

ECE4530

CONTROL-SYSTEMS LABORATORY

Fall 2015

Introductory experiments on response of control system components. Open-loop and closed-loop (feedback) response of servo systems. Simulation of systems on analog computer. Design of compensator systems. Coreq., ECE4510.

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Course web-page: <http://mocha-java.uccs.edu/>

Office Hours: TBD

Summary: This course deals with practical aspects of control engineering. It is intended as a companion course to *ECE4510: Feedback Control Systems*, and serves to augment and demonstrate concepts presented in the classroom.

Matlab/Simulink: You will use Matlab[®] and Simulink[®] extensively. Prior familiarity with Matlab is assumed, and prior exposure to Simulink is helpful. You are not required to purchase these packages. Matlab and Simulink run on the Control Systems Laboratory (EN129) computers and on the ECE Computation Laboratory (EN136) computers.

Grading: 10% of your course grade will depend on your prelab assignments and 90% of your course grade will depend on your lab reports. **No lab report will be given a grade better than “F” until you turn in an approved prelab assignment for that lab.** See further note regarding grading on the back of this page.

Lab Syllabus: The following topics will be covered during the fifteen lab meetings (Deliverables)

1. Introduction to the control-systems laboratory, Matlab, and Simulink.
2. The Real Time Workshop and QuaRC.
3. System identification: Sensors (Lab report for Labs 1 and 2 due.)
4. System identification: Actuators.
5. Small-signal linearization (Lab report for Labs 3 and 4 due.)
6. “Black-box” frequency response system identification.
7. Basic PID control: Design (Lab report for Labs 5 and 6 due.)
8. Basic PID control: Generalization.
9. Practical aspects of stability: Simulation (Lab report for Labs 7 and 8 due.)
10. Practical aspects of stability: Implementation.
11. Root-locus analysis and informal design: Simulation (Lab report for Labs 9 and 10 due.)
12. Root-locus analysis and informal design: Implementation.
13. Feedback linearization, pole-placement design: Lower disk (Lab report for Labs 11 and 12 due.)
14. Feedback linearization, pole-placement design: Upper disk.
15. *Instructor demonstration of two-disk control* (Lab report for Labs 13 and 14 due.)

Work Load: This is an aggressive lab course requiring weekly assignments. Expect to spend 3 to 4.5 hours per week *outside* of the lab preparing for the labs and completing lab writeups. This is in accord with UCCS policy relating credit hours for a laboratory course to student workload. More or less time may be required on an individual basis.

Grading: Due to space and equipment limitations, the labs must be performed in groups of two or three students per group. Your grade will be largely determined by the quality of your report. The reports will be graded based on organization, technical accuracy, neatness, grammar and spelling. If a lab-group submits a single report, then each student will receive the same grade for the lab report. If each student in a lab-group submits his or her own report, each student will receive an individual grade.

Lab Reports: The Department requires formal lab reports which must satisfy the following format rules:

1. Title page: This must include a title, group member names, course and section name and date.
2. Introduction: Explain the background and objective of the lab indicating requirements and desired results.
3. Discussion: Discuss the underlying applicable theory and concepts that support the measurements. Indicate and discuss the measurement set-up and equipment used.
4. Measurement data and/or Results: Present measurement results in tabular, graphical or numeric form. Present results from required lab exercises.
5. Discussion of Measurements: Discuss measured data in context of comparison to expectation, accuracy, difficulties, etc.
6. Summary and Conclusions: Discuss findings, explain errors and unexpected results; summarize and indicate conclusions.

Further requirements on the lab report are:

1. Correct spelling, grammar and punctuation is required.
2. Report must be typed; figures, drawings and equations may be handwritten.
3. Format of references must conform to IEEE (transactions) standards.

Prelab Assignments: Before you enter the laboratory each week, you are expected to thoroughly read and understand the corresponding lab writeup in this lab reader. Each lab has a prelab assignment. **This assignment must be completed and turned in to the lab instructor before you will be permitted to start the lab.** Each of your prelab assignments must contain:

- Answers to the specific questions given in the “Prelab Assignment” portion of the lab reader for that lab,
- A paragraph describing what you are supposed to do during the lab,
- A paragraph describing expected results.

A portion of your course grade will be assigned based on your prelab assignments. Furthermore, no grade better than an “F” is possible for any lab report until an approved prelab assignment has been turned in for that lab.

Missed Labs: Attendance is your responsibility. Missed labs will count as ZERO without a physician’s documentation of an illness, or other appropriate documentation of an emergency beyond your control and requiring your absence.

Drop Date: This course complies with the UCCS campus rules for drop date.

Please Note: The instructor reserves the right to change the syllabus and to add or delete lab assignments depending on circumstances. Changes will be announced ahead of time, and handouts will be given out at least one class ahead of the lab.

The Control-Systems Lab and the Lab Reader: This material is based upon work supported by the National Science Foundation under Grant No. 9981009. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation. The lab was upgraded in 2005 with the support of the EAS Instructional Fee.

The lab reader has been composed using the LyX document processing system and typeset with L^AT_EX₂ ϵ . All original diagrams have been created using either xfig or Matlab. Screen shots were made using IrfanView.

The lab reader has been prepared by Dr. Gregory Plett; however, much has been taken from the manual for the *Model 730 Magnetic Levitation* unit by Educational Control Products (ECP), <http://www.ecpsystems.com>, and the manual for the *WinCon* by Quanser Consulting, <http://www.quanser.com>. Text and diagrams have been copied from these manuals. Scanned photos from the ECP web site have also been included. All are used with permission of ECP.

Some material has been adapted from control labs developed by previous instructors at UCCS including: M.G. Larimore, Dr. E.J. Bauman, Dr. J.A. Magerko Jr., Dr. Charles Fosha, Gary Mansouri and Clinton Eads.

Introduction to the Control Systems Laboratory, Matlab, and Simulink

1.1 INTRODUCTION

During this lab period, several items will be addressed:

- Administration: A quick overview of the syllabus and expectations for lab reports.
- The laboratory: An introduction to the equipment in the lab. Some instructions in how to use it.
- Review: A short review of complex numbers, and a Simulink introduction.

This review has been constructed based on students' observed needs from past experience. Not all students will need all the review, but we feel that most will benefit from it.

1.2 ADMINISTRATION

1.2.1 *The Lab Reader*

This lab reader¹ contains fourteen labs. The first two labs are designed to familiarize you with the lab equipment, and how to use it. The remaining labs are divided into groups of two to cover a number of topics that include system identification, model verification, and several different control designs. Each laboratory exercise has a detailed discussion in this "lab reader" document, and a consistent format is used throughout:

Introduction: A short introduction to the lab experiment.

This Lab Is...: A summary of the main points of the lab.

Background: Theory and practical information required to perform the lab.

Prelab assignment: An assignment that *must* be completed and turned in before you will be permitted to start the lab.

Laboratory Experiment: The experiment to be performed during the lab period.

Assignment: Items that must be included in the formal lab writeup.

¹The reader has been entered using the L^AT_EX document processing system and typeset with L^AT_EX_{2 ϵ} on a Macintosh Core 2 Duo computer running the OS X operating system. All original diagrams have been created using either xfig or Matlab. Screen shots were made using xv and IrfanView.

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Some material has also been adapted from control labs developed by previous instructors at UCCS. Not all of the authors are known, but credit must be given to (chronologically) M.G. Larimore, Dr. E.J. Bauman, Dr. J.A. Magerko Jr., Dr. Charles Fosha, Ali (Gary) Mansouri and Clinton Eads.

1.2.2 Prelab Assignments

Before you enter the laboratory each week, you are expected to thoroughly read and understand the corresponding lab writeup in this lab reader. Each lab has a prelab assignment. This assignment must be completed and turned in to the lab instructor before you will be permitted to start the lab. Each of your prelab assignments must contain:

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- A paragraph describing expected results.

A portion of your course grade will be assigned based on your prelab assignments. Furthermore, no grade better than an “F” is possible for any lab report until an approved prelab assignment has been turned in for that lab.

1.2.3 Lab Reports

A syllabus of lab topics is listed on the front page of this lab reader. You will see that the semester is divided into seven pairs of labs. One report is required for each pair of labs (i.e., seven reports, total). Due dates for the lab reports is also listed on the syllabus (corresponding to the week in the semester this lab meets).

The Department requires formal lab reports. The format for formal lab reports must comply to the following:

1. Title page: This must include a title, group member names, course and section name and date.
2. Introduction: Explain the background and objective of the lab indicating requirements and desired results.
3. Discussion: Discuss the underlying applicable theory and concepts that support the measurements. Indicate and discuss the measurement set-up and equipment used.
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Further requirements on the lab report are:

1. Correct spelling, grammar and punctuation is required.
2. Report must be typed; figures, drawings and equations may be handwritten.
3. Format of references must conform to IEEE (transactions) standards.

1.3 THE LABORATORY

The Control Systems Laboratory has been made possible by a grant from the National Science Foundation directorate of undergraduate education in 2000. In 2005, the lab was updated with support from the EAS Instructional Fee. In 2008, the software in the lab was further updated with support from the ECE department. In 2015, the software and computers in the lab were updated with support from the ECE department. The lab comprises four student workstations, each having a computer and one or more devices to control.

1.3.1 The Lab Computers

The lab computers are standard PCs with additional special hardware for data I/O. The computers run Windows 7. You log in to the lab computers using your University (UFP account) user-name and password. You have access to all your files from these computers.

1.3.2 The Devices to Control

The lab experiments will focus on one physical device to control—the the Magnetic Levitation (MagLev) device. The lab also has Control Moment Gyroscope devices which are used in more advanced control courses. These two devices are depicted in Fig. 1.1.



Figure 1.1 The two lab devices. The MagLev device is to the left; the Gyro device is to the right.

Some cautions are in order for the safety of the equipment:

1. **Do not touch the glass rod on the MagLev system.** Oils break down the lubricant in the magnet bushings, increase friction and decrease the lifetime of the equipment.
2. **Do not touch the white surface of the magnets.** The surface must be clean for proper functioning of the laser sensor.
3. **Do not get magnetic (i.e., computer) media near the MagLev magnets.** The magnets are extremely strong and *will* erase computer media. You have been warned.
4. **Do not attempt to move the Gyro axes manually.** These devices have “brakes” which are activated when the device is off and when switches on the control box are turned on. These brakes will quickly wear out if forced.

1.4 REVIEW

1.4.1 Complex Numbers

A complex number has two parts: a “real” part, and an “imaginary” part. The real part is just a number, like 1.2 or π . The imaginary part is also a number. The complex number is formed by combining the real part and imaginary parts by multiplying the imaginary part by $j = \sqrt{-1}$ and adding the two parts. For example, $1.2 + 3.1j$. (Often,

engineers use the *notation* that $j = \sqrt{-1}$ and mathematicians use the *notation* that $i = \sqrt{-1}$. Both mean the same, and should not confuse you).

Complex numbers are very useful since they “carry around” two values at a time. We can plot the position of a complex number on a x - y (Cartesian) plane by plotting the real part on the x -axis and plotting the imaginary part on the y -axis. The distance from the point to the origin is the “magnitude” of the number, and the angle of the point from the x -axis is the “angle” or “phase” of the number. So, an equivalent representation of a complex number is in magnitude-phase (*a.k.a.*, “polar”) form. It is easy to convert back and forth:

$$x + jy = \sqrt{x^2 + y^2} \angle \left(\tan^{-1} \left(\frac{y}{x} \right) \right) \quad (\text{be careful about the quadrant of the } \tan^{-1}() \text{ fn!})$$

$$A \angle \theta = A \cos(\theta) + jA \sin(\theta).$$

Complex numbers are not used just because you can carry around two values with one number, but also because they have many practical applications. Mathematical operations can be performed on complex numbers which often have physical significance, such as the phase relationship between voltage and current.

Addition and Subtraction:

Two complex numbers are added by adding their real and imaginary parts separately.

$$(a + jb) + (c + jd) = (a + c) + j(b + d).$$

Subtraction is performed in the same way.

Multiplication:

Two complex numbers are multiplied using regular multiplication rules, and remembering that $j \cdot j = -1$.

$$\begin{aligned} (a + jb)(c + jd) &= ac + jbc + jad + j^2bd \\ &= (ac - bd) + j(bc + ad). \end{aligned}$$

Multiplication is easier if the numbers are in polar form:

$$(A \angle \theta)(B \angle \psi) = AB \angle (\theta + \psi).$$

Division:

Division of one complex number by another is accomplished by multiplying numerator and denominator by the “conjugate” of the denominator. The conjugate of $x + jy$ is $x - jy$; that is, the imaginary part is negated.

$$\begin{aligned} \frac{a + jb}{c + jd} &= \frac{a + jb}{c + jd} \cdot \frac{c - jd}{c - jd} \\ &= \frac{(ac + bd) + j(bc - ad)}{c^2 + d^2} \\ &= \frac{ac + bd}{c^2 + d^2} + j \frac{bc - ad}{c^2 + d^2}. \end{aligned}$$

Again, division of complex numbers is easier if they are in polar form:

$$\frac{A \angle \theta}{B \angle \psi} = \frac{A}{B} \angle (\theta - \psi).$$

The Complex Exponential

A special function which is seen *all the time* in linear systems and control systems is the complex exponential function. Euler's identity states

$$e^{j\theta} = \cos(\theta) + j \sin(\theta).$$

Therefore,

$$e^{\phi+j\theta} = e^{\phi}(\cos(\theta) + j \sin(\theta)).$$

We can unwrap Euler's identity and write the $\sin()$ and $\cos()$ functions in terms of the complex exponential.

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \quad \text{and} \quad \sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}.$$

1.4.2 Simulink

During the course of the semester you will perform a sequence of experiments treating the step-by-step design and test procedures for typical classical controllers. Here we encounter a valuable tool: simulation. Simulation is a means of predicting the quantitative and qualitative behavior of a physical system by using a mathematical model of the system. Often it is impractical to design a controller for a specific piece of equipment without careful preliminary testing. For example, an incorrectly designed process controller for a paper or chemical manufacturing plant could result in a cost of millions in damage. A faulty auto-pilot for a commercial aircraft could likewise cost in money as well as lives. Therefore, with the exception of simple isolated systems, we seldom design and test controllers on systems directly. Rather, a mathematical *model* of a dynamic system is constructed, and then somehow *simulated*. That is, the system's time response is predicted for arbitrary inputs.

For many cases the simulation phase can be done by means of digital computers, particularly when high numerical accuracy is required. You will be using Simulink: a simulation package that operates inside of Matlab. With Simulink, you first define a model of the system in block-diagram form, and then analyze the model by injecting signals and observing the outcome.

Starting Simulink

To start Simulink, you start Matlab, and then enter "simulink" at the Matlab ">>" prompt.

```
>> simulink
```

The Simulink block browser window appears, as shown in Fig. 1.2 on page 1-6 appears. (Note: the screen shots were taken for Simulink from Matlab 20014a, running on a Windows 7 workstation. Other versions of Simulink running on different platforms may not have identical appearance, but the methods for using the program should be the same).

From the Simulink **File** menu, you can either create a new model (i.e., a new block-diagram to simulink) or load an existing model. To create a new model, you will need to access the Simulink block library. By double-clicking on any of the icons in the Simulink window in Fig. 1.2 on page 1-6, the appropriate block-library is opened. Note that the blocks used in existing block diagram files may be copied by dragging and dropping into a new or existing file. Four block libraries, shown in Fig. 1.3 on page 1-7, are of particular interest. The "source" blocks are signal sources of various types; the "sink" blocks are signal sinks, which include plotting devices; the math blocks include gains and sums; and the continuous elements include integrators, differentiators and transfer-functions.

Existing block diagrams are saved as mdl-files. An example simulink block-diagram is shown in Fig. 1.4. This block diagram is saved as "test.mdl". The system may be opened by typing "test" at the Matlab prompt, and may be simulated by `sim('test')`. The block-diagram is generated by "dragging" blocks from the source, sink (etc) pallets, and by dragging wires between them. The parameters of a specific block (for example, the numerator and denominator polynomials for the transfer function block) can be modified by double-clicking on the block, and

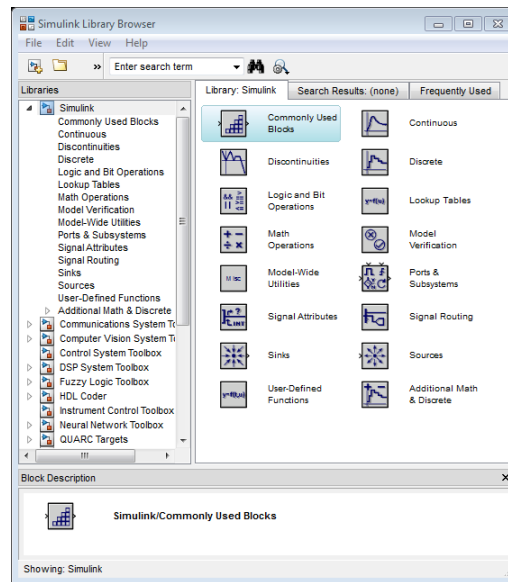


Figure 1.2 Simulink main window.

changing parameters in the dialog that appears. The simulation is started by selecting **Start** from the **Simulation** menu. (Note: different versions of Simulink may have shortcuts). In this block-diagram, the output is sent to a “scope” block, which may be viewed by double-clicking on the scope icon.

1.5 LABORATORY EXPERIMENT

1. In a number of labs, you will need to plot data similar to that shown in Table 1.1. Using Matlab on the lab computers, plot F_{m12} (on the vertical axis) versus y_{12} (on the horizontal axis). Include figure title, x - and y -labels, and a legend. You will need to know Matlab commands: `plot` `xlabel` `ylabel` `title` `legend`. If you are unsure as to how these work, type `help` and the command you wish to know more about (e.g., `help legend`). Matlab also has a more detailed documentation system, accessed via the `doc` command (e.g., `doc legend`).

Table 1.1 MAGNET-TO-MAGNET FORCE INTERACTION DATA.

Weight at Magnet #2 (F_{m12} , N)	Magnet #2 Position Relative to Magnet #1 at Equilibrium (y_{12} , cm)
0.402	11.73
0.612	10.30
0.735	9.57
0.998	8.80
1.190	8.09
2.171	6.72
3.152	5.90
4.133	4.78
6.095	4.25
11.00	2.74

2. Print your results to a file. (You may need to consult `help print`. The “png” format works well for inclusion of figures in Microsoft Word documents, and makes for professional looking lab reports.) Copy this file to your UFP account, and include it in your lab report writeup.

3. In Simulink, create a model with a sinusoidal source, a “scope” block as sink, and with the transfer function

$$H(s) = \frac{s + 10}{s + 100}.$$

Use the “transfer function” block from the “continuous” blockset library to implement the transfer function. Enter the numerator polynomial as the coefficients of decreasing powers of s in the numerator of $H(s)$. For this example as $[1 \ 10]$. Enter the denominator polynomial in the same way. Initial conditions should be zero. Use an input sinusoid of amplitude 1. The block diagram will look like the one in Fig. 1.4.

Simulate your system for the following frequencies: 1, 2, 4, 7, 10, 20, 40, 70, 100, 200, 400, 700 (all in radians per second). At steady-state, record the amplitude of the output sinusoid. Zoom in on the peaks for accurate results. You may want to either print your results and/or save to disk.

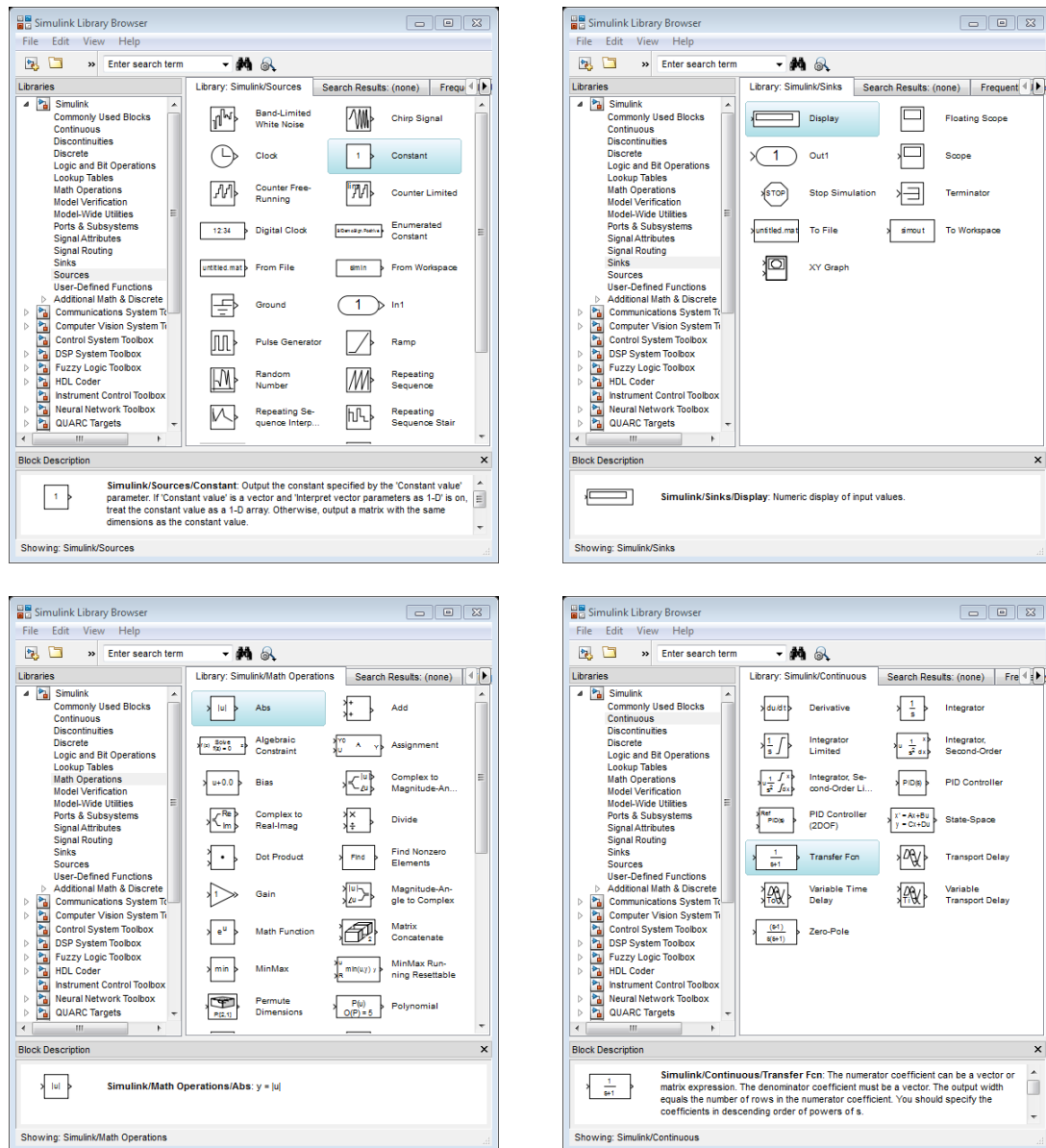


Figure 1.3 Some Simulink block libraries of interest: Sources, Sinks, Math, and Continuous.

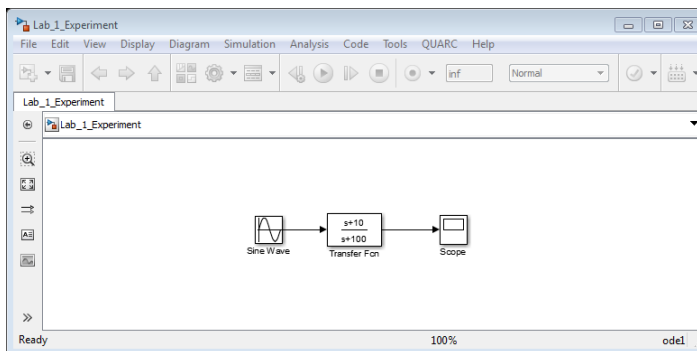


Figure 1.4 Example block diagram.

1.6 ASSIGNMENT

1. Include the results from the Laboratory Exercise.
2. Complex number exercises. Please work out by hand. You may check your answers with Matlab. Let $x_1 = 0.1 + 0.1j$; $x_2 = 0.1 - 0.1j$; $x_3 = -0.1 + 0.1j$; $x_4 = -0.1 - 0.1j$.
 - What are the magnitude and phases of x_1 , x_2 , x_3 and x_4 ?
 - What is $x_1 + x_2 + x_3 + x_4$?
 - What is $x_1 - x_2$?
 - What is $x_3 \cdot x_4$? (Give your answer both in rectangular and polar coordinates).
 - What is x_4/x_1 ? (Give your answer both in rectangular and polar coordinates).
 - What is e^{x_1} ?
3. For the transfer function in the Simulink exercise, $H(s) = (s + 10)/(s + 100)$, compute $|H(j\omega)|$. (One way is to compute $\sqrt{H(j\omega)H(-j\omega)}$). Then, plot $20 \log_{10}(|H(j\omega)|)$ versus ω on a semilog plot (Matlab: `log10` and `semilogx`). Overlay this plot with the data you recorded in part 3 of the Laboratory Experiment. Are the results what you expect? Why or why not?

