

Economics 448: Lecture 12

Measures of Inequality

October 11, 2012

Introduction

What is economic inequality?

- The context

- Economic inequality: Preliminary observations

Measuring economic inequality

- Introduction

- Four criteria for inequality measurement

- The Lorenz curve

- Complete measures of inequality

Summary

Inequality

Economic growth affects the level of income, wealth, well-being.

Also want to study the distribution of income, wealth, well-being.

Two dimensions:

- ▶ **efficiency**—for a given level of output, product at lowest cost.
- ▶ **equity**—distribution who receives what? Winners and losers.

Equity: Social Justice

In courses we spend probably 80–90% of time thinking (characterizing) economic efficiency. (Positive Economics; what is)

Equity involves notions of justice — what is a *fair* distribution of resources? (Normative economics; what should be)

Have mentioned Pareto Optimality Criterion: no one can be made better off without some else being made worse off.

Competitive equilibrium yields a Pareto outcome. But every point on the production possibility curve can support a competitive equilibrium.

Pareto optimality is a weak criterion need additional tools to make progress.

Two motivations to study equity

- ▶ Social Justice — philosophy. Rawls: veil of ignorance.
- ▶ Functional level — how the distribution of resources affects economic growth.

In Chapter 6 only study the **measurement** of inequality.

In chapter 7 studies functional level; consequences of inequality.

Context

Inequality is the fundamental disparity that permits one individual certain material choices, while denying another individual those very same choices.

Will not spend much time on notions of social justice (what is fair?).

Sen: **Commodities and Capabilities**. Oxford University Press. 1985.

Preliminary Observations

Distinguish stocks and flows.

- ▶ Income is a flow (per unit of time).
- ▶ Asset is a stock (at a point in time).

Assets the accumulation of past income flows.

Economic **Mobility** important. Can measure dispersion of income at point in time. But is there any churning in the distribution. “Musical chairs”? or “Make like a statue”?

Functional and Personal Distribution of Income

Functional income distribution — the returns to different factors of production (labor, capital, land).

Need to know who owns the different factors. Would like to know for each individual income from labor, profits from capital, land rents.

Understanding **sources** may affect how we judge the outcome.

Functional distribution income informative on features of development.

Challenge

How to compare many alternative distributions of income across many people?

- ▶ Pareto criterion gives us a partial ordering. May be able to rank order some distributions.
- ▶ But even that criterion does not offer much guidance.
- ▶ Will develop inequality indexes that collapse the distribution (income, assets, etc.) into a single number. Scalar measures easy to report and understand. And offers a **complete** ordering.

Criteria

The inequality measures are each a scalar x . But **many** scalar inequality measures exist and can and do give conflicting results.

Consider a set of four criteria of desirable properties of inequality measures.

Each criteria relates to ethical judgement. Need to be cognizant of the relationship.

Setup

Society has n individuals.

Index i stands for a generic individual, $i = 1, 2, \dots, n$.

An **income distribution** is a description of how much income y is received by individual i : (y_1, y_2, \dots, y_n) .

Compare **relative** “inequality” of two income distributions.

Anonymity principle

It does not matter **who** earns the income. Anonymity because we care about the ordering but **not** the identity of each earner. All that matters is the ranking from lowest to highest:

$y_1, y_2, \dots, y_n \rightarrow y_{(1)}, y_{(2)}, \dots, y_{(n)}$ order statistics

$$y_{(1)} \leq y_{(2)} \leq y_{(3)} \leq \dots \leq y_{(n)}$$

Since identity of person doesn't matter we can dispense with the order statistic notation $y_{(k)}$ and use the simpler notation where index i represents the level of the i^{th} income level in the society.

$$y_1 \leq y_2 \leq y_3 \leq \dots \leq y_n$$

Population Principle

If we compare an income distribution over n people and another population with $2n$ people with the same income pattern repeated twice, there should be no difference in inequality among the two income distributions.

Anonymity states that no information is lost by retaining only the sequence of individual incomes (and not the identities of each)

The population principle states it doesn't matter how large the population is, we can convert everything to percentiles (bottom 1%, lowest 20%, top 25%)

Relative Income Principle

Only the **relative** incomes should matter and the **absolute** levels of these incomes should not.

Thus if we transform one distribution by multiply by a positive constant (e.g., $Y^1 = \lambda Y^0$) then inequality should be the same for the two distributions.

