EE301 – Three Phase Sources and Loads

Learning Objectives

a. Review the induced AC voltage output for a three phase AC generator as a function of time and as phasors.
b. Define a three-wire Y-Y three phase circuit and a four-wire Y-Y three phase circuit
c. Define the symbols for line to neutral voltages, line to line voltages, line currents, and phase impedances that will be used in three phase circuits
d. Analyze Ohm's law in a three-wire Y-Y three phase circuit and in a four-wire Y-Y three phase circuit using a basic three phase generator that produces three balanced voltages which are connected to balanced loads
e. Analyze Kirchhoff's current law in a three-wire Y-Y three phase circuit and in a four-wire Y-Y three phase circuit using a basic three phase generator that produces three balanced voltages with a Y connected balanced purely resistive loads

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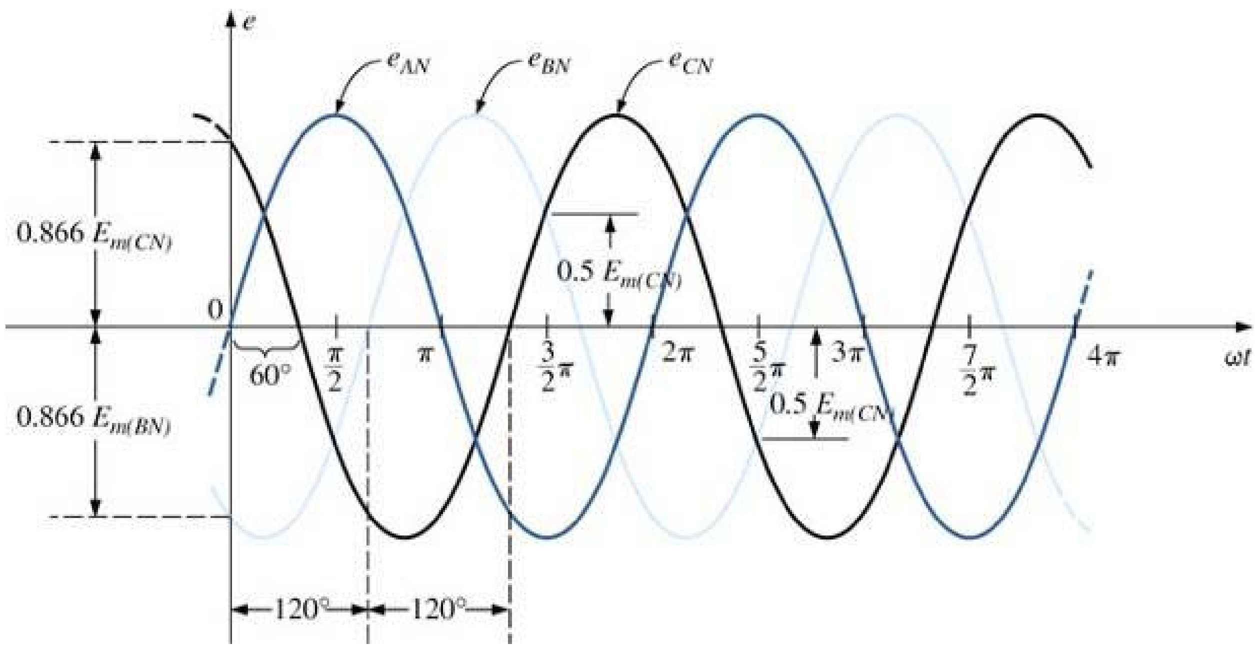
A three-phase source generates three separate ac voltage signals, all of the same peak value, but differing from each other by a phase separation of 120°. All commercial power in the U.S. is distributed and delivered as a three phase voltage. (Which raises the obvious question: *Why would we do such a thing?* -- We will defer that question until after we learn to analyze three-phase systems.)

