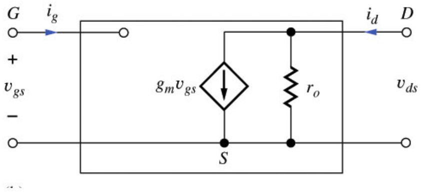
EE105 – Fall 2014
Microelectronic Devices and Circuits

Prof. Ming C. Wu

wu@eecs.berkeley.edu

511 Sutardja Dai Hall (SDH)

Small-Signal Operation
MOSFET Small-Signal Model - Summary



- **Since gate is insulated from channel by gate-oxide input resistance of transistor is infinite.**
- **Small-signal parameters are controlled by the Q-point.**
- **For the same operating point, MOSFET has lower transconductance and an output resistance that is similar to the BJT.**

$$I_G = 0$$

$$I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2\left(1 + \lambda V_{DS}\right)$$

Transconductance:

$$g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \sqrt{2K_n I_D}$$

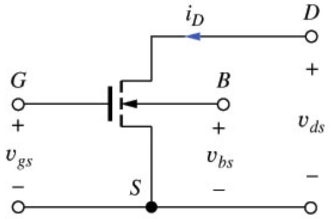
Output resistance:

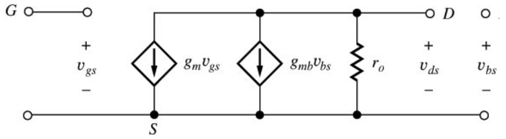
$$r_o = \frac{1}{g_o} = \frac{1 + \lambda V_{DS}}{\lambda I_D} \cong \frac{1}{\lambda I_D}$$

Amplification factor for IV_{DS}<<1:

$$\mu_f = g_m r_o = \frac{1 + \lambda V_{DS}}{\lambda I_D} \cong \frac{1}{\lambda}\sqrt{\frac{2K_n}{I_D}}$$

MOSFET Small-Signal Operation
Body Effect in Four-terminal MOSFETs





Drain current depends on threshold voltage which in turn depends on v_{SB}. Back-gate transconductance is:

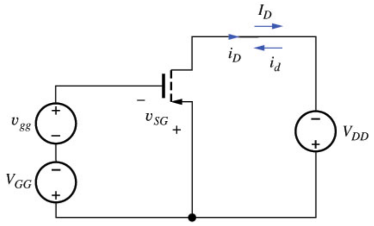
$$g_{mb} = \left.\frac{\partial i_D}{\partial v_{BS}}\right|_{\text{Q-point}} = -\left.\frac{\partial i_D}{\partial v_{SB}}\right|_{\text{Q-point}}$$

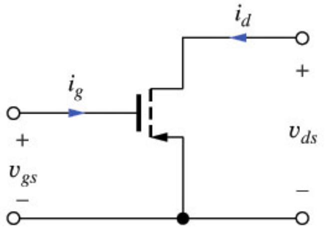
$$g_{mb} = -\left.\left(\frac{\partial i_D}{\partial V_{TN}}\right)\left(\frac{\partial V_{TN}}{\partial v_{SB}}\right)\right|_{\text{Q-point}} = -(-g_m \eta) = g_m \eta$$

$0 < \eta < 3$ is called the back-gate transconductance parameter.

bulk terminal is a reverse-biased diode. Hence, no conductance from the bulk terminal to other terminals.

MOSFET Small-Signal Operation
Small-Signal Model for PMOS Transistor



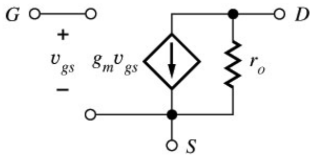


- **For a PMOS transistor**

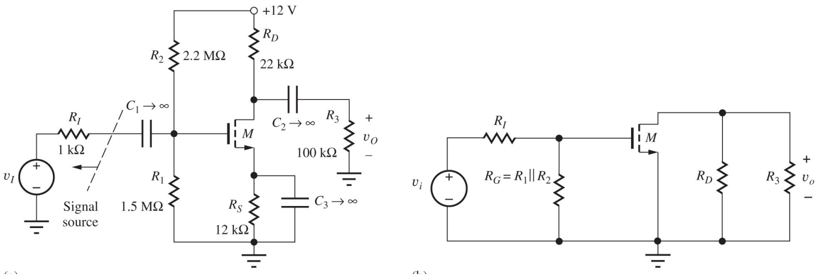
$$v_{SG} = V_{GG} - v_{gg}$$

$$i_D = I_D - i_d$$

- **Positive signal voltage v_{gg} reduces source-gate voltage of the PMOS transistor causing decrease in total current exiting the drain, equivalent to an increase in the signal current entering the drain.**
- **The NMOS and PMOS small-signal models are the same!**

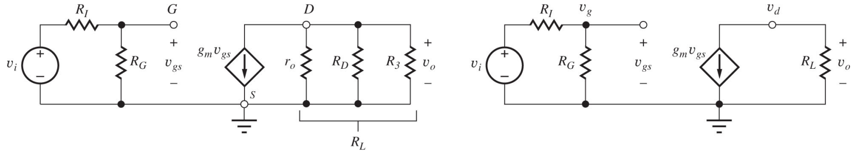


Common-Source Amplifiers
Small-Signal Analysis - ac Equivalent Circuit



- ac equivalent circuit is constructed by assuming that all capacitances have zero impedance at signal frequency and dc voltage sources are ac ground.

