Problem Set 5 Answers

1. **Ch 7, Problem 7.2**

A grocery shop is owned by Mr. Moore and has the following statement of revenues and costs:

Revenues	$250,000
Supplies	$25,000
Electricity	$6,000
Employee salaries	$75,000
Mr. Moore's salary	$80,000

Mr. Moore always has the option of closing down his shop and renting out the land for $100,000. **Also, Mr. Moore himself has job offers at a local supermarket at a salary of** $95,000 **and at a nearby restaurant at** $65,000. **He can only work one job, though. What are the shop's accounting costs? What are Mr. Moore's economic costs? Should Mr. Moore shut down his shop?**

The accounting costs are simply the sum: $25,000 + $6,000 + $75,000 +80,000 = $186,000. The shop's accounting profit is $64,000 which means that Mr. Moore's total gain from this venture is 80,000 + 64,000 = $144,000.

The economic costs also include the opportunity cost of the land rental ($100,000) and the opportunity cost of his time Mr. Moore would earn if he selected his next best alternative ($95,000). That is, Mr. Moore loses $15,000 by not choosing his next best alternative. So, the economic costs are $186,000 + $100,000 + $15,000 = $301,000, this exceed his revenue of 144,000 by $51,000.

Should Mr. Moore shut down his shop? If he shut down the shop he would early 100,000 + 95,000=195,000. This is more than the $144,000 he currently early, therefore he should shut down the shop.

2. **Ch 7, Problem 7.5**

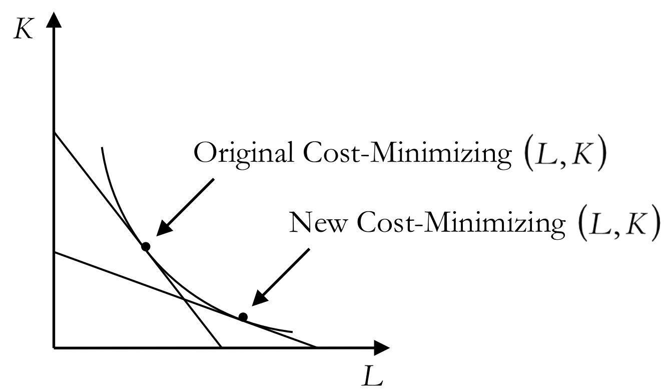
   A firm uses two inputs, capital and labor, to produce output. Its production function exhibits a diminishing marginal rate of technical substitution.

   a. **If the price of capital and labor services both increase by the same percentage amount (e.g., 20 percent), what will happen to the cost-minimizing input quantities for a given output level?**

      If the price of both inputs change by the same percentage amount, the slope of the isocost line ($-w/r$) will not change. Since we are holding the level of output fixed, the point at which the isocost line is tangent to the (fixed) isoquant does not change. Therefore, the cost-minimizing quantities of the inputs will not change.

   b. **If the price of capital increases by 20 percent while the price of labor increases by 10 percent, what will happen to the cost-minimizing input quantities for a given output level?**

      If the price of capital increases by a larger percentage than the price of labor, the isocost lines become flatter (since w/r decreases). This means that the point of tangency will move to the southeast as shown in the diagram below:

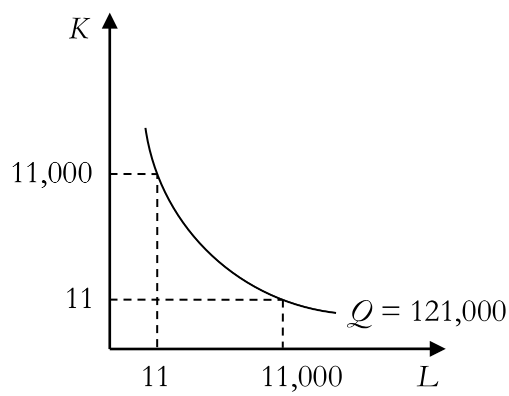


      Another way to think about this is to realize that when the price of capital increases by a larger percentage than the price of labor, labor has become cheaper relative to capital. The firm responds by substituting away from capital in favor of labor.