Consider a three phase source that produces the three separate sinusoidal voltages shown below:



The voltages have the same peak value, the same frequency, and are offset by 120°. For example, if the RMS value of each waveform is $E_{AN} = E_{BN} = E_{CN}$, then the three waveforms shown above are

$$e_{AN} = \sqrt{2}E_{AN}\sin(\omega t)$$
$$e_{BN} = \sqrt{2}E_{BN}\sin(\omega t - 120^\circ)$$
$$e_{CN} = \sqrt{2}E_{CN}\sin(\omega t - 240^\circ) \quad = \sqrt{2}E_{CN}\sin(\omega t + 120^\circ)$$

Our three voltages can be represented as phasors:

$$\overline{\mathrm{E}}_{\mathrm{AN}} = E_{AN}\angle 0^\circ$$

$$\overline{\mathrm{E}}_{\mathrm{BN}} = E_{BN}\angle -120^\circ$$
$$\overline{\mathrm{E}}_{\mathrm{CN}} = E_{CN}\angle 120^\circ$$

What is the sum of our three sinusoids $e_{AN} + e_{BN} + e_{CN}$ at any moment of time?

$$e_{AN} + e_{BN} + e_{CN} = \sqrt{2}E_{AN}\sin(\omega t) + \sqrt{2}E_{BN}\sin(\omega t - 120^\circ) + \sqrt{2}E_{CN}\sin(\omega t + 120^\circ)$$

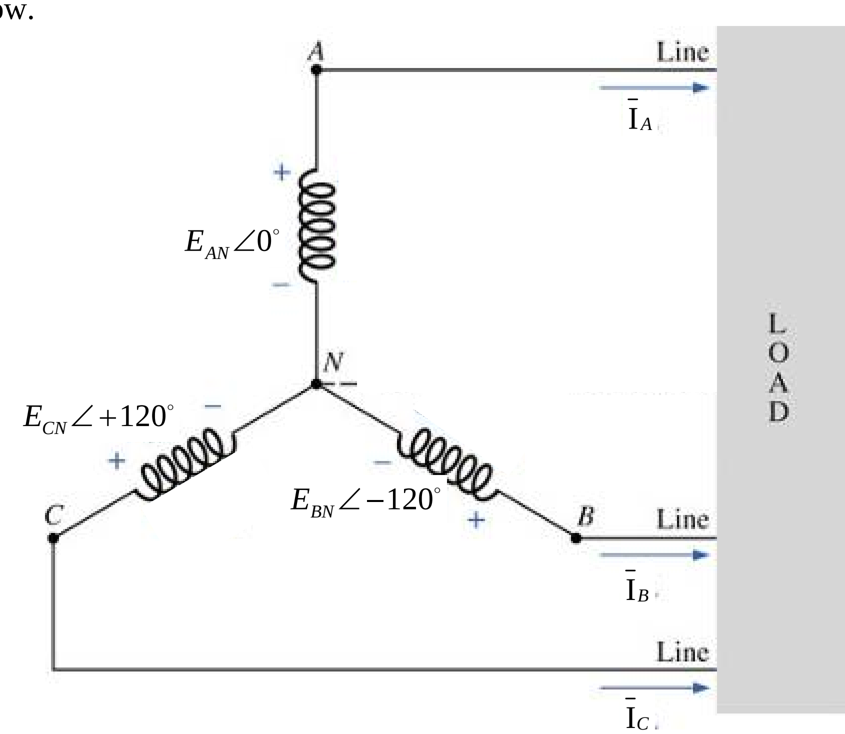
If $E_{AN} = E_{BN} = E_{CN} = E$ this becomes

$$e_{AN} + e_{BN} + e_{CN} = \sqrt{2}E\left(\sin(\omega t) + \sin(\omega t - 120^\circ) + \sin(\omega t + 120^\circ)\right)$$

Using trig identities, it is not hard to show that the sum on the right is always equal to zero. Thus,

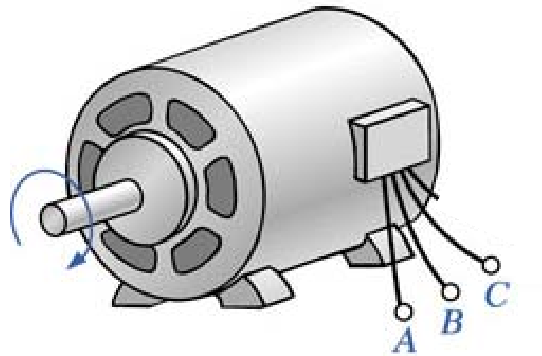
$$e_{AN} + e_{BN} + e_{CN} = 0$$

The Y-Connected Generator In the Y-connected generator, our three voltage sources above are connected as shown below.