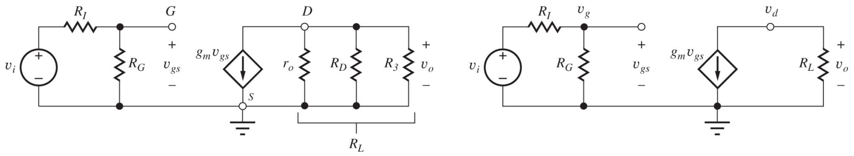
Common-Source Amplifiers
Small-Signal Equivalent Circuit



- Input voltage is applied to the gate terminal
- Output signal appears at the drain terminal
- Source is *common* to both input and output signals
  Thus circuit is termed a Common-Source (C-S) Amplifier.
- The terminal gain of the C-S amplifier is the gain from the gate terminal to the drain terminal

$$A_{vt}^{CE} = \frac{v_d}{v_g} = -g_m R_L \qquad R_L = r_o \| R_D \| R_3$$

Common-Source Amplifiers
Input Resistance and Signal-Source Gain



Define R_{iG} as the input resistance looking into the base of the transistor. R_{in} is the resistance presented to v_i

$$R_{iG} = \frac{v_g}{i_i} = R_G$$

$$R_{in} = R_I + R_G$$

The signal source voltage gain is:

$$A_v^{CS} = \frac{v_o}{v_i} = \frac{v_o}{v_g}\frac{v_g}{v_i} = A_{vt}^{CS}\frac{R_G}{R_I + R_G}$$

$$A_v^{CS} = -g_m R_L\left(\frac{R_G}{R_I + R_G}\right)$$

Common-Source Amplifiers
"Rule of Thumb" Design Estimate

$$A_v^{CS} = -g_m R_L\left(\frac{R_G}{R_I + R_G}\right) \cong A_{vt}^{CS} \qquad A_{vt}^{CS} = -g_m R_L \qquad R_L = r_o \| R_D \| R_3$$

Typically: $r_o >> R_D$ and $R_3 >> R_D$ $\qquad A_v^{CS} \cong -g_m R_D = -\frac{I_D R_D}{\left(\frac{V_{GS} - V_{TN}}{2}\right)}$

$I_D R_D$ represents the voltage dropped across drain resistor R_D

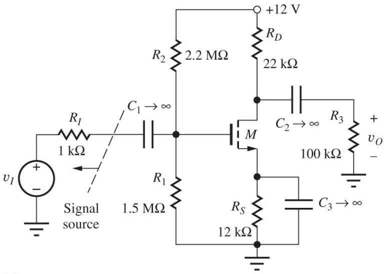
A typical design point is $I_D R_D = \frac{V_{DD}}{2}$ with $V_{GS} - V_{TN} = 1\ V$

$$\therefore A_v^{CS} \cong -V_{DD}$$

Our rule-of-thumb estimate for the C-S amplifier:
the voltage gain equals the power supply voltage.
Note that this is 10 times smaller than that for the BJT!

Common-Source Amplifiers
Voltage Gain Example

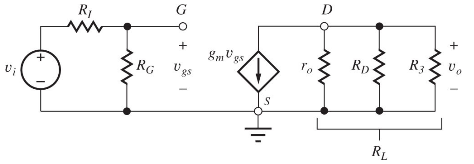
- **Problem: Calculate voltage gain, input resistance and maximum input signal level for a common-source amplifier with a specified Q-point**
- **Given data: K_n = 0.50 mA/V^2, V_{TN} = 1 V,**
- **λ = 0.0133V^{-1}, Q-point is (0.241 mA, 3.81 V)**
- **Assumptions: Transistor is in the active region. Signals are low enough to be considered small signals.**
- **Analysis:**

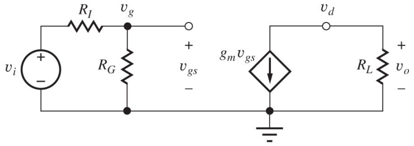


$$g_m = \sqrt{2K_n I_D\left(1+\lambda V_{DS}\right)} = 0.503\ mS \qquad r_o = \frac{\lambda^{-1}+V_{DS}}{I_D} = 328\ k\Omega$$

$$R_G = R_1 \| R_2 = 892\ k\Omega \qquad R_L = r_o \| R_D \| R_3 = 17.1\ k\Omega$$

