EARTHQUAKE LATERAL FORCE ANALYSIS

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Earthquake Lateral Force Analysis

- The design lateral force shall first be computed for the building as a whole.

- This design lateral force shall then be distributed to the various floor levels.

- There are two commonly used procedures for seismic design lateral forces:

1. Equivalent static force analysis

2. Dynamic analysis

Equivalent static force analysis

- The concept is a dynamic analysis into partly dynamic and partly static analyses for finding the maximum displacement.

- Is restricted only to a single mode of vibration of the structure.

Equivalent static lateral force analysis is based on the following assumptions,

1) Assume that structure is rigid.

2) Assume perfect fixity between structure and foundation.

3) During ground motion every point on the structure experience same accelerations

4) Dominant effect of earthquake is equivalent to horizontal force of varying magnitude over the height.

5) Approximately determines the total horizontal force (Base shear) on the structure.

The limitations of equivalent static lateral force analysis

- In the equivalent static force procedure, empirical relationships are used to specify dynamic inertial forces as static forces.

- These empirical formulas do not explicitly account for the dynamic characteristics of the particular structure being designed or analyzed.

- These formulas were developed to approximately represent the dynamic behavior of

  what are called *regular structures (Structures which have a reasonably uniform* distribution of mass and stiffness).

- Structures that are classified as *irregular violate the assumptions on which the* empirical formulas, used in the equivalent static force procedure.

Step by step procedure for Equivalent static force analysis

Step-1*: Depending on the location of the building site, identify the seismic zone and assign Zone factor (Z)*

Use Table 2 along with Seismic zones map or Annex of IS-1893 (2002)

Step-2: *Compute the seismic weight of the building (W)*

- As per Clause 7.4.2, IS-1893 (2002) – Seismic weight of floors
- As per Clause 7.4.3, IS-1893 (2002) – Seismic weight of the building

*:****Step-3*** *Compute the natural period of the building (Ta)*

- As per Clause 7.6.1 or Clause 7.6.2, IS-1893 (2002), as the case may be.

Step-4: *Obtain the data pertaining to type of soil conditions of foundation of the building*

- Assign type, I for hard soil, II for medium soil & III for soft soil

Step by step procedure for Equivalent static force analysis

Step-5: *Using Ta and soil type (I / II / III), compute the average spectral acceleration (Sa/g)*

• Use Figure 2 or corresponding table of IS-1893 (2002), to compute *(Sa/g)*

Step-6: *Assign the value of importance factor (I) depending on occupancy and/or* functionality of structure

• As per Clause 7.2 and Table 6 of IS-1893 (2002),

Step-7: *Assign the values of response reduction factor (R) depending on type of structure*

• As per Clause 7.2 and Table 7 of IS-1893 (2002)

Step-8: *Knowing Z, Sa/g, R and I compute design horizontal acceleration coefficient (Ah)*

using the relationship, $A_h = \dfrac{Z}{2} \dfrac{S_a}{g} \dfrac{I}{R}$ [Clause 6.4.2, IS-1893 (2002)]

Step by step procedure for Equivalent static force analysis

Step-9: *Using Ah and W compute design seismic base shear (V_B), from $V_B = A_hW$ [Clause*

7.5.3, IS-1893 (2002)]

Step-10: *Compute design lateral force (Q_i) of i_{th} floor by distributing the design seismic base*

shear (V_B) *as per the expression,*

$$Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2} \text{ [Clause 7.7.1, IS-1893 (2002)]}$$

Dynamic Analysis

Dynamic analysis is classified into two types, namely,

- *Response spectrum method and Time history method*

Time History Method*: Time history method of analysis, when used, shall be* based on an appropriate ground motion and shall be performed using accepted principles of dynamics.

Response Spectrum Method*: Response spectrum method of analysis shall be* performed using the design spectrum specified in Clause 6.4.2 or by a site specific design, spectrum mentioned in Clause 6.4.6 of IS 1893 (2002)

When dynamic analysis is carried out either by the Time History Method or by the Response Spectrum Method, the design base shear computed from dynamic analysis (V_B) shall be compared with a base shear calculated using a fundamental period T_a $(\overline{V}_B)$, where T_a is as per Clause 7.6. If base shear obtained from dynamic analysis (V_B) is less than base shear computed from equivalent static load method ($\overline{V}_B$ i.e., *using T_a as per Clause 7.6*), then as per Clause 7.8.2, all the response quantities (for example member forces, displacements, storey forces, storey shears and base reactions) shall be multiplied by ratio $\dfrac{\overline{V}_B}{V_B}$.

Step by step procedure for Response spectrum method

Step-1: *Depending on the location of the building site, identify the seismic zone and assign* Zone factor (*Z*)

- Use Table 2 along with Seismic zones map or Annex of IS-1893 (2002)

Step-2: *Compute the seismic weight of the building (W)*

- As per Clause 7.4.2, IS-1893 (2002) – Seismic weight of floors (W_i)

Step-3: *Establish mass [M] and stiffness [K] matrices of the building using system of* masses lumped at the floor levels with each mass having one degree of freedom.

