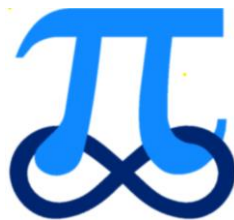


Edexcel
New GCE A Level Maths
workbook
Solving Linear and
Quadratic
Simultaneous
Equations.



Edited by: K V Kumaran

Solving linear simultaneous equations using the elimination method

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$\begin{array}{r} 3x + y = 5 \\ - \quad x + y = 1 \\ \hline 2x \quad = 4 \\ \text{So } x = 2 \end{array}$	1 Subtract the second equation from the first equation to eliminate the y term.
$\begin{array}{r} \text{Using } x + y = 1 \\ \quad 2 + y = 1 \\ \text{So } y = -1 \end{array}$	2 To find the value of y , substitute $x = 2$ into one of the original equations.
<p>Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES</p>	3 Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$\begin{array}{r} x + 2y = 13 \\ + \quad 5x - 2y = 5 \\ \hline 6x \quad = 18 \\ \text{So } x = 3 \end{array}$	1 Add the two equations together to eliminate the y term.
$\begin{array}{r} \text{Using } x + 2y = 13 \\ \quad 3 + 2y = 13 \\ \text{So } y = 5 \end{array}$	2 To find the value of y , substitute $x = 3$ into one of the original equations.
<p>Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES</p>	3 Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$$\begin{array}{r} (2x + 3y = 2) \times 4 \rightarrow 8x + 12y = 8 \\ (5x + 4y = 12) \times 3 \rightarrow \underline{15x + 12y = 36} \\ -7x = 28 \end{array}$$

So $x = 4$

Using $2x + 3y = 2$
 $2 \times 4 + 3y = 2$
So $y = -2$

Check:
equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES
equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES

1 Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term.

2 To find the value of y , substitute $x = 4$ into one of the original equations.

3 Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $4x + y = 8$
 $x + y = 5$

2 $3x + y = 7$
 $3x + 2y = 5$

3 $4x + y = 3$
 $3x - y = 11$

4 $3x + 4y = 7$
 $x - 4y = 5$

5 $2x + y = 11$
 $x - 3y = 9$

6 $2x + 3y = 11$
 $3x + 2y = 4$

Solving linear simultaneous equations using the substitution method

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Textbook: Pure Year 1, 3.1 Linear simultaneous equations

Key points

- The substitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 4 Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$$5x + 3(2x + 1) = 14$$

$$5x + 6x + 3 = 14$$

$$11x + 3 = 14$$

$$11x = 11$$

$$\text{So } x = 1$$

$$\text{Using } y = 2x + 1$$

$$y = 2 \times 1 + 1$$

$$\text{So } y = 3$$

Check:

$$\text{equation 1: } 3 = 2 \times 1 + 1 \quad \text{YES}$$

$$\text{equation 2: } 5 \times 1 + 3 \times 3 = 14 \quad \text{YES}$$

1 Substitute $2x + 1$ for y into the second equation.

2 Expand the brackets and simplify.

3 Work out the value of x .

4 To find the value of y , substitute $x = 1$ into one of the original equations.

5 Substitute the values of x and y into both equations to check your answers.

Example 5 Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$$y = 2x - 16$$
$$4x + 3(2x - 16) = -3$$

$$4x + 6x - 48 = -3$$

$$10x - 48 = -3$$

$$10x = 45$$

$$\text{So } x = 4\frac{1}{2}$$

$$\text{Using } y = 2x - 16$$

$$y = 2 \times 4\frac{1}{2} - 16$$

$$\text{So } y = -7$$

Check:

$$\text{equation 1: } 2 \times 4\frac{1}{2} - (-7) = 16 \quad \text{YES}$$

$$\text{equation 2: } 4 \times 4\frac{1}{2} + 3 \times (-7) = -3 \quad \text{YES}$$

1 Rearrange the first equation.

2 Substitute $2x - 16$ for y into the second equation.

3 Expand the brackets and simplify.

4 Work out the value of x .

5 To find the value of y , substitute $x = 4\frac{1}{2}$ into one of the original equations.

6 Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

7 $y = x - 4$

$$2x + 5y = 43$$

8 $y = 2x - 3$

$$5x - 3y = 11$$

9 $2y = 4x + 5$

$$9x + 5y = 22$$

10 $2x = y - 2$

$$8x - 5y = -11$$

11 $3x + 4y = 8$

$$2x - y = -13$$

12 $3y = 4x - 7$

$$2y = 3x - 4$$

13 $3x = y - 1$

$$2y - 2x = 3$$

14 $3x + 2y + 1 = 0$

$$4y = 8 - x$$

Extend

15 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \frac{3(y - x)}{4}$.

