

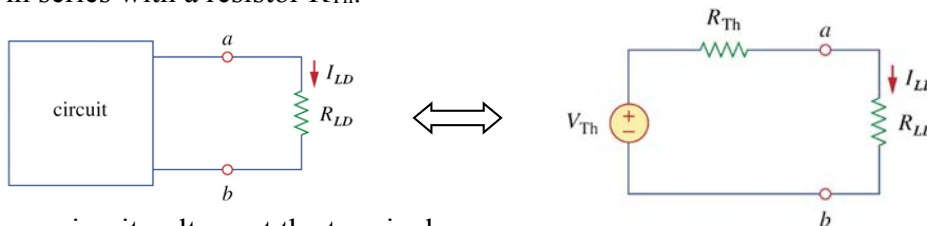
# EE301 – THÉVENIN'S THEOREM and MAX POWER TRANSFER

## Learning Objectives

- State and explain Thévenin's theorem
- List the procedure for determining the Thévenin equivalence of an actual circuit from the standpoint of two terminals
- Apply Thévenin's Theorem to simplify a circuit for analysis
- Analyze complex series-parallel circuits using Thévenin's theorem
- Apply the Maximum Power Transfer theorem to solve appropriate problems

**Thévenin's Theorem** Thévenin's theorem greatly simplifies analysis of complex circuits by allowing us to replace all of the elements with a combination of just one resistor and one voltage source.

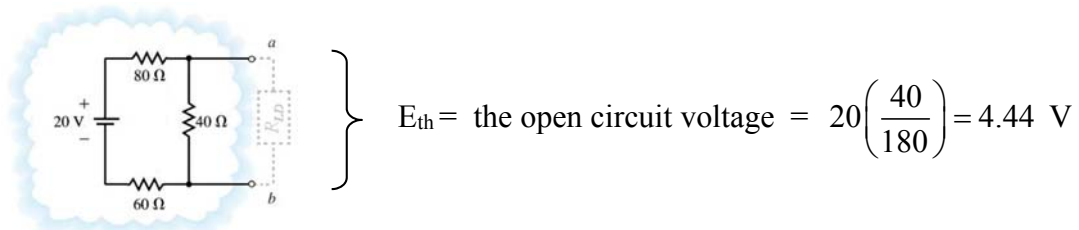
So, a complex two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source  $V_{Th}$  in series with a resistor  $R_{Th}$ .



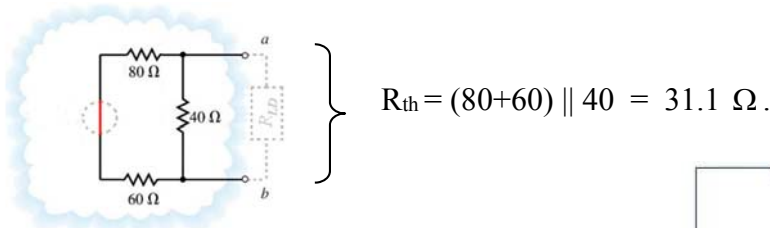
$E_{Th}$  is the open circuit voltage at the terminals.

$R_{Th}$  is the input or equivalent resistance at the terminals when the sources are turned off (zeroized).

**Determining  $E_{Th}$ :** Remove the load (open-circuit) and measure the resulting voltage.

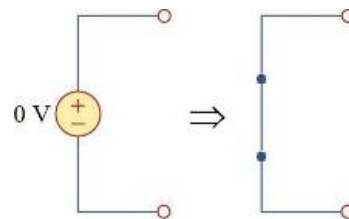


**Determining  $R_{Th}$ :** With the load disconnected, turn off all sources.  $R_{Th}$  is the equivalent resistance looking into the “dead” circuit through terminals a-b. To make a circuit dead we have to zeroize all the sources.

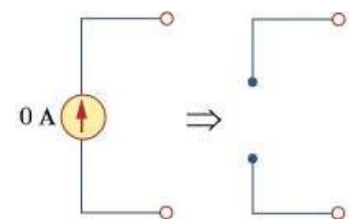


“Zeroizing” a source means setting its value equal to zero:

Voltage sources – 0 V is equivalent to a short circuit.

