edexcel
advancing learning, changing lives

# Edexcel AS Physics

STUDENTS' BOOK

This book also includes
Active Book

A PEARSON COMPANY

# How to use this book

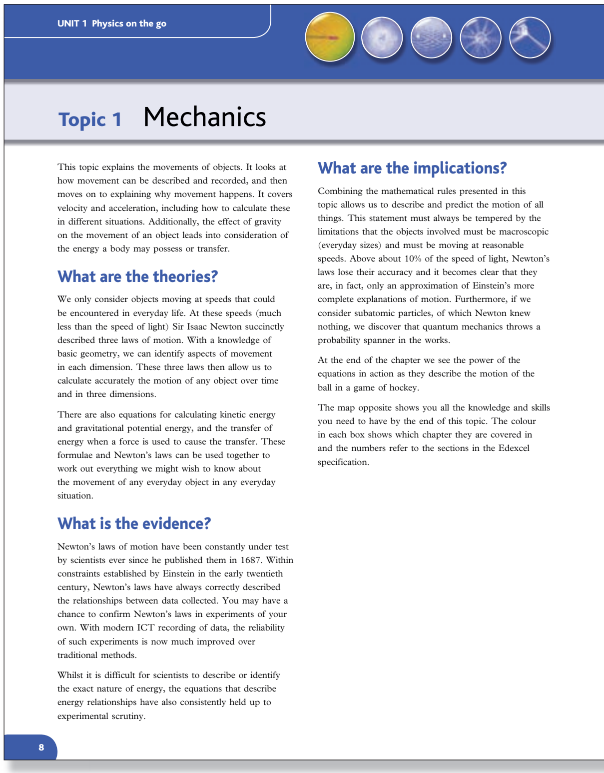
**This book contains a number of great features that will help you find your way around your AS Physics course and support your learning.**

## Introductory pages

Each topic has two introductory pages to help you identify how the main text is arranged to cover all that you need to learn. The left-hand page gives a brief summary of the topic, linking the content to three key areas of How Science Works:
What are the theories? What is the evidence? What are the implications?
The right-hand page of the introduction consists of a topic map that shows you how all the required content of the Edexcel specification for that topic is covered in the chapters, and how that content all interlinks. Links to other topics are also shown, including where previous knowledge is built on within the topic.

## Main text

The main part of the book covers all you need to learn for your course. The text is supported by many diagrams and photographs that will help you understand the concepts you need to learn.
Key terms in the text are shown in bold type. These terms are defined in the interactive glossary that can be found on the software using the 'search glossary' feature.



UNIT 1 Physics on the go

Topic 1 Mechanics

This topic explains the movements of objects. It looks at how movement can be described and recorded, and then moves on to explaining why movement happens. It covers velocity and acceleration, including how to calculate these in different situations. Additionally, the effect of gravity on the movement of an object leads into consideration of the energy a body may possess or transfer.

What are the theories?

We only consider objects moving at speeds that could be encountered in everyday life. At these speeds (much less than the speed of light) Sir Isaac Newton succinctly described three laws of motion. With a knowledge of basic geometry, we can identify aspects of movement in each dimension. These three laws then allow us to calculate accurately the motion of any object over time and in three dimensions.

There are also equations for calculating kinetic energy and gravitational potential energy, and the transfer of energy when a force is used to cause the transfer. These formulae and Newton's laws can be used together to work out everything we might wish to know about the movement of any everyday object in any everyday situation.

What is the evidence?

Newton's laws of motion have been constantly under test by scientists ever since he published them in 1687. Within constraints established by Einstein in the early twentieth century, Newton's laws have always correctly described the relationships between data collected. You may have a chance to confirm Newton's laws in experiments of your own. With modern ICT recording of data, the reliability of such experiments is now much improved over traditional methods.

Whilst it is difficult for scientists to describe or identify the exact nature of energy, the equations that describe energy relationships have also consistently held up to experimental scrutiny.

What are the implications?

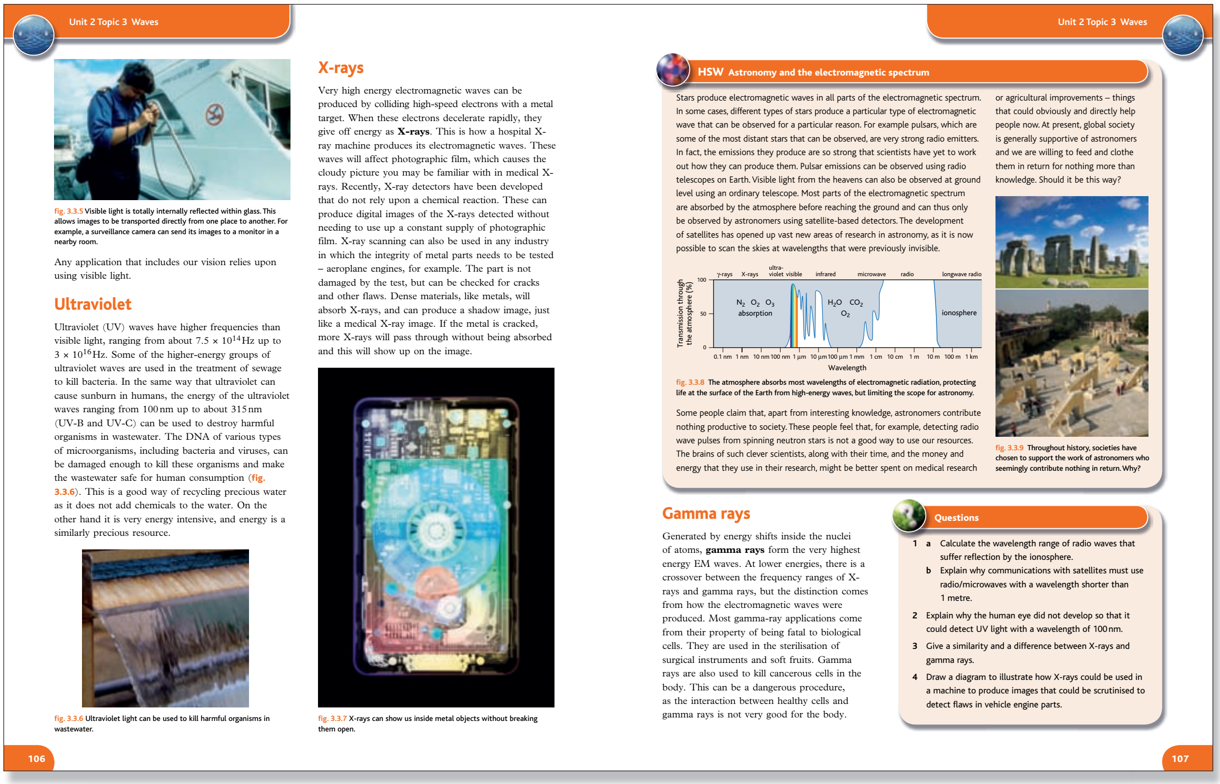
Combining the mathematical rules presented in this topic allows us to describe and predict the motion of all things. This statement must always be tempered by the limitations that the objects involved must be macroscopic (everyday sizes) and must be moving at reasonable speeds. Above about 10% of the speed of light, Newton's laws lose their accuracy and it becomes clear that they are, in fact, only an approximation of Einstein's more complete explanations of motion. Furthermore, if we consider subatomic particles, of which Newton knew nothing, we discover that quantum mechanics throws a probability spanner in the works.

At the end of the chapter we see the power of the equations in action as they describe the motion of the ball in a game of hockey.

The map opposite shows you all the knowledge and skills you need to have by the end of this topic. The colour in each box shows which chapter they are covered in and the numbers refer to the sections in the Edexcel specification.

8

Introductory pages



Unit 2 Topic 3 Waves

fig. 3.3.5 Visible light is totally internally reflected within glass. This allows images to be transported directly from one place to another. For example, a surveillance camera can send its images to a monitor in a nearby room.

Any application that includes our vision relies upon using visible light.

Ultraviolet

Ultraviolet (UV) waves have higher frequencies than visible light, ranging from about $7.5 \times 10^{14}$ Hz up to $3 \times 10^{16}$ Hz. Some of the higher-energy groups of ultraviolet waves are used in the treatment of sewage to kill bacteria. In the same way that ultraviolet can cause sunburn in humans, the energy of the ultraviolet waves ranging from 100 nm up to about 315 nm (UV-B and UV-C) can be used to destroy harmful organisms in wastewater. The DNA of various types of microorganisms, including bacteria and viruses, can be damaged enough to kill these organisms and make the wastewater safe for human consumption (fig. 3.3.6). This is a good way of recycling precious water as it does not add chemicals to the water. On the other hand it is very energy intensive, and energy is a similarly precious resource.

fig. 3.3.6 Ultraviolet light can be used to kill harmful organisms in wastewater.

X-rays

Very high energy electromagnetic waves can be produced by colliding high-speed electrons with a metal target. When these electrons decelerate rapidly, they give off energy as **X-rays**. This is how a hospital X-ray machine produces its electromagnetic waves. These waves will affect photographic film, which causes the cloudy picture you may be familiar with in medical X-rays. Recently, X-ray detectors have been developed that do not rely upon a chemical reaction. These can produce digital images of the X-rays detected without needing to use up a constant supply of photographic film. X-ray scanning can also be used in any industry in which the integrity of metal parts needs to be tested – aeroplane engines, for example. The part is not damaged by the test, but can be checked for cracks and other flaws. Dense materials, like metals, will absorb X-rays, and can produce a shadow image, just like a medical X-ray image. If the metal is cracked, more X-rays will pass through without being absorbed and this will show up on the image.

fig. 3.3.7 X-rays can show us inside metal objects without breaking them open.

106

Unit 2 Topic 3 Waves

HSW Astronomy and the electromagnetic spectrum

Stars produce electromagnetic waves in all parts of the electromagnetic spectrum. In some cases, different types of stars produce a particular type of electromagnetic wave that can be observed for a particular reason. For example pulsars, which are some of the most distant stars that can be observed, are very strong radio emitters. In fact, the emissions they produce are so strong that scientists have yet to work out how they can produce them. Pulsar emissions can be observed using radio telescopes on Earth. Visible light from the heavens can also be observed at ground level using an ordinary telescope. Most parts of the electromagnetic spectrum are absorbed by the atmosphere before reaching the ground and can thus only be observed by astronomers using satellite-based detectors. The development of satellites has opened up vast new areas of research in astronomy, as it is now possible to scan the skies at wavelengths that were previously invisible.



fig. 3.3.8 The atmosphere absorbs most wavelengths of electromagnetic radiation, protecting life at the surface of the Earth from high-energy waves, but limiting the scope for astronomy.

Some people claim that, apart from interesting knowledge, astronomers contribute nothing productive to society. These people feel that, for example, detecting radio wave pulses from spinning neutron stars is not a good way to use our resources. The brains of such clever scientists, along with their time, and the money and energy that they use in their research, might be better spent on medical research or agricultural improvements – things that could obviously and directly help people now. At present, global society is generally supportive of astronomers and we are willing to feed and clothe them in return for nothing more than knowledge. Should it be this way?

fig. 3.3.9 Throughout history, societies have chosen to support the work of astronomers who seemingly contribute nothing in return. Why?

Gamma rays

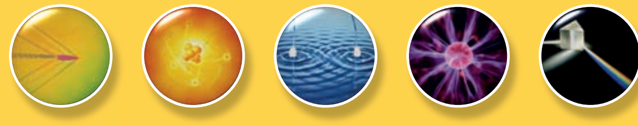
Generated by energy shifts inside the nuclei of atoms, **gamma rays** form the very highest energy EM waves. At lower energies, there is a crossover between the frequency ranges of X-rays and gamma rays, but the distinction comes from how the electromagnetic waves were produced. Most gamma-ray applications come from their property of being fatal to biological cells. They are used in the sterilisation of surgical instruments and soft fruits. Gamma rays are also used to kill cancerous cells in the body. This can be a dangerous procedure, as the interaction between healthy cells and gamma rays is not very good for the body.

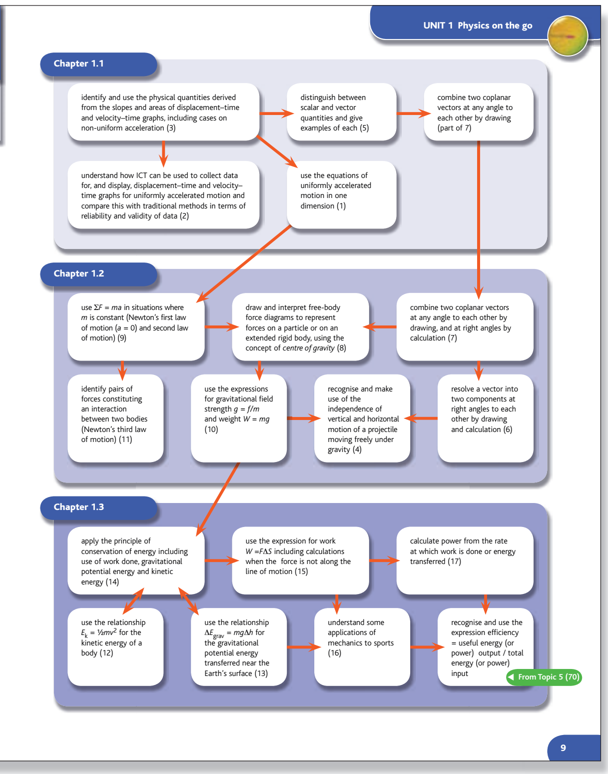
Questions

1 a Calculate the wavelength range of radio waves that suffer reflection by the ionosphere.
b Explain why communications with satellites must use radio/microwaves with a wavelength shorter than 1 metre.
2 Explain why the human eye did not develop so that it could detect UV light with a wavelength of 100 nm.
3 Give a similarity and a difference between X-rays and gamma rays.
4 Draw a diagram to illustrate how X-rays could be used in a machine to produce images that could be scrutinised to detect flaws in vehicle engine parts.

107

Main text




UNIT 1 Physics on the go

Chapter 1.1

identify and use the physical quantities derived from the slopes and areas of displacement–time and velocity–time graphs, including cases on non-uniform acceleration (3)

distinguish between scalar and vector quantities and give examples of each (5)

combine two coplanar vectors at any angle to each other by drawing (part of 7)

understand how ICT can be used to collect data for, and display, displacement–time and velocity–time graphs for uniformly accelerated motion and compare this with traditional methods in terms of reliability and validity of data (2)

use the equations of uniformly accelerated motion in one dimension (1)

Chapter 1.2

use $\Sigma F = ma$ in situations where $m$ is constant (Newton's first law of motion ($a = 0$) and second law of motion) (9)

draw and interpret free-body force diagrams to represent forces on a particle or on an extended rigid body, using the concept of *centre of gravity* (8)

combine two coplanar vectors at any angle to each other by drawing, and at right angles by calculation (7)

identify pairs of forces constituting an interaction between two bodies (Newton's third law of motion) (11)

use the expressions for gravitational field strength $g = f/m$ and weight $W = mg$ (10)

recognise and make use of the independence of vertical and horizontal motion of a projectile moving freely under gravity (4)

resolve a vector into two components at right angles to each other by drawing and calculation (6)

Chapter 1.3

apply the principle of conservation of energy including use of work done, gravitational potential energy and kinetic energy (14)

use the expression for work $W = F\Delta S$ including calculations when the force is not along the line of motion (15)

calculate power from the rate at which work is done or energy transferred (17)

use the relationship $E_k = ½mv^2$ for the kinetic energy of a body (12)

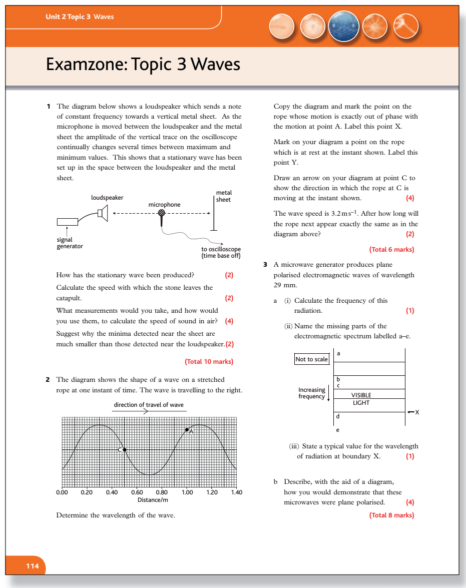
use the relationship $\Delta E_{grav} = mg\Delta h$ for the gravitational potential energy transferred near the Earth's surface (13)

understand some applications of mechanics to sports (16)

recognise and use the expression efficiency = useful energy (or power) output / total energy (or power) input

From Topic 5 (70)

9


Unit 2 Topic 3 Waves

Examzone: Topic 3 Waves

1 The diagram below shows a loudspeaker which sends a note of constant frequency towards a vertical metal sheet. As the microphone is moved between the loudspeaker and the metal sheet the amplitude of the vertical trace on the oscilloscope continually changes several times between maximum and minimum values. This shows that a stationary wave has been set up in the space between the loudspeaker and the metal sheet.



How has the stationary wave been produced? (2)

Calculate the speed with which the stone leaves the catapult. (2)

What measurements would you take, and how would you use them, to calculate the speed of sound in air? (4)

Suggest why the minima detected near the sheet are much smaller than those detected near the loudspeaker. (2)

(Total 10 marks)

2 The diagram shows the shape of a wave on a stretched rope at one instant of time. The wave is travelling to the right.



Determine the wavelength of the wave.

Copy the diagram and mark the point on the rope whose motion is exactly out of phase with the motion at point A. Label this point X.

Mark on your diagram a point on the rope which is at rest at the instant shown. Label this point Y.

Draw an arrow on your diagram at point C to show the direction in which the rope at C is moving at the instant shown. (4)

The wave speed is $3.2\,m\,s^{-1}$. After how long will the rope next appear exactly the same as in the diagram above? (2)

(Total 6 marks)

3 A microwave generator produces plane polarised electromagnetic waves of wavelength 29 mm.

a (i) Calculate the frequency of this radiation. (1)

(ii) Name the missing parts of the electromagnetic spectrum labelled a–e.



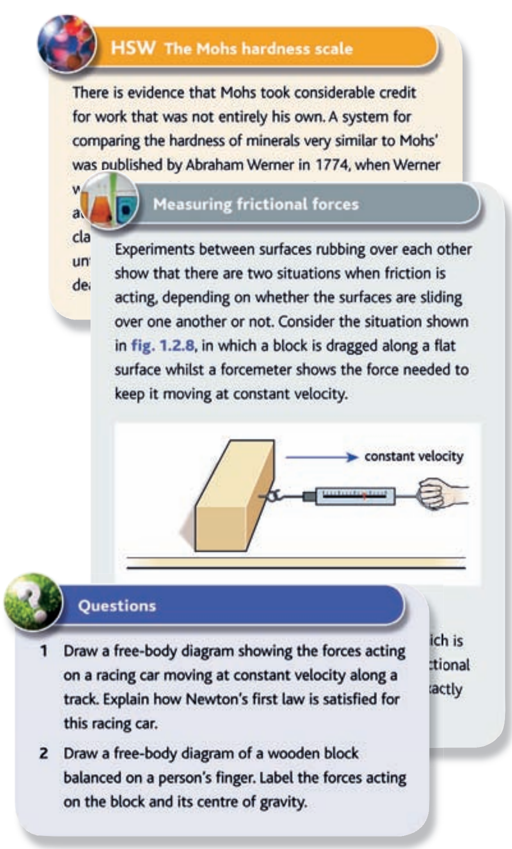
(iii) State a typical value for the wavelength of radiation at boundary X. (1)

b Describe, with the aid of a diagram, how you would demonstrate that these microwaves were plane polarised. (4)

(Total 8 marks)

114

Examzone page


HSW The Mohs hardness scale

There is evidence that Mohs took considerable credit for work that was not entirely his own. A system for comparing the hardness of minerals very similar to Mohs' was published by Abraham Werner in 1774, when Werner

Measuring frictional forces

Experiments between surfaces rubbing over each other show that there are two situations when friction is acting, depending on whether the surfaces are sliding over one another or not. Consider the situation shown in fig. 1.2.8, in which a block is dragged along a flat surface whilst a forcemeter shows the force needed to keep it moving at constant velocity.



Questions

1 Draw a free-body diagram showing the forces acting on a racing car moving at constant velocity along a track. Explain how Newton's first law is satisfied for this racing car.

2 Draw a free-body diagram of a wooden block balanced on a person's finger. Label the forces acting on the block and its centre of gravity.

## HSW boxes

How Science Works is a key feature of your course. The many HSW boxes within the text will help you cover all the new aspects of How Science Works that you need. These include how scientists investigate ideas and develop theories, how to evaluate data and the design of studies to test their validity and reliability, and how science affects the real world including informing decisions that need to be taken by individuals and society.

## Practical boxes

Your course contains a number of core practicals that you may be tested on. These boxes indicate links to core practical work. Your teacher will give you opportunities to cover these investigations.

## Question boxes

At the end of each section of text you will find a box containing questions that cover what you have just learnt. You can use these questions to help you check whether you have understood what you have just read, and whether there is anything that you need to look at again.

## Examzone pages

At the end of each topic you will find two pages of exam questions from past papers. You can use these questions to test how fully you have understood the topic, as well as to help you practise for your exams.