Step-4: *Using [M] and [K] of previous step and employing the principles of dynamics* compute the modal frequencies, {w} and corresponding mode shapes, [Φ] .

Step by step procedure for Response spectrum method

Step-5: Compute modal mass Mk of mode k using the following relationship with n being number of modes considered

$$M_k = \frac{\left[\sum_{i=1}^{n} W_i \phi_{ik}\right]^2}{g \sum_{i=1}^{n} W_i \phi_{ik}^2}$$ [Clause 7.8.4.5a of IS 1893 (2002)]

Step-6: Compute modal participation factors Pk of mode k using the following relationship with *n being number of modes considered*

$$P_k = \frac{\sum_{i=1}^{n} W_i \phi_{ik}}{\sum_{i=1}^{n} W_i \phi_{ik}^2}$$ [Clause 7.8.4.5b of IS 1893 (2002)]

Step by step procedure for Response spectrum method

Step-7: Compute design lateral force (Q_{ik}) at each floor in each mode (i.e., for i^{th} floor in mode k) using the following relationship,

$$Q_{ik} = A_{h(k)} \; \phi_{ik} \; P_k \; W_i \quad [\text{Clause 7.8.4.5c of IS 1893 (2002)}]$$

$A_{h(k)}$ = Design horizontal acceleration spectrum value as per Clause 6.4.2 of IS 1893

using the natural period $\left(T_k = \dfrac{2\pi}{\omega_k} \right)$ of vibration of mode k.

Step-8: Compute storey shear forces in each mode (V_{ik}) acting in storey i in mode k as given by,

$$V_{ik} = \sum_{i+1}^{n} Q_{ik} \qquad [\text{Clause 7.8.4.5d of IS 1893 (2002)}]$$

Step by step procedure for Response spectrum method

Step-9: Compute storey shear forces due to all modes considered, V_i in storey i, by combining shear forces due to each mode in accordance with Clause 7.8.4.4 of IS 1893 (2002). i.e., either CQC or SRSS modal combination methods are used.

Complete Quadratic Combination (CQC) method or Square Root of Square Sum (SRSS) method

Step-10: Finally compute design lateral forces at each storey as,

$$F_{roof} = V_{roof} \text{ and}$$

[Clause 7.8.4.5f of IS 1893 (2002)]

$$F_i = V_i - V_{i+1}$$

EXAMPLE: 1

Plan and elevation of a four-storey reinforced concrete office building is shown in Fig. 1.1. '
the building are as follows.

Number of Storey = 4

Zone = III

Live Load = 3 kN/m^2

Columns = 450 x 450 mm

Beams = 250 x 400 mm

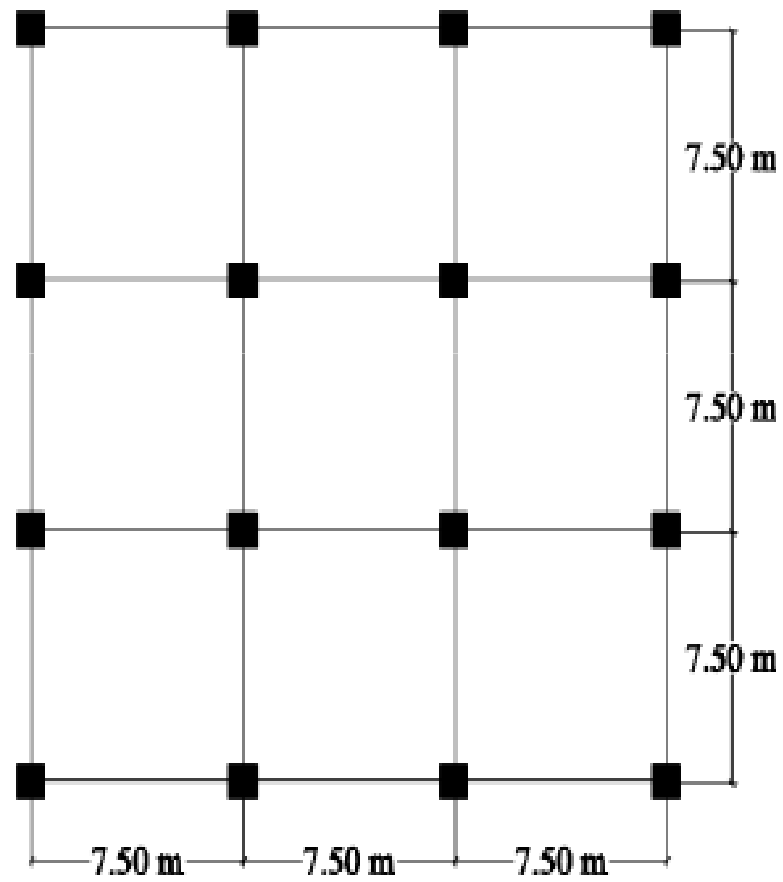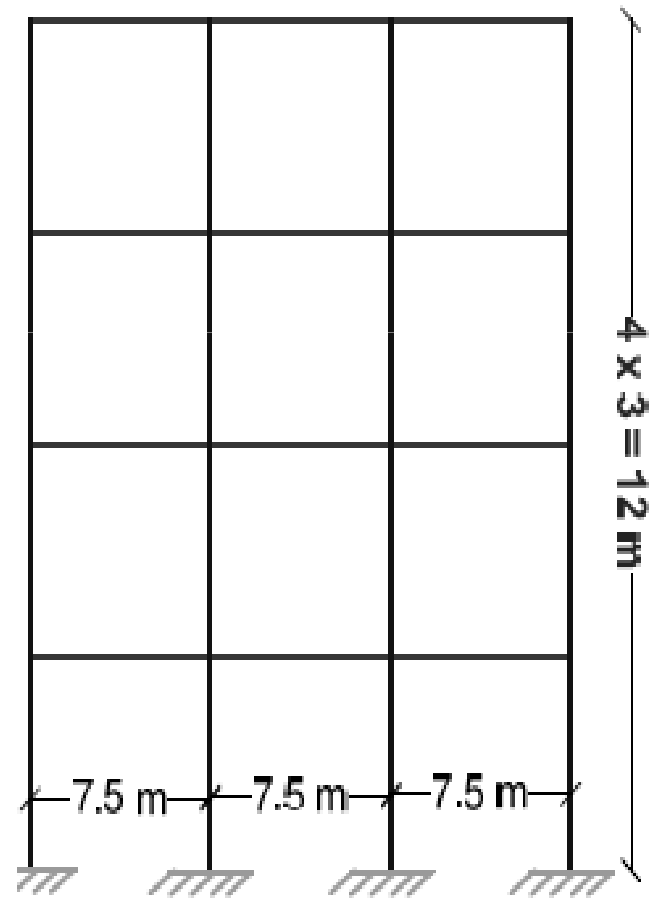Thickness of Slab = 150 mm