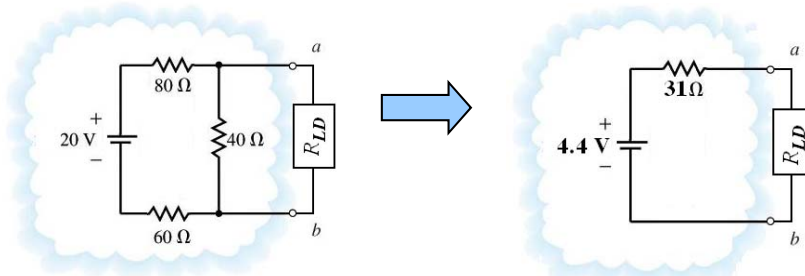


Current sources – 0 A is equivalent to an open circuit.



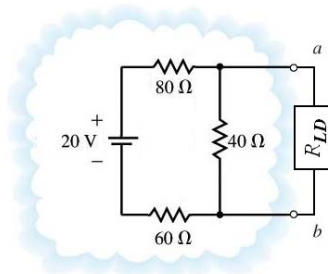
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Now we can redraw the circuit showing the Thévenin equivalent with the load installed.

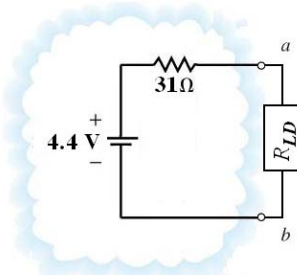


Now, you're probably thinking to yourselves: "Who cares. Big whoop. Another thing to study for the six-week exam. I wonder what's for lunch. And ... and ... and ... zzzzzz (sleeping)"

Well, consider this problem. Suppose you had to find the current and voltage across the load resistor  $R_{LD}$  in the circuit shown below:



BUT... you need to find the current and voltage across the load resistor  $R_{LD}$  for 20 different values of  $R_{LD}$ ! That would not be fun for the original circuit, but would not be so bad for the Thevenin equivalent circuit:



Repetitive solutions for various load resistances now becomes easy with the transformed circuit.

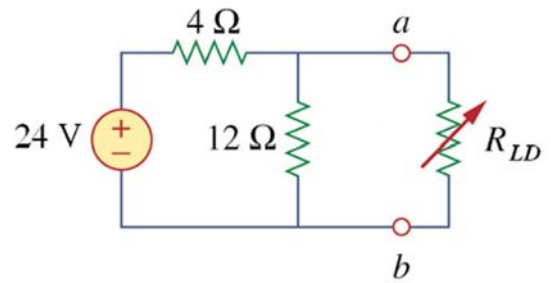
$$I_{LD} = \frac{E_{Th}}{R_{Th} + R_{LD}} \quad V_{LD} = \frac{R_{LD}}{R_{Th} + R_{LD}} E_{Th}$$

That's why studies have shown that all midshipmen love Thevenin's Theorem.

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1 Example: Find the Thévenin equivalent circuit external to  $R_{LD}$ . Determine  $I_{LD}$  when  $R_{LD} = 6\Omega$ .

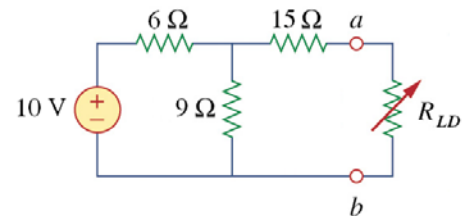
Solution:



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2 Example: Find the Thévenin equivalent circuit external to  $R_{LD}$ . Determine  $I_{LD}$  when  $R_{LD} = 2.5\Omega$ .

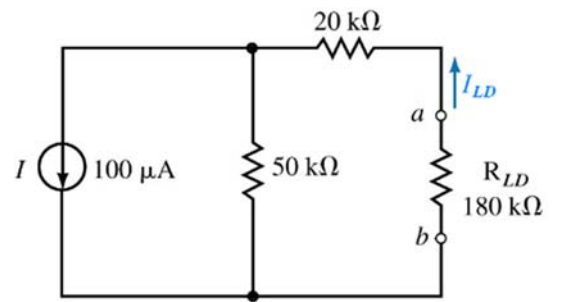
Solution:



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3 Example: Find the Thévenin equivalent circuit external to  $R_{LD}$ . Determine  $I_{LD}$ .