The contents list shows you that there are two units and five topics in the book, matching the Edexcel AS specification for physics. Page numbering in the contents list, and in the index at the back of the book, will help you find what you are looking for.

# How to use your ActiveBook

**The ActiveBook is an electronic copy of the book, which you can use on a compatible computer. The CD-ROM will only play while the disc is in the computer. The ActiveBook has these features:**

**Find Resources**

Click on this tab to see menus which list all the electronic files on the ActiveBook.

**Student Book tab**

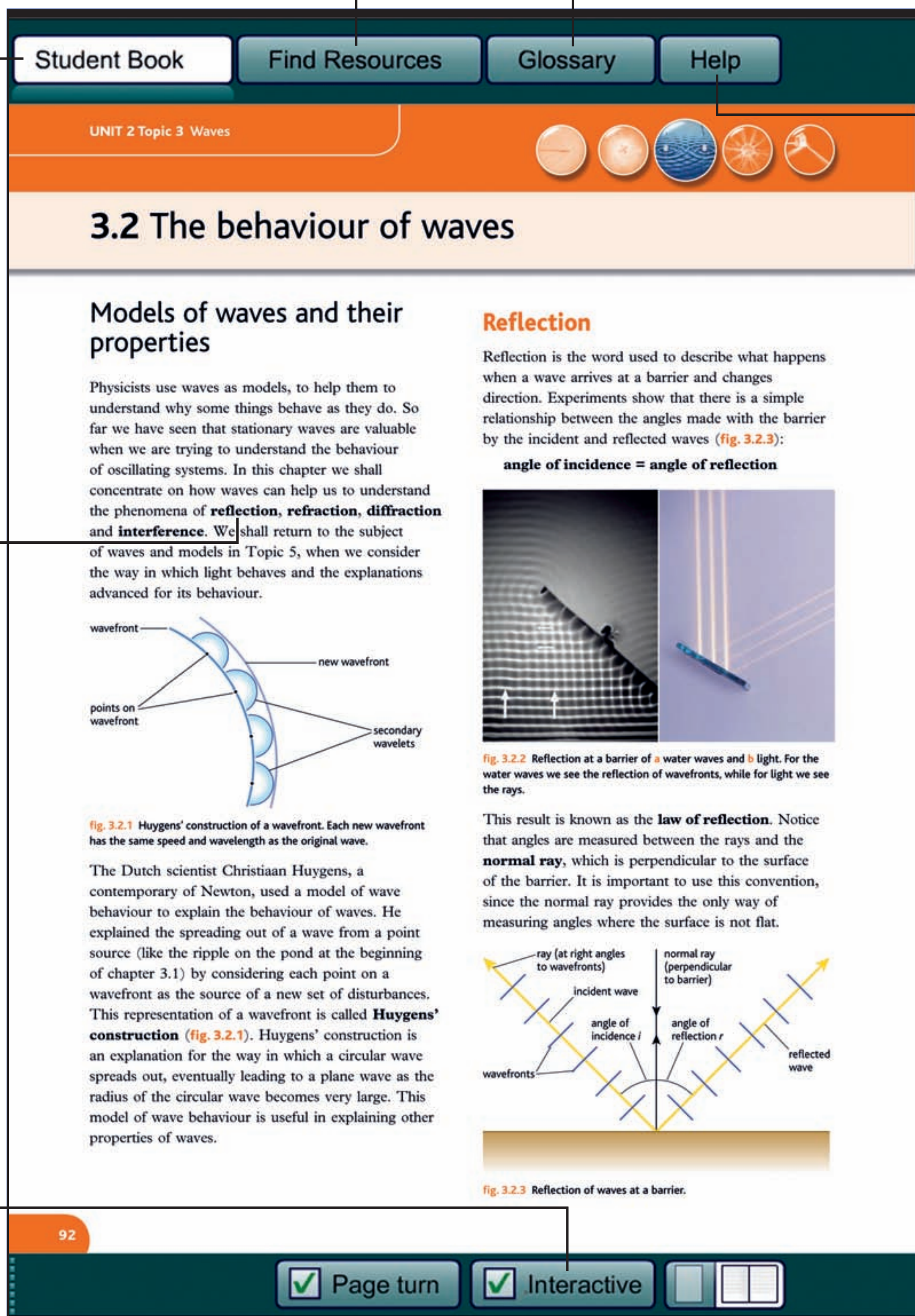
Click this tab at the top of the screen to access the electronic version of the book.

**Key words**

Click on any of the words in **bold** to see a box with the word and what it means. Click 'play' to listen to someone read it out for you to help you pronounce it.

**Interactive view**

Click this button to see all the icons on the page that link to electronic files, such as documents and spreadsheets. You have access to all of the features that are useful for you to use at home on your own. If you don't want to see these links you can return to **Book view**.



Student Book | Find Resources | Glossary | Help

UNIT 2 Topic 3 Waves

## 3.2 The behaviour of waves

### Models of waves and their properties

Physicists use waves as models, to help them to understand why some things behave as they do. So far we have seen that stationary waves are valuable when we are trying to understand the behaviour of oscillating systems. In this chapter we shall concentrate on how waves can help us to understand the phenomena of **reflection**, **refraction**, **diffraction** and **interference**. We shall return to the subject of waves and models in Topic 5, when we consider the way in which light behaves and the explanations advanced for its behaviour.



fig. 3.2.1 Huygens' construction of a wavefront. Each new wavefront has the same speed and wavelength as the original wave.

The Dutch scientist Christiaan Huygens, a contemporary of Newton, used a model of wave behaviour to explain the behaviour of waves. He explained the spreading out of a wave from a point source (like the ripple on the pond at the beginning of chapter 3.1) by considering each point on a wavefront as the source of a new set of disturbances. This representation of a wavefront is called **Huygens' construction** (fig. 3.2.1). Huygens' construction is an explanation for the way in which a circular wave spreads out, eventually leading to a plane wave as the radius of the circular wave becomes very large. This model of wave behaviour is useful in explaining other properties of waves.

### Reflection

Reflection is the word used to describe what happens when a wave arrives at a barrier and changes direction. Experiments show that there is a simple relationship between the angles made with the barrier by the incident and reflected waves (fig. 3.2.3):

**angle of incidence = angle of reflection**

fig. 3.2.2 Reflection at a barrier of **a** water waves and **b** light. For the water waves we see the reflection of wavefronts, while for light we see the rays.

This result is known as the **law of reflection**. Notice that angles are measured between the rays and the **normal ray**, which is perpendicular to the surface of the barrier. It is important to use this convention, since the normal ray provides the only way of measuring angles where the surface is not flat.



fig. 3.2.3 Reflection of waves at a barrier.

92

Page turn | Interactive

## Glossary

Click this tab to see all of the key words and what they mean. Click 'play' to listen to someone read them out to help you pronounce them.

## Help

Click on this tab at any time to search for help on how to use the ActiveBook.



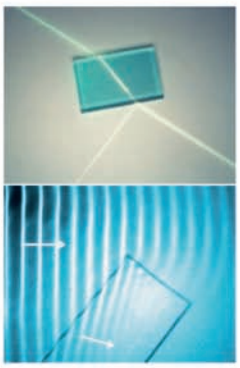


fig. 3.2.4 Refraction of a light passing through a glass block and b water waves at a change of depth. The water waves in part b are moving more slowly on the right of the photo, where the water is shallower.

### Refraction

**Refraction** is the change of direction of a wave that occurs when its speed changes. Refraction can be seen when light travels from one medium into another, say from air into glass or from glass into air (fig. 3.2.4a). Refraction can also be seen when water waves move from deeper water into shallower water, or vice versa (fig. 3.2.4b).

Experiments with light show that there is a straightforward relationship between the angle made by the incident ray with the normal ray (the angle of incidence $i$) and the angle made by the refracted ray with the normal ray (the **angle of refraction** $r$) (fig. 3.2.5). This relationship is known as **Snell's law**, and is expressed as:

$$\frac{\sin i}{\sin r} = \text{a constant}$$

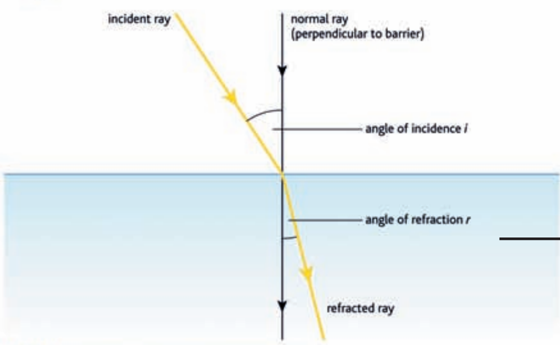



fig. 3.2.5 Refraction at a boundary between two media.

The constant is called the **refractive index** for the medium and is represented by the symbol $\mu$, the Greek letter mu (although sometimes you will see the letter $n$ used to represent refractive index). Table 3.2.1 shows values of refractive index for various substances.

The actual media involved in carrying the waves before and after refraction are very important. It is common to write the refractive index with subscripts indicating the medium in which the wave starts and finishes. In moving from medium 1 into medium 2, Snell's law is written as:

$${}_1\mu_2 = \frac{\sin i}{\sin r}$$

| Material | Crown glass | Diamond | Liquid water | Ice | Benzene | Air |
|---|---|---|---|---|---|---|
| Refractive index | 1.52 | 2.42 | 1.33 | 1.31 | 1.50 | 1.0003 |

table 3.2.1 Values of refractive index for different materials. Since refractive index varies with wavelength for many media, the values are quoted for light with a wavelength of $5.89 \times 10^{-7}$ m entering the medium from a vacuum (or air).



## Zoom feature

Just click on a section of the page and it will magnify so that you can read it easily on screen. This also means that you can look closely at photos and diagrams.

# CONTENTS

## Unit 1 Physics on the go

### Topic 1 Mechanics



### Topic 2 Materials

# Unit 2 Physics at work

## TOPIC 3 Waves



## TOPIC 4 DC electricity



## TOPIC 5 Nature of light

# Topic 1 Mechanics

This topic explains the movements of objects. It looks at how movement can be described and recorded, and then moves on to explaining why movement happens. It covers velocity and acceleration, including how to calculate these in different situations. Additionally, the effect of gravity on the movement of an object leads into consideration of the energy a body may possess or transfer.

## What are the theories?

We only consider objects moving at speeds that could be encountered in everyday life. At these speeds (much less than the speed of light) Sir Isaac Newton succinctly described three laws of motion. With a knowledge of basic geometry, we can identify aspects of movement in each dimension. These three laws then allow us to calculate accurately the motion of any object over time and in three dimensions.

There are also equations for calculating kinetic energy and gravitational potential energy, and the transfer of energy when a force is used to cause the transfer. These formulae and Newton's laws can be used together to work out everything we might wish to know about the movement of any everyday object in any everyday situation.

## What is the evidence?

Newton's laws of motion have been constantly under test by scientists ever since he published them in 1687. Within constraints established by Einstein in the early twentieth century, Newton's laws have always correctly described the relationships between data collected. You may have a chance to confirm Newton's laws in experiments of your own. With modern ICT recording of data, the reliability of such experiments is now much improved over traditional methods.

Whilst it is difficult for scientists to describe or identify the exact nature of energy, the equations that describe energy relationships have also consistently held up to experimental scrutiny.

## What are the implications?

Combining the mathematical rules presented in this topic allows us to describe and predict the motion of all things. This statement must always be tempered by the limitations that the objects involved must be macroscopic (everyday sizes) and must be moving at reasonable speeds. Above about 10% of the speed of light, Newton's laws lose their accuracy and it becomes clear that they are, in fact, only an approximation of Einstein's more complete explanations of motion. Furthermore, if we consider subatomic particles, of which Newton knew nothing, we discover that quantum mechanics throws a probability spanner in the works.

At the end of the chapter we see the power of the equations in action as they describe the motion of the ball in a game of hockey.

The map opposite shows you all the knowledge and skills you need to have by the end of this topic. The colour in each box shows which chapter they are covered in and the numbers refer to the sections in the Edexcel specification.

## Chapter 1.1

identify and use the physical quantities derived from the slopes and areas of displacement–time and velocity–time graphs, including cases on non-uniform acceleration (3)

distinguish between scalar and vector quantities and give examples of each (5)

combine two coplanar vectors at any angle to each other by drawing (part of 7)

understand how ICT can be used to collect data for, and display, displacement–time and velocity–time graphs for uniformly accelerated motion and compare this with traditional methods in terms of reliability and validity of data (2)

use the equations of uniformly accelerated motion in one dimension (1)

## Chapter 1.2

use $\Sigma F = ma$ in situations where $m$ is constant (Newton's first law of motion ($a = 0$) and second law of motion) (9)

draw and interpret free-body force diagrams to represent forces on a particle or on an extended rigid body, using the concept of *centre of gravity* (8)

combine two coplanar vectors at any angle to each other by drawing, and at right angles by calculation (7)

identify pairs of forces constituting an interaction between two bodies (Newton's third law of motion) (11)

use the expressions for gravitational field strength $g = f/m$ and weight $W = mg$ (10)

recognise and make use of the independence of vertical and horizontal motion of a projectile moving freely under gravity (4)

resolve a vector into two components at right angles to each other by drawing and calculation (6)

## Chapter 1.3

apply the principle of conservation of energy including use of work done, gravitational potential energy and kinetic energy (14)

use the expression for work $W = F\Delta S$ including calculations when the force is not along the line of motion (15)

calculate power from the rate at which work is done or energy transferred (17)

use the relationship $E_k = ½mv^2$ for the kinetic energy of a body (12)

use the relationship $\Delta E_{grav} = mg\Delta h$ for the gravitational potential energy transferred near the Earth's surface (13)

understand some applications of mechanics to sports (16)

recognise and use the expression efficiency = useful energy (or power) output / total energy (or power) input

From Topic 5 (70)

# 1.1 Motion


fig. 1.1.1 Average speed does not describe the speed at any particular instant.

## Describing motion

Movement is a central part of our world and the Universe in which we live, whether you look for it at the scale of atoms – around $10^{-9}$ metres – or at the scale of our planet orbiting the Sun – around $10^{11}$ metres. To understand movement is to understand one of the most fundamental aspects of us and our world. This is where we shall start.

### Speed and distance

Before we can attempt to begin to understand movement and what causes it, we need to be able to describe it. Let us look at the most obvious aspect of motion – **speed**.

How fast is something moving? An object's speed is calculated by dividing the distance moved by the time taken to move that distance. In the language of physics, we say that speed is *distance moved in unit time*, or:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

So what was your speed on your way to your home yesterday? If you travelled 3 km in 15 minutes (0.25 h), your answer to this might be 12 km h$^{-1}$ (3 km ÷ 0.25 h). But this isn't the whole story, as **fig. 1.1.1** shows.

It is clearly unlikely that anyone will cycle at a constant speed, even without hills and stops at a shop to cope with. So the calculation of distance ÷ time in the example above tells us simply the **average speed** for the journey. It doesn't tell us anything about the speed at any given instant, as would be measured by a speedometer, for example. In fact, **instantaneous speed** is often more important than average speed. If you drive at a speed of 40 mph along a street with a 30 mph speed limit and are stopped for speeding, the police officer will not be impressed by the argument that your average speed in the last 5 minutes was only 30 mph!

It is often very useful to represent motion using a graph. This graph could plot distance against time or it could plot speed against time. **Fig. 1.1.2** shows two graphs for a journey.

O – left home
O to A – accelerating
A to B – travelling at steady speed
B to C – slowing down as going uphill
C to D – travelling at steady speed, slower than between A and B
D to E – slowing down
E to F – stopped
F to G – accelerating again
G to H – travelling at steady speed, faster than between A and B
H to J – slowing down
J – reached destination

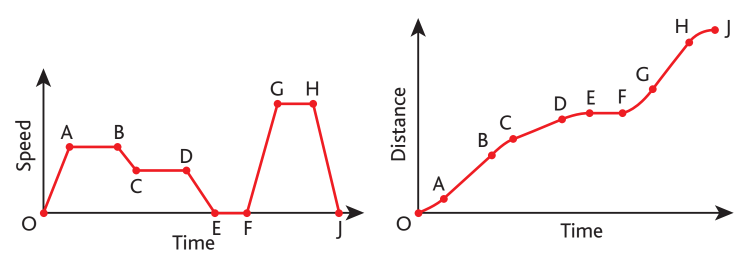



fig. 1.1.2 Speed–time and distance–time graphs for the same journey.

Look at the shapes of the two graphs very carefully. Remember that the second graph shows how far has been travelled from home, while the first graph shows the instantaneous speed of the person. Where the speed–time graph is horizontal between two points, the distance–time graph has a steady slope between the same points because the distance travelled per unit time is constant. Where the speed–time graph has a value of zero, the distance–time graph is horizontal because the person is stationary. In other words, the **slope** or **gradient** of a distance–time graph represents the speed at that particular point, the **instantaneous speed**.

Notice that steady speed corresponds to a straight line on the distance–time graph. Where an object has a steady speed, the slope of the distance–time graph is constant, and the object's average speed and its instantaneous speed are the same.

In the world of physics, speed is usually measured in metres per second ($m\,s^{-1}$) – although we are probably more used to using miles per hour (mph) in our everyday lives.

## HSW Straight line graphs

Graphs are extremely useful in physics for finding and confirming relationships between different variables (for example, the stretching of a piece of metal wire and the load applied to it). The simplest type of relationship is one which is **linear**, in which a graph of one variable against another is a straight line.

**Fig. 1.1.3** is an example of a linear relationship, showing how the speed of an object varies with time. Speed is plotted on the vertical axis (referred to as the '*y*-axis' or the 'ordinate') and time is plotted on the horizontal axis (referred to as the '*x*-axis' or the 'abscissa'). The straight line here shows that the speed of the object increases steadily with time.

The general form of the equation for a straight line is:

$y = mx + c$

where $m$ is the slope or gradient of the line

and $c$ is the **intercept** on the *y*-axis (the point where the line crosses the *y*-axis).

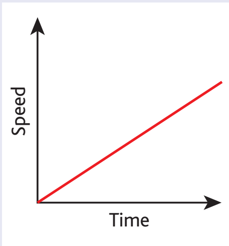



fig. 1.1.3 A speed–time graph where the speed increases steadily with time.

## Recording motion

Sensors connected to a computer can be used to record the position and velocity of an object over time. With these detailed and accurate measurements, computer software can produce graphs of the data automatically.

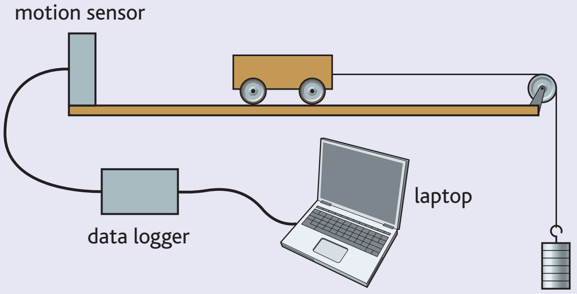



fig. 1.1.4 Data logging motion

Measurements made with electronic sensors will be more precise and more accurate than measurements made by people using stopwatches and rulers, for example. The electronic measurements will not suffer from human errors such as reaction time or misreading of scales. This means that the results and any conclusions drawn will be more reliable.

## Questions

1 A horse travels a distance of 500 m in 40 s. What is its average speed over this distance?

2 Nerve impulses travel at about $100\,m\,s^{-1}$. If a woman 1.8 m tall steps on a drawing pin:

  a roughly how long is it before she knows about it?

  b If she is walking along with a speed of $2\,m\,s^{-1}$, how far will she have travelled in this time?

3 **Fig. 1.1.3** shows a speed–time graph for an object which starts from rest and then steadily increases speed. Sketch speed–time graphs to show the motion of an object which:

  a has an initial speed of $5\,m\,s^{-1}$ at $t = 0$ and which then increases speed at a steady rate

  b starts at rest at $t = 0$, stays at rest for 5 s and then increases speed at a steady rate.

# Distance and displacement

Take a look at the map of the Southampton area in **fig. 1.1.5**. How far is it from Hythe to Southampton Town Quay? Of course, the distance depends on the route you take. By ferry the journey is only 1.5 miles, but by road you would need to travel 12 miles between the two places.

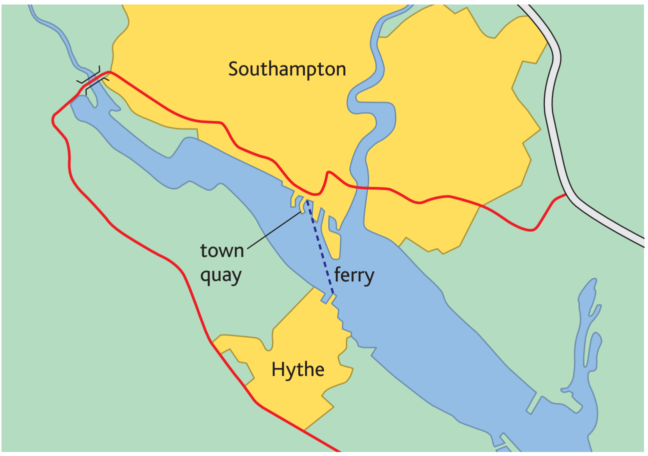



**fig. 1.1.5** The distance between two places depends on the route you take. The displacement of one relative to the other does not vary.

Clearly we need a way of distinguishing between these two meanings of distance. In physics we use the terms **distance** and **displacement** to do this. If you travel from Hythe to Town Quay by road, the *distance* you have travelled is defined as the *length of path* you have taken, as measured by the mileometer in your car. However, your *displacement* is defined as the *straight line distance* between Hythe and Town Quay, as if you had taken the ferry. To describe fully the distance travelled, we only need to say how far you have gone. To describe your displacement, we not only need to specify how far you are from where you started, but also in what direction you would need to travel to get there. Distance is a **scalar** quantity – it has only size or **magnitude**. Displacement is a **vector** quantity – as well as size, it has **direction**.

## Displacement and velocity

We have seen that speed is defined as distance moved in unit time. In the same way, we can now use the definition of displacement to calculate a new quantity, velocity:

$$\text{velocity} = \frac{\text{displacement}}{\text{time taken}}$$

This can also be written in symbol form as:

$$v = \frac{s}{t}$$

Like speed, velocity has magnitude. Like displacement, it also has a direction – it is a **vector**.

## Speeding up and changing velocity

We are quite used to saying that an object **accelerates** as its speed increases. However, the word accelerate has a very precise meaning in physics. As we have just seen, velocity is the rate of change of displacement. Acceleration is the rate of change of velocity, so it is a vector too.

A car moving away from rest increases its speed – it accelerates. Approaching some traffic lights at red, the car slows down – this is also acceleration, but a **negative** one, because speed is taken away as the car slows down.

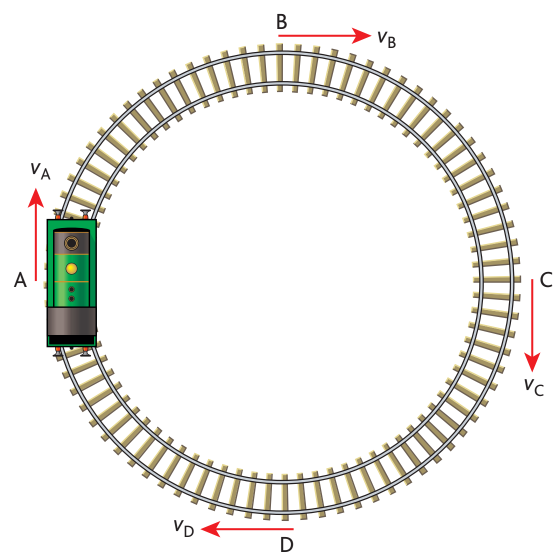



**fig. 1.1.6** Movement around a circular track.

Imagine the model train in **fig. 1.1.6** moving round the track from point A through points B, C and D and back to A again at a steady speed of $0.25\,\text{m}\,\text{s}^{-1}$. Although its speed is the same at A, B, C and D its velocity is not, because the **direction** in which it is moving is different. So although its speed is constant, the train's velocity is changing – and under these circumstances we also say that it is accelerating.

**Acceleration** is defined as **the rate of change of velocity with time**, and happens when there is:

a change in speed
or a change in direction
or a change in speed *and* direction.

$$\text{average acceleration} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken for change}}$$

This can also be written in symbol form as:

$$a = \frac{v - u}{t}$$

Notice one other thing about the model train. Although its average speed between two points is always $0.25\,\mathrm{m\,s^{-1}}$, its average velocity as it goes round the track from A and back to A again is zero, because its displacement is zero.

## HSW Units

A great deal of science is based on measuring physical quantities, such as length and mass. The value of a physical quantity consists of two things – a number, combined with a unit. For example, a length may be quoted as 2.5 km or 2500 m. In order that scientists and engineers can more easily exchange ideas and data with colleagues in other countries, a common system of units is now in use in the world of science. This system is called the **Système Internationale (SI)**, and consists of a set of seven **base units,** with other **derived units** obtained by combining these. The base SI units are the metre (m), the kilogram (kg), the second (s), the ampere (A), the kelvin (K), the candela (cd) and the mole (mol). Each of these base units relates to a standard held in a laboratory somewhere in the world, against which all other measurements are effectively being compared when they are made.

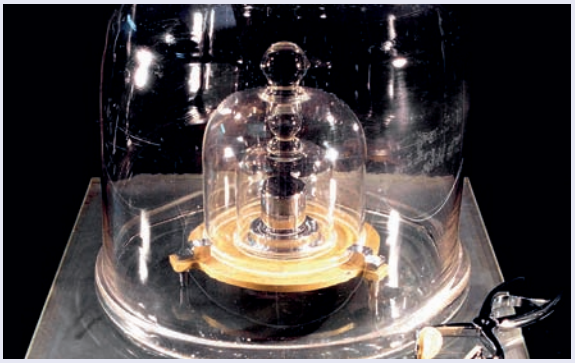


fig. 1.1.7 This platinum–iridium cylinder is the standard kilogram – it is defined as having a mass of exactly 1 kg. When you buy 1 kg of apples at the supermarket you are effectively comparing their mass with the mass of this cylinder!

The units of distance, speed and acceleration show how the base units and derived units are related:

- Distance is a length, and therefore has units of **metres** in the SI system.
- Speed is distance travelled in unit time – so the units of speed (and of velocity too) are **metres per second**. This may be written as metres/second, m/s or $\mathrm{m\,s^{-1}}$. Each of these means the same thing – metres ÷ seconds. Because $\mathrm{s^{-1}} = 1/\mathrm{s}$, $\mathrm{m\,s^{-1}}$ means the same as m/s.
- Acceleration is change in velocity in unit time, and is measured in (m/s)/s – written as $\mathrm{m/s^2}$ or $\mathrm{m\,s^{-2}}$.

For vector quantities like displacement, direction must also be considered. A direction in which displacement is to be measured in a given situation is decided and displacements in this direction are then taken as positive. Velocities and accelerations then take the same sign as displacement. The choice of direction is quite arbitrary – when solving problems, the direction is usually chosen so that the mathematics involved in the solution is as simple as possible.

## Questions

1 A travel brochure says that two airports are 34 km apart, and that airport A lies due south of airport B. The navigation system on board an aircraft travelling from airport A to airport B shows that it covers 380 km. Write down:

**a** the distance travelled by the aircraft as it flies from airport A to airport B

**b** the displacement of the aircraft at the end of the journey.

2 An athlete running in a sprint race crosses the finishing line and slows from a speed of $10\,\mathrm{m\,s^{-1}}$ to rest in 4 s. What is her average acceleration?

# More information from graphs of motion

Velocity–time graphs are particularly useful in providing information about motion. The slope of a distance–time graph gives us information about an object's speed, because speed is rate of change of distance with time. In the same way, the slope of a velocity time graph gives us information about an object's acceleration.

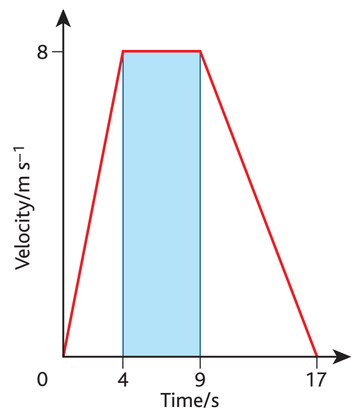



fig. 1.1.8 Velocity–time graph showing acceleration, a steady velocity, and then negative acceleration.

Look at the velocity–time graph in **fig. 1.1.8**. It tells us that:

- the object accelerates from rest to 8 m s$^{-1}$ from 0 s to 4 s. So:

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}$$
$$= \frac{8-0}{4-0} = 2\,\text{m s}^{-2}$$

- there is no change of velocity from 4 s to 9 s – the acceleration is zero
- the object accelerates to rest from 9 s to 17 s. So:

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}$$
$$= \frac{0-8}{17-9} = -1\,\text{m s}^{-2}$$

Where a velocity–time graph is a straight line, the acceleration is uniform. Acceleration may be represented as **the rate of change of velocity with time**.

The graph in **fig. 1.1.8** also gives information about the distance travelled. Between 4 s and 9 s the object travelled with a uniform (constant) velocity of 8 m s$^{-1}$.

We can use this information to work out how far it travelled (its change in displacement) in this time:

$$\text{velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$

so change in displacement = velocity × time taken
= 8 × 5
= 40 m

If you look carefully, you will see that this change in displacement represents the shaded area under the flat part of the graph. Because the area under the graph is calculated by multiplying together a velocity (in m s$^{-1}$) and a time (in s):

$$\frac{\text{m}}{\text{s}} \times \text{s} = \text{m}$$

the answer is in metres, and so represents a displacement.

In the same way, the area under the other parts of the graph represents displacement too:

change in displacement during initial acceleration = ½ × 8 × 4 = 16 m

change in displacement during final acceleration = ½ × 8 × 8 = 32 m

The total displacement for the whole 17 s is:

16 + 40 + 32 = 88 m

## Direction of acceleration

Velocity–time graphs can give information about more complicated situations too, as **fig. 1.1.9** shows.

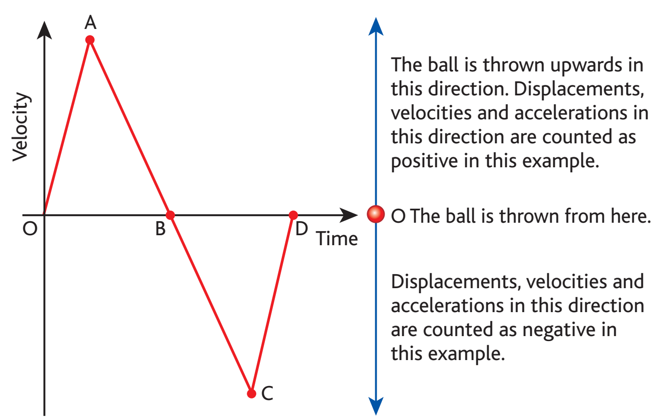



fig. 1.1.9 A highly simplified velocity–time graph for a ball being thrown upwards and then caught again.

The graph shows the motion of a ball thrown upwards, and falling back to Earth again to be caught. The ball starts from rest at time = 0. The graph is a straight line with a positive slope between O and A – this is because the person throwing the ball gives it a uniform upwards acceleration between these two points. The graph is a straight line with a negative slope between A and B – between these points the ball accelerates in a downward direction (slows down) at a steady rate, until it comes to rest at B, the highest point of its trajectory. Between B and C the graph has the same slope as it did between A and B, but its velocity is increasingly negative – it is steadily accelerating downwards (speeding up) between these points on the graph. At C the ball is caught. Between C and D the graph has a large positive slope as the person gives the ball a large upward acceleration to bring it back to rest.

Notice how the slope of each part of the graph tells us about the acceleration of the ball, while the line itself shows how the velocity of the ball changes. Careful measurement of the two areas of the graph OAB and BCD shows that they are equal, although area OAB is positive and area BCD is negative. Since the area under the line represents the ball's displacement, this shows that the ball's displacement upwards (in a positive direction) is equal to its displacement downwards (in a negative direction) – in other words, the ball falls back to Earth the same distance as it rises, and finishes up where it began.

## Non-linear graphs

Although it may not be as straightforward to do, the method of measuring the area under a graph to determine the distance travelled may be used for graphs which are non-linear (not straight lines) too.

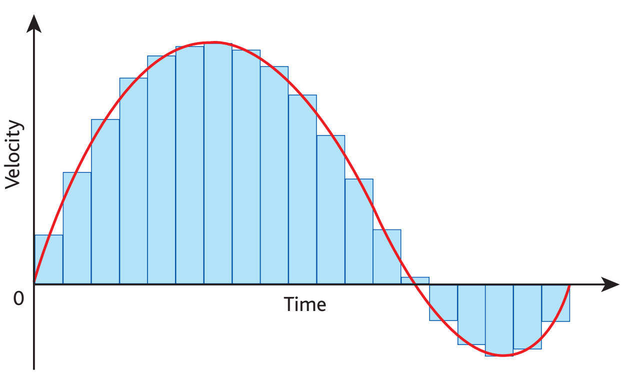



**fig. 1.1.10** Finding distance travelled from a non-linear velocity–time graph.

The area under the line in **fig. 1.1.10** can be calculated by adding together the area of all the strips under the line, each of which is a rectangle. The narrower the strips, the more accurately they represent the area under the line – but the more of them there are to add up. It is important to remember to take into account whether a strip is above or below the *x*-axis when adding its area to the area of the other strips between the line and the axis.

As an example, consider the graph in **fig. 1.1.10** again. The area between the *x*-axis and the line above it is the sum of the areas of all the strips – say 350 m. (Remember the area represents a displacement.) The area between the *x*-axis and the line below it is once more the sum of the strips – say –50 m. The area is negative because it represents a displacement in the opposite direction to the first displacement. So the total displacement is the total area between the *x*-axis and the line, which is 350 m + –50 m = 300 m.

### Questions

**1** A train travelling along a straight track starts from rest at point A and accelerates uniformly to 20 m $s^{-1}$ in 20 s. It travels at this speed for 60 s, then slows down uniformly to rest in 40 s at point C. It stays at rest at C for 30 s, then reverses direction, accelerating uniformly to 10 m $s^{-1}$ in 10 s. It travels at this speed for 30 s, then slows down uniformly to rest in 10 s when it reaches point B.

- **a** Plot a graph of the motion of the train.
- **b** Use your graph to calculate:
  - **i** the train's displacement from point A when it reaches point C
  - **ii** the train's displacement from point A when it reaches point B
  - **iii** the train's acceleration each time its speed changes.

# Equations of motion

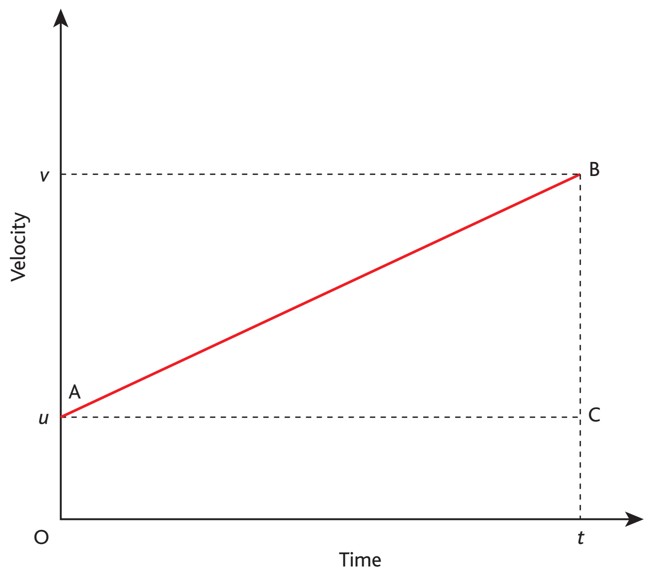



fig. 1.1.11 Velocity–time graph showing initial velocity $u$ and final velocity $v$ after time $t$.

The information shown in **fig. 1.1.11** enables us to write a set of four equations which can be applied in virtually all situations when objects are moving with constant acceleration, no matter what their size.

The slope of the graph tells us the acceleration of the object. If we use the symbol $a$ for acceleration, then we can write:

$$a = \frac{v - u}{t} \text{ i.e. } \frac{\text{(initial velocity – final velocity)}}{\text{time taken for change}}$$

which we can rewrite as:

$$v = u + at \qquad \text{(equation 1)}$$

The area under the graph is the area of the rectangle OACD (which has height $u$ and length $t$), plus the area of triangle ABC on top of it. This area is the object's displacement, $s$:

$$s = ut + ½(v - u)t$$

Equation 1 gives us a relationship between $u$, $v$, $t$ and $a$, so we can substitute $at$ for $(v - u)$ in the new equation:

$$s = ut + ½(at)t$$

or

$$s = ut + ½at^2 \qquad \text{(equation 2)}$$

Since the object's average speed can be calculated from its displacement and time, we can also calculate the object's displacement from its average velocity:

$$\text{average velocity} = \frac{v + u}{2}$$

So:

$$s = \frac{(v + u)\,t}{2} \qquad \text{(equation 3)}$$

Finally, equations 1 and 3 can be combined. Rearrange equation 1:

$$t = \frac{v - u}{a}$$

Substitute this expression for $t$ into equation 3:

$$s = \frac{(v + u)}{2}\,\frac{(v - u)}{a}$$

Multiply each side by $2a$:

$$2as = (v + u)(v - u)$$

Multiply out the brackets:

$$2as = v^2 - u^2$$

which gives:

$$v^2 = u^2 + 2as \qquad \text{(equation 4)}$$

## Using the equations of motion

The four equations of motion are used in a wide range of situations, and it is therefore very important that you know how to apply them. There are five symbols in the equations – if you know the numerical value of any three of these, the numerical value of the other two can always be found.

Always begin problems by writing down the numerical values you know.

## Worked examples

**Example 1**

A girl running in a race accelerates at $2.5\,\mathrm{m\,s^{-2}}$ for the first 4 s of the race. How far does she travel in this time?

Information known:

$u = 0\,\mathrm{m\,s^{-1}}$ (the athlete starts from rest)

$v = ?$

$a = 2.5\,\mathrm{m\,s^{-1}}$ $t = 4\,\mathrm{s}$

$s = ?$

Use equation 2:

$$s = ut + \tfrac{1}{2}at^2$$
$$= 0 \times 4 + \tfrac{1}{2} \times 2.5 \times (4)^2$$
$$= 20\,\mathrm{m}$$

**Example 2**

The driver of a train travelling at $40\,\mathrm{m\,s^{-1}}$ applies the brakes as the train enters a station. The train slows down at a rate of $2\,\mathrm{m\,s^{-2}}$. The platform is 400 m long. Will the train stop in time?

Information known:

$u = 40\,\mathrm{m\,s^{-1}}$ $v = 0\,\mathrm{m\,s^{-1}}$

$a = -2\,\mathrm{m\,s^{-2}}$ (the acceleration is negative as it is in the opposite direction to the velocity)

$t = ?\ \mathrm{s}$

$s = ?\ \mathrm{m}$ (the actual stopping distance of the train is not known – only the length of the platform)

Use equation 4:

$$v^2 = u^2 + 2as$$

Substitute values:

$$(0)^2 = (40)^2 + 2 \times -2 \times s$$

so $0 = 1600 - 4 \times s$

and $s = \dfrac{1600}{4} = 400\,\mathrm{m}$

The train stops just in time.

## Questions

1 A car is travelling along a road at $30\,\mathrm{m\,s^{-1}}$ when a pedestrian steps into the road 55 m ahead. The driver of the car applies the brakes after a reaction time of 0.5 s and the car slows down at a rate of $10\,\mathrm{m\,s^{-2}}$. What happens?

2 The cheetah is the fastest land animal in the world. It can accelerate from rest to $20\,\mathrm{m\,s^{-1}}$ in 2 s, and has a top speed of about $30\,\mathrm{m\,s^{-1}}$ although it can only maintain this for a distance of about 450 m before it has to stop to rest. In contrast, an antelope can run at around $22\,\mathrm{m\,s^{-1}}$ for long periods.

**a** What is a cheetah's average acceleration between rest and $20\,\mathrm{m\,s^{-1}}$?

**b** Assume that a cheetah accelerates up to its top speed with the acceleration in your answer to **a**.

**i** How far will the cheetah travel when it accelerates from rest up to its top speed?

**ii** How long does this acceleration take?

**c** If the cheetah continues at top speed, how long will it be before it has to stop to rest?

**d** If an antelope starts from rest and accelerates to its top speed at the same rate as a cheetah, how far will it travel in the time obtained in your answer to **d**?

**e** If a cheetah chases an antelope and both start from rest, what is the maximum head start the cheetah can allow the antelope?

# Moving in more than one direction – using vectors

So far we have confined ourselves to situations which are real enough, but which do not necessarily cover every type of motion found in our everyday lives. Think carefully about all the examples of motion you have seen so far and you will realise that they have all been concerned with things moving in a straight line. Whilst motion in a straight line does happen, it is usually more complex than that.

**Vectors** give us a fairly simple way of handling motion when it is not in a straight line. Vectors can be represented by arrows drawn to scale. The *length* of the arrow represents the **magnitude** of the vector, while the *direction* of the arrow represents the **direction** of the vector.

## Combining vectors – the triangle rule

The triangle rule can be applied whenever one vector acts followed by another. For example, suppose you travel 30 m due south, and then 40 m due east – what is your displacement from your starting position?

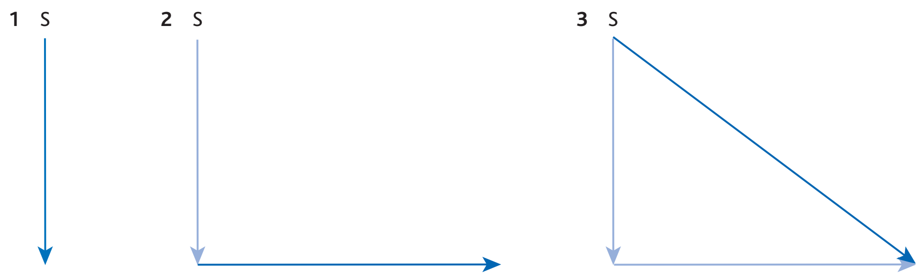



fig. 1.1.12 Adding displacement vectors using a scale diagram. 1 cm represents 10 m.

