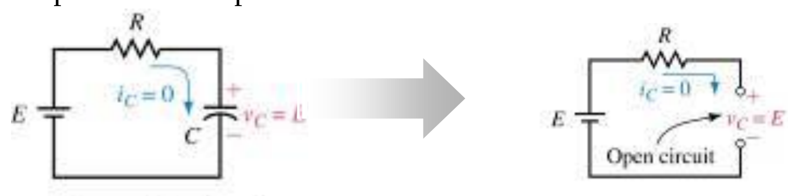
EE301 – CAPACITOR TRANSIENT ANALYSIS

Learning Objectives

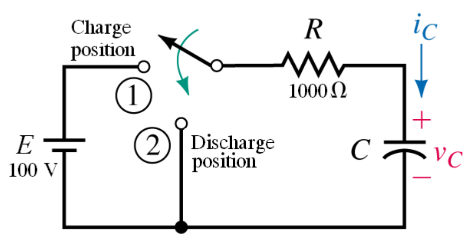
a. Calculate capacitor voltage and current as a function of time
b. Explain DC characteristics
c. Calculate capacitor energy stored

Capacitance and Steady State DC Recall from last lecture that a capacitor looks like an open circuit in steady state DC. In the circuit below, for example, presuming the circuit has been in the present state for a while, the capacitor reaches a voltage of *E* volts (and no more current flows). Since no current flows, we can *pretend* the capacitor is an open circuit.



(a) $v_C = E$ and $i_C = 0$ (b) Equivalent circuit for the capacitor

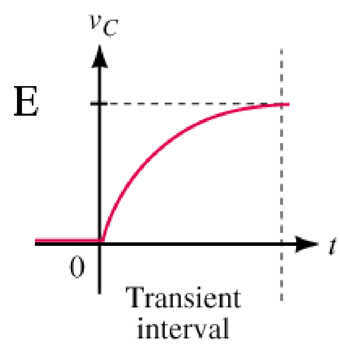
But what happens when the circuit is not in steady state? Suppose we have the circuit shown below, where the switch is not in either position 1 or 2, and the capacitor is fully (initially) discharged. When the switch then moves to position 1, what happens?



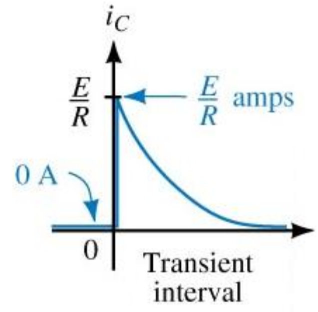
Capacitor Charging You should appreciate (from last lecture!) that when the switch moves to position 1, current will flow and eventually the voltage across the capacitor, v_c, will equal *E* and we will be in steady state. The period of time between when the switch shuts and when steady state is reached is referred to as a transient. Our goal is to analyze this transient quantitatively.

Initially, the capacitor is fully discharged, and so $v_c = 0$. Now, as soon as the switch moves to position 1, there will be a voltage drop across the resistor, and the current instantaneously jumps to

$$i_C = \frac{E}{R} = \frac{100}{1000} = 100 \text{ mA}.$$



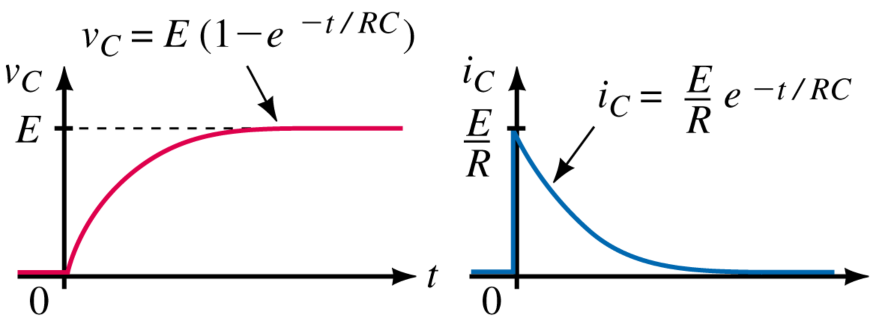
As charge is stored in the capacitor (i.e., as the capacitor charges up), the voltage across the capacitor starts to rise. This makes the voltage drop across the resistor decrease, so the current in the circuit also decreases.