Answers

1 $x = 1, y = 4$

2 $x = 3, y = -2$

3 $x = 2, y = -5$

4 $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = -1$

6 $x = -2, y = 5$

7 $x = 9, y = 5$

8 $x = -2, y = -7$

9 $x = \frac{1}{2}, y = 3\frac{1}{2}$

10 $x = \frac{1}{2}, y = 3$

11 $x = -4, y = 5$

12 $x = -2, y = -5$

13 $x = \frac{1}{4}, y = 1\frac{3}{4}$

14 $x = -2, y = 2\frac{1}{2}$

15 $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

Solving linear and quadratic simultaneous equations

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1 Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$$x^2 + (x + 1)^2 = 13$$

$$x^2 + x^2 + x + x + 1 = 13$$

$$2x^2 + 2x + 1 = 13$$

$$2x^2 + 2x - 12 = 0$$

$$(2x - 4)(x + 3) = 0$$

$$\text{So } x = 2 \text{ or } x = -3$$

$$\text{Using } y = x + 1$$

$$\text{When } x = 2, y = 2 + 1 = 3$$

$$\text{When } x = -3, y = -3 + 1 = -2$$

So the solutions are

$$x = 2, y = 3 \quad \text{and} \quad x = -3, y = -2$$

Check:

$$\text{equation 1: } 3 = 2 + 1 \quad \text{YES}$$

$$\text{and } -2 = -3 + 1 \quad \text{YES}$$

$$\text{equation 2: } 2^2 + 3^2 = 13 \quad \text{YES}$$

$$\text{and } (-3)^2 + (-2)^2 = 13 \quad \text{YES}$$

1 Substitute $x + 1$ for y into the second equation.

2 Expand the brackets and simplify.

3 Factorise the quadratic equation.

4 Work out the values of x .

5 To find the value of y , substitute both values of x into one of the original equations.

6 Substitute both pairs of values of x and y into both equations to check your answers.

Example 2 Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \frac{5-3y}{2}$ $2y^2 + \left(\frac{5-3y}{2}\right)y = 12$ $2y^2 + \frac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$ $y^2 + 5y - 24 = 0$ $(y+8)(y-3) = 0$ <p>So $y = -8$ or $y = 3$</p> <p>Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$</p> <p>So the solutions are $x = 14.5$, $y = -8$ and $x = -2$, $y = 3$</p> <p>Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2 \times (-8)^2 + 14.5 \times (-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES</p>	<ol style="list-style-type: none"> 1 Rearrange the first equation. 2 Substitute $\frac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y. 3 Expand the brackets and simplify. 4 Factorise the quadratic equation. 5 Work out the values of y. 6 To find the value of x, substitute both values of y into one of the original equations. 7 Substitute both pairs of values of x and y into both equations to check your answers.
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Practice

Solve these simultaneous equations.