You can find your final displacement by making a scale diagram of the vectors involved. The diagrams in **fig. 1.1.12** illustrate the process:

1. Draw an arrow 3 cm long from starting point S to show a displacement of 30 m south.
2. Draw an arrow 4 cm long at right angles to the first arrow to show a displacement of 40 m east.
3. Join the starting point S to the end of the second arrow. This vector is your displacement from your starting point.

You can then measure the distance and direction of the displacement from your scale diagram. Alternatively you can use trigonometry to calculate it. In the example in **fig. 1.1.12**, the final displacement is 50 m at an angle of 53° east of the first displacement.

The sum of two or more vector quantities is called their **resultant**.

fig. 1.1.13 Getting nowhere fast! $v_{\text{woman}} + v_{\text{walkway}} = 0$

## Combining vectors – the parallelogram rule

The parallelogram rule can be applied whenever vectors act at the same time or from the same point. If you have ever walked or run up a down escalator, you will have some idea of what **relative motion** is. When an object is moving, it is often very important to give some sort of information about what its motion is relative to. For example, someone running along a moving walkway may have a velocity of 2 m s$^{-1}$ relative to the walkway – but if the walkway has a velocity of –2 m s$^{-1}$ (note the negative sign, showing that the walkway is moving in the opposite direction to the person), the person will remain in the same position relative to the ground.

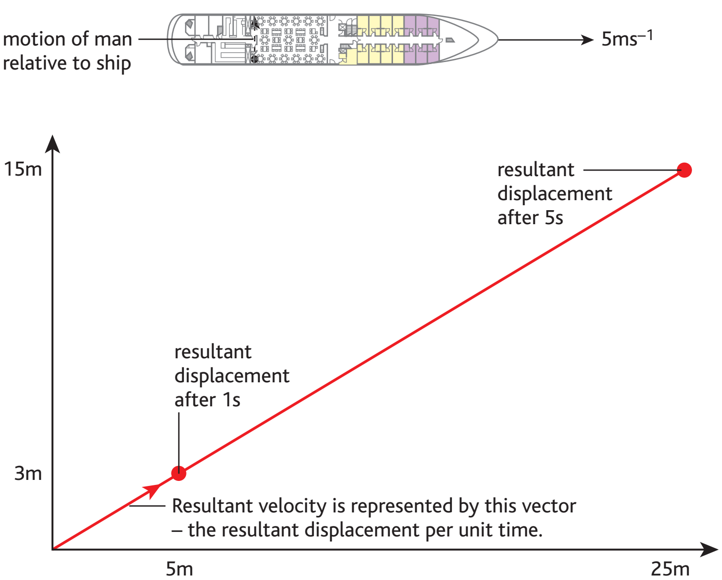



fig. 1.1.14 Adding velocity vectors to find the resultant velocity.

The resultant velocity of the woman in fig. 1.1.13 is the sum of the vectors for her velocity relative to the walkway and the velocity of the walkway relative to the ground. Adding vectors in this type of situation, when both vectors act along the same line is easy – but a slightly different method is needed when they act along different lines.

Think about a man on a ship walking from one side of the ship to the other. If the ship is steaming forwards with a speed of 5 m s$^{-1}$ and the man walks from one side to the other with a speed of 3 m s$^{-1}$, what will be the man's movement relative to the Earth's surface?

As the man walks across the ship in fig. 1.1.14, the ship carries him to the right. In 1 s the man moves 3 m across the boat, and in this time the ship carries him 5 m to the right. The vector diagram shows his displacement 1 s and 5 s after starting to walk. The man's resultant velocity relative to the Earth is the vector shown. This is the resultant of the ship's velocity relative to the Earth's surface and the man's velocity relative to the ship. The resultant velocity is 5.8 m s$^{-1}$ in a direction making an angle of 31° with the velocity of the ship.

### Questions

1. Why do aircraft take off and land into the wind?
2. A ball on a snooker table is hit by another ball and travels a distance of 50 cm due west. It is then hit again and travels a distance of 30 cm due north. Using a scale drawing, or by calculation, work out the snooker ball's displacement from its starting position.
3. A ship is travelling at 5 m s$^{-1}$ with a bearing of 20° east of north. There is a current of 1 m s$^{-1}$ flowing from the west. What is the resultant velocity of the ship?

# 1.2 Forces

## Causes of motion

Having looked at ways of describing motion, let us look at what causes motion. The Greek philosopher Aristotle, who lived from 384 to 322 BC, said that the answer to this question was simple – motion was maintained by forces. When the force which made something move stopped acting, the object came to a standstill. In modern contexts, this idea seems quite reasonable when you think about pulling a heavy box along the floor, or pushing a car along a flat road. But what about the situation when you kick a football, for example? Once your foot ceases to be in contact with the ball, it can no longer exert a force on it, and yet the ball carries on moving for some considerable time before it eventually comes to rest.

The Italian scientist Galileo Galilei thought about problems like this, nearly 2000 years after Aristotle (Galileo was born in 1564). Galileo understood that the idea of force is central to the understanding of motion, but realised that Aristotle's explanation was incomplete. According to one story, Galileo's interest in moving objects began as a result of attending a mass in the cathedral at Pisa. During the sermon, he noticed that a cathedral lantern suspended from the roof by a long chain always took the same time to swing, whether it was swinging through a large arc or a small one. (Not having a clock, he used his own heartbeat to time the swings.) Carrying out further experiments with a pendulum, Galileo noticed that a pendulum bob always rose to very nearly the same height as it had been released from on the opposite side of its swing. Carrying this investigation further, he fixed a pin below the point of support of a simple pendulum. He raised the bob to one side and released it. The bob still rose to the height from which it was released.

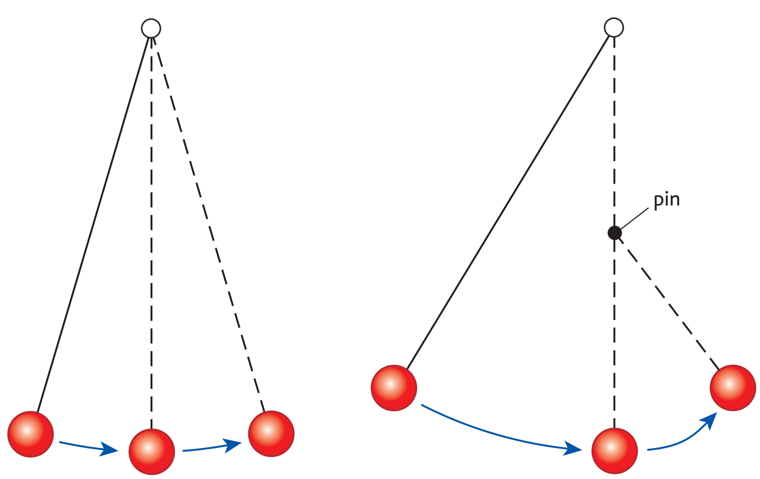



fig. 1.2.1 Galileo's pendulum experiment, which confirmed his ideas on forces and motion.

## The ball and the pendulum

Galileo extended his experiment with the pendulum by carrying out a 'thought experiment', that is, one which he carried out in his head. He reasoned that if a ball rolls down a slope onto an infinitely long flat surface, by simple analogy with the pendulum experiment it will continue moving until something else causes it to stop.

**Fig. 1.2.2** outlines Galileo's thought experiment.

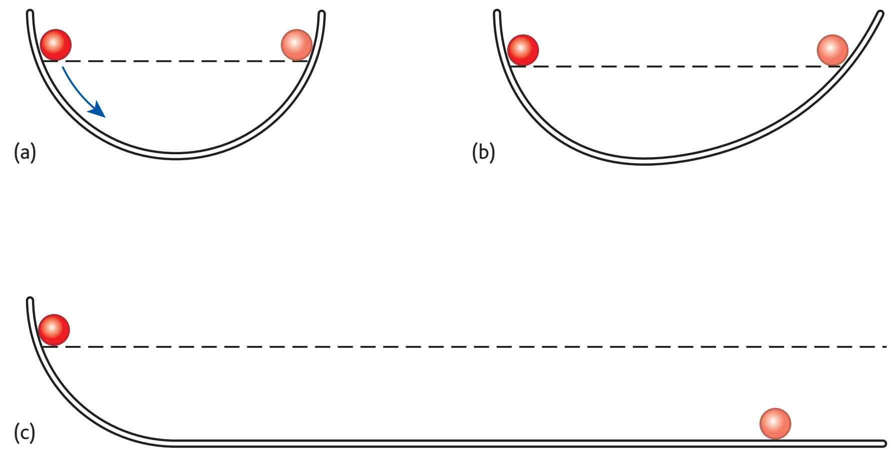



**fig. 1.2.2** The diagrams show Galileo's thought experiment. The track and balls are perfectly smooth, so that there are no frictional forces between them.

By careful analogy with his pendulum experiment, Galileo reasoned that the ball would always tend to rise to the same height as it had been released from, even if it had to travel a greater horizontal distance to do so – diagrams (a) and (b) in **fig. 1.2.2** show this happening. When the rising track on the right-hand side is replaced by a flat track (c), the ball carries on moving indefinitely in an attempt to rise to its original height.

This is in direct conflict with Aristotle's explanation of the motion of objects – although it took the work of Newton to carry forward Galileo's explanation and put it on a basis that we would today recognise as being 'scientific'. Galileo had realised the importance of distinguishing between motion horizontally and vertically in a gravitational field, and had laid the foundations of the journey to the Moon, over 300 years later.

### Questions

1. Aristotle argued that a force was needed in order to keep an object moving. Describe some everyday situations that are consistent with this argument. Suggest a more scientific explanation for each case that you describe.
2. 'Galileo had … laid the foundations of the journey to the Moon.' Write a short piece for a newspaper aimed at a non-scientific audience, showing why Galileo's work was so important.

# Newton's first law of motion

## Forces and changing motion

The key to understanding motion is to understand forces and their interactions. The reason why we appear to need to push something to keep it moving steadily is because the motion of any object here on Earth is opposed by friction forces – and in many cases these are quite considerable. If there were no friction forces, then one push would cause an object to move indefinitely along a flat surface at a steady speed. Galileo had noticed that the concept of force was important when thinking about *changing motion* rather than motion in its own right. Galileo's work was taken up and developed by Isaac Newton, born in Lincolnshire, England in 1642, the year of Galileo's death. Building on Galileo's work, Newton framed three simple rules governing the motion of objects, which he set out as his three laws of motion in his work the *Principia*, published in 1687.

Although we now know that Newton's laws of motion break down under certain conditions (in particular, as the velocity of an object approaches the velocity of light), the laws are very nearly correct under all common circumstances.

The *Principia* was written in Latin, the language of scholarship of the time. Translated into modern English, the first law can be stated as:

> **Every object continues in its state of rest or uniform motion in a straight line unless made to change by the total force acting on it.**

In other words, an object has a constant velocity (which may be zero) until a force acts on it. So the first law of motion defines for us what a force *is*, or rather what it *does* – a force is something which can cause **acceleration**.

## Newton's first law in mathematical terms

Since Newton's first law expresses motion in terms of the total force acting on a body, it can be written down involving mathematical terms. If we wish to write down 'the sum of all the forces acting on a body' we can use the mathematical expression $\Sigma F$ (sigma $F$) to do this. So to state Newton's first law we can say:

> **If a body has a number of forces $F_1, F_2 \dots F_n$ acting on it, it will remain in a state of constant motion only if:**
>
> $\Sigma F = 0$
>
> **(that is, the sum of all the forces from $F_1$ to $F_n$ is equal to zero).**

This can be calculated separately for horizontal and vertical forces. The effects of all horizontally acting forces are completely independent of those for all vertically acting forces. You will see later that this is also true for horizontal and vertical velocities, and for any pair of vectors at right angles to each other.

## Free-body diagrams

Before considering the first law further, it is worth looking at how we can represent clearly the forces acting on a body.

Because a force can cause acceleration, it is a vector quantity, with both magnitude and direction. It therefore requires a way of representing both magnitude and direction on a diagram. A diagram which shows all the forces acting on a body in a certain situation is called a **free-body diagram**. A free-body diagram does not show forces acting on objects other than the one being considered.

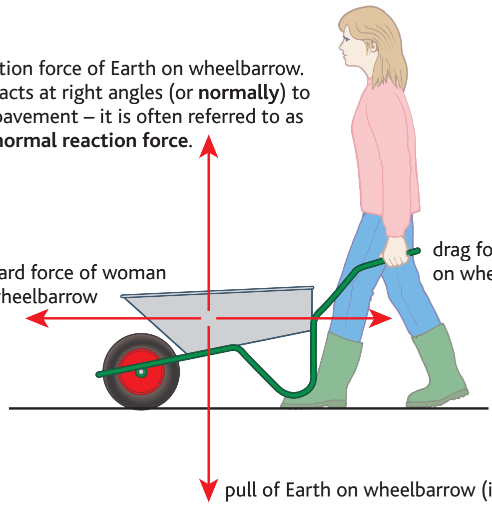



**fig. 1.2.3** A simplified free-body diagram of a wheelbarrow being pushed at a steady speed along a flat surface. Notice how each force acting is cancelled out by a force exactly equal in size but opposite in direction to it. This is what Newton's first law tells us – the resultant force acting on something with constant velocity is zero.

## Centre of gravity and centre of mass

In problems involving solid objects, we often draw the weight of an object as acting through a single point. This point is called the **centre of gravity**, and the justification for doing this is quite straightforward.

If we think of a ruler balanced at its midpoint, we would draw a free-body diagram of the forces acting on the ruler like that shown in **fig 1.2.4**.

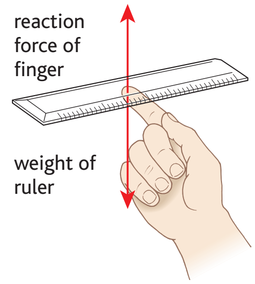



**fig. 1.2.4** A balanced ruler.

This diagram assumes that we can think of the weight of the ruler as acting at its midpoint. We can justify this is by thinking of the Earth pulling vertically downwards on each particle of the ruler. As each particle on one side of the ruler has a similar particle on the other side of the ruler exactly the same distance away from the ruler's centre, the ruler will balance when it is suspended at its midpoint.

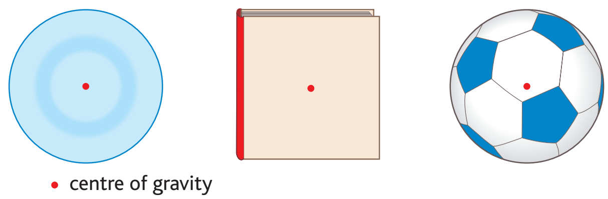



**fig. 1.2.5** The centre of gravity of some uniform objects.

The centre of gravity of an object is the point at which the weight of the object appears to act. An object's **centre of mass** has a similar definition – it is the point at which all the object's mass may be considered to be concentrated. In most common circumstances (in a uniform gravitational field) an object's centre of mass and centre of gravity are at the same place, although this is not always so.

For uniform objects, the centre of mass will be at the intersection of all lines of symmetry, essentially in the middle of the object.

### Questions

1 Draw a free-body diagram showing the forces acting on a racing car moving at constant velocity along a track. Explain how Newton's first law is satisfied for this racing car.

2 Draw a free-body diagram of a wooden block balanced on a person's finger. Label the forces acting on the block and its centre of gravity.

# Drag forces

Once we see the situation represented in a free-body diagram like that for the wheelbarrow in **fig. 1.2.3**, it becomes quite obvious why an object stops moving when you stop pushing it. Remove the forward force acting on it and the forces on an object are no longer balanced. The resultant force now acts backwards, so the wheelbarrow accelerates backwards – that is, it slows down and eventually stops.

So why doesn't an object start moving backwards once it has stopped if there is now a resultant force acting on it? The answer to this question is because of the way that drag forces work. Drag forces in an example like the wheelbarrow are made up of two types of force – friction and air resistance – both due to matter in contact with other matter.

Where two solid surfaces rub on each other (for example in a wheel bearing or axle) **friction** always occurs. Even though they may appear perfectly smooth, the surfaces in contact are slightly rough (**fig. 1.2.6**). It is this roughness that is the cause of friction, as the two surfaces rub over one another (**fig. 1.2.7**).

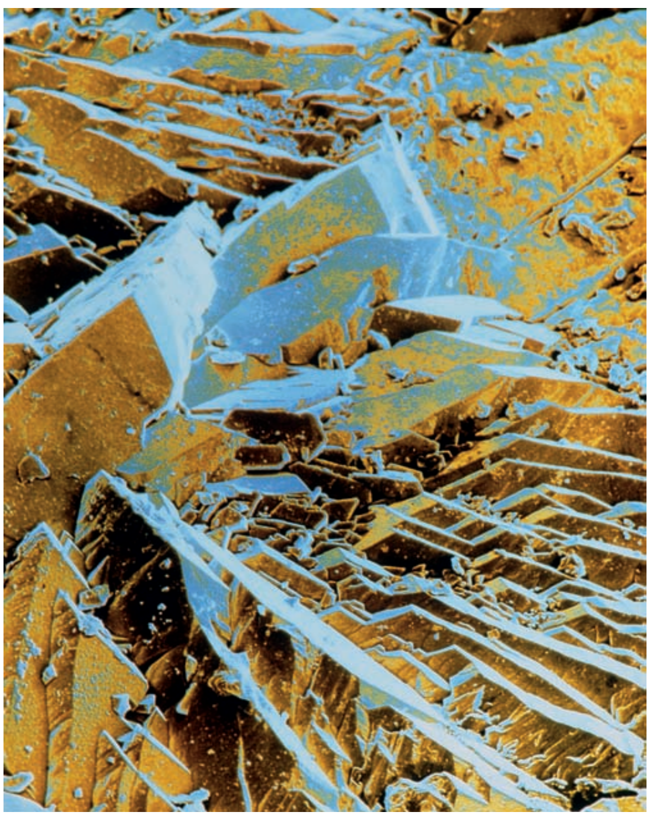


fig. 1.2.6 Even the smoothest of surfaces is rough, as this high magnification photograph of a metal surface shows.

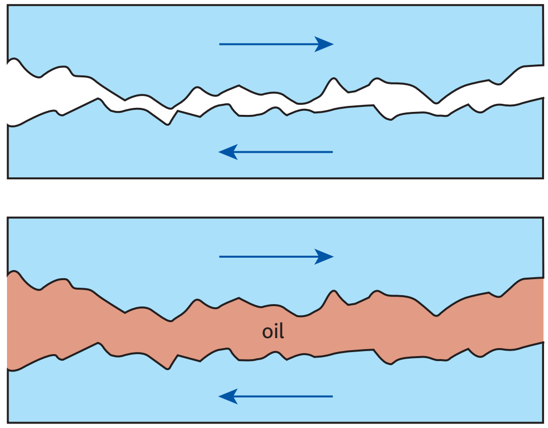



fig. 1.2.7 When two surfaces move over each other, this roughness makes it more difficult to move the surfaces – this is what we experience as friction. Oil between the surfaces pushes them apart and so reduces the frictional force between them.

This frictional force acts simply to *oppose* any motion that takes place – it cannot actually *cause* motion, as you will see if you think about the *cause* of the force. When an object comes to rest, the frictional force stops acting – it will only become important again when the object begins to move again.

## Measuring frictional forces

Experiments between surfaces rubbing over each other show that there are two situations when friction is acting, depending on whether the surfaces are sliding over one another or not. Consider the situation shown in **fig. 1.2.8**, in which a block is dragged along a flat surface whilst a forcemeter shows the force needed to keep it moving at constant velocity.

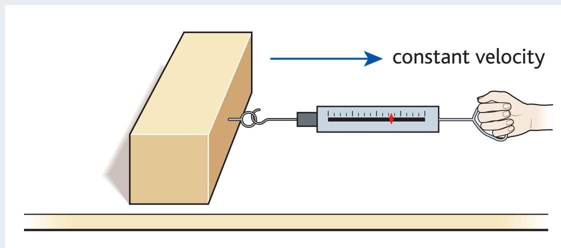



fig. 1.2.8 At constant velocity, the net force is zero.

Newton's first law tells us that, for an object which is not accelerating, $\Sigma F = 0$. This means that the frictional force resisting the motion of the box must be exactly balanced by the pulling force from the hand.

## Air resistance

The other drag force which acts in this example is not important – but it is very important in many other examples. Air resistance or **aerodynamic drag** is caused when a body moves through air. In the example with the wheelbarrow, this is so small as to be insignificant.

Aerodynamic drag is caused by the fact that an object has to push air out of the way in order to move through it – and this requires a force. The force that is exerted by two surfaces rubbing together does not depend on the speed at which the two surfaces move over each other. However, the aerodynamic drag caused by an object moving through air does depend on speed – the faster the object moves, the greater the aerodynamic drag. You will learn more about this in chapter 2.1.

fig. 1.2.9 Air resistance becomes more important the faster you want to go. Careful design can reduce aerodynamic drag, by producing shapes that can 'cut through' the air and cause as little disturbance to it as possible.

Because aerodynamic drag increases as an object's velocity increases, objects with a constant driving force tend to reach a maximum velocity when they accelerate – whether they are a parachutist falling through air or a car travelling along a race track.