Thickness of Wall = 120 mm

Importance factor = 1.0

Structure type = OMRF Building

Determine design seismic lateral load and storey shear force distribution.

Plan	Elevation

Plan dimensions: 7.50 m, 7.50 m, 7.50 m (horizontal) and 7.50 m, 7.50 m, 7.50 m (vertical)

Elevation: 7.5 m, 7.5 m, 7.5 m spans; 4 x 3 = 12 m height

Solution: Analysis considering stiffness of infill masonry

1. **Computation of Seismic weights**

   (Assuming unit weight of concrete as 25 kN/m^3 & 22.5 kN/m^3 for masonry)

   1) Slab:

      DL due to self weight of slab   = (22.5 x 22.5 x 0.15) x 25 = 1898.40 kN

   2) Beams:

      Self weight of beam per unit length = 0.25 x 0.4 x 25 = 2.5 kN/m

      Total length = 4x22.5x2 = 180 m

      DL due to self weight of beams = (2.5 x 22.5) x 4 x 2 = 450 kN

   3) Columns:

      Self weight of column per unit length = 0.45 x 0.45 x 25 = 5.0625 kN/m

      DL due to self weight of columns (16 No.s)   = 16 x 5.0625 x 3.0 = 243 kN

   4) Walls:

      Self weight of wall per unit length = 0.12 x 3 x 20 = 7.2 kN/m

      Total length = 4x22.5x2 = 180 m

      DL due to self weight of Walls = 7.2 x 22.5 x 4 = 648 kN

   5) Live Load [Imposed load] (25 %) = (0.25 x 3) x 22.5 x 22.5 = 380 kN

Load on all floors:

W1=W2=W3= 1898 + 380 + 450 + 243 + 648 = **3619 kN**

Load on roof slab (Live load on slab is zero)

W4 = 1898 + 0+450 + (243/2) + (648/2) = **2793.5 kN**

Total Seismic weight, W   = (3619 x 3) + 2793.5 = **13650.5 kN**

Fundamental period:

Natural period, $T_a = 0.09 \dfrac{h}{\sqrt{d}} = 0.09 \dfrac{12}{\sqrt{22.5}} = 0.2277$

(Moment resisting frame with in-fill walls)

Spectral acceleration:

Type of soil: Medium Soil

For T_a=0.2277 s

Sa/g   = 2.5

Zone factor: For Zone III, Z = 0.16

Importance Factor: I = 1.0

Response Reduction Factor: R = 3.0 (OMRF)

Horizontal acceleration coefficient (A_h):

$$A_h = \frac{Z}{2} \frac{S_a}{g} \frac{I}{R} = \frac{0.16}{2}(2.5)\left(\frac{1}{3}\right)$$

$$A_h = 0.0667$$

Base shear (V_B):

$$V_B = A_h W = 0.0667 \times 13650.50$$

$$V_B = 910.0333 \; kN$$

Storey lateral forces and shear forces are calculated and tabulated in the following table.