The Real Time Workshop, QuaRC

2.1 INTRODUCTION

This lab is an exercise to familiarize you with the Real Time Workshop and the QuaRC real-time target on the lab computers. You will be using these programs regularly during the semester.

2.2 THIS LAB IS...

- An introduction to the Real Time Workshop (RTW) to run hardware-in-the-loop (HIL) experiments.
- An HIL experiment using RTW and QuaRC to compare against last lab's software experiment.

2.3 BACKGROUND

2.3.1 *The Real Time Workshop*

Last lab period, we introduced how to use Simulink to simulate a dynamic system. The *Real Time Workshop (RTW)* is a part of Matlab that interfaces Simulink with the “real world.” Simulink by itself allows simulation of dynamic systems. With the Real Time Workshop, however, you can include physical devices inside the “simulation.” Signals from your Simulink block diagram may be output through digital to analog (D2A) converters, and external signals may be read into your Simulink block diagram through analog to digital (A2D) and optical encoder inputs.

The Real Time Workshop can operate with many different input-output (I/O) platforms, some having their own DSP on board to run the resulting code. The setup in the lab uses the *Quanser Consulting Q4* and *Q8* I/O boards, which include four/eight A2Ds, four/eight D2As, and four/eight quadrature optical encoder inputs, respectively (and a variety of digital I/O and other features we will not use). Many of these features are connected to the breakout box next to the computer so that you can simply wire up experiments using banana plugs. Three of the workstations use the Q4 board and the final workstation uses the Q8 board. The discussion in this lab reader will give instructions for using the Q4 board, but the Q8 board may be used in analogous ways.

For many control scenarios, the system will be configured as shown in Fig. 2.1 on page 2-2. A controller is connected to the MagLev through a power amplifier. The amplifier has input voltage range -10 to 10 volts, and produces output current in the appropriate range to drive the electromagnetic coils. The amplifier also buffers the sensor signal from the plant. When a computer is used as controller, it is connected to the amplifier box via a “breakout box” which provides the appropriate input and output signals via banana plugs. The Q4/Q8 board inside the PC is directly connected to the breakout box.

The Q4 I/O board does not have its own DSP, so the real-time software must run on the lab computer's own CPU. The Real Time Workshop, in conjunction with a piece of software called *QuaRC* (by Quanser Consulting), automatically generates C code to implement your Simulink diagram, compiles the C code into executable code, and runs the code on the CPU in your desktop computer.

The beauty of this arrangement, used frequently in industry to test and prototype control systems, is that you can test controllers in a “hardware in the loop” configuration without ever coding an interrupt service routine or writing a single line of C or assembler code! (Much less, debugging same.) Designs may be prototyped rapidly resulting in great economies of time and money. You will see that there is a learning curve the first time you use this particular scheme, but afterward it becomes completely natural.

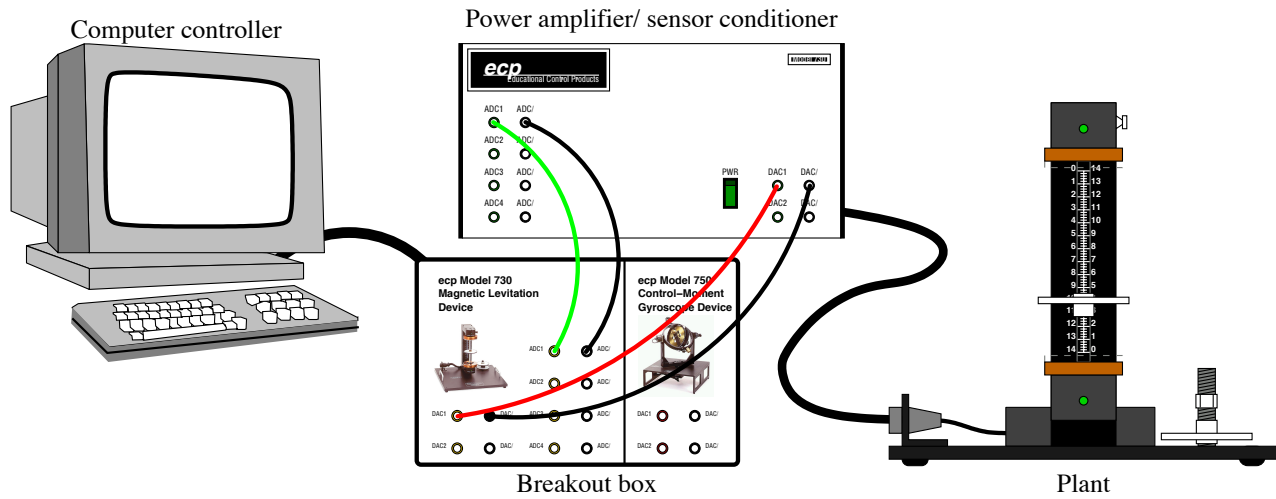


Figure 2.1 System setup for control.

2.3.2 Using the Real Time Workshop

In order to use the Real Time Workshop, you must know how to construct a Simulink block diagram containing RTW elements, be able to set all parameters for these elements and for Simulink/RTW, and be able to properly compile/build the code and run it. These topics are discussed in the following sections

Creating the Real Time Model

The HIL Initialize Block:

In order to start using the Real Time Workshop, Simulink will need to know that your model must be executed on external hardware, not via internal simulation. To configure the hardware inside your Simulink model, you will need to add a block “HIL Initialize”. You can find this block under the Simulink Library Browser via the path QuaRC Targets > Data Acquisition > Generic > Configuration (cf. Fig. 2.2). Drag this block into your model (it doesn’t matter where you put it in your model).

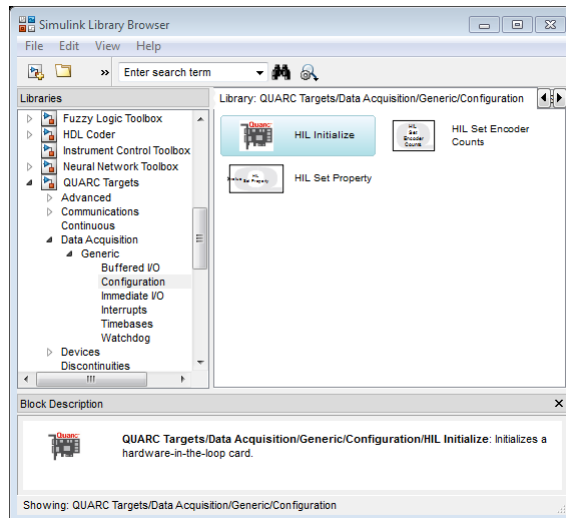


Figure 2.2 The hardware configuration block.

Double-click on this block in your model and set the hardware to correspond with the data acquisition board in your workstation. **Note that workstations 1–3 use the Quanser Q4 board, and workstation 4 uses the Quanser Q8 board.**

The A2D and D2A Converter Blocks:

Your block diagram will also need analog inputs and outputs (A2D and D2A blocks). You can find these blocks under the Simulink Library Browser via the path QuaRC Targets▷Data Acquisition▷Generic▷Immediate I/O (cf. Fig. 2.3). Drag the appropriate block(s) into your model, and connect them in the appropriate places. Note that the A2D operation that we will use is performed by the “HIL Read Analog” block, and the D2A operation that we will use is performed by the “HIL Write Analog” block.

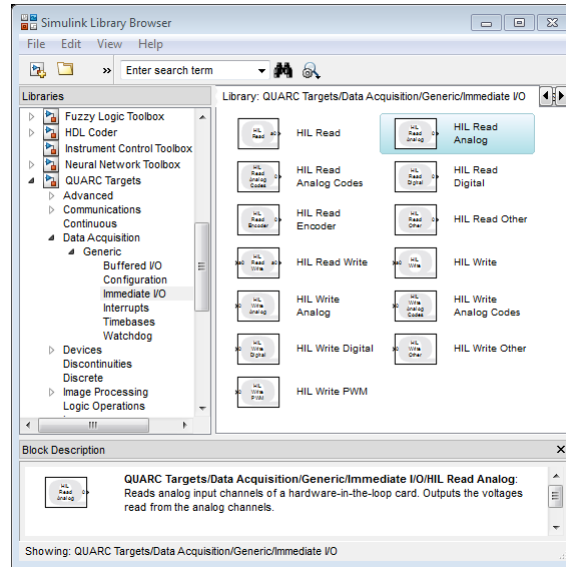


Figure 2.3 The hardware configuration block.

When you double-click on an HIL Read Analog block or an HIL Write Analog block, you can set the A2D or D2A channel that is accessed by that particular block, so that multiple A2D and D2A channels may be used in any particular block diagram. These channels correspond with the A2D and D2A banana plugs on the breakout box.

- The top set of banana plugs on the breakout box is A2D1 (channel 0) and so forth through to the bottom set which is A2D4 (channel 3). A2D1 is generally used as the input from the sensor on the bottom coil; A2D2 is generally used as the input from the sensor on the top coil. The other two inputs are the temperature readings of the two coils, and may be used to provide more accurate sensor calibration when the sensor heats up.
- D2A1 (channel 0) and D2A2 (channel 1) correspond to the MagLev drive coils (bottom and top, respectively). D2A3 (channel 2) and D2A4 (channel 3) correspond to the Gyro D2A1 and D2A2 on the breakout box, and are used to drive Axis 1 and Axis 2 of the Gyro.

For convenience, the associations between channel number and breakout-box labeling are repeated here:

Channel	Breakout Box	Typical Use
A2D Channel 0	MagLev A2D1	Bottom Coil Sensor
A2D Channel 1	MagLev A2D2	Top Coil Sensor
A2D Channel 2	MagLev A2D3	Bottom Temp Sensor
A2D Channel 3	MagLev A2D4	Top Temp Sensor

Channel	Breakout Box	Typical Use
D2A Channel 0	MagLev D2A1	Bottom Coil Drive
D2A Channel 1	MagLev D2A2	Top Coil Drive
D2A Channel 2	Gyro D2A1	Axis 1 Motor Drive
D2A Channel 3	Gyro D2A2	Axis 2 Motor Drive

Setting the Real-Time Options

There are a number of options that must be set before you can properly build and run your code. Most of these options will be automatically set properly for you. However, your model will not build if you have “discrete states” enabled. If you get an error message about your model having continuous states, navigate to the “solver” window via: `Simulation>Parameters` and clicking on the solver tab. **The simulation should be set to Fixed Step Size and an “ode1 (Euler)” method.** A sample period of 0.002s should work well for all labs in this course.

Building and Executing the Real-Time Model

Building the Model: From the prior discussion, you should be able to construct a Simulink block diagram including physical I/O and be able to set all necessary parameters. It remains to see how to run the resulting simulation and view the output.

Follow this procedure:

1. Be sure to enter your parameters in each block of your Simulink model. In particular you will need to set the required settings:
 - (a) Model execution, set from default “normal” to “external” (located in the upper right corner of your model window).
 - (b) Set the run time to the desired duration (default is inf = infinity, it will run forever).
 - (c) Go to Simulation->Model Configuration Parameters, a new window will open. Select Solver in this window and set the solver options to:
 - Type: Fixed-step
 - Solver: ode1 (Euler)
 - Fixed-step size (fundamental sample time): 0.002
2. Build the model by selecting the “Build” icon (looks like a keypad) or use keys Ctrl+B.
3. Before running the simulation, ensure that you have connected the breakout panel correctly, for the particular HIL operation you are performing. For example in the Lab 2 experiment you are simply connecting the corresponding analog channel out to the analog channel in: D2A1 -> A2D1.

The build process can take some time. The Matlab control GUI will display busy while the build is in progress.

If there are any errors in the Simulink model or a QuaRC setting, a dialog box will appear with warnings and errors. If there are no errors, then the code is built, and you are ready to execute the real-time target. **Note that every time you change your model you will need to re-build its code.** (If you just change a parameter value of a block, this may not be necessary. It is always safest to do so, however).

Executing the Model: When your code is successfully downloaded to the QuaRC client software, your diagram is ready to execute. From the QuaRC menu, select Start. To stop execution, select Stop from the QuaRC menu. Or, select the “connect” icon, then the “run” arrow to start. To stop before completion, select the “stop” icon. The simulation will normally terminate after the real time entered as part of step 1(b), above.

2.4 PRELAB ASSIGNMENT

This lab, for practice using Simulink, you will implement the following differential equation:

$$\dot{y}(t) + by(t) = \dot{x}(t) + ax(t),$$

where $x(t)$ is the input signal to a system, and $y(t)$ is the output signal. The Laplace transform (with zero initial conditions) of this differential equation is:

$$sY(s) + bY(s) = sX(s) + aX(s)$$

or

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s + a}{s + b}.$$

This transfer function is known as either a “lead” network or a “lag” network, depending on the relation of parameters a and b . (If $a < b$ the system is lead; if $a > b$ the system is lag). In ECE4510 you will discover that this transfer function is a fundamental building block when designing control systems.

The amplitude response of a system measures the scaling factor relating the output to the input at different frequencies. If ω is a frequency (in radians/second) then $|H(s)|_{s=j\omega} = |H(j\omega)|$ is the amplitude response of the system.

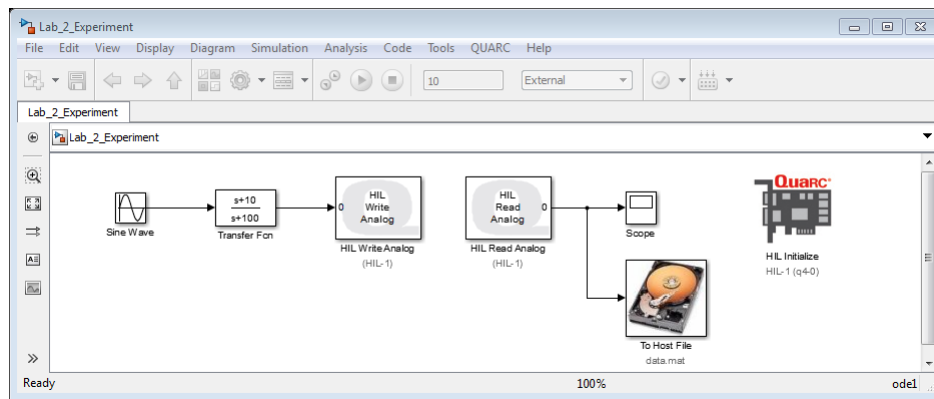
1. For $a = 10$ and $b = 100$, find the amplitude response of the lead network for the following frequencies (fill in the table). If this doesn't look remarkably similar to your results from Lab 1, something went wrong.

ω , rads/sec	Amplitude resp. $ H(j\omega) $
1	
2	
4	
7	
10	
20	
40	
70	
100	
200	
400	
700	

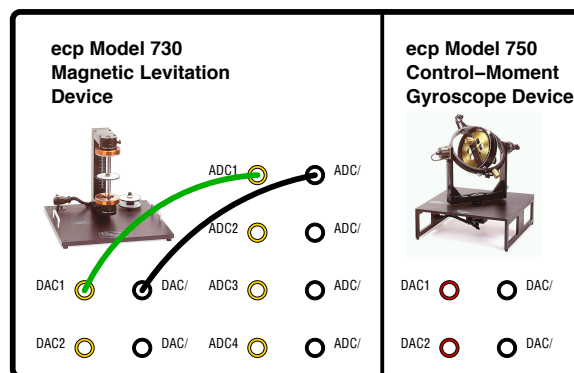
2. Using the values in your table, plot $20 \log_{10}(|H(j\omega)|)$ versus logarithmic ω (a Bode magnitude plot).

2.5 LABORATORY EXPERIMENT

1. For the laboratory exercise, you will repeat last week's test, but will force the simulation to include “real world” I/O. Construct a block diagram as shown below:



You need to wire the breakout box so that D2A Channel 0 is connected to A2D Channel 0. The appropriate wiring diagram is shown below:



Simulate and log results as done in Lab 1. Your results should be substantially the same, but there will be some differences. Note that it may be wise to increase the amplitude of the generated sinusoid for frequencies which are highly attenuated so that noise is not a dominating effect in your results. Be sure to divide the output magnitude by the input magnitude in order to arrive at the appropriate amplitude response.

2. Connect a (real!) oscilloscope to the D2A output to monitor the output. Is it what you expect?

2.6 ASSIGNMENT

1. Replot the prelab Bode magnitude plot. That is, plot $20 \log_{10}(|H(j\omega)|)$ versus logarithmic ω . Superimpose the results from Lab 1 and Step 1 above. Comment on similarities and/ or differences.
2. State some intelligent observations from Step 2 above. Is the (real) scope output the same as the (Simulink) scope output? How are they the same? How are they different?

System Identification: Sensors

3.1 INTRODUCTION

In the laboratory this semester, you will devote your effort to understanding and controlling the magnetic levitation system. In this particular lab you will perform a task called “system identification.” You will make a mathematical model of the dynamics of the device, including the nonlinearities of the actuator and sensor. So-called “white-box” system identification assumes that you have full knowledge of how the system operates and can use Newton’s laws, and so forth, in order to make the model. “Black-box” identification, the subject of a future lab, assumes no specific knowledge and uses generic techniques in order to make a system model.

3.2 THIS LAB IS...

- An introduction to the Magnetic Levitation device.
- An example of system identification—finding a mathematical model for the dynamics of a particular system.
- A brief introduction to Matlab for nonlinear optimization and function fitting.

The results from this lab are used throughout the rest of the semester, so it is in your best interest to be as accurate as you can!

3.3 BACKGROUND

3.3.1 The Magnetic Levitation System

Two views of the magnetic levitation system are depicted in Fig. 3.1 on page 3-2. The plant consists of upper and lower drive coils that produce a magnetic field in response to a dc current. One or two magnets travel along a precision ground Pyrex glass guide rod. By energizing the lower coil, a single magnet is levitated by a repulsive magnetic force. As current in the coil increases, the field strength increases and the levitated magnet height is increased. For the upper coil, the levitating force is attractive. Two magnets may be controlled simultaneously by stacking them on the glass rod. The magnets are of an ultra-high field strength rare earth (NeBFe) type and are designed to provide large levitated displacements to clearly demonstrate principles of levitation and motion control.

Two laser-based sensors measure the magnet positions. The lower sensor is typically used to measure a given magnet’s position in proximity ($\approx 8\text{ cm}$ range) to the lower coil, and the upper one for proximity to the upper coil. The sensor design utilizes light amplitude measurement and includes special circuitry to desensitize the signal to stray ambient light and thermal fluctuations.

For many control scenarios, the system will be configured as shown in Fig. 3.2 on page 3-2. A controller is connected to the MagLev through a power amplifier. The amplifier has input voltage range -10 to 10 volts, and produces an output current in the appropriate range to drive the electromagnetic coils. The amplifier also buffers the sensor signal from the plant. When a computer is used as controller, it is connected to the amplifier box via a “breakout box” which provides the appropriate input and output signals via banana plugs.

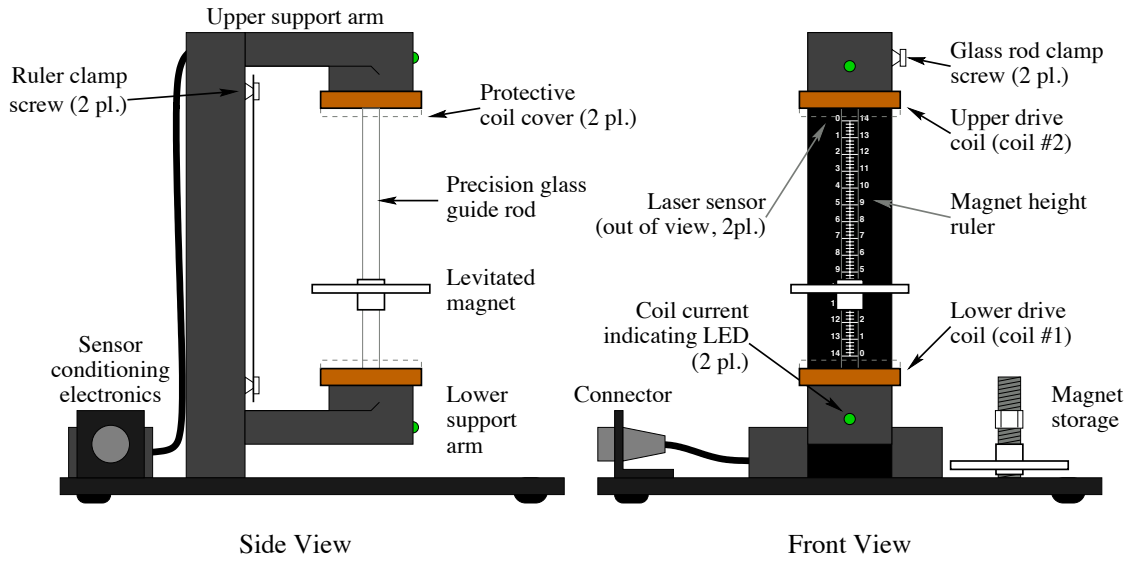


Figure 3.1 Two views of the magnetic levitation system.

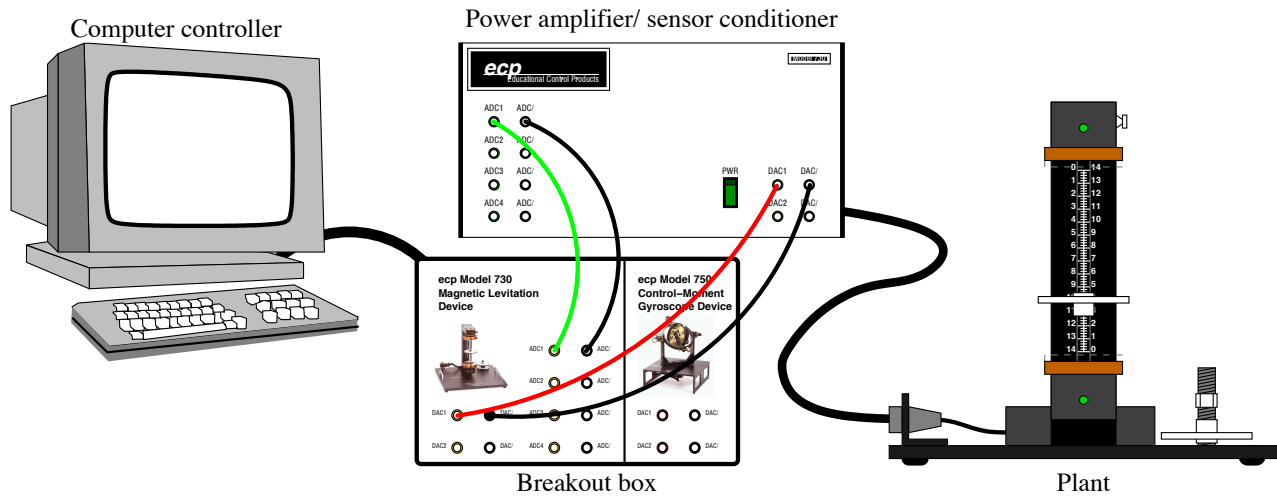


Figure 3.2 System setup for control.

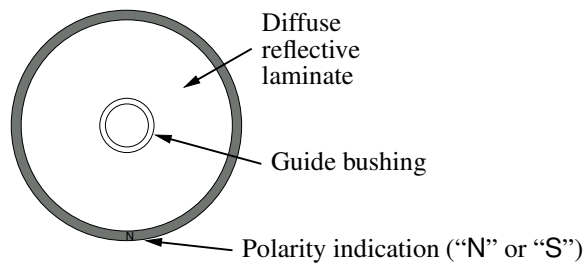


Figure 3.3 Magnetic disk.

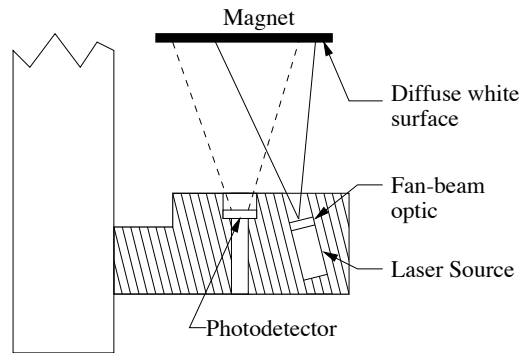


Figure 3.4 Functional diagram of laser sensor.

One or more magnetic disks may be placed on the Pyrex rod and levitated. Fig. 3.3 on page 3-2 shows a diagram of the disk. A dry-lubricated guide bushing at the center of the disk slides up and down the rod. A white reflective surface covers most of the disk. A “N” or “S” label identifies the polarity of a particular side of the disk.

