.
α_{E2} and α_G are curve fitting constant.
ρ is the normalized crack density function.

In this model, the non-linearity response as a result of matrix micro cracking only took place when the transverse stress in the ply reached the failure strength of the epoxy resin. Hence, the relationship between the crack density and applied stress can be derived [14] and given below;

$$\rho = \kappa\left[\frac{\sigma_2 - \sigma_2^{fail}}{\sigma_2^{fail}}\right] \tag{7}$$

Where; σ_2 is the limiting transverse stress in unidirectional ply
σ_2^{fail} is the failure strength of the matrix material
$K = \sqrt{\frac{(E_1 + E_2)G_1}{E_1E_2}}$, where K involves only the ply modulus constants

The estimation of effective transverse and shear modulus of the ply at every pressure group increment can then be calculated from equation (6). For close adaptation to the experimentally determined curve of all stress ratios, the curve fitting constants α_{E2} and α_G were fitted by optimising one constant at a time while retaining the value of the other. σ_2^{fail}, which is transverse failure stress was adjusted and assigned to a constant value hence demonstrating the effects of total stress on the laminate [14]. The effective modulus then applied with the laminate theory to determine the new corresponding axial and hoop modulus of the pipe after taken into account the effects of the matrix cracking. The gradually degraded stiffness calculated was later inserted into equation (2) establishing the nonlinear stress strain response.

Results and discussions

The modelled stress strain curves for a different ratio of fitting constants α_{E2}/α_G at various ratios of UEWS tests are shown in Figure 1-4. The calculations are based on equation (6-7) before subjected to the laminate theory to determine the corresponding strains in the pipe axes. Optimizations of the ratio of the fitting constant was carried out with the intention of getting the best possible match to the experimental strains of the 10th cycle obtained from UEWS test with axial strains superimposed at different α_{E2}/α_G ratios. Throughout the modeling work, σ_2^{fail} was chosen to be between 40-50MPa, since these values give the best fit for all loading conditions.

Through the model developed, suggests that they are closely conformed to the UEWS experimental data. In all loading conditions, the non-linearity modelled indicated slow change in the slope rather than abrupt change in response, which normally seen and described as the knee point. Figure 1 shows the model curve fitting, together with the actual findings for UEWS test conducted at 2:1 hoop to the axial stress ratio, within the room temperature environment. As we can see from the plot, the ratio of curve fitting constants between α_{E2} and α_G from equation (6) can be seen increasing from 0.8 to 1.28 to give the best fit of the stress strain response with the experimental results. At the ratio of 0.8, the stress strain behaviour showed an almost linear response. At $\alpha_{E2}/\alpha_G = 1.0$, the curve in the non-linear section showed an upward shift to a higher strain value. Further increase of α_{E2}/α_G = 1.28, at the end gives good agreement on the non-linearity response to the experimental result for the case of 2:1 loading condition. Here, it appears that by increasing the α_{E2} constant, which relates the effects of a matrix cracking to the deterioration in transverse modulus, the model's curve can be matched very well to the non-linear behaviour showed by the actual findings from UEWS tests.

Similar trend was also noted for the cases of 1:1 loading and pure axial (0:1) loading conditions illustrated in Figure 2 and 3, respectively. Though, the modelled strains are slightly higher to those obtained experimentally, especially within the linear region. Considerable increased values of

α_{E2}/α_G were attained for the case of these loadings. This implied that the non-linearity of the stress strain response during axial dominated loading much has been caused by the deterioration in transverse modulus. For 1:1 loading, the best fit was obtained at $\alpha_{E2}/\alpha_G = 3.0$. Whilst pure axial loading, which presumably more prone to transverse stiffness reduction by matrix cracking showed the closest fit to experimental data at $\alpha_{E2}/\alpha_G = 6.0$, which is the highest of the previous two modelling results.

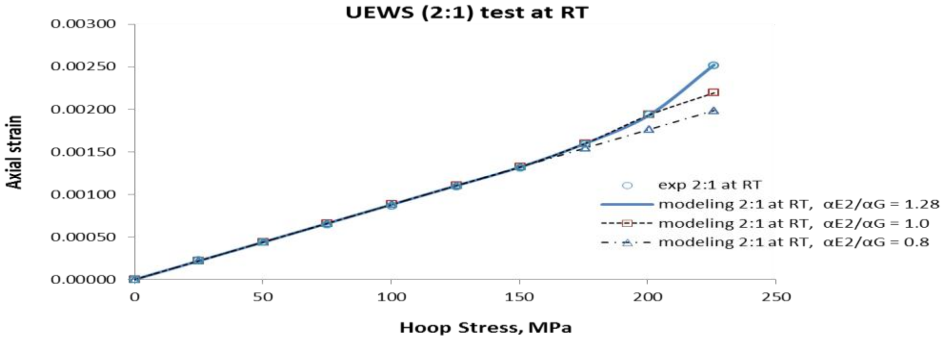

Figure 1. Experimental and model stress strain curve for UEWS test (2:1) at room temperature.