## Free fall and terminal velocity

Someone who jumps out of a tethered balloon some way above the ground accelerates towards the ground under the influence of their own weight. They will suffer air resistance which is not insignificantly small – ask any skydiver! The acceleration with which they start to fall is called the **acceleration of free fall** or the **acceleration due to gravity**. At the surface of the Earth the value of this acceleration is $9.81\,\text{m}\,\text{s}^{-2}$. The acceleration of such a falling object is not uniform, as **fig. 1.2.10** shows.

At the top of the jump, the man is instantaneously stationary, so his air resistance is zero. The resultant force acting on him is greatest at this point, so his acceleration at this point has its maximum value.

A little later the man is moving more rapidly and his air resistance is now significant. The magnitude of his weight is still greater than his air resistance, so he is still accelerating downwards, but not as quickly as at first.

Later still his velocity has reached a point where his air resistance is equal to his weight. Now the resultant force acting on him is zero – and he is no longer accelerating. The velocity at which this happens is called the **terminal velocity**. For a human being without a parachute, terminal velocity is about $56\,\text{m}\,\text{s}^{-1}$.

On opening the parachute, the air resistance increases dramatically due to the parachute's large surface area. Now the air resistance is greater than the weight – so the resultant force on the man is upwards. The man accelerates upwards and his velocity decreases.

Eventually the man's velocity decreases to a new terminal velocity. This terminal velocity is much lower than the previous terminal velocity – about $10\,\text{m}\,\text{s}^{-1}$. Hitting the ground at this speed still requires some care – it is like jumping off a wall 5 m high!

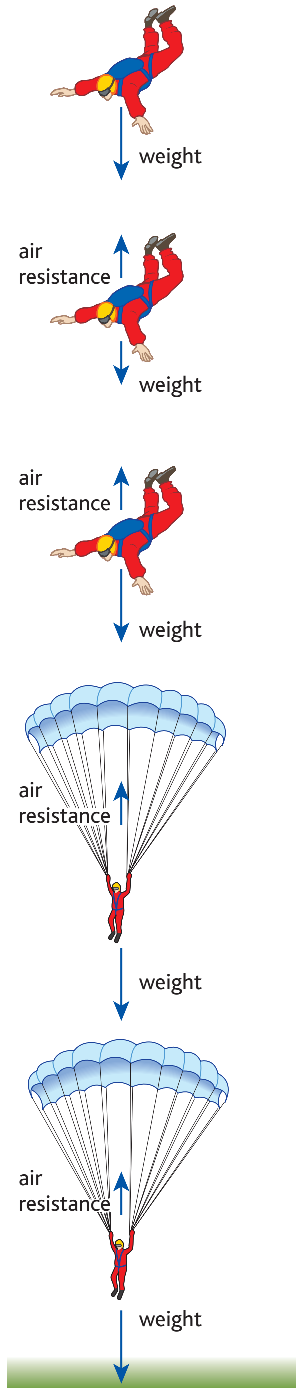



fig. 1.2.10 Free fall and terminal velocity.

### Questions

1 Draw a free-body diagram showing the forces acting on a skydiver at the instant they jump from a plane.

2 Describe and explain how the resultant force on a skydiver varies from the moment they jump from a plane.

# Newton's second law of motion

Having established a connection between force and acceleration which is qualitative, Newton went on to find a quantitative connection between these two. He claimed that:

$$\Sigma F = ma$$

## Investigating the relationship between *F*, *m* and *a*

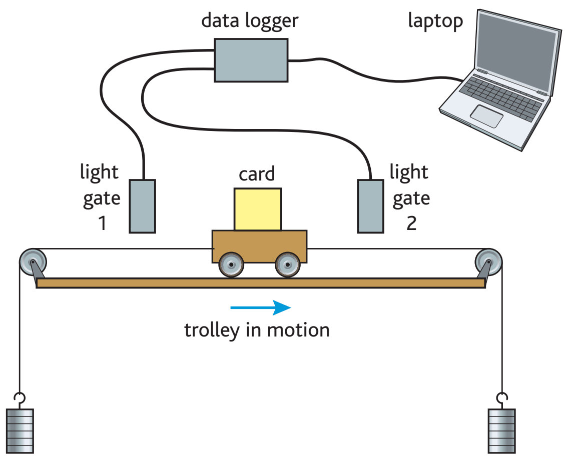



fig. 1.2.11 Experimental setup for investigating the relationship between *F*, *m* and *a*.

Using the setup shown in **fig. 1.2.11**, the acceleration can be measured for various values of the resultant force acting on the trolley while its mass is kept constant (**table 1.2.1**). By plotting a graph of acceleration against resultant force, a straight line will show that acceleration is proportional to the resultant force. A graph could also be plotted for varying masses of trolley while the resultant force is kept constant (**table 1.2.2**).

| Force *F*/N | Acceleration *a*/m s$^{-2}$ |
|---|---|
| 0.1 | 0.20 |
| 0.2 | 0.40 |
| 0.3 | 0.60 |
| 0.4 | 0.80 |
| 0.5 | 1.00 |
| 0.6 | 1.20 |

table 1.2.1 Values of acceleration for different forces acting on a glider of mass 0.5 kg.

| Mass/kg | Acceleration/m s$^{-2}$ |
|---|---|
| 0.5 | 1.00 |
| 0.6 | 0.83 |
| 0.7 | 0.71 |
| 0.8 | 0.63 |
| 0.9 | 0.55 |
| 1.0 | 0.50 |

table 1.2.2 Values of acceleration resulting from an applied force of 0.5 N when the mass of the glider is varied.

It is clear from **table 1.2.1** that there is a direct relationship between $F$ and $a$, and that $a$ is proportional to $F$ (i.e. as $F$ increases by a factor $x$, so does $a$). This can be represented as $F \propto a$.

The results in **table 1.2.2** show that there is a different relationship between $a$ and $m$. Here $a \propto 1/m$ (i.e. as $m$ increases by a factor $x$, $a$ changes by a factor of $1/x$). We say that $a$ is inversely proportional to $m$.

These two relationships can now be combined:

$$\left.\begin{array}{l} a \propto F \\ a \propto 1/m \end{array}\right\} \quad a \propto F/m \quad \text{or} \quad F \propto ma$$

Another way to express this is:

$$F = kma$$

where $k$ is a constant.

By using SI units for our measurements of mass and acceleration, the units of force become kg m s$^{-2}$ (the units of mass and acceleration multiplied together). If we **define** the unit of force in such a way that *one* unit of force accelerates a mass of *one* kilogram at a rate of *one* metre per second per second, then the constant in the equation must also have a value of *one*, and so:

$$F = ma$$

The unit of force in this system is of course better known as the **newton**. This equation defines the newton as being the resultant force which produces an acceleration of one metre per second per second when it acts on a mass of one kilogram. The mathematical statement $F = ma$ is sometimes referred to as **Newton's second law of motion**.

## Worked examples using $F = ma$

### Example 1

A runner in a sprint race reaches $9\,\mathrm{m\,s^{-1}}$ in 3 s from the start of the race. If her mass is 50 kg, what force must she exert in order to do this?

Information known:

$u = 0\,\mathrm{m\,s^{-1}}$ $v = 9\,\mathrm{m\,s^{-1}}$

$a = ?$ $t = 3\,\mathrm{s}$

$s = ?$

Use equation 1:

$$v = u + at$$

Substitute values:

$$9 = 0 + a \times 3$$

$$\text{so } a = \frac{9 - 0}{3}$$

$$= 3\,\mathrm{m\,s^{-2}}$$

Now apply $F = ma$:

$$F = 50 \times 3$$

$$= 150\,\mathrm{N}$$

So the athlete needs to exert a force of 150 N in order to accelerate at this rate. (Will she exert this force constantly over the first 3 s of the race? Why?)

### Example 2

An aeroplane lands with a velocity of $55\,\mathrm{m\,s^{-1}}$. 'Reverse thrust' from the engines is used to slow it to a velocity of $25\,\mathrm{m\,s^{-1}}$ in a distance of 240 m. If the mass of the aeroplane is $3 \times 10^4$ kg, what is the size of the reverse thrust supplied by the engines?

Information known:

$u = 55\,\mathrm{m\,s^{-1}}$ $v = 25\,\mathrm{m\,s^{-1}}$

$a = ?$ $t = ?$

$s = 240\,\mathrm{m}$

Use equation 4:

$$v^2 = u^2 + 2as$$

Substitute values:

$$(25)^2 = (55)^2 + 2 \times a \times 240$$

$$\text{so } a = \frac{625 - 3025}{2 \times 240}$$

$$= \frac{-2400}{480}$$

$$= -5\,\mathrm{m\,s^{-2}}$$

Now apply $F = ma$:

$$F = 3 \times 104 \times -5$$

$$= -1.5 \times 10^5\,\mathrm{N}$$

The reverse thrust of the engines is 150 000 N (or 150 kN). (Why was the answer obtained from the equations ***negative***?)

## Questions

1 a Use the results in **table 1.2.1** to plot a graph of acceleration against force.

b Calculate the value of 1/mass for each entry in the first column of **table 1.2.2**, and plot acceleration against (1/mass) for this set of results.

c Calculate the gradient of the best fit line for each graph.

d What conclusions can you draw from your graphs?

2 A railway locomotive with a mass of 70 tonnes accelerates at a rate of $1\,\mathrm{m\,s^{-2}}$. What force does the locomotive exert?

3 A 60 kg woman involved in a car accident is accelerated by her seatbelt from $14\,\mathrm{m\,s^{-1}}$ to rest in 0.15 s.

a What is the average horizontal force acting on her?

b How does this force compare with her weight?

# Inertia, mass and weight

Newton's first law of motion is useful in considering what we mean by the term force – but it can do more too.

The tendency of an object to stay in its state of rest or uniform motion is called its inertia. Inertia is something that we all experience in our everyday lives.

- A large, massive object like a car is harder to get moving than a relatively small, light one like a bicycle.
- Without the help of a seatbelt, it can be hard for someone sitting in a moving car to stop moving when the driver of the car applies the brakes sharply.

Both of these are examples of inertia.

An object's inertia depends only on its **mass**. The definition of mass is very difficult, and you will probably have met the idea that 'mass is a measure of the amount of matter in a body'. While this statement is not false, it is not the whole truth either. The most satisfactory definition of mass uses the idea of inertia. So if two objects A and B have the same acceleration, but the resultant force on object A is $2F$ while that on object B is $F$, then object A must have twice the mass of object B.

Mass has only size, with no direction; it is a scalar quantity.

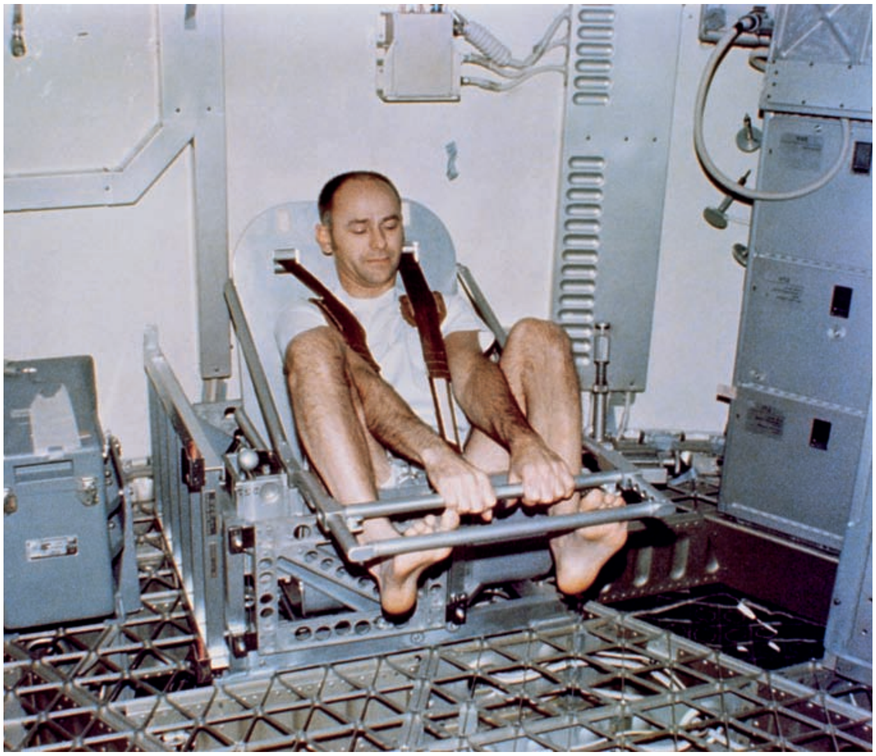


fig. 1.2.12 Newton's first law in action. It is important to know about changes in body mass happening to astronauts during long periods in orbit. Obviously bathroom scales are useless in this situation. This device uses the inertia of the astronaut's body to affect the way in which oscillations happen – the oscillations are then timed and used to calculate the mass of the astronaut.

We often use the term weight in everyday life – sometimes we mean mass, rather than weight, at other times we really do mean weight. An object's weight is a force acting on it. Following Galileo's work, in Book III of the *Principia* Newton set out his theory on how masses attract one another in a process termed **gravitation**. Newton argued that it was this attractive force that we call weight. Our modern interpretation of this theory says that all masses have a **gravitational field** around them. A field is a model which physicists use to explain 'action at a distance' – the way in which two objects not in contact exert a force on each other. Using this model, a mass is said to have a gravitational field around it which causes the mass to attract another mass which is close to it. The size of the field around a particular mass depends on the size of the mass and whereabouts in the field you are.

If another mass is brought into this field, it experiences a force which pulls it towards the first mass. Weight is thus the force which is caused by gravitation – the process which occurs when one mass is brought up to another mass. The size of this force varies with the strength of the gravitational field – and as we shall see later, this varies with the position of the mass in the field. So while mass is constant no matter where it is measured, weight varies according to the strength of the gravitational field an object is in. Because weight is a force, it has both magnitude and direction – it is a vector quantity.

## Gravitational field strength *g* and weight

Because the weight of an object varies according to the strength of the gravitational field it is in, this enables us to define the strength of a gravitational field.

**Gravitational field strength** $g$ at a point in a gravitational field is defined as the force per unit mass acting at that point. In mathematical terms:

$$g = \frac{F}{m}$$

where F is the force acting on the object with mass m. Gravitational field strength has SI units of $\mathrm{N\,kg^{-1}}$, and is a vector quantity. The weight of an object may be calculated from this relationship, giving an expression that you will certainly have used before:

$$W = mg$$

## Measuring mass and weight


fig. 1.2.13 You might think that both these objects are being weighed. In one case you would be wrong.

The digital balance in **fig. 1.2.13** relies on a piece of conducting material being compressed or deformed by a force which changes its shape and hence its electrical resistance. The reading given therefore depends on the force an object exerts on the pan of the balance, and will be different on the Earth from the reading on the Moon – in other words, it measures the object's **weight**.

The beam balance compares the force exerted by the object on one side of the beam with the force exerted by an object of known mass on the other side of the beam. This comparison does not depend on the strength of the gravitational field that the balance is in, and the balance will give the same reading whether it is used on Earth or on the Moon – so this instrument measures the object's **mass**.

## HSW Measuring $g$

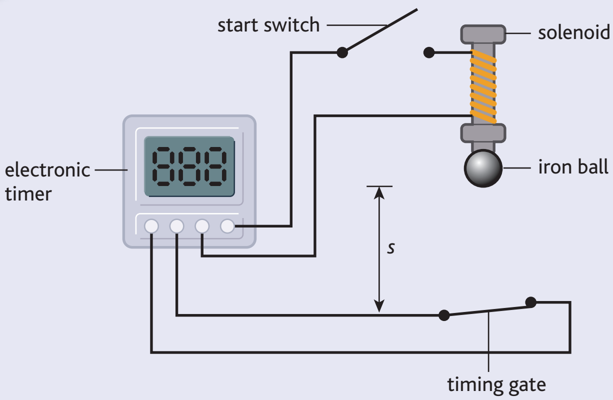



fig. 1.2.14 Apparatus for measuring $g$ by free fall.

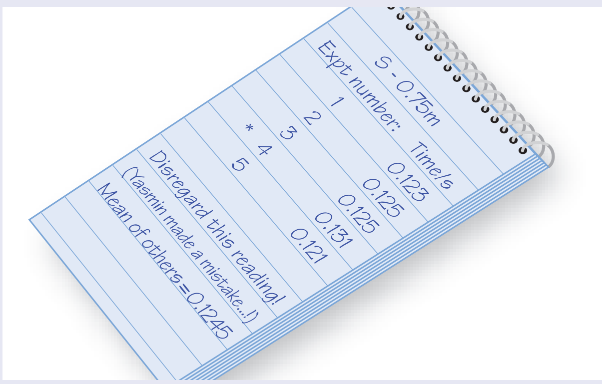



fig. 1.2.15 Free fall results

This is one way of measuring $g$, by analysing the free fall of an object. The iron ball is released by operating the switch, which also starts the timer. The time taken for the ball to fall the vertical distance $s$ is measured as it passes through the timing gate. Since:

$$s = ut + \tfrac{1}{2}at^2 \quad \text{and} \quad u = 0$$

we can write:

$$s = \tfrac{1}{2}gt^2$$

This equation can be used to calculate $g$ directly from one measurement of the falling ball. However, it is better to take *several* readings of the time to fall, take the most consistent ones, average these and then use the average to calculate $g$, since this should lead to a more reliable result (**fig 1.2.15**).

Even better is to notice that $s \propto t^2$. If we vary $s$, and plot the values of $t^2$ we obtain against $s$, the slope of the graph will be $\tfrac{1}{2}g$:

$$s = \tfrac{1}{2}gt^2$$

Compare this with $y = mx + c$ (the equation for a straight line graph). The graph will be a straight line through the origin with a gradient equal to $\tfrac{1}{2}g$.

## Questions

Assume $g = 9.81\,\text{m}\,\text{s}^{-2}$.

1 A person standing on a bus is thrown towards the rear of the bus as it starts to move forwards, and to the front as it slows down. Why?

2 A person standing on the side of a ship drops a coin and sees it splash into the water 2 s later. How far above the water is the person standing?

3 An astronaut on the Moon has a weight of 128 N and a mass of 80 kg. What is the gravitational field strength on the Moon?

4 In 2002 the Canary Wharf tower in London was scaled by French urban climber, Alain Robert, using no safety devices of any kind. The top of the tower is 235 m above street level. If Robert had dropped an apple as he reached the top of the tower:

a how long would it have taken the apple to fall to street level, assuming that air resistance is neglected?

b assuming that he could shout loudly enough, would it be any use if Robert had shouted to warn people below? (Speed of sound in air = $340\,\text{m}\,\text{s}^{-1}$.)

# Newton's third law of motion

## Forces come in pairs

Our everyday experience tells us that forces come in pairs. Push a laden supermarket trolley and you can feel it pushing backwards against you. Lift a heavy bag and you can feel it pulling down on you.

Newton realised this, and stated it in his **third law of motion**. This is probably the most widely known and quoted of his laws – and it is also the most widely misunderstood! Nowadays, the law is stated as:

> **If body A exerts a force on body B, then body B exerts a force of the same size on body A, but in the opposite direction.**

It is vital to realise that these third law pairs of forces act on *different* bodies, so that a free-body diagram will only ever contain one of a given pair.

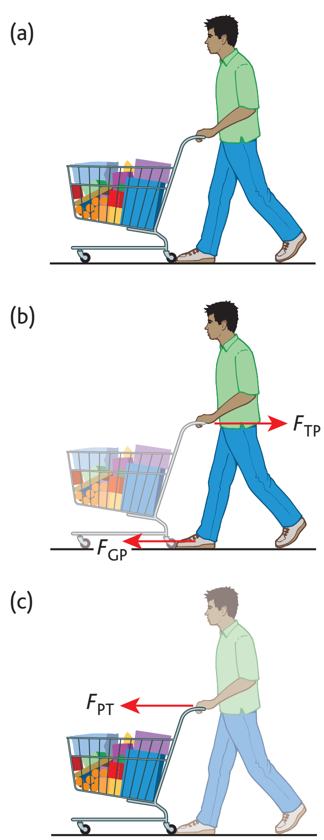



**fig. 1.2.16** Think about the forces acting as you do the weekly shopping. Diagram b shows the free-body diagram for *you* as you start the trolley moving, while diagram c shows the free-body diagram for the *trolley*. Only those forces which act in a horizontal direction have been shown.

In the situation shown in **fig. 1.2.16**, only the forces $F_{TP}$ and $F_{PT}$ (representing the force of the trolley on the person and the force of the person on the trolley, respectively) are a third law pair. The 'missing force' is the other member of the pair to which $F_{GP}$ (the force of the ground on the person) belongs. This would be shown on a free-body diagram for the Earth, as in **fig. 1.2.17** – and would be represented by $F_{PG}$ using this terminology.

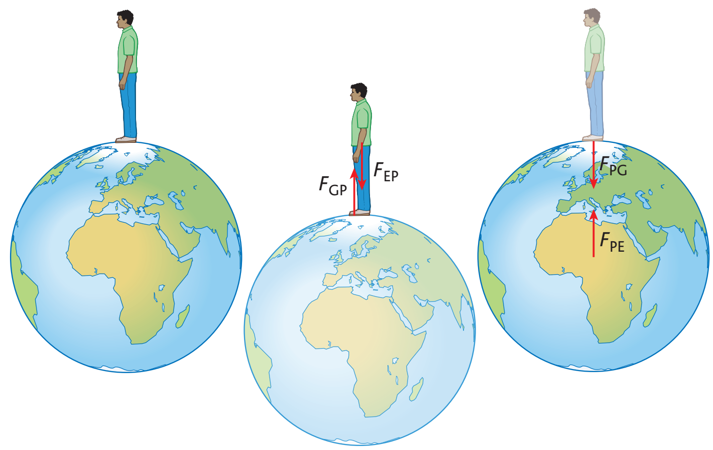



**fig. 1.2.17** These diagrams show the vertical forces acting on someone standing still on the Earth.

It is tempting to assume that the push of the ground upwards on our feet is the other member of the third law pair involving the pull of the Earth downwards on us – but the free-body diagrams show that this is not so. Notice that the two third law pairs in this case are different types of force pairs. One is a gravitational pair, the other is a pair caused by contact between two surfaces. If you jump in the air, the contact pair cease to exist while you are airborne – but the gravitational pair continue to exist, to bring you (literally) back to Earth. Third law pairs of forces are always of the same type – gravitational, electrostatic, contact, etc.

### Questions

1 A car is being towed by means of a rope connected to another car. Draw free-body diagrams showing the horizontal forces acting on:

- **a** the car being towed
- **b** the car doing the towing
- **c** the rope.

# Statics

So far we have looked at motion and the way in which forces cause it. There are many situations where motion and forces are connected in another way too – that is, where forces cancel each other out and the object on which they act is stationary or, more correctly, is in **equilibrium**. This situation is dealt with in a branch of physics sometimes referred to as **statics** – the study of bodies which are not moving.

Statics is obviously important to civil engineers designing a structure (for example, a large bridge), but surprisingly, it is also important to aeronautical engineers ensuring the stability of a new design of aircraft in flight. In both cases there needs to be a good understanding of the forces acting in order to ensure the strength and stability of the object being designed.

## Adding forces

Since force is a vector quantity, when we add forces together we must be careful to take into account both their magnitude and their direction. To do this easily we draw vector diagrams, like those used when dealing with velocities.

When adding two forces together the **parallelogram rule** is used, as the vectors act at the same point – like the example of adding two velocities in chapter 1.1. In this case we use a parallelogram of forces. **Fig. 1.2.18** shows a ball which has two forces acting on it at right angles to each other (the forces are not balanced, so the ball is accelerating).

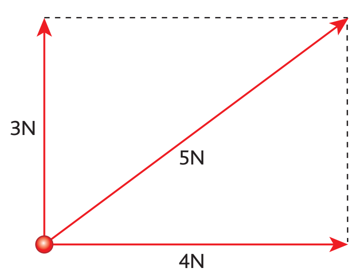



fig. 1.2.18 Example of a parallelogram of forces.

The sum of the 4 N force acting horizontally and the 3 N force acting vertically is the **resultant force** acting on the ball – in this case 5 N at an angle of 37° to the horizontal. You can find the resultant force by a carefully constructed scale drawing, from which the size of the resultant and its direction can be measured, or you can use trigonometry.

## Work it out!

In the example above, it is not too difficult to work out the resultant force, because the two forces acting are at right angles to each other and are simple numbers to use. We can find out the size and direction of the resultant force by using a scale drawing. A scale drawing with 1 N = 0.5 cm shows that the length of the line representing the resultant force is 2.5 cm, giving a resultant force of 5 N. The angle $\theta$ can be measured as 37°.

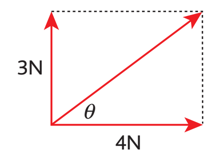



fig. 1.2.19 Scale drawing to find the resultant of two forces.

Alternatively, we can use trigonometry. Pythagoras' theorem relates the length of the hypotenuse to the length of the other two sides in a right-angled triangles. In this case it tells us that:

$$(\text{resultant force})^2 = (3)^2 + (4)^2 = 9^2 + 16^2 = 25\,\text{N}^2$$

so that:

$$\text{resultant force} = \sqrt{(25^2)} = 5\,\text{N}$$

To find out the angle $\theta$ we can use the fact that:

$\tan\theta = 3/4$ (from the property of the right-angled triangle)

so:

$$\theta = \tan^{-1}(0.75) = 37^\circ$$

Although it is not always as easy as this, the principle of adding two forces (or any other vector quantity) is always the same:

- draw the two forces acting at the same point
- construct the parallelogram
- draw in the diagonal from the point at which the forces act to the opposite corner of the parallelogram
- measure or calculate the size and direction of the resultant.

Where two forces act in the same direction, the parallelogram of forces 'collapses' to become a straight line, and the resultant force is simply the size of the two forces added together, acting in the same direction as it was in the case of the two velocities acting in the same direction on page 19. **Fig. 1.2.20** shows this happening.

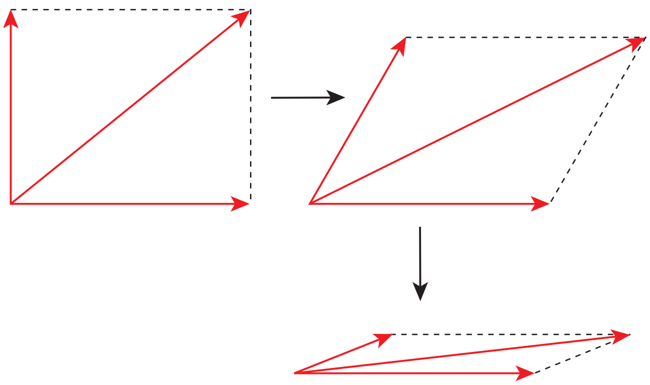


**fig. 1.2.20** The resultant of two forces acting in the same direction is simply the sum of the forces.

## Resolving forces

There are times when it is necessary to examine the way in which a force acts on something in a particular direction or pair of directions.

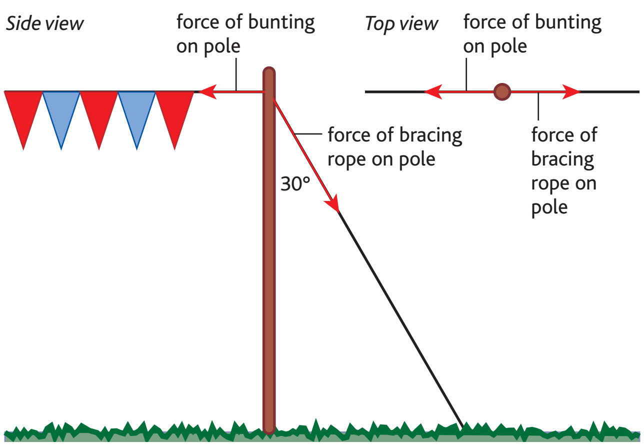



**fig. 1.2.21** Forces acting on a pole.

**Fig. 1.2.21** shows a pole with a line of bunting attached to it, together with a bracing rope. For simplicity we shall assume that the bunting is horizontal, even though this is actually impossible. The bunting and the rope pull on the pole as a result of being pulled tight. The bunting pulls the pole horizontally to the left, while the bracing rope pulls it downwards and to the right. This becomes obvious if we resolve the forces into their components acting at right angles to each other. In this case we shall use components which act horizontally and vertically (**fig. 1.2.22**).

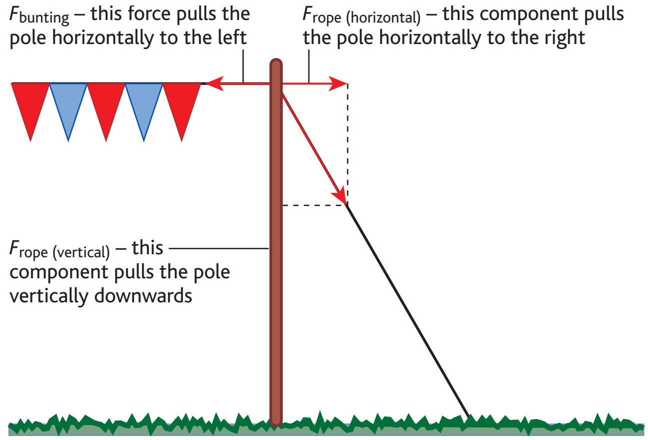



**fig. 1.2.22** The force is resolved into its components by drawing a parallelogram of forces in which the angles between the sides are all 90° – in other words, a rectangle.

By resolving the forces in this way it becomes clear that the tension in the bunting and the rope has two effects:

1. pulling the pole sideways
2. pulling the pole downwards.

The example of the bunting provides us with a good opportunity to gain an understanding of the idea of static equilibrium. Look again at **fig. 1.2.21** and **fig. 1.2.22**. As the pole is at rest, Newton's first law of motion tells us that the forces acting on it must be zero, that is:

$$\Sigma F = 0$$

If this were not the case, for example if one of the ropes attached to the pole were to snap, then the pole would be pulled to one side and would fall over. This means that $F_{\text{bunting}} = -F_{\text{rope (horizontal)}}$ and that $F_{\text{rope (vertical)}}$ must also be balanced by an equal and opposite force – supplied by the ground pushing upwards on the base of the pole.

This simple example illustrates the important physical idea of **equilibrium**, when an object has balanced forces acting on it (the word equilibrium comes from two Latin words meaning 'even balance') and is in a state of rest as a result.

### Question

1. If the bunting support rope from **fig 1.2.21** is at 30° to the vertical and its tension force is 400 N, what is vertical component of the tension, and what is the horizontal pull from the bunting?

# Projectiles

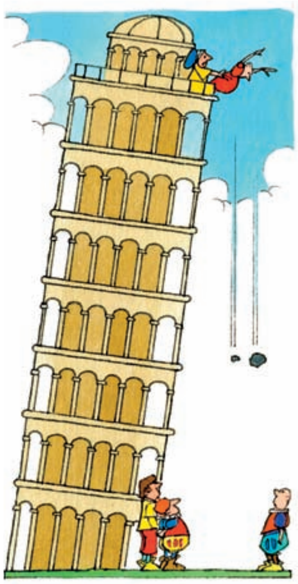


fig. 1.2.23 Galileo's famous experiment in which he dropped two unequal masses from the top of the leaning tower of Pisa. The independence of an object's mass and its acceleration in free fall was first deduced by Simon Stevinus in 1586, although this observation is usually attributed to Galileo. However, Galileo was the first to clearly state the need for the objects in question to be falling in a vacuum for this to be true rather than an approximation.

## Acceleration in the Earth's gravitational field

A **projectile** is an object which is *projected* – that is to say a force acts on it to start it moving and it is then subjected to a constant force while it moves. In most cases this will mean that the object is in free fall in the Earth's gravitational field.

An object which is dropped from rest a small distance above the surface of the Earth accelerates vertically downwards under the influence of its weight. Theoretically the acceleration of an object in free fall is independent of its mass, although this is strictly true only in a vacuum – air resistance affects the motion of objects unequally, according to their cross-sectional area.

At the surface of the Earth, the rate at which an object accelerates under the influence of the Earth's gravitational field is usually known as the 'acceleration due to gravity'. The Earth's gravitational field strength and the acceleration due to gravity are usually both represented by the symbol $g$, although the units of the two constants are different – $\mathrm{N\,kg^{-1}}$ and $\mathrm{m\,s^{-2}}$, respectively.

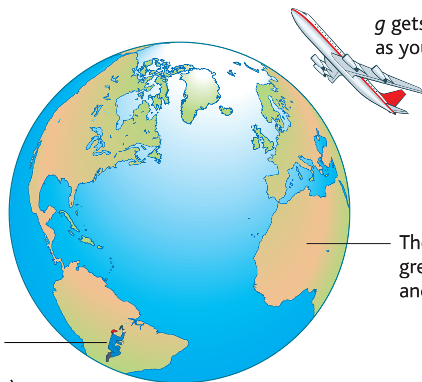



fig. 1.2.24 $g$ is generally taken to be about $9.81\,\mathrm{m\,s^{-2}}$, although it varies at different places on the Earth's surface.

## Vertical projection

What can the equations of motion (page 16) tell us about the motion of the ball in **fig. 1.2.25**? Taking $g$ to be $9.81\,\mathrm{m\,s^{-2}}$ and using the information we have been given, we can write:

$u = 20\,\mathrm{m\,s^{-1}}$

$v = 0\,\mathrm{m\,s^{-1}}$ (taking the ball's velocity as zero at the top of its trajectory)

$a = -9.81\,\mathrm{m\,s^{-2}}$ $\quad t = ?$ $\quad s = ?$

Using equation of motion 4:

$$v^2 = u^2 + 2as$$

Substitute values:

$$(0)^2 = (20)^2 + 2 \times -9.81 \times s$$

so:

$$0 = 400 - 2 \times 9.81 \times s$$

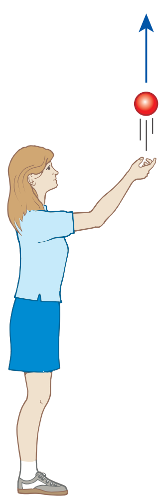



fig. 1.2.25 Vertical projection.

and:

$$s = \frac{400}{2 \times 9.81}$$
$$= 20.4\,\text{m}$$

The ball rises to a height of 20.4 m.

We can use equation 1 in the same way:

$$v = u + at$$

Substitute values:

$$0 = 20 + -9.81 \times t$$

so:

$$t = \frac{20}{9.81}$$
$$= 2.04\,\text{s}$$

The ball takes 2.04 s to rise to its maximum height.

Similar use of equation 2 and equation 1 should enable you to show that:

1 the ball takes 2.04 s to return to its initial height from the top of its trajectory

2 its final velocity before being caught again is $-20\,\text{m}\,\text{s}^{-1}$.

## Projection at an angle

Now consider an object projected at an angle rather than vertically upwards, as shown in **fig. 1.2.26**.

The velocity of the ball can be resolved into a horizontal and a vertical component. The force diagram shows the force acting on the ball. This acts only in a vertical direction, so the ball will accelerate in a vertical direction only. As a result, the *horizontal* motion of the ball is not subject to any acceleration, and so the horizontal component of the ball's velocity is constant. In this example the horizontal velocity is $u\cos\alpha$, where $\alpha$ is the angle with the horizontal at which the ball is projected. The vertical component of the ball's velocity is subject to an acceleration of $-g$.

The key variable that links the vertical and horizontal motion of the ball as it travels through the air is the *time*, which is normally measured from the instant of launching the object. If we represent the horizontal displacement by the symbol $s$, and the vertical displacement by the symbol $h$, the position of the ball can be plotted as the coordinates $(s, h)$ on a graph. The equations of motion enable us to express $s$ and $h$ in terms of the other variables $u$ and $\alpha$.

Using equation 2:

Horizontal motion:

$$s = (u\cos\alpha)t + \tfrac{1}{2}at^2$$

or:

$$s = ut\cos\alpha \quad \text{as} \quad a = 0$$

Vertical motion:

$h = (u\sin\alpha)\text{t} - \tfrac{1}{2}gt^2$ ($g$ is in the opposite direction to $h$ and $u$)

or:

$$h = ut\sin\alpha - \tfrac{1}{2}gt^2$$

The trajectory of a projectile thrown at an angle in this way is a parabola, as shown in **fig. 1.2.28**.

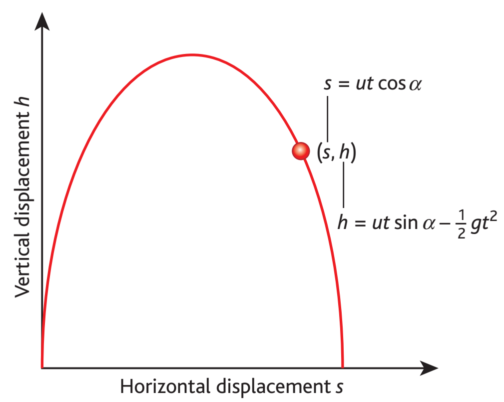



**fig. 1.2.27** Trajectory of a projectile thrown at an angle.

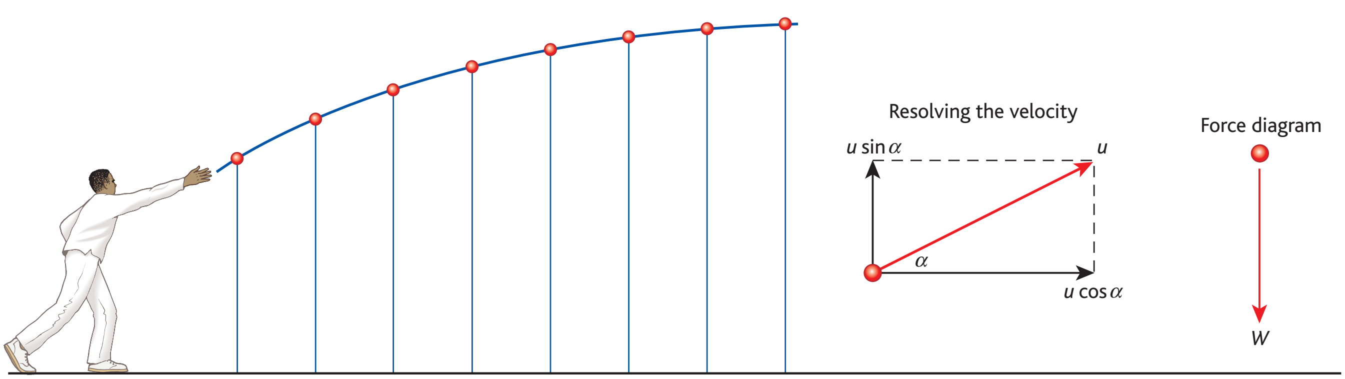
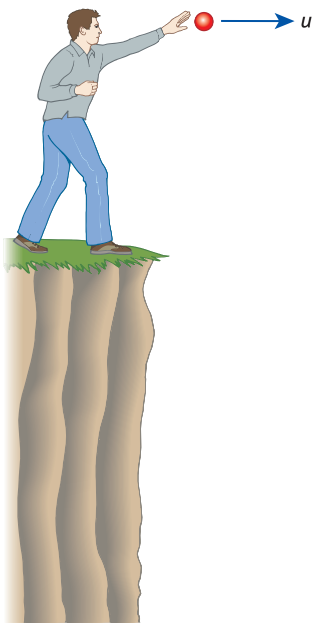



**fig. 1.2.26** In each time interval the ball travels a constant horizontal distance but a varying vertical distance.

## Projection horizontally



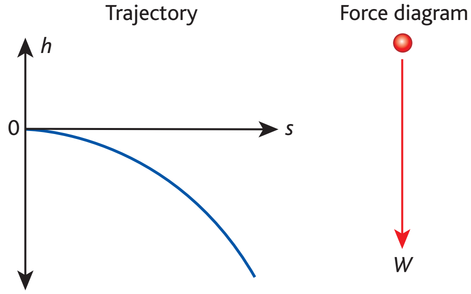




fig. 1.2.28 The trajectory and force diagram for a projectile thrown horizontally.

**Fig. 1.2.28** shows a projectile thrown horizontally. In this case $\alpha = 0°$, so $u \sin\alpha = 0$. This means that the equation for vertical displacement:

$$h = ut\sin\alpha - \tfrac{1}{2}gt^2$$

becomes:

$$h = -\tfrac{1}{2}gt^2$$

whilst the horizontal displacement is still:

$$s = ut\cos\alpha\ ut$$

The vertical motion and the horizontal motion can be considered quite separately. This leads to the surprising (but nevertheless true) conclusion that the time of flight (the time taken for a dropped object to reach the ground) of an object is the same whether it is dropped or projected horizontally – in both cases the initial vertical component of velocity is zero.

### Monkey and hunter demonstration

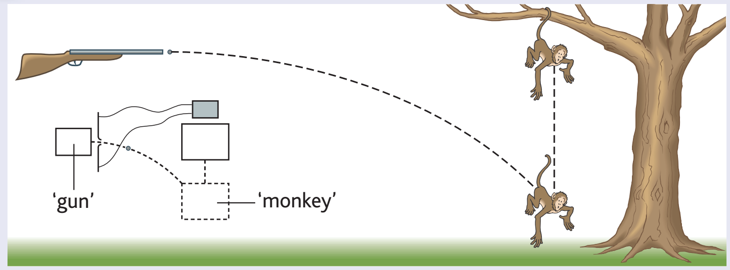



fig. 1.2.29 The 'monkey and hunter'. If the monkey drops from the branch at exactly the same time as a bullet is fired horizontally at it, the bullet will hit the monkey. This is because both monkey and bullet have the same downward acceleration.

You can set up a demonstration of the monkey and hunter thought experiment using a catapult which fires a ball bearing horizontally through a tin foil contact. Breaking the foil breaks a circuit, releasing a target which was held up electromagnetically. The 'bullet' should hit the 'monkey'. If you video record this from the side and play the footage back frame by frame, you can observe that the vertically falling monkey drops at exactly the same rate as the bullet.

## The range of a projectile

The equations which govern the vertical and horizontal displacements of a projectile can be combined to show how the angle of projection of a projectile affects its flight.

$$h = ut\sin\alpha - \tfrac{1}{2}gt^2$$

The projectile will hit the ground again at time $t$, when $h = 0$:

$$0 = ut\sin\alpha - \tfrac{1}{2}gt^2 \quad \text{or} \quad ut\sin\alpha - \tfrac{1}{2}gt^2$$

So: $$t = \frac{2u\sin\alpha}{g}$$

Since:

$$s = ut\cos\alpha$$

the projectile will have a horizontal displacement or range, $R$, when it hits the ground once more, given by:

$$R = u \times \frac{2u\sin\alpha}{g} \times \cos\alpha$$

$$= \frac{u^2 \times 2\sin a\cos\alpha}{g}$$

$$= \frac{u^2\sin 2\alpha}{g} \text{ (since } \sin 2\alpha = 2\sin\alpha\cos\alpha)$$

For a given initial velocity, the maximum range will be when $\sin 2\alpha$ has its maximum value. This is when $2\alpha = 90°$, that is, when $\alpha = 45°$.

## Worked example

A darts player throws a dart at the dartboard with an initial speed of 10 m s$^{-1}$. The distance from the player's hand to the board at the moment of releasing the dart is 2.0 m, and the dart strikes the bull's eye, which is at the same height as the player's hand as the dart leaves it. At what angle to the horizontal must the dart be thrown? (Take $g$ as 9.81 m s$^{-2}$.)

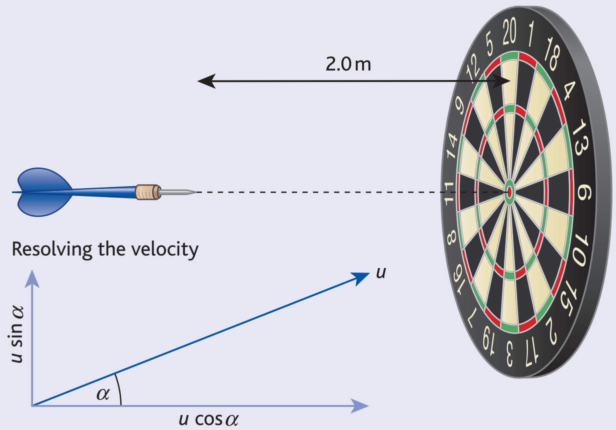



**fig. 1.2.31** Dart velocity

The horizontal component of the dart's initial velocity is 10 m s$^{-1}$ × cos$\alpha$. This velocity is constant throughout the flight of the dart, so:

$$2.0 = (10 \times \cos\alpha) \times t$$

and:

$$t = \frac{2.0}{10 \times \cos\alpha}$$

The vertical component of the dart's initial velocity is 10 × sin$\alpha$. This component is subject to a downward (and hence negative) acceleration of 9.81 m s$^{-2}$. The dart finishes at the same height as it started (that is, zero vertical displacement) and so:

$$s = ut + ½ at^2$$

$$0 = (10 \times \sin\alpha) \times t + ½ \times -9.81 \times t^2$$

That is:

$$10 \times \sin\alpha = ½ \times 9.81 \times t$$

But we know that:

$$t = \frac{2.0}{10 \times \cos\alpha}$$

so

$$10 \times \sin\alpha = ½ \times 9.81 \times \frac{2.0}{10 \times \cos\alpha}$$

Rearranging this, we get:

$$\sin\alpha\cos\alpha = 0.0981$$

Therefore:

$$2\sin\alpha\cos\alpha = 2 \times 0.0981$$

$$\sin 2\alpha = 0.1962$$

$$2\alpha = 11.3° \quad \text{and} \quad \alpha = 5.66°$$

So the dart must be thrown at an angle of just under 6° to the horizontal.

## Questions

1. A ball is thrown from ground level with a velocity of 15 m s$^{-1}$ at an angle of 30° to the horizontal. Calculate:
   - **a** its time of flight (the time between the point at which it leaves the thrower's hand and when it hits the ground) assuming that the ground is level
   - **b** its range
   - **c** its maximum height.
2. At Acapulco, divers jump from a cliff 36 m high into the sea. At the base of the cliff there is a ledge which sticks out a distance of 6.4 m. What must be a diver's minimum horizontal velocity in order to miss this ledge and enter the sea safely?
3. One record suggests that the maximum horizontal distance an arrow has been shot on level ground is 889 m. Assuming that the arrow was shot at an angle of 45°, at what speed was it launched?
4. An aeroplane carrying out a parcel drop releases a parcel while travelling at a steady speed of 90 m s$^{-1}$ at an altitude of 200 m. Calculate:
   - **a** the time between the parcel leaving the aeroplane and it striking the ground
   - **b** the horizontal distance travelled by the parcel in this time
   - **c** the speed at which the parcel strikes the ground.

# 1.3 Energy and power

## The concept of energy

So far we have examined the laws of motion described by Newton, and used these ideas together with those which began their development with Galileo to describe and explain the motion of objects.

The laws and rules which come from these sources are powerful tools to use in the investigation of the interactions between objects. No description of the tools that physicists use in this way would be complete without an introduction to one of the most powerful ideas that physicists have devised – the concept of **energy**.

## The law of conservation of energy

We commonly think of energy as coming in a number of forms, such as chemical, electrical, kinetic and so on. Investigations of energy and its forms lead us to conclude that although energy may appear in different forms when a change occurs (for example, when a battery – containing stored chemical energy – causes an electric current – electrical energy – to flow through a wire, lighting a light bulb – producing heat energy and light energy), the total amount of energy remains constant. This rule has the status of a **law** in physics, and is called the **law of conservation of energy**. This is often expressed as:

**Energy cannot be created or destroyed.**

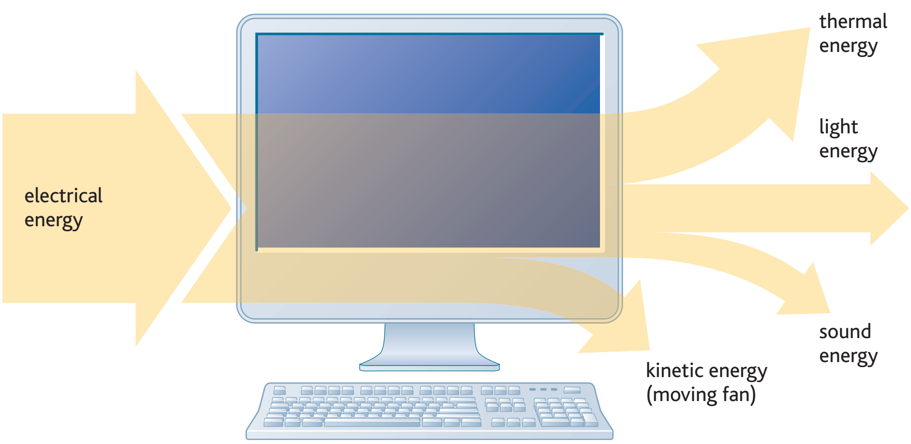



fig. 1.3.1 **The total amount of energy in a system remains constant.**

The law of conservation of energy, together with the idea of energy existing in different forms, make up an area of physics that is the most powerful and the most misunderstood of all. Essentially, the concept of energy helps us to understand and explain the way the Universe behaves – just like any other law or theory of physics.

## HSW Laws and theories

At its simplest, a law of physics can be thought of as a summary of observations that physicists have made. A law is of course built on a limited number of observations, so that it is necessary for physicists to *assume* that it is possible to predict the behaviour of the Universe in the future based on observations made in the past. This is an assumption common to the whole of science, and the success of science as an area of study certainly seems to justify it.

A law supplies us with information about *how* the physical world may be expected to behave based on past experience, but it does not tell us *why* the physical world behaves in this way. For example, the 'energy law' we have just met says that we cannot create or destroy energy, but does not explain very much about what energy actually is or why it cannot be created or destroyed.

It is the job of **theories** in science to tell us about the behaviour of things in rather more depth. Strangely, scientists and philosophers find it very hard to agree on the way in which scientific theories come about, although most would accept that they are best described as being 'invented' (that is, made up by people) rather than simply 'discovered'.

One way in which theories are produced, based on the ideas of the philosopher Sir Karl Popper, runs as follows. A scientist wishing to explain something about the behaviour of the world will formulate a tentative idea, called at this stage a **hypothesis**. This hypothesis must be based on existing scientific knowledge, but as long as it does not contradict this knowledge, it can be as bold and imaginative as the scientist wishes. The scientist then tests the hypothesis by carrying out a number of *experiments*. If it survives this testing, an account of the hypothesis and the tests it has survived is then published in a scientific journal so that other scientists can read about it and decide whether it is true or not – a process called peer-review. Hypotheses that survive the process of **peer-review** become accepted scientific knowledge – that is, they become theories.

It is important not to confuse use of the term 'theory' in a scientific context with its use in everyday life, where it is often used to mean 'guess'. (For example, I may have a 'theory' about why a particular football team is unlikely to win any competitions next year.) A theory in science is not a guess, but a very well-established framework that explains many observations and experimental results. A good theory will produce new hypotheses that can be tested, and so on.

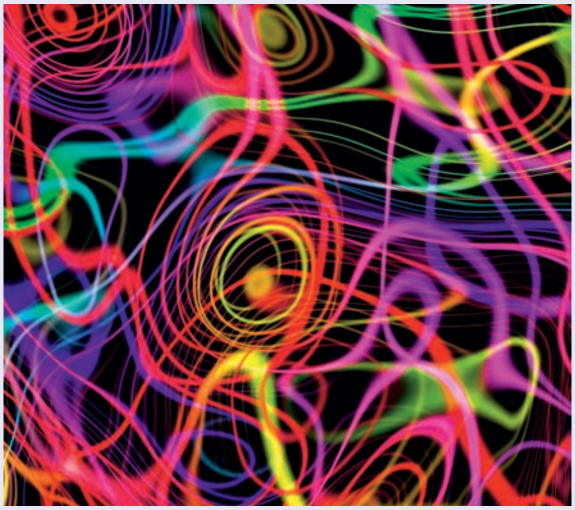


fig. 1.3.2 Superstring theory suggests that we live in a Universe in which there are ten dimensions, six of which are 'coiled up' on each other. This is a very bold and imaginative theory, but it is based on far more than simple guesswork.

## Questions

1 We have seen that when a driving force is removed, a moving object's drag will slow it down to a stop. This means it loses all its kinetic energy. Explain how this disappearance of energy can be in keeping with the law of conservation of energy.

2 One way of storing surplus electrical energy from a power station is to use it to raise water from a lower reservoir to a higher one. This water can then be released to generate electricity again later. (This system is referred to as 'pumped storage'.) Explain how this system might be limited by the law of conservation of energy.

# Energy transformations

The ideas behind the concept of energy represent a model for understanding the way the world (and the whole Universe) behaves. We talk about chemical energy being transferred to kinetic energy by a petrol engine. This way of talking is a quick way of saying that the petrol – because of the way the bonds in its molecules are arranged – may be burned in an engine to produce movement, and that in doing this there is a relationship between the amount of petrol used and the movement produced. We do not mean that the engine literally, as if by magic, takes something out of the petrol and uses it to turn the wheels.

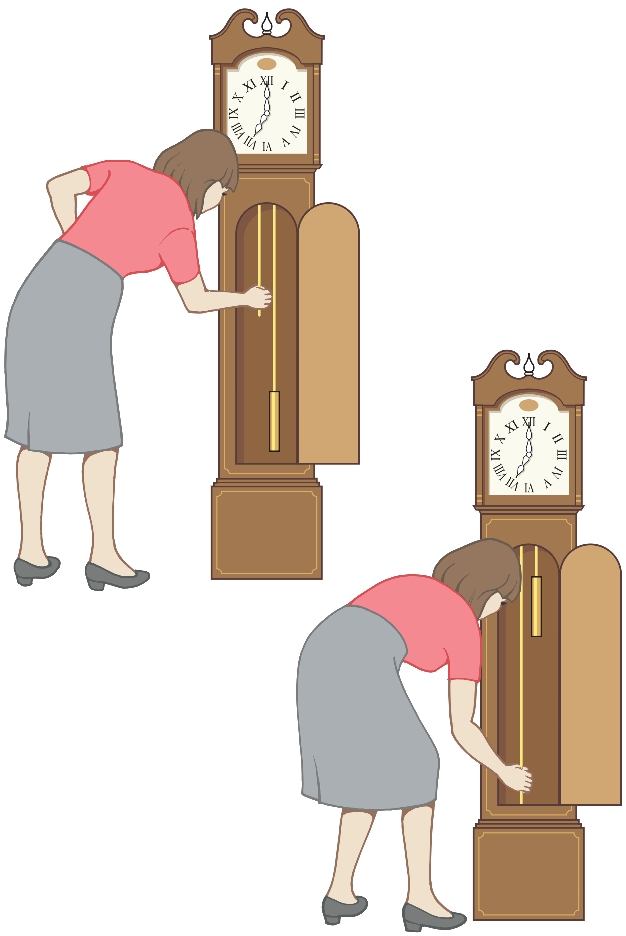


fig. 1.3.3 Winding a grandfather clock.

Consider the example of winding a grandfather clock shown in **fig. 1.3.3**. The clock is driven by a falling weight. This weight can be regarded as a source of energy, in just the same way as the spring in an old-fashioned watch or the battery in a modern digital watch. Where does this energy come from in the case of the grandfather clock?

A scientific understanding of the situation (that is, one that fits with physicists' current understanding of the Universe) is that once the weight has been raised the person's muscles contain *less* of the chemicals that can be used to do something useful (like raise a weight). The raised weight, however, now has something which will let it do *more* than it could before. Energy is simply the 'accounting system' which we use to keep track of a system's ability to do something useful – rather like we use money to keep track of our ability to buy things.

An alternative explanation says that **chemical potential energy** in the person's muscles has been used to lift the weight and that, as a result, the chemical energy has been transformed into **gravitational potential energy** stored in the raised weight. This implies that something has gone from the person's muscles into the weight and has changed in some way – 'chemical energy has been transformed into potential energy'. Whilst this might *seem* to be a satisfactory explanation, it is not, because of the idea of energy transformation that it uses – a much better term to use is **energy transfer**. **Fig. 1.3.4** shows how energy transfers can be compared to transferring money.

fig. 1.3.4 When you pay a cheque into the bank you increase your ability to buy things by transferring money into your account! When you pay for something using your debit card later, you decrease it again – but no actual money changes hands in the transfer. In the same way, energy may be transferred. The energy in a system can be changed without anything actually flowing into or out of it. This is why it is better to talk about energy being *transferred* rather than to say that it is *transformed*.

## Transferring energy – heating and working

Transferring energy is compared to transferring money in **fig. 1.3.4**. There are many ways of transferring money (cash, cheque, credit card, postal order, direct debit …), but far fewer ways of transferring energy. In this section of the book we are concerned only with two of these – the ways which physicists call **heating** and **working**.

The difference between heating and working lies in the way that energy is transferred. If we heat an object, we transfer energy to it using a temperature difference – perhaps by means of a flame. If we wish to transfer energy without making use of a temperature difference, we do it by doing work – for example, by lifting an object off the floor onto a table. The terms 'heat' and 'work' therefore describe energy which has been transferred in a particular way – by means of a temperature gradient or by means of moving a force.

**fig. 1.3.5** In many cases it is much easier to say 'heat energy' than 'energy transferred by means of a temperature difference' or 'work done' rather that 'energy transferred by means of moving a force'.

## Work and the units of energy

As we have just seen, when energy is transferred it may be transferred by doing **work**, for example when you push a supermarket trolley across a car park. The amount of work done is calculated simply as:

work done = force × distance moved in direction of force

This simple relationship leads to the definition of the unit of energy, in the same way as Newton's second law of motion led to the definition of the unit of force. In this case, the SI unit of energy, the **joule** (J), is defined as being the energy transferred when a force of 1 newton is displaced a distance of 1 metre, that is, $1\,\text{J} = 1\,\text{N}\,\text{m}$.

Although in calculating work we are multiplying two vectors together (force and displacement), the result is a scalar. Energy has magnitude only.

If the force and the displacement are in different directions, the force must be resolved in order to calculate the work done. **Fig. 1.3.6** shows how this is done. In this figure the force is resolved so that one component ($F\cos\theta$) lies in the same direction as the displacement. This is the component of the force that is involved in transferring energy. The component of the force perpendicular to the displacement does *no* work, as it does not move in the direction in which it is acting.

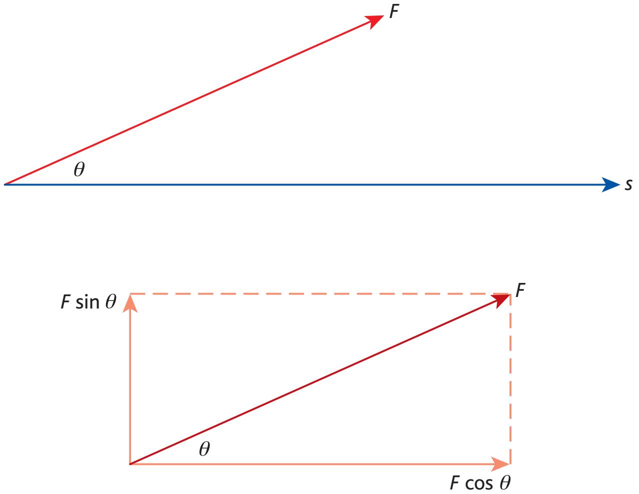



**fig. 1.3.6** Resolving a force to calculate work done. In this case, work done = $F\cos\theta \times s$.

### Questions

1. Give an example of a transfer of energy which could be classified as:
   **a** heating **b** working.
2. A delivery driver lifts a carton with a mass of 6.5 kg onto the back of the lorry, a height of 1.5 m from the ground. How much work is done in this energy transfer?

# Energy and efficiency

## Kinetic and potential energy

We are used to saying that an object has gravitational potential energy when it is raised through a distance $\Delta h$. In this case we write:

$$\Delta E_{grav} = m \times g \times \Delta h$$

where $m$ is the object's mass and $g$ is the gravitational field strength of the Earth. (Strictly, $mg\Delta h$ is the *change* in something's potential energy.)

In the same way, we talk about a moving object possessing **kinetic energy**, and write the amount of this energy as:

$$E_k = ½mv^2$$

where $m$ is the object's mass and $v$ its velocity.

The idea of work as a means of transferring energy by moving a force lies behind both of these expressions for the energy an object has in a particular situation. The grandfather clock example (see **fig. 1.3.3** on page 40) can show us this.

When the weight in the clock is raised, a force equal and opposite to the gravitational force of attraction on the weight is applied to it. Therefore, the work done on it ('the energy transferred by means of moving a force') is equal to the weight $W$ of the weight multiplied by the distance it is raised.

$$\begin{aligned} \text{work done} &= W \times \Delta h \\ &= mg\Delta h \\ &= \text{gravitational potential energy stored in weight } (\Delta E_{grav}) \end{aligned}$$

This potential energy may be released slowly as the weight does work on the clock mechanism ('transfers energy to the mechanism by means of a moving force'), driving the hands round.

If the wire supporting the weight breaks, the weight will fall. Instead of doing work on the clock mechanism, the weight now does work on itself (because the weight is not connected to anything, the energy transferred by the moving force has only one place to go – it stays with the weight), and its potential energy will be transferred to kinetic energy. In this case:

$$v^2 = u^2 + 2as = 2a\Delta h \quad \text{(as the weight falls from rest, } u = 0\text{, and it falls a distance } \Delta h\text{)}$$

so

$$\Delta h = \frac{v^2}{2a}$$

Now:

$$\begin{aligned} \text{kinetic energy gained} &= \text{work done on falling weight} \\ &= F \times \Delta h \\ &= F \times \frac{v^2}{2a} \end{aligned}$$

Since $F = ma$, we have:

$$\begin{aligned} \text{kinetic energy gained} &= ma \times \frac{v^2}{2a} \\ &= ½mv^2 \end{aligned}$$

### Pendulum energy exchange

Galileo's pendulum experiment in which he determined that the pendulum would always return to the same height illustrates the conservation of gravitational and potential energy nicely.

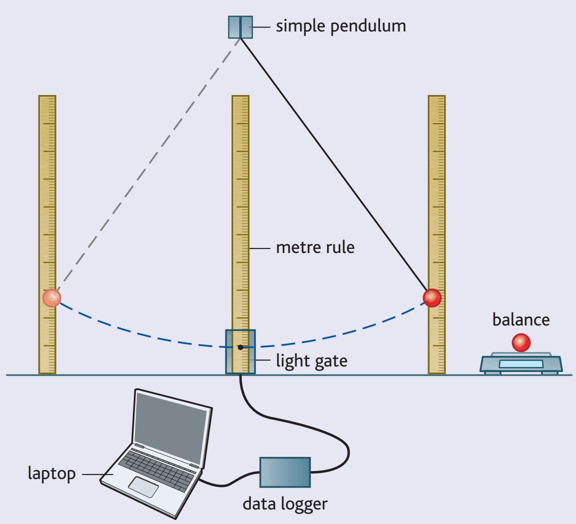



fig. 1.3.7 You can investigate energy changes for a swinging pendulum in a school laboratory.

By careful measurement of the height a pendulum rises and falls through its swing, we can determine the gravitational potential energy it loses and gains throughout one oscillation. This can then be compared with the kinetic energy it has as it passes through the lowest point. This experiment will show that the energy is constantly being transferred from kinetic to gravitational potential and back again.

