EDEXCEL INTERNATIONAL A LEVEL

MATHEMATICS
MECHANICS 1

Student Book

3 CONSTANT ACCELERATIONS

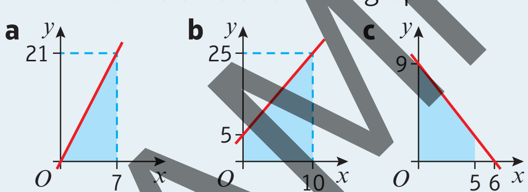
Learning objectives

After completing this chapter you should be able to:

- Understand and interpret displacement–time graphs → pages 31–33
- Understand and interpret velocity–time graphs → pages 33–36
- Derive the constant acceleration formulae and use them to solve problems → pages 37–46
- Use the constant acceleration formulae to solve problems involving vertical motion under gravity → pages 46–52

Prior knowledge check

1 For each graph find:
 i the gradient
 ii the shaded area under the graph.



2 A car travels for 45 minutes at an average speed of 35 mph. Find the distance travelled.

3 **a** Solve the simultaneous equations:

$$3x - 2y = 9$$
$$x + 4y + 4 = 0$$

b Solve $2x^2 + 3x - 7 = 0$. Give your answers to 3 s.f.

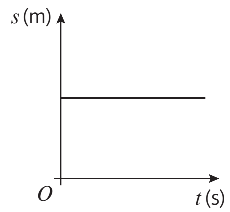
A body falling freely under **gravity** can be modelled as having **constant acceleration**. You can use this to estimate the time it will take a cliff diver to reach the water.

→ Exercise 9E Q1

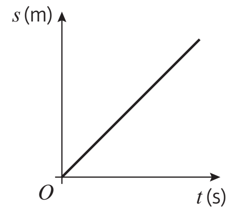
9.1 Displacement–time graphs

You can represent the motion of an object on a displacement–time graph. Displacement is always plotted on the vertical axis and time on the horizontal axis.

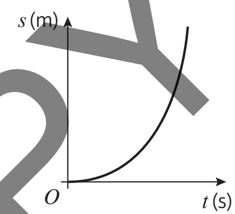
In these graphs s represents the displacement of an object from a given point in metres and t represents the time taken in seconds.



There is no change in the displacement over time and the object is stationary.



The displacement increases at a constant rate over time and the object is moving with constant velocity.



The displacement is increasing at a greater rate as time increases. The velocity is increasing and the object is accelerating.

- **Velocity is the rate of change of displacement.**
  - **On a displacement–time graph the gradient represents the velocity.**
  - **If the displacement–time graph is a straight line, then the velocity is constant.**
- **Average velocity** $= \dfrac{\textbf{displacement from starting point}}{\textbf{time taken}}$
- **Average speed** $= \dfrac{\textbf{total distance travelled}}{\textbf{time taken}}$

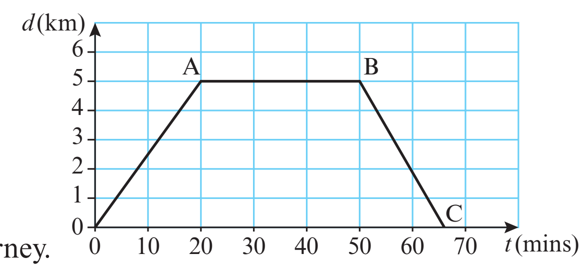
Example 1

A cyclist rides in a straight line for 20 minutes. She waits for half an hour, then returns in a straight line to her starting point in 15 minutes. This is a displacement–time graph for her journey.

a Work out the average velocity for each stage of the journey in km h^{-1}.

b Write down the average velocity for the whole journey.

c Work out the average speed for the whole journey.



a Journey from O to A: time = 20 mins; displacement = 5 km

Average velocity $= \dfrac{5}{20} = 0.25$ km min^{-1}

$0.25 \times 60 = 15$ km h^{-1}

Journey from A to B: no change in displacement so average velocity = 0

Journey from B to C: time = 15 mins; displacement = −5 km

Average velocity $= \dfrac{-5}{15} = -\dfrac{1}{3}$ km min^{-1}

$-\dfrac{1}{3} \times 60 = -20$ km h^{-1}

To convert from km min^{-1} to km h^{-1} multiply by 60.

A horizontal line on the graph indicates the cyclist is stationary.

The cyclist starts with a displacement of 5 km and finishes with a displacement of 0 km, so the change in displacement is −5 km, and velocity will be negative.

b The displacement for the whole journey is 0 so average velocity is 0.

c Total time = 65 mins

Total distance travelled is 5 + 5 = 10 km

Average speed = $\frac{10}{65} = \frac{2}{13}$ km min^{-1}

$\frac{2}{13} \times 60 = 9.2$ km h^{-1} (2 s.f.)

At C the cyclist has returned to the starting point.

The cyclist has travelled 5 km away from the starting point and then 5 km back to the starting point.

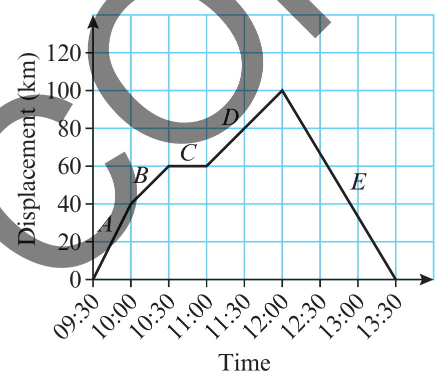
Exercise 9A

1 This is a displacement–time graph for a car travelling along a straight road. The journey is divided into 5 stages labelled A to E.

a Work out the average velocity for each stage of the journey.

b State the average velocity for the whole journey.

c Work out the average speed for the whole journey.



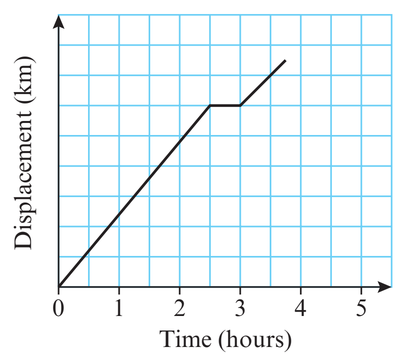
(P) **2** Khalid drives from his home to a hotel. He drives for $2\frac{1}{2}$ hours at an average velocity of 60 km h^{-1}. He then stops for lunch before continuing to his hotel. The diagram shows a displacement–time graph for Khalid's journey.

a Work out the displacement of the hotel from Khalid's home.

b Work out Khalid's average velocity for his whole journey.

Problem-solving

You need to work out the scale on the vertical axis.



(P) **3** Sarah left home at 10:00 and cycled north in a straight line. The diagram shows a displacement–time graph for her journey.

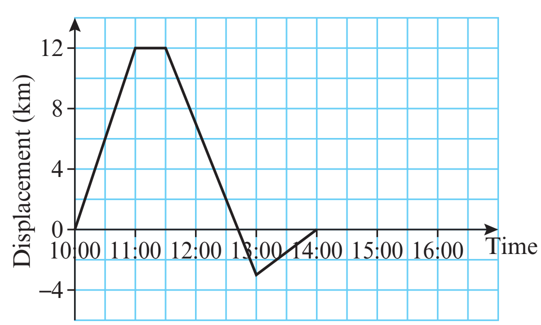
a Work out Sarah's velocity between 10:00 and 11:00.

On her return journey, Sarah continued past her home for 3 km before returning.

b Estimate the time that Sarah passed her home.

c Work out Sarah's velocity for each of the last two stages of her journey.

d Calculate Sarah's average speed for her entire journey.



(P) **4** A ball is thrown vertically up in the air and falls to the ground. This is a displacement–time graph for the motion of the ball.

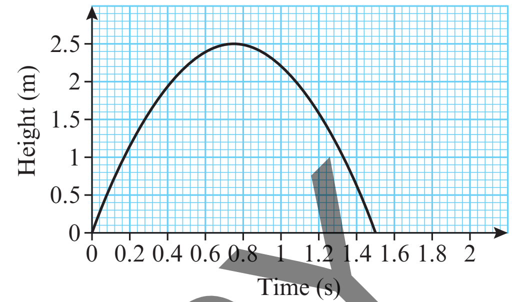
a Find the maximum height of the ball and the time at which it reaches that height.

b Write down the velocity of the ball when it reaches its highest point.

c Describe the motion of the ball:

i from the time it is thrown to the time it reaches its highest point

ii after reaching its highest point.

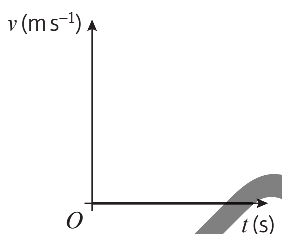


Hint To describe the motion you should state the direction of travel of the ball and whether it is accelerating or decelerating.