The sensor makes use of the reflective properties of the disk surface. The laser sensor is depicted functionally in Fig. 3.4. The monochromatic, coherent laser beam is spread via a single hemicylindrical optical element into a fan shape. This beam is projected on the diffuse white surface of the magnet. A photodetector views the beam and generates a voltage proportional to the amount of beam power incident on it. The white surface properties and laser/detector view factors and geometries are designed to maximize the sensor gain slope (change in output versus change in position) through the sensor operational range of about 8 cm.

Because of the fan beam shape and the fact that the reflected light is a combination of specular and diffuse, the power on the detector diminishes with the inverse and inverse square of the magnet distance. Additionally, the geometry of the laser, detector, and magnet and the resulting cutoff of the detector’s view of the beam as a function of magnet height give rise to a linear dependency at small magnet distance. These relationships are used to motivate the general form of the sensor linearization function as discussed later.

Electronic circuitry makes the design immune to stray light noise, such as turning room lights on and off, and rejects most induced electronic disturbances. Thus a relatively low noise signal is output from the amplifier box.¹

3.3.2 Making a Mathematical Model

We wish to determine a mathematical model of the MagLev system. To do so, free-body diagrams for the MagLev apparatus in two configurations are shown in Fig. 3.5 on page 3-4. We will first consider the double-magnet configuration, and then specialize to the single-magnet configuration. Each magnet is acted on by forces from both drive coils, from the other magnet, from gravity, and from friction (modeled as viscous). In the diagram, y_1 is the position of the lower magnet, y_2 is the position of the upper magnet, y_c is the maximum magnet separation, i_1 is the current into the lower coil and i_2 is the current into the upper coil. All other labels refer to forces operating on the magnets. From the figure, we have an equation of motion for the first magnet

$$\begin{aligned} m\ddot{y}_1 &= \sum \text{Forces on disk 1} \\ m\ddot{y}_1 &= F_{u_{11}} + F_{u_{21}} - F_{m_{12}} - c_1\dot{y}_1 - mg. \end{aligned}$$

Similarly for the second magnet

$$\begin{aligned} m\ddot{y}_2 &= \sum \text{Forces on disk 2} \\ m\ddot{y}_2 &= F_{m_{12}} - F_{u_{22}} - F_{u_{12}} - c_2\dot{y}_2 - mg. \end{aligned}$$

¹A laser temperature signal is also output. This may be used to compensate for the laser’s inherent reduction in emitted power as a function of temperature (approx. 15% maximum for operational temperatures of the MagLev device).

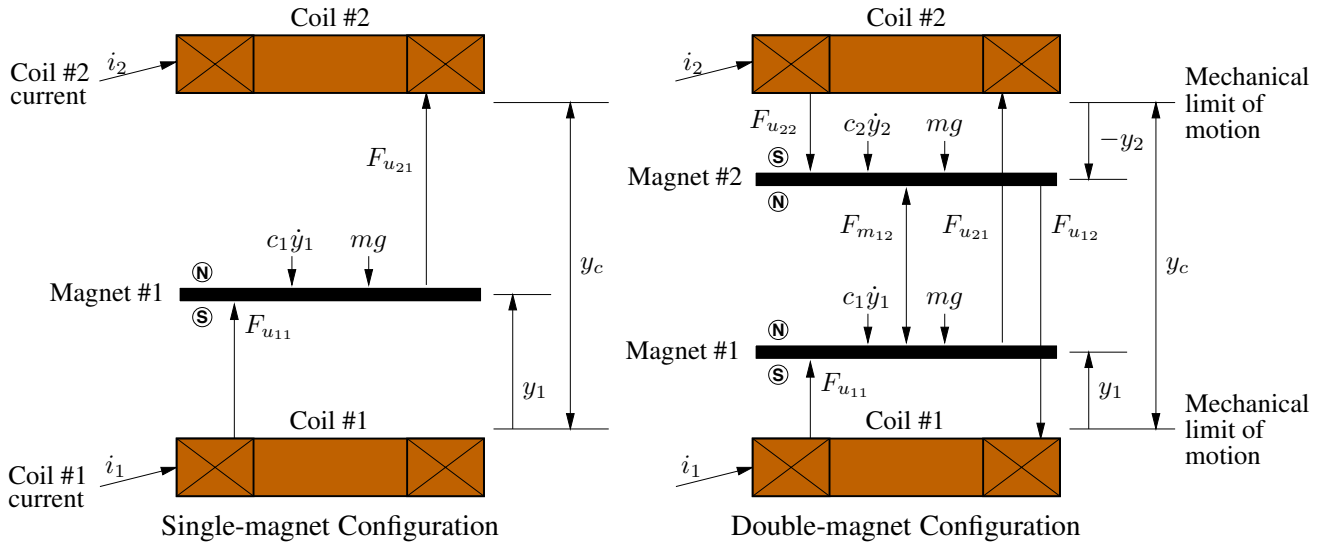


Figure 3.5 Force diagrams for the MagLev device in two configurations.

The magnetic force terms are modeled as having the following forms (see Appendix A for details)

$$\begin{aligned}
 F_{u11} &= \frac{i_1}{a(y_1 + b)^N} & F_{u12} &= \frac{i_1}{a(y_c + y_2 + b)^N} & F_{m12} &= \frac{c}{(y_{12} + d)^N} \\
 F_{u22} &= \frac{i_2}{a(-y_2 + b)^N} & F_{u21} &= \frac{i_2}{a(y_c - y_1 + b)^N} & &
 \end{aligned}$$

where the actual magnet separation y_{12} is

$$y_{12} = y_c + y_2 - y_1, \quad y_c \approx 13.0 \text{ cm}$$

and a , b , c , d , and N are constants which may be determined by numerical modeling of the magnetic configuration or by empirical methods. Typically $3 < N < 4.5$. The maximum magnet separation may be found by measurement to be $y_c \approx 13.0 \text{ cm}$.

3.3.3 Equations of Motion for the Double-magnet Configuration

For the MagLev apparatus, the coefficients of friction affecting both disks are the same, and may be written as $c_1 = c_2 = m\beta$. In addition, a value of $N = 4$ has been shown empirically to yield a close approximation of the force/distance relationships. Finally, since current in a coil is proportional to the voltage applied to the power amplifier for that coil, we find that the following equations are useful for design purposes:

$$m\ddot{y}_1 = F_{u11} + F_{u21} - F_{m12} - m\beta\dot{y}_1 - mg \quad (3.1)$$

$$m\ddot{y}_2 = F_{m12} - F_{u22} - F_{u12} - m\beta\dot{y}_2 - mg \quad (3.2)$$

$$F_{u11} = \frac{u_1}{a(y_1 + b)^4} \quad (3.3)$$

$$F_{u12} = \frac{u_1}{a(13 + y_2 + b)^4} \quad (3.4)$$

$$F_{u22} = \frac{u_2}{a(-y_2 + b)^4} \quad (3.5)$$

$$F_{u21} = \frac{u_2}{a(13 - y_1 + b)^4} \quad (3.6)$$

$$F_{m12} = \frac{c}{(y_{12} + d)^4} \quad (3.7)$$

3.3.4 Equation of Motion for the Single-magnet Configuration (Magnet in Lower Position)

This semester, you will be using the MagLev only in a single-disk configuration. Sometimes, the magnet will be controlled by the lower coil and sometimes the magnet will be controlled by the upper coil. When controlled by the lower coil, the above equations may be modified to describe the single-magnet configuration by deletion of the appropriate terms:

$$m\ddot{y}_1 = F_{u_{11}} - m\beta\dot{y}_1 - mg. \quad (3.8)$$

3.3.5 Equation of Motion for the Single-magnet Configuration (Magnet in Upper Position)

When a single magnet is controlled only by the upper coil, the equations are modified as follows:

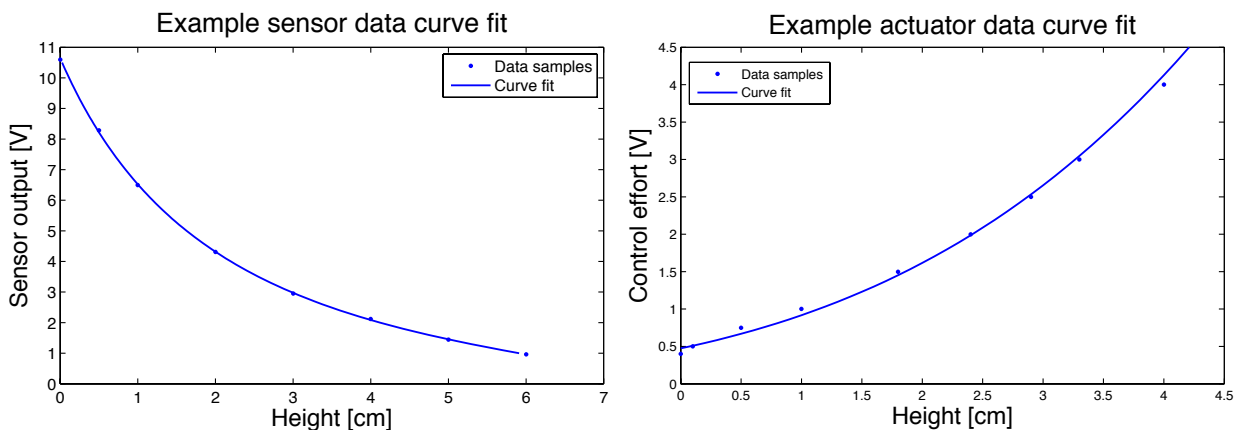
$$m\ddot{y}_2 = -F_{u_{22}} - m\beta\dot{y}_2 - mg. \quad (3.9)$$

Note that sometimes we will use $-F_{u_{22}}$ in this equation, and sometimes we will use $+F_{u_{22}}$, depending on whether the “N” or “S” face of the magnet is pointing upward.

3.3.6 Nonlinear Function Optimization in Matlab

In this and the next lab you perform, you will make measurements and will need to fit a nonlinear curve to these measurements. You will be given the form of the curve that needs to be fit to the data. In each case, the form will have two or more unknown constant parameters which must be chosen in order to make the fit.

For example, we denote the true magnet position of magnet k by $y_k(t)$ [cm] and the sensor voltage for this position as $y_{k_{raw}}(t)$ [V]. (For clarity, we often drop the dependence on time t .) We can make measurements of the relationship between y_k and $y_{k_{raw}}$ and plot them (as in the dots in the leftmost figure below). These discrete measurements are not as useful as a mathematical equation (curve fit) that describes the measurements with a low degree of error. We are trying to find this curve fit in this section. (The rightmost figure below shows the same idea for fitting an actuator curve, which you will do in the next lab.)



The curve fit for the sensor data “calibrates” the measurements. It takes a measurement in volts and converts it to an estimate of magnet height in centimeters. A relationship that works quite well for this lab is

$$y_{k_{cal}} = \frac{e_k}{y_{k_{raw}}} + \frac{f_k}{\sqrt{y_{k_{raw}}}} + g_k + h_k \cdot y_{k_{raw}},$$

where e_k , f_k , g_k , and h_k are parameters which are chosen to make $y_{k_{cal}}$ as “close” to y_k as possible. We make the calibrated curve fit the measurements by minimizing the distance between the measurements and the curve. Specifically, you will choose these parameters to minimize the sum-squared difference between y_k and $y_{k_{cal}}$ for your

measured data. This is the same as minimizing J in the summation

$$J = \sum_{\text{data}} (y_k - y_{k_{\text{cal}}})^2.$$

In Matlab, we can easily find a set of $\{e_k, f_k, g_k, h_k\}$ which are good parameters to fit such a function using the `fminsearch` nonlinear optimization routine from the optimization toolbox. This routine calls a user-written function which must return the sum-squared error J . For example, the following code segment computes the error squared for a specific set of $\{e_k, f_k, g_k, h_k\}$ where $e_k = \text{efgh}(1)$; $f_k = \text{efgh}(2)$; $g_k = \text{efgh}(3)$; and $h_k = \text{efgh}(4)$. The variables `yk` and `ykraw` are the measured data values. We type in the following code segment and save it to disk as `fitfun.m`.

```
function J = fitfun(efgh)
    yk = [0 0.5 1 2 3 4 5 6]; % actual magnet height, [cm]
    ykraw = [10.4 8.7 6.9 4.6 3.5 2.3 1.6 1.1]; % measured sensor voltage, [V]
    ykcal = efgh(1)./ykraw+efgh(2)./sqrt(ykraw)+efgh(3)+efgh(4).*ykraw;
    curve_fit_error = yk - ykcal;
    J = norm(curve_fit_error);
end
```

In order to find the set of $\{e_k, f_k, g_k, h_k\}$ which is optimum (which minimizes J), we call `fminsearch`:

```
efgh=fminsearch('fitfun',[e0 f0 g0 h0])
```

where `e0`, `f0`, `g0`, and `h0` are (user-specified) initial guesses for $\{e_k, f_k, g_k, h_k\}$. The `fminsearch` routine executes your `fitfun.m` function using your initial guesses, and through repeated calls to `fitfun.m` it intelligently modifies the $\{e_k, f_k, g_k, h_k\}$ parameters based on the J value the function returns in order to minimize J . The solution that Matlab finds will in general be different for different initial guesses. Some solutions are better than others, but you should find that any solution Matlab returns will be “good enough.”

In this lab, you will need to make fitting functions for two sets of sensor measurements. In the next lab you will need to make one fitting function for a set of actuator measurements. Use the `edit` command in Matlab to start the editor. The menus of the editor will allow you to save the file to disk and to exit.

3.4 PRELAB ASSIGNMENT

Construct tables similar to Table 3.1 on page 3-7 to record data. Carefully read and understand the procedure outlined below. Bring a small plastic ruler or other straightedge to the lab to assist in making accurate measurements.

3.5 LABORATORY EXPERIMENT

This section gives a procedure for identifying the plant parameters and nonlinear characteristics. The approach will be to measure the specific input/output characteristics of the laser sensor as they vary with relative position. The laser position nonlinearity results from the nature of the change of light intensity on the detector with magnet position and is associated with the particular sensor used.

3.5.1 Sensor Characteristic Procedure

1. When you arrive at the lab, the mechanism should already be set up with a single magnet on the Pyrex rod, resting on the lower drive coil. Make certain that your hands are clean at all times when touching the magnet—avoid touching the white surface. Power to the Control Box should be off.

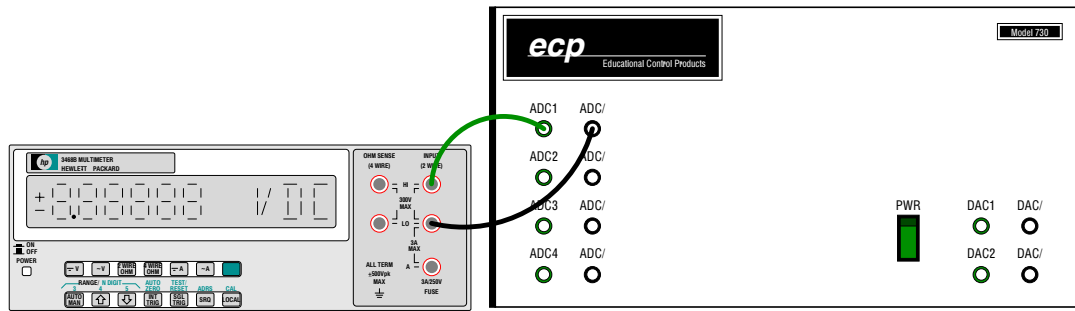


Figure 3.6 Configuration for measuring sensor characteristic.

2. Connect the digital multimeter (DMM) to ADC1 and ADC/ (ground) on the Control Box as shown in Fig. 3.6. Turn on the DMM, configured to measure dc voltage.
3. Turn on power to the Control Box. You should see the laser light beam on the upper and lower magnet surfaces. Move the magnet manually up and down to verify that the sensor output voltage is changing.
4. Sight along the top of the magnet and adjust the “magnet height ruler” (see Fig. 3.1 on page 3-2) so that the 0 cm position precisely matches the plane of the *top* of the magnet. Loosen the thumbscrews, move the ruler up or down as necessary, and re-tighten the thumbscrews.
5. Record the sensor output voltage at the 0 cm position. Refer to the notes below regarding moving the magnet to the desired calibration positions and reading the sensor voltage. Manually move the magnet to the 0.50 cm position and read and record the raw sensor output. Continue moving and recording the sensor data for the magnet positions in Table 3.1.
6. Repeat Steps 1 through 5 for the upper sensor. You will need to change the leads connected to the Model 730 Control Box from ADC1 and ADC/ to ADC2 and the corresponding ADC/. Here you will need to sight off the *bottom* surface of the magnet. Again, adjust the “magnet height ruler” to do so. In this case “Magnet Position” is the distance below its uppermost travel. Note that by convention, the displacements of the upper magnet are negative-going as the magnet moves downward.

Notes on taking sensor calibration data:

1. Hold the magnet only by the edges and do not block the laser light.
2. Apply light finger pressure radially inward so that the magnet bushing is snug against one side of the glass rod. This will assure that the magnet is not tilted relative to the rod and is approximately horizontal. It should also assure that the magnet does not move during the measurement. Any movement can lead to inaccurate results.

Table 3.1 SAMPLE TABLE FOR EXPERIMENTAL RESULTS OF SENSOR DATA.

Magnet Position for Sensor 1	$y_{1,raw}$ (V) for Sensor 1	Magnet Position for Sensor 2	$y_{2,raw}$ (V) for Sensor 2
0.0		0.0	
0.5		-0.5	
1.0		-1.0	
2.0		-2.0	
3.0		-3.0	
4.0		-4.0	
5.0		-5.0	
6.0		-6.0	

- When the magnet is at the close end of travel relative to the sensor (at the bottom for sensor #1, at the top for sensor #2) you should rotate it and find the “clocking” position where the sensor output is approximately the average of the highest and lowest values. There may be as much as $\pm 0.3V$ difference between the highest and lowest values. This amounts to an equivalent position measurement error of less than 1mm and becomes much less for positions farther away from the coil. You should try to maintain this approximate clocking for the remainder of the magnet positions at which you take sensor data in Step 5 (*i.e.*, keep the magnet polarity “N” label pointed in the same direction). There may be some noise in the sensor reading; you should “visually average” the values displayed. You need only read the DMM voltage to three significant digits.

3.6 ASSIGNMENT

- Plot the sensor #1 output $y_{1_{raw}}$ as a function of magnet height. Use curve fitting techniques to find a set of coefficients $\{e_1, f_1, g_1, h_1\}$ in

$$y_1 \approx y_{1_{cal}} = \frac{e_1}{y_{1_{raw}}} + \frac{f_1}{\sqrt{y_{1_{raw}}}} + g_1 + h_1 \cdot y_{1_{raw}},$$

that yields values of $y_{1_{cal}}$ that closely match your data y_1 (within 1 mm for all points) at the locations measured in Table 3.1. Using these coefficients, plot y_1 and $y_{1_{cal}}$ versus $y_{1_{raw}}$ on the same plot as the test data to show the closeness of the match. Repeat this procedure for the Sensor #2 output to find a different set $\{e_2, f_2, g_2, h_2\}$ to fit

$$y_2 \approx y_{2_{cal}} = \frac{e_2}{y_{2_{raw}}} + \frac{f_2}{\sqrt{y_{2_{raw}}}} + g_2 + h_2 \cdot y_{2_{raw}}.$$

Determine the units of e_k , f_k , g_k , and h_k .

System Identification: Actuators

4.1 INTRODUCTION

This laboratory exercise is a continuation of the last lab. We now focus on making a curve-fit (a mathematical model) of the actuator characteristics of the MagLev device. When you are finished with this, you will have a complete nonlinear model for how the MagLev works.

4.2 THIS LAB IS...

- A continuing exploration into system identification—finding a mathematical model for the dynamics of a particular system.
- A more challenging task of using Matlab for nonlinear optimization and function fitting.

The results from this lab are used throughout the rest of the semester, so it is in your best interest to be as accurate as you can!

4.3 BACKGROUND

The background information is the same as for the prior lab. Please review that material before starting this lab.

We will not be constructing a model suitable for two-disk control, so we will not need to estimate the c and d terms for the $F_{m_{12}}$ term. (However, for those curious about how to do this, the necessary data is listed in Table 1.1, to which Eq. (3.7) is fit to find the c and d parameters.)

The final salient properties of the magnet in the MagLev apparatus and its dynamic environment are its inertia (or mass) and the associated force (weight) due to the earth's gravitational field. You are not required to measure the mass of the magnet. It is approximately $120 \pm 4 g$. The weight of the magnet is therefore approximately $1.18 N$.

4.4 PRELAB ASSIGNMENT

1. Construct a table similar to Table 4.1 on page 4-2 to record data.
2. Create a Matlab function, similar to the `fitfun` function in the prior lab, to be used with `fminsearch` to optimize the actuator data that you will measure. This function should have as input the a and b values to try, and should compute as output

$$J = \sum_{\text{data}} (y_m - y_1)^2,$$

where y_m comprises measured magnet heights and y_1 is the predicted magnet height variable in the equation

$$F_w = \frac{u_1}{a(y_1 + b)^4}.$$

(Hint: In order to levitate a magnet to a certain height using control effort $u_1 [V]$, it is necessary to exactly cancel the force on that magnet due to gravity.)

3. Carefully read and understand the procedure outlined below. Bring a small plastic ruler or other straightedge to the lab to assist in making accurate measurements.

Table 4.1 ACTUATOR CALIBRATION/ LINEARIZATION DATA.

Magnet Position [<i>cm</i>]	Control effort $u_{1_{raw}}$ [<i>V</i>]
0.0 ⁺	
	0.50
	0.75
	1.00
	1.50
	2.00
	2.50
	3.00
	4.00

4.5 LABORATORY EXPERIMENT

This section gives a procedure for identifying the plant parameters and nonlinear characteristics. The approach will be to measure the specific input/output characteristics of the magnet/coil actuators as they vary with relative position. These strong magnetic field nonlinearities are inherent in this class of magnetic systems.

4.5.1 Actuator Characteristic

The mechanism should be in the same configuration as in the last section. Assure that the north pole of the magnet is facing upward. The “magnet height ruler” should again be adjusted to measure off the **upper** surface of the magnet when it is resting in its lowermost position.

1. Connect the dc power supply and the DMM to DAC1 and DAC/ (ground) on the Control Box as in Fig. 4.1 on page 4–3. Power to the Control Box should be off. Turn on power to the dc power supply and to the DMM. Adjust the dc power supply from 0 *V* to 10 *V* and ensure (using the DMM) that the correct polarity and voltage is being supplied to the Control Box. Adjust the dc power supply back to 0 *V*. **Only then**, turn on the power to the Control Box.
2. Record the height of the magnet corresponding to 4 *V* in a table similar to Table 4.1. You should gently spin the magnet (again touching only its edge) for all measurements to reduce the effects of friction so that the true equilibrium height is observed.
3. Record the height of the magnet corresponding to 3 *V*. You should notice the magnet height become lower.
4. Repeat step 3 to find the control effort value at which the magnet is lifted only a very slight amount above the support pads (*i.e.*, the 0⁺ position).
5. Repeat step 3 for the remaining control effort values of Table 4.1. *Turn voltage of the dc power supply down immediately after measuring magnet height to minimize heat build-up in the coil and servo amplifiers during exposure to the higher control effort values (those greater than 2 V). Do not exceed 5 V of control effort.*

4.6 ASSIGNMENT

1. The experiments in Section 4.5.1 found the equilibrium height of the magnet for a given control effort (in Volts or, proportionally, current in the coil). Equivalently, the same information establishes the control effort u_1 , required to produce a constant force on the magnet F_w (equal to the magnet’s weight) as a function of magnet height. Plot your data from Table 4.1 showing this required control effort u_1 as a function of magnet height. Solve for coefficients $\{a, b\}$ to fit this data with a function

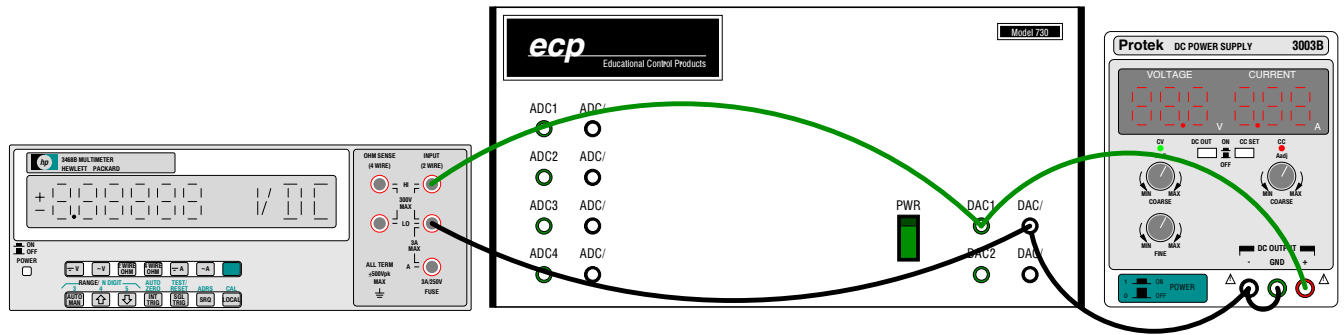


Figure 4.1 Configuration for measuring actuator characteristic.

$$F_w = \frac{u_1}{a(y_1 + b)^4} \quad (4.1)$$

within a tolerance of $0.1 V$ at a given height. From the closeness of the fit to your data, does this curve fit capture the nonlinearity of the actuator well? (As a point of reference, typical servo amplifier nonlinearities are on the order of 2–10%) Determine the units of a and b .