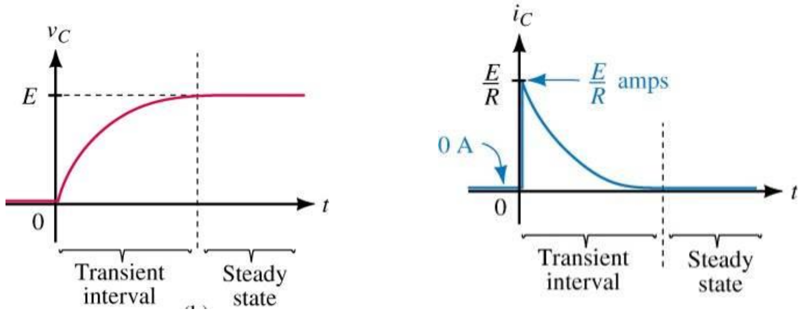
So, the voltage across the capacitor v_c starts at 0 and builds up to E.

The current through the capacitor starts out at E/R and decays to 0.

Capacitor Charging Equations It turns out that the voltage and current in a charging circuit change exponentially over time:



The circuit is at steady state when the voltage and the current reach their final values and stop changing. In steady state, the capacitor has a voltage across it, but no current flows through the circuit: the capacitor acts like an open circuit.



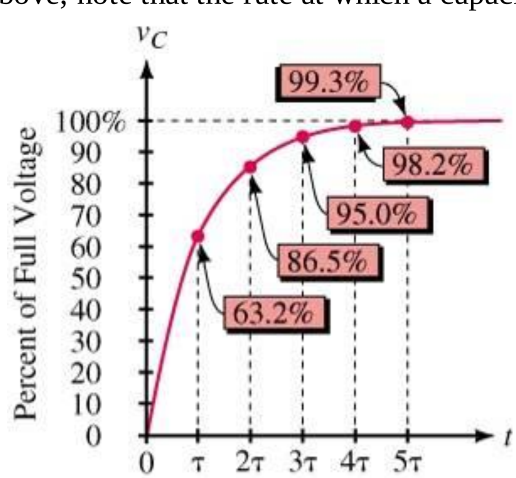
The Time Constant From the capacitor charging equation above, note that the rate at which a capacitor charges depends on R and C, which is called the **TIME CONSTANT**, and is denoted as τ :

$$\tau = RC$$

We can then re-write our charging equation for $v_c(t)$ as

$$v_C(t) = E(1 - e^{-t/\tau})$$

Note that the exponential function's argument must be unit-less so the units for τ must be seconds. If we plot $v_c(t)$



versus *t* while noting increments of τ on the time axis (as shown on the right), we see that the transient can be considered to last for 5 time constants. Stated another way, we can presume that the charging transient is over after time $t = 5\tau$.

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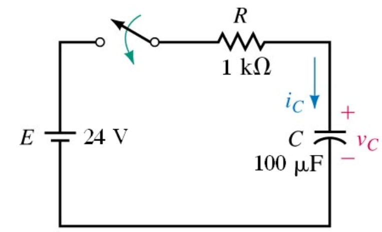
1 Example: In the circuit shown below, the capacitor is initially uncharged and the switch is open.

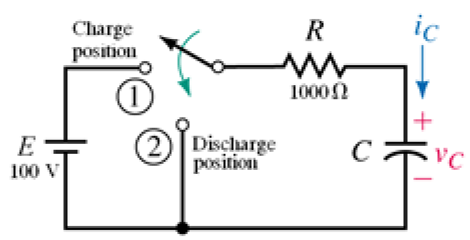


The switch is then shut at time $t = 0$.

a. Determine how long it will take for the circuit to reach a steady-state conditions.
b. Write the equation for $v_c(t)$.
c. Sketch the transient.

Solution:

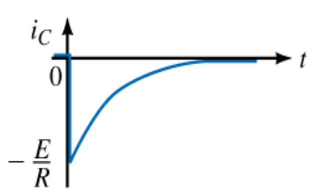
Capacitor Discharging Now, considering the circuit below, suppose the switch has been in position 1 for a long time. The capacitor is initially fully charged and acts like an open circuit.

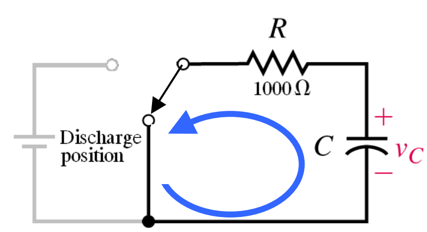


What happens when the switch is then moved to position 2?