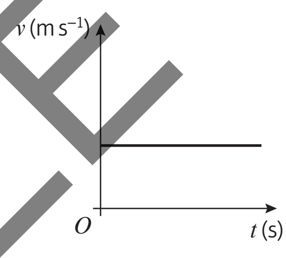
9.2 Velocity–time graphs

You can represent the motion of an object on a velocity–time graph. Velocity is always plotted on the vertical axis and time on the horizontal axis.

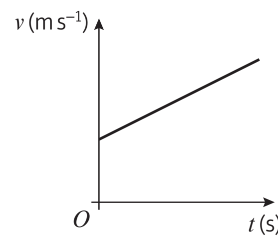
In these graphs v represents the velocity of an object in metres per second and t represents the time taken in seconds.



The velocity is zero and the object is stationary.



The velocity is unchanging and the object is moving with constant velocity.

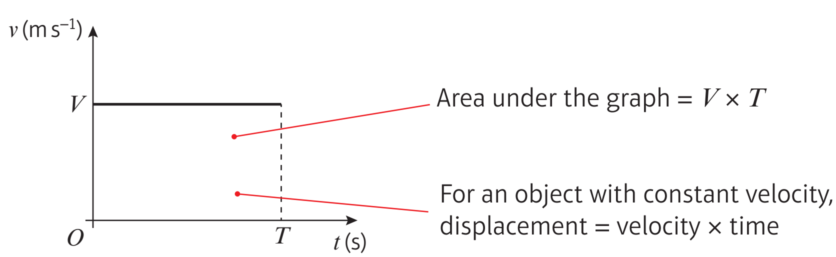


The velocity is increasing at a constant rate and the object is moving with constant acceleration.

- **Acceleration is the rate of change of velocity.**
  - **In a velocity–time graph the gradient represents the acceleration.**
  - **If the velocity–time graph is a straight line, then the acceleration is constant.**

Notation Negative acceleration is sometimes described as deceleration or retardation.

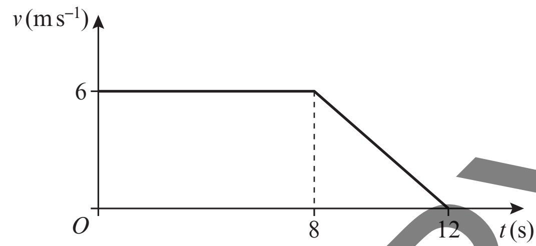
This velocity–time graph represents the motion of an object travelling in a straight line at constant velocity V m s^{-1} for time T seconds.



- **The area between a velocity–time graph and the horizontal axis represents the distance travelled.**
  - **For motion in a straight line with positive velocity, the area under the velocity–time graph up to a point t represents the displacement at time t.**

Example 2

The figure shows a velocity–time graph illustrating the motion of a cyclist moving along a straight road for a period of 12 seconds. For the first 8 seconds, she moves at a constant speed of 6 m s^{-1}. She then decelerates at a constant rate, stopping after a further 4 seconds.



a Find the displacement from the starting point of the cyclist after this 12 second period.

b Work out the rate at which the cyclist decelerates.

a The displacement s after 12 s is given by the area under the graph.

$$s = \frac{1}{2}(a + b)h$$
$$= \frac{1}{2}(8 + 12) \times 6$$
$$= 10 \times 6 = 60$$

The displacement of the cyclist after 12 s is 60 m.

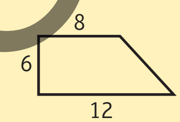
b The acceleration is the gradient of the slope.

$$a = \frac{-6}{4} = -1.5$$

The deceleration is 1.5 m s^{-2}.

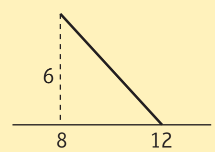
Model the cyclist as a particle moving in a straight line.

The displacement is represented by the area of the trapezium with these sides.



You can use the formula for the area of a trapezium to calculate this area.

The gradient is given by $\frac{\text{difference in the } v\text{-coordinates}}{\text{difference in the } t\text{-coordinates}}$



Example 3

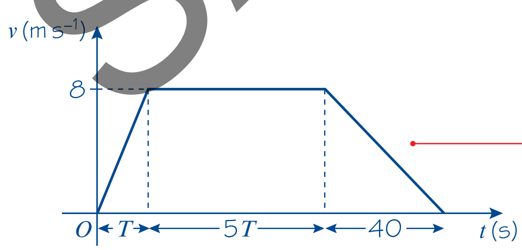
A particle moves along a straight line. The particle accelerates uniformly from rest to a velocity of 8 m s^{-1} in T seconds. The particle then travels at a constant velocity of 8 m s^{-1} for $5T$ seconds. The particle then decelerates uniformly to rest in a further 40 s.

a Sketch a velocity–time graph to illustrate the motion of the particle.

Given that the total displacement of the particle is 600 m:

b find the value of T.

a



If the particle accelerates from rest and decelerates to rest this means the initial and final velocities are zero.

b The area of the trapezium is:

$s = \frac{1}{2}(a + b)h$

$= \frac{1}{2}(5T + 6T + 40) \times 8$

$= 4(11T + 40)$

The displacement is 600 m.

$4(11T + 40) = 600$

$44T + 160 = 600$

$T = \frac{600 - 160}{44} = 10$

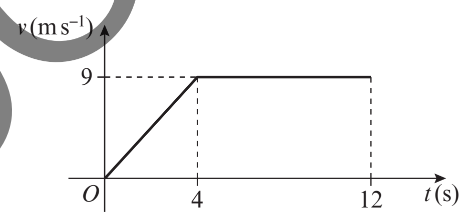
The length of the shorter of the two parallel sides is $5T$. The length of the longer side is $T + 5T + 40 = 6T + 40$.

Problem-solving

The displacement is equal to the area of the trapezium. Write an equation and solve it to find T.

Exercise 9B

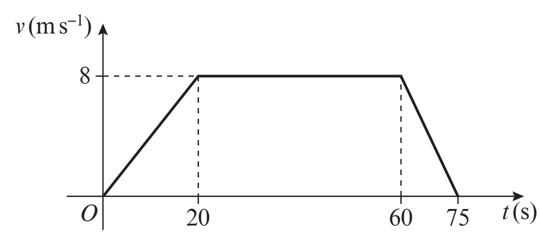
1 The diagram shows the velocity–time graph of the motion of an athlete running along a straight track. For the first 4 s, he accelerates uniformly from rest to a velocity of 9 m s^{-1}.
This velocity is then maintained for a further 8 s. Find:



a the rate at which the athlete accelerates
b the displacement from the starting point of the athlete after 12 s.

2 A car is moving along a straight road. When $t = 0$ s, the car passes a point A with velocity 10 m s^{-1} and this velocity is maintained until $t = 30$ s. The driver then applies the brakes and the car decelerates uniformly, coming to rest at the point B when $t = 42$ s.
a Sketch a velocity–time graph to illustrate the motion of the car.
b Find the distance from A to B.

(E) **3** The diagram shows the velocity–time graph of the motion of a cyclist riding along a straight road. She accelerates uniformly from rest to 8 m s^{-1} in 20 s. She then travels at a constant velocity of 8 m s^{-1} for 40 s. She then decelerates uniformly to rest in 15 s. Find:



a the acceleration of the cyclist in the first 20 s of motion **(2 marks)**
b the deceleration of the cyclist in the last 15 s of motion **(2 marks)**
c the displacement from the starting point of the cyclist after 75 s. **(2 marks)**

(E) **4** A motorcyclist starts from rest at a point S on a straight race track. He moves with constant acceleration for 15 s, reaching a velocity of 30 m s^{-1}. He then travels at a constant velocity of 30 m s^{-1} for T seconds. Finally he decelerates at a constant rate coming to rest at a point F, 25 s after he begins to decelerate.
a Sketch a velocity–time graph to illustrate the motion. **(3 marks)**
Given that the distance between S and F is 2.4 km:
b calculate the time the motorcyclist takes to travel from S to F. **(3 marks)**

E **5** A train starts from a station X and moves with constant acceleration of $0.6\,\text{m}\,\text{s}^{-2}$ for 20 s. The velocity it has reached after 20 s is then maintained for T seconds. The train then decelerates from this velocity to rest in a further 40 s, stopping at a station Y.

a Sketch a velocity–time graph to illustrate the motion of the train. **(3 marks)**

Given that the distance between the stations is 4.2 km, find:

b the value of T **(3 marks)**

c the distance travelled by the train while it is moving with constant velocity. **(2 marks)**