The attractive magnetic characteristic (force versus distance) of the upper coil/magnet is essentially the same as the repulsive one of the lower coil/magnet. Assuming that the upper actuator is configured so that a positive control effort imparts an attractive force on the magnet, what are $\{a, b\}$ for the upper actuator?

Record your values of a , b , and both sets of e , f , g , and h (including units) somewhere where you can easily refer to them in future labs. Also record and report which MagLev device (1–4) you used. Use this same device for all future experiments unless instructed otherwise.

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Small-Signal Linearization

5.1 INTRODUCTION

In Labs 3 and 4, you performed system identification of the magnetic levitation device. That is, you learned its dynamic equations of motion and you determined the unknown parameters to these equations by measurement and optimization. You discovered that its dynamics are very nonlinear. This might seem unfortunate as all the tools for control-system analysis and design you will learn this semester apply only to linear systems. However, we can *linearize* many nonlinear systems around a certain operating point. That is, we make an approximate linear model of the system in the region of operation space we plan to use.

You will make linearized models of the MagLev system using two different methods. In order to test your linear models, you will simulate their response to various inputs using Simulink. You will also drive the real system with the same inputs and compare the output of the real system with the output of your linear models. You will then make conclusions regarding the suitability of using your linear models to design and analyze control systems.

5.2 THIS LAB IS...

- An introduction to small-signal linearization. This topic will **not** be covered in much detail in ECE4510, so read the background section carefully. If you have questions, feel free to ask Dr. Plett or the lab instructor substantially *before* the lab period.

5.3 BACKGROUND

A “standard” feedback control system is configured as shown in Fig. 5.1 where $D(s)$ is the transfer function of the controller, and $G(s)$ is the transfer function of the plant. Note that both $D(s)$ and $G(s)$ are linear systems. Our desire is that $y(t)$ match $r(t)$ as closely as possible, and this is achieved by designing an appropriate $D(s)$.

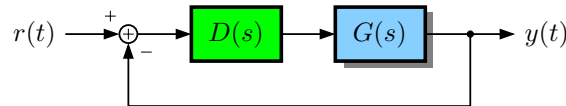


Figure 5.1 Standard feedback control system.

The problem that we have when trying to fit the MagLev device into this framework is that its dynamics are not linear. In Labs 3 and 4, the equations of motion for the MagLev system were found to be:

$$m\ddot{y}_1 = F_{u_{11}} + F_{u_{21}} - F_{m_{12}} - m\beta\dot{y}_1 - mg \quad (5.1)$$

$$m\ddot{y}_2 = F_{m_{12}} - F_{u_{22}} - F_{u_{12}} - m\beta\dot{y}_2 - mg \quad (5.2)$$

$$F_{u_{11}} = \frac{u_1}{a(y_1 + b)^4} \quad (5.3)$$

$$F_{u_{12}} = \frac{u_1}{a(13 + y_2 + b)^4} \quad (5.4)$$

$$F_{u_{22}} = \frac{u_2}{a(-y_2 + b)^4} \quad (5.5)$$

$$F_{u_{21}} = \frac{u_2}{a(13 - y_1 + b)^4} \quad (5.6)$$

$$F_{m_{12}} = \frac{c}{(y_{12} + d)^4}. \quad (5.7)$$

Furthermore, the sensor output voltage $y_{k_{raw}}$ for sensor k is related to the true magnet position y_k through the nonlinear relationship

$$y_k \approx \frac{e_k}{y_{k_{raw}}} + \frac{f_k}{\sqrt{y_{k_{raw}}}} + g_k + h_k \cdot y_{k_{raw}}. \quad (5.8)$$

For the purpose of the present discussion we will consider the case where there is one magnet on the device. The equation of motion for that magnet is then

$$m\ddot{y}_1 = \frac{u_1}{a(y_1 + b)^4} - m\beta\dot{y}_1 - mg. \quad (5.9)$$

These dynamics are represented schematically as in Fig. 5.2. In the diagram, y_1 is the true position of the magnet and $y_{1_{raw}}$ is the measured voltage corresponding to that position. $G(s) = 1/(ms^2 + m\beta s)$.

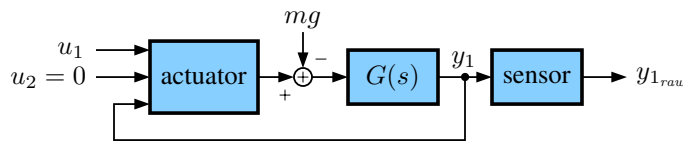


Figure 5.2 Block diagram of MagLev dynamics, single-magnet configuration (lower position).

Even in Eq. (5.9) with one magnet on the device we see that the dynamics are nonlinear. (It is not a linear constant coefficient ordinary differential equation). There are three sources of nonlinearity: the actuator contributes a nonlinearity via Eq. (5.3), the sensor contributes another nonlinearity via (the inverse of) Eq. (5.8), and gravity contributes a nonlinearity via the constant added force mg (LCCODEs do not have constant terms).

In the following sections you will learn a method to make a linearized model useful for designing a controller for the MagLev system.

5.3.1 Small-Signal Linearization Using Calibrated Sensor Data

►► Model predicts height of magnet y_k .

The concept we will use is shown in Fig. 5.3 on page 5–3. The constant force due to gravity is *canceled* by applying a constant steady-state voltage u_{1o} to the controller output.¹ This alone has the effect of levitating the magnet to a constant height, in the absence of disturbance. This removes the nonlinear gravity term from the equations by canceling it.

The voltage $y_{1_{raw}}$ corresponding to the magnet's height is measured. We compute a calibrated estimate $y_{1_{cal}}$ of the actual magnet position y_1 using the measured sensor voltage $y_{1_{raw}}$ via the relation

$$y_{1_{cal}} = \frac{e_1}{y_{1_{raw}}} + \frac{f_1}{\sqrt{y_{1_{raw}}}} + g_1 + h_1 \cdot y_{1_{raw}}. \quad (5.10)$$

¹This value is a function of the desired steady-state output y_{1o} .

In the diagram, this operation is performed by the “sensor inverse” block. It effectively inverts or cancels the sensor nonlinearity. We have $y_{1cal} \approx y_1$ and have therefore removed the sensor nonlinearity term from the equations by canceling it.

The actuator nonlinearity is not explicitly canceled. Rather, the nonlinear portion of the block diagram inside the thick gray rectangle is “linearized” using methods described below. A linear approximation of the nonlinear function inside the gray box is used as the plant model.

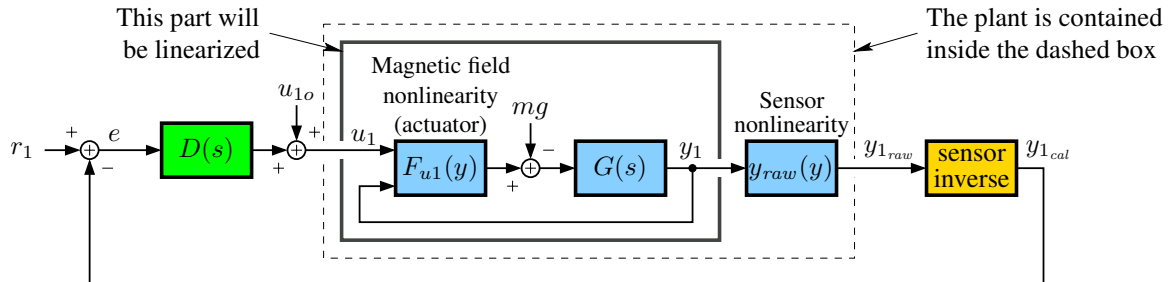


Figure 5.3 Method 1 for compensating nonlinearities. The linear model predicts the actual magnet height. Your controller must specifically invert the sensor nonlinearity.

Taylor-Series Function Expansion

Nonlinear equations may be linearized using Taylor-series expansion. You should recall from your calculus courses that a function may be expanded via a Taylor-series expansion to be represented as a polynomial. For example, the $\sin(x)$ function may be expanded around $x = 0$ and represented as

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \dots$$

The expansion is valid for all x . (x in radians).

For small values of x , the sinusoidal function may be approximated as linear. We can say

$$\sin(x) \approx x$$

for $|x| \ll 1$. In general, linearization of a function is done by keeping the zeroth- and first-order terms of the Taylor-series expansion. For some functions, linearization gives a good approximation to its form in a small operating region. Other times, linearization does not work well at all. We are fortunate with the MagLev device to discover that linearization works well.

Linearization

Using the method to be presented, a nonlinear system is linearized around an “operating point”. For example, suppose we apply a constant control effort u_{1o} to the bottom coil to keep the bottom magnet levitated to a height of 2.5 cm in the absence of disturbances. Then, small changes in the control effort u_1 around the constant value of u_{1o} will lead to linear behavior in the magnet height, plus the constant offset of 2.5 cm. In this example, the operating “point” consists of the constant u_{1o} and the constant $y_{1o} = 2.5$ cm.

The linearized equations of motion are found by solving for the zeroth- and first-order terms of the Taylor-series expansion of the respective equations about the operating point. For example, the full set of equations of motion for the MagLev system are listed in Eqs. (5.1) to (5.7).

We customarily re-arrange the equations of motion so that one side of the equation contains the linear terms, and the other side of the equation contains the nonlinear terms. So, Eqs. (5.1) and (5.2) become (when considered in single-disk configurations only):

$$\begin{aligned} m\ddot{y}_1 + m\beta\dot{y}_1 &= F_{u_{11}} - mg \\ m\ddot{y}_2 + m\beta\dot{y}_2 &= -F_{u_{22}} - mg. \end{aligned}$$

Calling right-hand-side of the first equation $\alpha(y_1, u_1, t)$, and calling the right-hand-side of the second equation $\beta(y_2, u_2, t)$, we have

$$\alpha(y_1, u_1, t) \approx \alpha(y_{1o}, u_{1o}, t) + \left. \frac{\partial \alpha}{\partial y_1} \right|_{y_{1o}, y_{2o}, u_{1o}, u_{2o}} \cdot (y_1 - y_{1o}) + \left. \frac{\partial \alpha}{\partial u_1} \right|_{y_{1o}, y_{2o}, u_{1o}, u_{2o}} \cdot (u_1 - u_{1o}), \quad (5.11)$$

where y_{1o} and u_{1o} are the respective magnet positions and control efforts that define the operating point. For the purposes of control design we choose the operating point to be at an equilibrium (the system is at rest and not accelerating) so that

$$\alpha(y_{1o}, u_{1o}, t) = (F_{u_{11}} + -mg)|_{y_{1o}, u_{1o}} = 0; \quad (5.12)$$

$$\text{also, } \beta(y_{2o}, u_{2o}, t) = (-F_{u_{22}} - mg)|_{y_{2o}, u_{2o}} = 0. \quad (5.13)$$

Evaluating Eq. (5.11) and using Eq. (5.12) we have

$$m\ddot{y}_1 + m\beta\dot{y}_1 + \frac{4u_{1o}}{a(y_{1o} + b)^5}(y_1 - y_{1o}) = \frac{1}{a(y_{1o} + b)^4}(u_1 - u_{1o}),$$

which may be rewritten as (where $y_1^* = y_1 - y_{1o}$, $u_1^* = u_1 - u_{1o}$ and so forth; note also that $\ddot{y}_1 = \ddot{y}_1^*$ and $\dot{y}_1 = \dot{y}_1^*$)

$$m\ddot{y}_1^* + m\beta\dot{y}_1^* + k_{11}y_1^* = k_{u11}u_1^*. \quad (5.14)$$

Similarly for the second differential equation

$$m\ddot{y}_2^* + m\beta\dot{y}_2^* + k_{22}y_2^* = k_{u22}u_2^*, \quad (5.15)$$

where

$$k_{11} = \frac{4u_{1o}}{a(y_{1o} + b)^5}, \quad k_{u11} = \frac{1}{a(y_{1o} + b)^4}, \quad k_{22} = \frac{4u_{2o}}{a(-y_{2o} + b)^5}, \quad k_{u22} = \frac{-1}{a(-y_{2o} + b)^4},$$

When the gravity and sensor nonlinearities are canceled, and the remaining system in the gray box is linearized, we achieve the block diagram in Fig. 5.4, suitable for designing a controller.

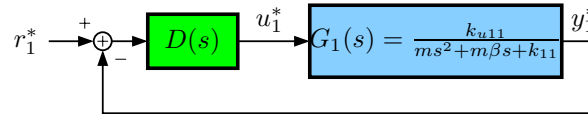


Figure 5.4 First linearized system for design: single-magnet, lower actuator only.

5.4 PRELAB ASSIGNMENT

- Consider passing a positive signal $x(t)$ through the operation $y(t) = \sqrt{x(t)}$. Next, compute $z(t) = [y(t)]^2$. What is the relationship between $x(t)$ and $z(t)$? This is the same idea behind “inverting the sensor.” The MagLev computes its sensor output $y_{k_{raw}}$ to be a function of y_k . You will compute $y_{k_{cal}}$ to be a function of $y_{k_{raw}}$ to recover y_k .
- In this lab the MagLev will be configured with a single magnet. The operating point for the magnet will be 2.5 cm. Determine the steady-state control effort u_{1o} to operate at this level. Hint: Consider Eq. (5.1) with the magnet at rest.
- Determine $G_1(s)$; that is, fill in the numbers. **Be very careful to use compatible units!** Note that $1 N = 1 kg m/sec^2$ and that the units of the Laplace operator “s” are $1/sec$. For now, assume $\beta = 0$. Note that your transfer function $G_1(s)$ should have units $[cm/V]$ (since the input is in V and the output is in cm).

- Make a Simulink subsystem to cancel the sensor nonlinearity. That is, implement Eq. (5.8). Save this subsystem on your UFP account for easy access when you enter the lab.

5.5 LABORATORY EXPERIMENT

Figure 5.5 shows a sample Simulink diagram to test your plant model $G_1(s)$ using small-signal linearization. The actual MagLev output is compared to the output predicted by your models. The steady-state operating point will be at $y_{1o} = 2.5\text{ cm}$. The “Fix Bottom Sensor MagLev n” block represents the Simulink subsystem you created in the fourth prelab exercise to cancel the sensor nonlinearity.

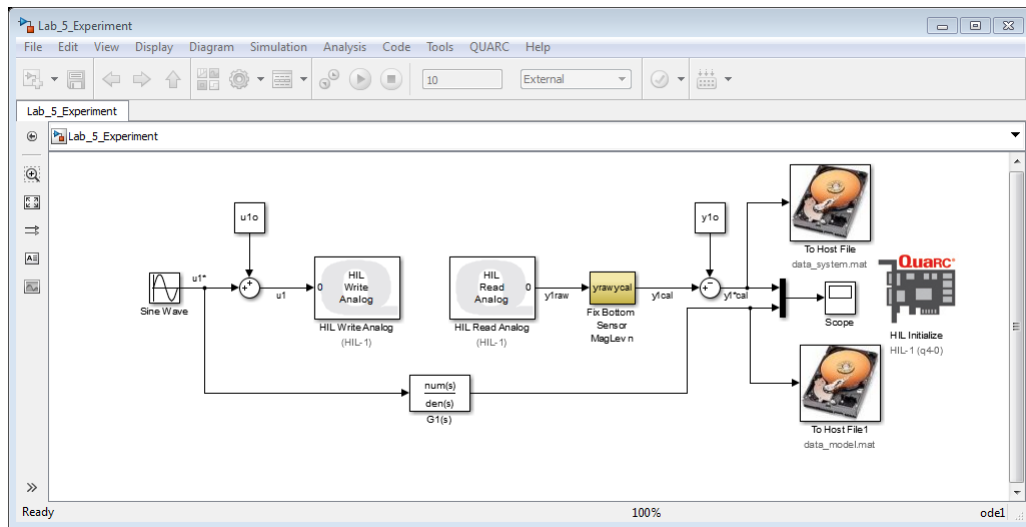


Figure 5.5 Experimental setup.

1. Configure the lab equipment as shown in Fig. 5.6. The computer output to the MagLev is on D2A channel 0 and the computer input from the MagLev sensor is on A2D channel 0.

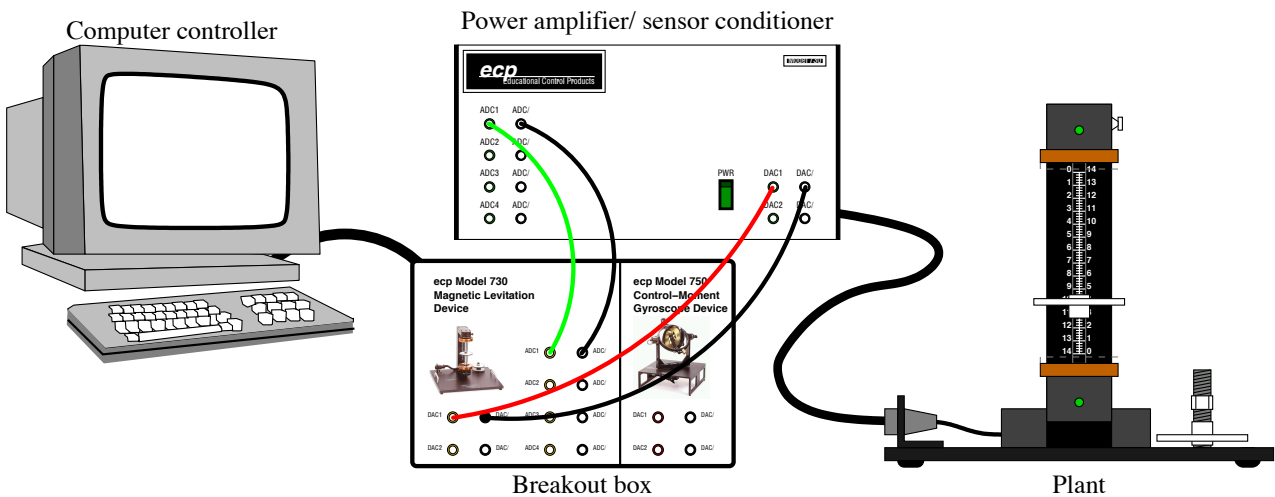


Figure 5.6 Setup of the lab equipment.

2. Create a Simulink block diagram to execute Fig. 5.5. The signal source will be a sine wave with magnitude 0.5. You will use frequencies 5, 10, and 15 radians per second to judge the accuracy of your model at different frequencies. For now, assume that $\beta = 0$.

3. Run your simulation for this block diagram. ***Be prepared to stop the simulation immediately upon observing unstable behavior!*** If you do observe unstable behavior, something is wrong with your simulation because it *should* work! The idea here is that the output of the “G1(s)” block should match the “y1*cal” signal very closely. The purpose of this lab is to compare the linear model to the actual nonlinear system and see how close the results actually are.
4. Your linear model assumes negligible friction. You may find that this assumption is not valid. Experiment with a (viscous) friction term $m\beta s$ in the denominator of your transfer function to find a good value for β . It will probably be in the range $[0, 10)$.

If you have time, experiment with different sources. Try to determine experimentally the range of frequencies and amplitudes that the two models approximate well. Print all results.

5.6 ASSIGNMENT

In the lab writeup, include printouts of your Simulink block diagrams and printouts of all experimental results. Include approximate friction coefficients β . Comment on valid operation ranges for the two small-signal linearization models.

5.7 APPENDIX: SUMMARY OF NOTATION USED

Symbol	Meaning (Lower magnet is magnet 1; Upper magnet is magnet 2)
$y_k, [cm]$	The actual position of magnet k . Positive for magnet 1; negative for magnet 2.
$y_{k_{raw}}, [V]$	The sensor output corresponding to magnet k . Always positive.
$y_{k_{cal}}, [cm]$	You compute $y_{k_{cal}}$ using measured $y_{k_{raw}}$ to predict y_k .
$y_{k_o}, [cm]$	The equilibrium magnet position of magnet k .
$y_{k_{raw,o}}, [V]$	The equilibrium sensor output corresponding to the equilibrium position for magnet k .
$y_k^*, [cm]$	$y_k^* = y_k - y_{k_o}$. The difference between y_k and its equilibrium position.
$y_{k_{raw}}^*, [V]$	$y_{k_{raw}}^* = y_{k_{raw}} - y_{k_{raw,o}}$. The difference between $y_{k_{raw}}$ and its equilibrium value.
$u_k, [V]$	The control effort applied to coil k .
$u_{k_o}, [V]$	The equilibrium control effort applied to coil k .
$u_k^*, [V]$	$u_k^* = u_k - u_{k_o}$. The difference between u_k and its equilibrium value.
$r_k, [cm]$	The reference input signal for magnet k . The <i>desired</i> magnet position.
$r_{k_{raw}}, [V]$	The <i>desired</i> output sensor reading for magnet k .
$r_k^*, [cm]$	$r_k^* = r_k - r_{k_o} = r_k - y_{k_o}$. The difference between desired magnet height and equilibrium height.
$r_{k_{raw}}^*, [V]$	$r_{k_{raw}}^* = r_{k_{raw}} - y_{k_{raw,o}}$. The difference between desired sensor reading and equilibrium value.

“Black-Box” Frequency-Response System Identification

6.1 INTRODUCTION

If the input to a linear time-invariant system is a sinusoid, then the output (in steady state) is a sinusoid of the same frequency, but with different amplitude and phase. The system may then be characterized by observing how it responds to different input frequencies. The magnitude response relates how the system amplifies or attenuates certain frequencies. The phase response relates how the system delays certain frequencies.

In Lab 5, you determined a transfer-function model of the MagLev, linearized around a certain operating point. From the transfer function you found, $G(s)$, you may determine the magnitude response as $|G(j\omega)|$ and the phase response as $\angle G(j\omega)$. Similarly, if you were to find magnitude and phase responses of the system in some other way, you may predict a transfer function of the system. This is one form of system identification. In this lab, you will make frequency-response measurements to develop a second transfer-function model of the MagLev. These measurements and the model you develop will be used in later labs to aid control-system design.

6.2 THIS LAB IS...

- A deeper look at frequency-response measurement;
- A preview of Bode plots and Nyquist plots;
- Some musings on frequency response of a nonlinear system;
- A second method to perform system identification.

6.3 BACKGROUND

Frequency-response design methods will be discussed in ECE4510 later on this semester. They are perhaps the most important methods available since they allow very similar design procedures for continuous-time and discrete-time (digital) systems, and may be extended to consider the currently-popular field of “robust” control. These methods rely on an accurate frequency-response model of the plant to be controlled.

This model may be derived by first finding the equations of motion for the system (Labs 3 and 4) and then linearizing the equations to find a linear model around a certain operating point (Lab 5). The transfer function thus found, $G(s)$, may be used to predict the magnitude response $|G(j\omega)|$ and the phase response $\angle G(j\omega)$.

A second method to find the frequency response of the system is much more direct. Input a sinusoidal signal of a certain frequency (plus a dc offset to achieve the desired equilibrium solution). The output will be a sinusoidal signal (plus a dc offset). By measuring the ratio of magnitudes relating the input to the output, and by measuring the phase difference of the input and output sinusoids, we may determine the frequency response of the system at the frequency of the input sinusoid. Repeat for all frequencies of interest.