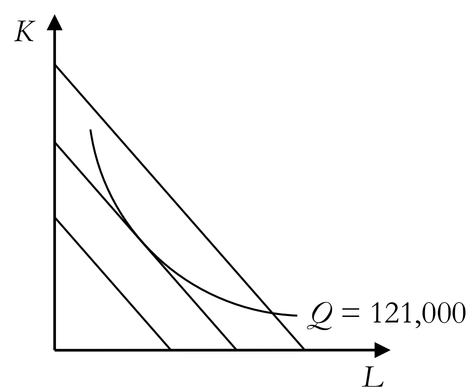
      In sum, the cost-minimizing quantity of labor should increase, and the cost-minimizing quantity of capital should decrease.

3. Suppose the production of digital cameras is characterized by the production function $Q = LK$, where Q represents the number of digital cameras produced. Suppose that the price of labor is \$10 per unit and the price of capital is \$1 per unit.

a. Graph the isoquant for $Q = 121{,}000$.



b. On the graph you drew for part a, draw several isocost lines including one that is tangent to the isoquant you drew. What is the slope of the isocost lines?



The slope of the isocost lines is $-w/r = -10/1 = -10$.

c. Find the cost-minimizing combination of labor and capital for a manufacturer that wants to produce 121,000 **digital cameras. Mark this point on the graph you drew for part a.**

We recognize that the production function $Q = LK$ is Cobb-Douglas; therefore, we will have an interior solution to this problem.

We can begin by setting up the minimization problem:

$$\underset{L,K}{Min}\ TC = wL + rK$$
$$s.t.\ Q_0 = LK$$

d. What are the input demand functions in terms of w and r (i.e. not subbing in prices).

To determine these we need to use the regular solution technique but not sub in for w and r yet. Since we are asked to find the labor demand function for L, I want to solve the constraint for K in terms of L and sub that back into the objective function.

$$K=Q_0/L$$

Then I will take the derivative of the cost function with respect to L and set that equal to zero. Rearranging I get the Labor demand function.

$$\underset{L}{Min}\quad TC = wL + r\frac{Q_0}{L}$$
$$\frac{\partial TC}{\partial L} = w - \frac{rQ_0}{L^2} = 0$$
$$solving$$
$$L = \sqrt{\frac{rQ_0}{w}}\quad \text{This is the labor demand function}$$

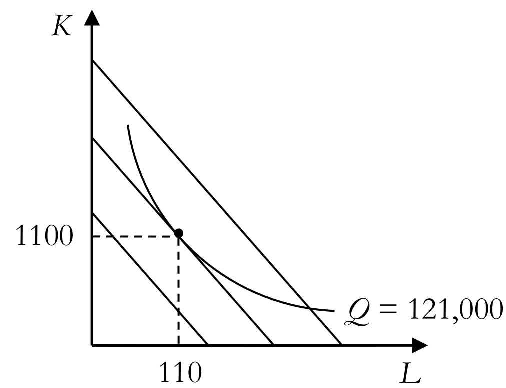
e. Find the cost-minimizing combination of labor and capital for a manufacturer that wants to produce 121,000 digital cameras. Mark this point on the graph you drew for part a.

We can sub in for r , w and Q to find L*=110. Note you could have subbed in for w, r and Q_0 first to find L*. I did not do this because first I needed to find the demand function for L.

Subbing the labor demand into quantity constraint we can obtain the capital demand function.

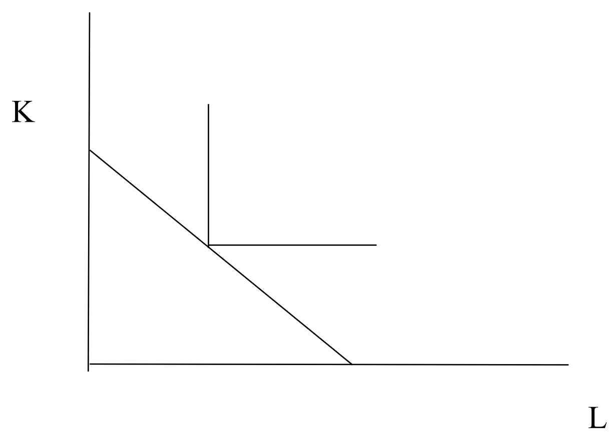
$$K^* = 121{,}000/110 = 1100$$

Therefore, the cost-minimizing combination of labor and capital is $(L^*, K^*) = (110, 1100)$. This point is marked in the graph below:



4. **Ch 7, Problem 7.14**

Suppose a production function is given by $Q = \min\{L, K\}$**. Draw a graph of the demand curve for labor when the firm wants to produce** 10 **units of output (** $Q = 10$**).**



Remember these are L shaped isoquants and to cost minimize we need to be at the corner of the isoquant.

Lets remember what the firm's problem is:

$$\underset{L,K}{Min}\ TC = wL + rK$$
$$s.t.\ Q = \min\{L, K\}$$

First we want to recognize that the production function is one of perfect complements or fixed proportion. There are L shaped isoquants and we need to produce were we are the corner of an isoquant. We find the corner of an isoquant when what is in the two parts of the brackets equals each other.

L=K this has to $=Q_0$. So we know that $L=K=Q_0$, where our Q_0 will be 10 but lets leave it as Q_0 for now to be general.

Now to determine the demand curve for labor we just need to rewrite this equation in terms of L.

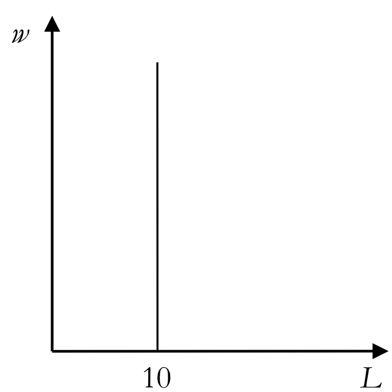
$L=Q_0$ is the demand function for labor
$K=Q_0$ is the demand function for capital.

Note that in the case of fixed proportions, we have to use L and K at a specific ratio to cost minimize, the prices of the inputs w and r are not in the demand functions. This is because the price doesn't matter, we need to be at the corner point to cost minimize.

Since this is the constraint we know Q=L=K

The demand function for labor is L=Q
The demand function for capital is K=Q

Now lets graph the labor demand curve. It is a demand curve so we need the price of labor on the vertical axis and the quantity of labor on the horizontal axis. Note that the demand for labor does not depend on the price of labor w. In particular, if the firm wants to produce 10 units of output, its demand for labor is simply $L = 10$. We sketch this demand curve below:



To reiterate, the demand curve is vertical because the demand for labor does not vary with the price of labor w.

5. Ch 7 problem 7.19

A plant's production function is Q=2L+K. The price of labor services w is $4 and of capital services r is $5 per unit.

a) **In the short-run, the plant's capital is fixed at $\bar{K}$ =9. Find the amount of labor it must employ to produce Q=45 units of output.**

Since we know K and Q we can use the quantity constraint to find the short-run amount of labor needed to produce Q=45. 45 = 2L + 9, so L*=18.

b) **How much money is the firm sacrificing by not having the ability to choose its level of capital optimally?**

The optimal solution is the long-run cost minimizing amounts of L and K. In the long-run capital is not fixed. Note here that the production function is linear so the firm will use all L or all K. When we have a linear production function we can compare the bang for the buck for each input. So we compare

$$\frac{MP_L}{w} = \frac{2}{4} = \frac{1}{2} \ \ and \ \ \frac{MP_K}{r} = \frac{1}{5}$$

For each dollar spent on an input, based on the production function, there is more output produced when it is spent on Labor than Capital. So the firm will minimize costs when it uses all labor and no capital.

To determine the optimal amount of labor we can use the constraint. We have to produce 45 units of output and we will use K=0. So subbing these values into the production function:

45=2L + 0, L*=45/2=22.5

Therefore the cost minimizing amount of Labor is L = 22.5 and capital is K=0.

Now to see how much money the firm is sacrificing:

TC in short-run are: TC=4*18+5*9=117
TC in the long-run are: TC=4*22.5 + 5*0=90

The firm sacrifices 117-90=27