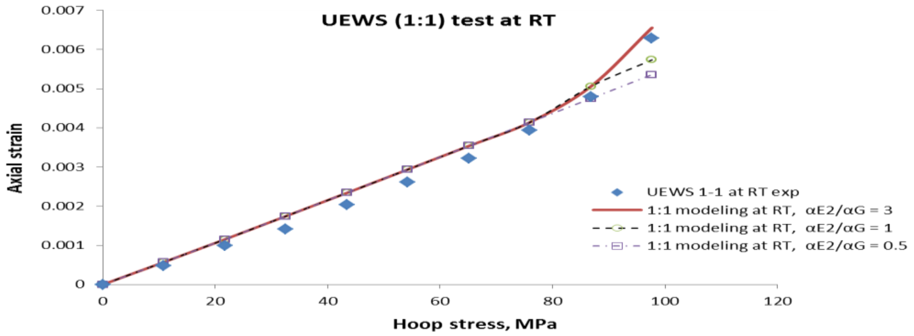

Figure 2. Experimental and model stress strain curve for UEWS test (1:1) at room temperature.

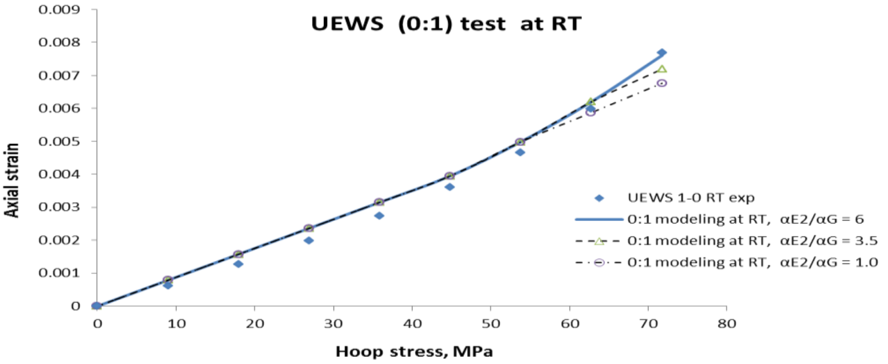

Figure 3. Experimental and model stress strain curve for UEWS test (0:1) at room temperature.

On contrary to previous results, for pure hoop loading (1:0), the ratio between α_{E2} and α_G showed a reduction from 1.0 to 0.625 to achieve the best fit. As shown in Figure 4, at $\alpha_{E2}/\alpha_G=1.0$, a practically linear stress strain behaviour was established. Reducing the ratio of the fitting constant to 0.8 caused a downshift of the hoop strains indicating the starts of the non-linear response, closer to the experimental results. Finally, optimizations is achieved at $\alpha_{E2}/\alpha_G = 0.625$. This suggests that, unlike previous results, for hoop dominated loading the fitting constant α_G that relates the deterioration of shear modulus, is more sensitive in causing the non-linear response outcome of the strains. UEWS points for this loading was taken at σ_H = 220MPa, which later transformed to the ply stresses and resulted in τ_{12}=220MPa, It is believed that at this stress, it is sufficient to cause shear failure in the resin system.

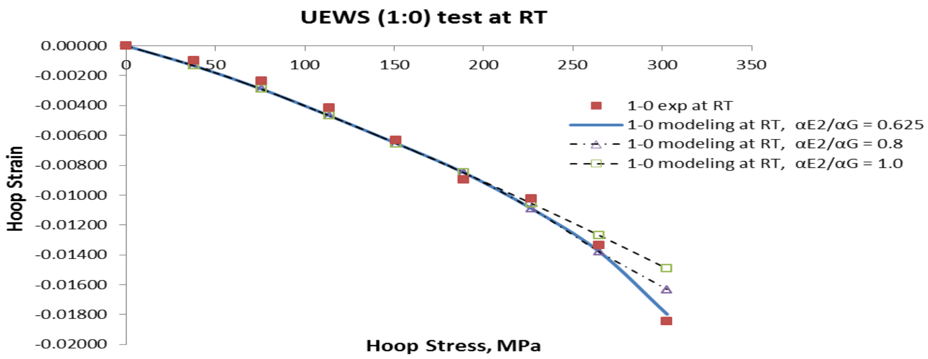

Figure 4. Experimental and model stress strain curve for UEWS test (1:0) at room temperature.

Conclusions

The stress-strain response as results of increasing transverse matrix cracking of GRE composite pipes under multiaxial UEWS tests is presented in this paper. The results from the model for all stress ratios showed a good agreement with the experimental data. The ratio of curve fitting constants between α_{E2} and $\alpha_{G,}$ which relates the effects of a matrix cracking to the deterioration in transverse modulus for hydrostatic loading (2:1) and axial dominated loadings (1:1 and 0:1) were found to increase and noted to become more pronounced at axial dominated of pure axial loading (0:1). On the contrary, modeling for pure hoop loading (1:0) showed a reduction in the ratio between α_{E2} and α_G from 1.0 to 0.625 to achieve the closest agreement to experimental data. This seems to indicate that the fitting constant α_G that describes the degradation of shear modulus is more sensitive in causing the non-linear response outcome of the strains.

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