# Efficiency

The idea of **efficiency** is a useful one when we are considering energy transfers. With some thought it comes as little surprise to find that it is often impossible to take all the energy in an energy store (for example a litre of petrol) and transfer it so that it does something useful. **Fig. 1.3.8** shows the energy transfers in this situation.


**fig. 1.3.8** Energy transfer in a car.

Clearly this process is not 100% efficient, since in burning petrol in a car engine we want to end up with as much kinetic energy as possible – we certainly do not want thermal energy. The efficiency of this process can be calculated as:

$$\text{efficiency} = \frac{\text{useful energy got out (the kinetic energy)}}{\text{energy put in}}$$
$$= \frac{35}{100} \times 100\%$$
$$= 35\%$$

If we think of a machine or a process as a box which has energy going into it and energy coming out of it, as in **fig. 1.3.9**, then we define efficiency in the way we have just seen, that is:

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{energy input}} \times 100\%$$

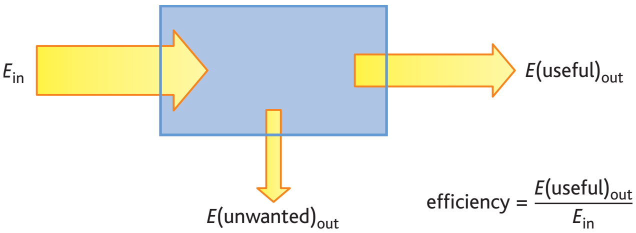



**fig. 1.3.9** Energy transfer by a machine

Since efficiency is calculated by dividing one quantity in joules by another also in joules, it has no units – it is simply a ratio. It may, of course, be expressed as a percentage by multiplying the ratio by 100%.

## Worked examples

A central heating boiler supplies 250 kJ of energy to hot water flowing through the boiler. In order to supply this energy to the water, the gas burned in the boiler must produce 400 kJ of energy. What is the efficiency of the boiler?

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{energy input}} \times 100\%$$
$$= \frac{250 \times 10^3}{400 \times 10^3} \times 100\%$$
$$= 62.5\%$$

## Questions

1 Legend has it that Galileo was only 17 years old when he started thinking about pendulum movements, whilst watching a lamp hanging on a long cable in the cathedral in Pisa, Italy. If the lamp had a mass of 1.2 kg and a draught imparted 10 J of kinetic energy to it:

   a how fast would the lamp move initially?

   b how high could the lamp rise in a swing?

2 A cricketer hits a ball straight up in the air. It leaves the bat at 16.8 m s$^{-1}$ and has a mass of 160 g.

   a What is the kinetic energy of the ball as it leaves the bat?

   b Assuming air resistance is negligible, what is the maximum height the ball reaches above the point it left the bat?

   c If, in reality, flying up in the air and back down again is a process with an efficiency of 88%, then how fast will the ball be travelling when it returns to the start point?

# Power


**fig. 1.3.10** Which car has more power? The idea of power is common in everyday life. Physicists use the word 'power' to mean much the same thing as we understand when comparing the performance of cars.

In physics, the definition of power relates energy transferred to the time taken to do it. So:

$$\text{power} = \frac{\text{energy transferred}}{\text{time taken}}$$

In symbols:

$$P = \frac{E}{t}$$

Power may be measured in joules per second ($\text{J s}^{-1}$) in the SI system. The unit $\text{J s}^{-1}$ is also known as the watt (W).

Since we often refer to the power developed when work is done, power may also be defined as:

$$\text{power} = \frac{\text{work done}}{\text{time taken}}$$

If a force is being moved at a steady rate, provided we know the velocity at which the force is moving and the size of the force, we can calculate the power:

$$\text{work done} = \text{force} \times \text{distance moved in direction of force}$$

and

$$\text{power} = \frac{\text{work done}}{\text{time taken}}$$

so

$$\text{power} = \text{force} \times \frac{\text{distance moved in direction of force}}{\text{time taken}}$$

$$= \text{force} \times \text{velocity at which force is moving}$$

In symbols:

$$P = Fv$$

## Worked examples

**Example 1**

An athlete of mass 75 kg runs up a steep slope, rising a vertical distance of 30 m in 50 s. Neglecting the effect of any drag forces acting on the athlete, what power must his leg muscles develop in order to do this? (Assume $g = 9.81\ \text{N kg}^{-1}$.)

The athlete's muscles must supply a force to lift the athlete through a vertical distance of 30 m. Assuming that this is done at a steady rate, then the force exerted will be equal in size to the athlete's weight.

So:

$$\text{work done by athlete} = \text{weight} \times \text{vertical distance raised}$$
$$= 75 \times 9.81 \times 30$$
$$= 22\,100\ \text{J (3 s.f.)}$$

This work was done in a time of 50 s, so:

$$\text{power developed} = \frac{22\,100}{50}$$
$$= 442\ \text{W}$$

**Example 2**

A car towing a caravan travels along at a steady speed of $20\ \text{m s}^{-1}$. If the force exerted by the engine is 2 kN, what is the power output of the engine?

$$\text{power} = \text{force} \times \text{velocity}$$
$$= 2000 \times 20$$
$$= 40\,000\ \text{W}$$

The power output of the engine is 40 kW.

## Horsepower

The power output of car engines is often expressed in **horsepower** – a method of measuring the rate of doing work dating from before the industrial revolution. $1\,\text{HP} \approx 750\,\text{W}$, so the power output of the car engine in Example 2 above is about 53 HP. In Example 1, what is the power output of the athlete's leg muscles in HP?

### Investigating power and efficiency

You could investigate your own power by running up a hill of known height like the athlete in our example. A bit less tiring would be a lab investigation in which a brick is pulled up a ramp (**fig. 1.3.11**).

By pulling the brick up at a constant velocity, and using a constant force, you could calculate the power from $P = Fv$. If you also measured the time taken and the height the brick rises vertically, the efficiency of the process could be calculated.

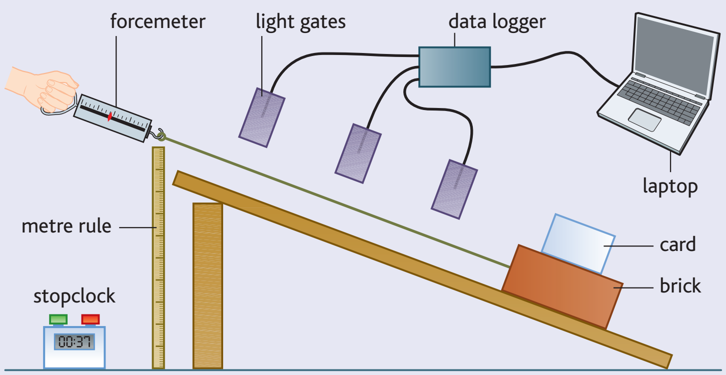



fig. 1.3.11 Experimental setup for investigating power and efficiency.

## Relating power and efficiency

We saw previously that:

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{energy input}} \times 100\%$$

This means that in a given time, comparing the energy output with the energy input would be the same as comparing the power output with the power input. Thus:

$$\text{efficiency} = \frac{(\text{useful energy output/time})}{(\text{energy input/time})} \times 100\%$$

Note that mathematically these two equations are the same – dividing the first equation by time/time (which is the same as dividing by one) gives us the second equation.

As $E/t = P$:

$$\text{efficiency} = \frac{\text{useful energy output/time}}{\text{power input}} \times 100\%$$

### Questions

1 What is the power of a kettle which transfers 264 kJ of energy in two minutes?

2 What is the power of a luxury motorboat which is moving at a constant $22\,\text{m}\,\text{s}^{-1}$ if the total drag forces on it are 123 kN?

3 A crane lifts a steel girder with a mass of 800 kg. The girder rises 21 metres in 6 s. What is the power of the crane?

## HSW The mechanics of hockey

Hockey is the second most popular sport on Earth in terms of numbers of people playing. It is a fast-moving, skilful sport in which the ball may fly at 100 miles per hour. Here we will consider how the mechanics covered in Unit 1 can apply to events in hockey.


fig. 1.3.12 Great Britain vs. Mexico

### The shot at goal

If we want to calculate how fast a hockey ball would be moving after being hit hard at goal from rest, we need to think about its acceleration through Newton's second law. A standard hockey ball has a mass of 0.14 kg. If the stick applies a force of 70 N for a twentieth of a second, we can work out the answer:

$$F = ma \quad \text{so} \quad a = \frac{F}{m}$$

$$= \frac{70}{0.14}$$

$$= 500\,\text{m}\,\text{s}^{-2}$$

$v = u + at$ (the ball is initially stationary, so $u = 0\,\text{m}\,\text{s}^{-1}$, $a = 500\,\text{m}\,\text{s}^{-2}$, $t = 0.05\,\text{s}$)

$$= 0 + 500 \times 0.05$$

$$= 25\,\text{m}\,\text{s}^{-1}$$

A goal only counts if the ball is hit from inside the shooting circle. This means the ball must be no more than about 14 metres from the goal. Assuming zero drag forces, what would be the longest time the goalkeeper has to react to this shot?


fig. 1.3.13 The goalkeeper has to react quickly to save the goal.

We know the start velocity and the distance, so this is a straightforward question.

$$v = \frac{s}{t} \qquad \text{so} \quad t = \frac{v}{s}$$

$$t = \frac{15}{14}$$

$$= 1.07\,\text{s}$$

The protective padding that goalkeepers wear has developed rapidly with improved materials technology. It is now much lighter and yet provides better protection than ever before. If the goalkeeper reacts quickly enough to get a leg pad in the way, how deeply will this shot squash into the foam padding which is normally around 8 cm thick? The maximum decelerating force the foam can provide to slow the ball is 1000 N.

To answer this we should consider the removal of all the ball's kinetic energy as work being done.

$$E_k = \tfrac{1}{2}mv^2$$

$$= \tfrac{1}{2} \times 0.14 \times 25^2$$

$$= 43.8\,\text{J}$$

$$W = F\Delta s$$

$$\Delta s = \frac{\Delta W}{F}$$

$$= \frac{43.8}{1000}$$

$$= 4.38 \times 10^{-2}\,\text{m}$$

So the pad depresses by just over 4 cm. The goalkeeper is likely to feel this but it should not be painful. According to Newton's third law, when the pad exerts a force of 1000 N on the ball, the ball exerts a force of 1000 N on the pad. If the goalkeeper's leg were not protected, this force could cause a serious injury.

### The aerial ball

A modern development in hockey is the aerial ball, a long pass in which a player flicks the ball high in the air in order to avoid it being intercepted. If successful, the ball may fly for four seconds and land 50 metres further downfield. To maximise the range, the player will attempt to send the ball up at an angle of 45° to the pitch surface.

With this information, we can calculate many things about the flight of this long pass. What is the initial velocity the player gives the ball; and how high does the ball travel vertically upwards?

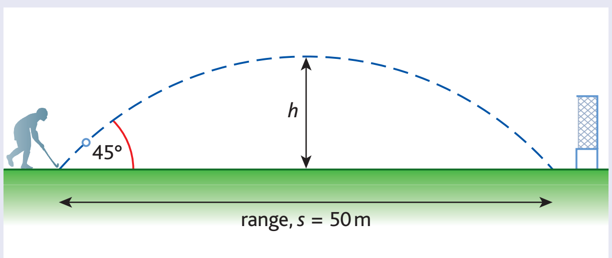



**fig. 1.3.14** **Trajectory of the aerial ball.**

The horizontal velocity is easily found from the distance and time:

$$v_H = \frac{s}{t}$$

$$= \frac{50}{4} = 12.5\,\text{m s}^{-1}$$

As the angle of flight is 45° from the ground, this means the overall initial velocity can be found by trigonometry:

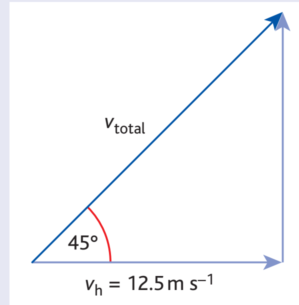



**fig. 1.3.15** **Finding the horizontal component of the velocity.**

In this case:

$$\cos 45° = \frac{v_h}{v_{\text{total}}}$$

$$v_{\text{total}} = \frac{v_h}{\cos 45°}$$

$$= \frac{12.5}{0.71}$$

$$= 17.6\,\text{m s}^{-1}$$

With a 45° initial flight angle, the vertical velocity starts the same as the horizontal velocity: $v_V = 12.5\ \text{ms}^{-1}$. If we consider only the vertical motion, we can find the kinetic energy due to $v_V$:

$$E_k = ½ m v_v^2$$

$$= ½ \times 0.14 \times 12.5^2$$

$$= 10.9\,\text{J}$$

All the kinetic energy due to the vertical velocity of the ball is transferred into gravitational potential energy at the highest point in its flight. Remember the horizontal velocity remains constant throughout, so its contribution to the total kinetic energy is always present as kinetic energy and can be ignored in this calculation.:

$$\Delta E_{\text{grav}} = mg\Delta h$$

$$\Delta h = \frac{\Delta E_{\text{grav}}}{mg}$$

$$= \frac{10.9}{0.14 \times 9.81}$$

$$= 7.96\,\text{m}$$

## Questions

1 A hockey player passes the ball at $15\,\text{m s}^{-1}$ to a team mate who stops it completely in 0.1 s. What force does the receiver's stick have to apply to the ball?

2 A penalty stroke is flicked from a distance of 6.40 m from the goal line. The striker scoops it so that the ball leaves the ground at a 45° angle and a speed of $8\,\text{m s}^{-1}$. How long does the goalkeeper have to make a save before the ball crosses the goal line?

3 The study of mechanics in sport is a popular and often profitable new area of scientific study. Describe how a sports scientist could use ICT to collect data to study the movement of players and equipment over time. Explain why technological developments have made the data collected more valid and reliable than with traditional methods of studying mechanics.

# Examzone: Topic 1 Mechanics

**1** A man is pushing a shopping trolley at a speed of 1.10 m s$^{-1}$ along a horizontal surface. There is a constant frictional force opposing the motion. The man stops suddenly, letting go of the trolley, which rolls on for a distance of 1.96 m before coming to rest. Show that the deceleration of the trolley is approximately 0.3 m s$^{-2}$. **(3)**

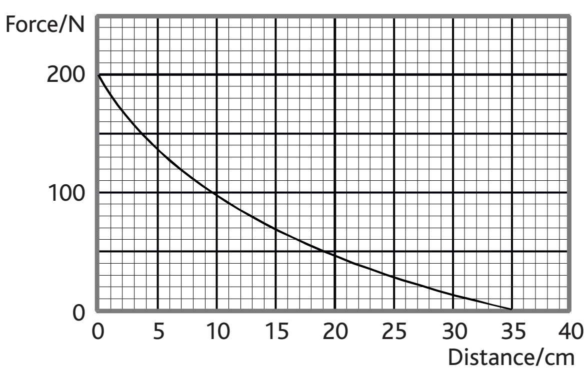
The total mass of the trolley and its contents is 28.0 kg. Calculate the frictional force opposing its motion. **(2)**

Calculate the power needed to push the trolley at a steady speed of 1.10 m s$^{-1}$. **(2)**

The man catches up with the trolley. Calculate the steady force he must now apply to it to accelerate it from rest to 1.10 m s$^{-1}$ in 0.900 s. **(3)**

**(Total 10 marks)**

**2** A catapult fires an 80 g stone horizontally. The graph shows how the force on the stone varies with distance through which the stone is being accelerated horizontally from rest.

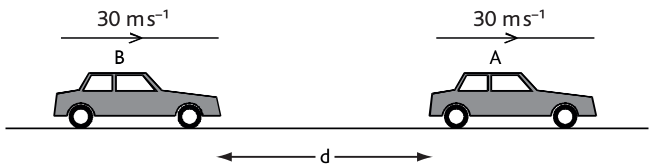



Use the graph to estimate the work done on the stone by the catapult. **(4)**

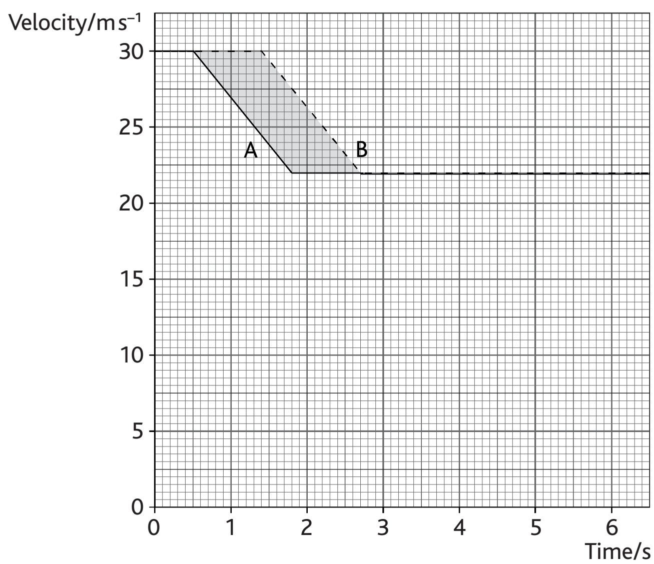
Calculate the speed with which the stone leaves the catapult. **(2)**

**(Total 6 marks)**

**3** Two cars, A and B, are travelling along the outside lane of a motorway at a speed of 30.0 m s$^{-1}$. They are a distance $d$ apart.




The driver of car A sees a slower vehicle move out in front of him, and brakes hard until his speed has fallen to 22.0 m s$^{-1}$. The driver of car B sees car A brake and, after a reaction time of 0.900 s, brakes with the same constant deceleration as A.




The diagram above shows velocity–time graphs for car A (solid line) and car B (broken line).

Find the deceleration of the cars whilst they are braking. **(3)**

What does the area under a velocity–time graph represent? **(1)**

Determine the shaded area. **(2)**

Suppose that, instead of only slowing down to 22.0 m s$^{-1}$, the cars had to stop. Copy the graph above and add lines to the grid above to show the velocity–time graphs in this case. (Assume that the cars come to rest with the same constant deceleration as before.) **(1)**

Explain why a collision is now more likely. **(2)**

**(Total 9 marks)**

**4** Explain how a body moving at constant speed can be accelerating. **(3)**

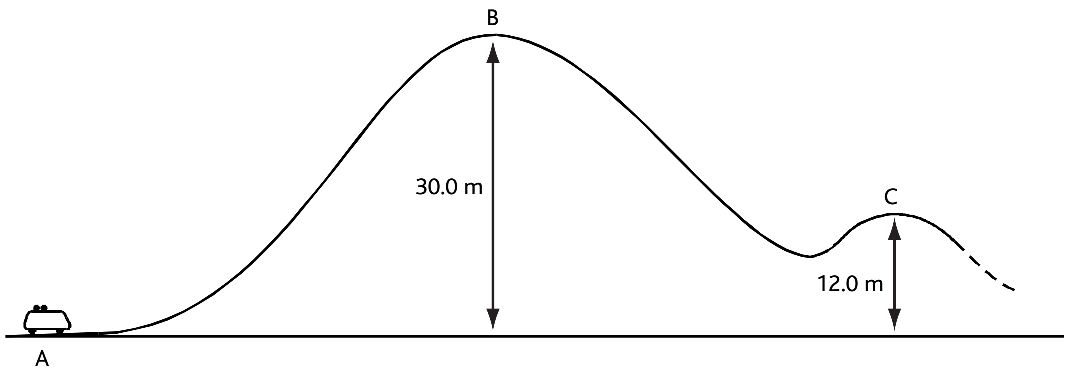
The Moon moves in a circular orbit around the Earth. The Earth provides the force which causes the Moon to accelerate. In what direction does this force act? **(1)**

There is a force which forms a Newton's third law pair with this force on the Moon.

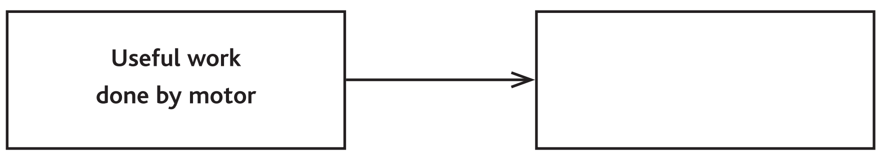
On what body does this force act and in what direction? **(2)**

**(Total 6 marks)**

**5** The diagram shows part of a roller coaster ride. In practice, friction and air resistance will have a significant effect on the motion of the vehicle, but you should ignore them throughout this question.




The vehicle starts from rest at A and is hauled up to B by a motor. It takes 15.0 s to reach B, at which point its speed is negligible. Copy and complete the box in the diagram below, which expresses the conservation of energy for the journey from A to B. **(1)**




The mass of the vehicle and the passengers is 3400 kg. Calculate

(i) The useful work done by the motor.

(ii) The power output of the motor. **(4)**

At point B the motor is switched off and the vehicle moves under gravity for the rest of the ride. Describe the overall energy conversion which occurs as it travels from B to C. **(1)**

Calculate the speed of the vehicle at point C. **(3)**

On another occasion there are fewer passengers in the vehicle; hence its total mass is less than before. Its speed is again negligible at B. State with a reason how, if at all, you would expect the speed at C to differ from your previous answer. **(2)**

**(Total 11 marks)**