| Floor level (i)	$W_i\,(kN)$	$h_i\,(m)$	$W_i\,h_i^2$ (kN-m2)	Storey forces $Q_i = V_B \dfrac{W_i h_i^2}{\sum\limits_{j=1}^{n} W_j h_j^2}$	Storey shear forces $[V_i]$ (Cumulative sum) (kN)
4	2793.5	12.0	402264	426.53	426.53
3	3619	9.0	293139	310.83	737.35
2	3619	6.0	130284	138.14	875.50
1	3619	3.0	32571	34.54	910.03

Storey shear forces are calculated as follows (last column of the table),

$$V_4 = Q_4 = 426.53 \ kN$$
$$V_3 = V_4 + Q_3 = 426.53 + 310.82 = 737.35 \ kN$$
$$V_2 = V_3 + Q_2 = 737.35 + 138.14 = 875.50 \ kN$$
$$V_1 = V_2 + Q_1 = 875.50 + 34.54 = 910.03 \ kN = V_B$$

Later force and shear force distribution is shown in the Figure-EX1.
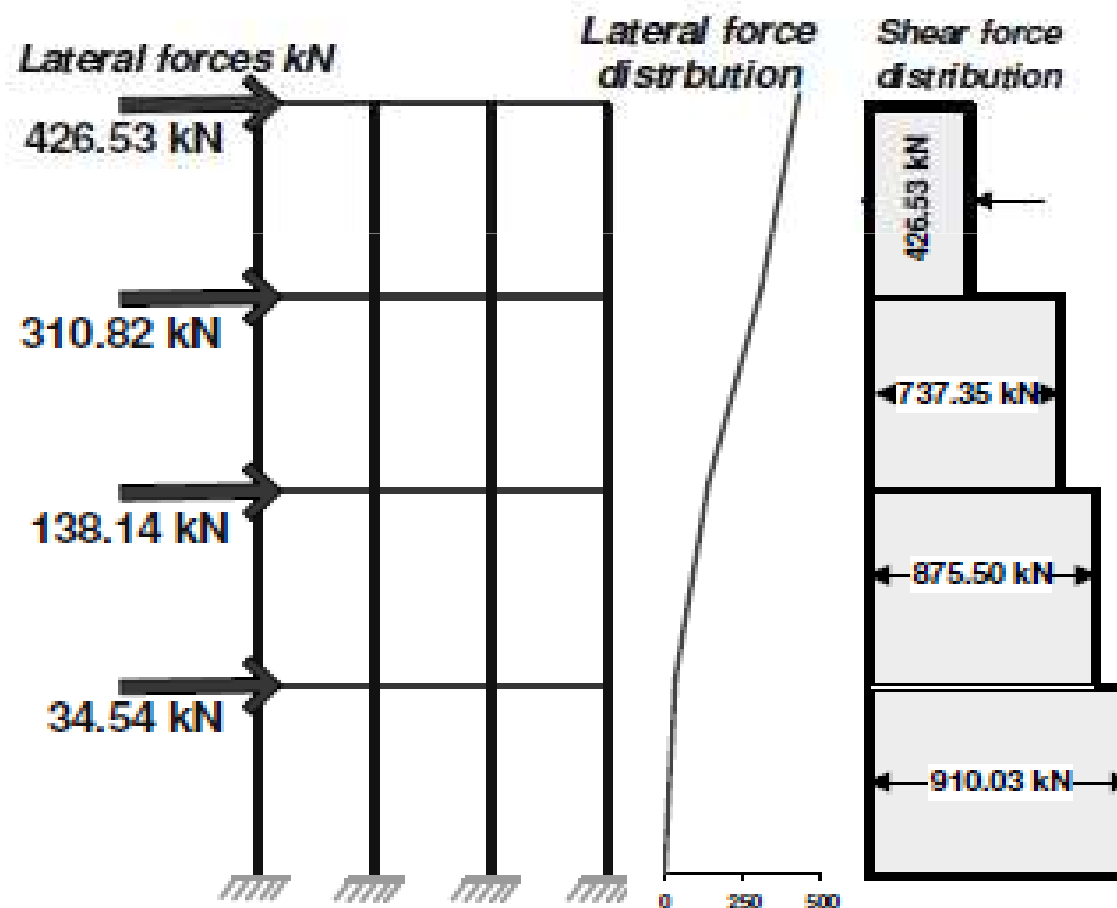
Solution: Analysis without considering stiffness of infill masonry

Fundamental period:

Natural period, $T_a = 0.075 h^{0.75} = 0.075 \times 12^{0.75} = 0.4836$

(Moment resisting frame without in-fill walls)

Spectral acceleration:

Type of soil: Medium Soil

For $T_a = 0.4836$ s

$Sa/g = 2.5$ (because, $T_a = 0.4836$ s, i.e., $0.10 \leq T_a \leq 0.55$)

Zone factor: For Zone III, $Z = 0.16$

Importance Factor: $I = 1.0$

Response Reduction Factor: $R = 3.0$ (OMRF)

Horizontal acceleration coefficient (A_h):

$$A_h = \frac{Z}{2} \frac{S_a}{g} \frac{I}{R} = \frac{0.16}{2}(2.5)\left(\frac{1}{3}\right)$$

$$A_h = 0.0667$$

Base shear (V_B):

$$V_B = A_h W = 0.0667 \times 13650.50$$

$$V_B = 910.0333 \ kN$$

EXAMPLE: 2

Analyse the building frame considered in Example-1 using response spectrum method (Dynamic analysis) with all other data being same.