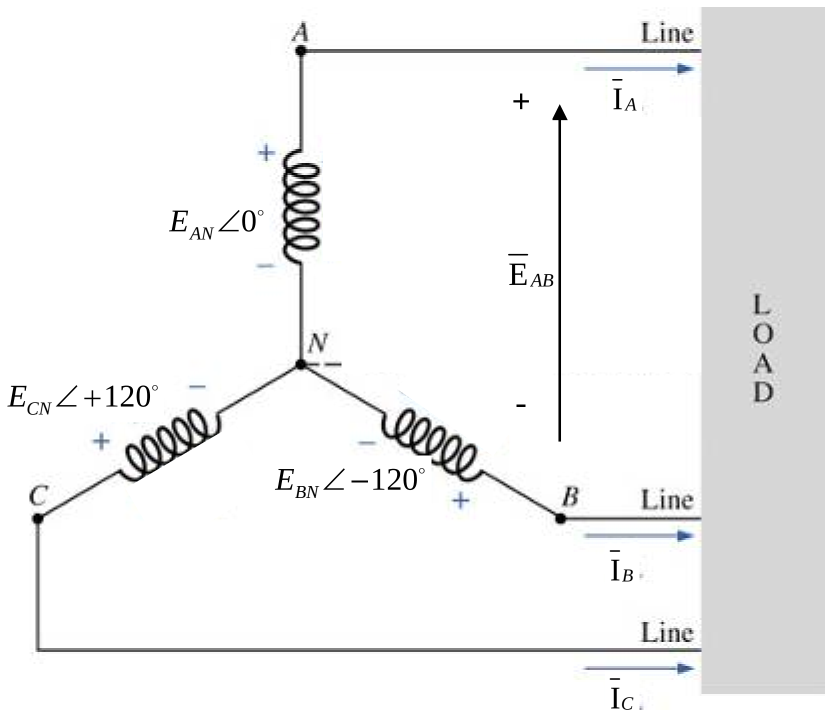


Note that the negative terminals of the three sources are connected together at the point labeled *N*, called *the neutral*. Each of our three voltages in the arrangement above is termed a *phase voltage*. Our three phase Y-connected generator is connected to a load via three *lines*. We have labeled the phasor currents on each of these three lines as $\overline{\mathrm{I}}_A$, $\overline{\mathrm{I}}_B$ and $\overline{\mathrm{I}}_C$.

Note the labeling of points *A*, *B* and *C* in the picture above. You might have the physical three-phase Y-connected generator shown below, where you have access only to the lines labeled *A*, *B* and *C*. Specifically, it might be the case that you do not have access to the neutral point.



Since we only have access to the lines *A*, *B* and *C*, we are interested in the line voltages – the voltages between the lines – which we denote $\overline{\mathrm{E}}_{AB}$, $\overline{\mathrm{E}}_{BC}$ and $\overline{\mathrm{E}}_{CA}$. So, in the picture below, let's calculate the line voltage $\overline{\mathrm{E}}_{AB}$.



By KVL we have

$$\overline{\mathrm{E}}_{AB} = E_{AN}\angle 0^\circ - E_{BN}\angle -120^\circ .$$

Since the RMS phase voltages are equal, let $E_{AN} = E_{BN} = E$, and our equation above becomes

$$\overline{\mathrm{E}}_{AB} = E\angle 0^\circ - E\angle -120^\circ$$

Shifting the right-hand side of the equation to rectangular coordinates, we have

$$\overline{\mathrm{E}}_{AB} = E - \left\{ E\cos\left(-120^\circ\right) + jE\sin\left(-120^\circ\right) \right\}$$

Simplifying, we have

$$\overline{\mathrm{E}}_{AB} = E - \left\{ -E\left(\frac{1}{2}\right) + jE\left(\frac{\sqrt{3}}{2}\right) \right\}$$

and simplifying further,

$$\overline{\mathrm{E}}_{AB} = \left(\frac{3}{2}\right)E + j\left(\frac{\sqrt{3}}{2}\right)E$$

Converting this back to polar form yields the important result:

$$\overline{\mathrm{E}}_{AB} = \sqrt{3}E \angle 30^\circ$$

Using the same technique, we can solve for the other two line voltages:

$$\overline{\mathrm{E}}_{BC} = \sqrt{3}E \angle -90^\circ$$
$$\overline{\mathrm{E}}_{CA} = \sqrt{3}E \angle 150^\circ$$

So, for a Y-connected generator, if we know the phase voltages we can immediately get the line voltages, and vice versa:

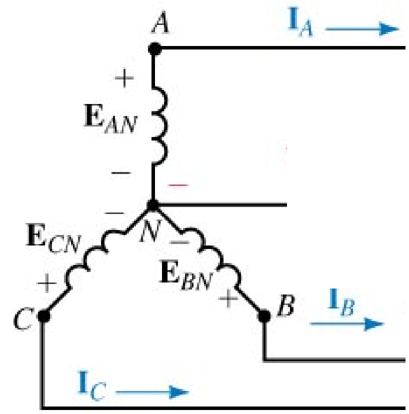
Phase Voltages		**Line Voltages**
$\overline{\mathrm{E}}_{AN} = E\angle 0^\circ$	$\Rightarrow$	$\overline{\mathrm{E}}_{AB} = \sqrt{3}E \angle 30^\circ$
$\overline{\mathrm{E}}_{BN} = E\angle -120^\circ$	$\Rightarrow$	$\overline{\mathrm{E}}_{BC} = \sqrt{3}E \angle -90^\circ$
$\overline{\mathrm{E}}_{CN} = E\angle 120^\circ$	$\Rightarrow$	$\overline{\mathrm{E}}_{CA} = \sqrt{3}E \angle 150^\circ$

Furthermore, there is no reason that the phase for $\overline{\mathrm{E}}_{\mathrm{AN}}$ be fixed at zero. If the phase of $\overline{\mathrm{E}}_{\mathrm{AN}}$ is θ, the relationships between phase and line voltages are:

Phase Voltages		**Line Voltages**
$\overline{\mathrm{E}}_{AN} = E\angle \theta$	$\Rightarrow$	$\overline{\mathrm{E}}_{AB} = \sqrt{3}E \angle \theta + 30^\circ$
$\overline{\mathrm{E}}_{BN} = E\angle -120^\circ + \theta$	$\Rightarrow$	$\overline{\mathrm{E}}_{BC} = \sqrt{3}E \angle -90^\circ + \theta$
$\overline{\mathrm{E}}_{CN} = E\angle 120^\circ + \theta$	$\Rightarrow$	$\overline{\mathrm{E}}_{CA} = \sqrt{3}E \angle 150^\circ + \theta$

1 Example: Consider the three phase system shown on the left.
You are given one of the phase voltages:

$$\mathbf{E}_{AN} = 7620\angle{-18°}\text{ V}$$



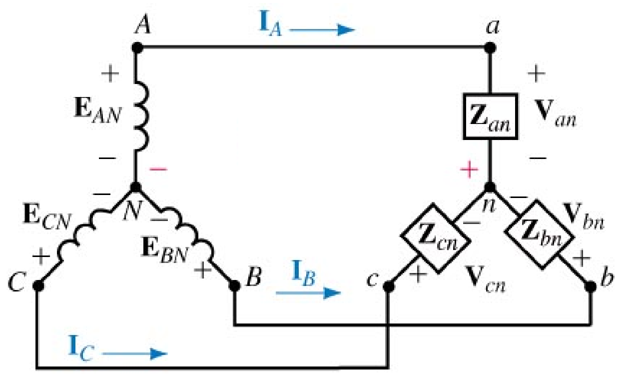
Determine:

a. $\mathbf{E}_{BN}$
b. $\mathbf{E}_{CN}$
c. The three line-to-line voltages

Solution:

The Y-Connected Generator With a Y-Connected Load

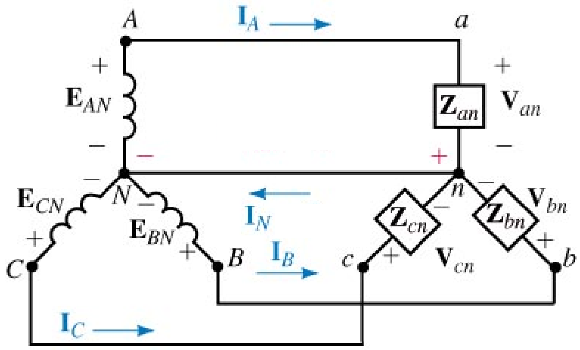
Suppose we connect a load to our Y-connected generator as shown below. A load in this configuration is, for obvious reasons, called a Y-connected load. We would like to analyze this circuit; i.e., we would like to determine the line currents and the various voltages.