1 $y = 2x + 1$
 $x^2 + y^2 = 10$

2 $y = 6 - x$
 $x^2 + y^2 = 20$

3 $y = x - 3$
 $x^2 + y^2 = 5$

4 $y = 9 - 2x$
 $x^2 + y^2 = 17$

5 $y = 3x - 5$
 $y = x^2 - 2x + 1$

6 $y = x - 5$
 $y = x^2 - 5x - 12$

7 $y = x + 5$
 $x^2 + y^2 = 25$

8 $y = 2x - 1$
 $x^2 + xy = 24$

9 $y = 2x$
 $y^2 - xy = 8$

10 $2x + y = 11$
 $xy = 15$

Extend

11 $x - y = 1$
 $x^2 + y^2 = 3$

12 $y - x = 2$
 $x^2 + xy = 3$

Answers

1 $x = 1, y = 3$

$$x = -\frac{9}{5}, y = -\frac{13}{5}$$

2 $x = 2, y = 4$

$$x = 4, y = 2$$

3 $x = 1, y = -2$

$$x = 2, y = -1$$

4 $x = 4, y = 1$

$$x = \frac{16}{5}, y = \frac{13}{5}$$

5 $x = 3, y = 4$

$$x = 2, y = 1$$

6 $x = 7, y = 2$

$$x = -1, y = -6$$

7 $x = 0, y = 5$

$$x = -5, y = 0$$

8 $x = -\frac{8}{3}, y = -\frac{19}{3}$

$$x = 3, y = 5$$

9 $x = -2, y = -4$

$$x = 2, y = 4$$

10 $x = \frac{5}{2}, y = 6$

$$x = 3, y = 5$$

11 $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$

$$x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$$

12 $x = \frac{-1+\sqrt{7}}{2}, y = \frac{3+\sqrt{7}}{2}, \quad x = \frac{-1-\sqrt{7}}{2}, y = \frac{3-\sqrt{7}}{2}$

Q1.

Solve the simultaneous equations

$$x + y = 2$$

$$4y^2 - x^2 = 11$$

(7)

Q2.

Solve the simultaneous equations

$$y - 3x + 2 = 0$$

$$y^2 - x - 6x^2 = 0$$

(7)

Q3.

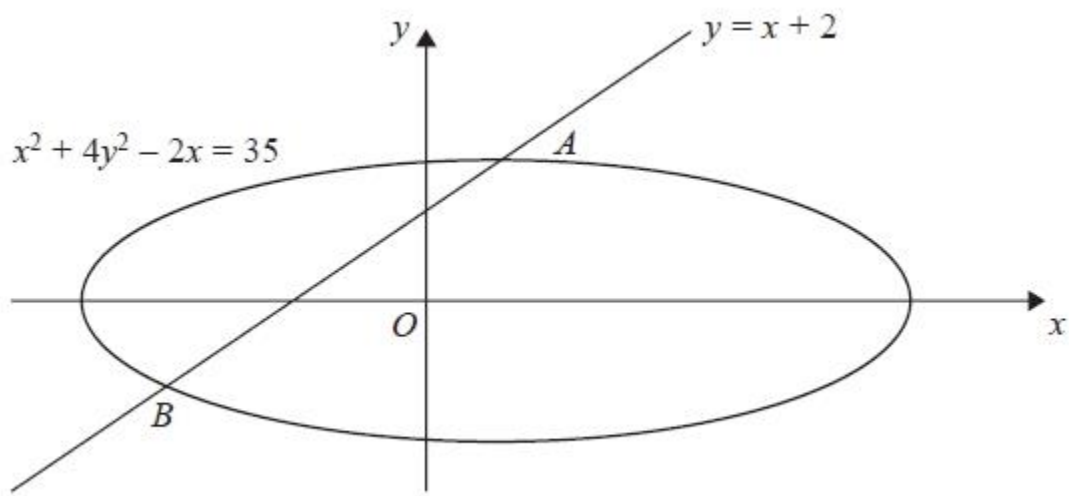


Figure 2

The line $y = x + 2$ meets the curve $x^2 + 4y^2 - 2x = 35$ at the points A and B as shown in Figure 2.

Find the coordinates of A and the coordinates of B .

(6)

Q4.

Solve the simultaneous equations

$$y - 2x - 4 = 0$$

$$4x^2 + y^2 + 20x = 0$$

(7)

Q5.

Solve the simultaneous equations

$$x + y = 2$$

$$x^2 + 2y = 12.$$

(6)

Q6.

(a) By eliminating y from the equations

$$y = x - 4$$

$$2x^2 - xy = 8,$$

show that

$$x^2 + 4x - 8 = 0$$

(2)

(b) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

(5)

Q7.

Solve the simultaneous equations

$$y = x - 2,$$

$$y^2 + x^2 = 10.$$

(7)

Q8.

(a) By eliminating y from the equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

show that

$$x^2 + 4x - 8 = 0. \tag{2}$$

(b) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers. (5)

Q9.

Solve the simultaneous equations

$$y - 3x + 2 = 0$$

$$y^2 - x - 6x^2 = 0$$

(7)

Q10.

Solve the simultaneous equations

$$x + y = 2$$

$$4y^2 - x^2 = 11$$

(7)