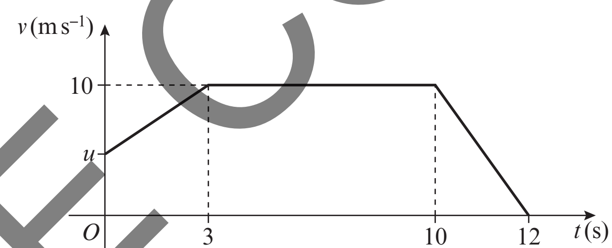
E **6** A particle moves along a straight line. The particle accelerates from rest to a velocity of $10\,\text{m}\,\text{s}^{-1}$ in 15 s. The particle then moves at a constant velocity of $10\,\text{m}\,\text{s}^{-1}$ for a period of time. The particle then decelerates uniformly to rest. The period of time for which the particle is travelling at a constant velocity is 4 times the period of time for which it is decelerating.

a Sketch a velocity–time graph to illustrate the motion of the particle. **(3 marks)**

Given that the displacement from the starting point of the particle after it comes to rest is 480 m

b find the total time for which the particle is moving. **(3 marks)**

E **7** A particle moves 100 m in a straight line. The diagram is a sketch of a velocity–time graph of the motion of the particle. The particle starts with velocity $u\,\text{m}\,\text{s}^{-1}$ and accelerates to a velocity of $10\,\text{m}\,\text{s}^{-1}$ in 3 s. The velocity of $10\,\text{m}\,\text{s}^{-1}$ is maintained for 7 s and then the particle decelerates to rest in a further 2 s. Find:



a the value of u **(3 marks)**

b the acceleration of the particle in the first 3 s of motion. **(3 marks)**

E **8** A motorcyclist M leaves a road junction at time $t = 0$ s. She accelerates from rest at a rate of $3\,\text{m}\,\text{s}^{-2}$ for 8 s and then maintains the velocity she has reached. A car C leaves the same road junction as M at time $t = 0$ s. The car accelerates from rest to $30\,\text{m}\,\text{s}^{-1}$ in 20 s and then maintains the velocity of $30\,\text{m}\,\text{s}^{-1}$. C passes M as they both pass a pedestrian.

a On the same diagram, sketch velocity–time graphs to illustrate the motion of M and C. **(3 marks)**

b Find the distance of the pedestrian from the road junction. **(3 marks)**

Challenge

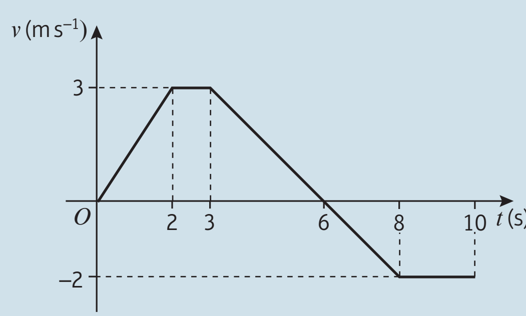
The graph shows the velocity of an object travelling in a straight line during a 10-second time interval.

a After how long did the object change direction?

b Work out the total distance travelled by the object.

c Work out the displacement from the starting point of the object after:

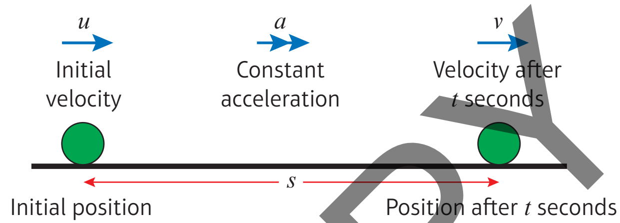
i 6 seconds **ii** 10 seconds.



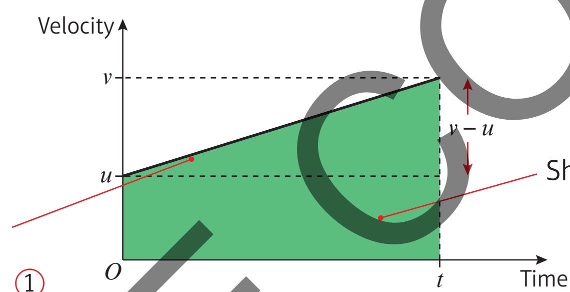
9.3 Constant acceleration formulae 1

A standard set of letters is used for the motion of an object moving in a straight line with constant acceleration.

- s is the displacement.
- u is the initial velocity.
- v is the final velocity.
- a is the acceleration.
- t is the time.



You can use these letters to label a velocity–time graph representing the motion of a particle moving in a straight line accelerating from velocity u at time 0 to velocity v at time t.



Gradient of line $= \frac{v-u}{t}$

$a = \frac{v-u}{t}$

Rearranging $v = u + at$ ①

Shaded area $= \left(\frac{u+v}{2}\right)t$

$s = \left(\frac{u+v}{2}\right)t$ ②

- $\boldsymbol{v = u + at}$ ①
- $\boldsymbol{s = \left(\frac{u+v}{2}\right)t}$ ②

You need to know how to derive these formulae from the velocity–time graph.

Hint Formula ① does not involve s and formula ② does not involve a.

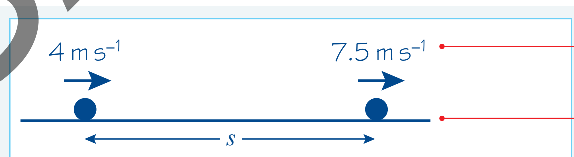
Links These formulae can also be derived using calculus. → Chapter 11

Example 4

A cyclist is travelling along a straight road. She accelerates at a constant rate from a velocity of $4\,\text{m s}^{-1}$ to a velocity of $7.5\,\text{m s}^{-1}$ in 40 seconds. Find:

a the distance she travels in these 40 seconds

b her acceleration in these 40 seconds.



$u = 4,\ v = 7.5,\ t = 40,\ s = ?,\ a = ?$

Start by drawing a diagram.

Model the cyclist as a particle.

Write down the values you know and the values you need to find.

a $s = \left(\frac{u+v}{2}\right)t$

$= \left(\frac{4+7.5}{2}\right) \times 40$

$= 230$

The distance the cyclist travels is 230 m.

b $v = u + at$

You need a and you know v, u and t so you can use $v = u + at$.

$7.5 = 4 + 40a$

Substitute the values you know into the formula. You can solve this equation to find a.

$a = \frac{7.5 - 4}{40} = 0.0875$

You could rearrange the formula before you substitute the values:

$a = \frac{v - u}{t}$

The acceleration of the cyclist is $0.0875\ \text{m s}^{-2}$.

In real-life situations values for the acceleration are often quite small.
Large accelerations feel unpleasant and may be dangerous.

Example 5

A particle moves in a straight line from a point A to a point B with constant deceleration $1.5\ \text{m s}^{-2}$. The velocity of the particle at A is $8\ \text{m s}^{-1}$ and the velocity of the particle at B is $2\ \text{m s}^{-1}$. Find:

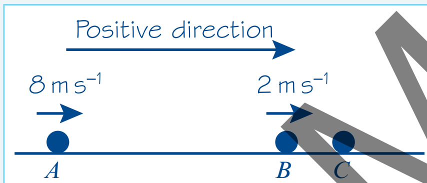
a the time taken for the particle to move from A to B

b the distance from A to B.

After reaching B the particle continues to move along the straight line with constant deceleration $1.5\ \text{m s}^{-2}$. The particle is at the point C 6 seconds after passing through the point A. Find:

c the velocity of the particle at C

d the distance from A to C.



$u = 8,\ v = 2,\ a = -1.5,\ t = ?,\ s = ?$

Problem-solving

It's always a good idea to draw a sketch showing the positions of the particle. Mark the positive direction on your sketch, and remember that when the particle is **decelerating**, your value of a will be **negative**.

a $v = u + at$

$2 = 8 - 1.5t$

$1.5t = 8 - 2$

$t = \frac{8 - 2}{1.5} = 4$

The time taken to move from A to B is 4 s.

b $s = \left(\frac{u+v}{2}\right)t$

You can use your answer from part **a** as the value of t.

$= \left(\frac{8+2}{2}\right) \times 4 = 20$

The distance from A to B is 20 m.

c $u = 8,\ a = -1.5,\ t = 6,\ v = ?$

$v = u + at$

$= 8 + (-1.5) \times 6$

$= 8 - 9 = -1$

> The velocity at C is negative. This means that the particle is moving from right to left.

The velocity of the particle is $1\,\text{m s}^{-1}$ in the direction $\overrightarrow{BA}$.

> Remember that to specify a velocity it is necessary to give speed and direction.

d $s = \left(\dfrac{u+v}{2}\right)t$

$= \left(\dfrac{8 + (-1)}{2}\right) \times 6$

> Make sure you use the correct sign when substituting a negative value into a formula.