Income **levels** have no meaning for **inequality measurement**.
Absolute measure matters for assessing economic development.
We will see that level matters for the measurement of **poverty**.

Roughly think of poverty as a measure of location (level) and inequality as a measure of dispersion.

Parsimony

Relative income principle means that data can be further collapsed.

Population and income can be expressed as shares of the total.

This means we can compare income distributions for countries with different average income levels.

Need to record only income shares to measure inequality. Ordered incomes from poorest to riches. So if put into five income categories (quintiles) report the share of income by each fifth of the population.

For example, Figure 6.3 (p. 177) graphs the share of income of the (richest) fifth quintile (31%), share of the fourth quintile (25%), share of the third quintile (20%), share of the second quintile (15%), and first quintile (poorest) (9%).

Dalton Principle

The two principles not controversial, third more difficult (well-being is proportional to income). This is fundamental to the construction of inequality measures.

Let (y_1, y_2, \dots, y_n) be an income distribution and consider two incomes y_i and y_j with $y_i \leq y_j$.

A transfer of income from individual i to individual j is called a **regressive** transfer.

If inequalities is strict $y_i < y_j$ the regressive transfer is from the poorer individual to the richer individual.

With weak inequality (\leq) use the language “not richer” to “not poorer”

Dalton Principle

Our inequality index as a function of the form:

$$I = I(y_1, y_2, \dots, y_n)$$

with I defined over all conceivable distributions of income (y_1, y_2, \dots, y_n) .

Dalton principle: if one income distribution can be achieved from another by constructing a sequence of regressive transfers, then the former distribution must be deemed more **unequal** than the latter.

If for every income distribution (y_1, y_2, \dots, y_n) and every transfer $\delta > 0$,

$$I(y_1, \dots, y_i, \dots, y_j, \dots, y_n) < I(y_1, \dots, y_i - \delta, \dots, y_j + \delta, \dots, y_n)$$

The Lorenz Curve

Lorenz curve is a simple diagrammatic way to depict the distribution of income.

On the horizon axis we list the **cumulative** percentage of the population arranged in increasing order of income.

Thus point on the axis refer to the poorest 20% of the population, the poorest half, etc.

On the vertical axis we measure the percentage of the national income accruing to any particular fraction of the population thus arranged.

The diagonal line (45°) represents equal distribution income.

Lorenz Curve Properties

The slope of the Lorenz curve is the contribution of the person at that point to the cumulative share of national income.

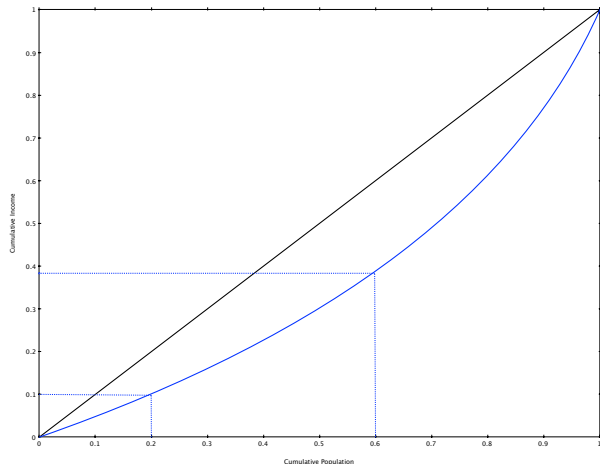
Ordered from poorest to richest the “marginal contribution” can never fall.

Equivalently, the Lorenz curve can never get flatter as we move from left to right.

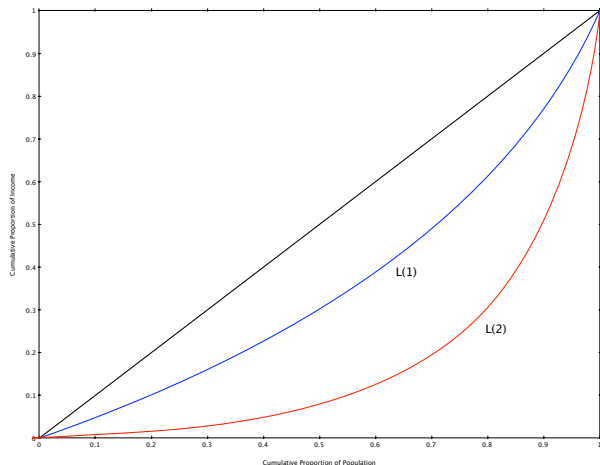
Lorenz Curve

The overall distance between the 45° and the Lorenz curve represents the amount of inequality present in the society.