Solution:

Note: In plan structure is symmetrical about both X and Y directions)

1) Seismic weights:

$W1=W2=W3= 1898 + 380 + 450 + 243 + 648 = \textbf{3619 kN}$

$W4 = 1898 + 0 + 450 + (243/2) + (648/2) = \textbf{2793.5 kN}$

Therefore, seismic masses are,

$M1 = M2 = M3 = 368.91 \times 10^3$ kg.

$M4 = 284.76 \times 10^3$ kg

2) Floor stiffness (Without considering stiffness of infill wall):

MI of columns, $I_C = (0.45)^4/12 = 3.1417875 \times 10^{-3}$ m^4

Young's Modulus, $E_C = 5000(f_{ck})^{0.5} = 25000$ MPa $= 25 \times 10^9$ N/m^2

(Assuming M25 concrete for columns)

K1 = K2 = K3 = K4 = 16 x (12 x 25 x 10^9 x 3.1417875x 10^{-3})/(3^3) = 0.6075 x 10^9 N/m

3) Natural frequencies and Mode shapes:

Mass matrix,

$$
M = \begin{bmatrix}
M1 & 0 & 0 & 0 \\
0 & M2 & 0 & 0 \\
0 & 0 & M3 & 0 \\
0 & 0 & 0 & M4
\end{bmatrix}
= \begin{bmatrix}
368.91 & 0 & 0 & 0 \\
0 & 368.91 & 0 & 0 \\
0 & 0 & 368.91 & 0 \\
0 & 0 & 0 & 284.76
\end{bmatrix} \times 10^3 kg
$$

Stiffness Matrix,

$$
K = \begin{bmatrix}
K1+K2 & -K2 & 0 & 0 \\
-K2 & K2+K3 & -K3 & 0 \\
0 & -K3 & K3+K4 & -K4 \\
0 & 0 & -K4 & -K4
\end{bmatrix}
= \begin{bmatrix}
1.215 & -0.6075 & 0 & 0 \\
-0.6075 & 1.215 & -0.6075 & 0 \\
0 & -0.6075 & 1.215 & -0.6075 \\
0 & 0 & -0.6075 & 0.6075
\end{bmatrix} \times 10^9 N/m
$$

Solving the Eigen equation, $\left|K - M\omega^2\right| = 0$, we get Eigen value and corresponding Eigen vectors as,

Eigen values, $\omega^2 = \begin{Bmatrix} 219.9 \\ 1793.2 \\ 4079.8 \\ 5920.9 \end{Bmatrix}$ $\therefore$ the natural frequencies are, $\omega = \begin{Bmatrix} 14.83 \\ 42.35 \\ 63.87 \\ 76.95 \end{Bmatrix}$ rad / s

The mode shapes are,

$$\phi_1 = \begin{Bmatrix} 1.00 \\ 1.87 \\ 2.48 \\ 2.77 \end{Bmatrix}, \ \phi_2 = \begin{Bmatrix} 1.00 \\ 0.91 \\ -0.17 \\ -1.07 \end{Bmatrix}, \ \phi_3 = \begin{Bmatrix} 1.00 \\ -0.48 \\ -0.77 \\ 0.85 \end{Bmatrix}, \ \& \ \phi_4 = \begin{Bmatrix} 1.00 \\ -1.60 \\ 1.55 \\ -0.87 \end{Bmatrix}$$

The natural periods are, $T = 2\pi / \{\omega\} = \begin{Bmatrix} 0.424 \\ 0.148 \\ 0.098 \\ 0.082 \end{Bmatrix}$ seconds

Calculation of modal participation factor

Storey Level	Seismic weight (W_i), kN	MODE-1		
		ϕ_{i1}	$W_i\,\phi_{i1}$	$W_i\,\phi_{i1}^{\;2}$
4	2793.5	2.77	7737.995	21434.25
3	3619	2.48	8975.12	22258.3
2	3619	1.87	6767.53	12655.28
1	3619	1.00	3619.00	3619.00
Σ	**13650.5**		**27099.65**	**59966.82**
Modal mass $M_1 = \dfrac{\left[\sum W_i\phi_{i1}\right]^2}{g\sum W_i\phi_{i1}^{2}}$		$= \dfrac{27099.65^2}{59966.82\,g} = $ 12246.62 kN/g		
% of Total weight		89.72 %		
Modal participation factor, $P_1 = \dfrac{\sum W_i\phi_{i1}}{\sum W_i\phi_{i1}^{2}}$		$= \dfrac{27099.65}{59966.82} = $ **0.452**		

Storey Level	Seismic weight (W_i), kN	MODE-2		
		ϕ_{i2}	$W_i\,\phi_{i2}$	$W_i\,\phi_{i2}{}^2$
4	2793.5	-1.07	-2989.05	3198.278
3	3619	-0.17	-615.23	104.5891
2	3619	0.91	3293.29	2996.894
1	3619	1.00	3619.00	3619.00
Σ	13650.5		3308.015	9918.761
Modal mass, $M_2 = \dfrac{\left[\sum W_i\phi_{i2}\right]^2}{g\sum W_i\phi_{i2}^2}$		$= \dfrac{3308.015^2}{9918.761g} = 1103.26$ kN/g		
% of Total weight		8.08 %		
Modal participation factor, $P_2 = \dfrac{\sum W_i\phi_{i2}}{\sum W_i\phi_{i2}^2}$		$= \dfrac{3308.015}{9918.761} = 0.334$		

Storey Level	Seismic weight (W_i), kN	MODE-3		
		ϕ_{i3}	$W_i\,\phi_{i3}$	$W_i\,\phi_{i3}^{\,2}$
4	2793.5	0.85	2374.475	2018.304
3	3619	-0.77	-2786.63	2145.705
2	3619	-0.48	-1737.12	833.8176
1	3619	1.00	3619.00	3619.00
Σ	13650.5		1469.725	8616.826
Modal mass, $M_3 = \dfrac{\left[\sum W\phi_{i3}\right]^2}{g\sum W\phi_{i3}^2}$		$= \dfrac{1469.725^2}{8616.826\,g} = 250.683 \text{ kN/g}$		
% of Total weight		1.84 %		
Modal participation factor, $P_3 = \dfrac{\sum W_i\phi_{i3}}{\sum W_i\phi_{i3}^2}$		$= \dfrac{1469.725}{8616.826} = 0.171$		

Storey Level	Seismic weight (W_i), kN	MODE-4		
		ϕ_{i4}	$W_i\,\phi_{i4}$	$W_i\,\phi_{i4}^{\,2}$
4	2793.5	-0.87	-2430.35	2114.4
3	3619	1.55	5609.45	8694.648
2	3619	-1.60	-5790.4	9264.64
1	3619	1.00	3619.00	3619.00
Σ	13650.5		1007.705	23692.69
Modal mass, $M_4 = \dfrac{\left[\sum W_i\phi_{i4}\right]^2}{g\sum W_i\phi_{i4}^2}$		$= \dfrac{1007.705^2}{23692.69\,g} = 42.86$ kN/g		
% of Total weight		0.314 %		
Modal participation factor, $P_4 = \dfrac{\sum W_i\phi_{i4}}{\sum W_i\phi_{i4}^2}$		$= \dfrac{1007.705}{23692.69} = 0.043$		

The lateral load Q_{ik} acting at i^{th} floor in the kth mode is,

$Q_{ik} = A_{h(k)}\ \phi_{ik}\ P_k\ W_i$ .... (Clause 7.8.4.5c of IS: 1893 Part 1)

The value of $A_{h(k)}$ for different modes is obtained from clause 6.4.2.

MODE-1:

$$T_1 = 0.424 \ s$$

$$\frac{S_a}{g} = 2.5....(0.10 \leq T_1 \leq 0.55 - \text{Medium soil})$$

$$A_{h(1)} = \frac{Z}{2} \frac{S_a}{g} \frac{I}{R} = \frac{0.16}{2}(2.5)\frac{1}{3} = 0.0667$$

$$Q_{i1} = A_{h(1)} \ \phi_{i1} \ P_1 \ W_i = 0.0667 \times 0.452 \times (\phi_{i1} \ W_i) = 0.03015(\phi_{i1} \ W_i)$$

MODE-2:

$$T_2 = 0.148 \ s$$

$$\frac{S_a}{g} = 2.5....(0.10 \leq T_2 \leq 0.55 - \text{Medium soil})$$

$$A_{h(2)} = \frac{Z}{2} \frac{S_a}{g} \frac{I}{R} = \frac{0.16}{2}(2.5)\frac{1}{3} = 0.0667$$

$$Q_{i2} = A_{h(2)} \ \phi_{i2} \ P_2 \ W_i = 0.0667 \times 0.334 \times (\phi_{i2} \ W_i) = 0.0223(\phi_{i2} \ W_i)$$

MODE-3:

$$T_3 = 0.098 \ s$$

$$\frac{S_a}{g} = 1 + 15T_3 = 2.47 .... (0.00 \le T_3 \le 0.10 - \text{Medium soil})$$

$$A_{h(3)} = \frac{Z}{2} \frac{S_a}{g} \frac{I}{R} = \frac{0.16}{2} (2.47) \frac{1}{3} = 0.0659, \text{ But, } T_3 \le 0.10,$$

$$\therefore A_{h(3)} = \frac{Z}{2} = 0.08 > 0.0659$$

$$Q_{i3} = A_{h(3)} \ \phi_{i3} \ P_4 \ W_i = 0.08 \times 0.171 \times (\phi_{i3} \ W_i) = 0.01368(\phi_{i3} \ W_i)$$

