



EE 232 Lightwave Devices Lecture 3: Basic Semiconductor Physics and Optical Processes

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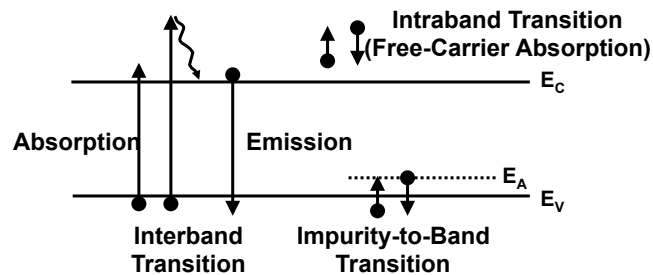
University of California, Berkeley
Electrical Engineering and Computer Sciences Dept.

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Optical Properties of Semiconductors



• Optical transitions

- Absorption: exciting an electron to a higher energy level by absorbing a photon
- Emission: electron relaxing to a lower energy state by emitting a photon

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Band-to-Band Transition

- Since most electrons and holes are near the band edges, the photon energy of band-to-band (or interband) transition is approximately equal to the bandgap energy:

$$h\nu = E_g$$

- The optical wavelength of band-to-band transition can be approximated by

$$\lambda = \frac{c}{\nu} = \frac{hc}{E_g} \approx \frac{1.24}{E_g}$$

λ : wavelength in μm

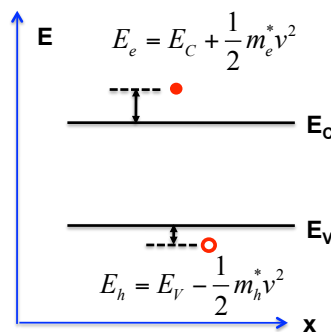
E_g : energy bandgap in eV

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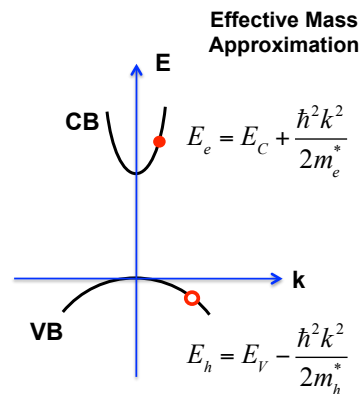
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Energy Band Diagram in Real Space and k-Space



Momentum:
 $\hbar k = m_e^*v_e$

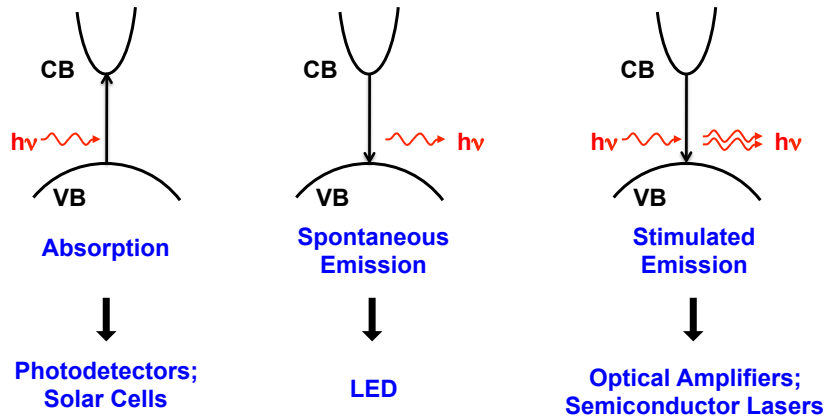


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Band-to-Band Transition

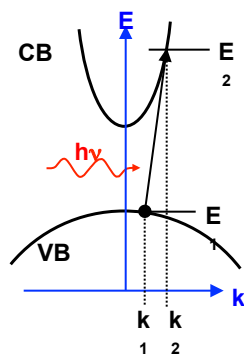


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Conservation of Energy and Momentum



Optical transitions are "vertical" lines

- **Conditions for optical absorption and emission:**
 - Conservation of energy

$$E_2 - E_1 = h\nu$$

- Conservation of momentum

$$k_2 - k_1 = k_{h\nu}$$

$$k_2, k_1 \sim \frac{2\pi}{a}$$

$$k_{h\nu} \sim \frac{2\pi}{\lambda}$$

$$(a \sim 0.5 \text{ nm}) \ll (\lambda \sim 1 \mu\text{m})$$

Lattice Constant

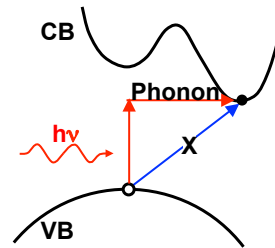
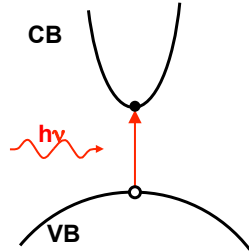
$$\Rightarrow k_2 = k_1$$

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Direct vs Indirect Bandgaps



- **Direct bandgap materials**
 - CB minimum and VB maximum occur at the same k
 - **Examples**
 - GaAs, InP, InGaAsP
 - $(\text{Al}_x\text{Ga}_{1-x})\text{As}$, $x < 0.45$

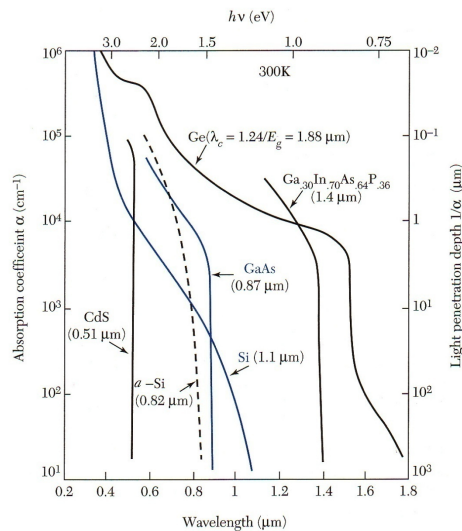
- **Indirect bandgap materials**
 - CB minimum and VB maximum occur at different k
 - **Example**
 - Si, Ge
 - $(\text{Al}_x\text{Ga}_{1-x})\text{As}$, $x > 0.45$
 - Not “optically active”

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Absorption Coefficient



- **Light intensity decays exponentially in semiconductor:**

$$I(x) = I_0 e^{-\alpha x}$$

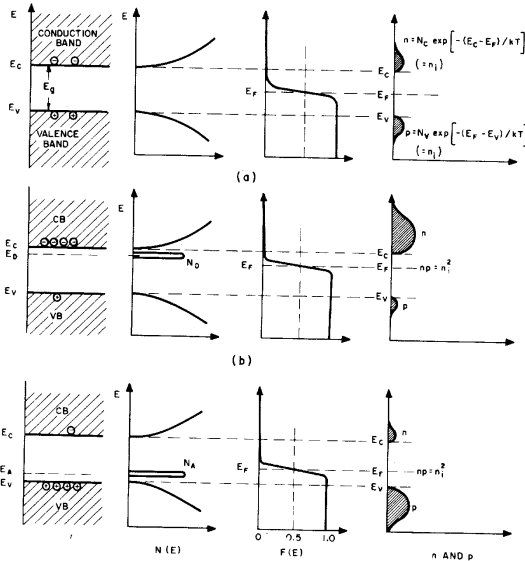
- **Direct bandgap semiconductor has a sharp absorption edge**
- **Si absorbs photons with $h\nu > E_g = 1.1 \text{ eV}$, but the absorption coefficient is small**
 - Sufficient for CCD
- **At higher energy ($\sim 3 \text{ eV}$), absorption coefficient of Si becomes large again, due to direct bandgap transition to higher CB**

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Review of Semiconductor Physics



Electron and hole concentrations:

$$n = \int_{E_c}^{\infty} f_n(E) \rho_e(E) dE$$

$$p = \int_{-\infty}^{E_v} f_p(E) \rho_h(E) dE$$

Fermi-Dirac distributions:

$$f_n(E) = \frac{1}{1 + \exp\left(\frac{E - F_n}{k_B T}\right)}$$

$$f_p(E) = \frac{1}{1 + \exp\left(\frac{F_p - E}{k_B T}\right)}$$

F_n : electron quasi-Fermi level

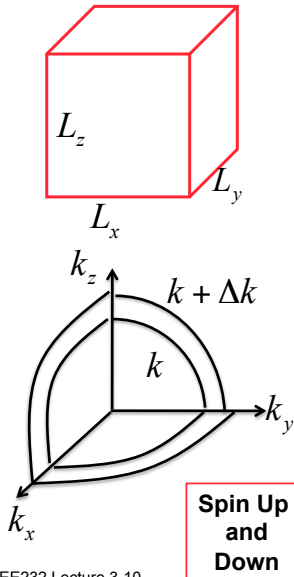
F_p : hole quasi-Fermi level

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Electron/Hole Density of States (1)



- Electron wave with wavevector \vec{k}

$$e^{i\vec{k}\cdot\vec{r}}$$

- Periodic boundary conditions

$$e^{i\vec{k}\cdot\vec{r}} = e^{i\vec{k}\cdot(\vec{r}+L_x\vec{x})} = e^{i\vec{k}\cdot(\vec{r}+L_y\vec{y})} = e^{i\vec{k}\cdot(\vec{r}+L_z\vec{z})}$$

- An electron state is defined by

$$(k_x, k_y, k_z) \uparrow \downarrow = \left(m \frac{2\pi}{L_x}, n \frac{2\pi}{L_y}, l \frac{2\pi}{L_z} \right) \uparrow \downarrow$$

- Number of electron states between \vec{k} and $\vec{k} + \Delta\vec{k}$ in k -space per unit volume

$$\frac{2}{V} \cdot \frac{4\pi k^2 dk}{2\pi \cdot 2\pi \cdot 2\pi} = \frac{k^2}{\pi^2} dk = \rho_k(k) dk$$

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Electron/Hole Density of States (2)

- Number of electron states between E and E + ΔE per unit volume

$$E = E_C + \frac{\hbar^2 k^2}{2m_e^*} \Rightarrow dE = \frac{\hbar^2 k}{m_e^*} dk$$

$$\frac{k^2}{\pi^2} dk = \frac{m_e^*}{\hbar^2 \pi^2} \frac{\sqrt{2m_e^*(E - E_C)}}{\hbar} dE = \rho_e(E) dE$$

$$\rho_e(E) = \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2} \right)^{3/2} \sqrt{E - E_C}$$

- Likewise, hole density of states

$$\rho_h(E) = \frac{1}{2\pi^2} \left(\frac{2m_h^*}{\hbar^2} \right)^{3/2} \sqrt{E_V - E}$$

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Electron and Hole Concentrations

$$n = \int_{E_C}^{\infty} f_n(E) \rho_e(E) dE = \int_{E_C}^{\infty} \frac{1}{1 + \exp\left(\frac{E - F_n}{k_B T}\right)} \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2} \right)^2 \sqrt{E - E_C} dE$$

$$n = N_C \cdot F_{1/2} \left(\frac{F_n - E_C}{k_B T} \right)$$

$$N_C = 2 \left(\frac{\pi m_e^* k_B T}{2\pi^2 \hbar^2} \right)^{3/2}$$

$$p = N_V \cdot F_{1/2} \left(\frac{E_V - F_p}{k_B T} \right)$$

$$N_V = 2 \left(\frac{\pi m_h^* k_B T}{2\pi^2 \hbar^2} \right)^{3/2}$$

Fermi-Dirac Integral

$$F_j(\eta) = \frac{1}{\Gamma(j+1)} \int_0^{\infty} \frac{x^j}{1 + e^{x-\eta}} dx$$

Gamma Function

$$\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$$

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Approximation of Electron/Hole Concentration

$$F_j(\eta) = \frac{1}{\Gamma(j+1)} \int_0^\infty \frac{x^j}{1+e^{x-\eta}} dx \approx \begin{cases} e^\eta & \text{when } \eta \ll 1 \\ \frac{4}{3} \left(\frac{\eta^3}{\pi} \right)^{1/2} & \text{when } \eta \gg 1 \end{cases}$$

When $F_n \ll E_C$ (Boltzmann approximation)

$$n \approx N_C \cdot e^{-\frac{E_C - F_n}{k_B T}}$$

When $F_n \gg E_C$ (Degenerate)

$$n \approx N_C \cdot \frac{4 \left(\frac{F_n - E_C}{k_B T} \right)^{3/2}}{3\sqrt{\pi}}$$