In EE301 the load will always be balanced. That means, in the picture above, $Z_{an} = Z_{bn} = Z_{cn}$.

Hmm… as far as analysis goes, this doesn't look too pretty, does it?

Suppose we could connect a wire between the points labeled *N* and *n* above, resulting in this circuit:



Would this circuit be easier to analyze? Yes! Much easier! For example, $I_A = \dfrac{V_{an}}{Z_{an}}$. But we can't just put wires into our circuit to make analysis easier. Or can we?

Let's consider a concrete example. Suppose, in the figure above, with a wire connected between the points labeled *N* and *n*, we have $Z_{an} = Z_{bn} = Z_{cn} = 12\,\Omega$ and $\mathbf{E}_{AN} = 120\angle 0^\circ$. Then we can calculate $\bar{I}_A$, $\bar{I}_B$ and $\bar{I}_C$ as:

$$\bar{I}_A = 10\angle 0^\circ$$

$$\bar{I}_B = 10\angle -120^\circ$$

$$\bar{I}_C = 10\angle 120^\circ$$

Then what is $\bar{I}_N$?

$$\bar{I}_N = \bar{I}_A + \bar{I}_B + \bar{I}_C = 10\angle 0^\circ + 10\angle 120^\circ + 10\angle -120^\circ$$

But this is the same calculation we did at the top of page 2! Thus, $\bar{I}_N = 0$!!!

So, what's the point?

We can connect the wire from *N* to *n* without changing the circuit since no current flows in this wire. But the addition of this wire makes circuit analysis easier!

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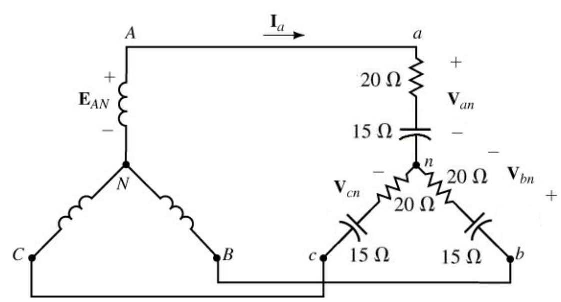
2 Example: In the circuit on the right, $\mathbf{E}_{AN}$ = 277∠-30° V

a. Determine all of the phase voltages and line voltages

b. Determine the line currents

Solution:

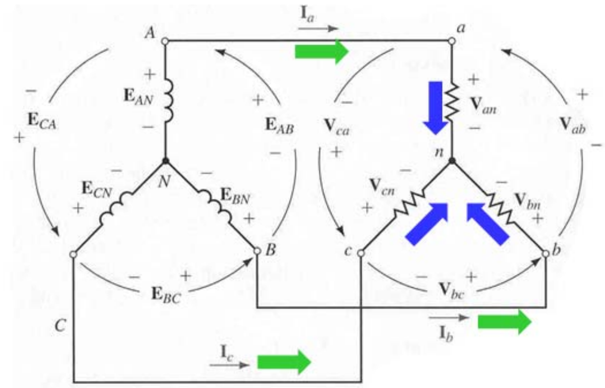


Y-Y system definitions

Line voltages are the voltages between the lines A, B and C (E_{CA}, E_{AB}, E_{BC}, V_{ca}, V_{ab}, V_{bc}).
Phase voltages are the voltages across phases (E_{AN}, E_{BN}, E_{CN}, V_{AN}, V_{BN}, V_{CN}).

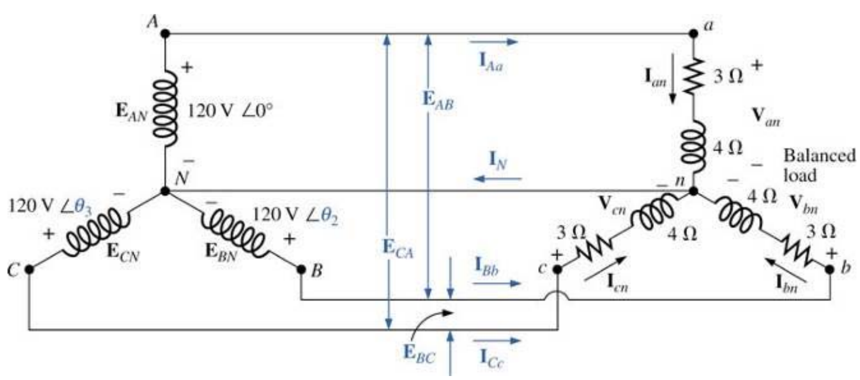
For the generator, E_{AN}, E_{BN}, E_{CN} are the phase voltages.