In this lab, you will implement the Simulink block diagram in Fig. 6.1 on page 6-2. A sinusoidal signal is sent to DAC1 (channel 0) after u_{1o} is added to it to achieve a steady-state magnet height of 2.5 cm. DAC1 is wired to

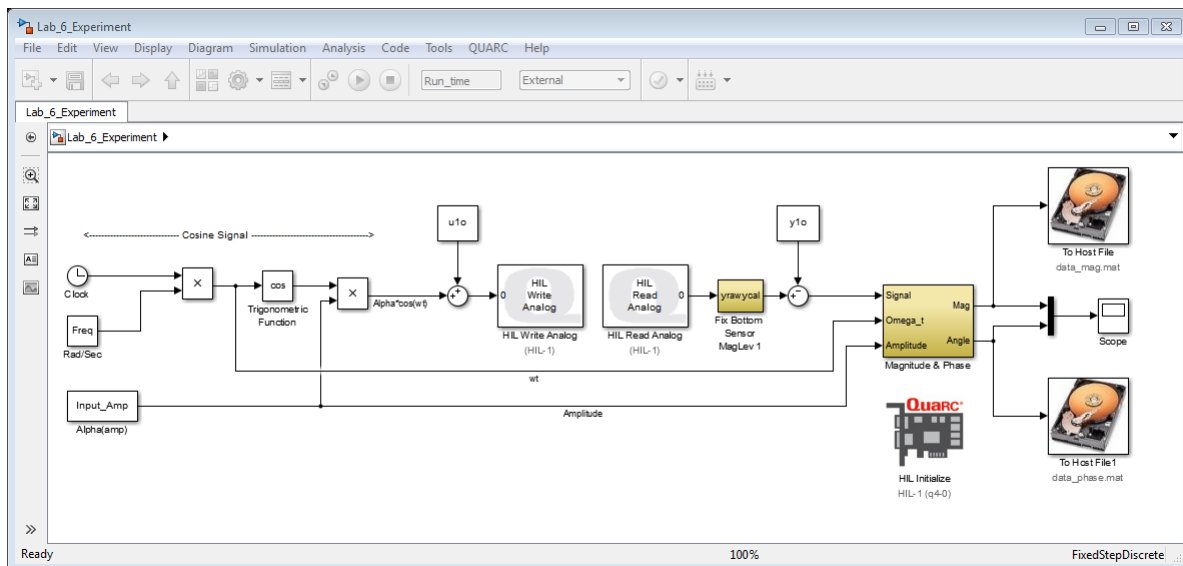


Figure 6.1 Setup for measuring frequency response of the MagLev.

the MagLev, and the sensor output is wired to ADC1. The “FixSensor” block converts the raw voltage $y_{1_{raw}}$ into a height $y_{1_{cal}}$. The equilibrium height is subtracted from this signal. This is the test signal. The test signal is input to a magnitude and phase detection block, described below.¹

6.3.1 Measuring Frequency Response

If the input to a linear time-invariant system is $u(t) = \alpha \cos(\omega t)$, then the output is

$$y(t) = \alpha |G(j\omega)| \cos(\omega t + \arg G(j\omega)) + v(t) + \text{transient},$$

where $v(t)$ is noise, and $G(j\omega)$ is the frequency response. This result leads to a simple procedure for finding the frequency response, known as frequency analysis:

- Excite the system with a sinusoidal input at frequency ω_0 ; wait for transient to die out; collect N time samples.
- Measure (graphically) the amplification factor and phase shift at that frequency, yielding $\hat{G}_N(j\omega)$.
- Repeat for all frequencies of interest.

ISSUE: The frequency function is continuous in the frequency domain. This means that for a complete identification of the function, all frequencies from 0 to ∞ must be tested. However, in a practical implementation we would need to select only a few frequencies and be willing to interpolate between the measured data.

ISSUE: With the presence of noise, $v(t)$, it may be difficult to accurately determine amplification and phase-shift factors. We can use correlations to average out the noise. Define (using discrete-time signals now)

$$I_c(N) = \frac{1}{N} \sum_{k=1}^N y[k] \cos(\omega k) \quad \text{and} \quad I_s(N) = \frac{1}{N} \sum_{k=1}^N y[k] \sin(\omega k).$$

We insert the formula for $y[k]$, ignoring the transient term (can wait until it dies out before collecting data),

¹Note that this procedure will measure the frequency response of the MagLev *combined with* the DAC/ADC pair that drive the MagLev. To be more accurate, we would repeat this procedure to find the magnitude and phase response of the DAC/ADC pair by themselves, and subtract this from the overall frequency response.

$$\begin{aligned}
I_c(N) &= \frac{1}{N} \sum_{k=1}^N (\alpha |G(e^{j\omega})| \cos(\omega k + \arg G(e^{j\omega})) + v[k]) \cos(\omega k) \\
&= \frac{1}{N} \sum_{k=1}^N (\alpha |G(e^{j\omega})| \cos(\omega k + \arg G(e^{j\omega}))) \cos(\omega k) \\
&\quad + \frac{1}{N} \sum_{k=1}^N v[k] \cos(\omega k) \\
&= \frac{\alpha}{2} |G(e^{j\omega})| \cos(\arg G(e^{j\omega})) \\
&\quad + \frac{\alpha}{2N} |G(e^{j\omega})| \sum_{k=1}^N \cos(2\omega k + \arg G(e^{j\omega})) \\
&\quad + \frac{1}{N} \sum_{k=1}^N v[k] \cos(\omega k),
\end{aligned}$$

noting that $\cos(a)\cos(b) = \frac{1}{2}(\cos(a+b) + \cos(a-b))$.

The second term tends to zero as $N \rightarrow \infty$. The third term also tends to zero unless $v[k]$ has a pure periodic frequency component at frequency ω . Similarly, we can find that

$$\begin{aligned}
I_s(N) &= -\frac{\alpha}{2} |G(e^{j\omega})| \sin(\arg G(e^{j\omega})) \\
&\quad + \frac{\alpha}{2N} |G(e^{j\omega})| \sum_{k=1}^N \sin(2\omega k + \arg G(e^{j\omega})) \\
&\quad + \frac{1}{N} \sum_{k=1}^N v[k] \sin(\omega k),
\end{aligned}$$

noting that $\cos(a)\sin(b) = \frac{1}{2}(\sin(a+b) - \sin(a-b))$. By combining $I_c(N)$ and $I_s(N)$, we can come up with a frequency-response estimate

$$\begin{aligned}
|\hat{G}_N(e^{j\omega})| &= \frac{\sqrt{I_c^2(N) + I_s^2(N)}}{\alpha/2} \\
\arg \hat{G}_N(e^{j\omega}) &= -\text{atan}(I_s(N)/I_c(N)).
\end{aligned}$$

This is known as the frequency analysis by correlation method. A block diagram of the magnitude and phase detector is shown in Fig. 6.2 on page 6-4.

6.3.2 Nonlinear System Effects

The MagLev is not a linear device. Sinusoidal inputs do not produce purely sinusoidal outputs. Harmonics and sub-harmonics of the input frequency will be present. You should realize from your knowledge of Fourier Series that the output will still be periodic, but that the shape of the output waveform will appear as a distorted sinusoid. We wish to measure the magnitude- and phase-response of the nonlinear MagLev system at the input frequency, and ignore the magnitude- and phase-response at harmonics and sub-harmonics of the input frequency.

The output waveform is distorted in such a way that it is not symmetric. The upper travel of the magnet is compressed by gravity forcing the magnet downward. The downward travel of the magnet is extended by gravity. This effect can be minimized by using a low-magnitude input sinusoid so that the magnitude of the output sinusoid will also be small.

The distorted shape of the output can confuse the magnitude and (especially) the phase detectors. Canceling the *apparent* dc value of the test signal is not identical to subtracting the equilibrium value since the output wave is

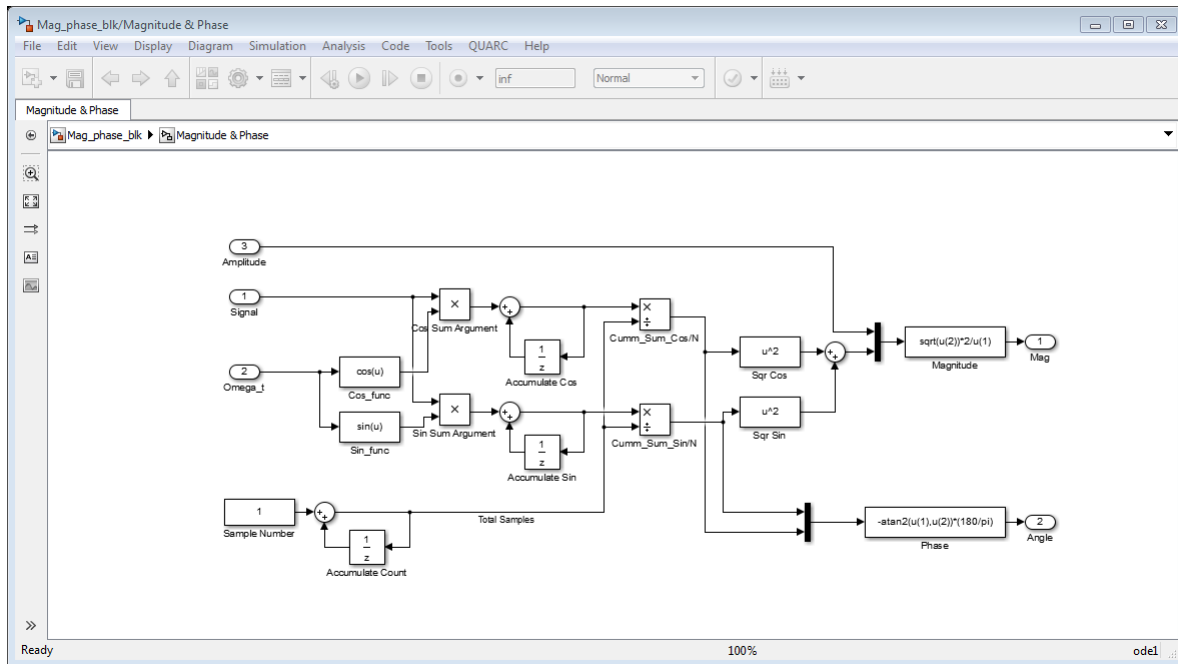


Figure 6.2 Magnitude detector circuit in Simulink.

not symmetric. Phases around 0° are especially sensitive to this effect, and the output of the phase detector may bounce between 0° and -360° (the “same” phase, but very different on the plot!). You may need to run several data-collection simulations at frequencies with low phase in order to get an acceptable value.

Perhaps a more important question is that if the output of the MagLev is not a sinusoid when the input is a sinusoid, does measuring the frequency response (of the fundamental frequency) make sense? There is some literature on this subject: Technically, you are not measuring a frequency response but a “describing function” of the nonlinear system. The correct way to do this is to keep the input signal magnitude constant, and to sweep it across the frequencies of interest (the describing functions are a function of both frequency and input signal magnitude). We cannot do this on the MagLev system. Small input magnitudes must be used near the resonant frequency since the sinusoid is amplified and the system output must remain limited to avoid damage. Larger input magnitudes are needed at higher and lower frequencies to overcome “stiction” (nonlinear friction). Suggested input-signal magnitudes for each frequency are given in the Laboratory Experiment section.

6.4 PRELAB ASSIGNMENT

Second-order transfer functions have the generic form

$$\frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2},$$

where ω_n is the natural frequency, ζ is the damping ratio, and K is the dc gain. The magnitude response (normalized to its dc value) and phase response of generic second-order systems are plotted in Fig. 6.3 on page 6–5 for different values of ζ .

You have already determined that the transfer function of the MagLev is second order, and have a linearized model of the system around the 2.5 cm point.

1. Plot the magnitude and phase response of your linearized transfer function of the MagLev, determined in Lab 5. Note that it is critical to include your estimate of friction term $m\beta s$ in the denominator to get a meaningful plot. Also, record the magnitude and phase response in the following table for the specified

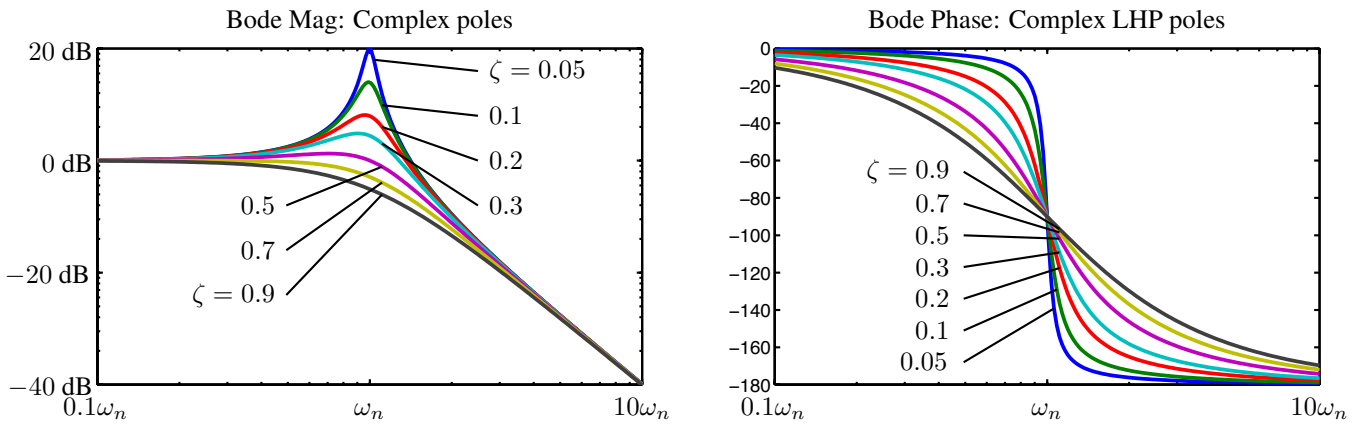


Figure 6.3 Magnitude- and phase-response of second order systems (stable).

frequencies. Hint: `sys = tf(ku11, [m m*beta k11]); [m,p] = bode(sys, omega)`. You will need to `squeeze(m)` and `squeeze(p)` to get results.

ω [rads/sec]	$ G(j\omega) $	$\angle G(j\omega)$
1		
2		
4		
7		
10		
15		
18		
19		
20		
22		
25		
30		
40		
70		
100		

2. Show that at ω_n

$$|H(j\omega_n)| = \frac{K}{2\zeta}.$$

This provides a means of estimating the damping factor:

$$\zeta = \frac{1}{2} \frac{|H(0)|}{|H(j\omega_n)|}.$$

The peak frequency is approximately ω_n .

6.5 LABORATORY EXPERIMENT

1. For the frequencies in the following table, use input signal

$$A_i \sin(\omega_i t).$$

Use this in block diagram in Fig. 6.1 on page 6–2. Record the magnitude and phase response measured from the MagLev. Run each simulation for 120 *sec* to ensure that the magnitude- and phase-detectors have settled to (nearly) final values. **Note that you do not have to create the magnitude and phase detectors in Simulink yourself!** You can find the detectors in the `utility` library. That is, at the Matlab prompt, type “utility” and press enter. You will be able to use the blocks from the Simulink library that appears on your display.

ω [rads/sec]	A	$ G(j\omega) $	$\angle G(j\omega)$
1	1.5		
2	1.5		
4	1		
7	1		
10	1		
15	0.75		
18	0.5		
19	0.4		
20	0.4		
22	0.4		
25	0.75		
30	0.75		
40	0.75		
70	0.75		
100	0.75		

Your data may contain “observation errors” because of nonlinear effects, and will probably not fall along nice straight asymptotes. You must use your judgment in disregarding outlying points. It is very helpful to bring some graph paper with you to make a rough plot of the data as you make measurements. This will help you determine if you have bad measurements, and need to re-measure data points.

6.6 ASSIGNMENT

1. Plot, on the same pair of axes, the Bode plot of your linearized transfer function of the system from Lab 5, and the Bode magnitude and phase points of your measured data from this lab. Note that a Bode magnitude plot is $20 \log_{10} |G(j\omega)|$ plotted versus $\log_{10}(\omega)$ and that the Bode phase plot is $\angle G(j\omega)$ in degrees plotted versus $\log_{10}(\omega)$.
2. Estimate the transfer function of the MagLev from your frequency-response data. Match the dc gain, the peak frequency, the peak height and phase response as best you can. Plot the Bode magnitude and phase plots of your new transfer function on the same pair of axes.
3. The frequency response may be plotted as a polar plot. Each frequency-response data pair is interpreted as the magnitude and phase of a complex number on the complex plane. Plot your results in this form. That is, plot as points in the complex plane $|G(j\omega)| \cos(\angle G(j\omega))$ on the x -axis (Real axis) and $\pm |G(j\omega)| \sin(\angle G(j\omega))$ on the y -axis (Imaginary axis). For the new system transfer function you estimated in the previous step, superimpose the `nyquist()` plot of the system. Compare the two plots. (Don’t forget that Matlab’s `sin` and `cos` functions require inputs in radians, not degrees.)
4. In your writeup be sure to discuss any unexpected effects and outlying data points. Comment on the similarity and differences between the frequency response of the system from Lab 5, and the frequency response of the system found in this lab. Can you account for any differences? Which is more accurate? Why?

5. We cannot *directly* measure the frequency response of the MagLev set up in an unstable configuration. Why? Suggest a simple way to measure the frequency response of the upper coil/ magnet position.

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Basic PID Control: Design

7.1 INTRODUCTION

Feedback (closed-loop) controllers have great advantages over open-loop controllers. By actually measuring system response (and using it as an input to the controller) the controller is able to reject disturbances, eliminate steady-state errors and have low sensitivity to modeling errors.

In this lab you will use the Ziegler-Nichols “Method II” to design P, PI and PID controllers. You will compare the response of the system in open-loop and closed-loop control scenarios.

7.2 THIS LAB IS...

- Your first implemented control design, and
- A demonstration of the advantages of feedback.

7.3 BACKGROUND

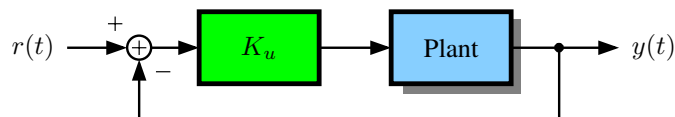
It has been estimated that up to 90% of all commercial control system applications use PID-type controllers. Recall that a PID controller has transfer function of the form:

$$D(s) = K \left(1 + \frac{1}{T_I s} + T_D s. \right)$$

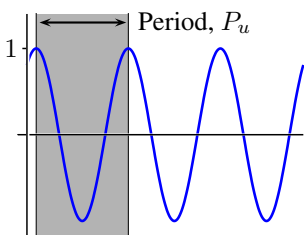
7.3.1 PID Parameter Tuning

Ziegler and Nichols proposed “rules of thumb” for selecting K , T_I , and T_D for a PID controller. You have seen these two methods in ECE4510. Neither method is optimal in any sense—they just provide good performance for a wide variety of plants. In this lab, you will use “Method II” to design a feedback controller for the MagLev system.

METHOD II: To use Ziegler and Nichols’ second tuning method, configure system as



Turn up the gain K_u until system produces oscillations (on stability boundary). The value of gain K_u which produces marginal stability is termed the “ultimate gain.” The PID controller may then be tuned using this value of K_u and the period of oscillations, P_u (as shown below).



RESULTING TUNING RULES:

P	PI	PID
$K = 0.5K_u$	$K = 0.45K_u$	$K = 0.6K_u$
$T_I = \infty$	$T_I = \frac{1}{1.2}P_u$	$T_I = 0.5P_u$
$T_D = 0$	$T_D = 0$	$T_D = \frac{P_u}{8}$

7.3.2 Practical Considerations

The PID controller just investigated has the form of Fig. 7.1(a) when inserted in a control system. A practical difficulty when using this form is that the reference input signal $r(t)$ is differentiated by the KT_Ds term. If we have a rapidly-changing reference input, such as a step input, this will lead to excessive control effort. A practical work-around is to use the circuit of Fig. 7.1(b) instead.

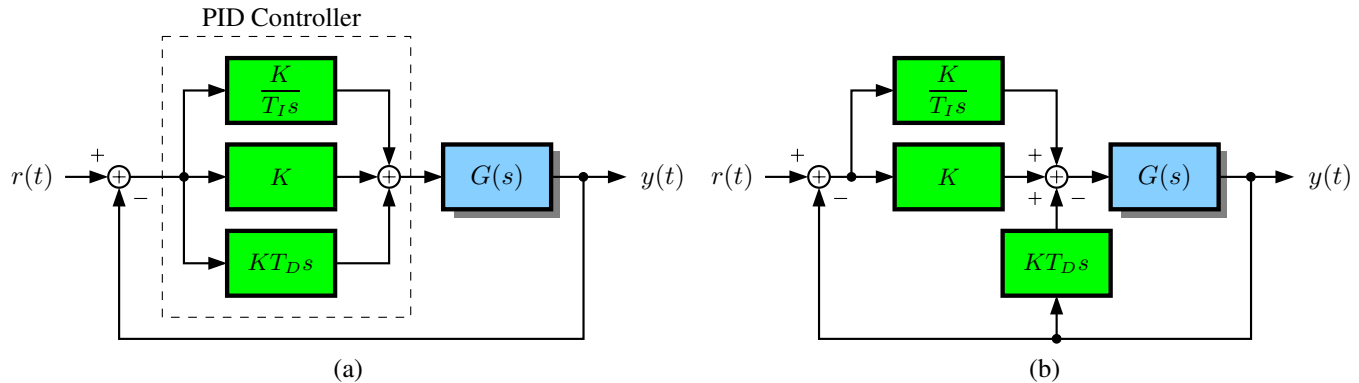


Figure 7.1 PID control; (a) Standard scenario, (b) Modified scenario.

The closed-loop transfer function of this system is slightly different from before, but it has the same poles, and the reference-input is no longer differentiated. The (more-slowly moving) plant output is the only signal being differentiated.

7.4 PRELAB ASSIGNMENT

1. Consider the claim made that the transfer functions of Figs. 7.1(a) and (b) are different, but have the same poles (*i.e.*, the denominators of the two transfer functions are the same). Prove this for a general plant $G(s)$.
2. Recall from Lab 5 (or calculate anew) the value u_{1o} that levitated the magnet to a height of 2.5 cm and record it for use in this lab. Also compute the value $u_{3.5cm}$ for a constant height of 3.5 cm.

7.5 LABORATORY EXPERIMENT

In the lab you will compare open-loop versus closed-loop control using P, PI and PID controllers. You will use four metrics for comparison: Steady-state error, disturbance rejection, sensitivity and transient response. In all cases you will have the system track a reference-input which will be a unit-step at time 5 sec. That is, the output of the magnet is desired to be at height 2.5 cm from 0 sec until 5 sec and then jump to 3.5 cm until the simulation ends.

1. Configure the system as in Fig. 7.2 on page 7-3. Make a Simulink model file as in Fig. 7.3. Use the appropriate “FixSensor” routine from the `utility.mdl` library file to convert y_{1raw} to y_{1cal} .
2. Execute this model file for a simulation time of 20 sec, (starting with a gain of $K_u \approx 0.25$). Use the desired equilibrium height of 2.5 cm. Increase K_u in small increments until critical stability is reached. Record this value of K_u and the period of oscillations P_u . Always be prepared to abort the simulation or to turn off power to the MagLev to avoid damaging the equipment if the system is badly unstable.
3. **Open-loop control:** Create a Simulink block diagram to output u_{1o} for 5 sec and then $u_{3.5cm}$ for the remaining simulation time. This is the open-loop controller. One possible Simulink implementation is shown in Fig. 7.4 on page 7-4.

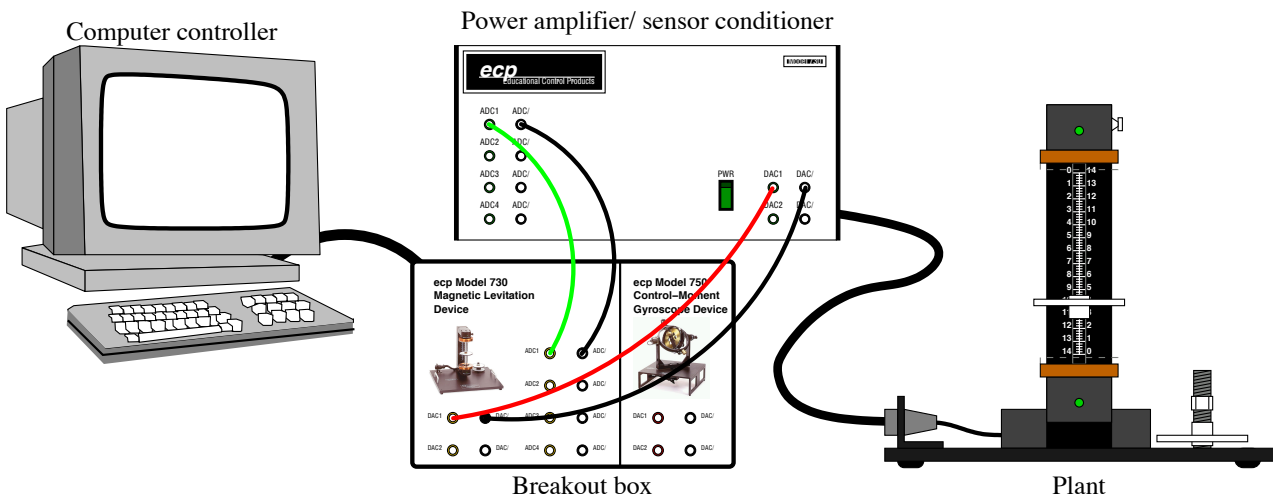


Figure 7.2 System setup for control.

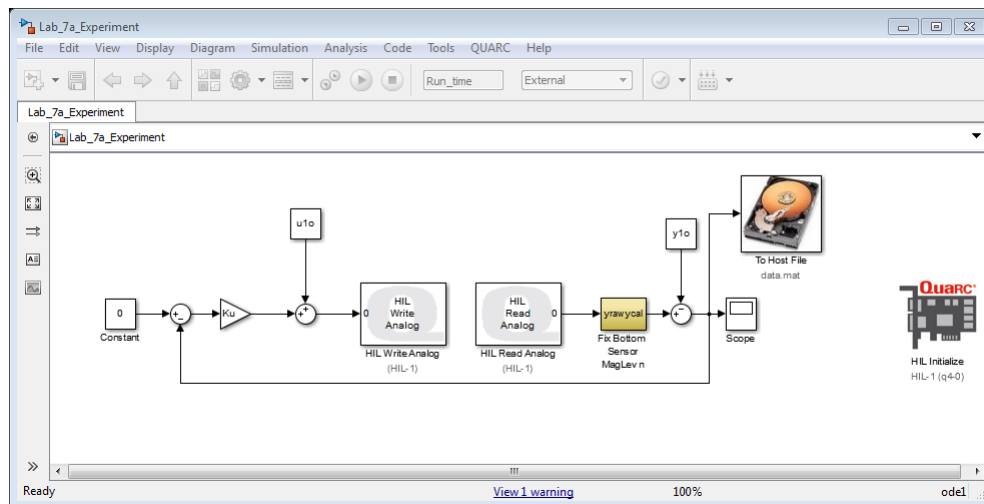


Figure 7.3 Simulink diagram to determine K_u .

4. **PID control:** Create Simulink block diagrams for the P, PI and PID controllers. The feedback will be $y_{1_{cal}}$. One possible Simulink implementation is shown in Fig. 7.5. You will need to change the K , K/T_I and KT_D settings to achieve P, PI, or PID control. For the feedback control system, $r(t)$ will be a unit step starting at 5 sec. Record system response for a total of 20 sec.
5. Simulate and record the response of the MagLev device using the four (open-loop, P, PI, and PID) controllers.
6. Place a small but non-negligible mass on top of the magnet (e.g., six pennies). This will act as a “disturbance force”. Repeat step 5.
7. You will likely find that the P/PI/PID controllers tuned using Ziegler-Nichols Method II will give stable performance, but will otherwise be pretty poor on the MagLev device. For each controller type (P/PI/PID), hand optimize the parameters K , T_I , and T_D , starting from the Ziegler-Nichols values, to achieve the “best” performance you can find. This will be a somewhat subjective exercise, but by best we mean the fastest rise time with the lowest steady-state error and the lowest peak overshoot. In your report be sure to compare your hand-optimized output to the no-brainer Ziegler-Nichols output.

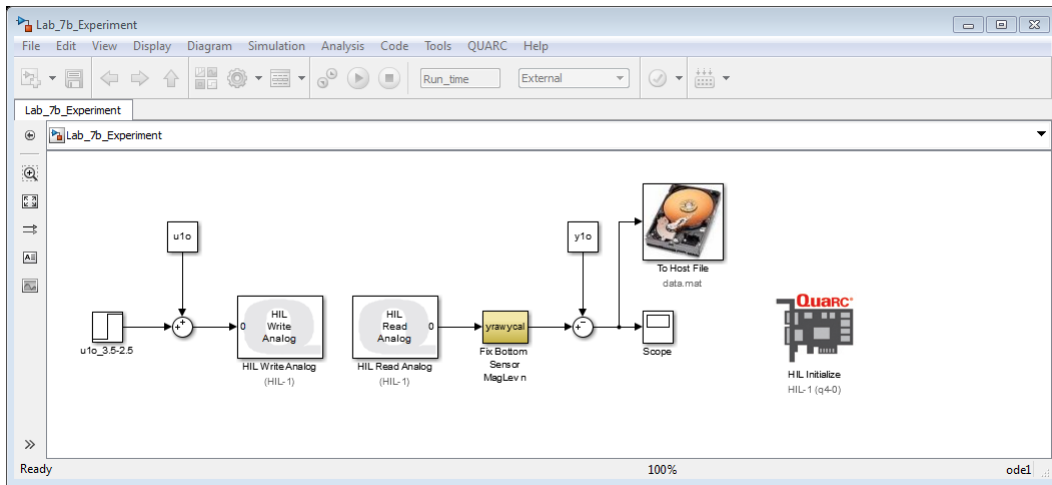


Figure 7.4 Simulink diagram for open-loop control.

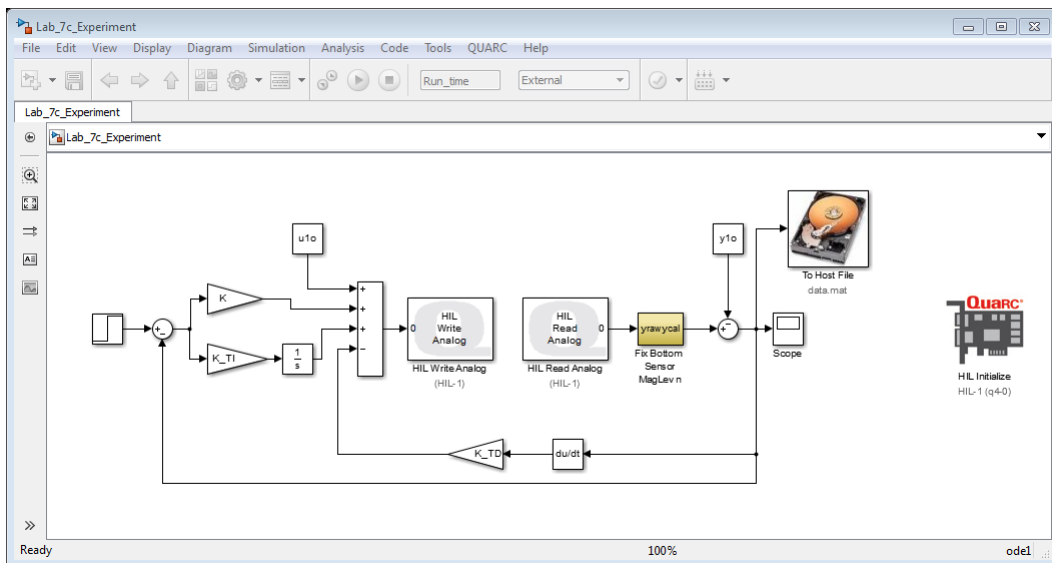


Figure 7.5 Simulink diagram for PID control.

7.6 ASSIGNMENT

In your write-up describe the results you obtained and be sure to compare and contrast the performance of the four different controllers with regards to steady-state errors, disturbance rejection, sensitivity to modeling errors and transient response.

Basic PID Control: Generalization

8.1 INTRODUCTION

Most times you design a controller, your intention is to use it on more than a single device. For example, you might want to make a single control design that works well on all four MagLevs in the lab. (Or, you might want to make a flight controller that works well on all Boeing 777s, and so forth.) There are specific challenges associated with making controllers that work for more than a single device. This is the topic of a field of study called “robust control design.” The basic issue is that you may create the best controller in the world for MagLev 1, but that controller may not work very well for MagLev 2. If you “de-tune” the performance of your controller a little bit on MagLev 1, you may find that it works well on all MagLevs in the lab.

In this laboratory exercise, you will test your control design from the last lab period on a different MagLev.

8.2 THIS LAB IS...

- A quick look at the generalization performance of your PID control design from Lab 7.

8.3 BACKGROUND

The background information is the same as for Lab 7. Please review this material if necessary.

8.4 PRELAB ASSIGNMENT

1. Record the K , T_I , and T_D values that were computed in Lab 7 for the P/PI/PID controllers using the Ziegler-Nichols method. Also, record the hand-optimized values. You will need them again for this lab.

8.5 LABORATORY EXPERIMENT

In this lab, you will work on a different MagLev from your typical one. If you started with MagLev 1, use MagLev 2 for this lab; if you started with MagLev 2, use MagLev 3; if you started with MagLev 3, use MagLev 4; and if you started with MagLev 4, use MagLev 1. Recall that MagLevs 1–3 use the Q4 I/O board while MagLev 4 uses the Q8 I/O board. Therefore, if you will use a different I/O board in this lab from the one you used in Lab 7, you will need to change the corresponding parameter in the “HIL Initialize” block.

The objectives of this lab are essentially the same as for Lab 7. You will compare open-loop versus closed-loop control using P, PI and PID controllers. You will use four metrics for comparison: Steady-state error, disturbance rejection, sensitivity and transient response. In all cases you will have the system track a reference-input which will be a unit-step at time 5 *sec*. That is, the output of the magnet is desired to be at height 2.5 *cm* from 0 *sec* until 5 *sec* and then jump to 3.5 *cm* until the simulation ends.

Note that the sensors and actuators of the new MagLev system (the “native” system) are different from the sensors and actuators of the MagLev system that you used to design your controller last week (the “design” system). Your

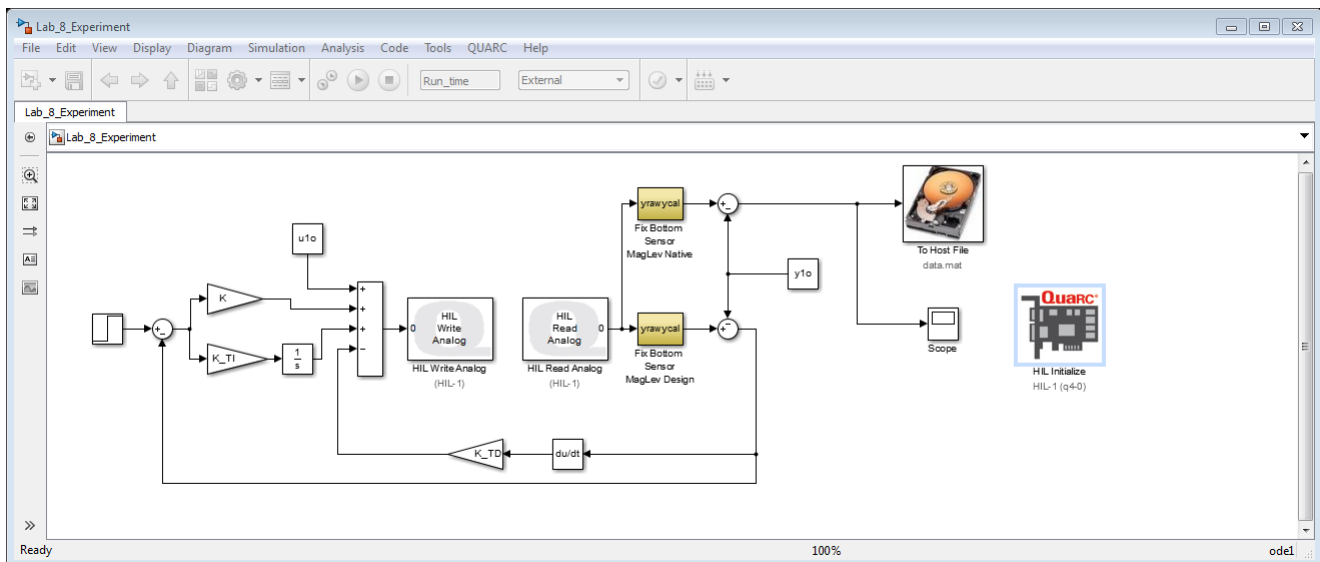


Figure 8.1 Simulink diagram for PID control on a second MagLev.

feedback controller needs to still use the “Fix Sensor” blocks from the design system of Lab 7, but in order to evaluate the controller’s performance, you will need to know the real magnet height on the “native” system. Is this real magnet height what we want? One possible implementation is shown in Fig. 8.1. The feedback path should be $y_{1_{cal}}$ using the “FixSensor” block for the MagLev of your original design. This is labeled “FixSensor Design” in the figure. However, you should plot $y_1 \approx y_{cal}$ using the “FixSensor” block for the MagLev you are using. This is labeled “FixSensor Native” in the figure.

The following steps are basically the same as last week, except implemented on a different MagLev, with the two “FixSensor” blocks as indicated above.

1. **Open-loop control:** Create a Simulink block diagram to output u_{1o} for 5 sec and then $u_{3.5cm}$ for the remaining simulation time. This is the open-loop controller.
2. **PID control:** Create Simulink block diagrams for the P, PI and PID controllers. The feedback will be $y_{1_{cal}}$. You will need to change the K , K/T_I and KT_D settings to achieve P, PI, or PID control. For the feedback control system, $r(t)$ will be a unit step starting at 5 sec. Record system response for a total of 20 sec.
3. Simulate and record the response of the MagLev device using the four (open-loop, P, PI, and PID) controllers, using the default K , T_I , and T_D values produced by Ziegler-Nichols.
4. Place a small but non-negligible mass on top of the magnet (*e.g.*, six pennies). This will act as a “disturbance force”. Repeat step 3.
5. Repeat steps 3–4 using the values of parameters K , T_I , and T_D that you hand-optimized last lab.

8.6 ASSIGNMENT

In your write-up describe the results you obtained and be sure to compare and contrast the performance of the four different controllers with regards to steady-state errors, disturbance rejection, sensitivity to modeling errors and transient response. Compare the performance of your controllers on the “design” system to the performance on the “generalization” system.

Practical Aspects of Stability: Simulation

9.1 INTRODUCTION

A very important aspect of control-system design is to ensure that the resulting system is stable. There is a very precise mathematical description of stability that states that a system is (BIBO) stable if and only if every bounded input produces a bounded output. (By “bounded” we mean that there is a finite limit to the maximum value of the signal.) This definition is useful because it leads to mathematical tests for stability such as the Routh test you are familiar with. However, there are limitations.

For example, consider the MagLev system. Every input (bounded or not) will produce a bounded output since the magnet is constrained to move on the Pyrex rod and has a limited range of motion. Is the system stable? Not necessarily. Consider trying to levitate a magnet by powering the upper electro-magnetic coil and using an attractive magnetic force. If the magnet falls below a certain distance away from the coil, it will not be possible to generate a large enough magnetic field to attract it again, and the system ceases to operate normally. We say that this configuration is unstable.

What the issue boils down to is that *nonlinear* system stability is different than *linear* system stability. The MagLev is a nonlinear system. Fortunately, BIBO stability methods applied to a linearized model of the MagLev gives us a fairly good indication of the actual stability from a practical point of view.

9.2 THIS LAB IS...

- An investigation of stability using the MagLev device.
- Practice designing a feedback stabilizer circuit using the Routh test.
- An introduction to design validation via simulation.

9.3 BACKGROUND

For this lab, the MagLev will be set up with a single magnet on the Pyrex rod, and you will levitate it a constant distance from the *upper* magnetic coil. This is an unstable configuration. If you apply *exactly* the correct voltage to attract the magnet to the upper coil such that it is levitated at the desired height, everything will work correctly until an external disturbance influences the magnet . . . and it plummets to the bottom of the rod! We would like to protect the system for posterity, and so use a plastic clip to limit the distance the magnet can fall. See Fig. 9.1 on page 9-2.

9.3.1 System Model

In order to investigate the stability of the MagLev in this configuration, and to find a stabilizing controller, we must make a mathematical model of the system. In this lab, you will control a single disk in the upper position using the top actuator. From Lab 5 we know that the equation of motion for this configuration is

$$m\ddot{y}_2^* + m\beta\dot{y}_2^* - k_{22}y_2^* = -k_{u22}u_2^*,$$

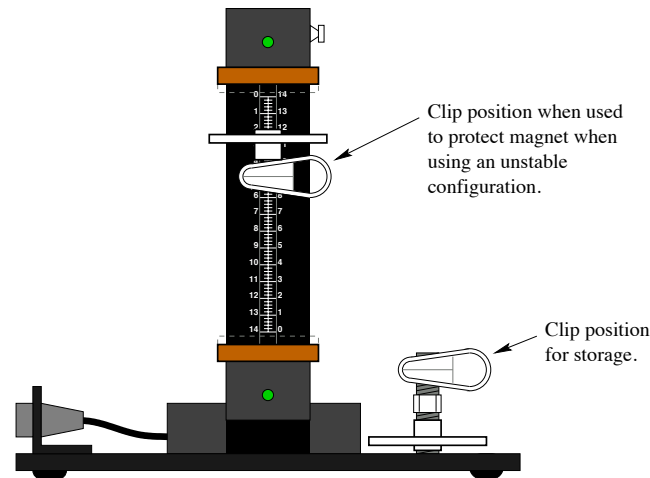


Figure 9.1 MagLev unit showing plastic clip used to protect the device.

where the minus signs differ from what we saw before because the “N” side of the magnet will be facing upward in this lab as opposed to the “S” side that is assumed in the equation in Lab 5. When we take the Laplace transform, we get the transfer function

$$G(s) = \frac{Y_2(s)}{U_2(s)} = \frac{-k_{u22}}{ms^2 + m\beta s - k_{22}},$$

where

$$k_{u22} = \frac{-1}{a(-y_{2o} + b)^4} \quad \text{and} \quad k_{22} = \frac{4u_{2o}}{a(-y_{2o} + b)^5}.$$

We can determine the dc value u_{2o} to be $u_{2o} = mga(-y_{2o} + b)^4$. Note that by the Routh test, “case 0,” we immediately see that $G(s)$ is unstable.

9.3.2 “Desktop Validation” of Controller Design

Next week, you will implement your controller on the unstable MagLev configuration and demonstrate that it is able to stabilize the disk in the upper configuration. However, it is always wise when trying a new control design to first see how it works on a simulated version of the plant, rather than the real plant. That way, if you are wrong, no damage is done, and changing your design is generally faster as well. This is especially true when attempting to control an unstable system.

We would like the simulation of the controller combined with the plant to be as accurate as possible. We could put our controller $D(s)$ in a feedback loop with our linear plant model $G(s)$, and that is a good thing to try. However, back in Labs 3 and 4, we made a very good nonlinear model of the plant. While this nonlinear model is not very helpful for control design, it is very helpful for controller validation. It is the best we can do without using the real plant, since it is the best model that we have of the true plant. Note that since this design validation approach can be done “on the desktop” via simulation, it is often called a “desktop validation” approach.

The nonlinear model that was developed in Labs 3 and 4 can be converted into Simulink diagrams. Figure 9.2 on page 9–3 shows an example diagram that models the single disk MagLev 1, with the disk in the lower position. Figure 9.3 on page 9–3 shows an example diagram that models the single disk MagLev 1, with the disk in the upper position. Both diagrams have similarities. The control voltage input is on the left. This voltage is limited to be in the range of $\pm 10 V$ by a saturation block. The actuator dynamics are modeled next—either $F_{u_{11}}$ or $F_{u_{22}}$, depending on whether we are modeling the top or bottom disk. Note that the actuator dynamics are a function of control voltage as well as disk position, so the MagLev dynamics have internal feedback. Per the linear part of the relationships, we then compute $\ddot{y}_1(t)$ or $\ddot{y}_2(t)$ as a combination of the applied force, gravity, and a friction feedback term. These accelerations are then integrated twice to get position. The position is limited to be between 0 and 13 cm for the

lower disk and between -13 and 0 cm for the upper disk. The actual height of the magnet is one of the outputs of the model. The second output, the sensor voltage reading, is computed by table lookup using the values measured in Lab 3.

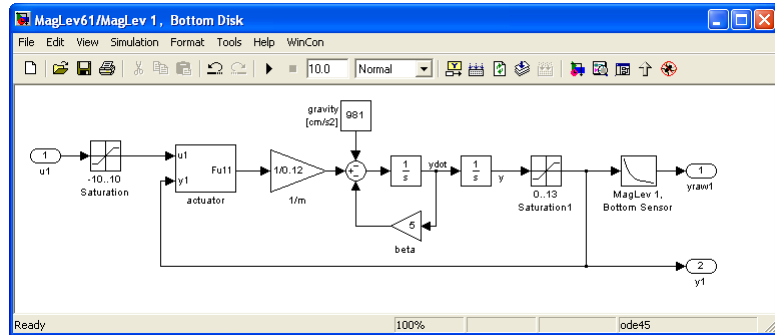


Figure 9.2 Simulink nonlinear model of single disk in lower configuration.

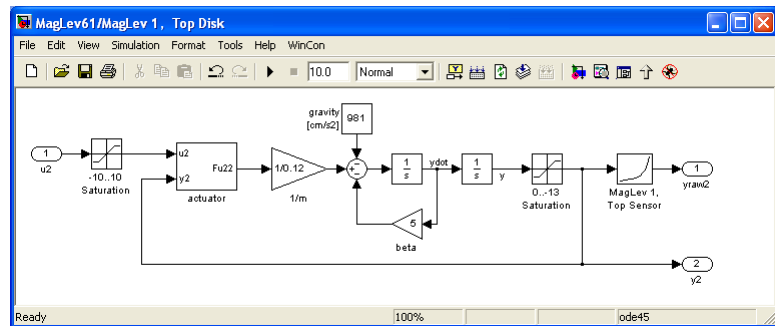


Figure 9.3 Simulink nonlinear model of single disk in upper configuration.

To perform design verification using these models, first select which model applies: lower disk or upper disk. Then, the D2A and A2D blocks in any diagram are replaced by this Simulink model.

9.4 PRELAB ASSIGNMENT

1. Determine $G(s)$ for an operating point of $y_{2o} = -2.5\text{ cm}$; fill in all numeric values. As always, be careful with units. Determine u_{2o} for the same operating point.
2. Verify, using the Routh test, that $G(s)$ is open-loop unstable.
3. Assume that we will configure the MagLev in a feedback loop with a lead controller $D(s) = k\frac{s+a}{s+b}$ where $0 < a < b$. See Fig. 9.4. Determine the closed-loop transfer function of the system.
4. For $k = 10$ and $b = 100$, what range of a results in a stable system (Use the Routh test)?
5. For $a = 10$ and $b = 100$, what range of k results in a stable system (Use the Routh test)?

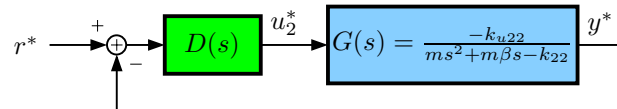


Figure 9.4 Configuration for analyzing system performance.

9.5 LABORATORY EXPERIMENT

This experiment uses the lab computer and Simulink to implement a lead controller to stabilize the MagLev in simulation. Next week, the same steps will be performed with the actual MagLev hardware. The setup (for the hardware case) is drawn in Fig. 9.5.

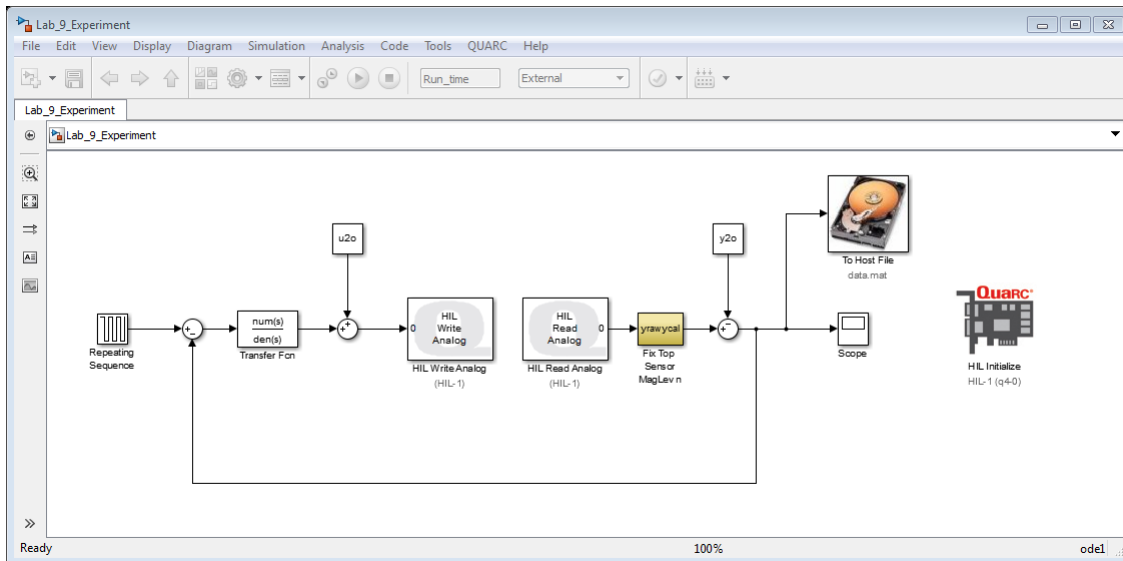


Figure 9.5 Configuration for implementing control.

1. In Simulink, create a feedback control system which uses a lead controller with transfer function $D(s) = 15 \frac{s+10}{s+100}$. See Fig. 9.5, but replace the D2A and A2D blocks with an appropriate nonlinear Simulink MagLev model, which will be made available to you.
2. You will start the magnet at the $y_2(0) = -2 \text{ cm}$ position. This is accomplished in Simulink by double-clicking on the rightmost integrator in the nonlinear model and setting its initial output to -2 .
3. Start the program operating. What happens to the output? Is it stable?
4. Experiment with the “ a ” term in the lead controller. Use different values for different program runs. That is, implement $D(s) = 10 \frac{s+a}{s+100}$. Record the range of this variable that seems to give stable response.
5. Experiment with the “ k ” term in the lead controller. That is, implement $D(s) = k \frac{s+15}{s+100}$. Record the range of this variable that seems to give stable response.
6. Use $D(s) = 15 \frac{s+10}{s+100}$ and execute a repeating bidirectional step move. See Fig. 9.6 on page 9-5. The bidirectional step may be created using the “Repeating Sequence” source block with time vector $[0 \ 2 \ 2.002 \ 4]$ and output vector $[0.1 \ 0.1 \ -0.1 \ -0.1]$. Run the output of the sequence through an amplifier so that you can vary the amplitude of the square wave easily. For amplitudes $\{0, 0.1, 0.2, 0.3, 0.4\}$ plot y_{cal}^* . Does the magnitude of the reference command $r(t)$ affect the stability of the output? Should it? Explain the shape of the output curves.

9.6 ASSIGNMENT

Include all derived stability results in your report. Compare theoretical versus observed results regarding stability. Comment on system response to the bidirectional step moves for various amplitude inputs.

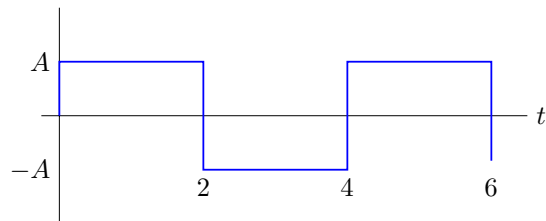


Figure 9.6 Bidirectional step move of amplitude A (repeats periodically).

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Practical Aspects of Stability: Implementation

10.1 INTRODUCTION

This lab is an extension of last week's lab. We continue to investigate the ability to stabilize an unstable MagLev configuration. Having already verified our design in software, we move to "hardware-in-the-loop" (HIL) validation.

10.2 THIS LAB IS...

- A continuing investigation of stability using the MagLev device.
- An introduction to design validation via hardware-in-the-loop (HIL) testing.

10.3 BACKGROUND

The background for this lab is the same as for Lab 9. Please refer to the writeup there with any questions. As a reminder, for this lab, the MagLev will be set up with a single magnet on the Pyrex rod, and you will levitate it a constant distance from the *upper* magnetic coil. We would like to protect the system for posterity, and so use a plastic clip to limit the distance the magnet can fall. See Fig. 10.1.

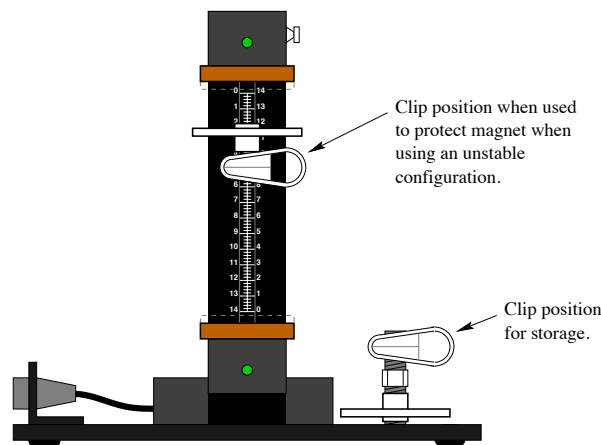


Figure 10.1 MagLev unit showing plastic clip used to protect the device.

10.4 PRELAB ASSIGNMENT

1. What differences do you expect to see between the desktop validation performed last week and the hardware-in-the-loop validation you will perform this week?

10.5 LABORATORY EXPERIMENT

This experiment uses the lab computer and Simulink to implement a lead controller to stabilize the MagLev. The setup is drawn in Fig. 10.2.

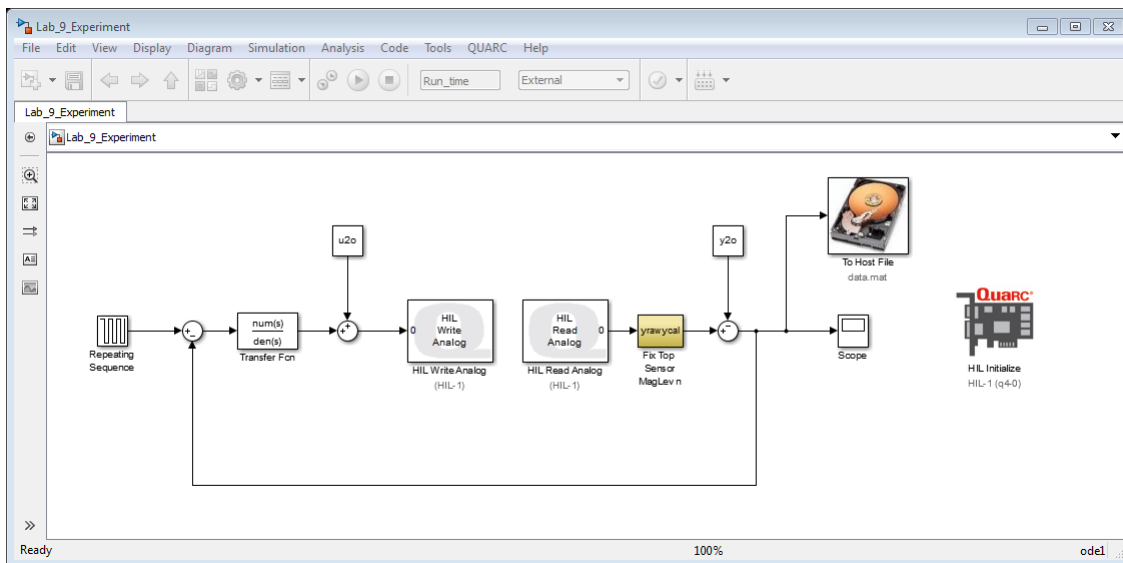


Figure 10.2 Configuration for implementing control.

1. In Simulink, create a feedback control system which uses a lead controller with transfer function $D(s) = 15 \frac{s+10}{s+100}$. See Fig. 10.2.
2. Set the plastic clip on the glass rod so that the magnet cannot fall below 5 cm from the upper mechanical stop. While holding the magnet at the approximate $y = -2 \text{ cm}$ position, start the program operating. Gently move the magnet up and down by touching only the edges of the magnet. Can you feel the opposing force supplied by your controller?
3. Experiment with the “ a ” term in the lead controller. Use different values for different program runs. That is, implement $D(s) = 10 \frac{s+a}{s+100}$. Record the range of this variable that seems to give stable response. Spin the magnet gently to release friction.
4. Experiment with the “ k ” term in the lead controller. That is, implement $D(s) = k \frac{s+15}{s+100}$. Record the range of this variable that seems to give stable response. Spin the magnet gently to release friction.
5. Use $D(s) = 15 \frac{s+10}{s+100}$ and execute a repeating bidirectional step move. See Fig. 10.3. The bidirectional step may be created using the “Repeating Sequence” source block with time vector $[0 \ 2 \ 2.002 \ 4]$ and output vector $[1 \ 1 \ -1 \ -1]$. Run the output of the sequence through an amplifier so that you can vary the amplitude of the square wave easily. For amplitudes $\{0, 0.1, 0.2, 0.3, 0.4\}$ plot y_{cal}^* . Does the magnitude of the reference command $r(t)$ affect the stability of the output? Should it? Explain the shape of the output curves.

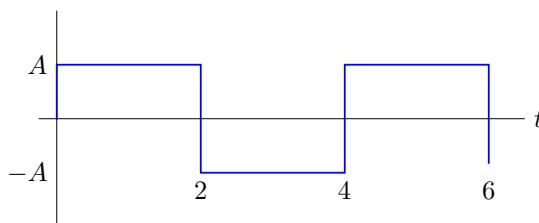


Figure 10.3 Bidirectional step move of amplitude A (repeats periodically).

10.6 ASSIGNMENT

Include all derived stability results in your report. Compare theoretical versus observed results regarding stability. Comment on system response to the bidirectional step moves for various amplitude inputs. Comment on the similarities and differences between your desktop validation results and hardware-in-the-loop validation results.

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Root-Locus Analysis and Informal Design: Simulation

11.1 INTRODUCTION

By now, you have created a number of models of the MagLev system. These models are used to analyze system behavior and to design control systems which use feedback. One powerful tool we can use is the root locus method, which tells us all possible closed-loop pole locations given a certain $D(s)$ and $G(s)$ and a variable gain K (see Fig. 11.1).

11.2 THIS LAB IS...

- An informal design approach using root-locus methods

11.3 BACKGROUND

Control systems are often configured as shown in Fig. 11.1. The plant is $G(s)$, the dynamics of the controller are $D(s)$ and the controller gain is K . The control design objective is to choose $D(s)$ and K in order that the closed-loop system has a certain desired response.

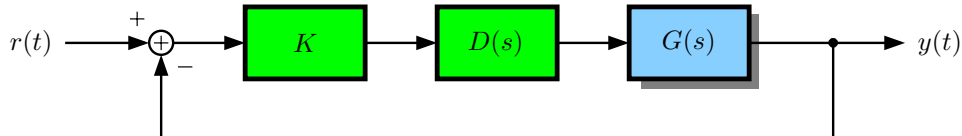


Figure 11.1 Generic feedback system.

If $D(s)$ and $G(s)$ are fixed, the root locus method can be used to determine all possible closed-loop pole locations of the system if K is varied $0 \leq K < \infty$.

11.4 PRELAB ASSIGNMENT

1. First, consider your model of the lower coil of the MagLev with $y_{1_{cal}}$ output, in an open-loop stable configuration.
 - (a) Investigate Matlab's `rlocus()` command. `rlocus()` uses the *open-loop* transfer function of the system to plot the locus of the roots if the loop is closed and the gain K is varied, $0 \leq K < \infty$. Plot the root locus of the system for $D(s) = 1$. Comment on the stability of the closed-loop system.
 - (b) Recall that as the feedback gain K is increased, $K \rightarrow \infty$, the closed-loop poles of the 180° root locus migrate toward the open-loop zeros. Use this knowledge to design a controller $D(s) = s + a$ to make the system stable and to achieve a damping ratio between 0.7 and 0.8. State the controller $D(s)$ you made, and the feedback gain K used. Plot the root locus and step-response of your compensated system. (See the Matlab `rlocfind`, `cloop`, `feedback`, and `step` commands).

- (c) A physically realizable (and reasonable) controller has at least as many poles as it has zeros. Redesign your controller to be realizable and meet the specs of part (b). Again, plot the root locus, state the value of gain K used, show the compensator you use, and plot the closed-loop step response. ***You should be able to meet specs with $D(s)$ a lead controller.***
2. Secondly, consider your model of the upper coil of the MagLev with $y_{2_{cal}}$ output, in an open-loop unstable configuration (model from Lab 9). Repeat steps (a) and (c), above.
 3. Add an integrator to the feedback path to eliminate steady-state errors. Can you still achieve the desired specifications with a lead controller? Come up with a $D(s)$ that stabilizes both single-disk systems (i.e., upper and lower configurations) that also has an integrator in the feedback loop.

11.5 LABORATORY EXPERIMENT

For this lab, you will again test your control designs first in simulation: a desktop validation approach. Next lab, you will implement your designs in hardware.

1. Control of single disk, lower position using lower actuator (open-loop stable system): Record the step response of your controlled system. Did you meet spec? If not, redesign and repeat.
2. Control of single disk, upper position using the upper actuator (open-loop unstable system): Record the step response of your controlled system. Did you meet spec? If not, redesign and repeat.

11.6 ASSIGNMENT

In your writeup, be sure to include all results and reasoning from the Prelab section. Also include the step responses of the MagLev as recorded from desktop validation. Do the experimental and predicted step responses agree? Comment on any disagreement.

Root-Locus Analysis and Informal Design: Implementation

12.1 INTRODUCTION

This is a continuation of your design from the prior lab: you will now implement your controllers on the MagLev hardware and test them.

12.2 THIS LAB IS...

- Hardware validation of your root-locus inspired designs

12.3 BACKGROUND

The background for this lab is the same as for Lab 11. Please refer back to that lab with any questions.

12.4 PRELAB ASSIGNMENT

1. How do you think your control designs might behave differently on the real hardware from your desktop validation results? What steps might you need to take to change your controllers to work on the hardware?

12.5 LABORATORY EXPERIMENT

This week, you implement your designs in hardware.

1. Control of single disk, lower position using lower actuator (open-loop stable system): Record the step response of your controlled system. Did you meet spec? If not, redesign and repeat.
2. Control of single disk, upper position using the upper actuator (open-loop unstable system): Record the step response of your controlled system. Did you meet spec? If not, redesign and repeat. Use the plastic clip under the magnet to keep it from falling below a height of approximately -4 cm .

12.6 ASSIGNMENT

In your writeup, be sure to include all results and reasoning from the Prelab section. Also include the step responses of the MagLev as recorded experimentally. Do the experimental and predicted step responses agree? Comment on any disagreement. Compare desktop validation systems with hardware performance.

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Feedback Linearization and Pole-Placement Design: Lower Disk

13.1 INTRODUCTION

In Labs 9 through 12 you designed and implemented controllers for the MagLev system using calibrated sensor output and linearized system models found by Taylor-series expansion in Lab 5. These models were only approximate representations of the true nonlinear dynamics of the MagLev. This sometimes gives us unrealistic expectations as to how these controllers might work in practice, as you have experienced by now. In this lab you will explore a second linearization method which does not make *approximate* models of nonlinear dynamics, but explicitly *cancels* all nonlinear dynamics. Root-locus pole-placement design will be used to control the system.

13.2 THIS LAB IS...

- An introduction to feedback linearization.
- Practice with pole placement using root-locus methods.

13.3 BACKGROUND

13.3.1 Feedback linearization

The small-signal linearization method encountered in Lab 5 (and used subsequently) creates approximate linear models of a nonlinear system operating around a specific equilibrium point. Another approach is called *feedback linearization* and is valid for any operating point. The method is shown in Fig. 13.1. The gravity, sensor and actuator nonlinearities are exactly canceled, leaving only the linear part of the system. This is the most accurate linearization method but also the most computationally demanding in a real-time application.

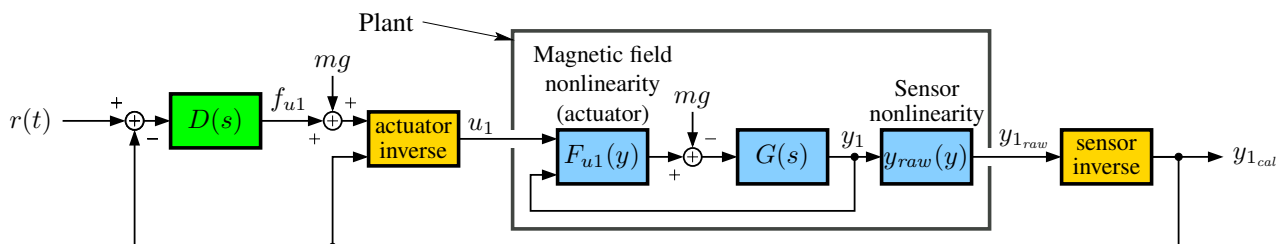


Figure 13.1 Another method for compensating nonlinearities.

The sensor nonlinearity is canceled as before (*i.e.*, as in method 1). An estimate y_{k_cal} of the true magnet position y_k is made using measurements y_{k_raw} and the relationship

$$y_k \approx y_{k_cal} = \frac{e_k}{y_{k_raw}} + \frac{f_k}{\sqrt{y_{k_raw}}} + g_k + h_k \cdot y_{k_raw}, \quad k = 1, 2.$$

This computation may be done as a subsystem in Simulink, as you have already seen. It is represented in Fig. 13.1 on page 13–1 as the “sensor inverse” block.

The gravity nonlinearity may also be canceled. We assume that the output of the controller $D(s)$ is an upward force f_{u1} (in Newtons), to which we must add the constant force mg to cancel the downward gravity force on the magnet. Note that mg is not a function of magnet height, as u_{1o} was in previous methods.

Finally, the actuator nonlinearity is canceled. For example, we know that the upward force provided by the bottom actuator is

$$F_{u_{11}} = \frac{u_1}{a(y_1 + b)^4}.$$

In order to compute u_1 to produce a desired force $f_{1_{des}}$ we re-arrange the equation (and assume that $y_{1_{cal}} \approx y_1$) to get

$$u_1 = f_{1_{des}} a(y_{1_{cal}} + b)^4. \quad (13.1)$$

This equation may be implemented as a Simulink block. It is represented in Fig. 13.1 on page 13–1 as the “actuator inverse” block. Notice that in order to compute u_1 we need knowledge of $y_{1_{cal}}$. This forms a feedback loop—hence the name “feedback linearization.” The equivalent equation for the top disk is $u_2 = f_{2_{des}} a(-y_{2_{cal}} + b)^4$.

Considering for now the dynamics of a single-disk, lower position, lower actuator system, we achieve the equations of motion:

$$\begin{aligned} m\ddot{y}_1 + m\beta\dot{y}_1 - F_{u_{11}} + mg &= 0 \\ m\ddot{y}_1 + m\beta\dot{y}_1 - \frac{u_1}{a(y_1 + b)^4} + mg &= 0. \end{aligned}$$

Let the output of the controller be f_{u1} , and let u_1 be calculated using Eq. (13.1) where $f_{1_{des}} = f_{u1} + mg$. Then,

$$m\ddot{y}_1 + m\beta\dot{y}_1 - (f_{u1} + mg) \frac{a(y_{1_{cal}} + b)^4}{a(y_1 + b)^4} + mg = 0.$$

Assuming that $y_1 \approx y_{1_{cal}}$ we get

$$m\ddot{y}_1 + m\beta\dot{y}_1 = f_{u1}$$

or

$$\frac{Y_1(s)}{F_{u1}(s)} = \frac{1}{ms^2 + m\beta s}.$$

When the gravity, sensor and actuator nonlinearities are canceled we achieve the equivalent block diagram in Fig. 13.2, suitable for designing a controller.

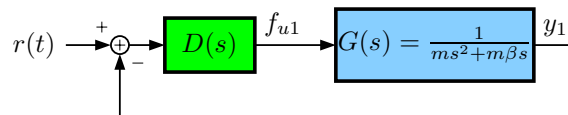


Figure 13.2 Third linearized system for design.

13.4 PRELAB ASSIGNMENT

Consider the MagLev configured as a single-disk system (lower position). You will use feedback linearization around the MagLev to determine the equivalent plant.

- Determine $G_1(s) = \frac{1}{ms^2 + m\beta s}$; that is, fill in the numbers. Be careful to use compatible units! Does $G_1(s)$ vary from one MagLev to another?
- Make a Simulink subsystem to cancel the lower actuator nonlinearity; that is, to implement Eq. (13.1).

- If $D_1(s) = 1$ consider the open-loop transfer function $KG_1(s)$. What can we say about the closed-loop system's steady-state response, assuming that the system is stable and nonlinearities are perfectly canceled?
- Select desired pole locations s_0 and s_0^* such that the step response of the closed-loop system has settling time faster than 1 second, overshoot of less than 25% and rise time of less than 0.2 seconds. Design a lead controller

$$D_1(s) = \frac{a_1s + a_0}{b_1s + 1}$$

using root-locus methods to place the poles of the feedback-linearized MagLev system at these locations. Start with an arbitrary guess for a_0 .

- Plot the root locus of $D_1(s)G_1(s)$ to ensure that your compensator has placed all poles in the left-half s -plane (stable). If your system is unstable, try reducing a_0 and re-design your lead compensator.

13.5 LABORATORY EXPERIMENT

1. Implement your feedback control system for the lower disk in Simulink. See Fig. 13.3. You will create the “Subsystem to fix Actuator” block to implement Eq. (13.1). Despite your earlier prediction, steady-state error will be a problem since nonlinearities will not be *perfectly* canceled. Therefore, you will need a lag network of the form

$$D_3(s) = \frac{s + 2}{s + 0.001}$$

and the entire controller will be $D(s) = D_1(s)D_3(s)$. (Does the presence of the lag network change the root locus much?)

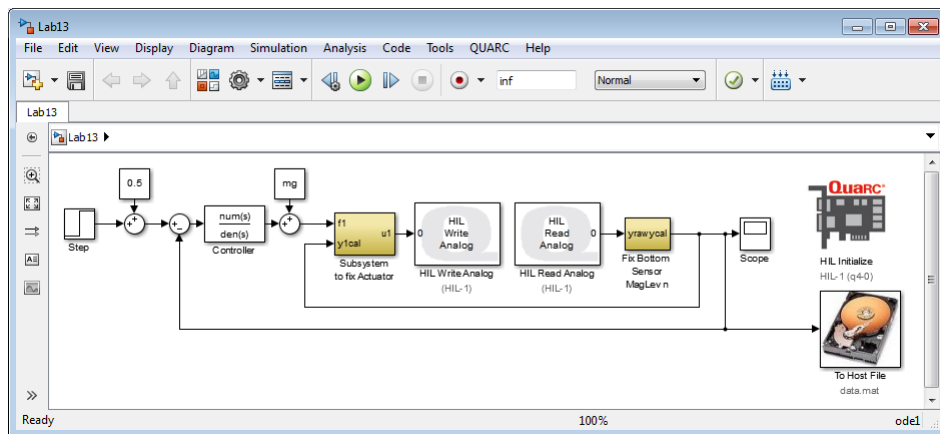


Figure 13.3 Block diagram to implement feedback-linearized control on the lower disk.

2. Control the lower disk. For a reference input equal to 0.5 plus a unit step at time 5 sec, record your system output. Is the system stable? If not, redesign your controller.
 - (a) Implement your control design using the desktop validation approach first;
 - (b) Secondly, implement your design on the actual hardware.
3. Repeat for reference input signals equal to 0.5 plus step signals of magnitude 1.5, 2.0, 2.5 and 3.0. Does the amplitude of the step affect response?

13.6 ASSIGNMENT

Report your controller designs, and all Prelab information. Discuss the transient and steady-state response of your closed-loop systems. Did you meet specifications? Discuss all observations made.

Feedback Linearization and Pole Placement Design: Upper Disk

14.1 INTRODUCTION

This final lab is a continuation of Lab 13, where you now use feedback linearization to control a single disk in the upper (unstable) configuration.

14.2 THIS LAB IS...

- A continuing look at feedback linearization.
- More practice with pole placement using root-locus methods, specifically for an unstable plant.

14.3 BACKGROUND

The background information for this lab is the same as for Lab 13. Please refer back to that discussion as necessary.

14.4 PRELAB ASSIGNMENT

Consider the MagLev configured as a single-disk system, upper position, upper actuator. Again, you will use feedback linearization around the MagLev to determine the equivalent plant.

- Determine $G_2(s) = Y_2(s)/U_2(s)$.
- Make a Simulink subsystem to cancel the upper actuator nonlinearity. Make a Simulink subsystem to cancel the upper sensor nonlinearity.
- Select desired pole locations s_0 and s_0^* such that the step response of the closed-loop system has settling time faster than 1 second, overshoot of less than 25% and rise time of less than 0.2 seconds. Design a lead controller

$$D_2(s) = \frac{a_1s + a_0}{b_1s + 1}$$

using root-locus methods to place the poles of the feedback-linearized MagLev system at these locations. The a_1 , a_0 and b_1 coefficients will be different from those in $D_1(s)$. Start with an arbitrary guess for a_0 (You might need to consider negative values!).

- Plot the root locus of $D_2(s)G_2(s)$ to ensure that your compensator has placed all poles in the left-half s -plane (stable). If your system is unstable, try reducing the magnitude of a_0 and re-design your lead compensator.

14.5 LABORATORY EXPERIMENT

1. Move the plastic clip to about the -4 cm position. Allow the upper magnet to drop to this height. Implement your feedback control system for the upper disk in Simulink. Again, steady-state error will be a problem. Therefore, you will need a lag network and the entire controller will be $D(s) = D_2(s)D_3(s)$.
2. For a reference input of -0.5 minus a unit step at time 5 sec , record your system output. Is the system stable? If not, redesign your controller.
 - (a) Implement your control design using the desktop validation approach first;
 - (b) Secondly, implement your design on the actual hardware.
3. Repeat for reference input signals of -0.5 minus step signals of magnitude 1.5, 2.0, 2.5, and 3.0. Does the amplitude of the step affect response?

14.6 ASSIGNMENT

Report your two controller designs, and all Prelab information. Discuss the transient and steady-state response of your closed-loop systems. Did you meet specifications? Discuss all observations made. Be sure to compare the performance of the lower-disk controller versus the upper-disk controller, and compare performance of the desktop validation system to the actual implementation.

App. A: Magnetic Levitation Principles

This appendix gives a brief overview of key concepts in magnetism in general and magnetic levitation in particular. It includes equations and illustrations that show how magnetic fields and forces are generated within the Model 730 apparatus and concludes with a reference section. For a more rigorous treatment of these topics, the reader is referred to the literature listed at the end of the appendix.