Common-Source Amplifiers
Voltage Gain Example (cont.)





$$g_m = 0.503\ mS \qquad r_o = 328\ k\Omega \qquad R_G = 892\ k\Omega \qquad R_L = 17.1\ k\Omega$$

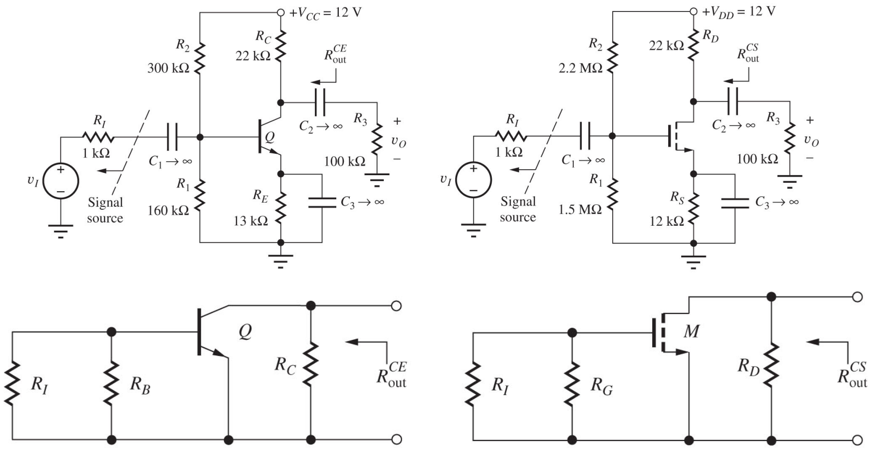
$$A_v^{CS} = -g_m R_L\left(\frac{R_G}{R_I+R_G}\right) = -0.503mS\left(17.1k\Omega\right)\left(\frac{892k\Omega}{1k\Omega+892k\Omega}\right) = -8.60\left(0.999\right) = -8.59$$

$$R_{in} = R_I + R_G = 893\ k\Omega \qquad v_{gs} = v_i\left(\frac{R_G}{R_I+R_G}\right) \to \left|v_i\right|\left(\frac{R_G}{R_I+R_G}\right) \le 0.2\left(V_{GS}-V_{TN}\right)$$

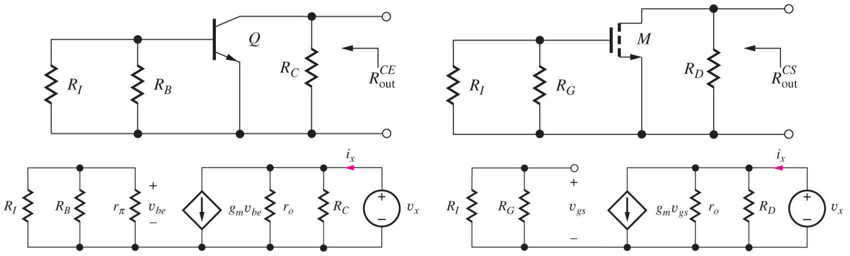
$$V_{GS}-V_{TN} \cong \sqrt{\frac{2I_D}{K_n}} = 0.982\ V \qquad \therefore \left|v_i\right| \le 0.2\left(0.982V\right)\left(\frac{893k\Omega}{892k\Omega}\right) = 0.197\ V$$

Check the rule-of-thumb estimate: $A_v^{CS} \cong -V_{DD} = -12\ V$ (ballpark estimate)

C-E and C-S Amplifiers Output Resistance



C-E and C-S Amplifiers Output Resistance (cont.)



Apply test source v_x and find i_x (with $v_i = 0$)

$v_{be} = 0 \rightarrow g_m v_{be} = 0$

$$\therefore R_{out} = \frac{v_x}{i_x} = R_C \| r_o$$

$R_{out} \cong R_C$ for $r_o >> R_C$

$v_{gs} = 0 \rightarrow g_m v_{gs} = 0$

$$\therefore R_{out} = \frac{v_x}{i_x} = R_D \| r_o$$

$R_{out} \cong R_D$ for $r_o >> R_D$

For comparable bias points, output resistances of C-S and C-E amplifiers are similar.