The distance from A to C is 21 m.

Convert all your measurements into base SI units before substituting values into the formulae.

Example 6

A car moves from traffic lights along a straight road with constant acceleration. The car starts from rest at the traffic lights and 30 seconds later the car passes a speed-trap where it is registered as travelling at $45\,\text{km h}^{-1}$. Find:

a the acceleration of the car **b** the distance between the traffic lights and the speed-trap.

$45\,\text{km h}^{-1} = 45 \times \dfrac{1000}{3600}\,\text{m s}^{-1} = 12.5\,\text{m s}^{-1}$

> Convert into SI units, using:
> 1 km = 1000 m
> 1 hour = 60 × 60 s = 3600 s

> Model the car as a particle and draw a diagram.

$u = 0,\ v = 12.5,\ t = 30,\ a = ?,\ s = ?$

> The car starts from rest, so the initial velocity is zero.

a $v = u + at$

$12.5 = 0 + 30a$

$a = \dfrac{12.5}{30} = \dfrac{5}{12}$

The acceleration of the car is $\dfrac{5}{12}\,\text{m s}^{-2}$

> This is an exact answer. If you want to give an answer using decimals, you should round to three significant figures.

b $s = \left(\dfrac{u+v}{2}\right)t$

$= \left(\dfrac{0 + 12.5}{2}\right) \times 30 = 187.5$

The distance between the traffic lights and the speed-trap is 187.5 m.

Exercise 9C

1 A particle is moving in a straight line with constant acceleration $3\,\text{m}\,\text{s}^{-2}$. At time $t = 0$, the velocity of the particle is $2\,\text{m}\,\text{s}^{-1}$. Find the velocity of the particle at time $t = 6\,\text{s}$.

2 A car is approaching traffic lights. The car is travelling with velocity $10\,\text{m}\,\text{s}^{-1}$. The driver applies the brakes to the car and the car comes to rest with constant deceleration in 16 s. Modelling the car as a particle, find the deceleration of the car.

3 A car accelerates uniformly while travelling on a straight road. The car passes two signposts 360 m apart. The car takes 15 s to travel from one signpost to the other. When passing the second signpost, it has velocity $28\,\text{m}\,\text{s}^{-1}$. Find the velocity of the car at the first signpost.

4 A cyclist is moving along a straight road from A to B with constant acceleration $0.5\,\text{m}\,\text{s}^{-2}$. Her velocity at A is $3\,\text{m}\,\text{s}^{-1}$ and it takes her 12 seconds to cycle from A to B. Find:

a her velocity at B

b the distance from A to B.

5 A particle is moving along a straight line with constant acceleration from a point A to a point B, where $AB = 24\,\text{m}$. The particle takes 6 s to move from A to B and the velocity of the particle at B is $5\,\text{m}\,\text{s}^{-1}$. Find:

a the velocity of the particle at A

b the acceleration of the particle.

6 A particle moves in a straight line from a point A to a point B with constant deceleration $1.2\,\text{m}\,\text{s}^{-2}$. The particle takes 6 s to move from A to B. The speed of the particle at B is $2\,\text{m}\,\text{s}^{-1}$ and the direction of motion of the particle has not changed. Find:

a the speed of the particle at A

b the distance from A to B.

(P) 7 A train, travelling on a straight track, is slowing down with constant deceleration $0.6\,\text{m}\,\text{s}^{-2}$. The train passes one signal with speed $72\,\text{km}\,\text{h}^{-1}$ and a second signal 25 s later. Find:

> **Hint** Convert the speeds into $\text{m}\,\text{s}^{-1}$ before substituting.

a the velocity, in $\text{km}\,\text{h}^{-1}$, of the train as it passes the second signal

b the distance between the signals.

8 A particle moves in a straight line from a point A to a point B with a constant deceleration of $4\,\text{m}\,\text{s}^{-2}$. At A the particle has velocity $32\,\text{m}\,\text{s}^{-1}$ and the particle comes to rest at B. Find:

a the time taken for the particle to travel from A to B

b the distance between A and B.

(E) 9 A skier travelling in a straight line up a hill experiences a constant deceleration. At the bottom of the hill, the skier has a velocity of $16\,\text{m}\,\text{s}^{-1}$ and, after moving up the hill for 40 s, he comes to rest. Find:

a the deceleration of the skier **(2 marks)**

b the distance from the bottom of the hill to the point where the skier comes to rest. **(4 marks)**

E **10** A particle is moving in a straight line with constant acceleration. The points A, B and C lie on this line. The particle moves from A through B to C. The velocity of the particle at A is $2\,\text{m s}^{-1}$ and the velocity of the particle at B is $7\,\text{m s}^{-1}$. The particle takes 20 s to move from A to B.

a Find the acceleration of the particle. **(2 marks)**

The velocity of the particle is C is $11\,\text{m s}^{-1}$. Find:

b the time taken for the particle to move from B to C **(2 marks)**

c the distance between A and C. **(3 marks)**

E **11** A particle moves in a straight line from A to B with constant acceleration $1.5\,\text{m s}^{-2}$. It then moves along the same straight line from B to C with a different acceleration. The velocity of the particle at A is $1\,\text{m s}^{-1}$ and the velocity of the particle at C is $43\,\text{m s}^{-1}$. The particle takes 12 s to move from A to B and 10 s to move from B to C. Find:

a the velocity of the particle at B **(2 marks)**

b the acceleration of the particle as it moves from B to C **(2 marks)**

c the distance from A to C. **(3 marks)**

E/P **12** A cyclist travels with constant acceleration $x\,\text{m s}^{-2}$, in a straight line, from rest to $5\,\text{m s}^{-1}$ in 20 s. She then decelerates from $5\,\text{m s}^{-1}$ to rest with constant deceleration $\frac{1}{2}x\,\text{m s}^{-2}$. Find:

a the value of x **(2 marks)**

b the total distance she travelled. **(4 marks)**

Problem-solving

You could sketch a velocity–time graph of the cyclist's motion and use the area under the graph to find the total distance travelled.

E/P **13** A particle is moving with constant acceleration in a straight line. It passes through three points, A, B and C, with velocities $20\,\text{m s}^{-1}$, $30\,\text{m s}^{-1}$ and $45\,\text{m s}^{-1}$ respectively. The time taken to move from A to B is t_1 seconds and the time taken to move from B to C is t_2 seconds.

a Show that $\frac{t_1}{t_2} = \frac{2}{3}$. **(3 marks)**

Given also that the total time taken for the particle to move from A to C is 50 s:

b find the distance between A and B. **(5 marks)**

Challenge

A particle moves in a straight line from A to B with constant acceleration. The particle moves from A with velocity $3\,\text{m s}^{-1}$. It reaches point B with velocity $5\,\text{m s}^{-1}$ t seconds later.

One second after the first particle leaves point A, a second particle also starts to move in a straight line from A to B with constant acceleration. Its velocity at point A is $4\,\text{m s}^{-1}$ and it reaches point B with velocity $8\,\text{m s}^{-1}$ at the same time as the first particle.

Find:

a the value of t

b the distance between A and B.

Problem-solving

The time taken for the second particle to travel from A to B is $(t - 1)$ seconds.

9.4 Constant acceleration formulae 2

You can use the formulae $v = u + at$ and $s = \left(\frac{u+v}{2}\right)t$ to work out three more formulae.

You can eliminate t from the formulae for constant acceleration.

$t = \frac{v-u}{a}$ — Rearrange the formula $v = u + at$ to make t the subject.

$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$ — Substitute this expression for t into $s = \left(\frac{u+v}{2}\right)t$.

$2as = v^2 - u^2$

- $\boldsymbol{v^2 = u^2 + 2as}$ — Multiply out the brackets and rearrange.

You can also eliminate v from the formulae for constant acceleration.

$s = \left(\frac{u+u+at}{2}\right)t$ — Substitute $v = u + at$ into $s = \left(\frac{u+v}{2}\right)t$.

$s = \left(\frac{2u}{2} + \frac{at}{2}\right)t$

$s = \left(u + \frac{1}{2}at\right)t$ — Multiply out the brackets and rearrange.

- $\boldsymbol{s = ut + \frac{1}{2}at^2}$

Finally, you can eliminate u by substituting into this formula:

$s = (v - at)t + \frac{1}{2}at^2$ — Substitute $u = v - at$ into $s = ut + \frac{1}{2}at^2$.