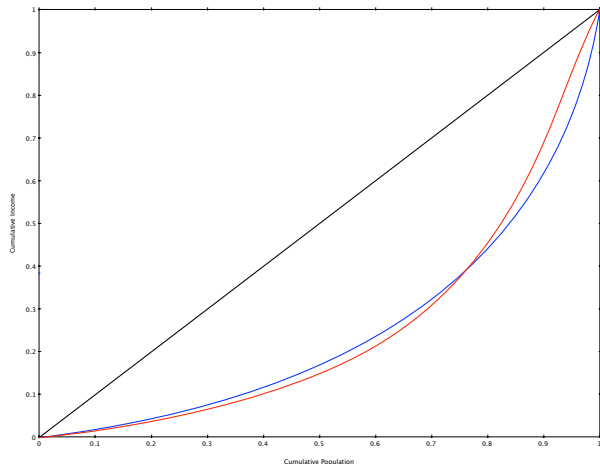
Lorenz Curve Figure 6.4



Lorenz Curve Comparisons 6.5



Lorenz Curve: Crossing



Complete Measures of Inequality

Lorenz curves offer a visual representation of inequality.

But the curves offer only a **partial** ranking: when Lorenz curves cross they offer no additional information to prefer one distribution over another.

The (scalar) numerical measures provide a complete ranking of alternative distributions.

However, these quantitative measures sometimes disagree.

Scalar Measures of Inequality: Setup

Assume we have a population census.

There are N units (households or individuals) in the population.

Let there be m income groups, e.g., [10000, 20000) with representative value y_g for group g .

y_g could be the **mean** income (within the interval), or the **median**, or the interval midpoint.

Define the mean as

$$\mu = \frac{1}{N} \sum_{g=1}^m y_g$$

Will divide by μ to make the inequality measures independent of the units of income.

Three simple measures of Inequality

Range $R = \mu^{-1}(y_G - y_1)$, crude but sometimes useful.

Kuznets Ratio Richest $x\%$ to the poorest $y\%$ where x and y stand for numbers such as 10, 20, or 40. Also crude, focuses on ends of distribution.

Mean Absolute Deviation $M = \frac{1}{N\mu} \sum_{g=1}^m n_g |y_g - \mu|$. Do not use.

Three Measures that Satisfy all Four Principles

Coefficient of Variation $C = \sqrt{\sum_{g=1}^m \frac{n_g}{N} \left(\frac{y_g - \mu}{\mu}\right)^2}$.

Gini Coefficient $G = \frac{1}{2n^2\mu} \sum_{j=1}^m \sum_{k=1}^m n_j n_k |y_j - y_k|$.

Theil Index $T = \frac{1}{N\mu} \sum_{g=1}^m y_g \ln(y_g/\mu)$

Comments on Measures

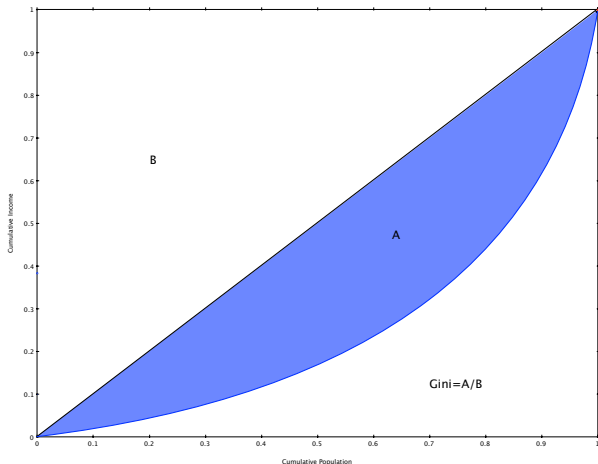
CV This is a standard measure of relative dispersion.

Gini It is the work horse of inequality studies.

The Gini Coefficient is the ratio of the **area** between the Lorenz curve and the 45° line to the **area of the triangle** below the 45° line.

$$G = \frac{\text{area between the Lorenz Curve and the } 45^\circ \text{ line}}{\text{area of triangle below } 45^\circ \text{ line}}$$

Graphical Display of Gini



Comments on Measures

Theil The Theil Index can be used to exactly decompose overall inequality into between-group and within-group components.

Useful for descriptive studies to disaggregate inequality by race, gender, education, etc.

Measures from World Bank Report

Will put on course web page for your review.