A.1 Introduction

Magnetic fields are used to describe forces at a distance from electric currents. These currents are of two types:

1. Free, or Amperian, currents as drawn from a battery pack, power supply, or an electrical outlet, and
2. Bound currents as in permanent magnet materials.

The forces come in three variations:

1. An electrical current feels a force from another current,
2. A current feels a force from a permanent magnet, and
3. A permanent magnet feels a force from another permanent magnet.

This action at a distance is described by saying a magnetic field exists created by one of the bodies at the location of the other body. The magnetic field is the medium by which the force is transferred.

In this section, a brief discussion concerning the magnetic fields caused by magnetized materials (*i.e.*, permanent magnets) is presented. By demonstrating that magnetic materials can be reduced to effective current distributions, this discussion forms the basis for calculating the forces on permanent magnets. The magnetic fields due to free current distributions are calculated next. These fields are used to calculate the forces felt by current-carrying conductors. Time-varying currents cause time-varying magnetic fields. These changing magnetic fields induce electric currents that, in turn, experience a force.

Maglev systems utilize the fundamental physics of electric currents experiencing forces-at-a-distance. These systems are most often described in terms of the interaction of electrical current with magnetic fields. Because the masses of the vehicles are large, large forces are required for magnetic suspension. These large forces are provided by the high magnetic fields of either large superconducting currents or small air gaps in normal ferromagnetic circuits.

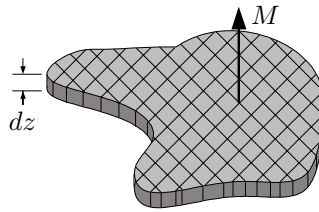
A dictionary of common terms is provided at the end of the section and a reference list is included featuring books, Maglev studies and Maglev URLs.

A.2 Magnetic Fields & Forces

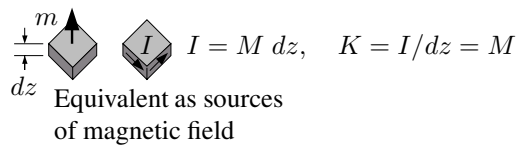
A.2.1 Magnetic Fields Caused by Magnetized Materials

Electron spin is a quantum mechanical phenomenon. Its significance here is the fact that there is associated with the electron spin a magnetic moment of fixed magnitude. To determine the forces on magnetic materials, we use the fact that the magnetic moment of the electron spin *acts as if* it is a current loop.

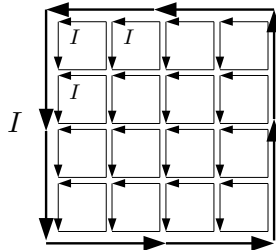
A volume of magnetized material contains a very large number of aligned electron spins. This is illustrated schematically in the figure below. A grid pattern was superposed onto the material to indicate small volumes of materials which can be analyzed discretely.



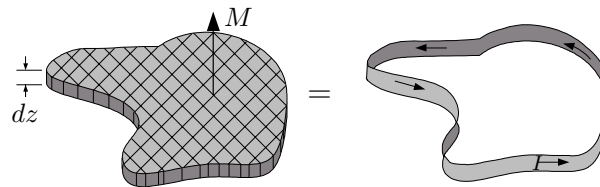
The figure below isolates two small volumes. The one on the left generates a magnetic field due to its magnetic moment, m . The one on the right generates a magnetic field by virtue of the current wrapping the volume (this is commonly referred to as a “current sheet.”), and otherwise ignores the presence of the magnetic material. **The two magnetic fields are entirely equivalent** (external to the material). The material property M describes the strength of the magnetic material and the amount of current required per unit distance of height. The parameter K describes the current per unit height within the current sheet. For modern high performance neodymium-iron-boron permanent magnets, $K \approx 900,000$ amps/meter.



Assembling many small volumes in juxtaposition, one can see in the figure below the result of substituting the current loop for the magnetic material; the internal currents cancel while the surface currents do not cancel. Hence, no matter the shape of the material, the external magnetic field can be exactly reproduced by a current stripe along the material perimeter. (This is true if the magnetization is uniform. If the magnetization is uniform then the currents are equal and cancel. If the magnetization is not uniform, then the equivalent current distribution can be calculated by $J = \nabla \times M$, where J is the volume current density.)



In summary, the magnetic field of a uniformly magnetized permanent magnet can be exactly reproduced by a current sheet along the perimeter. This equality is indicated graphically in the figure below. The magnitude of the current is proportional to the material thickness with the constant of proportionality dependent upon the material itself. This equivalence of magnetic fields is commonly exploited to calculate the forces on permanent magnet materials.



A.2.2 Calculating Magnetic Fields

The Biot-Savart Law is the fundamental relationship between current and magnetic field:

$$d\vec{B} = \frac{\mu}{4\pi} \frac{I d\vec{l} \times \vec{r}}{r^3}, \quad (\text{A.1})$$

where,

$d\vec{B}$ = differential magnetic field, tesla,

μ = permeability, Henry/m,

I = current, amps,

$d\vec{l}$ = differential length of current-carrying element, m,

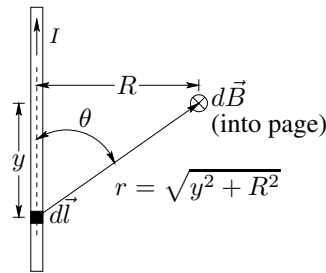
\vec{r} = vector distance from current element to field point, m.

Two simple geometries for calculating the magnetic field are an infinitely long, straight conductor and a circular loop of conductor.

Straight Conductor

For the straight conductor carrying current upwards along the y-direction, the magnetic field can be calculated at any point due to a differential current element as:

$$d\vec{B} = \frac{\mu}{4\pi} \frac{I d\vec{l} \sin \theta}{r^2}, \quad (\text{A.2})$$



By integrating along y from $-\infty$ to ∞ , the result is:

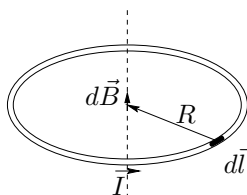
$$B = \frac{\mu I}{2\pi R},$$

where the direction of the magnetic field is determined by the right hand rule (point the thumb of the right hand along the direction of current and the fingers curl in the direction of the magnetic field).

Circular Loop at Center

The magnetic field at the center of a circular loop of wire can also be calculated from the Biot-Savart Law. In this case, the angle between the current element and field point is a constant (90°) so the vector cross product always yields unity. Then the magnetic field is:

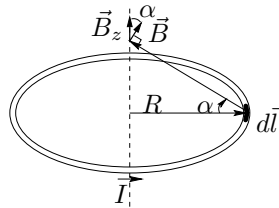
$$d\vec{B} = \frac{\mu}{4\pi} \frac{I 2\pi R}{R^2} = \frac{\mu I}{2R}. \quad (\text{A.3})$$



Circular Loop Anywhere Along Axis

The magnetic field along the axis of a circular loop of wire can also be calculated from the Biot-Savart Law. In this case, the angle between the current element and field point is still a constant (90°) so the vector cross product always yields unity. The net magnetic field from any current element has vertical and horizontal components. However, as the current element follows the conductor path, the horizontal components of the field cancel while the vertical components add. Then the axial magnetic field is:

$$B_z = \frac{\mu}{4\pi} \frac{I 2\pi R}{z^2 + R^2} \cos \alpha = \frac{\mu I R}{2(z^2 + R^2)} \frac{R}{\sqrt{z^2 + R^2}} = \frac{\mu I R^2}{2(z^2 + R^2)^{3/2}}. \quad (\text{A.4})$$



Due to the common occurrence of circular coils, this relationship is very important. Along the axis the magnetic field is purely vertical. For values of height, z , above the plane of the coil, which are small compared to the radius, R , ($z \ll R$) the vertical magnetic field is insensitive to z . This is due to the finite radius of the coil. For heights z much larger than R ($z \gg R$) the axial field decreases as the reciprocal third power of height. Off-axis, the radial magnetic field decreases as the reciprocal fourth power of height.

Calculating Magnetic Forces

The force between current carrying conductors is given by the Lorentz Law:

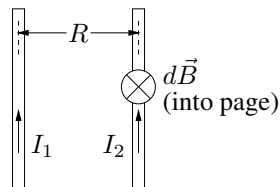
$$d\vec{F} = I d\vec{l} \times \vec{B}, \quad (\text{A.5})$$

where, $d\vec{F}$ = differential force, in Newtons.

One simple geometry for calculating the magnetic force per unit length is two infinitely long, straight conductors parallel to each other.

$$\frac{F}{l} = -\frac{\mu I_1 I_2}{2\pi R}.$$

The negative sign means the two parallel currents attract one another. If the direction of one of the currents were to be reversed (anti-parallel currents), the force would also reverse, and the currents would repel. Reversing *both* currents, of course, once again produces an attractive force.



To evaluate the magnetic field at off-axis points, the same formula (A.1) can be used. The mathematics quickly becomes intractable and the solution is usually implemented numerically. Alternatively, specialized computer aided design software can be used to calculate the magnetic field at arbitrary points in space. The two methods approach the calculation differently (the former is the idealized magnetic field numerically approximated while the other is the high fidelity numerical model of the detailed system). Either method will, of course, yield the same result.

The force due to a current-field interaction off-axis can be calculated according to (A.5). Thus, in the case of a Neodymium Iron Boron permanent magnet positioned above a coil, the magnet can be modeled as a current sheet of thickness equal to that of the magnet. This “current” creates a magnet field at the location of the coil conductors.

The force on the coil and, by Newton's third law, the force on the magnet is simply the product of the magnetic field, current and conductor path length.

Since the equivalent current of the permanent magnet is in the theta direction (circumferential around the magnet), the vector cross product in the force equation suggests that to get an axial force, F_z we must have a radial magnetic field, B_r ($-\hat{z} = \hat{\theta} \times \hat{r}$). In developing the control system, it is convenient to express the radial magnetic field equation in the following form:

$$B_r = \frac{c_1}{(z + c_2)^N},$$

where c_1 and c_2 are constants depending upon the geometry of the permanent magnets and N is a parameter describing the decrease in magnetic field with increasing axial distance. For the case of the axial magnetic field, we saw above c_1 is the surface current density multiplied by the magnet thickness, c_2 is related to the square of the magnet radius, and N is three or less, depending upon the relative axial distance. The radial magnetic field decreases more rapidly with distance than the axial magnetic field by approximately one more power of the denominator: $3 < N < 4$.¹

In order to suit the control law, the following form of the force equation for a magnet-coil interaction is sought:

$$F = \frac{K}{(z + D)^N} \cdot \frac{I}{I_{\max}}.$$

The various magnet-coil interactions have been analyzed and the appropriate constants have been determined. Because of the inherent non-linearity of magnetic fields, these constants can vary when the excursion from the calculated system is large; that is, the equations have been approximated by constant parameters in the region of interest. That is, at very large axial heights or very low heights the constants will differ from those calculated.

The form of the force is the same for interactions between permanent magnets due to the equivalent current concept discussed above. In this case, however, the current is not a free parameter for control but is determined by the geometry.

Calculating Induced Currents

A time-varying magnetic field induces voltages in a closed loop according to Faraday's Law:

$$V = -N_t \frac{\partial \phi}{\partial t},$$

where,

V is the induced voltage around a closed loop,

N_t is the number of turns of conductor around the loop, and,

$\partial \phi / \partial t$ is the flux rate of change through that loop.

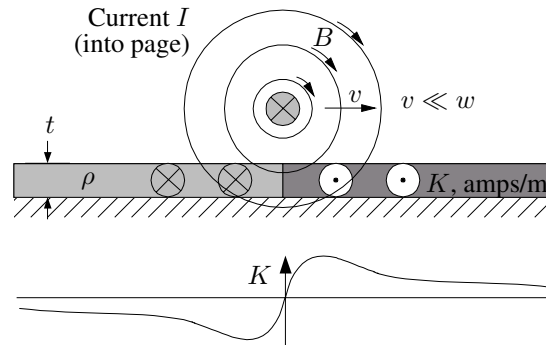
The negative sign in Faraday's Law is the manifestation of Lenz's Law. Lenz's Law states that when currents are induced in bodies due to a changing magnetic field, the currents are in such a direction as to cancel the change in magnetic field experienced by the body. For instance, if no field is present and suddenly a field is applied, the induced currents tend to circulate to cancel the magnetic field. If, however, the magnetic field has previously existed, removal of the magnetic field causes currents to flow in an attempt to maintain the field.

Consider a single current-carrying conductor moving relative to a conductive sheet. It can be shown [Magneto-Solid Mechanics, pg. 339 ff] that there is a characteristic velocity of the motion, w :

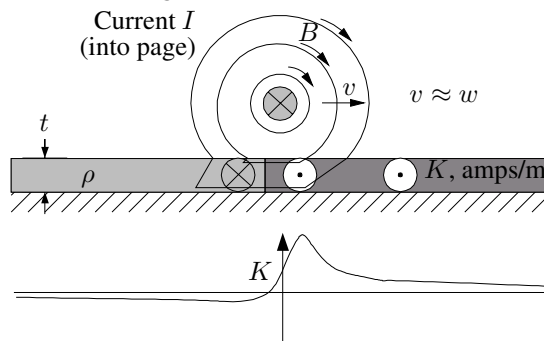
$$w = \frac{2\rho}{\mu_0 t},$$

¹In evaluating specific geometries and using numerical curve fitting, exponents of value $N > 4.0$ can result. For the Model 730 Apparatus, $N \leq 4.3$.

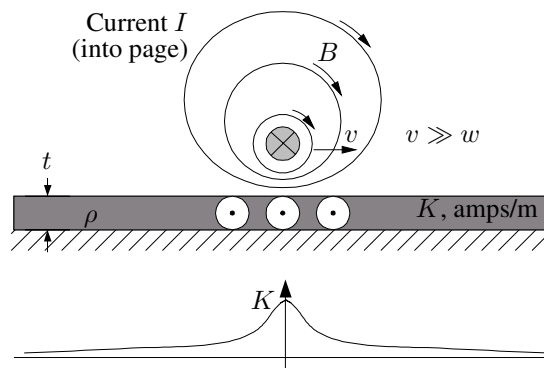
where, ρ is the sheet material resistivity, t is the thickness of the sheet and μ_0 is the permeability of free space. At standstill, the magnetic field of the current will fully permeate the sheet conductor. The magnetic field lines will be perfect circles about the current center. At very low speeds, ($v \ll w$) the field still permeates the sheet and the field lines will still be very nearly circular. This situation is shown in the figure below, the induced current in the sheet is K amps/meter.



As the conductor speed is increased to approximately the characteristic velocity ($v \approx w$), the movement of the magnetic field through the sheet causes induced currents to flow. According to Lenz's Law, these currents flow in such a manner so as to cancel the effect of the approaching field. Nevertheless, due to finite resistance, the magnetic field will still penetrate the conductive sheet to an extent and as the conductor leaves the region of magnetic field additional currents are induced to maintain the presence of the field. This situation is shown in the figure below. Notice the shear effect of the motion on the magnetic field.



When the speed is increased to substantially above the characteristics velocity, the conductivity of the sheet prevents the magnetic field from any significant penetration. The conductor is moving sufficiently fast that significant resistive dissipation does not occur. Each section of sheet generates the exact required current to perfectly shield the interior of the conductive sheet from the magnetic field. This situation is shown in the figure below. Notice that the magnetic field lines do not enter the sheet.



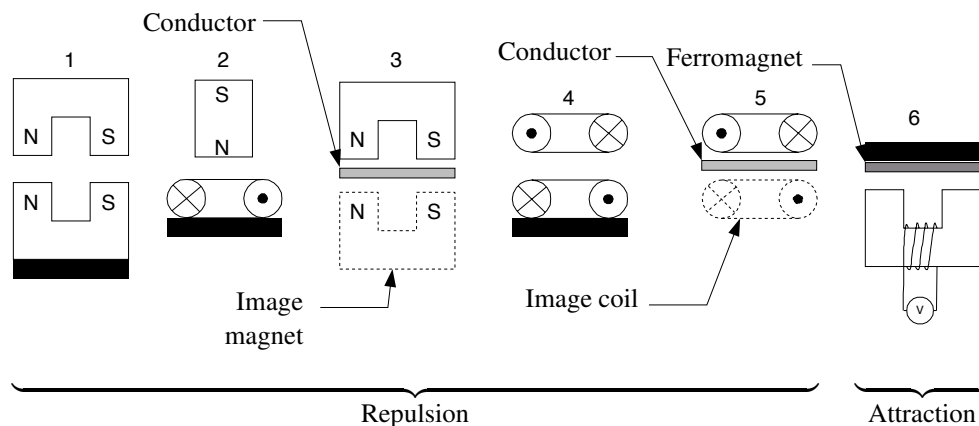
For application to the demonstration unit, the current element is the equivalent current of the permanent magnet edge. The permanent magnet flying above a rotating conductive sheet introduces several details different from that

described above. The physical principle remains the same; that is, the currents are induced in the sheet in response to the apparent change in magnetic field, and these currents tend to cancel the change in magnetic field. The interaction of the (magnetic field from) the induced currents and the original permanent magnetic field produce a repulsion force which levitates the magnet. These fundamentals are the basis of several different configurations of Maglev systems.

A.3 Maglev Applications

The figure below shows six arrangements used in magnetic levitation of moving vehicles. Five of the arrangements rely on repulsive forces. The lower elements are fixed (say, with respect to the earth) and the upper elements levitate. The first arrangement (permanent magnet like poles) is the common one for demonstrating like magnetic poles repel. The second and fourth arrangements (permanent magnet and/or superconducting magnet flying over a normal copper lower coil) is similar to that used for the Japanese superconducting Maglev system. The third and fifth systems are similar to the Magneplane system where permanent magnets or superconducting coils fly over normal sheet conductor. Notably, this has been variously proposed as an inexpensive method to attain levitation.

The sixth arrangement (electromagnet under a ferromagnet) is quite different and is the basis for the German Transrapid Maglev system. The fixed element is the upper ferromagnetic material and the lower electromagnet is actively controlled. If the current in the electromagnet is too large, the electromagnet feels a net upward force until it contacts the ferromagnetic material. If the current is too small, then insufficient force is available and the electromagnet falls. Hence, the current in the electromagnet must be continuously adjusted to enable levitation without contact.



For applications involving moving vehicles, all maglev designs share a common trait: while generating magnetic lift (in the direction perpendicular to travel) there is also generated magnetic drag (opposing the direction of travel). The details of the lift and drag forces, of course, depend upon the configuration.

A.4 Reference

News Item: 14 April 1999 Japanese Maglev vehicle demonstrates top speed of 552 km/hr (345 mph).

A.4.1 Current Status of Maglev in the United States and Internationally

Superconducting Maglev technology was initiated in the late 1960s and early 1970s in the United States in 1969 when Drs. James Powell and Gordon Danby of New York's Brookhaven National Laboratory invented the concept of a repulsive magnetic suspension using superconducting magnets. In the mid-1970s the US stopped Maglev development due to funding problems. Other countries, however, continued to develop Maglev and today have viable systems. In the early 1990s Maglev research was rekindled at a Federal government level. At various times, the Department of Transportation's Federal Railroad Administration and Federal Transit Administration, NASA, Department of the Air Force, and Department of the Navy have joined resources for the purpose of developing

Maglev and Linear Motor technology. Although each agency had its own specific application in mind, a loose consortium seemed to provide the best bang for the buck. The situation has progressed where today there are several high speed (≈ 300 mph) and low speed (≈ 100 mph) regions in this country where Maglev is thought to be a viable alternative means of rapid public transportation—yet it is still unproven in the United States.

Germany's Transrapid vehicle has been extensively tested and has been proposed for use on several projects in this country. The Germans are presently constructing a Transrapid route from Hamburg to Berlin.

Japan is developing a system that uses superconducting magnets and is currently constructing a major test route that will ultimately be incorporated into a revenue-producing route. Approximately 80% of this system will be in tunnels cut into mountains. This has greatly increased the construction cost but has decreased the cost of land acquisition for the Maglev right-of-way.

A.4.2 Glossary

Eddy Currents: Induced currents in conductors by changing magnetic fields. Since the currents flow in closed paths within the materials, they are similar to eddies in rivers. In accordance with Lenz's Law, these currents flow in such a manner as to oppose the change in magnetic field inducing them. Frequently, eddy currents are undesirable, but they can be desired in cases of induction and microwave heating for industrial and consumer purposes.

EDS: Electrodynamic System

Electrodynamic System: Magnetic forces based upon repulsion from induced currents. Inherently stable can be lightly damped or even negatively damped at sufficiently high frequencies.

Electromagnetic System: Magnetic forces based upon attraction from applied currents. Inherently unstable and currents must be controlled.

EMS: Electromagnetic System

HSST: High Speed Surface Transportation (not as high speed as the name sounds). Japanese EMS Maglev.

Inductance, L : $L = N_t \phi / I$, Total flux linked per unit current. Units are Henries, H.

Maglev: Generic term for magnetic suspension. Sometimes refers to both levitation (vertical forces) and guidance (side-to-side forces).

Magnetic Drag: Magnetic force opposing propulsion, due to resistive dissipation. Units are Newtons.

Magnetic Field, H : Three-dimensional vector used to calculate the enclosed current as used in Ampere's Law $\nabla \times H = \mu_0 J_{free}$. This parameter is independent of the material properties. Units are Amperes/meter, A/m.

Magnetic Flux Density, B : Three-dimensional vector used to calculate the force on conductors and magnetic materials. Units are Wb/m² or Tesla, T. $\nabla \times B = \mu_0 (J_{free} + J_{bound})$. Recall also $F = I dl \times B$.

Magnetic Flux, ϕ : $\phi = \iint \vec{B} \cdot d\vec{A}$, the integral of the vector dot product of the flux density and area. Units are Webers, Wb.

Magnetic Lift: Magnetic force opposing gravity.

Permeability: $\mu = B/H$, Ratio of flux density to magnetic field. Units are Henries/meter, H/m.

Reluctance: Ratio of total magnetomotive force to total flux through a circuit. Units are inverse Henries, H⁻¹.

RTRI: Railway Technical Research Institute. Japanese research arm to develop EDS Maglev.

Superconductivity: The material property of the complete lack of electric resistance, obtained for special materials only under conditions of refrigeration. Two classes of materials are low critical temperature superconductors ($T \leq 10K$) and high critical temperature superconductors (HTSC) ($T \leq 77K$).

Transrapid: German EMS Maglev, Hamburg to Berlin route under construction.

Magnetomotive force, mmf: $MMF = \oint \vec{H} \cdot d\vec{l} = N_t I$. Intermediate quantity used for magnetic circuits. Conceptually similar to the electromotive force or voltage of an electric circuit. Units of mmf are Ampere-turns.

Lenz's Law: A law stating that induced currents resulting from a change in magnetic field are in such a direction as to oppose the change in magnetic field. Hence, if a magnetic field is increased, current is induced to cancel it. If the magnetic field is decreased, current is induced to preserve it.

A.4.3 Reference Literature:

Books

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2. *Case Studies in Superconducting Magnets: Design and Operational Issues*, Yukikazu Iwasa, Plenum Press, New York, 1994.
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2. "New York State Technical and Economic Maglev Evaluation," Michael Proise, Grumman Space and Electronics Division, New York, June 1991.
3. "Electrodynamic Suspension and Linear Synchronous Motor Propulsion for High Speed Guided Ground Transportation," David Atherton, Canadian Maglev Group, CIGGT Report No. 77-13, September 1977.
4. "Conceptual Design and Analysis of The Tracked Magnetically Levitated Vehicle Technology Program Repulsion Scheme, Volume I—Technical Studies, Ford Motor Co., US DOT/FRA, Report PB247931, Feb. 1975.

Maglev Web Sites

1. <http://bmes.ece.utexas.edu/~jcamp/physics/index.html> Descriptive link of Maglev technologies and basic physics.
2. <http://eb-p5.eb.uah.edu/maglev/maglev.html> Univ. of Alabama Huntsville College of Engineering WWW Archive and Information service for High Speed and Automated Transportation Technologies. (This site is relatively new and has unfinished links.)
3. <http://www.rtri.or.jp/> RTRI Home page. Japanese EDS Maglev.
4. <http://www.mvp.de/english.htm> Transrapid Official Homepage. German EMS Maglev.

5. <http://weber.u.washington.edu/~jbs/itrans/hsst.htm> **HSST** unofficial descriptive page.
6. <http://www.meitetsu.co.jp/chsst/mecha.html> **Mechanism of HSST**. Japanese EMS.
7. <http://popularmechanics.com/popmech/sci/9805STTRP.html> “Track to the Future” article. Descriptive article of one recent Maglev approach.
8. <http://www.llnl.gov/str/Post.html> “A New Approach to Levitating Trains—and Rockets.” Elaboration of the previous link for a Maglev approach.