For Δ loads, **phase voltage** and **line voltage** are the same thing.



Line currents are the currents in the line conductors.
Phase currents are the currents through phases.
For Y loads, line current and phase current are the same.

3 Example: Consider the balanced circuit shown below.

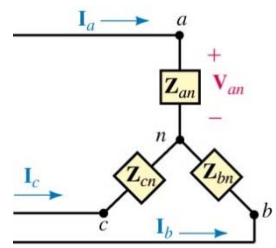


a. Determine θ_2 and θ_3
b. Determine the line voltages
c. Determine the line currents
d. Repeat the problem if we remove the wire connecting *N* to *n*

It is worth reiterating again:

For Y loads, line current and phase current are the same.

$$\boxed{\mathbf{I}_a = \mathbf{V}_{an} / \mathbf{Z}_{an}}$$

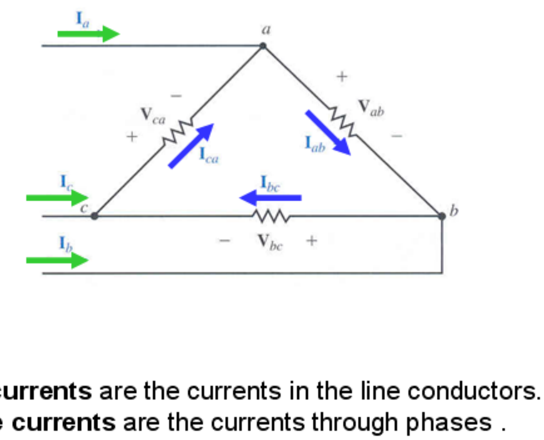


You should have gathered from the symmetry that for a Y-Y balanced circuit, if we know one current, say $\bar{I}_a$, then we can immediately write down $\bar{I}_b$ and $\bar{I}_c$. Specifically, if $\bar{I}_a = I\angle\theta$ then

$$\bar{I}_b = I\angle\theta - 120^\circ$$

$$\bar{I}_c = I\angle\theta + 120^\circ$$

<u>The Δ-connected Load</u> For fun and excitement, we sometimes connect out balanced load in a delta configuration:



For EE301, the impedances on each branch of the delta are the same; i.e., the load is balanced..

Notice that in this configuration, the line currents (shown in green above) that appear at the load are **<u>not</u>** equal to the phase currents within the load.

Note that the line voltages that appear at the load **<u>are equal</u>** to the phase voltages within the load.

Again, note the distinction between Y-Y and Y-Δ configurations:

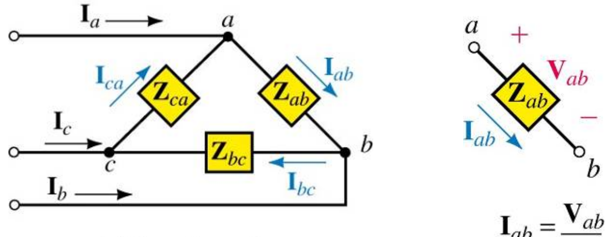
For Y-Y: the line currents equal the phase currents; the line voltages do not equal the phase voltages.

For Y-Δ: the line voltages equal the phase voltages; the line currents do not equal the phase currents.

Line/Phase Currents for a Delta Circuit

Line/Phase Currents for a Delta Circuit
Relationship between line and phase currents

$$\boxed{\begin{aligned}\mathbf{I}_{ab} &= \mathbf{V}_{ab} / \mathbf{Z}_{ab} \\ \mathbf{I}_{a} &= \mathbf{I}_{ab} - \mathbf{I}_{ca} \\ \mathbf{I}_{a} &= \sqrt{3}\mathbf{I}_{ab}\angle -30^\circ\end{aligned}}$$



(a) $\mathbf{I}_a = \mathbf{I}_{ab} - \mathbf{I}_{ca}$

For a balanced Δ system, *the magnitude of line current is 1.732 times the magnitude of the phase current* and *line current lags phase current by 30°.* Therefore, given any current at a point in a balanced, 3-phase Δ system, *you can determine the remaining 5 currents by inspection*.