MODE-4:

$$T_4 = 0.082 \ s$$

$$\frac{S_a}{g} = 1 + 15T_4 = 2.23 .... (0.00 \le T_4 \le 0.10 - \text{Medium soil})$$

$$A_{h(4)} = \frac{Z}{2} \frac{S_a}{g} \frac{I}{R} = \frac{0.16}{2} (2.23) \frac{1}{3} = 0.0595, \text{ But, } T_3 \le 0.10,$$

$$\therefore A_{h(4)} = \frac{Z}{2} = 0.08 > 0.0595$$

$$Q_{i4} = A_{h(4)} \ \phi_{i4} \ P_4 \ W_i = 0.08 \times 0.043 \times (\phi_{i4} \ W_i) = 0.00344(\phi_{i4} \ W_i)$$

Lateral load calculation by modal analysis – SRSS method

Storey level	Weight W_i (kN)	Mode – 1 [$Q_{i1} = 0.03015(\phi_{i1}\,W_i)$]		
		ϕ_{i1}	Q_{i1}	V_{i1}
4	2793.5	2.77	233.30	**233.30**
3	3619	2.48	270.60	**503.90**
2	3619	1.87	204.04	**707.94**
1	3619	1.00	109.11	**817.05**

Storey level	Weight W_i (kN)	Mode – 2 [$Q_{i2} = 0.0223(\phi_{i2}\,W_i)$]		
		ϕ_{i2}	Q_{i2}	V_{i2}
4	2793.5	-1.07	-66.66	**-66.66**
3	3619	-0.17	-13.72	**-80.38**
2	3619	0.91	73.44	**-6.93**
1	3619	1.00	80.70	73.77

Storey level	Weight W_i (kN)	Mode – 3 $[Q_{i3} = 0.01368(\phi_{i3} W_i)]$		
		ϕ_{i3}	Q_{i3}	V_{i3}
4	2793.5	0.85	32.48	**32.48**
3	3619	-0.77	-38.12	**-5.64**
2	3619	-0.48	-23.76	**-29.40**
1	3619	1.00	49.51	**20.11**

Storey level	Weight W_i (kN)	Mode – 4 $[Q_{i4} = 0.0034(\phi_{i4} W_i)]$		
		ϕ_{i4}	Q_{i4}	V_{i4}
4	2793.5	-0.87	-8.26	**-8.26**
3	3619	1.55	19.07	**10.81**
2	3619	-1.6	-19.69	**-8.88**
1	3619	1.00	12.30	3.43

<u>SRSS method (Clause 7.8.4.4 – IS1893-2002):</u>

The contribution of different modes are combined by Square Root of the Sum of the Squares

(SRSS) using the following relationship, $V_i = \sqrt{V_{i1}^2 + V_{i2}^2 + V_{i3}^2 + V_{i4}^2}$

Then, storey lateral forces are calculated by, $F_i = V_i - V_{i+1}$.

The results obtained are tabulated in the following table.

Storey level	V_{i1}	V_{i2}	V_{i3}	V_{i4}	Combined shear force (SRSS) V_i (kN)	Combined lateral force (SRSS) F_i (kN)
4	233.30	-66.66	32.48	-8.26	244.94	244.94
3	503.90	-80.38	-5.64	10.81	510.42	265.48
2	707.94	-6.93	-29.40	-8.88	708.64	198.22
1	817.05	73.77	20.11	3.43	820.63	111.99

CQC method (Clause 7.8.4.4 – IS1893-2002):

(**Important note:** *Since modal frequencies are well separated in this example, the SRSS modal combination method is sufficient to combine contribution of each mode. For the purpose of demonstration CQC method of modal combination and to compare SRSS and CQC methods following calculations are carried out. However, CQC method is preferred when modal frequencies are closely spaced*)

The contributions of different modes are combined by Complete Quadratic Combination (CQC) method as demonstrated in the following calculations. Shear force quantities in each of the four modes can be expressed as,

$$\lambda_4 = \{V_{41} \quad V_{42} \quad V_{43} \quad V_{44}\} = \{233.30 \quad -66.66 \quad 32.48 \quad -8.26\}$$

$$\lambda_3 = \{V_{31} \quad V_{32} \quad V_{33} \quad V_{34}\} = \{503.90 \quad -80.38 \quad -5.64 \quad 10.81\}$$

$$\lambda_2 = \{V_{21} \quad V_{22} \quad V_{23} \quad V_{24}\} = \{707.94 \quad -6.93 \quad -29.40 \quad -8.88\}$$

$$\lambda_1 = \{V_{11} \quad V_{12} \quad V_{13} \quad V_{14}\} = \{817.05 \quad 73.77 \quad 20.11 \quad 3.43\}$$

Where λ_i is the shear force in the ith mode.