- $\boldsymbol{s = vt - \frac{1}{2}at^2}$

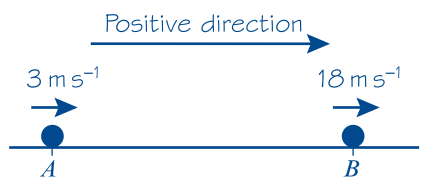
■ **You need to be able to use and to derive the five formulae for solving problems about particles moving in a straight line with constant acceleration.**

- $\boldsymbol{v = u + at}$
- $\boldsymbol{s = \left(\frac{u+v}{2}\right)t}$
- $\boldsymbol{v^2 = u^2 + 2as}$
- $\boldsymbol{s = ut + \frac{1}{2}at^2}$
- $\boldsymbol{s = vt - \frac{1}{2}at^2}$

Watch out These five formulae are sometimes referred to as the **kinematics formulae** or ***suvat* formulae**. They are given in the formulae booklet.

Example 7

A particle is moving along a straight line from A to B with constant acceleration $5\,\text{m}\,\text{s}^{-2}$. The velocity of the particle at A is $3\,\text{m}\,\text{s}^{-1}$ in the direction $\overrightarrow{AB}$. The velocity of the particle at B is $18\,\text{m}\,\text{s}^{-1}$ in the same direction. Find the distance from A to B.



$a = 5,\ u = 3,\ v = 18,\ s = ?$

Write down the values you know and the values you need to find. This will help you choose the correct formula.

$v^2 = u^2 + 2as$

t is not involved so choose the formula that does not have t in it.

$18^2 = 3^2 + 2 \times 5 \times s$

$324 = 9 + 10s$

$s = \dfrac{324 - 9}{10} = 31.5$

Substitute in the values you are given and solve the equation for s. This gives the distance you were asked to find.

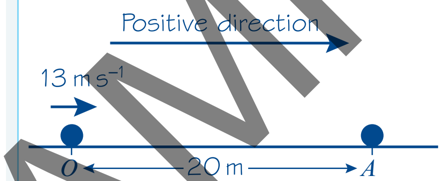
$AB = 31.5\text{ m}$

Example 8

A particle is moving in a straight horizontal line with constant deceleration $4\,\text{m}\,\text{s}^{-2}$. At time $t = 0$ the particle passes through a point O with speed $13\,\text{m}\,\text{s}^{-1}$ travelling towards a point A, where $OA = 20\,\text{m}$. Find:

a the times when the particle passes through A

b the value of t when the particle returns to O.



a $a = -4,\ u = 13,\ s = 20,\ t = ?$

The particle is decelerating so the value of a is negative.

$s = ut + \frac{1}{2}at^2$

You are told the values of a, u and s and asked to find t. You are given no information about v and are not asked to find it so you choose the formula without v.

$20 = 13t - \frac{1}{2} \times 4t^2$

$= 13t - 2t^2$

$2t^2 - 13t + 20 = 0$

$(2t - 5)(t - 4) = 0$

$t = \frac{5}{2}$, or $t = 4$

There are two answers. Both are correct. The particle moves from O to A, goes beyond A and then turns round and returns to A.

The particle moves through A twice, $2\frac{1}{2}$ seconds and 4 seconds after moving through O.

Problem-solving

When you use $s = ut + \frac{1}{2}at^2$ with an unknown value of t you obtain a quadratic equation in t. You can solve this equation by factorising, or using the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

b The particle returns to O when $s = 0$.

$s = 0, u = 13, a = -4, t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 13t - 2t^2$

$= t(13 - 2t)$

$t = 0$, or $t = \frac{13}{2}$

The particle returns to O 6.5 seconds after it first passed through O.

When the particle returns to O, its displacement (distance) from O is zero.

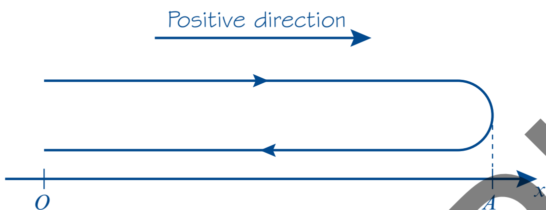
The first solution ($t = 0$) represents the starting position of the particle. The other solution ($t = \frac{13}{2}$) tells you when the particle returns to O.

Example 9

A particle P is moving on the x-axis with constant deceleration $2.5\,\text{m s}^{-2}$. At time $t = 0$, the particle P passes through the origin O, moving in the positive direction of x with speed $15\,\text{m s}^{-1}$. Find:

a the time between the instant when P first passes through O and the instant when it returns to O

b the total distance travelled by P during this time.



a $a = -2.5, u = 15, s = 0, t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 15t - \frac{1}{2} \times 2.5 \times t^2$

$0 = 60t - 5t^2$

$= 5t(12 - t)$

$t = 0, t = 12$

The particle P returns to O after 12 s.

b $a = -2.5, u = 15, v = 0, s = ?$

$v^2 = u^2 + 2as$

$0^2 = 15^2 - 2 \times 2.5 \times s$

$5s = 15^2 = 225$

$s = \frac{225}{5} = 45$

The distance $OA = 45$ m.

The total distance travelled by P is $2 \times 45\text{ m} = 90\text{ m}$.

Problem-solving

Before you start, draw a sketch so you can see what is happening. The particle moves through O with a positive velocity. As it is decelerating it slows down and will eventually have zero velocity at a point A, which you don't yet know. As the particle is still decelerating, its velocity becomes negative, so the particle changes direction and returns to O.

When the particle returns to O, its displacement (distance) from O is zero.

Multiply by 4 to get whole-number coefficients.

At the furthest point from O, labelled A in the diagram, the particle changes direction. At that point, for an instant, the particle has zero velocity.

In the 12 s the particle has been moving it has travelled to A and back. The total distance travelled is twice the distance OA.

Exercise 9D

1 A particle is moving in a straight line with constant acceleration $2.5\,\text{m s}^{-2}$. It passes a point A with velocity $3\,\text{m s}^{-1}$ and later passes through a point B, where $AB = 8\,\text{m}$. Find the velocity of the particle as it passes through B.

2 A car is accelerating at a constant rate along a straight horizontal road. Travelling at $8\,\text{m s}^{-1}$, it passes a pillar box and 6 s later it passes a sign. The distance between the pillar box and the sign is 60 m. Find the acceleration of the car.

3 A cyclist travelling at $12\,\text{m s}^{-1}$ applies her brakes and comes to rest after travelling 36 m in a straight line. Assuming that the brakes cause the cyclist to decelerate uniformly, find the deceleration.

4 A train is moving along a straight horizontal track with constant acceleration. The train passes a signal with a velocity of $54\,\text{km h}^{-1}$ and a second signal with a velocity of $72\,\text{km h}^{-1}$. The distance between the two signals is 500 m. Find, in m s^{-2}, the acceleration of the train.

5 A particle moves along a straight line, with constant acceleration, from a point A to a point B where $AB = 48\,\text{m}$. At A the particle has velocity $4\,\text{m s}^{-1}$ and at B it has velocity $16\,\text{m s}^{-1}$. Find:

a the acceleration of the particle

b the time the particle takes to move from A to B.

6 A particle moves along a straight line with constant acceleration $3\,\text{m s}^{-2}$. The particle moves 38 m in 4 s. Find:

a the initial velocity of the particle

b the final velocity of the particle.

7 The driver of a car is travelling at $18\,\text{m s}^{-1}$ along a straight road when she sees an obstruction ahead. She applies the brakes and the brakes cause the car to slow down to rest with a constant deceleration of $3\,\text{m s}^{-2}$. Find:

a the distance travelled as the car decelerates

b the time it takes for the car to decelerate from $18\,\text{m s}^{-1}$ to rest.

8 A stone is sliding across a frozen lake in a straight line. The initial speed of the stone is $12\,\text{m s}^{-1}$. The friction between the stone and the ice causes the stone to slow down at a constant rate of $0.8\,\text{m s}^{-2}$. Find:

a the distance moved by the stone before coming to rest

b the speed of the stone at the instant when it has travelled half of this distance.

9 A particle is moving along a straight line OA with constant acceleration $2.5\,\text{m s}^{-2}$. At time $t = 0$, the particle passes through O with speed $8\,\text{m s}^{-1}$ and is moving in the direction OA. The distance OA is 40 m. Find:

a the time taken for the particle to move from O to A

b the speed of the particle at A. Give your answers to one decimal place.

10 A particle travels with uniform deceleration $2\,\text{m s}^{-2}$ in a horizontal line. The points A and B lie on the line and $AB = 32\,\text{m}$. At time $t = 0$, the particle passes through A with velocity $12\,\text{m s}^{-1}$ in the direction $\overrightarrow{AB}$. Find:

a the values of t when the particle is at B

b the velocity of the particle for each of these values of t.