BJT and FET Small-Signal Model Summary

TABLE 13.3

Small-Signal Parameter Comparison

PARAMETER	BIPOLAR TRANSISTOR	MOSFET
Transconductance g_m	$\frac{I_C}{V_T}$	$\frac{2I_D}{V_{GS}-V_{TN}} \cong \sqrt{2K_n I_D}$
Input resistance	$r_\pi = \frac{\beta_o}{g_m} = \frac{\beta_o V_T}{I_C}$	∞
Output resistance r_o	$\frac{V_A+V_{CE}}{I_C} \cong \frac{V_A}{I_C}$	$\frac{\frac{1}{\lambda}+V_{DS}}{I_D} \cong \frac{1}{\lambda I_D}$
Intrinsic voltage gain μ_f	$\frac{V_A+V_{CE}}{V_T} \cong \frac{V_A}{V_T}$	$\frac{2\left(\frac{1}{\lambda}+V_{DS}\right)}{V_{GS}-V_{TN}} \cong \frac{1}{\lambda}\sqrt{\frac{2K_n}{I_D}}$
Small-signal requirement	$v_{be} \le 0.005\ \text{V}$	$v_{gs} \le 0.2(V_{GS}-V_{TN})$

dc i-v active region expressions for use with Table 13.3:

BJT: $I_C = I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right)-1\right]\left[1+\frac{V_{CE}}{V_A}\right] \qquad V_T = \frac{kT}{q}$

MOSFET: $I_D = \frac{K_n}{2}(V_{GS}-V_{TN})^2(1+\lambda V_{DS}) \qquad K_n = \mu_n C_{\text{ox}}\frac{W}{L}$

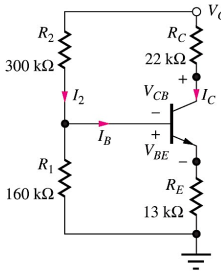
JFET: $I_D = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2(1+\lambda V_{DS})$

Common-Emitter / Common-Source Amplifiers Summary

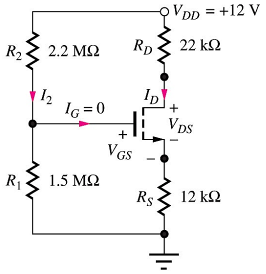
	COMMON-EMITTER AMPLIFIER	COMMON-SOURCE AMPLIFIERS
Terminal gain A_{vt}	$-g_m R_L$	$-g_m R_L$
Rule-of-thumb estimate for $g_m R_L$	$-10V_{CC}$	$-V_{DD}$
Voltage gain A_v	$A_v = \frac{\mathbf{v}_o}{\mathbf{v}_i} = -g_m(R_{\text{out}} \| R_3)\left(\frac{R_{\text{in}}}{R_I+R_{\text{in}}}\right)$	
Input resistance R_{in}	$R_B \| r_\pi$	R_G
Output resistance R_{out}	$R_C \| r_o \cong R_C$	$R_D \| r_o \cong R_D$
Input signal phase	0.005 V	$0.2(V_{GS}-V_{TN})$ or $0.2(V_{GS}-V_P)$

Amplifier Power Dissipation

Static power dissipation in amplifiers is found from their dc equivalent circuits.



(a)



(b)

(a) Total power dissipated in BJT:

$$P_D = V_{CE} I_C + V_{BE} I_B$$

Total power supplied is:

$$P_S = V_{CC} \ (I_C + I_2)$$

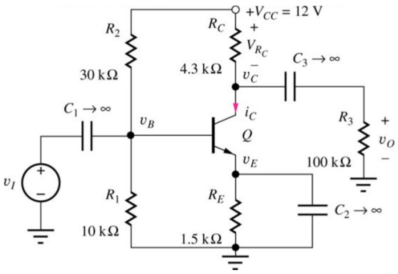
(b) Total power dissipated in MOSFET:

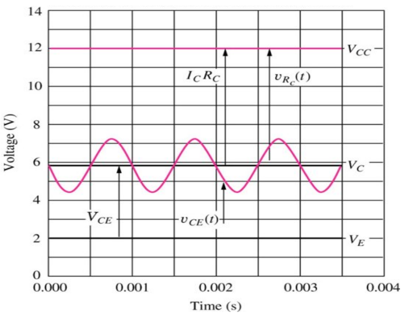
$$P_D = V_{DS} I_D$$

Total power supplied is:

$$P_S = V_{DD} \ (I_D + I_2)$$

Amplifier Signal Range





$v_{CE} = V_{CE} - V_m \sin \omega t$ where V_m is the output signal. Active region operation requires $v_{CE} \geq v_{BE}$ So: $V_m \leq V_{CE} - V_{BE}$

Also: $v_{R_c}(t) = I_C R_C - V_m \sin \omega t \geq 0$

$\therefore V_m \leq \min\left[I_C R_C, \left(V_{CE} - V_{BE}\right)\right]$

Similarly for MOSFETs:

$$V_M \leq \min\left[I_D R_D, (V_{DS} - (V_{GS} - V_{TN}))\right]$$