β_{ij} is the frequency ratio between i^{th} and the jth mode is, $\beta_{ij} = \dfrac{\omega_j}{\omega_i} = \dfrac{T_j}{T_i}$,

the four modes, β_{ij} may be expressed in matrix form as,

$$\beta_{ij} = \begin{bmatrix} T_1/T_1 & T_2/T_1 & T_3/T_1 & T_4/T_1 \\ T_1/T_2 & T_2/T_2 & T_3/T_2 & T_4/T_2 \\ T_1/T_3 & T_2/T_3 & T_3/T_3 & T_4/T_3 \\ T_1/T_4 & T_2/T_4 & T_3/T_4 & T_4/T_4 \end{bmatrix} = \begin{bmatrix} 1.00 & 0.35 & 0.23 & 0.19 \\ 2.86 & 1.00 & 0.66 & 0.55 \\ 4.33 & 1.51 & 1.00 & 0.84 \\ 5.17 & 1.80 & 1.20 & 1.00 \end{bmatrix}$$

Where natural periods of different modes are, $T = \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} 0.424 \\ 0.148 \\ 0.098 \\ 0.082 \end{Bmatrix}$ sec

Now calculate cross modal coefficient ρ_{ij},

$$\rho_{ij} = \frac{8\varsigma^2 (1+\beta_{ij}) \beta_{ij}^{1.5}}{(1-\beta_{ij}^2)^2 + 4\varsigma^2 \beta_{ij} (1+\beta_{ij})^2}$$

Taking damping ratio, $\varsigma = 0.05$ and β_{ij} values computed above, cross modal coefficient ρ_{ij} may be computed and expressed in matrix form as,

$$\rho_{ij} = \begin{bmatrix} 1.0000 & 0.01294 & 0.00460 & 0.00311 \\ 0.09597 & 1.0000 & 0.13526 & 0.06039 \\ 0.07799 & 0.26212 & 1.0000 & 0.51229 \\ 0.07494 & 0.17222 & 0.59962 & 1.0000 \end{bmatrix}$$

For example in the above matrix ρ_{12} & ρ_{34} are computed as,

$$\rho_{12} = \frac{8(0.05)^2(1+0.35)0.35^{1.5}}{(1-0.35^2)^2 + 4(0.05)^2(0.35)(1+0.35)^2} = 0.01294$$

$$\rho_{34} = \frac{8(0.05)^2(1+0.84)0.84^{1.5}}{(1-0.84^2)^2 + 4(0.05)^2(0.35)(1+0.84)^2} = 0.51229$$

Storey shear forces are computed by combining shear forces of different modes as follows,

$$V_4 = \sqrt{\{\lambda_4\}\left[\rho_{ij}\right]\{\lambda_4\}^T}$$

$$\lambda_4 = \{233.30 \quad -66.66 \quad 32.48 \quad -8.26\}$$

$$\{\lambda_4\}^T = \{233.30 \quad -66.66 \quad 32.48 \quad -8.26\}^T$$

$$\therefore V_4 = \sqrt{\{\lambda_4\}\begin{bmatrix} 1.0000 & 0.01294 & 0.00460 & 0.00311 \\ 0.09597 & 1.0000 & 0.13526 & 0.06039 \\ 0.07799 & 0.26212 & 1.0000 & 0.51229 \\ 0.07494 & 0.17222 & 0.59962 & 1.0000 \end{bmatrix}\{\lambda_4\}^T}$$

$$V_4 = \sqrt{57747}$$

$$V_4 = 240.31 \ kN$$

Simillarly,

$$V_3 = 532.85 \ kN$$

$$V_2 = 706.97 \ kN$$

$$V_1 = V_{Base} = 826.01 \ kN$$

Now, storey lateral forces are computed from storey shear forces

$$Q_4 = V_4 = 240.31 \ kN$$
$$Q_3 = V_3 - V_4 = 532.85 - 240.31 = 292.31 \ kN$$
$$Q_2 = V_2 - V_3 = 706.97 - 532.85 = 174.12 \ kN$$
$$Q_1 = V_1 - V_2 = 826.01 - 706.97 = 119.04 \ kN$$

Table: Summary of results from different methods of analyses

Storey level	Equivalent static Method		CQC Method		SRSS Method	
	Shear force V_i (kN)	Lateral force F_i (kN)	Shear force V_i (kN)	Lateral force F_i (kN)	Shear force V_i (kN)	Lateral force F_i (kN)
4	426.53	426.53	240.31	240.31	244.94	244.94
3	737.35	310.83	532.85	292.31	510.42	265.48
2	875.50	138.14	706.97	174.12	708.64	198.22
1	910.03	34.54	826.01	119.04	820.63	111.99

In the above example, $\bar{V}_B = 910.03 \ kN$

Base shear as calculated by response spectrum method (SRSS) is, $V_B = 820.63 \ kN$

$$\therefore \ \frac{\bar{V}_B}{V_B} = \frac{910.03}{820.63} = 1.109$$

Thus, the seismic forces obtained above by dynamic analysis should be scaled up as

$Q_4 = 244.94 \times 1.109 = 271.64 \ kN$

$Q_3 = 265.48 \times 1.109 = 294.42 \ kN$

$Q_2 = 198.22 \times 1.109 = 219.83 \ kN$

$Q_1 = 111.99 \times 1.109 = 124.20 \ kN$

THANK YOU