E/P **11** A particle is moving along the x-axis with constant deceleration $5\,\text{m s}^{-2}$. At time $t = 0$, the particle passes through the origin O with velocity $12\,\text{m s}^{-1}$ in the positive direction. At time t seconds the particle passes through the point A with x-coordinate 8. Find:

Problem-solving

The particle will pass through A twice. Use $s = ut + \frac{1}{2}at^2$ to set up and solve a quadratic equation.

a the values of t **(3 marks)**

b the velocity of the particle as it passes through the point with x-coordinate -8. **(3 marks)**

E **12** A particle P is moving on the x-axis with constant deceleration $4\,\text{m s}^{-2}$. At time $t = 0$, P passes through the origin O with velocity $14\,\text{m s}^{-1}$ in the positive direction. The point A lies on the axis and $OA = 22.5\,\text{m}$. Find:

a the difference between the times when P passes through A **(4 marks)**

b the total distance travelled by P during the interval between these times. **(3 marks)**

E/P **13** A car is travelling along a straight horizontal road with constant acceleration. The car passes over three consecutive points A, B and C where $AB = 100\,\text{m}$ and $BC = 300\,\text{m}$. The speed of the car at B is $14\,\text{m s}^{-1}$ and the speed of the car at C is $20\,\text{m s}^{-1}$. Find:

a the acceleration of the car **(3 marks)**

b the time take for the car to travel from A to C. **(3 marks)**

E/P **14** Two particles P and Q are moving along the same straight horizontal line with constant accelerations $2\,\text{m s}^{-2}$ and $3.6\,\text{m s}^{-2}$ respectively. At time $t = 0$, P passes through a point A with speed $4\,\text{m s}^{-1}$. One second later Q passes through A with speed $3\,\text{m s}^{-1}$, moving in the same direction as P.

a Write down expressions for the displacements of P and Q from A, in terms of t, where t seconds is the time after P has passed through A. **(2 marks)**

b Find the value of t where the particles meet. **(3 marks)**

c Find the distance of A from the point where the particles meet. **(3 marks)**

Problem-solving

When P and Q meet, their displacements from A are equal.

E/P **15** In an orienteering competition, a competitor moves in a straight line past three checkpoints, P, Q and R, where $PQ = 2.4\,\text{km}$ and $QR = 11.5\,\text{km}$. The competitor is modelled as a particle moving with constant acceleration. She takes 1 hour to travel from P to Q and 1.5 hours to travel from Q to R. Find:

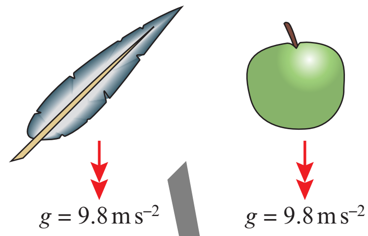
a the acceleration of the competitor

b her speed at the instant she passes P. **(7 marks)**

9.5 Vertical motion under gravity

You can use the formulae for constant acceleration to model an object moving vertically under gravity.

- **The force of gravity causes all objects to accelerate towards the earth. If you ignore the effects of air resistance, this acceleration is constant. It does not depend on the mass of the object.**



As the force of gravity does not depend on mass, this means that in a vacuum an apple and a feather would both accelerate downwards at the same rate.

On earth, the acceleration due to gravity is represented by the letter g and is approximately 9.8 m s^{-2}.

The actual value of the acceleration can vary by very small amounts in different places due to the changing radius of the earth and height above sea level.

- **An object moving vertically under gravity can be modelled as a particle with a constant downward acceleration of $g = 9.8\text{ m s}^{-2}$.**

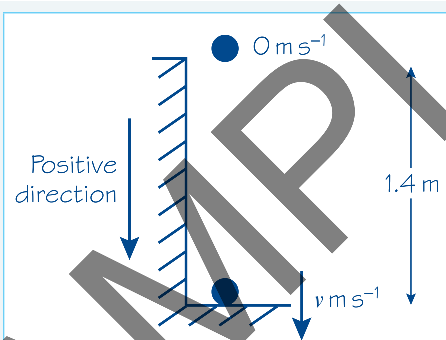
Watch out In mechanics questions you will always use $g = 9.8\text{ m s}^{-2}$ unless a question specifies otherwise. However, if a different value of g is specified (e.g. $g = 10\text{ m s}^{-2}$ or $g = 9.81\text{ m s}^{-2}$) the degree of accuracy in your answer should be chosen to be consistent with this value.

When solving problems about vertical motion you can choose the positive direction to be either upwards or downwards. Acceleration due to gravity is always downwards, so if the positive direction is upwards then $g = -9.8\text{ m s}^{-2}$.

Notation The total time that an object is in motion from the time it is projected (thrown) upwards to the time it hits the ground is called the **time of flight**. The initial speed is sometimes called the **speed of projection**.

Example 10

A book falls off the top shelf of a bookcase. The shelf is 1.4 m above a wooden floor. Find: **a** the time the book takes to reach the floor, **b** the speed with which the book strikes the floor.



Model the book as a particle moving in a straight line with a constant acceleration of magnitude 9.8 m s^{-2}.

As the book is moving downwards throughout its motion, it is sensible to take the downwards direction as positive.

a $s = 1.4$

$a = +9.8$ — You have taken the downwards direction as positive and gravity acts downwards. Here the acceleration is positive.

$u = 0$ — Assume the book has an initial speed of zero.

$t = ?$

$s = ut + \frac{1}{2}at^2$ — Choose the formula without v.

$1.4 = 0 + \frac{1}{2} \times 9.8 \times t^2$

$t^2 = \frac{1.4}{4.9} = 0.2857\ldots$ — Solve the equation for t^2 and use your calculator to find the positive square root.

$t = \sqrt{0.2857\ldots} = 0.5345\ldots$

The time taken for the book to reach the floor is 0.53 s, to two significant figures. — Give the answer to two significant figures to be consistent with the degree of accuracy used for the value of g.

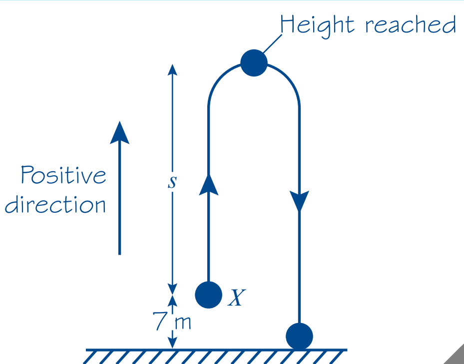
b $s = 1.4$
$a = 9.8$
$u = 0$
$v = ?$
$v^2 = u^2 + 2as$
$= 0^2 + 2 \times 9.8 \times 1.4 = 27.44$
$v = \sqrt{27.44} = 5.238\ldots \approx 5.2$
The book hits the floor with speed $5.2\,\text{m s}^{-1}$, to two significant figures.

Choose the formula without t.

Use unrounded values in your calculations, but give your final answer correct to two significant figures.

Example 11

A ball is projected vertically upwards, from a point X which is 7 m above the ground, with speed $21\,\text{m s}^{-1}$. Find: **a** the greatest height above the ground reached by the ball, **b** the time of flight of the ball.



Problem-solving

In this sketch the upward and downwards motion have been sketched side by side. In reality they would be on top of one another, but drawing them separately makes it easier to see what is going on. Remember that X is 7 m above the ground, so mark this height on your sketch.

a $u = 21$
$v = 0$
$a = -9.8$
$s = ?$
$v^2 = u^2 + 2as$
$0^2 = 21^2 + 2 \times (-9.8) \times s = 441 - 19.6s$
$s = \frac{441}{19.6} = 22.5$
$(22.5 + 7)\,\text{m} = 29.5\,\text{m}$
Greatest height is 30 m (2 s.f.)

At its highest point, the ball is turning round. For an instant, it is neither going up nor down, so its speed is zero.

22.5 m is the distance the ball has moved above X but X is 7 m above the ground. You must add on another 7 m to get the greatest height above the ground reached by the ball.

b $s = -7$
$u = 21$
$a = -9.8$
$t = ?$
$s = ut + \frac{1}{2}at^2$
$-7 = 21t - 4.9t^2$
$4.9t^2 - 21t - 7 = 0$

The time of flight is the total time that the ball is in motion from the time that it is projected to the time that it stops moving. Here the ball will stop when it hits the ground. The point where the ball hits the ground is 7 m below the point from which it was projected so $s = -7$.

$$t = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$= \frac{-(-21) \pm \sqrt{((-21)^2 - 4 \times 4.9 \times (-7))}}{2 \times 4.9}$$

$$= \frac{21 \pm \sqrt{578.2}}{9.8} \approx \frac{21 \pm 24.046}{9.8}$$

$t \approx 4.5965,$

or $t \approx -0.3108$

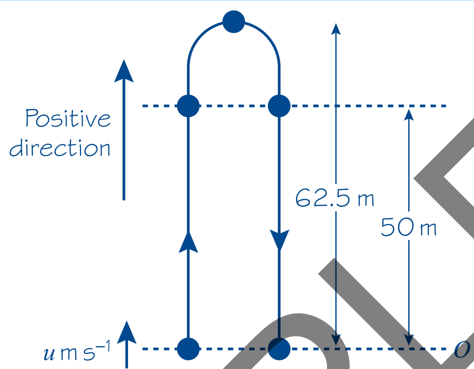
Time of flight is 4.6 s (2 s.f.)

Rearrange the equation and use the quadratic formula.

Take the positive answer and round to two significant figures.

Example 12

A particle is projected vertically upwards from a point O with speed $u\ \text{m s}^{-1}$. The greatest height reached by the particle is 62.5 m above O. Find: **a** the value of u, **b** the total time for which the particle is 50 m or more above O.



The particle will pass through the point 50 m above O twice: once on the way up and once on the way down.

a $v = 0$

$s = 62.5$

$a = -9.8$

$u = ?$

$v^2 = u^2 + 2as$

$0^2 = u^2 + 2 \times (-9.8) \times 62.5$

$u^2 = 1225$

$u = \sqrt{1225} = 35\ \text{m s}^{-1}$

There is no t, so you choose the formula without t.

In this part, you obtain an exact answer, so there is no need for approximation.

b $s = 50$

$u = 35$

$a = -9.8$

$t = ?$

$s = ut + \frac{1}{2}at^2$

$50 = 35t - 4.9t^2$

$4.9t^2 - 35t + 50 = 0$

$$t = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$= \frac{35 \pm \sqrt{(35^2 - 4 \times 4.9 \times 50)}}{9.8}$$

Two values of t need to be found: one on the way up and one on the way down.

Write the equation in the form $ax^2 + bx + c = 0$ and use the quadratic formula.

$= \frac{35 \pm \sqrt{245}}{9.8} \approx \frac{35 \pm 15.6525}{9.8}$

$t \approx 5.1686...$, or $t \approx 1.9742...$

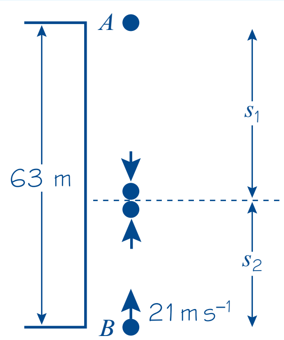
$(5.1686...) - (1.9742...) \approx 3.194$

Particle is 50 m or more above O for 3.2 s (2 s.f.)

Between these two times the particle is always more than 50 m above O. You find the total time for which the particle is 50 m or more above O by finding the difference of these two values.

Example 13

A ball A falls vertically from rest from the top of a tower 63 m high. At the same time as A begins to fall, another ball B is projected vertically upwards from the bottom of the tower with speed 21 m s^{-1}. The balls collide. Find the distance of the point where the balls collide from the bottom of the tower.



Problem-solving

You must take special care with problems where objects are moving in different directions. Here A is moving downwards and you will take the acceleration due to gravity as positive. However, B is moving upwards so for B the acceleration due to gravity is negative.

For A, the motion is downwards

$u = 0$

$a = 9.8$

$s = ut + \frac{1}{2}at^2$

$s_1 = 4.9t^2$

You cannot find s_1 at this stage. You have to express it in terms of t.

For B, the motion is upwards

$u = 21$

$a = -9.8$

As B is moving upwards, the acceleration due to gravity is negative.

$s = ut + \frac{1}{2}at^2$

$s_2 = 21t - 4.9t^2$

You now have expressions for s_1 and s_2 in terms of t.

The height of the tower is 63 m.

$s_1 + s_2 = 63$

$4.9t^2 + (21t - 4.9t^2) = 63$

$21t = 63$

$t = 3$

Adding together the two distances gives the height of the tower. You can write this as an equation in t.

$s_2 = 21t - 4.9t^2$

$= 21 \times 3 - 4.9 \times 3^2 = 18.9$

You have found t but you were asked for the distance from the bottom of the tower. Substitute your value for t into your equation for s_2.

The balls collide 19 m from the bottom of the tower, to two significant figures.

Exercise 9E

1 A cliff diver jumps from a point 28 m above the surface of the water. Modelling the diver as a particle moving freely under gravity with initial velocity 0, find:

a the time taken for the diver to hit the water

b the speed of the diver when he hits the water.

2 A particle is projected vertically upwards with speed 20 m s^{-1} from a point on the ground. Find the time of flight of the particle.

3 A ball is thrown vertically downward from the top of a tower with speed 18 m s^{-1}. It reaches the ground in 1.6 s. Find the height of the tower.

4 A pebble is catapulted vertically upwards with speed 24 m s^{-1}. Find:

a the greatest height above the point of projection reached by the pebble

b the time taken to reach this height.

5 A ball is projected upwards from a point which is 4 m above the ground with speed 18 m s^{-1}. Find:

a the speed of the ball when it is 15 m above its point of projection

b the speed with which the ball hits the ground.

6 A particle P is projected vertically downwards from a point 80 m above the ground with speed 4 m s^{-1}. Find:

a the speed with which P hits the ground

b the time P takes to reach the ground.

7 A particle P is projected vertically upwards from a point X. Five seconds later P is moving downwards with speed 10 m s^{-1}. Find:

a the speed of projection of P

b the greatest height above X attained by P during its motion.

8 A ball is thrown vertically upwards with speed 21 m s^{-1}. It hits the ground 4.5 s later. Find the height above the ground from which the ball was thrown.

9 A stone is thrown vertically upward from a point which is 3 m above the ground, with speed 16 m s^{-1}. Find:

a the time of flight of the stone

b the total distance travelled by the stone.

P 10 A particle is projected vertically upwards with speed 24.5 m s^{-1}. Find the total time for which it is 21 m or more above its point of projection.

E/P 11 A particle is projected vertically upwards from a point O with speed u m s^{-1}. Two seconds later it is still moving upwards and its speed is $\frac{1}{3}u$ m s^{-1}. Find:

a the value of u **(3 marks)**

b the time from the instant that the particle leaves O to the instant that it returns to O. **(4 marks)**

Problem-solving

Use $v = u + at$ and substitute $v = \frac{1}{3}u$.

E/P **12** A ball A is thrown vertically downwards with speed $5\,\text{m s}^{-1}$ from the top of a tower block 46 m above the ground. At the same time as A is thrown downwards, another ball B is thrown vertically upwards from the ground with speed $18\,\text{m s}^{-1}$. The balls collide. Find the distance of the point where A and B collide from the point where A was thrown. **(5 marks)**

E/P **13** A ball is released from rest at a point which is 10 m above a wooden floor. Each time the ball strikes the floor, it rebounds with three-quarters of the speed with which it strikes the floor. Find the greatest height above the floor reached by the ball

a the first time it rebounds from the floor **(3 marks)**

b the second time it rebounds from the floor. **(4 marks)**

Problem-solving

Consider each bounce as a separate motion.

Challenge

1 A particle P is projected vertically upwards from a point O with speed $12\,\text{m s}^{-1}$. One second after P has been projected from O, another particle Q is projected vertically upwards from O with speed $20\,\text{m s}^{-1}$. Find: **a** the time between the instant that P is projected from O and the instant when P and Q collide, **b** the distance of the point where P and Q collide from O.

2 A stone is dropped from the top of a building and two seconds later another stone is thrown vertically downwards at a speed of $25\,\text{m s}^{-1}$. Both stones reach the ground at the same time. Find the height of the building.

Chapter review 9

1 A car accelerates in a straight line at a constant rate, starting from rest at a point A and reaching a velocity of $45\,\text{km h}^{-1}$ in 20 s. This velocity is then maintained and the car passes a point B 3 minutes after leaving A.

a Sketch a velocity–time graph to illustrate the motion of the car.

b Find the displacement of the car from its starting point after 3 minutes.

2 A particle is moving on an axis Ox. From time $t = 0$ s to time $t = 32$ s, the particle is travelling with constant velocity $15\,\text{m s}^{-1}$. The particle then decelerates from $15\,\text{m s}^{-1}$ to rest in T seconds.

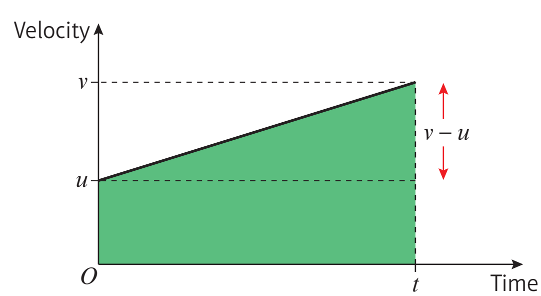
a Sketch a velocity–time graph to illustrate the motion of the particle.

The total distance travelled by the particle is 570 m.

b Find the value of T.

c Sketch a displacement–time graph illustrating the motion of the particle.

P **3** The velocity–time graph represents the motion of a particle moving in a straight line accelerating from velocity u at time 0 to velocity v at time t.



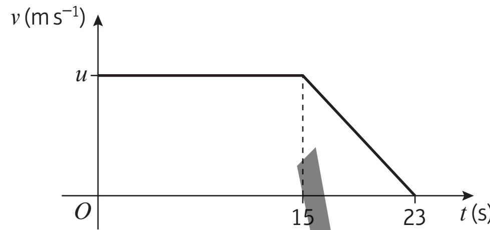
a Use the graph to show that:

i $v = u + at$ **ii** $s = \left(\frac{u+v}{2}\right)t$

b Hence show that:

i $v^2 = u^2 + 2as$ **ii** $s = ut + \frac{1}{2}at^2$ **iii** $s = vt - \frac{1}{2}at^2$

P **4** The diagram is a velocity–time graph representing the motion of a cyclist along a straight road. At time $t = 0$ s, the cyclist is moving with velocity u m s^{-1}. The velocity is maintained until time $t = 15$ s, when she slows down with constant deceleration, coming to rest when $t = 23$ s. The total distance she travels in 23 s is 152 m. Find the value of u.



5 A car travelling on a straight road slows down with constant deceleration. The car passes a road sign with velocity 40 km h^{-1} and a post box with velocity of 24 km h^{-1}. The distance between the road sign and the post box is 240 m. Find, in m s^{-2}, the deceleration of the car.

6 A particle P is moving along the x-axis with constant deceleration 2.5 m s^{-2}. At time $t = 0$ s, P passes through the origin with velocity 20 m s^{-1} in the direction of x increasing. At time $t = 12$ s, P is at the point A. Find:

a the distance OA

b the total distance P travels in 12 s.

7 A ball is thrown vertically downward from the top of a tower with speed 6 m s^{-1}. The ball strikes the ground with speed 25 m s^{-1}. Find the time the ball takes to move from the top of the tower to the ground.

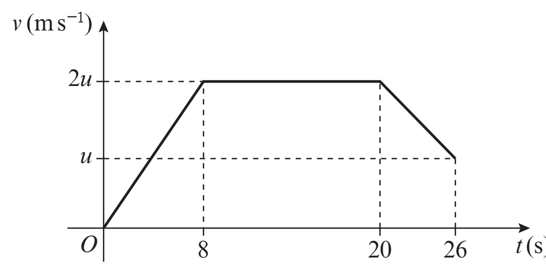
8 A child drops a ball from a point at the top of a cliff which is 82 m above the sea. The ball is initially at rest. Find:

a the time taken for the ball to reach the sea

b the speed with which the ball hits the sea.

c State one physical factor which has been ignored in making your calculation.

P **9** A particle moves 451 m in a straight line. The diagram shows a speed–time graph illustrating the motion of the particle. The particle starts at rest and accelerates at a constant rate for 8 s reaching a speed of $2u$ m s^{-1}. The particle then travels at a constant speed for 12 seconds before decelerating uniformly, reaching a speed of u m s^{-1} at time $t = 26$ s. Find:



a the value of u

b the distance moved by the particle while its speed is less than u m s^{-1}.

E/P **10** A train is travelling with constant acceleration along a straight track. At time $t = 0$ s, the train passes a point O travelling with velocity 18 m s^{-1}. At time $t = 12$ s, the train passes a point P travelling with velocity 24 m s^{-1}. At time $t = 20$ s, the train passes a point Q. Find:

a the speed of the train at Q **(5 marks)**

b the distance from P to Q. **(2 marks)**

E **11** A particle moves along a straight line, from a point X to a point Y, with constant acceleration. The distance from X to Y is 104 m. The particle takes 8 s to move from X to Y and the speed of the particle at Y is 18 m s^{-1}. Find:

a the speed of the particle at X **(3 marks)**

b the acceleration of the particle. **(2 marks)**

The particle continues to move with the same acceleration until it reaches a point Z. At Z the speed of the particle is three times the speed of the particle at X.

c Find the distance XZ. **(4 marks)**

E **12** A pebble is projected vertically upwards with speed 21 m s^{-1} from a point 32 m above the ground. Find:

a the speed with which the pebble strikes the ground **(3 marks)**

b the total time for which the pebble is more than 40 m above the ground. **(4 marks)**

c Sketch a velocity–time graph for the motion of the pebble from the instant it is projected to the instant it hits the ground, showing the values of t at any points where the graph intercepts the horizontal axis. **(4 marks)**

E **13** A car is moving along a straight road with uniform acceleration. The car passes a checkpoint A with speed 12 m s^{-1} and another checkpoint C with speed 32 m s^{-1}. The distance between A and C is 1100 m.

a Find the time taken by the car to move from A to C. **(2 marks)**

b Given that B is the midpoint of AC, find the speed with which the car passes B. **(2 marks)**

E/P **14** A particle is projected vertically upwards with a speed of 30 m s^{-1} from a point A. The point B is h metres above A. The particle moves freely under gravity and is above B for a time 2.4 s. Calculate the value of h. **(5 marks)**

E/P **15** Two cars A and B are moving in the same direction along a straight horizontal road. At time $t = 0$, they are side by side, passing a point O on the road. Car A travels at a constant speed of 30 m s^{-1}. Car B passes O with a speed of 20 m s^{-1}, and has constant acceleration of 4 m s^{-2}. Find:

a the speed of B when it has travelled 78 m from O **(2 marks)**

b the distance from O of A when B is 78 m from O **(3 marks)**

c the time when B overtakes A. **(4 marks)**

E/P **16** A car is being driven on a straight stretch of motorway at a constant velocity of 34 m s^{-1}, when it passes a velocity restriction sign S warning of road works ahead and requiring speeds to be reduced to 22 m s^{-1}. The driver continues at her velocity for 2 s after passing S. She then reduces her velocity to 22 m s^{-1} with constant deceleration of 3 m s^{-2}, and continues at the lower velocity.

a Draw a velocity–time graph to illustrate the motion of the car after it passes S. **(2 marks)**

b Find the shortest distance before the road works that S should be placed on the road to ensure that a car driven in this way has had its velocity reduced to 22 m s^{-1} by the time it reaches the start of the road works. **(4 marks)**

E/P **17** A train starts from rest at station A and accelerates uniformly at $3x\text{ m s}^{-2}$ until it reaches a velocity of 30 m s^{-1}. For the next T seconds the train maintains this constant velocity. The train then decelerates uniformly at $x\text{ m s}^{-2}$ until it comes to rest at a station B. The distance between the stations is 6 km and the time taken from A to B is 5 minutes.

a Sketch a velocity–time graph to illustrate this journey. **(2 marks)**

b Show that $\frac{40}{x} + T = 300$. **(4 marks)**

c Find the value of T and the value of x. **(2 marks)**

d Calculate the distance the train travels at constant velocity. **(2 marks)**

e Calculate the time taken from leaving A until reaching the point halfway between the stations. **(3 marks)**

Challenge

A ball is projected vertically upwards with speed 10 m s^{-1} from a point X, which is 50 m above the ground. T seconds after the first ball is projected upwards, a second ball is dropped from X. Initially the second ball is at rest. The balls collide 25 m above the ground. Find the value of T.

Summary of key points

1 Velocity is the **rate of change** of displacement.
On a displacement–time graph the **gradient** represents the velocity.
If the displacement–time graph is a straight line, then the velocity is constant.

2 Average velocity $= \dfrac{\text{displacement from starting point}}{\text{time taken}}$

3 Average speed $= \dfrac{\text{total distance travelled}}{\text{time taken}}$

4 Acceleration is the **rate of change** of velocity.
In a velocity–time graph the **gradient** represents the acceleration.
If the velocity–time graph is a straight line, then the acceleration is constant.

5 The area between a velocity–time graph and the horizontal axis represents the distance travelled.
For motion in a straight line with positive velocity, the area under the velocity–time graph up to a point t represents the displacement at time t.

6 You need to be able to use and to derive the five formulae for solving problems about particles moving in a straight line with constant acceleration.

- $v = u + at$
- $s = \left(\frac{u + v}{2}\right)t$
- $v^2 = u^2 + 2as$
- $s = ut + \frac{1}{2}at^2$
- $s = vt - \frac{1}{2}at^2$

7 The force of **gravity** causes all objects to accelerate towards the earth. If you ignore the effects of air resistance, this acceleration is constant. It does not depend on the mass of the object.

8 An object moving vertically in a straight line can be modelled as a particle with a constant downward acceleration of $g = 9.8\text{ m s}^{-2}$.