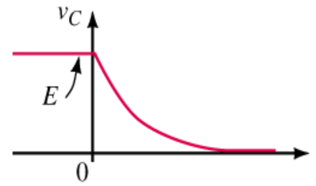
The voltage across the capacitor cannot change instantaneously (although its current can). Thus, when the switch is moved to the discharge position (position 2), the current instantaneously jumps to:

$$i_C = -\frac{E}{R} = -\frac{100V}{1000\Omega} = -100mA$$



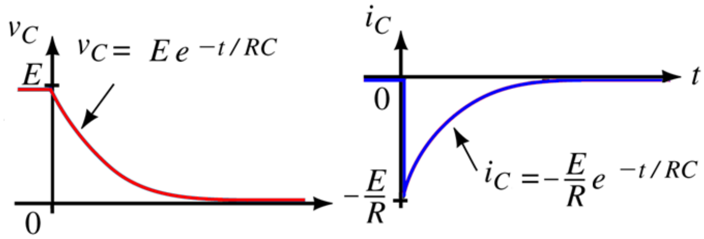


As charge flows out of the capacitor, the voltage across the capacitor drops.



This makes the voltage drop across the resistor drop, so current in the circuit drops until the capacitor is fully discharged.

<u>Capacitor Discharging Equations</u> Voltages and currents in a discharging circuit also change exponentially over time



As with the charging case, here again the transients can be considered to last for 5 time constants, where the time constant τ is given by $\tau = RC$. Stated another way, we can presume that the discharge transient is over after time $t = 5\tau$.

<u>More complex circuits</u> As a Naval Academy midshipman, you look at the above problems and you say: "This is too easy. Can't we make these problems more challenging?" The answer is: Yes (we aim to please). If the circuit does not look like the simple charge circuit or simple discharge circuit, then you will need to use the Thèvenin equivalent circuit to make it into the simple circuit.

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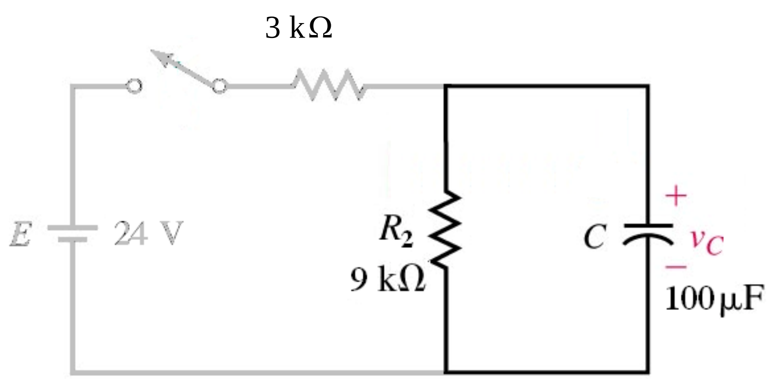
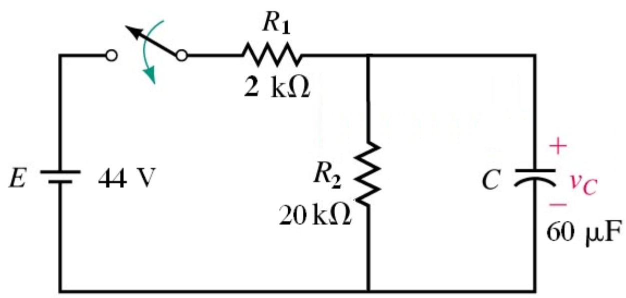
2 <u>Example</u>: In the circuit below, the switch was shut for a long time, then opens at time t = 0. Determine the equation for $v_c(t)$.

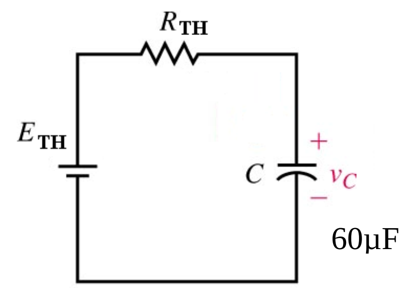
<u>Solution</u>:



3 Example: The circuit below does not have the same charging equation as the previous circuits, since the voltage drop across the capacitor is controlled by the voltage divider circuit.



a. Transform the circuit into the Thevenin equivalent circuit as seen by the capacitor.



b. Determine the charging time constant.

c. Determine how long it will take for the capacitor to fully charge (>99% of final voltage).

d. Write the charging equation.

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4 Example: Continuing with the problem above, the capacitor is now fully charged. The switch is opened to start the discharge cycle.

a. Determine how long it will take for the capacitor to fully discharge.

b. Identify the direction of current flow.

c. Write the equation for $v_c(t)$.

d. Sketch the transient.

Solution:

5 Example: The switch has been in position a for a long time. At time t=0, it moves to position b.

a. Calculate the steady state voltage across the capacitor at position a
b. Calculate the steady state voltage across the capacitor at position b
c. Calculate the time constant (tau) in position b
d. Find the voltage v(t) across the capacitor for t>0
e. Find the energy stored by the capacitor at t=60 secs