Solution:



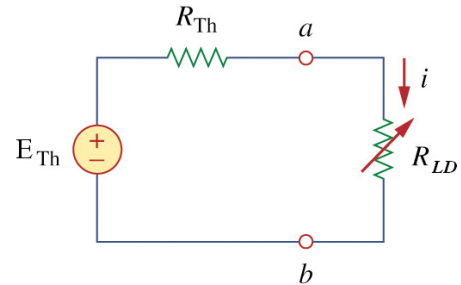
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### MAX POWER TRANSFER

In some applications, the purpose of a circuit is to provide maximum power to a load. Some examples:

- Stereo amplifiers
- Radio transmitters
- Communications equipment

Our question is: If you have a system, what load should you connect to the system in order that the load receives the maximum power that the system can deliver?



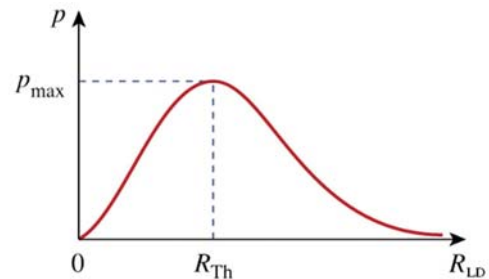
The Thévenin equivalent circuit makes this analysis simpler. Consider the case that  $E_{Th}$  and  $R_{Th}$  are fixed and the load resistance  $R_{LD}$  is variable. How might we determine  $R_{LD}$  such that  $P_{LD}$  is maximized? Solving the circuit above for the power delivered to the load, we find

$$P_{LD} = I_{LD}^2 R_{LD} = \left( \frac{V_{Th}}{R_{Th} + R_{LD}} \right)^2 R_{LD}$$

The right side of the equation for  $P_{LD}$ ,  $P_{LD} = \left( \frac{V_{Th}}{R_{Th} + R_{LD}} \right)^2 R_{LD}$ , has only one unknown:  $R_{LD}$ . To

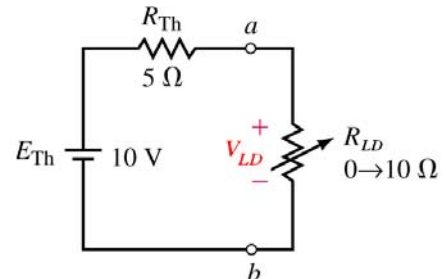
find the value of  $R_{LD}$  that maximizes  $P_{LD}$ , we take the derivative of the equation above, set it equal to zero and solve for  $R_{LD}$ . We find that  $R_{LD} = R_{Th}$ .

Thus, the maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load. This famous result is termed the *Maximum Power Transfer Theorem*. When  $R_{LD} = R_{Th}$ , the source and load are said to be matched.



**4 Example:** Consider the circuit on the right, which has a variable load resistor. If you want to maximize the power delivered to the load, what value should you choose for the load resistor?

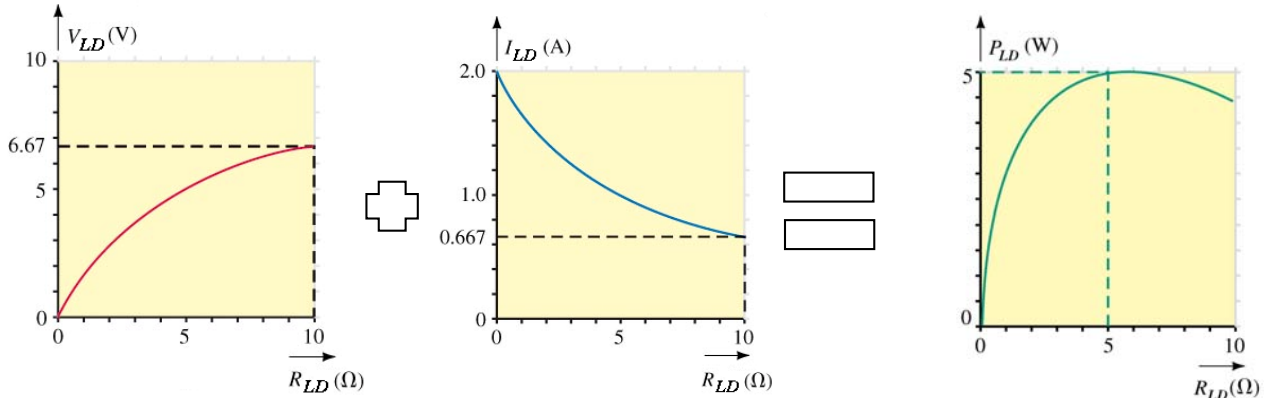
Solution:



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Notice the trade-off in the prior example:

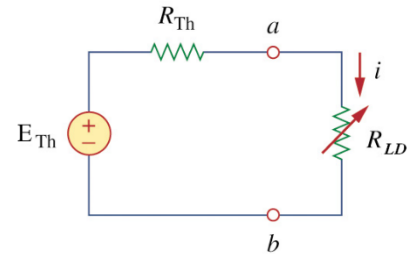
- As  $R_{LD}$  increases, a higher percentage of the total power is dissipated in the load resistor.
- But since the total resistance is increasing, the total current is dropping, and a point is reached where the total power dissipated by the entire circuit starts dropping.



So, the maximum power is delivered when  $R_{LD} = R_{Th}$ , but what is the value of this power? The power delivered when  $R_{LD} = R_{Th}$  is

$$P_{LD} = I_{LD}^2 R_{LD} = \left( \frac{V_{Th}}{R_{Th} + R_{LD}} \right)^2 R_{LD} = \left( \frac{V_{Th}}{R_{Th} + R_{Th}} \right)^2 R_{Th}$$

$$= V_{Th}^2 \frac{R_{Th}}{4R_{Th}^2} = \frac{V_{Th}^2}{4R_{Th}} = P_{MAX}$$



Note that this value of max power occurs when  $R_{LD} = R_{Th}$ . It is not true that  $P_{LD} = P_{MAX} = \frac{V_{Th}^2}{4R_{Th}}$  when  $R_{LD}$  does not equal  $R_{Th}$ .

**Efficiency** When maximum power is delivered to  $R_{LD}$ , the efficiency is a mere 50%.

$$\eta = \frac{p_{out}}{p_{in}} = \frac{i^2 R_{LD}}{i^2 R_{Th} + i^2 R_{LD}} \times 100\% = \frac{i^2 R_{Th}}{i^2 R_{Th} + i^2 R_{Th}} \times 100\% = 50\%$$

For communication circuits and amplifiers, attaining the max power transferred is more desirable than attaining a high efficiency.

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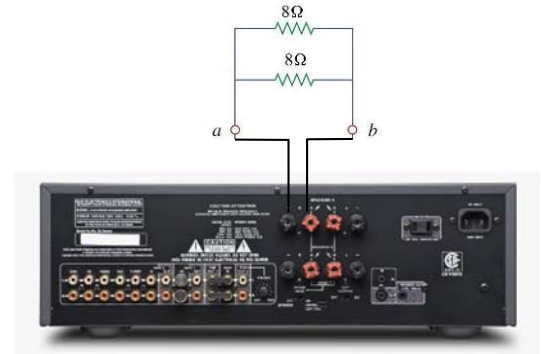
On the other hand, for power transmission (115 VAC 60 Hz Power ), attaining a high efficiency is more desirable than attaining the max power transfer. For this reason, in these circuits, the load resistance is kept much larger than the internal resistance of the voltage source. So, in this latter case, the circuit operates much more efficiently, but delivers less power than it otherwise might.

4 Example: A stereo is rated for maximum output power of 150 W per channel when  $R_{LD} = 8\Omega$ .

(a) Sketch the Thévenin Equivalent Circuit

(b) What would the output power be if two  $8\Omega$  speakers were connected in parallel to one of the channels?

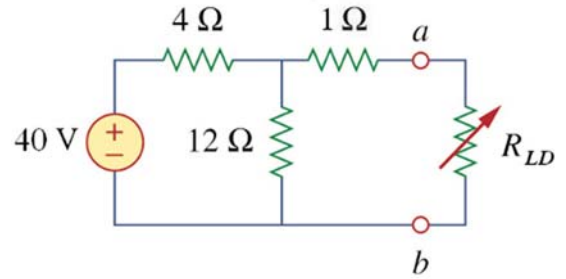
Solution:



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5 Example: Consider the circuit shown on the right.

- Find the Thévenin equivalent circuit to the left of terminals a-b.
- Calculate the maximum power transfer to the load if  $R_{LD} = R_{TH}$ .
- Determine the power dissipated by  $R_{LD}$  for load resistances of  $2\Omega$  and  $6\Omega$ .



Solution: